Introduction Into Probability Theory

MTH 231

Lecture 7

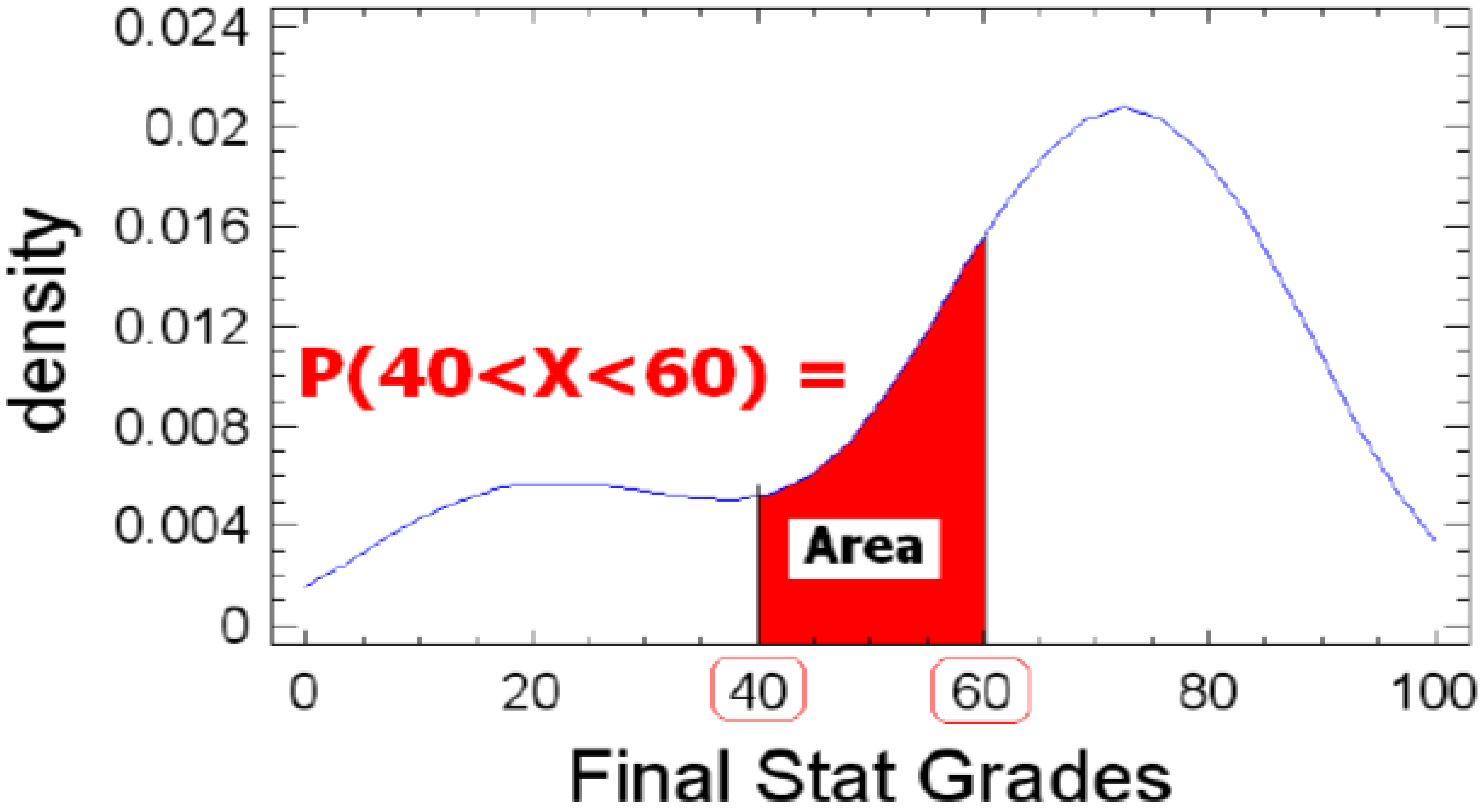
Chapter 5

Some Useful Continuous Distributions

Today’s lecture

- Some Important Continuous Distributions:
  - Uniform Distribution
  - Exponential Distribution
  - Normal, or Gaussian, Distribution

Continuous Distribution



1- The Uniform Distribution

- The uniform distribution is a probability distribution that has equal probabilities for all possible outcomes of the random variable. It is also called the rectangular distribution. The density function of the continuous uniform random variable X on the interval [α, β] is:

$$f(X) = \begin{cases} \dfrac{1}{\beta - \alpha} & \text{if } \alpha \le X \le \beta \\ 0 & \text{otherwise} \end{cases}$$

where

f(X) = value of the density function at any X value

α = minimum value of X

β = maximum value of X

- The **mean** of a uniform distribution is

$$\mu = \frac{\alpha + \beta}{2}$$

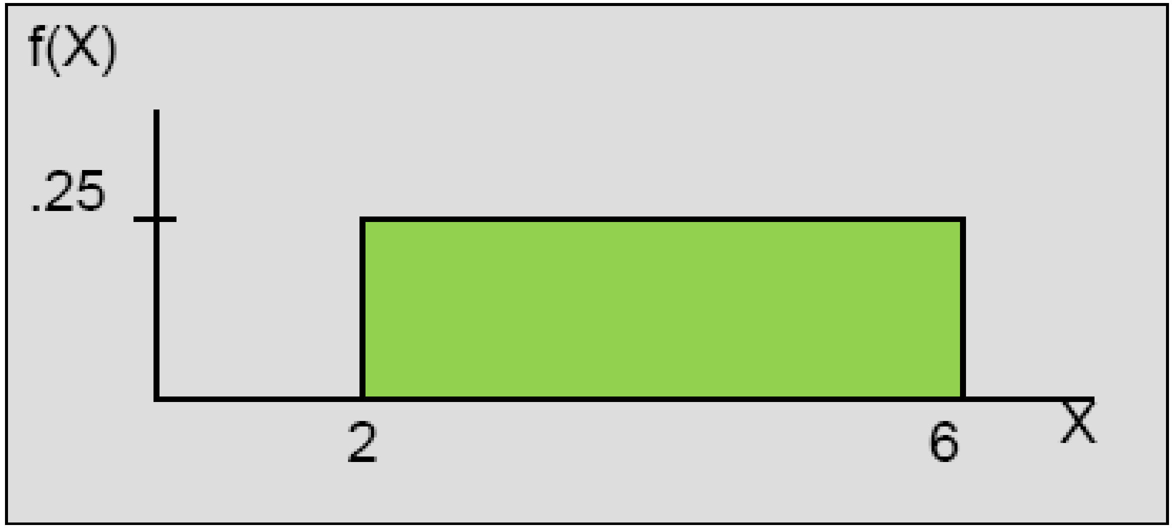
- The **standard deviation** is

$$\sigma = \sqrt{\frac{(\beta - \alpha)^2}{12}}$$

Example: Uniform probability distribution

over the range $2 \le X \le 6$:

$$f(X) = \frac{1}{6 - 2} = .25 \quad \text{for } 2 \le X \le 6$$



$$\mu = \frac{2+6}{2} = 4$$

$$\sigma = \sqrt{\frac{(\beta - \alpha)^2}{12}} = \sqrt{\frac{(6 - 2)^2}{12}} = 1.1547$$

➢**Example** : If X is uniformly distributed over [0, 10], calculate the probability that **(a) $X < 3$, (b) $X > 6$, (c) $3 < X < 8$, (d) μ and σ^2**

