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Definition 1: statment (propositions)

is a sentence that is true or false but not both

For example 1:

1. Two plus Two equal four.
2. Washington, D.C, is the Capital of United state, of America.
3. $2+2=5$
4. Ismailia is the Capital of Egypt.

propositions 1 and 2 are true, whereas 3 and 4 false.

on the other hand,

Example 2.

1. He is a College student ?
2. $x+y>0$
3. $x+1=2$
4. $x+y=z$

not proposition (statment)

where ① not statment, since the truth of ① depends on the relference for the pronoun. he. For some value of he the sentence is true; for other it is false.

② Sentence (2), (3) and (4) are not statment because they neither true nor false. Note that each of sentence Can be turned into a proposition if we assign value to the variable.

Note: we use letters to denote statement variables. that is, variables that represent propositions such as $p, q, r, s, \dots$

Definition 2: let p be a statment. the negation of p, denoted by $\neg p$ or $\sim p$ (also denoted by $\bar{p}$), is the statment "It's not the Case that p" (not p)
the truth value of the negation of p, $\sim p$, is the opposite of the truth value of p.

negation operator

p	$\sim p$
T	F
F	T

The truth table for the negation of proposition.

This table has a row for each of the two possible truth values of statement p. Each row show the truth value of $\sim p$ Corresponding to the truth value of p.

Example: Find the negation of the proposition.

- Micheal's PC runs Linux.

Solution:

Micheal's PC does not run linux.

- Vandana's smart phone has at least 32GB of memory.

Solution: Vanda's smart phone has less than 32GB of memory

or Vanad's smart phone does not have at least 32GB.

Definition 3: If p and q is a statement variables, the Conjunction of P and q is "P and q", denoted by $p \wedge q$. It's true when, and only when, both P and q are true. If either P or q is false or if both are false, $p \wedge q$ is false.

P	q	$P \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Definition 4: If p and q are statment variables, the disjunction of P and q is "P or q", denoted by $(P \vee q)$. $(P \vee q)$ is false when both P and q are false and is true otherwise.

P	q	$P \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Note: A variety of English words translate into logic as $\wedge$, $\vee$ or $\sim$. For instance, the word but translates the same as and when it links to independent clauses, as in "Jim is tall but he is not heavy".

table

p but q mens b and q
neither p nor q " $\sim p$ and $\sim q$
$p \downarrow q$

Example: write each of the following sentences symbolic letting h = "It's hot" and s = "It's sunny".

(a) It's not hot but It's sunny.

(b) It's neither hot nor sunny.

Solution: (a) $\sim h \wedge s$
(b) $\sim h \wedge \sim s$

Example Suppose x is particular real number. let p, q, r be symbolize "$0 < x$", "$x < 3$" and "$x = 3$" respectivel, write the following inequalities symbolically

(a) $x \le 3$
(b) $0 < x < 3$
(c) $0 < x \le 3$

Solution (a) $q \vee r$ (b) $p \wedge q$ (c) $p \wedge (q \vee r)$

Example. let h = "John is healthy", w = "John is wealthy" and s = "John is wise".

(a) John is healty and wealthn but notwise.
(b) John is not wealthy but he is healty and wise.
(c) John is neigther healty, wealthy, nor wise.
(d) " " neither wealthy nor wise, but he is healty
(e) " " wealthy, but he is not both healty and wise.

Solution

(a) $h \wedge w \wedge \sim s$
(b) $\sim w \wedge (h \wedge s)$
(c) $\sim h \wedge \sim w \wedge \sim s$
(d) $\sim w \wedge \sim s \wedge h$
(e) $w \wedge \sim(h \wedge s)$

Note) In ordinary language or is sometime used in exculsive sense (p or q but not both) and sometimes in an inclusive sense (p or q or both). For example, waiter who says you may have "Coffee, tea, or milk" uses the word or in an exculsive sense. on other hand, a waiter who offes "Cream or sugar" uses the word or in an inculsive sense

p	q	$p \vee q$	$p \wedge q$	$\sim(p \wedge q)$	$(p \vee q) \wedge \sim(p \wedge q)$
T	T	T	T	F	F
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	F	T	F

Truth table for Exclusive or: $(p \vee q) \wedge (\sim(p \wedge q))$

$p \oplus q \rightarrow$ XOR

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The order of operations:

In expressions that includ the symbol $\sim$ as well as $\wedge$ or $\vee$, the order of operations is that

① $\sim$ is performed first.
② the symbols $\wedge$ and $\vee$ are Considered Coequal in order of operation, and an expression such as $p \wedge q \vee r$ is Considered ambiguous, must be written either

$(p \wedge q) \vee r$ or $p \wedge (q \vee r)$ to have mening.

