AST 4414/5416 Cosmology & Structure Formation, Spring 2021

Problem Set #4

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This homework deals with measuring distances in astronomy and cosmology. Let's define the very basics prerequisite concepts before beginning the questions. We will be dealing with the distance ladder, which defines the method of measuring stellar distances based on the scale of the distances

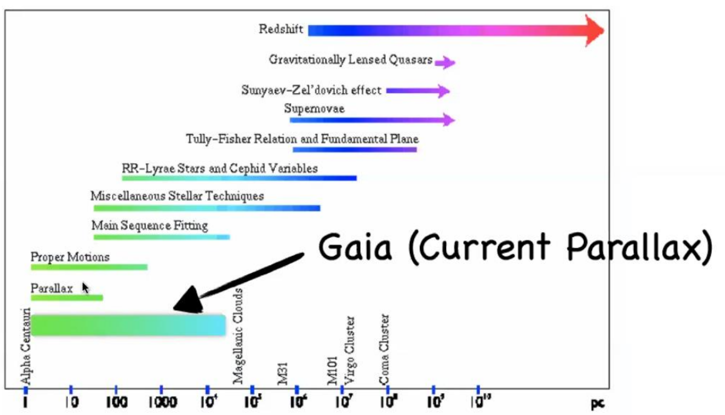


Figure 3.2: *The different distance estimators. This seemingly simple plot shows a grand overview of our efforts to measure distances in the Universe. Adapted from [Rowan-Robinson, 1985] and [Roth and Primack, 1996].*

Parallax is good for measuring distances for close stars, where the parallax is relative to earth in its two sides of the orbit around the sun, if we see one arcsec difference then this corresponds to a distance of 1 parsec. For stars farther away, however, parallax is not a good measure of distance, as the parallax by very far away objects is much less than 1 arcsec.

One challenge of using parallax is that these angles are incredibly small (remember that an arcsec is 1/3600 of a degree, which is like seeing a quartes 5 km away!). This means that earth's atmosphere introduces all kinds of irregularities and disturbances that will distort the light coming from these objects and hence the angles of these objects, and this makes it very difficult to measure these very small angles for far away objects. The logical next step is to use telescopes outside of earth's atmosphere, which was done with the Hipparcos satellite which measured the distance of 100000 relatively close by stars. Also Gaia measured the parallax of 1 billion stars in the milky way galaxy.

HR Diagram

If you measure the color of a star and its luminosity, then you plot it on the HR diagram (luminosity on the y-axis and color on the x-axis). There are very distinct trends that the stars follow on this diagram which helps us determine the stellar evolution, etc. By measuring the color of globular clusters we can infer its luminosity and hence its distance, this is called spectroscopic parallax.

Cepheid variable (or variable stars) are stars are pulsating, and the period of the pulsating (getting brighter and dimmer) is related to its distance. Henrietta Leavitt found that the period of these variable stars was related to the luminosity, then if we have luminosity we can get the distance (from the flux equation)

Type 1a supernova is one of the brightest things in the universe (about 1000 times btighter than our sun, they have brightness that is comparable to the brightness of a galaxy's core.) In a star we have gravitywhich pushes inwards, and the stars core which has nuclear fusion which points outward. When the star runs out of fuelsable material (fuel) to burn, so the pressure forces will fail and the star will collapse (by gravity), then due to the high pressures and densities of the collapse they will allow a new set of nuclear reactions to occur. These nuclear reactions will push the outer layer incredibly far and bright. Type 1a result when a white dwarf (essentially a degenerate ball of electrons) ignites and converts the carbon and oxygen to things like cobalt, iron and that makes a huge bright expanding light. Also the white dwarf is part of a binary star which accetes material to it and causing the reaction. You can measure the distance with these to about 6-7 percent. But they are very rare so its hard to calibrate its distance.

Redshift z is related to the observed wavelength λ_o and emitted wavelength λ_e by

$$z = \frac{\lambda_o - \lambda_e}{\lambda_e} = \frac{v}{c}$$

Which for relatively close by objects (100s of millions of ly's as opposed to billions of ly's) is related to the distance d and Hubble constant H_0

$$z \approx \frac{dH_0}{c}$$

Hence if we measure the redshift and we have a way of determining H_o then we can measure the distance to the object. In order to measure the redshift we have to measure the emitted wavelength also. This can be done by measuring the spectrum of the light, and by the spectral lines, particular elements produce very specific wavelengths. So when we measure the observed wavelength we notice a shift from the the spectral lines we observe and the ones we expect based on the elements.