Solution

$$f(x) = \frac{1}{10}, \qquad 0 < X < 10$$

$$\text{(a)} \quad P(X < 3) = \int_0^3 \frac{1}{10}\,dx = \frac{3}{10}$$

$$\text{(b)} \quad P(X > 6) = \int_6^{10} \frac{1}{10}\,dx = \frac{4}{10} = \frac{2}{5}$$

$$\text{(c)} \quad P(3 < X < 8) = \int_3^8 \frac{1}{10}\,dx = \frac{5}{10} = \frac{1}{2}$$

$$\text{(d)} \quad \mu = \frac{\alpha + \beta}{2} = \frac{10}{2} = 5 \qquad \sigma^2 = \frac{(\beta - \alpha)^2}{12} = \frac{(10)^2}{12} = \frac{100}{12} = \frac{25}{3}$$

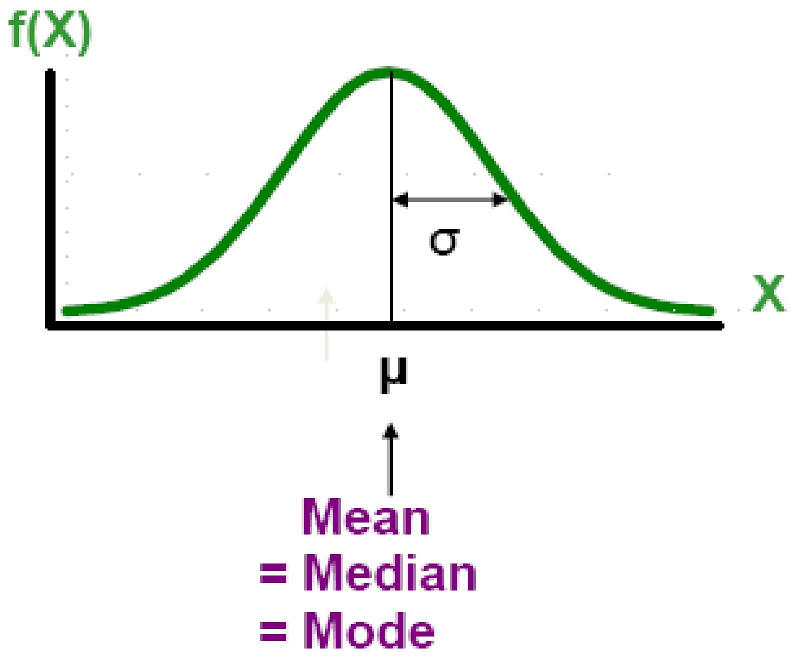
2- The Normal Distribution $N(\mu , \sigma^2)$

The probability density function of the normal distribution or Gaussian distribution is defined by the equation:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$-\infty < x < \infty \;, -\infty < \mu < \infty \;, \sigma > 0$$

where μ = mean, σ = standard deviation,
$\pi = 3.14159\ldots$, $e = 2.71828\ldots$

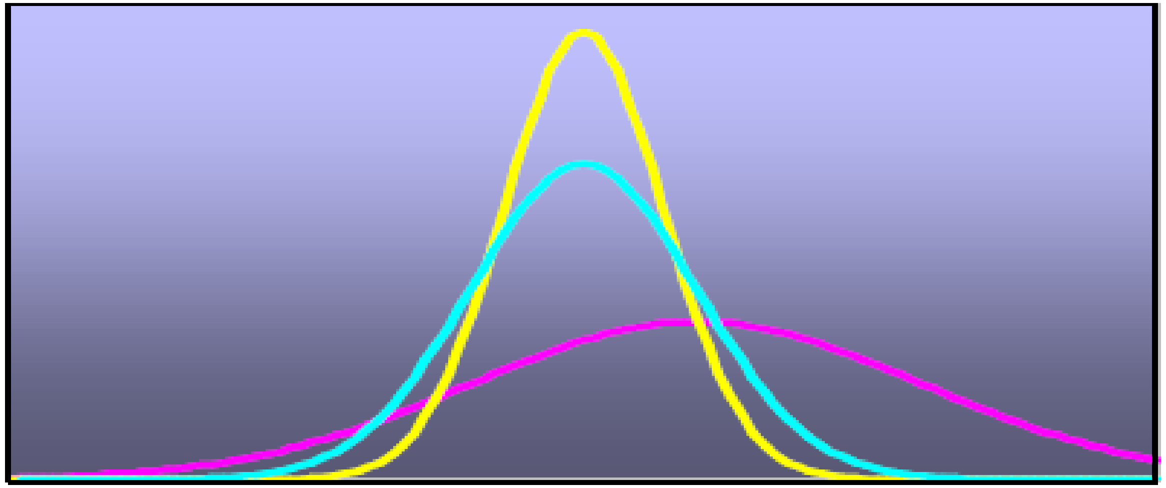


The normal curve.

- ✓**Bell Shaped**
- ✓**Symmetrical**
- ✓**Location is determined by the mean, μ**
- ✓**Spread is determined by the standard deviation, σ**
- ✓**The random variable has an infinite theoretical range: $-\infty$ to $+\infty$**

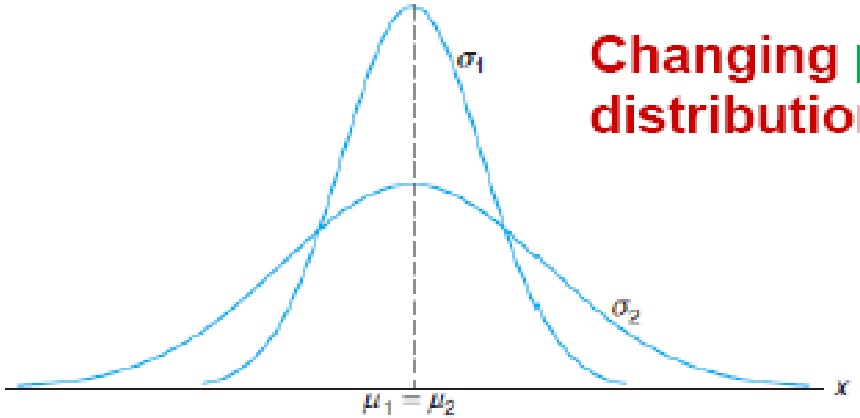
Properties of the normal curve:

1. The mode, which is the point on the horizontal axis where the curve is a maximum, occurs at $x = \mu$.
2. The curve is symmetric about a vertical axis through the mean μ.
3. The curve has its points of inflection at $x = \mu \pm \sigma$; it is concave downward if $\mu - \sigma < X < \mu + \sigma$ and is concave upward otherwise.
4. The normal curve approaches the horizontal axis asymptotically as we proceed in either direction away from the mean.
5. The total area under the curve and above the horizontal axis is equal to 1.



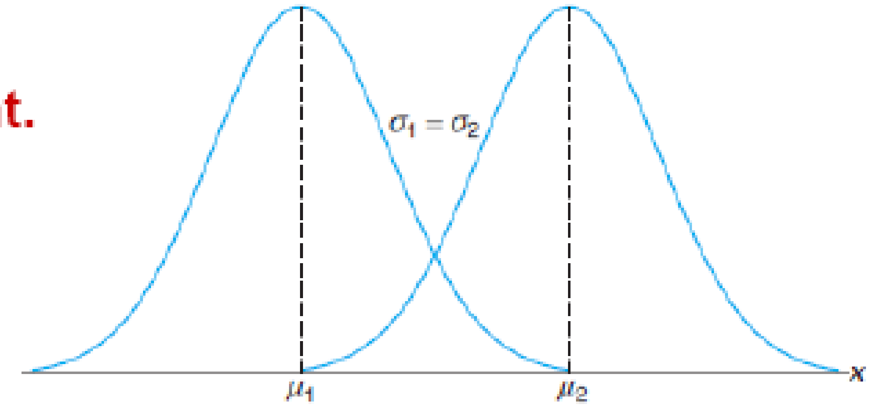
By varying the parameters μ and σ, we obtain different normal distributions

Note that the normal distribution is defined by two parameters, μ and σ. You can draw a normal distribution for any μ and σ combination. There is one normal distribution, Z, that is special. It has a $\mu = 0$ and a $\sigma = 1$. This is the Z distribution, also called the standard normal distribution. It is one of trillions of normal distributions we could have selected.



