

# \* Exercise set 1.1

→ write truth tables for the statement forms in 14-18.

14.  $\sim p \wedge q$

P	q	$\sim p$	$\sim p \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

15.  $\sim(p \wedge q) \vee (p \vee q)$

P	q	$p \wedge q$	$\sim(p \wedge q)$	$p \vee q$	$\sim(p \wedge q) \vee (p \vee q)$
T	T	T	F	T	T
T	F	F	T	T	T
F	T	F	T	T	T
F	F	F	T	F	T

16.  $p \wedge (q \wedge r)$

P	q	r	$q \wedge r$	$p \wedge (q \wedge r)$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	T	F
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F



17.  $P \wedge (\sim q \vee r)$

P	q	r	$\sim q$	$\sim q \vee r$	$P \wedge (\sim q \vee r)$
T	T	T	F	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	F	F	F
F	F	T	T	T	F
F	F	F	T	T	F

18.  $(P \vee (\sim P \vee q)) \wedge \sim (q \wedge \sim r)$  → sheet

→ Determine which of the Pairs of statement forms are logically equivalent. Justify your answers using truth tables and include a few words of explanation. Read (t) to be a tautology and (c) to be a Contradiction:

19.  $P \vee (P \wedge q)$  and  $P$  → logically equivalent.

P	q	$P \wedge q$	$P \vee (P \wedge q)$
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F



21.  $P \vee t$  and  $t \rightarrow$  logically equivalent.

P	t	$P \vee t$
T	T	T
F	T	T

23.  $(P \wedge q) \wedge r$  and  $P \wedge (q \wedge r) \rightarrow$  logically equivalent.

P	q	r	$P \wedge q$	$q \wedge r$	$(P \wedge q) \wedge r$	$P \wedge (q \wedge r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	F	T	F	F
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

25.  $(P \wedge q) \vee r$  and  $P \wedge (q \vee r) \rightarrow$  not logically equivalent.

P	q	r	$P \wedge q$	$q \vee r$	$(P \wedge q) \vee r$	$P \wedge (q \vee r)$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	F	T	T	F
T	F	F	F	F	F	F
F	T	T	F	T	T	F
F	T	F	F	T	F	F
F	F	T	F	T	T	F
F	F	F	F	F	F	F



→ use Demorgan's law to write negations for the statements

29. Hal is a math major and Hal's sister is a Computer science major.

answer: Hal is not a math major or Hal's sister is not a Computer science major.

31. The Connector is loose or the machine is unplugged.

answer: The Connector is not loose and the machine is not unplugged.

→ Assume  $x$  is a Particular real number and use De Morgan's Laws to write negations for the statements

35.  $-2 < x < 7$

answer:  $-2 \geq x$  or  $x \geq 7$

37.  $1 > x \geq -3$

$1 \leq x$  or  $x < -3$

→ Use truth tables to establish which of the statement forms are tautologies and which are Contradictions.

41.  $(P \wedge q) \vee (\sim P \vee (P \wedge \sim q)) \rightarrow$  Tautology.

P	q	$\sim P$	$\sim q$	$P \wedge q$	$P \wedge \sim q$	$\sim P \vee (P \wedge \sim q)$	$(P \wedge q) \vee (\sim P \vee (P \wedge \sim q))$
T	T	F	F	T	F	F	T
T	F	F	T	F	T	T	T
F	T	T	F	F	F	T	T
F	F	T	T	F	F	T	T



42.  $(P \wedge \sim q) \wedge (\sim P \vee q) \rightarrow \text{Contradiction}$

P	q	$\sim P$	$\sim q$	$P \wedge \sim q$	$\sim P \vee q$	$(P \wedge \sim q) \wedge (\sim P \vee q)$
T	T	F	F	F	T	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	F	T	F

→ Rewrite the statements in if-then form:

1. This loop will repeat exactly N times if it does not contain a stop or a go to.

answer: if this loop does not contain a stop or a go to, then it will repeat exactly N times.

3. Freeze or I'll shoot.

answer: if you do not freeze, then I'll shoot.

→ use theorem 1.1.1 to verify the logical equivalences.

Supply a reason for each step.

47.  $(P \wedge \sim q) \vee P \equiv P$

answer:  $(P \wedge \sim q) \vee P \equiv P \vee (P \wedge \sim q) \rightarrow \text{by the Commutative law for } \vee$   
 $\equiv P \rightarrow \text{by the absorption law}$

49.  $\sim(P \vee \sim q) \vee (\sim P \wedge \sim q) \equiv \sim P$

answer:  $\sim(P \vee \sim q) \vee (\sim P \wedge \sim q) \equiv (\sim P \wedge \sim(\sim q)) \vee (\sim P \wedge \sim q) \rightarrow \text{by De Morgan's law}$   
 $\equiv (\sim P \wedge q) \vee (\sim P \wedge \sim q) \rightarrow \text{by double negation law}$   
 $\equiv \sim P \wedge (q \vee \sim q) \rightarrow \text{by distributive law}$   
 $\equiv \sim P \wedge t \rightarrow \text{by negation law}$   
 $\equiv \sim P \rightarrow \text{by identity law}$