Ali Alkadhim

Quantum Computing HW 2

2.1 a) If Eve (E) knows that Alice (A) and Bob (B) are using one of the two basis: Haddamard (H) or standard (S), then E randomly picks one of the two bases for each photon polarization she measures. If she measures in the correct basis, she is sure she has gotten the correct bit, but if she measures in the wrong basis, she gets the correct bit only half of the time. Thus if she listens in when A & B reveal their basis sequence, she only knows that one half of all photon measurements encode the correct bit that A sent, i.e. those which E measured with the same basis as A.

b) Similarly, only one half of the bits is E sure to have correct, the other half of Eve's string of bits will have a corresponding 50% probability to have the correct bits, since that is the $|\text{Amplitude}|^2$ of her getting the correct bit whilst using a different

Scanned with CamScanner

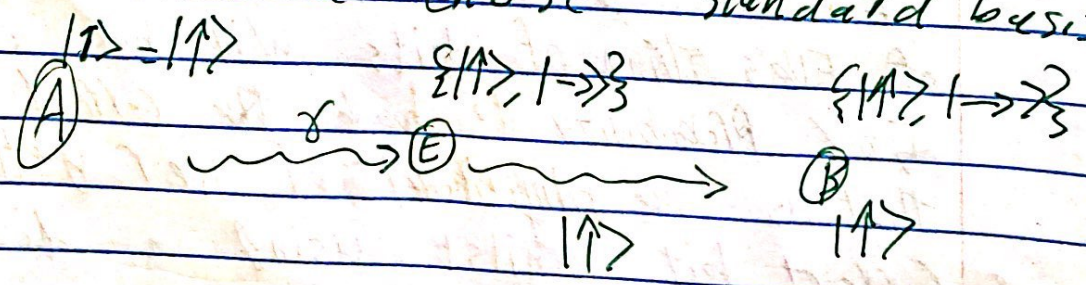
basis. Hence she will have a fraction

$$\frac{1}{2} + \frac{1}{2}\frac{1}{2} = \boxed{\frac{3}{4} = 75\%}$$

of her bit sequence being the correct bits that A sent.

c) Let's start by computing the probability for one photon measurement. The question asks to generalize the probability of of B measuring the correct bit for n measurements. A can send bits in either the H or S basis, but they throw away those measurements in which their basis choice does not match. of the remaining sequence, we have to consider the two cases for Eve's choice of the basis. Let's suppose that for one measurement, A and B kept their matching basis, let's suppose it's in the standard (S) basis:

Case 1: Eve chose standard basis



Case 2: Eve chose a different basis (Hadamard)

$\{|\nearrow\rangle, |\nwarrow\rangle\}$; $|\nearrow\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\rightarrow\rangle)$

$$|\nwarrow\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\rightarrow\rangle)$$

Then E could choose $|\nearrow\rangle$ or $|\nwarrow\rangle$, so the probability that B receives the correct bit is $P = P_1 + P_2$, where

$$P_1 = \left| |\langle\uparrow|\nearrow\rangle|^2 |\langle\nearrow|\uparrow\rangle|^2 \right|$$

A's original Photon Polarization

Bob's measurement Polarization

E's transmitted Photon Polarization

E's measurement Polarization

$$P_1 = \left| |\langle\uparrow|(\tfrac{1}{\sqrt{2}}(|\uparrow\rangle + |\rightarrow\rangle))\rangle|^2 \left|(\tfrac{1}{\sqrt{2}}(\langle\uparrow| + \langle\rightarrow|))|\uparrow\rangle\right|^2 \right|$$

$$= \left| \frac{1}{2} \cdot \frac{1}{2} \right| = \frac{1}{4}$$

$$P_2 = \left| |\langle\uparrow|\nwarrow\rangle|^2 |\langle\nwarrow|\uparrow\rangle|^2 \right|$$

$$= \left| \left| \langle\uparrow|(\tfrac{1}{\sqrt{2}}(|\uparrow\rangle - |\rightarrow\rangle))\right|^2 \left|(\tfrac{1}{\sqrt{2}}(|\uparrow\rangle + |\rightarrow\rangle)|\uparrow\rangle\right|^2 \right|$$

$$= \frac{1}{4}$$

Hence $P = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

Scanned with CamScanner

Hence the total Probability B measures the correct bit is $1 + \frac{1}{2} = \frac{3}{2}$.