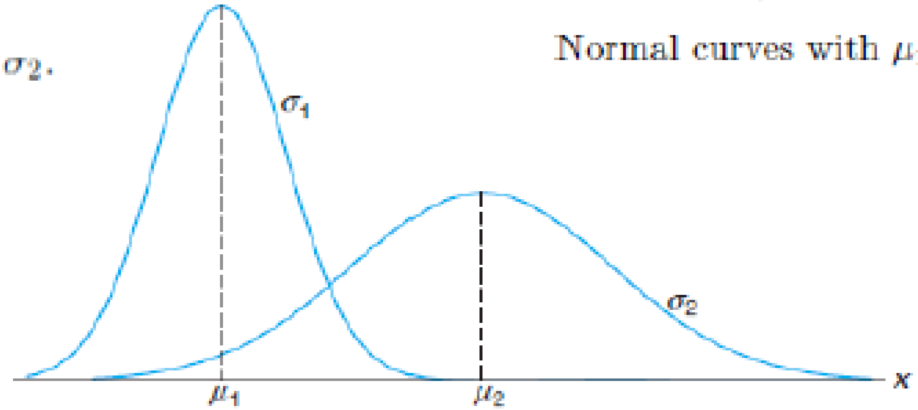
Normal curves with $\mu_1 = \mu_2$ and $\sigma_1 < \sigma_2$.

Changing μ shifts the distribution left or right.



Normal curves with $\mu_1 < \mu_2$ and $\sigma_1 = \sigma_2$.

Changing σ increases or decreases the spread.



Normal curves with $\mu_1 < \mu_2$ and $\sigma_1 < \sigma_2$.

Translation to the Standardized Normal Distribution N(0, 1)

- Translate from X to the standardized normal (the "Z" distribution) by subtracting the mean of X and dividing by its standard deviation:

Z always has mean = 0 and standard deviation = 1

$$Z = \frac{X - \mu}{\sigma}$$

Statistical table concerning area under the standard normal curve is available. By $\Phi(a)$ we mean the area under the standard normal curve that is less than a, that is

$$\Phi(a) = P[Z < a]$$

Note that:

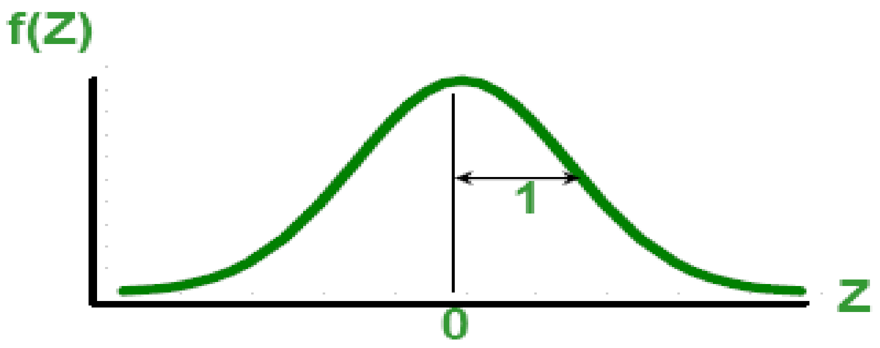
(i) $P[Z < a] = \Phi(a)$
(ii) $P[Z > a] = 1 - \Phi(a)$
(iii) $P[a < Z < b] = \Phi(b) - \Phi(a)$
(iv) $\Phi(-a) = 1 - \Phi(a)$

The Standardized Normal Probability Density Function

- The formula for the standardized normal probability density function is

$$f(Z) = \frac{1}{\sqrt{2\pi}} e^{-(1/2)Z^2}$$

Where e = the mathematical constant approximated by 2.71828
π = the mathematical constant approximated by 3.14159
Z = any value of the standardized normal distribution



Values above the mean have positive Z-values, values below the mean have negative Z-values

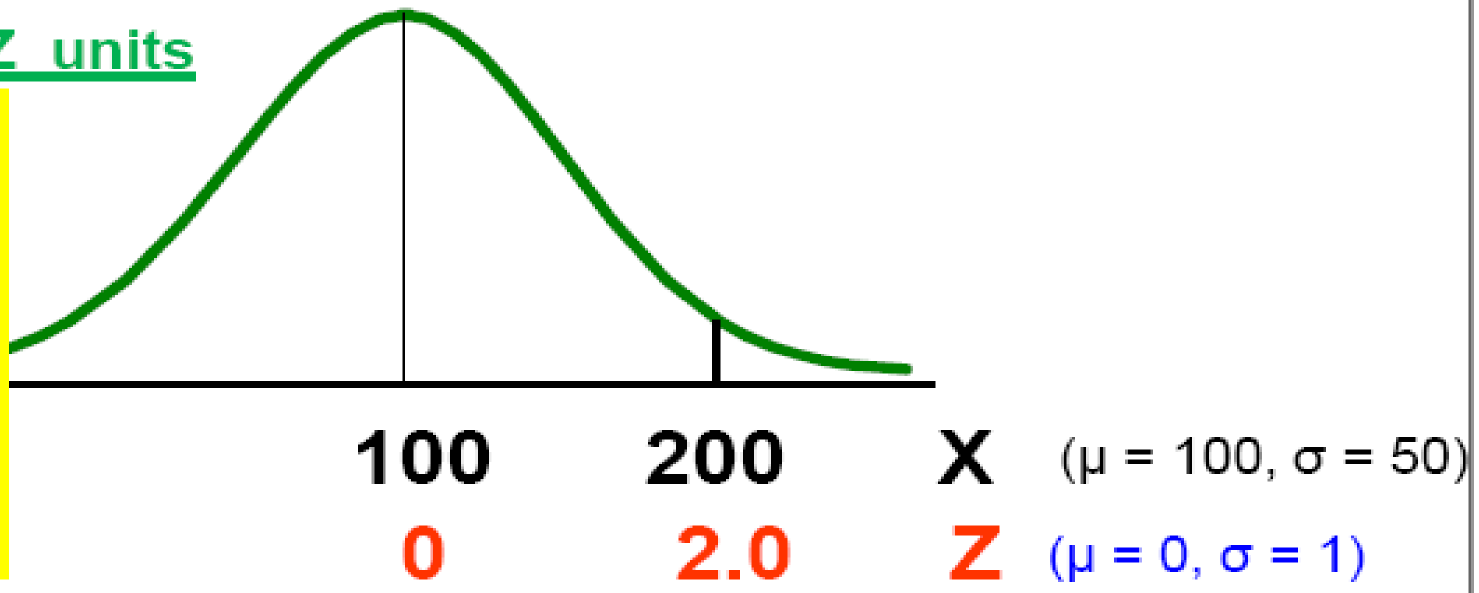
➢**Example** : If X is distributed normally with mean of 100 and standard deviation of 50, the Z value for X = 200 is

$$Z = \frac{X - \mu}{\sigma} = \frac{200 - 100}{50} = 2.0$$

- This says that X = 200 is two standard deviations (2 increments of 50 units) above the mean of 100.