Hubble's Law

Hubble discovered that the spectral lines (wavelengths) of galaxies were shifted towards red by an amount that was proportional to their distance, this means the are moving away from us by an amount proportional to their distance.

$$z = \frac{H_0}{c} r$$

Orr with $z \approx v/c$, Hubble's law takes the form

$$v = H_0 r$$

More recent results show that $H_0 = 500\ km\, s^{-1} Mpc^{-1}$. But galaxies do not follow Hubble's law exactly. In addition to the expansion of the universe, galaxy motions are affected by gravity of nearby objects. Each galaxy has therefore some "peculiar motion" that affects , hence the recession of a galaxy is given by

$$v = H_0 d + v_{pec}$$

Where v_{pec} is the peculiar velocity of a galaxy along the line of sight. Peculiear velocities are typically about 300 km/s and they very rarely exceed 1000 km/s.

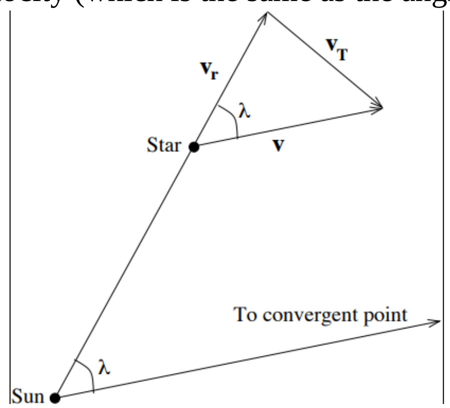
Peculiar velocity (peculiar motion) is motion that doesn't fit in with Hubble's Law. This means that there are things going towards you, things going away, etc, not all uniformly expanding according to Hubble law. This happens when galaxies are close together, ie when the force of gravity is stronger than the force of the expanding space between them. For example, Andromeda is approaching towards us.

QUESTIONS

1)

a) Proper motion $\mu = 0.1223'' yr^{-1}$, $\lambda = 34^0.44$, $v_r = 39.3\ km\ s^{-1}$.

We can find the distance in parsecs to BD +15 612 using the moving cluster method. We can draw the figure below, where the stars in the cluster are moving towards a convergent point with a velocity v. We can decompose the velocity to radial velocity v_r and tangential velocity v_T via the angle between the convergent point and the radial velocity (which is the same as the angle between v and v_r) λ.



Then the distance d to the star can easily be found with geometry, since the tangential velocity is related to the proper motion as

$$v_T = \mu\, d$$

So

$$\tan(\lambda) = \frac{v_T}{v_r} = \frac{\mu\, d}{v_r}$$

If v_r is in km/s and μ is in arcsec/yr, then d will be in parsecs

$$d(pc) = \frac{v_r \tan(\lambda)}{4.74\, \mu}$$

Where the 4.74 factor is in converting $1''\ yr^{-1}\ at\ 1\ pc = 4.74\ km/s$.

In [5]:
```python
import numpy as np
phi = 34.44
mu = 0.1223 #arcsec/yr
vr = 39.3 #km/s
D = (vr*np.tan(np.radians(phi)))/(4.74*mu)
D
```
Out[5]:
```
46.48869704850459
```

Plugging in the values in the equation above, I attain a distance of

$$d = 46.49\ pc$$

Which is close to the distance to the center of the cluster, a good sign.

b) The distance modulus is modulus $m - M$ is the difference between the apparent magnitude m and the absolute magnitude M of and astronomical object. If the distance D is in parsecs, then it is related to the distance by

$$\log_{10}(D) = 1 + \frac{m - M}{5}$$

$$Distance\ Modulus \equiv m - M = 5\log_{10}(D) - 5$$

Since we have our answer for distance, calculating the distance modulus is straight forward

```
In [6]:  d_mod = 5*np.log10(D) -5
         d_mod

Out[6]: 3.336736871313974
```

