Introduction Into Probability Theory

MTH 231
Lecture 8
Chapter 6

Fundamental of Statistical Analysis
Parameter Estimation, Confidence Intervals & Test of Hypothesis

Today’s lecture

- Parameter Estimation
- Confidence Intervals
- Sampling Error
- Test of Hypothesis

Parameter Estimation

Populations and Samples

The terms population and sample are defined in statistics as follows:

Population: It is a collection of all possible individuals, about which we require information. A population may be finite or infinite. For example the population consisting of all bolts produced in a factory on a given day is finite; the population consisting of all grains of sand in the world is infinite.

For a population of size **N**, its mean, **μ**, is given by

$$\mu = \frac{\sum_{i=1}^{N} x_i}{N} = \sum_{i=1}^{N} x_i \frac{1}{N}$$

Sample: A sample is a portion of the population of interest.
For a sample of size **n**, the sample mean, $\overline{X}$, is given by

$$\overline{X} = \frac{\sum_{i=1}^{n} x_i}{n}$$

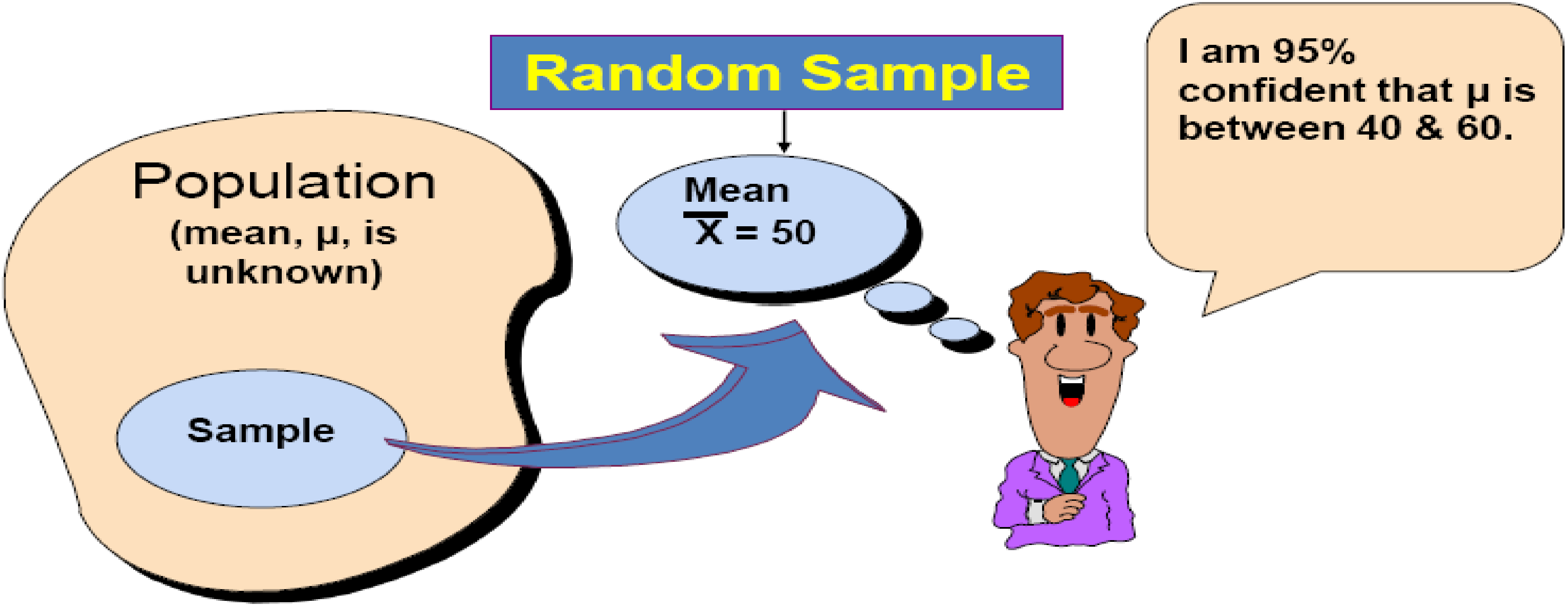
Point and Interval Estimates

- A **parameter** is a population measure (e.g. μ, σ^2).
- A **statistic** is a sample function (e.g. $\overline{X}$, S^2).
- Hence statistics may be regarded as random variables .
- Statistics are used to estimate parameters and are called **point estimators**.
- A **point estimate** of a parameter is a single numerical value of a respective estimator.
- The standard deviation of an estimator is called the **standard error**.

