Ex. if (t) is a tautology and (c) is a Contradiction, show that

$p \wedge t \equiv p$ and $p \wedge c \equiv c$

p	t	$p \wedge t$	p	c	$p \wedge c$
T	T	T	T	F	F
F	T	F	F	F	F

$p \wedge t \equiv p$ $\qquad$ $p \wedge c \equiv c$

Theorem. logical equivalences:

Given any statment variables p, q, r, a tautology t and a Contradiction c the following logical equalient hold.

1. Commutative laws: $p \wedge q \equiv q \wedge p$ $\qquad$ $p \vee q \equiv q \vee p$
2. Associative laws: $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
   $(p \vee q) \vee r \equiv p \vee (q \vee r)$
3. Distributive laws: $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
   $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
4. Identity law: $p \wedge t \equiv p$ $\qquad$ $p \vee c \equiv p$
5. Negation law: $p \vee \sim p \equiv t$ $\qquad$ $p \wedge \sim p \equiv c$
6. Double negation law: $\sim(\sim p) \equiv p$

[7] Idempotent laws: $p \wedge p \equiv p$ $\quad p \vee p \equiv p$

[8] Universal bound laws $p \vee t \equiv t$ $\quad p \wedge c \equiv c$

[9] De Morgan's laws: $\sim(p \wedge q) \equiv \sim p \vee \sim q$

$\sim(p \vee q) \equiv \sim p \wedge \sim q$

[10] Absorption laws $p \vee (p \wedge q) \equiv p$

$p \wedge (p \vee q) \equiv p$

[11] Negation of t and c: $\sim t \equiv c$, $\quad \sim c \equiv t$

* Simplifying statment Forms :-

verify the logical equivalence

$\sim(\sim p \wedge q) \wedge (p \vee q) \equiv p$ without using Thruth tabl

Solution:

$\sim(\sim p \wedge q) \wedge (p \vee q) \equiv (\sim(\sim p) \vee \sim q) \wedge (p \vee q)$ (De Mor

$\equiv (p \vee \sim q) \wedge (p \vee q)$ Double negative law.

$\equiv p \vee (\sim q \wedge q)$ distributive law.

$\equiv p \vee (q \wedge \sim q)$ Commutative law.

$\equiv p \vee c$ negation law.

$\equiv p$ identity law

Ex. show that $\sim(p \vee (\sim p \wedge q))$ and $\sim p \wedge \sim q$ are logically equivalent by developing series of logical equivalences.

$\sim(p \vee (\sim p \wedge q)) \equiv \sim p \wedge \sim(\sim p \wedge q)$ De Morgan's law.

$\equiv \sim p \wedge [\sim(\sim p) \vee \sim q]$ De Morgan's law

$\equiv \sim p \wedge (p \vee \sim q)$ Double negation law.

$\equiv (\sim p \wedge p) \vee (\sim p \wedge \sim q)$ distributive law.

$\equiv F \vee (\sim p \wedge \sim q)$ Negation law

$\equiv \sim p \wedge \sim q$ Identity law.

Ex. $(p \wedge (\sim(\sim p \vee q))) \vee (p \wedge q) \equiv p$

$(p \wedge (\sim(\sim p \vee q))) \vee (p \wedge q)$

$\equiv (p \wedge (\sim(\sim p) \wedge \sim q)) \vee (p \wedge q)$ De Morgan's law.

$\equiv (p \wedge (p \wedge \sim q)) \vee (p \wedge q)$ Double negation law.

$\equiv ((p \wedge p) \wedge \sim q) \vee (p \wedge q)$ associative law.

$\equiv (p \wedge \sim q) \vee (p \wedge q)$ Idempotent laws.

$\equiv p \wedge (\sim q \vee q)$ distributive law.

$\equiv p \wedge t$ Negation law.

$\equiv p$ Identity law.

Definition: (Conditional)

If p and q are statement variables, the Conditional of q by p is "If p then q" or "p implies q" and is denoted by $p \to q$.

It's false when p is true and q is false; otherwise it's true.

we Call p the hypothesis (or antecedent) of the Conditional and q Conclusion (or Consequent).

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

* logical equivalence involving $\to$:

For example: $p \vee q \to r \equiv (p \to r) \wedge (q \to r)$

↑ hypothesis   Conclusion

p	q	r	$p \vee q$	$p \to r$	$q \to r$	$p \vee q \to r$	$(p \to r) \wedge (q \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	T	T
T	F	F	T	F	T	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	F	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

are logically equivalent.

* Representation if then As or:

$$p \to q \equiv \sim p \vee q$$

Ex. rewrite the following statment in if-then form:
Either you get to work on time or you are fired.

Soln.
let $\sim p$ be you get to work on time

q: you are fired.

then solution. If you don't get to work on time then you are fired.

* Negation of a Conditional statement.

$$\sim(p \to q) \equiv p \wedge \sim q$$

The negation of "if p then q" is logically equivalent to "p and not q"

$$\sim(p \to q) \equiv \sim(\sim p \vee q)$$
$$\equiv \sim(\sim p) \wedge \sim q$$
$$\equiv p \wedge \sim q$$

Ex. write negations for each of the following statements:
① IF my Car is in the repair shop, then I Can not get to class.

ans: My Car is in the repair shop and I Can get to class.

Contrapositive.

The Contrapositive of a Conditional statement of the form "IF p then q" is.

IF $\sim q$ then $\sim p$.

Symbolically

The Contra Positive of $p \rightarrow q$ is $\sim q \rightarrow \sim p$

Note

Conditional statment is logically equivalent to it's Contrapositive.

Ex. Write Contrapositive form of the following statement:-

IF today is Easter, then tomorrow is Monday.

ans. IF tomorrow is not Monday then today is not Easter.

Converse and Inverse

suppose that Conditional statement of the form 'IF p then q'

1. Converse is if q then p
2. inverse is if $\sim p$, then $\sim q$

Symbolically

The Converse of $p \rightarrow q$ is $q \rightarrow p$
The inverse of $p \rightarrow q$ is $\sim p \rightarrow \sim q$

Note ① Conditional statement and it's Converse are not logically equivalent.

② Conditional statement and it's inverse are not logically equivalent.

Ex. Write Converse and inverse of ~~ex~~ the following statement

if today is Easter, then tomorrow is Monday.

ans-

Converse. if tomorrow is Monday then today is Easter.

Inverse. If today is not Easter, then tomorrow is not Monday.

Definition. BiConditional

given statement variables p and q. the biConditional of p and q is "p if and only if q" is denoted by $p \Leftrightarrow q$.

it's true if both p and q have the same truth values otherwise false. the word if and only if are sometimes abbreviated iff.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

$$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$$ (Excersie for student byusing truth table.)

$$\equiv (\sim p \vee q) \wedge (\sim q \vee p)$$

Note that :

p is necessary and sufficient condition of q means "p iff q" which is equivalent to BiConditional.