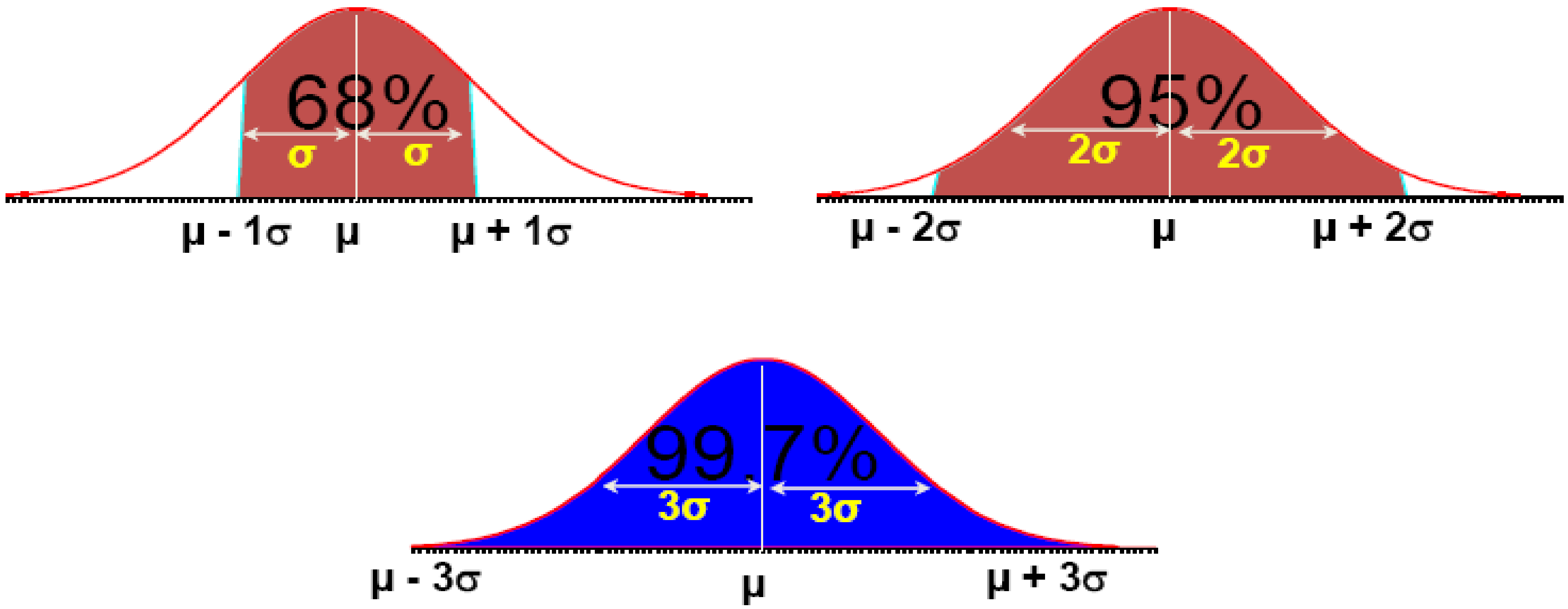
Comparing X and Z units

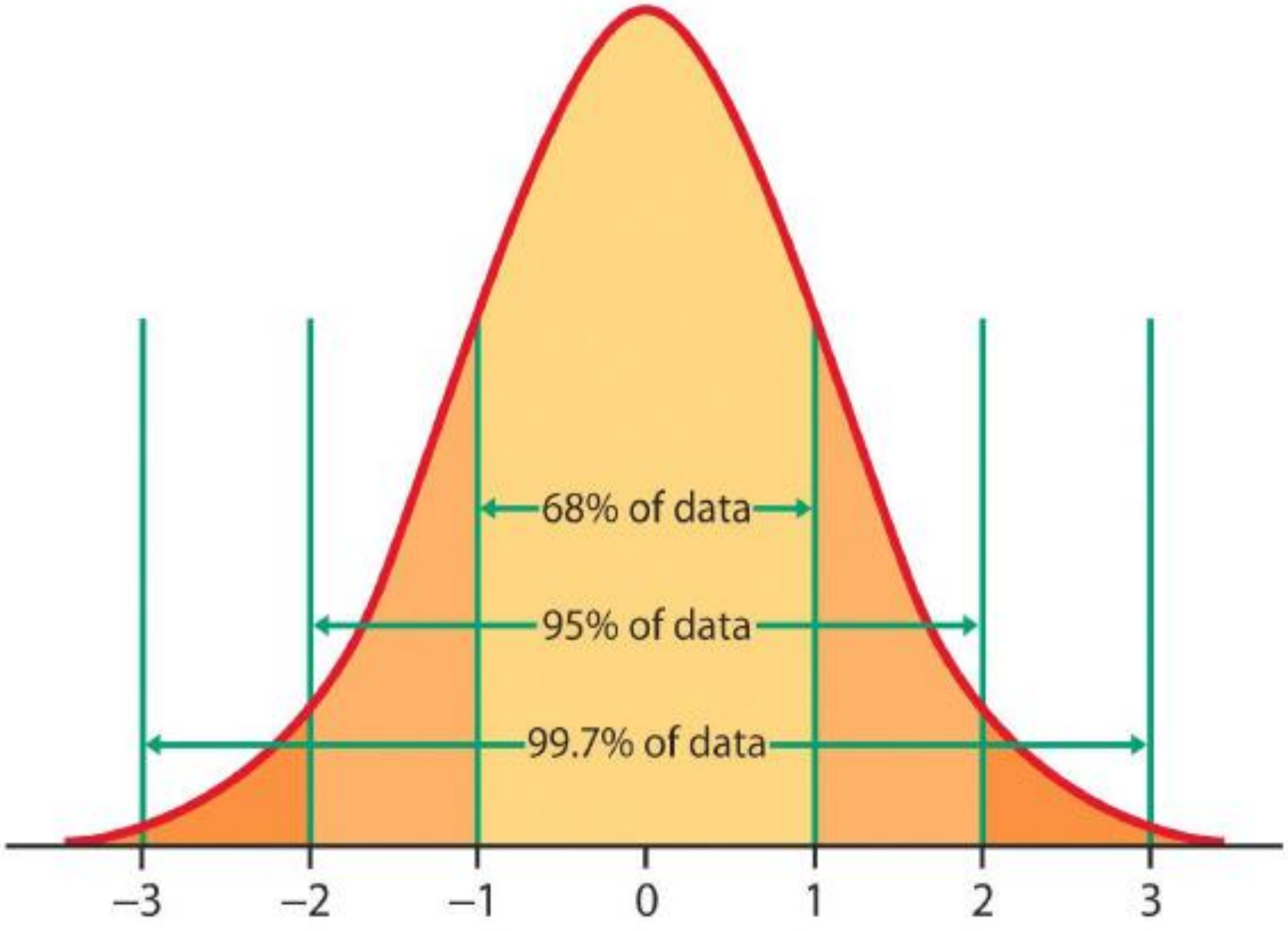
Note that the distribution is the same, only the scale has changed. We can express the problem in original units (X) or in standardized units (Z)