In plugging in the equation above, I attain a distance modulus

$$\boldsymbol{m - M = 3.337}$$

C) The absolute magnitude M is defined as the apparent magnitude of an object when seen at a distance of 10 parsecs. If we now suppose that BD +15 612 has an apparent visual magnitude $m = 3.64$ we can find the absolute visual magnitude by the equation above, which relates the distance modulus (let's denote it by D_{mod}) and m and M:

```
In [7]:  d_mod = 5*np.log10(D) -5
         M = 3.64-d_mod
         M

Out[7]: 0.3032631286860261
```

$$M = m - D_{mod} = 3.64 - (m - M) = 0.3033$$

$$\boldsymbol{M = 0.3033}$$

2)

a) Intrinsic luminosity is another name for absolute luminosity. Intrinsic luminosity is related to apparent luminosity by

$$l = \frac{L_{int}}{4\,\pi d^2}$$

Where d is the distance to the object, and l is the apparent luminosity (flux). Intrinsic luminosity is also related to the luminosity distance by

$$l = \frac{L_{int}}{4\,\pi d_L^2}$$

Hence $d_L = \sqrt{\frac{4\pi l}{L_{int}}}$. Therefore if we have Intrinsic luminosity of Type Ia supernovae of about $\pm 20\%$ i.e. $\delta L_{int} = \pm 0.2$ we can find the corresponding accuracy of the supernova's luminosity distance d_L by propagation of errors:

$$\delta d_L(L_{int}) = \frac{d\ d_L}{dL_{int}} \cdot \delta L_{int}$$

$$\frac{d\ d_L}{dL_{int}} = \sqrt{4\pi l}\frac{\partial L_{int}^{-\frac{1}{2}}}{\partial L_{int}} = -\sqrt{4\pi l}\frac{1}{2}\ \ L_{int}^{-\frac{3}{2}} = -\frac{\sqrt{\pi l}}{L_{int}^{3/2}}$$

$$\rightarrow \boldsymbol{\delta d_L} = \frac{\sqrt{\boldsymbol{\pi l}}}{\boldsymbol{L_{int}^{3/2}}} \times \mathbf{0.2}$$

Where we have assumed that there is no error in l (the flux) and d_L has uncorrelated Gaussian error propagation.. So for any given intrinsic (aka absolute) luminosity, the accuracy of its associated distance luminosity will be given by the equation above.

However, this is not such a useful equation since we don't know l or L_{int} therefore we would not be able to get a numerical δd_L. If we want to get a numerical answer, we can take ratios of the fluxes of two objects. We know that the brightness decreases as inverse square of the distance. So for two objects

$$\frac{l_1}{l_2} = \left(\frac{d_2}{d_1}\right)^2$$

This leads to the very useful distance formula

$$\log d = \frac{m - M + 5}{5}$$

Where d is the distance measured in pc. Note that m is measured, then we can determine the distance to some object if we know the absolute magnitude M.

b) Peculiar velocities v_{pec} make Hubble's law take the form

$$v = H_0 d + v_{pec}$$

We can assume that we are dealing with the local universe so that the Hubble parameter stays constant. So assuming $H_0 = 70\ kms^{-1}Mpc^{-1}$ and $v_{pec} = 600\ km\ s^{-1}$

I have spent forever on this question and couldn't get to a satisfactory solution, I apologize for misunderstanding what it means exactly.

3) Note that the surface brightness is usually denoted as B, not Σ!

a) If we consider a small patch of a galaxy, that is the part contained in a solid angle $d\Omega$, then we can measure the flux, dF , (where $F = \frac{L}{4\pi r^2}$ for a luminosity L and distance r) of light coming from the part of the galaxy contained within $d\Omega$. This quantity $\frac{dF}{d\Omega}$ is defined as the surface brightness B

$$B = \frac{dF}{d\Omega}$$

The claim is that surface brightness of an object (like a galaxy) is independent to the distance to that object. To see this, consider the following argument: assume that a patch of galaxy contained within $d\Omega_1$ contains N_1 stars, with a mean luminosity L , when it is a distance r_1. Hence the surface brightness is

$$B_1 = \frac{N_1\ L}{4\pi\ r_1^2\ d\Omega_1}$$

Now *increase the distance* to the galaxy by a factor of 2 (say $r_2 = 2r_1$). The *flux from each star will now decrease* by a factor of 2^2 since $F \propto \frac{1}{r^2}$ (so $dF_2 = 2^2 dF_2$) . However, the number of stars within $d\Omega$ will increase by a factor of 2^2 (say $N_2 = 2^2 N_1$), since the physical size of the patch enclosed by $d\Omega$ will double when r doubles (of course $d\Omega$ stays the same since it's independent of distance). Hence the new surface brightness will equal the old one

$$B_2 = \frac{N_2\ L}{4\pi\ r_2^2\ d\Omega_2} = \frac{2^2 N_1 L}{4\pi\ (2r_1)^2 d\Omega_1} = \frac{N_1 L}{4\pi\ r_1^2\ d\Omega_1} = B_1$$