Point Estimates

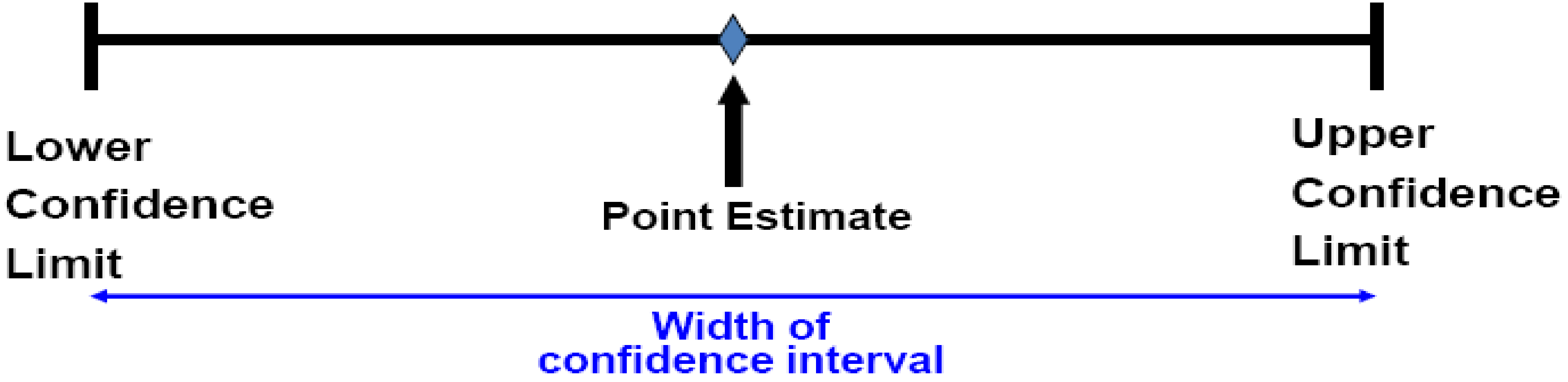
We can estimate a Population Parameter …		with a Sample Statistic (a Point Estimate)
Mean	μ	$\overline{X}$