68-95-99.7 Rule for Any Normal Curve

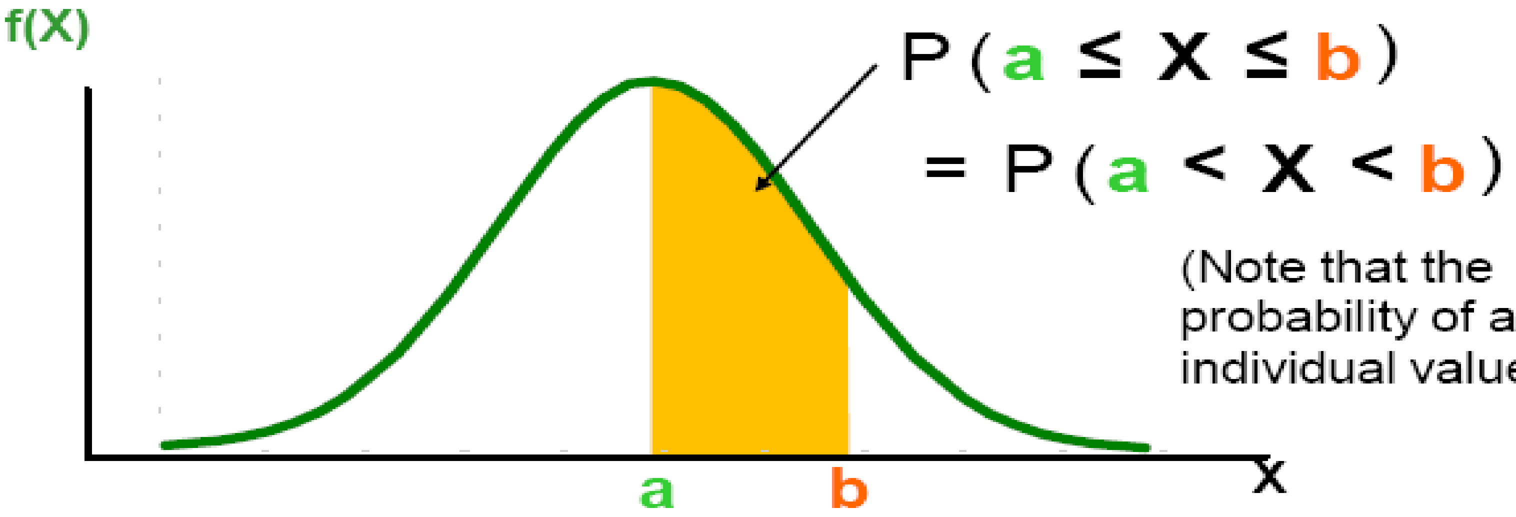
- 68% of the observations fall within one standard deviation of the mean
- 95% of the observations fall within two standard deviations of the mean
- 99.7% of the observations fall within three standard deviations of the mean in either direction



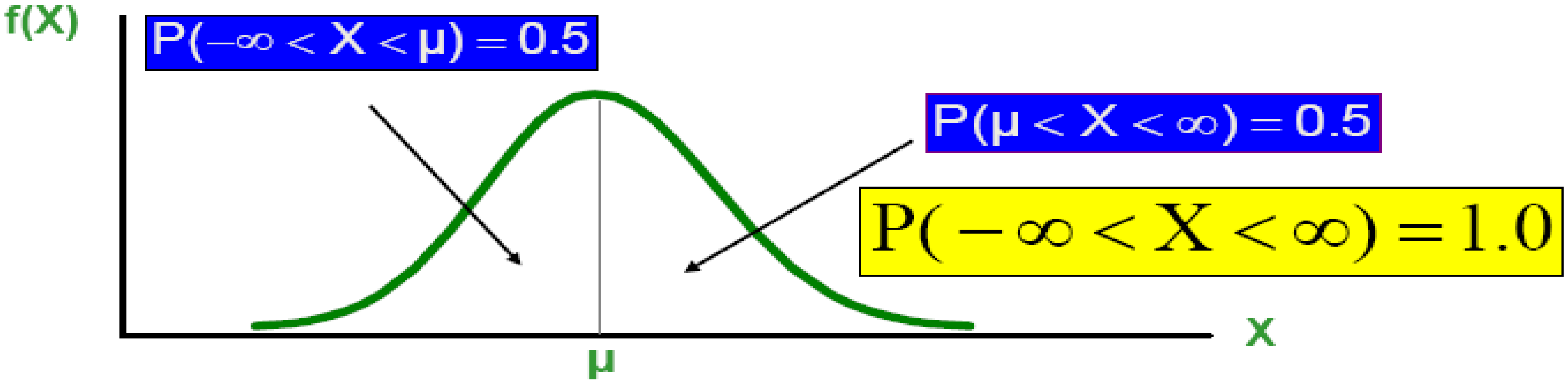

68% of data
95% of data
99.7% of data
−3
−2
−1
0
1
2
3

Finding Normal Probabilities

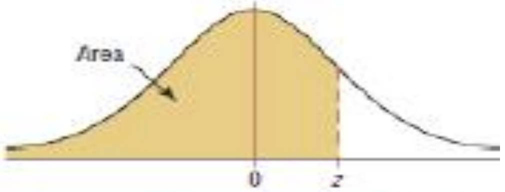
- Probability is measured by the area under the curve



(Note that the probability of any individual value is zero)



The Cumulative Standardized Normal Table



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

The Normal Table

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-y^2/2}\, dy$$

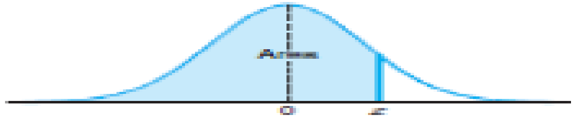
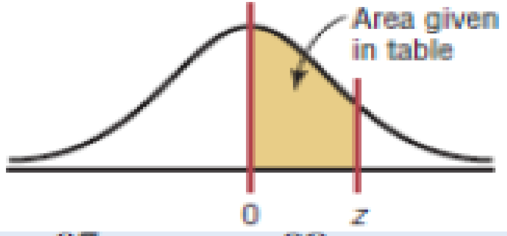


Table A.3 Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

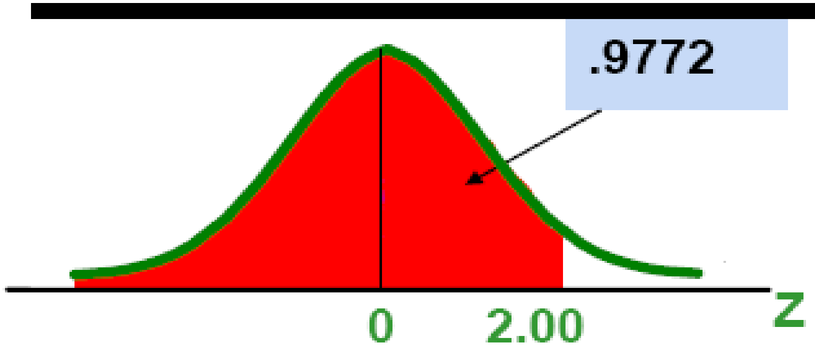
The Standardized Normal Table



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998

The Standardized Normal Table

(continued)



➢ **Example (10):**

Find P(Z < 2.00)

The **column** gives the value of Z to the second decimal point

Z	0.00	0.01	0.02 …
0.0			
0.1			
.			
.			
.			
2.0	.9772		

The **row** shows the value of Z to the first decimal point

The value within the table gives the probability from Z = – ∞ up to the desired Z value