Now let's check this argument more rigorously. Assume that a face-on galaxy at distance r has surface density Σ (in stars/pc^2), all with the same luminosity L. The surface area of the galaxy dS, contained within solid angle $d\Omega$ is $dS = r^2\ d\Omega$. Therefore the number of stars within $d\Omega$ is dN, which will be

$$dN = \Sigma\ dS = \Sigma\ r^2\ d\Omega$$

The flux received from one single star is $F = \frac{1}{4\pi\, r^2}$ therefore the flux received from all N stars contained within $d\Omega$ id dF and will be given by

$$dF = \left(\frac{1}{4\pi\, r^2}\right) \times dN = \frac{1}{4\pi\, r^2} \times \Sigma\, r^2\, d\Omega = \frac{\Sigma\, \mathrm{d}\Omega}{4\pi}$$

And since, by definition, $B = \frac{dF}{d\Omega}$, we have

$$\boldsymbol{B \equiv \frac{dF}{d\Omega} = \frac{\Sigma}{4\pi}}$$

Which is independent of distance, like we expected!!!!!!

b) using d_L(luminosity distance) and d_A (angular diameter distance), we can see that if galaxies are cosmologically distant but otherwise identical, the surface brightness B will go like $B \propto (1+z)^{-4}$ where z is the redshift. This can be seen from the definition of the angular diameter distance (in Euclidean geometry) which is

$$d_A \equiv \frac{s}{\theta} = \frac{a(t_1) r_1 \theta}{\theta} = a(t_1)\, r_1$$

Where s is the proper distance perpendicular to the line of sight, which subtends and angle θ. This proper distance is the distance to a source comoving at radial coordinate r_1 that emits light at time t_1.

Now let's look at the luminosity distance. We have apparent luminosity l related to d_L by

$$l \equiv \frac{L}{4\pi\, r_1^2\, a^2(t_0)(1+z)^2} = \frac{L}{4\pi d_L^2}$$

So

$$d_L \equiv a(t_0) r_1\, (1+z)$$

Where here, the light leaves the source at distance r_1 , and *arrives* to the origin at time t_0. The two terms of expansion at the two times, $a(t_0)$ and $a(t_1)$ can be related to each other via the important equation (W.1.2.5):

$$1 + z = \frac{a(t_0)}{a(t_1)}$$

Hence $a(t_0) = a(t_1)(1+z)$. Hence we can get a ratio between d_A and d_L that depends only on the redshift z !

$$\frac{d_A}{d_L} = \frac{a(t_1)\, r_1}{a(t_0)\, r_1 (1+z)} = \frac{a(t_1) r_1}{a(t_1)(1+z)^2\, r_1} = \frac{1}{(1+z)^2}$$

So

$$\frac{d_A}{d_L} = (1+z)^{-2}$$

This is useful since now if we consider a light source that has absolute luminosity per unit area $\mathcal{L}$, then the apparent luminosity of a patch of area A is

$$l = \frac{\mathcal{L}A}{4\pi\, d_L^2}$$

This patch will subtend a solid angle $\Omega = \frac{A}{d_A^2}$. The surface brightness B is defined as the apparent luminosity l (or equivalently, the flux F) per solid angle, so

$$B \equiv \frac{l}{\Omega} = \frac{\mathcal{L}A}{4\pi\, d_L^2}\frac{d_A^2}{A} = \frac{\mathcal{L}}{4\pi}\left(\frac{d_A}{d_L}\right)^2$$

And above we found this ratio $\frac{d_A}{d_L} = (1+z)^{-2}$ Hence

$$B = \frac{\mathcal{L}}{4\pi}\,(1+z)^{-4}$$

Hence if for cosmologically distant galaxies that are otherwise identical (i.e. have the same luminosity per area $\mathcal{L}$), the surface brightness, by the equation above, should decrease with the redshift as $B \propto (1+z)^{-4}$.

