

\* Exercise set 1.2 :-

→ Construct truth tables for the statement forms in 5-11 :-

5.  $\sim p \vee q \rightarrow \sim q$

| p | q | $\sim p$ | $\sim q$ | $\sim p \vee q$ | $\sim p \vee q \rightarrow \sim q$ |
|---|---|----------|----------|-----------------|------------------------------------|
| T | T | F        | F        | T               | F                                  |
| T | F | F        | T        | F               | T                                  |
| F | T | T        | F        | T               | F                                  |
| F | F | T        | T        | T               | T                                  |

7.  $p \wedge \sim q \rightarrow r$

| p | q | r | $\sim q$ | $p \wedge \sim q$ | $p \wedge \sim q \rightarrow r$ |
|---|---|---|----------|-------------------|---------------------------------|
| T | T | T | F        | F                 | T                               |
| T | T | F | F        | F                 | T                               |
| T | F | T | T        | T                 | T                               |
| T | F | F | T        | T                 | F                               |
| F | T | T | F        | F                 | T                               |
| F | T | F | F        | F                 | T                               |
| F | F | T | T        | F                 | T                               |
| F | F | F | T        | F                 | T                               |



$$9. P \wedge \sim r \leftrightarrow q \vee r$$

| P | q | r | $\sim r$ | $P \wedge \sim r$ | $q \vee r$ | $P \wedge \sim r \leftrightarrow q \vee r$ |
|---|---|---|----------|-------------------|------------|--|
| T | T | T | F        | F                 | T          | F  |
| T | T | F | T        | T                 | T          | T  |
| T | F | T | F        | F                 | T          | F  |
| T | F | F | T        | T                 | F          | F  |
| F | T | T | F        | F                 | T          | F  |
| F | T | F | T        | F                 | T          | F  |
| F | F | T | F        | F                 | T          | F  |
| F | F | F | T        | F                 | F          | T  |

Sheet 10.  $(P \rightarrow r) \leftrightarrow (q \rightarrow r)$

12. use the logical equivalence established in Example 1.2.3,  $P \vee q \rightarrow r \equiv (P \rightarrow r) \wedge (q \rightarrow r)$ , to rewrite the following statement. (Assume that  $x$  represents a fixed real number.)  
if  $x > 2$  or  $x < -2$ , then  $x^2 > 4$

Solution  $\rightarrow$  if  $x > 2$  then  $x^2 > 4$ , and if  $x < -2$  then  $x^2 > 4$ .

13. use truth tables to verify the following logical equivalences. Include a few words of explanation with your answers.

$$a. P \rightarrow q \equiv \sim P \vee q$$

| P | q | $\sim P$ | $P \rightarrow q$ | $\sim P \vee q$ | $P \rightarrow q$ and $\sim P \vee q$ always have the same truth values so they are logically equivalent. |
|---|---|----------|-------------------|-----------------|---|
| T | T | F        | T                 | T               |   |
| T | F | F        | F                 | F               |   |
| F | T | T        | T                 | T               |   |
| F | F | T        | T                 | T               |   |



→ In 16 and 17, write each of the two statements in symbolic form and determine whether they are logically equivalent. Include a truth table and a few words of explanation.

16. if you paid full price, you didn't buy it at Crown books.  
you didn't buy it at Crown books or you paid full price.

**Solution** → let  $P$  represent "you paid full price" and  $q$  represent "you didn't buy it at Crown books". Thus, "if you paid full price, you didn't buy it at Crown books" has the form  $P \rightarrow q$ . and "you didn't buy it at Crown books or you paid full price" has the form  $q \vee P$ .

| $P$ | $q$ | $P \rightarrow q$ | $q \vee P$ |  |
|-----|-----|-------------------|------------|--|
| T   | T   | T                 | T          | These two statements aren't logically equivalent because their forms have different truth values in row 2 and 4. |
| T   | F   | F                 | T          |  |
| F   | T   | T                 | T          |  |
| F   | F   | T                 | F          |  |

19. True or False? The negation of "If Sue is Luiz's mother, then Deana is his Cousin" is "If Sue is Luiz's mother, then Deana is not his Cousin".

**Solution** → False. The negation of an if-then statement is not an if-then statement. it is an and statement.

"Sue is Luiz's mother, and Deana is not his Cousin."



20. write negations for each of the following statements.  
(assume that the variables represent fixed quantities or entities as appropriate.)

a. if  $P$  is a square, then  $P$  is a rectangle.

solution  $\rightarrow P$  is a square and  $P$  is not a rectangle.

d. if  $n$  is Prime, then  $n$  is odd or  $n$  is 2.

solution  $\rightarrow n$  is Prime and both  $n$  is not odd and  $n$  is not 2.

OR  $\rightarrow n$  is Prime and  $n$  is neither odd nor 2.

P. If Tom is Ann's father, then Jim is her uncle and Sue is her aunt.

solution  $\rightarrow$  Tom is Ann's father and either Jim is not her uncle or Sue is not her aunt.

23. write the Converse and inverse for each statement of exercise 20.

solution  $\rightarrow$  a. Converse: if  $P$  is a rectangle, then  $P$  is a square.  
inverse: if  $P$  is not a square, then  $P$  is not a rectangle.

d. Converse: if  $n$  is odd or  $n$  is 2, then  $n$  is Prime.  
inverse: if  $n$  is not Prime, then  $n$  is not odd and  $n$  is not 2.

P. Converse: if Jim is Ann's uncle and Sue is her aunt, then Tom is her father.

Inverse: if Tom is not Ann's father, then Jim is not her uncle or Sue is not her aunt.