Averaging over all the Possible bases choices (2) the total probability is

$$\frac{1}{2}\,\frac{3}{2} = \frac{3}{4}.$$

In order to have 90% confidence that Eve is undetected, i.e. at least 90% chance of detecting Eve, we need

$$\left(\frac{3}{4}\right)^n \leq 0.1 \quad \text{for } n \text{ measurements}$$

$$n \log \frac{3}{4} \leq \log 0.1$$

$$\Rightarrow \quad n \geq 8$$

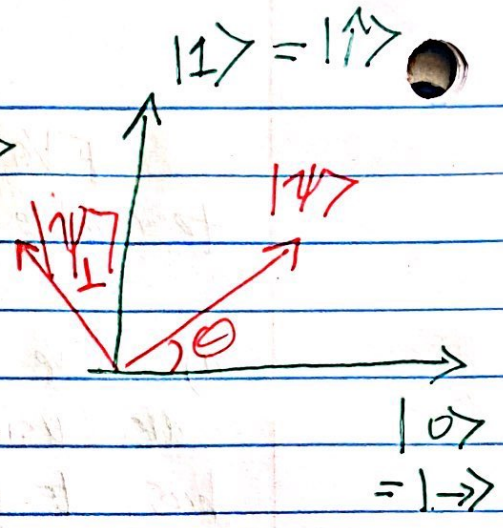
Scanned with CamScanner

Eve does <u>not</u> know which two bases to choose from

a) Eve does not know that A and B are using the S and H bases, she has to choose from an infinite number of possible bases each time she makes a measurement. Hence for her to know that a bit she measured is the correct bit, she has to be sure that her basis is correct, and she is picking from an infinite number of bases, the percentage of bit values that E will know for sure are the correct bits is $\frac{1}{\infty} = \boxed{0\%}$.

b) In the same manner as part b in the previous question, only this time there are infinitely many possible choices for E's basis, meaning an infinite number of possible angles between A's basis and E's basis. suppose that for this one photon, A and B are keeping the measurement where their bases agree, suppose their basis is standard (S) in this instance, then Eve makes a measurement

in an arbitrary basis $|\psi\rangle$ which is a superposition of $|1\rangle$ and $|0\rangle$



$$|\psi\rangle = \cos\theta\,|0\rangle + \sin\theta\,|1\rangle$$

where $0 \leq \theta \leq \pi/2$

Then suppose Alice encodes $|0\rangle$, then the probability that E measures $|0\rangle$ is

$$|\langle 0|\psi\rangle|^2 = |\langle 0|\cos^2\theta|0\rangle|^2 = \cos^2\theta$$

Averaging this probability over all possible angles, we get

$$\frac{1}{\pi/2}\int_0^{\pi/2} \cos^2\theta \, d\theta = \frac{2}{\pi}\int_0^{\pi/2} \frac{1+\cos 2\theta}{2}\, d\theta$$

$$= \frac{1}{\pi}\left[\theta + \frac{\sin 2\theta}{\theta}\right]_0^{\pi/2}$$

$$= \frac{1}{\pi}\left[\frac{\pi}{2} + 0\right] = \boxed{\frac{1}{2} = 50\%}$$

Scanned with CamScanner

c) In a similar fashion we have to consider the two cases. suppose again that Alice chose $|0\rangle$, then Eve measures in her $\{|\psi\rangle, |\psi_\perp\rangle\}$ basis, where

$$|\psi\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle \; ; \; |\psi_\perp\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$$

Case 1: Eve chooses $|\psi\rangle$:

$P_1(B \text{ still gets } |0\rangle) =$

$$\left| |\langle 0|\psi\rangle|^2 |\langle\psi|0\rangle|^2 \right|$$

Bob's measured Pol. Eve's transm. Pl tm Pol. Eve's measurement basis Alice's orig. Polariz.