Estimation Process



Interval Estimates

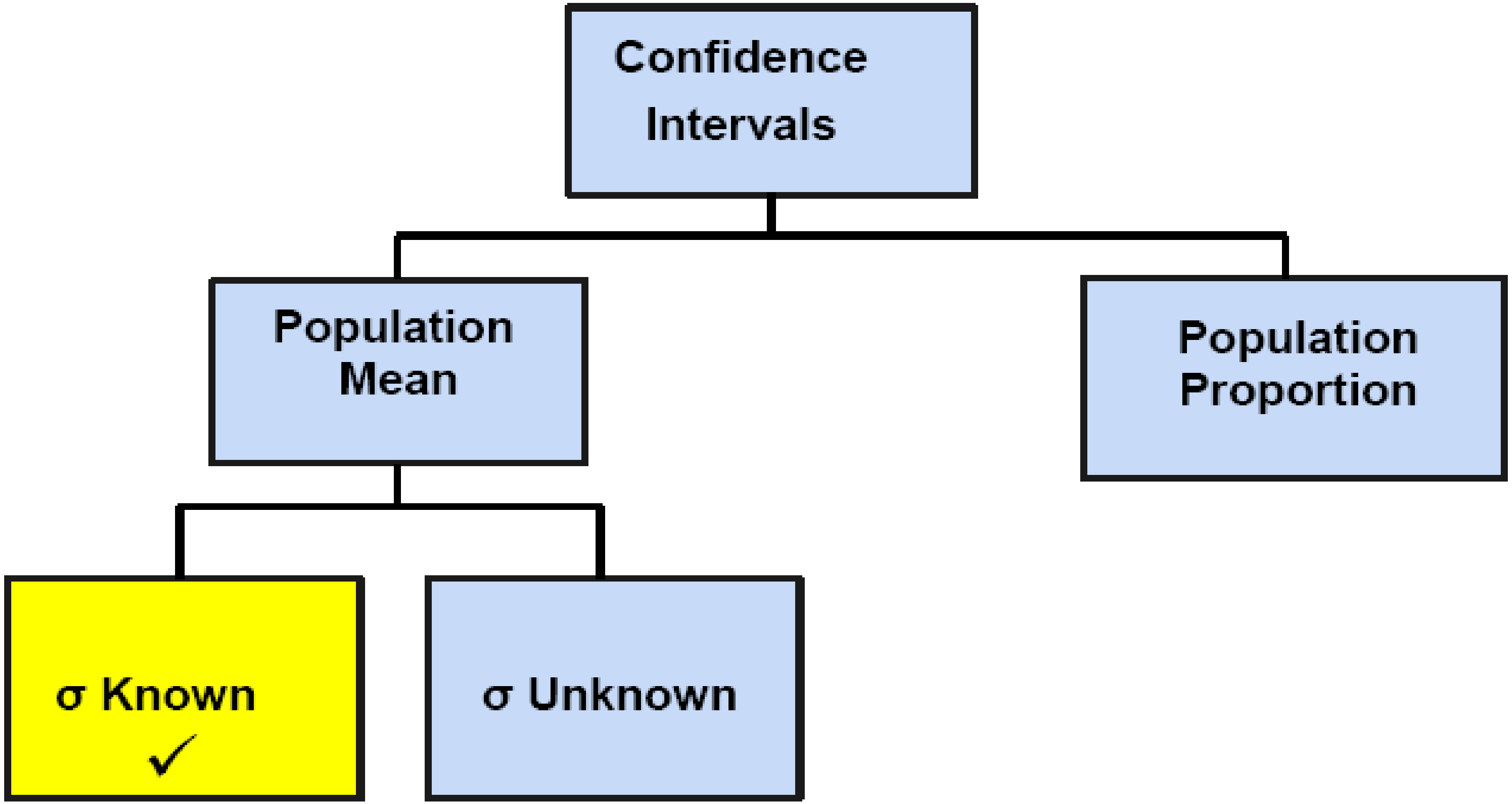
- An **interval estimate of a parameter** is an interval within which the parameter is estimated to exist.
- The **confidence level of an interval estimate** is the probability that the interval contains the parameter.
- Notation: An interval estimate with a confidence level 1- α, is referred to as a **1- α confidence interval**.



Confidence Interval Estimate

- How much uncertainty is associated with a point estimate of a population parameter?
- An **interval estimate** provides more information about a population characteristic than does a **point estimate**
- Such interval estimates are called **confidence intervals**
- An interval gives a **range** of values:
- Based on observation from 1 sample
- Gives information about closeness to unknown population parameters
- Stated in terms of level of confidence
- Can never be 100% confident

