CPSC 449 Assignment #3 **Fall 2020**

Ali Akbari
30010402
Tutorial #03

Question #2)

Definition:

```
Data Polynomial =     PConst Integer |
                      PVar |
                      PAdd Polynomial Polynomial |
                      PMul Polynomial Polynomial

degree :: Polynomial -> Integer
degree (PConst n)   = 0                                 (degree.1)
degree PVar         = 1                                 (degree.2)
degree (PAdd p1 p2) = max (degree p1) (degree p2)       (degree.3)
degree (PMul p1 p2) = (degree p1) + (degree p2)         (degree.4)

d (PConst n)   = PConst 0                               (d.1)
d PVar         = PConst 1                               (d.2)
d (PAdd p1 p2) = PAdd (d p1) (d p2)                     (d.3)
d (PMul p1 p2) = PAdd (PMul p1 (d p2))
                  (PMul (d p1) p2)                      (d.4)
```

a) Principle of Structural Induction for Polynomial:
To prove that P(p) holds for all finite Polynomials prove the following:

1) P(PConst n)
2) P(PVar)
3) P(p1) and P(p2) ⇒ P(PAdd p1 p2)
4) P(p1) and P(p2) ⇒ P(PMul p1 p2)

Proof Goals:

1) degree PConst n ≥ degree (d PConst n) (Base.1)
2) degree PVar ≥ degree (d PVar) (Base.2)

Assume:

3) degree p1 ≥ degree (d p1) (hyp.1)
4) degree p2 ≥ degree (d p2) (hyp.2)

Prove:

5) degree PAdd p1 p2 ≥ degree (d PAdd p1 p2) (Ind.1)
6) degree PMul p1 p2 ≥ degree (d PMul p1 p2) (Ind.2)

```
b) Base Case:
   Want:
   degree PConst n ≥ degree (d PConst n)                     (Base.1)
   degree PVar ≥ degree (d PVar)                             (Base.2)

      1) degree PConst n ≥ degree (d PConst n)               (Base.1)
         LHS: degree PConst n = 0                            (degree.1)
         RHS: degree (d PConst n) =
         degree (PConst 0) =                                 (d.1)
         degree (PConst 0) = 0                               (degree.1)
         LHS ≥ RHS
         So base case holds for:
         degree PConst n ≥ degree (d PConst n)
      2) degree PVar ≥ degree (d PVar)                       (Base.2)
         LHS: degree PVar = 1                                (degree.2)
         RHS: degree (d PVar) =
         degree (PConst 1) =                                 (d.2)
         degree (PConst 1) = 0                               (degree.1)
         LHS ≥ RHS
         So base case holds for:
         degree PVar ≥ degree (d PVar)

c) Induction Step:
   Want:
   degree PAdd p1 p2 ≥ degree (d PAdd p1 p2)                 (Ind.1)
   degree PMul p1 p2 ≥ degree (d PMul p1 p2)                 (Ind.2)
   Assume:
   degree p1 ≥ degree (d p1)                                 (hyp.1)
   degree p2 ≥ degree (d p2)                                 (hyp.2)

      1) degree PAdd p1 p2 ≥ degree (d PAdd p1 p2)           (Ind.1)
         LHS: degree PAdd p1 p2 =
         max (degree p1) (degree p2) =                       (degree.3)
         Case1: degree p1 = max
         Case2: degree p2 = max

         RHS: degree (d PAdd p1 p2) =
         degree(PAdd(d p1)(d p2)) =                          (d.3)
         max degree((d p1)) degree((d p2)) =                 (degree.3)
         Case1: degree (d p1)= max
         Case2: degree (d p2) = max
```

Case 1: degree p1 ≥ degree (d p1) (hyp.1)
If the degree of polynomial 1 is > than degree of polynomial 2 then by hyp.2 the degree of polynomial 2 is greater than the derivative of polynomial 2, thus by the transitive property we will have that the degree polynomial 1 will be greater than the derivative of polynomial 2.
So, LHS≥RHS.
Case 2: degree p2 ≥ degree (d p2) (hyp.2)
If the degree of polynomial 2 is > than degree of polynomial 1 then by hyp.1 the degree of polynomial 1 is greater than the derivative of polynomial 1, thus by the transitive property we will have that the degree polynomial 2 will be greater than the derivative of polynomial 1.
So, LHS≥RHS.

2) degree PMul p1 p2 ≥ degree (d PMul p1 p2) (Ind.2)
LHS: degree PMul p1 p2 =
(degree p1) + (degree p2) (degree.4)

RHS: degree (d PMul p1 p2) =
degree(PAdd (PMul p1 (d p2))(PMul (d p1) p2)) = (d.4)
max (degree (PMul p1 (d p2) degree(PMul (d p1) p2))= (degree.3)
max (((degree p1)+(degree (d p2)))((degree (d p1))+(degree p2)))= (degree.4)
Case1: (degree p1)+ (degree (d p2)) = max
Case2: (degree (d p1))+ (degree p2) = max

Case 1: (degree p1) + (degree p2)≥(degree p1) + (degree (d p2))
(degree p2)≥(degree (d p2))
Both sides have the degree p1 we can remove it from both sides and the resulting inequality depends on degree p2.
The degree of polynomial 2 is greater than the degree of the derivative of polynomial 2 by hyp.2.
So, LHS≥RHS.
Case 2: (degree p1) + (degree p2)≥(degree (d p1))+ (degree p2)
Both sides have the degree p2 we can remove it from both sides and the resulting inequality depends on degree p1.
The degree of polynomial 1 is greater than the degree of the derivative of polynomial 1 by hyp.1.
So, LHS≥RHS.

Thus, in general the LHS≥RHS, so
degree PMul p1 p2 ≥ degree (d PMul p1 p2) .