QOSF_task2

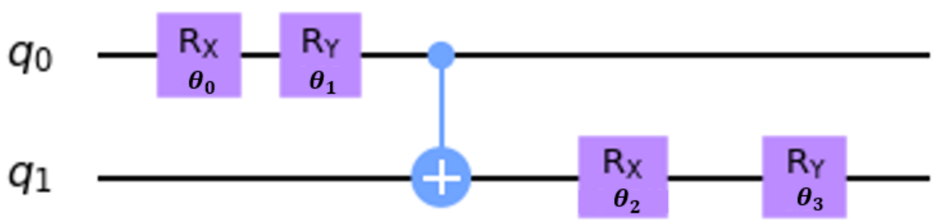
1 Solution

In solving this task I applied the following circuit architecture:

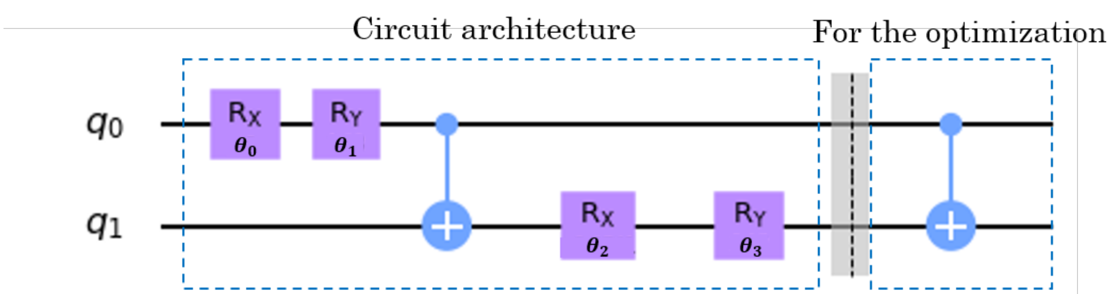
R_x followed by R_y appliled on qubit 0

A controlled-not gate between qubit 0 and qubit 1

R_x followed by R_y appliled on qubit 1



This was the circuit architicture itself but to do the optimization needed to generate the bell state $|\Psi^{01}\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$ I applied one more controlled-not gate between qubit 0 and qubit 1. This will help identify that we truely have the intended Bell state and not any other state with a relative phase.



Once we apply the controlled-not gate we can see that if we want to have the $|\Psi^{01}\rangle$ state then we have to measure -1 on qubit 1 (which tells us that the two qubits are different). But that is not all, we also have to check that qubit 0 is in the $|+\rangle$ state by measuring along the x and obtaining a +1. This will ensure that we have a $|\Psi^{01}\rangle$ and not any other state.

The mathmatics will clear things up. Our target is to obtain the state $\left|\Psi^{01}\right\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$. Now if we apply a CNOT, we will get the following:

$$\begin{aligned} CNOT\left|\Psi^{01}\right\rangle &= \frac{1}{\sqrt{2}}(CNOT\,|01\rangle + CNOT\,|10\rangle) \\ &= \frac{1}{\sqrt{2}}(|01\rangle + |11\rangle) \\ &= |+\rangle \otimes |1\rangle \end{aligned}$$

Now once we add the CNOT gate our target would be to find +1 for the expectation value of the PauliX operator on qubit 0 (measurement along x should ideally yield +1) and we should ideally get -1 for the expectaion value of the PauliZ operator. We can do this by minimizing the the following cost:

$$cost = \langle Z\rangle - \langle X\rangle$$

I did the optimization on two simulators, A noiseless one and another one with noise.

[1]:
```
import pennylane as qml
from pennylane import numpy as np
import matplotlib.pyplot as plt
```

Do the optimization on a noiseless simulator

[2]:
```
dev_noiseless = qml.device('default.qubit', wires=2, shots=1024, analytic=False)

@qml.qnode(dev_noiseless)
def circuit(thetas):

    # Apply parametric gates on qubit 0
    qml.RX(thetas[0], wires = 0)
    qml.RY(thetas[1], wires = 0)

    # Apply a contolled not gate bewtween qubit 0 and qubit 1
    qml.CNOT(wires = [0, 1])

    # Apply parametric gates on qubit 1
    qml.RX(thetas[2], wires = 1)
    qml.RY(thetas[3], wires = 1)

    # Apply CNOT for the optimization step
    qml.CNOT(wires=[0, 1])

    return qml.expval(qml.PauliX(0)), qml.expval(qml.PauliZ(1))
```