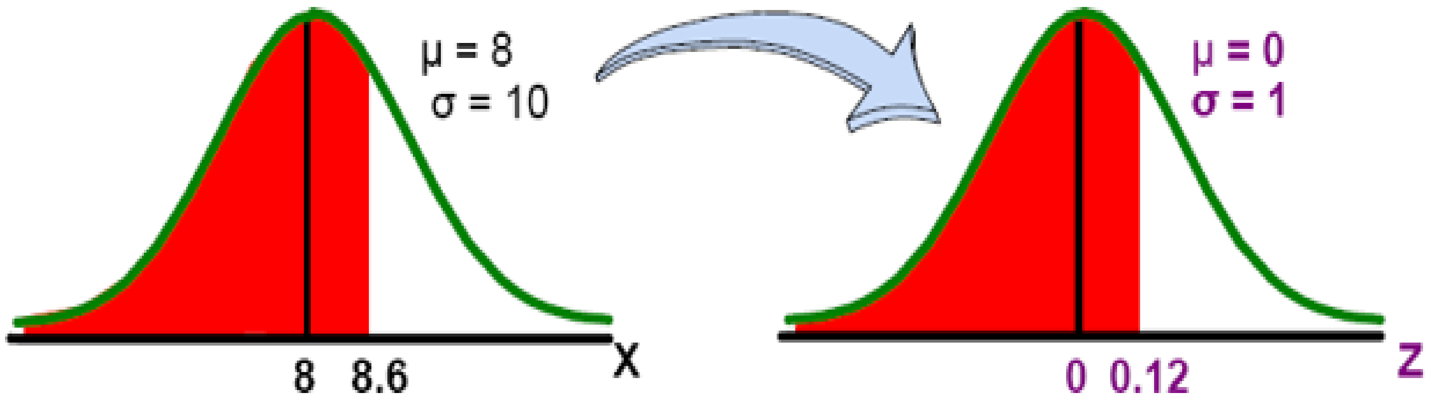
P(Z < 2.00) = .9772

Finding Normal Probabilities

(continued)

- **Example** Suppose X is normal with mean 8.0 and standard deviation 5.0. Find P(X < 8.6)

$$Z = \frac{X - \mu}{\sigma} = \frac{8.6 - 8.0}{5.0} = 0.12$$



P(X < 8.6)

P(Z < 0.12)

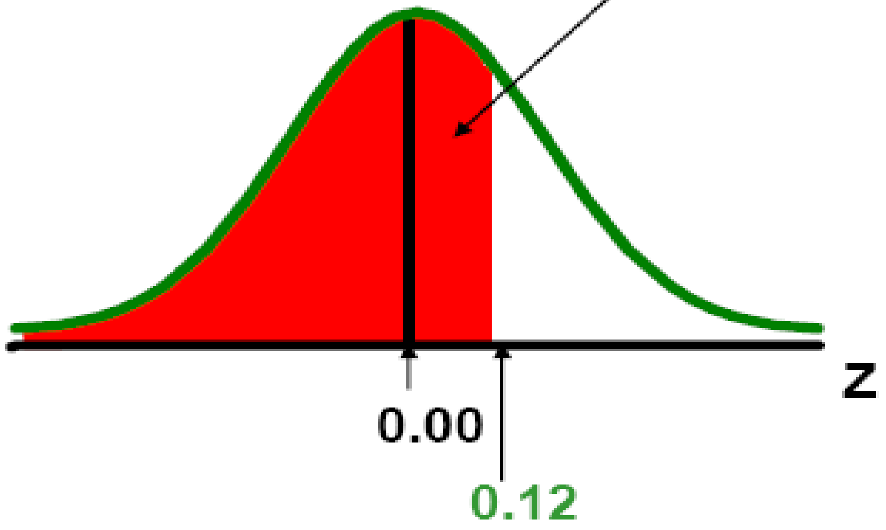
Solution:

Standardized Normal Probability Table (Portion)

Z	.00	.01	.02
0.0	.5000	.5040	.5080
0.1	.5398	.5438	.5478
0.2	.5793	.5832	.5871
0.3	.6179	.6217	.6255

$P(X < 8.6)$
$= P(Z < 0.12)$
$= \Phi(0.12)$

.5478

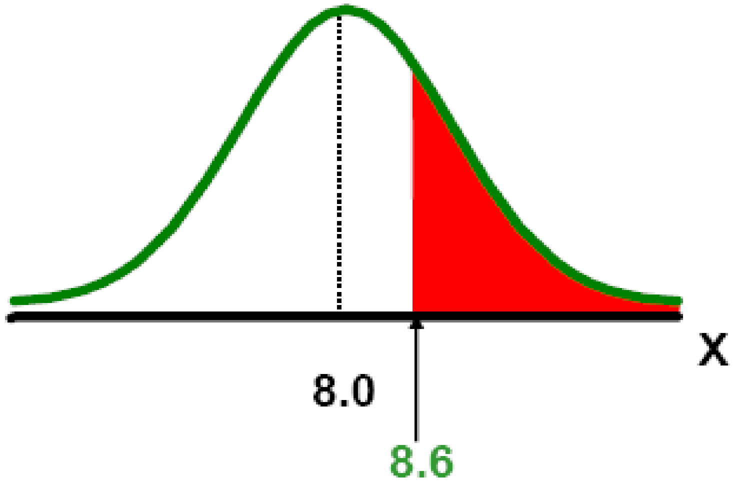


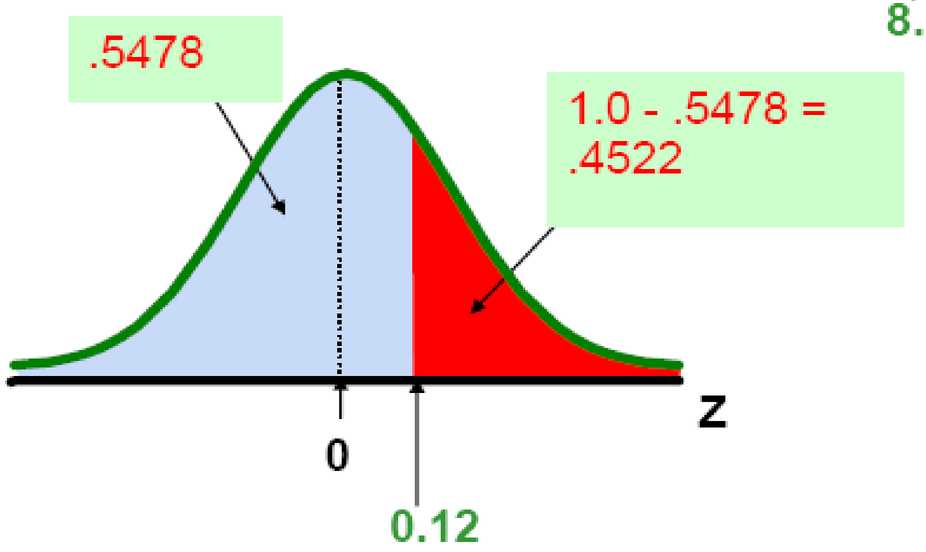
Upper Tail Probabilities

- **Example** : Suppose X is normal with mean 8.0 and standard deviation 5.0.
- Now Find P(X > 8.6)

Solution

$$P(X > 8.6) = P(Z > 0.12) = 1.0 - \mathbf{\Phi(0.12)}$$
$$= 1.0 - .5478$$
$$= .4522$$