$$= \cos^2\theta \cdot \cos^2\theta = \cos^4\theta$$

Case 2: Eve chooses $|\psi_\perp\rangle$:

$P_2(B \text{ still gets } |0\rangle) =$

$$|\langle 0|\psi_\perp\rangle|^2 |\langle\psi_\perp|0\rangle|^2$$

$$= \sin^2\theta \cdot \sin^2\theta = \sin^4\theta$$

so $P(B \text{ still gets } |0\rangle) = P_1(B \text{ still gets } |0\rangle) + P_2(B \text{ still gets } |0\rangle)$

$$= \sin^2\theta + \cos^2\theta$$

Scanned with CamScanner

Averaging over the range of all possible angles; we get

$$P = \frac{1}{\pi/2}\int_0^{\pi/2} \cos^4\theta + \sin^4\theta \, d\theta$$

$$\int_0^{\pi/2} \cos^4\theta \, d\theta = \int \cos^2\theta \cos^2\theta \, d\theta$$

$$= \int \frac{1+\cos(2\theta)}{2} \cdot \frac{1+\cos(2\theta)}{2} \, d\theta$$

$$= \frac{1}{4}\int_0^{\pi/2} 1 + 2\cos 2\theta + \cos^2(2\theta) \, d\theta$$

$$= \frac{1}{4}\int_0^{\pi/2} 1 + 2\cos 2\theta + \frac{1+\cos(4\theta)}{2} \, d\theta$$

$$= \left[\frac{3}{8}\theta + \cancel{\frac{1}{4}\sin(2\theta)} + \cancel{\frac{1}{32}\sin(4\theta)}\right]_0^{\pi/2}$$

$$= \frac{3}{8}\frac{\pi}{2} = \frac{3\pi}{16}$$

Similarly,

$$\int_0^{\pi/2} \sin^4\theta \, d\theta = \left[\frac{3}{8}\theta - \cancel{\frac{1}{4}\sin 2\theta} + \cancel{\frac{1}{32}\sin(4\theta)}\right]_0^{\pi/2}$$

$$= \frac{3}{8}\frac{\pi}{2} = \frac{3\pi}{16}$$

Scanned with CamScanner

Hence

$$P = \frac{2}{\pi}\left(\frac{3\pi}{16} + \frac{3\pi}{16}\right)$$

$$= 2\left(\frac{6}{16}\right) = \frac{12}{16} = \frac{3}{4}$$

In order for A & B to have at least 90% chance of detecting E, we need

$$\left(\frac{3}{4}\right)^n \leq 0.1$$

$$n \log \frac{3}{4} \leq \log 0.1$$

$$\Rightarrow \boxed{n > 8}$$

Scanned with CamScanner

$\boxed{2.2}$ P. 2.11

a) Let's prove this with an illustrative example. Suppose Alice generates a random string x of 1's and 0's, and she encodes

$0 \longmapsto |0\rangle$ and $1 \longmapsto |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

e.g. $x = \{1, 0, 0, 1, 0, 1, 1\}$

$= \{|+\rangle, |0\rangle, |0\rangle, |+\rangle, |0\rangle, |+\rangle, |+\rangle\}$

She sends this string to Bob, who generates a random string y composed of 1's and 0's.

e.g. $y = \{0, 0, 0, 1, 0, 1, 0\}$

If $y_i = 0$, B measures in the Haddamard basis

$H: \{|+\rangle, |-\rangle\} = \{\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\}$

If $y_i = 1$, B measures in the standard basis

$S: \{|0\rangle, |1\rangle\}$.

Note that if B measures in the H basis a state $|0\rangle$ from Alice, he has a 50% chance of getting a $|+\rangle$ and a 50% chance of getting $|-\rangle$.

If he measures in the S basis a state $|0\rangle$ from Alice, he has 100% chance of measuring the correct state $|0\rangle$.

If Bob's measurement resulted in $|+\rangle$ or $|0\rangle$, he sends 0,

Scanned with CamScanner

If Bob's measurement resulted in $|1\rangle$ or $|-\rangle$, he sends 1. Let's see what he measures.