c) The question (I believe) asks us to compare the surface brightness B to the total flux (apparent brightness) from an unresolved point source F using the luminosity distance d_L. Recall that $d_L = a(t_0)\, r_1\, (1+z)$ and the total flux

$$F_{d_L} = \frac{L}{4\pi\, d_L^2}$$

Where I denote F_{d_L} as the total flux using the luminosity distance definition. Hence we are to compare the total flux using the surface brightness definition, which I denote as F_B

$$B = \frac{\mathcal{L}}{4\pi}\,(1+z)^{-4} \;\Rightarrow\; F_B = \frac{L}{4\pi}(1+z)^{-4}$$

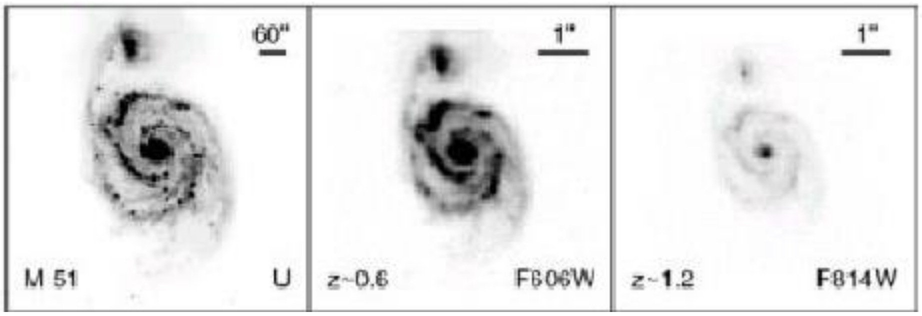
(where in the above step is clear since by $B \equiv \frac{dF}{d\Omega}$ and $\mathcal{L} = \frac{L}{d\Omega}$ hence $F = B\, d\Omega$).

To

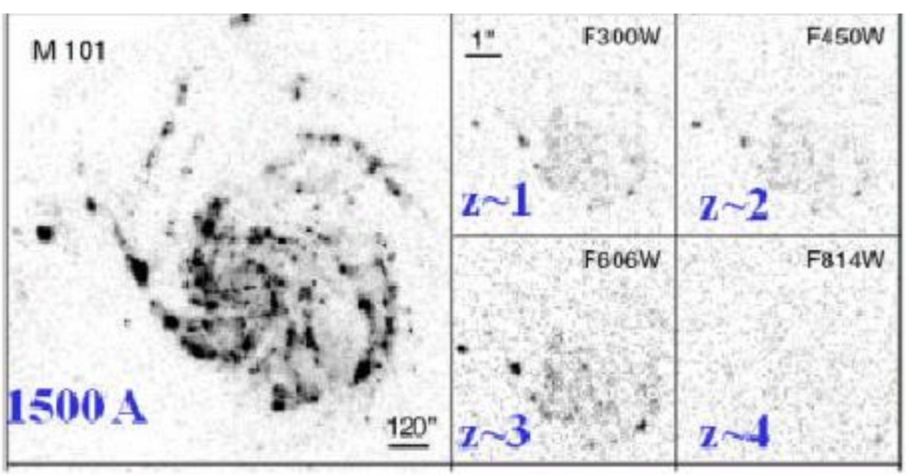
$$F_{d_L} = \frac{L}{4\pi\, a^2(t_0) r_1^2}(1+z)^{-2}$$

Hence there are differences to the total flux when the point source (galaxy) is unresolved, using the luminosity distance, F_{d_L} , to the total flux from a resolved point source, which uses both the luminosity distance and the angular diameter distance.

The provided figures from the paper by Kuchinski et al. shows exactly just that, the surface brightness B appears to decrease with increasing z just like we expect. This is very clear to see for the $M51$ objects.



The $M101$ objects show further complications. According to the paper, these show "peculiar galaxies" which are irregular galaxies that may have tidal features, mergers or other obvious disruptions. The paper shows apparent increase in the fraction of irregular/peculiar features at high redshifts. This is logical since according to Hubble's law, galaxies at higher redshifts correspond to objects at further away distances, which also corresponds to the surface brightness dimming and loss of special resolution. Therefore one might expect such morphological irregularities in the surface brightness or flux measurements that we observe from such dim and highly redshifted objects.



For more about this please refer to the cited paper by Kuchinski et al. 2001. Other interesting things to note about this $M101$ system is that only bright star-forming regions in the spiral arms are visible at high redshift; the nucleus is quickly lost below the detection threshold. Therefore there is a threshold for the redshift, beyond which it would be very difficult to detect.