Probability Between Two Values

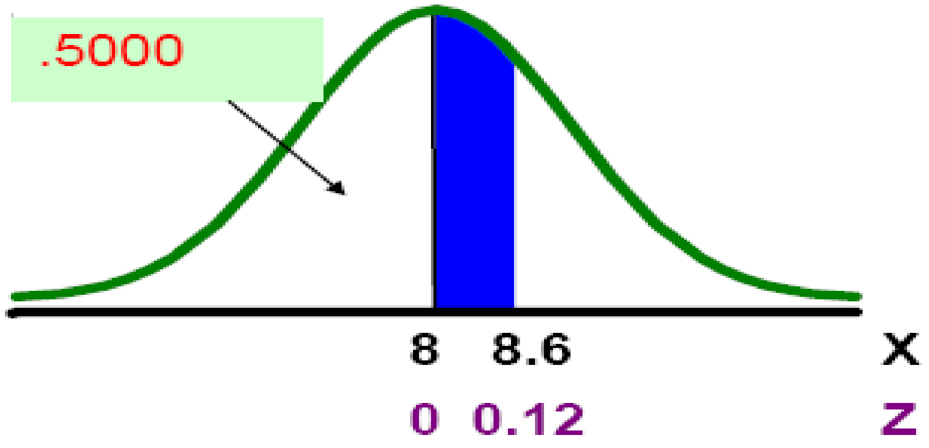
➢ **Example** : Suppose X is normal with mean 8.0 and standard deviation 5.0. Find P(8 < X < 8.6).

Solution: Calculate Z-values

$$Z = \frac{X - \mu}{\sigma} = \frac{8 - 8}{5} = 0$$

$$Z = \frac{X - \mu}{\sigma} = \frac{8.6 - 8}{5} = 0.12$$

Z	.00	.01	.02
0.0	.5000	.5040	.5080
0.1	.5398	.5438	.5478
0.2	.5793	.5832	.5871
0.3	.6179	.6217	.6255



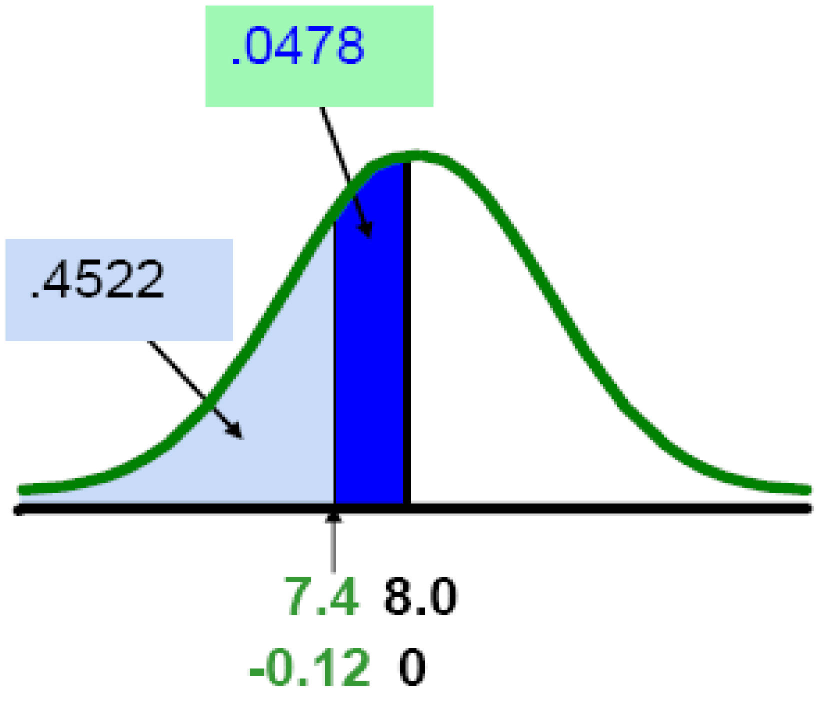
$$\begin{aligned} & P(8 < X < 8.6) \\ &= P(0 < Z < 0.12) \\ &= P(Z < 0.12) - P(Z \le 0) \\ &= \Phi(0.12) - .5000 \\ &= .5478 - .5000 = .0478 \end{aligned}$$

Probability in the Lower Tail

➢ **Example** : Find $P(7.4 < X < 8)$…

Solution:

$$
\begin{aligned}
& P(7.4 < X < 8) \\
&= P(-0.12 < Z < 0) \\
&= P(Z < 0) - P(Z \le -0.12) \\
&= .5000 - \Phi(-0.12) \\
&= .5000 - [1 - \Phi(0.12)] \\
&= .5000 - .4522 = .0478
\end{aligned}
$$

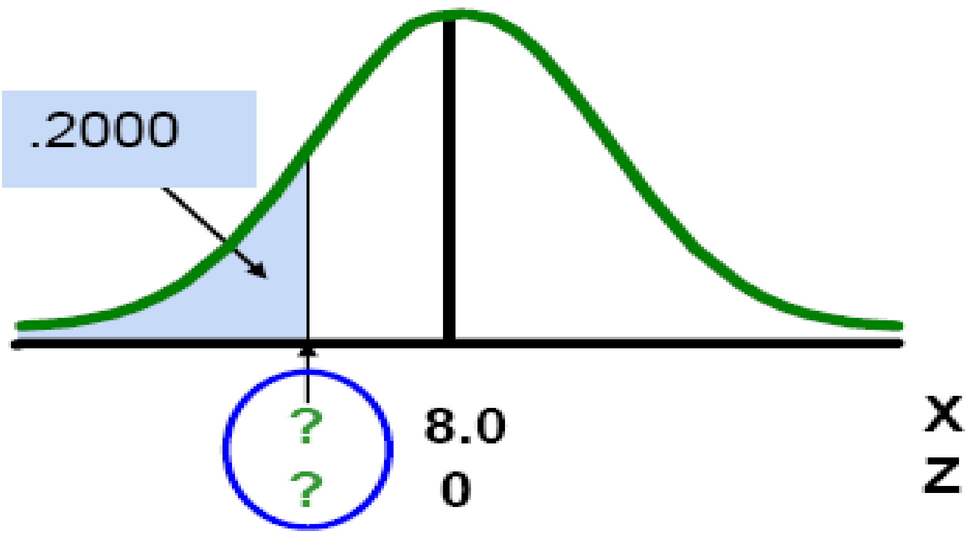


The Normal distribution is symmetric, so this probability is the same as $P(0 < Z < 0.12)$

Finding the X value for a Known Probability

➢ **Example**

- Suppose X is normal with mean 8.0 and standard deviation 5.0.
- Now find the X value so that only 20% of all values are below this X



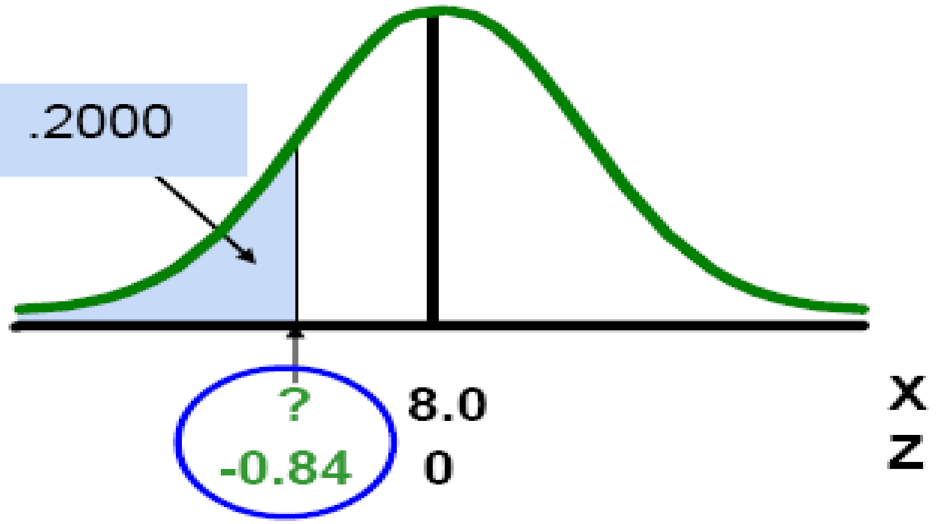
Find the Z value for 20% in the Lower Tail *(continued)*