Let p = result of his measurement

q = the string of bit values that he sends as a result of his measurement.

$x =$	1,	0,	0,	1,	0,	1,	1
$=$	$\lvert+\rangle$,	$\lvert0\rangle$,	$\lvert0\rangle$,	$\lvert+\rangle$,	$\lvert0\rangle$,	$\lvert+\rangle$,	$\lvert+\rangle$
$y =$	0,	0,	0,	1,	0,	1,	0
$=$	H,	H,	H,	S,	H,	S,	H
$p =$	$\lvert+\rangle$,	$\lvert1\rangle$ or $\lvert0\rangle$,	$\lvert1\rangle$ or $\lvert0\rangle$,	$\lvert1\rangle$ or $\lvert0\rangle$,	$\lvert1\rangle$ or $\lvert0\rangle$,	$\lvert1\rangle$ or $\lvert0\rangle$,	$\lvert+\rangle$
$q =$	0,	1 or 0,	1 or 0,	1 or 0,	1 or 0,	1 or 0,	0

Now, they discard all bits from strings x and y for which Bob's bit value from measurement (strig q) yielded 0. These bits are circled above, yielding strings x' and y':

$$\boxed{\begin{aligned} x' &= \{0, 0, 1, 0, 1\} \\ y' &= \{0, 0, 1, 0, 1\} \end{aligned}} \Rightarrow x' \text{ and } y' \text{ are identical}$$

This algorithm works!!! One can test it with longer random strings, and following exactly the same logic we see that it works in any general setting.

Scanned with CamScanner

b) Since Alice encodes $0 \to |0\rangle$ and $1 \to \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, supposing Bob picks the standard Basis, then he knows for sure the state is $|0\rangle$, but this could correspond to either a bit 0 or 1 from Alice, hence he does not know in certainty the original bit Value. The same thing would apply if he had picked the Haddamard basis. If however Bob's bit value from measurement, $q_i = 1$, then the state is $|1\rangle$ or $|-\rangle$, which means that his measurement <u>must</u> be $|1\rangle$ if Alice sends $|+\rangle$ which correspond to a bit Value 1 (hence their bit Values agree), since Alice does not even send $|-\rangle$ states. Furthermore, if he measures $|-\rangle$, then the bit Value encoded by Alice <u>must</u> be 0 (since she encodes $0 \to |0\rangle$, which is a super Position of $|+\rangle$ and $|-\rangle$, whereas $1 \to |+\rangle$ is not). In either case, removing the 0's from Bob's measurement string q ensures that they have the same remaining bit Values.

Scanned with CamScanner

c) If Eve is listening in and she uses either the H or S basis for each qubit measurement before sending them to Bob, then she can either know that they are discarding the 0's from x and y or not.

Case 1: she knows to discard the 0's from her measurement.

this means she discards the $|+\rangle$ and $|0\rangle$ states from her measurement.

$\Longrightarrow$
- If she measures $|-\rangle$, she can be sure that Alice's bit value was 0.
- If she measures $|1\rangle$, she can be sure that Alice's bit value was 1.

(by the same logic as the previous parts of the question).

Case 2: she doesn't know to discard the 0's from measurement.

If she doesn't discard the 0's from her measurement (her measurement is a string analogous to Bob's string q), then she will have incorrect measurements, since the bit values of x and y will be different for those measurements where she keeps the 0's.

Scanned with CamScanner

In order to detect Eve's presence, Bob and Alice must compare their chances of getting the correct measurement with and without Eve's presence.

If Eve is not there, Bob can be 100% sure that he measures the correct bit value if his $y_i \neq x_i$.

For example, if Alice encodes 0 (in $|0\rangle$) and Bob generates 1 (i.e. he uses $\{|0\rangle$ or $|1\rangle\}$, then he has 100% chance of getting 0 as his measurement.

Now, if Eve interferes, then she could generate either 0 or 1 (50% chance of each), which correspond to using either the Haddamard or standard basis, respectively. Hence Bob will have another 50% chance of having an error, associated with Eve choosing the basis that is inconsistent with Alice's, which will yield the wrong state 50% of the time.