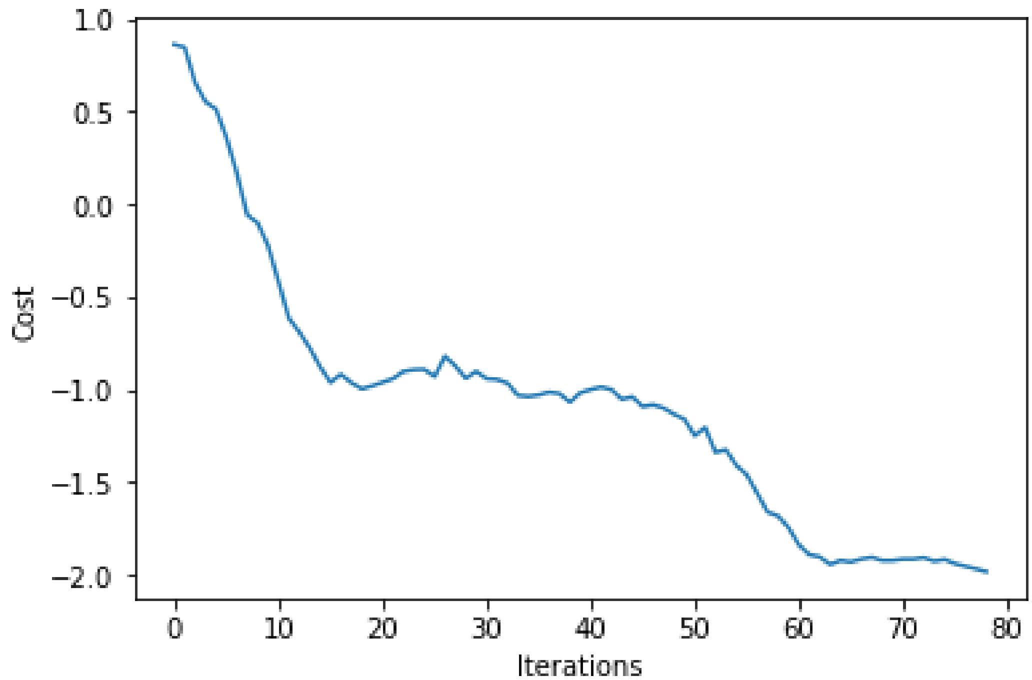
[3]:
```
def cost(thetas):
    # The cost function defined so that when it is minimized we ideally should␣
↪get the desired Bell state
    C = circuit(thetas)
```

```
    return C[1] - C[0]
```

[4]:
```
# We use the Adam optimizer with step size 0.1 and we use a tolerance of 0.2 as␣
↪a stopping criteria

tol = 0.02

opt = qml.AdamOptimizer(0.1)

# Initialize the parameters to zero
thetas_noiseless = np.zeros(4)

# A list containing the costs in each iteration
costs_noiseless = []

while cost(thetas_noiseless) > -2 + tol:
    thetas_noiseless = opt.step(cost, thetas_noiseless)
    costs_noiseless.append(cost(thetas_noiseless))
```

[5]:
```
# The optimum values of the parameters
print('The optimum values for the parameters are ' +  str(thetas_noiseless))

# Plot the cost vs interations p
plt.plot(costs_noiseless)
plt.ylabel('Cost');
plt.xlabel('Iterations');
```

```
The optimum values for the parameters are [-0.08781541  1.5778328  -3.24938139
0.11317726]
```



[6]:
```
# Show the expextation values of the Paulix on qubit 0 and the Pauliz on qubit 1
# Ideally we should get [1, -1]

print(circuit(thetas_noiseless))
```

```
[ 0.99414062 -0.99023438]
```

Do the optimization on a qiskit noise model from the backend properities of the device 'IBMQ Vigo'

[7]:
```
from qiskit import QuantumCircuit, execute, Aer, IBMQ
from qiskit.visualization import plot_histogram
from qiskit.providers.aer.noise import NoiseModel
provider = IBMQ.load_account()
```

```
C:\Users\Ali\anaconda3\lib\site-
packages\qiskit\providers\ibmq\ibmqfactory.py:192: UserWarning: Timestamps in
IBMQ backend properties, jobs, and job results are all now in local time instead
of UTC.
  warnings.warn('Timestamps in IBMQ backend properties, jobs, and job results '
```