Confidence Intervals



Intervals and Level of Confidence

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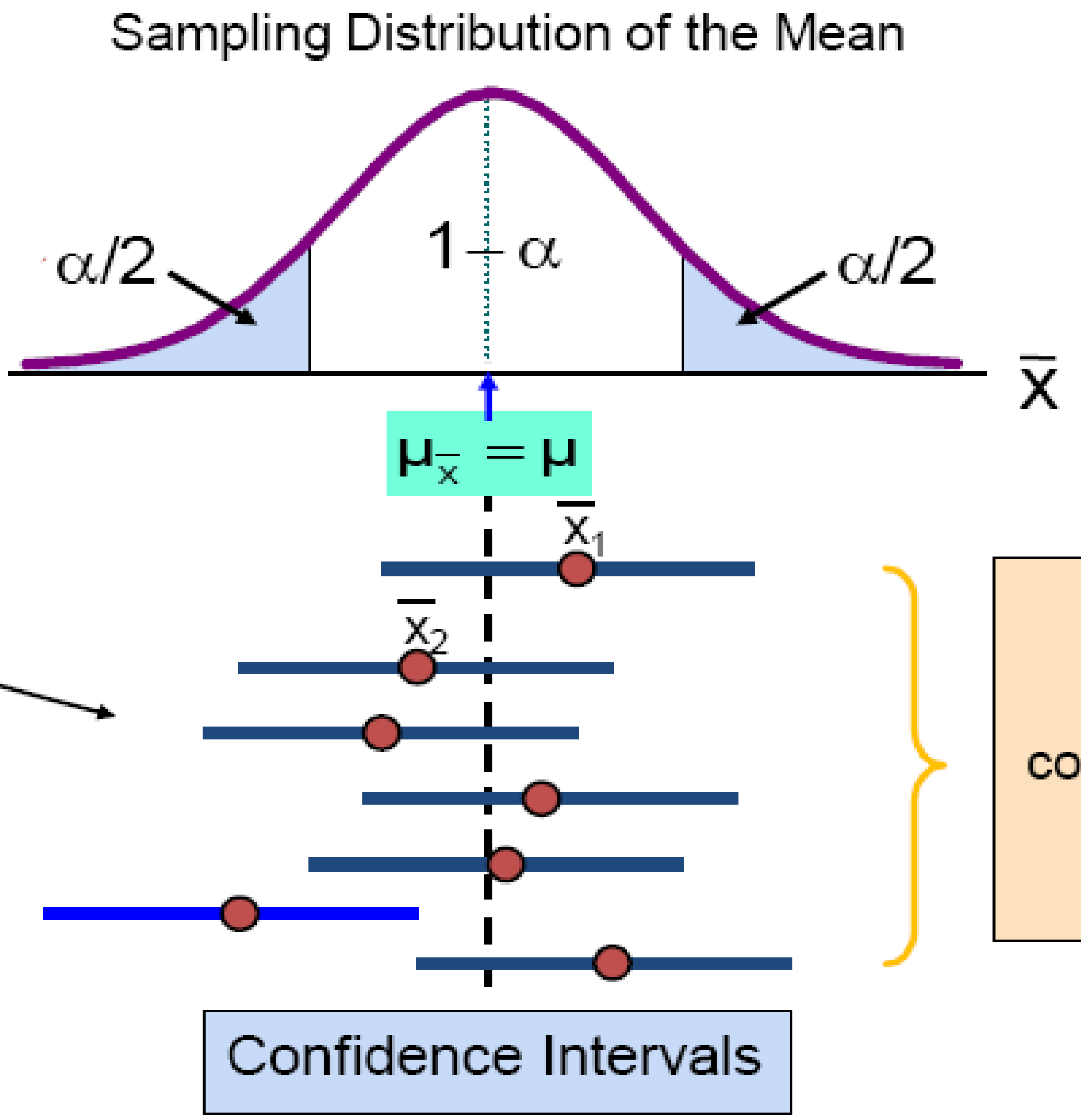


Intervals extend from

$$\bar{X} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

to

$$\bar{X} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

(1-α)x100% of intervals constructed contain μ

Interval Estimates of the Population Mean

- An interval estimate on the mean is an interval centered at the sample mean is: $$(\bar{x}-\varepsilon, \bar{x}+\varepsilon)$$
- ε is the **maximum error of estimation**.
- Saying that $\mu \in (\bar{x}-\varepsilon, \bar{x}+\varepsilon)$ is equivalent to saying that $\mu = \bar{x} \pm \varepsilon$
- How confident we are in this statement depends on $(1 - \alpha)$ (the degree of confidence).

The general formula for all confidence intervals is:

Point Estimate ± (Critical Value)(Standard Error)

Standard Error of the Mean

- Different samples of the same size from the same population will yield different sample means
- A measure of the variability in the mean from sample to sample is given by the **Standard Error of the Mean**:
- Note that the standard error of the mean decreases as the sample size increases

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

If the Population is Normal

If a population is **normal** with mean μ and standard deviation σ, the sampling distribution of $\bar{X}$ is **also normally distributed** with

$$\mu_{\bar{X}} = \mu$$

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

① A (1-α) 100% two-sided confidence interval of μ

$$\left(\overline{X} - Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}, \quad \overline{X} + Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}\right)$$

Or;

$$\overline{X} - Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} < \mu < \overline{X} + Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$

Where $Z_{\frac{\alpha}{2}}$ is the Z- value above which we find an area of $\frac{\alpha}{2}$ that is $P(Z > Z_{\frac{\alpha}{2}}) = \frac{\alpha}{2}$

