Sieve of Eratosthenes and Euclidean Algorithm

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Sieve of Eratosthenes Example

- **Problem**: Find primes up to 10: [2, 3, 4, 5, 6, 7, 8, 9, 10].
- Mark multiples of primes as non-prime.
- Step-by-Step:
 - 1. Start with 2, mark 4, 6, 8, 10.
 - 2. Next 3, mark 6, 9.
 - 3. Result: [2, 3, 5, 7].

Sieve of Eratosthenes

- ▶ Mark multiples of each prime starting from i^2 .
- ▶ Only check PLEASE check up to \sqrt{n} .
- ▶ Time Complexity: $O(n \log \log n)$.
- **Space Complexity**: O(n).

Euclidean Algorithm Example

- Problem: Find GCD of 48 and 18.
- Use division to reduce numbers.
- Step-by-Step:

1.
$$48 = 2 \times 18 + 12$$

2.
$$18 = 1 \times 12 + 6$$

3.
$$12 = 2 \times 6 + 0$$

4.
$$GCD = 6$$

Euclidean Algorithm

- Repeatedly divide and take remainder until remainder is 0.
- GCD is last non-zero remainder.
- ▶ Time Complexity: $O(\log \min(a, b))$.
- **Space Complexity**: O(1).