[8]:
```
# The noise model from the backend properties
backend = provider.get_backend('ibmq_vigo')
noise_model = NoiseModel.from_backend(backend)
```

```
# Get coupling map from backend
coupling_map = backend.configuration().coupling_map

# Get basis gates from noise model
basis_gates = noise_model.basis_gates

dev_noise = qml.device('qiskit.aer', wires=2, noise_model=noise_model, shots =␣
↪1024)

@qml.qnode(dev_noise)
def circuit(thetas):


    qml.RX(thetas[0], wires = 0)
    qml.RY(thetas[1], wires = 0)

    qml.CNOT(wires = [0, 1])

    qml.RX(thetas[2], wires = 1)
    qml.RY(thetas[3], wires = 1)

    qml.CNOT(wires=[0, 1])

    return qml.expval(qml.PauliX(0)), qml.expval(qml.PauliZ(1))
```

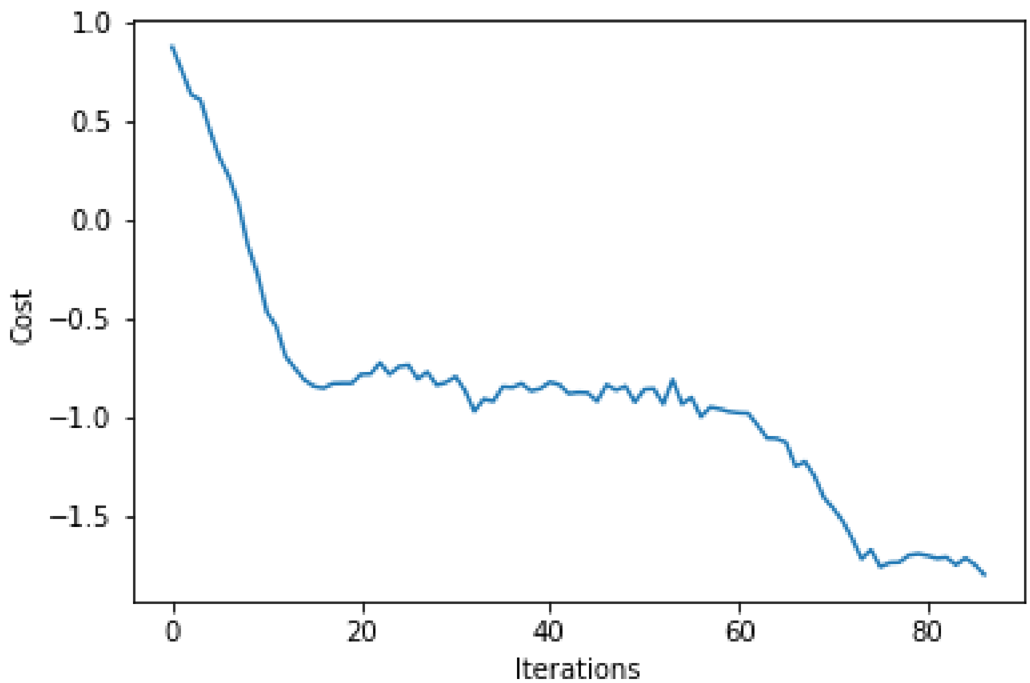
[9]:
```
def cost(thetas):
    # The cost function defined so that when it is minimized we ideally should␣
↪get the desired Bell state
    C = circuit(thetas)
    return C[1] - C[0]
```

[10]:
```
# We use the Adam optimizer with step size 0.1 and we use a tolerance of 0.2 as␣
↪a stopping criteria

tol = 0.2

opt = qml.AdamOptimizer(0.1)

# Initialize the parameters to zero
thetas_noise = np.zeros(4)

# A list containing the costs in each iteration
costs_noise = []

while cost(thetas_noise) > -2 + tol:
    thetas_noise = opt.step(cost, thetas_noise)
    costs_noise.append(cost(thetas_noise))
```

[11]:

```
# The optimum values of the parameters
print('The optimum values for the parameters are ' +  str(thetas_noise))

# Plot the cost vs interations
plt.plot(costs_noise)
plt.ylabel('Cost');
plt.xlabel('Iterations');
```

```
The optimum values for the parameters are [-3.30201918  1.50608131  0.22837535
-3.030718  ]
```



[12]:

```
# Show the expextation values of the Paulix on qubit 0 and the Pauliz on qubit 1
# Ideally we should get [1, -1]

print(circuit(thetas_noise))
```

```
[ 0.88476562 -0.85546875]
```