1. Find the Z value for the known probability

Standardized Normal Probability Table (Portion)

Z	…	.03	.04	.05
-0.9	…	.1762	.1736	.1711
-0.8	…	.2033	.2005	.1977
-0.7	…	.2327	.2296	.2266

- 20% area in the lower tail is consistent with a Z value of -0.84



Finding the X value

2. Convert to X units using the formula:

$$
\begin{aligned}
X &= \mu + Z\sigma \\
&= 8.0 + (-0.84)5.0 \\
&= 3.80
\end{aligned}
$$

So 20% of the values from a distribution with mean 8.0 and standard deviation 5.0 are less than 3.80

➢**Example** A monthly amount of newspaper for garbage or recycling is normally distributed with a mean of 28 and a standard deviation of 2 pounds. If a household is selected at random. Find the probability it generates

More than 30.2 pounds per month;

Solution: Let X be the amount of newspaper for garbage or recycling per month . $\mu = 28$, and $\sigma = 2$

$$P(X > 30.2) = P\left(\frac{X-\mu}{\sigma} > \frac{30.2-28}{2}\right) = P(Z > 1.1)$$

$$= 1 - \Phi(1.1) = 1 - 0.8643 = 0.1357$$

Normal Approximation to the Binomial

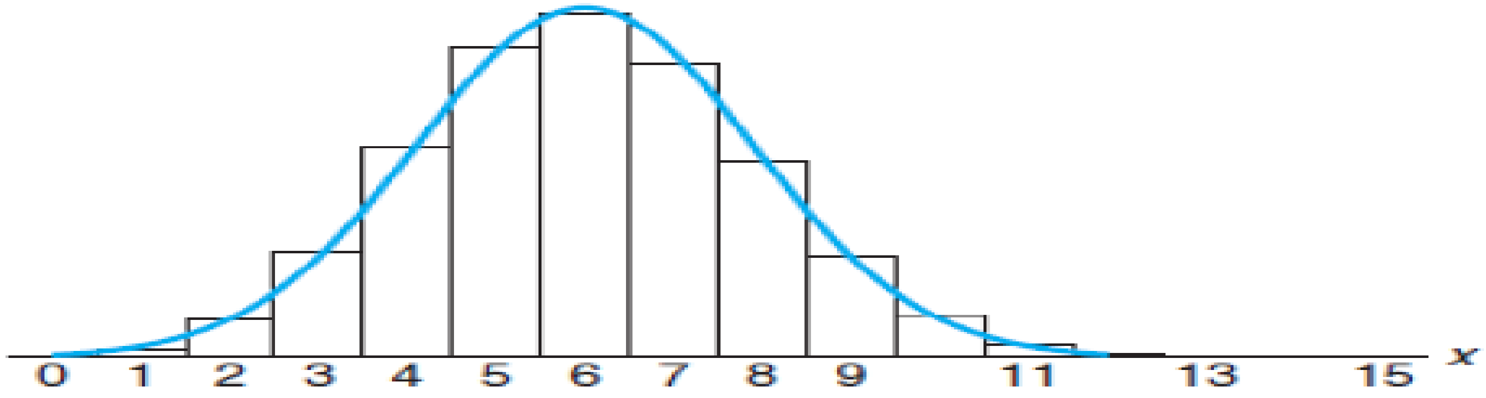
If n is large, ($n \geq 30$), the binomial distribution can be closely approximated by the standard normal distribution with standardized variable given by

$$Z = \frac{X - np}{\sqrt{npq}}$$

If a binomial probability distribution "**which is discrete**" satisfies the requirements that $n.p \geq 5$ and $n.q \geq 5$, then the binomial probability distribution can be approximated by a normal distribution "**which is continuous**" with

(i) mean $\mu = n.p$ and

(ii) variance (npq)



- **Example** The probability that a patients recover from a rare blood diseases is 0.4. If 100 people are known to have contracted this diseases . What the probability that less than 30 survive .

- **Solution** p=0.4, n=100 >30,

  b(n,p)≈ N(np,$\sqrt{npq}$)

$$P(X<30)=P\left(Z<\frac{30-np}{\sqrt{npq}}\right)$$

$$=P\left(Z<\frac{30-40}{\sqrt{24}}\right)=P(Z<-2.04)$$

$$=0.0162$$

3- The Exponential Distribution

A continuous random variable whose probability density function is given, for some parameter $\lambda > 0$, by

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

- Used to model the length of time between two occurrences of an event (the time between arrivals)
- Examples:

  Time between trucks arriving at an unloading dock
  Time between transactions at an ATM Machine
  Time between phone calls to the main operator

$$\textbf{(a)}\ \text{mean} = E(X) = \mu = \frac{1}{\lambda}$$

$$\textbf{(b)}\ \text{variance} = Var(X) = \sigma^2 = \frac{1}{\lambda^2}$$

The cumulative Distribution function of the Exponential Distribution

$$F(x) = P(X \le x) = \int_0^x f(t)dt = \int_0^x \lambda e^{-\lambda t} dt$$

$$= -e^{-\lambda t}\Big|_0^x = e^{-\lambda t}\Big|_x^0 = 1 - e^{-\lambda x}$$

Then ,

$$F(x) = \begin{cases} 1 - e^{-\lambda x} & x \ge 0 \\ 0 & x < 0 \end{cases}$$

The probability that an arrival time is less than some specified time X is

$$\boxed{P(\text{arrival time} < X) = 1 - e^{-\lambda X}}$$

➢**Example** : The life time in hours that a certain part remains operational is a random variable with exponential distribution with mean 500 hours. What is the probability that the part remains operationally for..

(a) more than 1000 hours?

(b) less than 500 hours?

(c) greater than 500 but less than 1000 hours?

Solution

(a) $$P(X > 1000) = \int_{1000}^{\infty} \frac{1}{500} e^{-\frac{t}{500}} dt = e^{-2} = 0.1353 \quad \boxed{\lambda = 1/500}$$

Or

$$P(X > 1000) = 1 - F(1000) = e^{-1000\lambda} = e^{-2}$$

(b) $P(X < 500) = \int_0^{500} \frac{1}{500} e^{-\frac{t}{500}} dt = 1 - e^{-1} = 0.6321$

Or

$P(X < 500) = F(500) = 1 - e^{-500\lambda} = 1 - e^{-1}$

(c) $P(500 < X < 1000) = F(1000) - F(500)$

$= (1-e^{-2}) - (1-e^{-1}) = e^{-1} - e^{-2}$

$= 0.368 - 0.135 = 0.233$

Questions!


THANK YOU


Probability Of Success
I won't - 0%
I can't - 10%
I don't know how - 20%
I wish I could - 30%
I want to - 40%
I think I might - 50%
I might - 60%
I think I can - 70%
I can - 80%
I am - 90%
I did - 100%