Hence Bob will have a chance of errors

$$P(\text{Error}) = P(\text{Eve is undetected}) = \frac{1}{2}\frac{1}{2} + \frac{1}{2}\cdot 1 = \frac{3}{4}$$

for each measurement. Hence if there are N uncorrelated measurements, $P(\text{Eve is there but undetected}) = \left(\frac{3}{4}\right)^N$

Hence the probability to detect Eve is

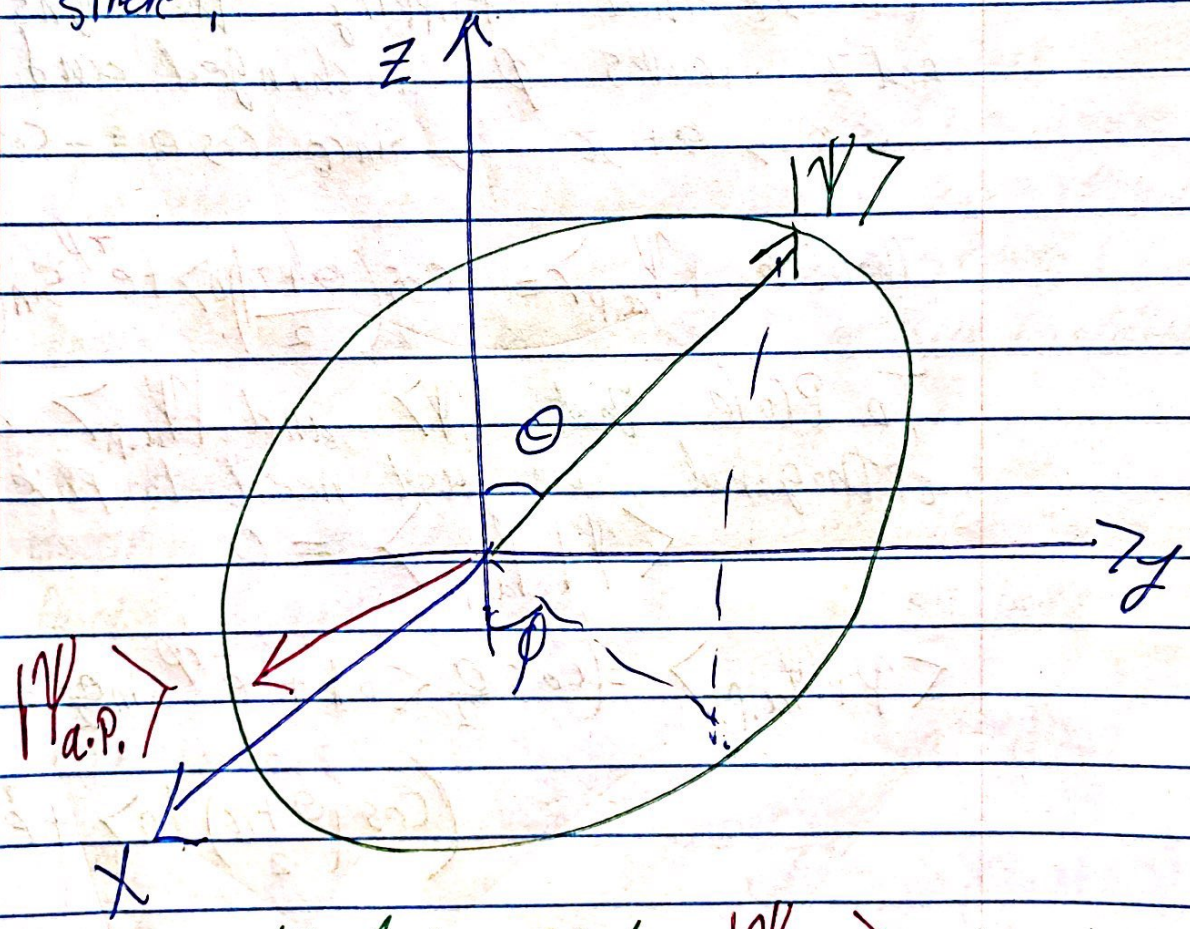
$$\boxed{P(\text{E is detected}) = 1 - \left(\frac{3}{4}\right)^N}$$