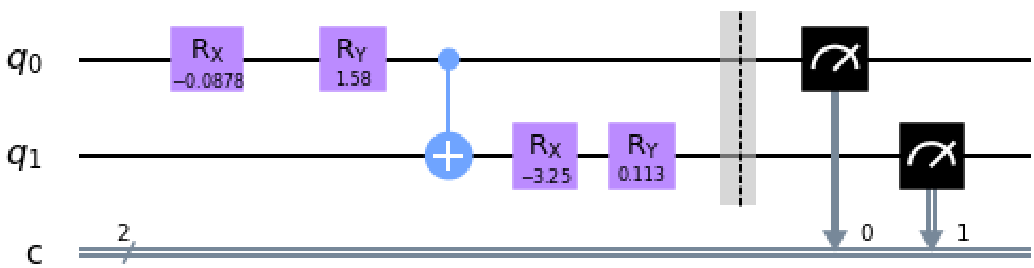
2 Results

We show the results form toptimizing on a noiseless simulator here.

[13]:

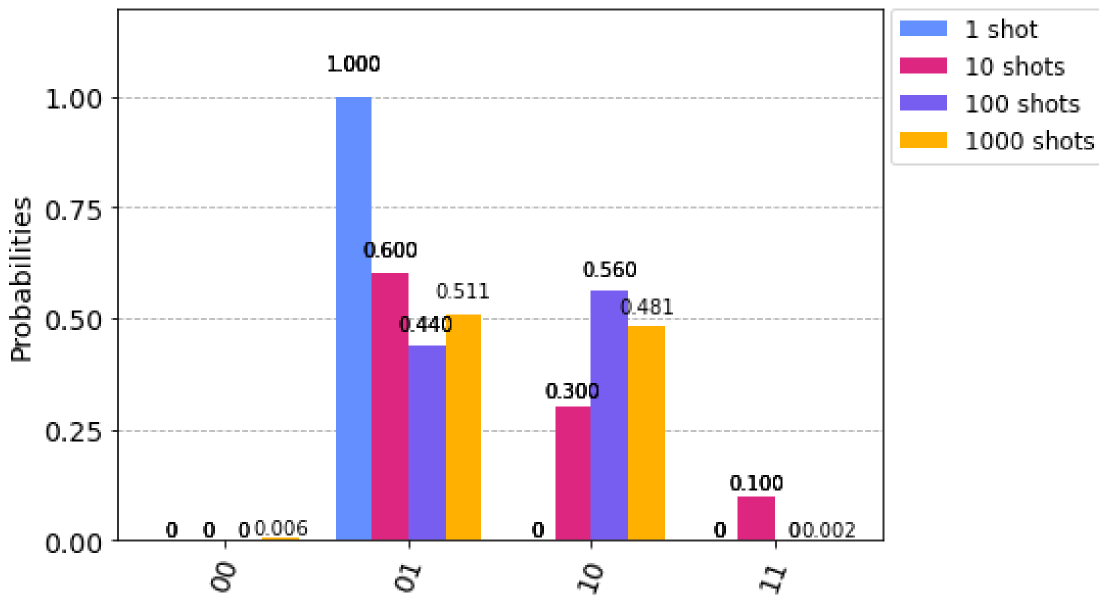
```python
# Here we show the results of applying the circuit architecture with the optimum␣
↪values obtained from the noiseless simulator

thetas = thetas_noiseless


qc = QuantumCircuit(2, 2)

qc.rx(thetas[0], 0)
qc.ry(thetas[1], 0)

qc.cx(0, 1)

qc.rx(thetas[2], 1)
qc.ry(thetas[3], 1)
qc.barrier()

qc.measure([0,1], [0,1])


qc.draw('mpl')
```

[13]:



[14]:
```python
shots = [1, 10, 100, 1000]
counts = []
for i in shots:

    result = execute(qc, Aer.get_backend('qasm_simulator'), shots = i).result()
    counts.append(result.get_counts(qc))

plot_histogram(counts, legend=["1 shot", "10 shots", "100 shots", "1000 shots"])
```

[14]:

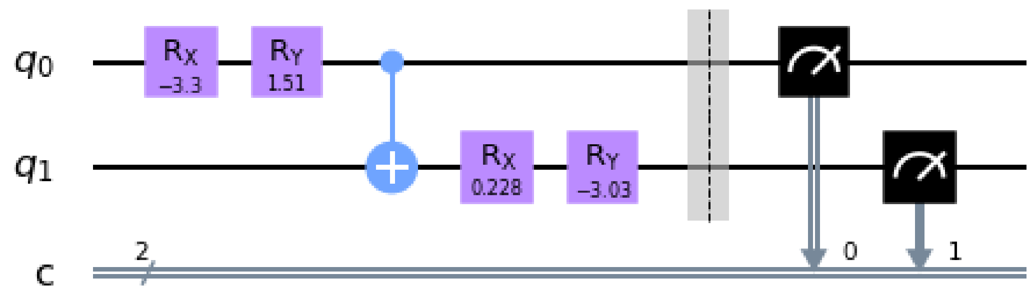


We also show the results form optimizing on a noisy simulator here.

[15]:
```
# Here we show the results of applying the circuit architecture with the optimum␣
↪values obtained from the noisy simulator

thetas = thetas_noise

# The noise model from the backend properties
backend = provider.get_backend('ibmq_vigo')
noise_model = NoiseModel.from_backend(backend)

# Get coupling map from backend
coupling_map = backend.configuration().coupling_map

# Get basis gates from noise model
basis_gates = noise_model.basis_gates

qc = QuantumCircuit(2, 2)

qc.rx(thetas[0], 0)
qc.ry(thetas[1], 0)

qc.cx(0, 1)

qc.rx(thetas[2], 1)
qc.ry(thetas[3], 1)
qc.barrier()
```

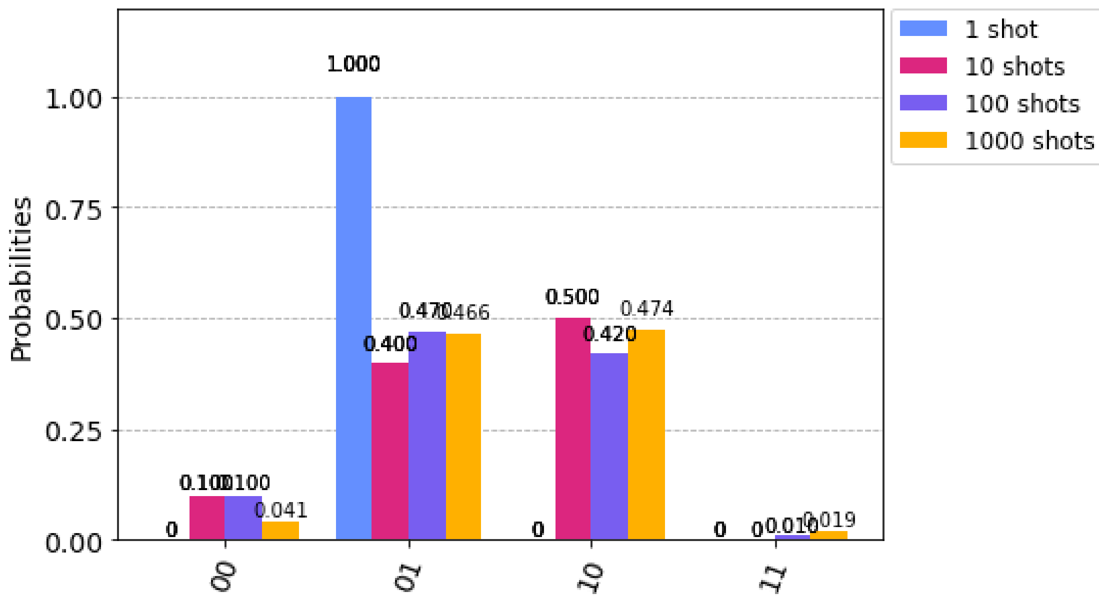
```
qc.measure([0,1], [0,1])


qc.draw('mpl')
```

[15]:




```
[16]: shots = [1, 10, 100, 1000]
counts = []
for i in shots:

    result = execute(qc, Aer.get_backend('qasm_simulator'),␣
 ↪coupling_map=coupling_map, basis_gates=basis_gates,
                     noise_model=noise_model, shots = i).result()
    counts.append(result.get_counts(qc))


plot_histogram(counts, legend=["1 shot", "10 shots", "100 shots", "1000 shots"])
```

[16]:



3 Bonus

The method I used here already makes sure that we have the state $\left|\Psi^{01}\right\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$ and not any other combination with a non-zero relative phase.