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Prove, by structural induction, for all finite lists xs, we have:

palindrome xs = xs ++ reverse xs **(EQ)**

You may assume the following equations for reverse and length:

Definitions:

```
palindrome :: [a]->[a]
Palindrome [] = []                                        (PAL.1)
palindrome (x:xs) = [x] ++ palindrome xs ++ [x]           (PAL.2)

reverse :: [a] -> [a]
reverse [] = []                                           (REV.1)
reverse (x:xs) = reverse xs ++ [x]                        (REV.2)

[] ++ zs = zs                                             (++.1)
(w:ws) ++ zs = w:(ws ++ zs)                               (++.2)
```

a. **Proof Goals**:
   We want to prove that for all finite lists:
   First, we are going to prove the base case:
   palindrome [] = [] ++ reverse [] (BASE)
   Then, we are going to prove the induction step:
   palindrome (x:xs) = (x:xs) ++ reverse (x:xs) (IND)
   Assuming the hypothesis:
   palindrome xs = xs ++ reverse xs (HYP)

b. **Proving the base case**:
   We have to prove that **palindrome [] = [] ++ reverse []** (BASE)
   Let xs = []
   Then,
   Left-hand side:
   palindrome[] = [] by length.1
   Right-hand side:
   reverse [] = [] by reverse.2
   [] ++ [] = by ++.1
   Left-hand side = Right-hand side

c. Proving the Induction step:

```
palindrome (x:xs) = (x:xs) ++ reverse (x:xs)            (IND)
palindrome xs = xs ++ reverse xs                        (HYP)

Left-hand side:
      palindrome (x:xs) =
      [x] ++ palindrome xs ++ [x]                       by (PAL.2)
      [x] ++ xs ++ reverse xs ++ [x]              by (HYP)
      = (x:xs)++ reverse xs ++ [x]                      by (++.2)

Right-hand side:
      (x:xs) ++ reverse (x:xs)
      = (x:xs) ++ reverse xs ++ [x]                     by (REV.2)
Left-hand side = Right-hand side
End of Proof.
```