Scanned with CamScanner

$\boxed{2.3}$

a) $|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}e^{i\phi}|1\rangle$

Suppose we have a state $|\psi\rangle$ drawn below, oriented at angles θ and ϕ in the unit sphere;



an antipodal state $|\psi_{a.p.}\rangle$ is a state in which the signs of the cartesian coordinates of $|\psi\rangle$ have been flipped. The angles θ and ϕ can be parameterized by the cartesian coordinates by:

$$\theta = \arccos(z)$$

$$\phi = \arctan\left(\frac{y}{x}\right)$$

Now, using $\cos(-x) = \cos x$ and $\sin(-x) = -\sin(x)$, and $\tan(-x) = -\tan(x)$ we see that flipping the signs of x, y and z leaves ϕ unchanged and takes $\theta \to \theta + \pi$, since $\cos\theta = -\cos(\theta+\pi)$

Therefore $|\psi_{a.p.}\rangle = \cos\left(\frac{\theta+\pi}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta+\pi}{2}\right)|1\rangle$

To prove that $|\psi\rangle$ and $|\psi_{a.p.}\rangle$ are orthogonal, we just need to prove that

$$\langle\psi|\psi_{a.p.}\rangle = 0$$

$$\langle\psi|\psi_{a.p.}\rangle = \left(\cos\frac{\theta}{2}\langle 0| + e^{-i\phi}\sin\frac{\theta}{2}\langle 1|\right)\left(\cos\left(\frac{\theta+\pi}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta+\pi}{2}\right)|1\rangle\right)$$

$$= \cos\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta+\pi}{2}\right) + \sin\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta+\pi}{2}\right)$$

using $\cos(a-b) = \cos a\cos b + \sin a\sin b$

where $a = \frac{\theta}{2}$, $b = \frac{\theta+\pi}{2}$, we get

Scanned with CamScanner

$$\langle \psi | \psi_{ap.} \rangle = \cos\left(\frac{\theta - \theta - \pi}{2}\right)$$

$$= \cos\left(\frac{-\pi}{2}\right)$$

$$= \boxed{0}$$

b) Here we want to prove the inverse of Part (a), i.e. we want to prove that for any two orthogonal states (i.e. those whose inner product is zero), they correspond to antipodal points on the Bloch sphere (i.e. points that have the opposite Cartesian coordinates)

To make things more general, we can multiply each state by a global phase factor, say $e^{i\delta}$. So if we have

$$|\psi_1\rangle = e^{i\delta_1}\left(\cos\left(\frac{\theta_1}{2}\right)|0\rangle + e^{i\phi_1}\sin\left(\frac{\theta_1}{2}\right)|1\rangle\right)$$

$$|\psi_2\rangle = e^{i\delta_2}\left(\cos\left(\frac{\theta_2}{2}\right)|0\rangle + e^{i\phi_2}\sin\left(\frac{\theta_2}{2}\right)|1\rangle\right)$$

and requiring that they are orthogonal:

$$\langle \psi_1 | \psi_2 \rangle = 0$$

Scanned with CamScanner

$$\langle \psi_1 | \psi_2 \rangle = \left[e^{-i\delta_1} \left(\cos\left(\frac{\theta_1}{2}\right) \langle 0| + e^{-i\phi_1} \sin\left(\frac{\theta_1}{2}\right) \langle 1| \right) \right]$$

$$\left[e^{i\delta_2} \left(\cos\left(\frac{\theta_2}{2}\right) |0\rangle + e^{i\phi_2} \sin\left(\frac{\theta_2}{2}\right) |1\rangle \right) \right]$$

$$= e^{i(\delta_2 - \delta_1)} \left(\cos\left(\frac{\theta_1}{2}\right) \cos\left(\frac{\theta_2}{2}\right) + e^{i(\phi_2 - \phi_1)} \sin\left(\frac{\theta_1}{2}\right) \sin\left(\frac{\theta_2}{2}\right) \right)$$

$$= 0$$

$e^{i(\delta_2 - \delta_1)}$ cannot be zero, so

$$\cos\frac{\theta_1}{2} \cos\frac{\theta_2}{2} + e^{i(\phi_2 - \phi_1)} \sin\frac{\theta_1}{2} \sin\frac{\theta_2}{2} = 0$$