* Evaluating the truth of more General Compound statmen

- Truth table for $(p \wedge q) \vee \sim r$ $\rightarrow$ statment form

p	q	r	$p \wedge q$	$\sim r$	$(p \wedge q) \vee \sim r$
T	T	T	T	F	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	F	T	T
F	T	T	F	F	F
F	T	F	F	T	T
F	F	T	F	F	F
F	F	F	F	T	T

Example. Truth table of $(p \vee (\sim p \vee q)) \wedge \sim (q \wedge \sim r)$
(For student)

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Definition: logicall equivalent

Two statment form are called logically equivalent if and only if, they have identical truth values for each possible substitution of statments for thier statement variables.

The logical equivalence of statment forms P and Q is denoted by $P \equiv Q$.

Ex. Double negative property: $\sim(\sim p) \equiv p$

p	$\sim p$	$\sim(\sim p)$
T	F	T
F	T	F

p and $\sim(\sim p)$ always have the same truth values, so the are logically equivalent.

Ex. $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ (distributive law)

p	q	r	$q \vee r$	$p \wedge q$	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	F	T	T
T	F	F	F	F	F	F
F	T	T	T	F	F	F
F	T	F	T	F	F	F
F	F	T	T	F	F	F
F	F	F	F	F	F	F

Same truth values.

not the same Case as $(p \vee q) \vee (p \wedge r) \not\equiv (p \vee q) \wedge r$

Ex. Show that statment form $\sim(p \wedge q)$ and $\sim p \wedge \sim q$ are not logically equivalent (for student)

De Morgan's laws :-

$$\sim(p \wedge q) \equiv \sim p \vee \sim q$$
$$\sim(p \vee q) \equiv \sim p \wedge \sim q$$

The negation of an and statement is logically equivalent to the or statement in which each component is negated.
The negation of an or statment is logically equivalent to the and statement in which each component is negate.

- Applying De Morgan's laws, write negation for each statement.

① John is 6 feet tall and he weight at least 200 Pound
② The bus was late or Tom's watch was slow.
③ This Computer program has logical error in the first ten lines or it is being run with in Complete data set.
④ Sam is an orange belt and kate is a red belt.

Solutions:
① John is not 6 feet tall or he weight less than 200 Pound
② The bus wasn't late and Tom's watch wasn't slow.
③ This Computer Program ~~has~~ does not have logical err. and it's not run with in Complete data set.
④ Sam is not an orange belt or kate is not a red belt

Ex. Use De Morgan's laws to write the negation of $-1 < X \leq 4$

Solution since $-1 < X \leq 4$ means $-1 < X$ and $X \leq 4$

Then negation is $X \leq -1$ ~~on~~ or $X > 4$

not $-1 \geq X > 4$ ⇒ This wrong

Definition Tautologies and Contradictions

A tautolgies is a statement form that is always true regardless of the thruth ~~table~~ values of the individual statments substituted for it's statment variables. A statement whose form is a tautology is a tautological statment.

A Contradiction is a statement form that is always false regardless of the truth values of the individual statments substituted for it's statement variables. A statement whose form is a Contradication is a Contrdication statement.

Ex. show that the statment for $p \vee \sim p$ is a tautology and that statment from $p \wedge \sim p$ is a Contraduction

p	$\sim p$	$p \vee \sim p$	$p \wedge \sim p$
T	F	T	F
F	T	T	F

all T's so $p \vee \sim p$ is tautology

all F's so $p \wedge \sim p$ is Contradication.

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Ex. if t is a tautology and c is a Contradiction, show that

$p \wedge t \equiv p$ and $p \wedge c \equiv c$

p	t	$p \wedge t$	p	c	$p \wedge c$
T	T	T	T	F	F
F	T	F	F	F	F

$p \wedge t \equiv p$ $\qquad$ $p \wedge c \equiv c$

Definition: (Conditional)

If p and q are statement variables, the Conditional of q by p is "If p then q" or "p implies q" and is denoted by $p \rightarrow q$.
It's false when p is true and q is False; otherwise it's true.
we Call p the hypothesis (or antecedent) of the Conditional and q Conclusion (or Consequent).

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

* logical equivalence involving $\rightarrow$:

For example: $p \vee q \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$

hypothesis Conclusion.

p	q	r	$p \vee q$	$p \rightarrow r$	$q \rightarrow r$	$p \vee q \rightarrow r$	$(p \rightarrow r) \wedge (q \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	T	T
T	F	F	T	F	T	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	F	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

are logically Equivalent.

* Representation if then As or:-

$$p \to q \equiv \sim p \vee q$$

Ex. rewrite the following statment in if-then form:-

Either you get to work on time or you are fired.

Soln.

let $\sim p$ be you get to work on time.

q : you are fired.

then solution. If you don't get to work on time then you are fired.

* Negation of a Conditional statement.

$$\sim(p \to q) \equiv p \wedge \sim q$$

The negation of "if p then q" is logically equivalent to "p and not q"

$$\sim(p \to q) \equiv \sim(\sim p \vee q)$$

$$\equiv \sim(\sim p) \wedge \sim q$$

$$\equiv p \wedge \sim q$$

Ex. write negations for each of the following statements:-

① IF my Car is in the repair shop, then I Can not get to class.

ans: My Car is in the repair shop and I Can get to class.