**Ali Akbari Tutorial 03** **30010402**

Structural Induction Proof that sum(reverse xs) == sum xs

Proof:

We want to prove two goals of the induction proof:

**Base Case:**

(base) We have to prove that sum(reverse []) == sum []

Let xs = []

Then,

Left-hand side:

reverse [] = [] -by reverse .1

sum(reverse []) = sum([]) = 0 -by sum .1

Right-hand side:

sum xs = sum [] = 0 -by s1

sum(reverse []) == sum []

0 = 0

**Induction Step:**

(ind) We have to prove that sum(reverse (x : xs)) == sum (x : xs) on the assumption that:

(hyp) sum(reverse xs) == sum xs.

Left-hand side:

sum(reverse (x : xs)) =

sum(reverse xs ++ [x]) = -by reverse.2

sum(reverse xs) + sum [x] = -by sum and ++3 proven below

sum(reverse xs) + x =

x + sum(reverse xs)

Right-hand side:

sum (x : xs) = x + sum xs -by sum.2

= x + sum(reverse xs) -by hyp assumption

x + sum(reverse xs) = x + sum(reverse xs) QED

**Sum and ++3:**

Proof of  sum (xs ++ ys) = sum xs + sum ys

**Base Case:**
Let xs = []
Then,
Left-hand side:
sum ([] ++ ys) = sum ys -by ++1
Right-hand side:
sum [] + sum ys = 0 + sum ys -by sum.1
= sum ys

**Induction Step:**
Left-hand side:
sum ((x:xs) ++ ys) = sum (x:(xs ++ ys)) -by ++2
= x + sum (xs ++ ys) -by sum.2
= x + sum xs + sum ys -by hyp

Rigth-hand side:
sum (x:xs) + sum ys = x + sum xs + sum ys -by s2