There are the following ways to satisfy the above equation:

1. $\cos\frac{\theta_1}{2} \cos\frac{\theta_2}{2} = 0 = \sin\frac{\theta_1}{2} \sin\frac{\theta_2}{2}$

using the identity

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

We have:

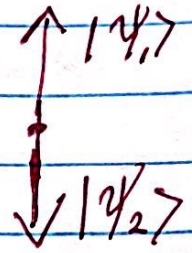
$$\cos\frac{\theta_1}{2} \cos\frac{\theta_2}{2} - \sin\frac{\theta_1}{2} \sin\frac{\theta_2}{2} = \cos\left(\frac{\theta_1 + \theta_2}{2}\right)$$

$\cos\left(\frac{\theta_1 + \theta_2}{2}\right) = 0$ only if

$\theta_1 = \pi$ and $\theta_2 = 0$, or $\theta_1 = 0$ and $\theta_2 = \pi$

Scanned with CamScanner

Hence, one state is at $\theta_1 = 0$ and the other at $\theta = \pi$. These states are clearly antipodal, regardless of φ_1 and φ_2.



2. $e^{i(\varphi_2 - \varphi_1)} = 1$ and

$$\cos\frac{\theta_1}{2}\cos\frac{\theta_2}{2} + \sin\frac{\theta_1}{2}\sin\frac{\theta_2}{2} = 0.$$

This requires $\varphi_1 = \varphi_2$ and

$$\cos\left(\frac{\theta_1 - \theta_2}{2}\right) = 0,$$ therefore, by the same argument, $\theta_1 = \theta_2 + \pi$.

This is the same condition for antipodal states that we derived in part (a).

3. $e^{i(\varphi_2 - \varphi_1)} = -1$ and

$$\cos\frac{\theta_1}{2}\cos\frac{\theta_2}{2} - \sin\frac{\theta_1}{2}\sin\frac{\theta_2}{2} = 0.$$

This requires $\varphi_2 = \varphi_1 + \pi$ and

$$\cos\left(\frac{\theta_1 + \theta_2}{2}\right) = 0,$$ therefore $\theta_2 = \pi - \theta_1$.

Let $|\psi_1\rangle$ have cartesian coordinates (x_1, y_1, z_1),

Scanned with CamScanner

and let $|\psi_2\rangle$ have Cartesian coordinates (x_2, y_2, z_2). Therefore

$$x_1 = \sin\theta_1 \cos\varphi_1$$

$$y_1 = \sin\theta_1 \sin\varphi_1$$

$$z_1 = \cos\theta_1$$

$$x_2 = \sin\theta_2 \cos\varphi_2$$

$$= \sin(\pi - \theta_1)\cos(\varphi_1 + \pi)$$

$$= -\sin\theta_1 \cos\varphi_1 = \boxed{-x_1}$$

$$y_2 = \sin\theta_2 \sin\varphi_2$$

$$= \sin(\pi - \theta_1)\sin(\varphi_1 + \pi)$$

$$= -\sin\theta_1 \sin\varphi_1 = \boxed{-y_1}$$

$$z_2 = \cos\theta_2$$

$$= \cos(\pi - \theta_1) = \boxed{-z_1}$$

Thus the orthogonal state $|\psi_2\rangle$ has opposite signs of its Cartesian coordinates to $|\psi_1\rangle$, i.e. its antipodal to $|\psi_1\rangle$. $\neq$

Scanned with CamScanner

$\boxed{2.4}$ P. 3.5 is

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle|+\rangle + |1\rangle|-\rangle\right) \quad \text{entangled?}$$

using $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) + |1\rangle \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\right)$$

$$= \frac{1}{2}\left(|0\rangle|0\rangle + |0\rangle|1\rangle + |1\rangle|0\rangle - |1\rangle|1\rangle\right)$$

Now, if we have two states

$$(a_1|0\rangle + b_1|1\rangle) \otimes (a_2|0\rangle + b_2|1\rangle)$$

$$= a_1 a_2 |0\rangle|0\rangle + a_1 b_2 |0\rangle|1\rangle + b_1 a_2 |1\rangle|0\rangle + b_1 b_2 |1\rangle|1\rangle$$

matching the coefficients, we see that

$$a_1 a_2 = \frac{1}{2}$$

$$a_1 b_2 = \frac{1}{2}$$

$$a_2 b_1 = \frac{1}{2}$$

$$b_1 b_2 = -\frac{1}{2}$$

Scanned with CamScanner

since $a_1 a_2 = a_1 b_2 = \frac{1}{2} \Rightarrow \boxed{a_2 = b_2}$

$a_1 a_2 = a_2 b_1 = \frac{1}{2} \Rightarrow a_1 = b_1$

If this system has no solution the state is entangled. But if the system has a solution, $a_1 a_2 = b_1 b_2$. But

$a_1 a_2 = b_1 a_2$

$\Rightarrow b_1 a_2 = b_1 b_2$

So $a_2 = b_2$, so this is not the case, therefore the system has no solution,

so the state <u>is entangled.</u>

Scanned with CamScanner

$\boxed{2.5}$ P. 37

The Bell basis states appear in Page 36 of the book.

a) $|00\rangle$, The $|00\rangle$ state appears in $|\phi^+\rangle$ and $|\phi^-\rangle$

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$|\phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

$$\Rightarrow |\phi^+\rangle + |\phi^-\rangle = \frac{?}{\sqrt{2}}|00\rangle$$

$$\Rightarrow |00\rangle = \frac{\sqrt{2}}{2}(|\phi^+\rangle + |\phi^-\rangle)$$

$$\boxed{|00\rangle = \frac{1}{\sqrt{2}}(|\phi^+\rangle + |\phi^-\rangle)}$$

b) $|+\rangle|-\rangle$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \quad |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

$$\Rightarrow |+\rangle|-\rangle = \frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle)$$

$$= \frac{1}{2}(\underbrace{|00\rangle - |11\rangle}_{|\phi^-\rangle} \underbrace{- |01\rangle - |11\rangle}_{-|\psi^-\rangle})$$

Scanned with CamScanner

$$\Rightarrow \boxed{|+\rangle|-\rangle = \frac{1}{\sqrt{2}}(|\varphi^-\rangle - |\psi^-\rangle)}$$

c) $\frac{1}{\sqrt{3}}(|00\rangle + |01\rangle + |10\rangle)$

from (a), $|00\rangle = \frac{1}{\sqrt{2}}(|\varphi^+\rangle + |\varphi^-\rangle)$

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

$$\Rightarrow |\psi^+\rangle + |\psi^-\rangle = \frac{2}{\sqrt{2}}|01\rangle$$

$$\Rightarrow |01\rangle = \frac{1}{\sqrt{2}}(|\psi^+\rangle + |\psi^-\rangle)$$

$$|10\rangle = \frac{1}{\sqrt{2}}(|\psi^+\rangle - |\psi^-\rangle)$$

$$\Rightarrow \frac{1}{\sqrt{3}}(|00\rangle + |01\rangle + |10\rangle)$$

$$= \frac{1}{\sqrt{3}}\left[\frac{1}{\sqrt{2}}(|\varphi^+\rangle + |\varphi^-\rangle) + \frac{1}{\sqrt{2}}(|\psi^+\rangle + |\psi^-\rangle) + \frac{1}{\sqrt{2}}(|\psi^+\rangle - |\psi^-\rangle)\right]$$

$$= \frac{1}{\sqrt{3}}\frac{1}{\sqrt{2}}\left[|\varphi^+\rangle + |\varphi^-\rangle + 2|\psi^+\rangle\right]$$

$$\boxed{= \frac{1}{\sqrt{6}}(|\varphi^+\rangle + |\varphi^-\rangle + 2|\psi^+\rangle)}$$

Scanned with CamScanner