- Assumptions

  Population standard deviation σ is known

  Population is normally distributed

2 A (1-α) 100% one-sided confidence interval of μ

(i) A (1 - α) 100% One-sided Lower interval for μ

$$\left(\overline{X} - Z_{\alpha}\frac{\sigma}{\sqrt{n}}, \quad \infty\right)$$

Or,

$$\overline{X} - Z_{\alpha}\frac{\sigma}{\sqrt{n}} < \mu < \infty$$

(ii) A (1 - α) 100% One-sided Upper interval for μ

$$\left(-\infty, \quad \overline{X} + Z_{\alpha}\frac{\sigma}{\sqrt{n}}\right)$$

Or,

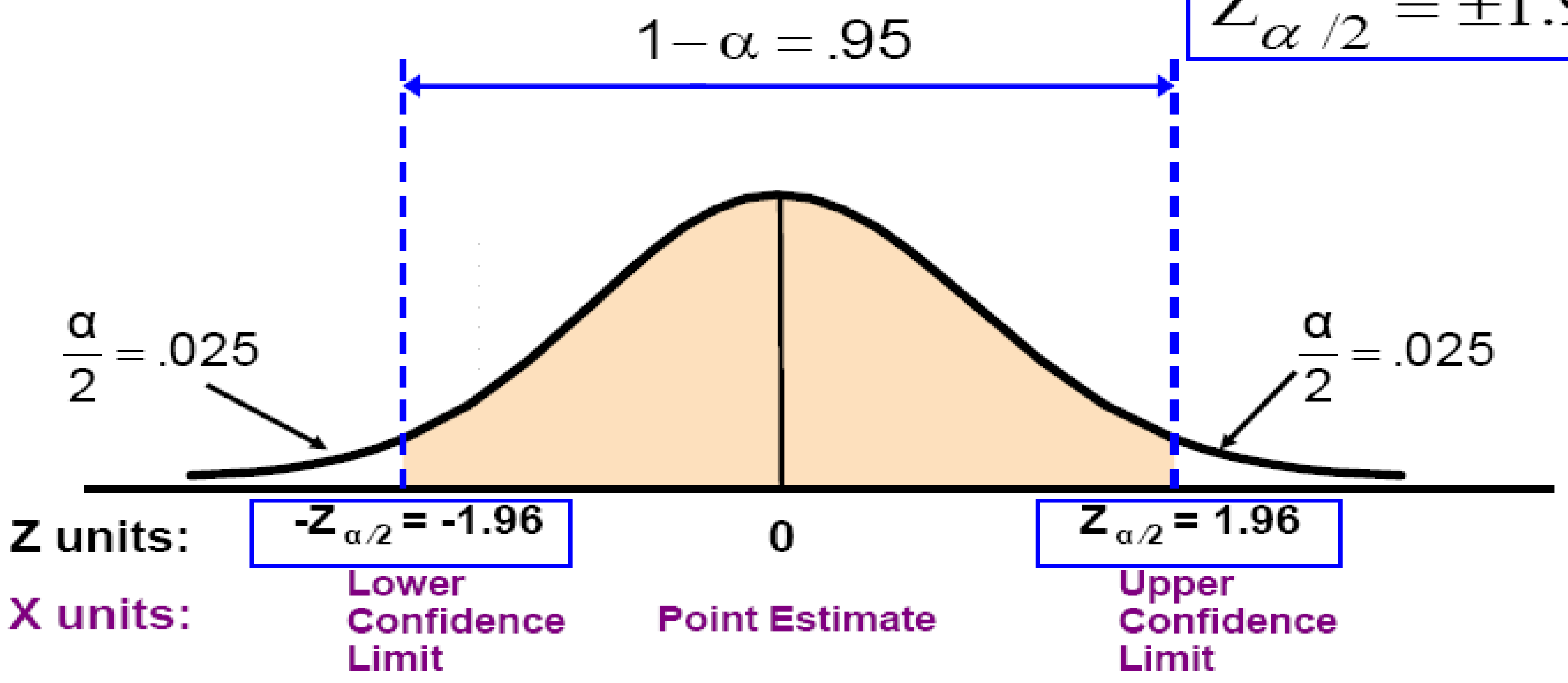
$$-\infty < \mu < \overline{X} + Z_{\alpha}\frac{\sigma}{\sqrt{n}}$$

Finding the Critical Value, Z

- Consider a 95% confidence interval:

$\Phi(z_{\alpha/2}) = 0.975$

$\boxed{Z_{\alpha/2} = \pm 1.96}$



$P(Z > Z_{\frac{\alpha}{2}}) = 1 - \Phi(Z_{\frac{\alpha}{2}}) = \frac{\alpha}{2}$ Then, $\Phi(Z_{\frac{\alpha}{2}}) = 1 - \frac{\alpha}{2}$ Similarly, $\Phi(Z_{\alpha}) = 1 - \alpha$

Common Levels of Confidence

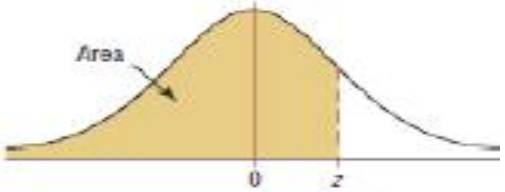
- The Commonly used confidence levels are 90%, 95%, and 99%

Confidence Level	*Confidence Coefficient,* $1-\alpha$	*Z value*
80%	.80	1.28
90%	.90	1.645
95%	.95	1.96
98%	.98	2.33
99%	.99	2.57
99.8%	.998	3.08
99.9%	.999	3.27

that is

1. **90% (with $\alpha = 0.1$) and critical value $z_{\alpha/2} = 1.645$.**
2. **95% (with $\alpha = 0.05$) and critical value $z_{\alpha/2} = 1.96$.**
3. **99% (with $\alpha = 0.01$) and critical value $z_{\alpha/2} = 2.575$.**

The Cumulative Standardized Normal Table



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

➢**Example (1)**: Suppose that when a signal having value μ is transmitted from location A the value received at location B is normally distributed with mean μ and variance 4. That is , if μ is sent , then the value is μ + N where N, representing noise, is normal with mean 0 and variance 4. To reduce error, suppose the same value is sent 9 times . If the successive values received are 5, 8.5, 12, 15, 7, 9, 7.5, 6.5, and 10.5.

(a) Construct a 95% confidence interval for μ

(b) Determine the lower and upper 95 percent confidence interval estimates of μ.

Solution

$$\sigma^2 = 4 \text{ , so } \sigma = 2$$

$$\overline{X} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{81}{9} = 9$$

(a) A 95% confidence interval for μ, $\alpha = 5\% = 0.05$

$$\left(\overline{X} - Z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}, \quad \overline{X} + Z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}\right)$$

$$Z_{\frac{\alpha}{2}} = Z_{0.025} = 1.96$$

Then ,95 percent confidence interval for μ is ,

$$\left(9 - 1.96\frac{2}{\sqrt{9}}, \quad 9 + 1.96\frac{2}{\sqrt{9}}\right) = (7.69, \; 10.31)$$

(b) $\alpha = 0.05$, and $Z_{\alpha} = Z_{0.05} = 1.645$

(i) the lower 95 percent confidence interval estimates of μ is

$$\left(\overline{X} - Z_{\alpha}\frac{\sigma}{\sqrt{n}}, \quad \infty\right) = (7.903\;, \infty)$$

(ii) the upper 95 percent confidence interval estimates of μ is

$$\left(-\infty\;, \quad \overline{X} + Z_{\alpha}\frac{\sigma}{\sqrt{n}}\right) = (-\infty\;, 10.097\;)$$

➢**Example (2)**: The average zinc concentration recovered from a sample of zinc measurements in 36 different locations is found to be 2.6 grams per milliliter. Find the 95% and 99% confidence interval for the mean zinc concentration in the river. Assume that the population standard deviation is 0.3.

Solution: $\overline{X} = 2.6 \quad , \sigma = 0.3, n = 36$

(i) 95% confidence interval of μ, $\alpha = 0.05$

$$\left(\overline{X} - Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}, \quad \overline{X} + Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}\right), \; Z_{\frac{\alpha}{2}} = Z_{0.025} = 1.96$$

Then, the 95 percent confidence interval for μ is,

$$\left(2.6 - 1.96\frac{0.3}{\sqrt{36}}, \quad 2.6 + 1.96\frac{0.3}{\sqrt{36}}\right) = (2.50, \; 2.70)$$

(ii) 99% confidence interval of μ, $\alpha = 0.01$, $Z_{\frac{\alpha}{2}} = Z_{0.005} = 2.575$

Then, the 99 percent confidence interval for μ is,

$$\left(2.6 - 2.575\frac{0.3}{\sqrt{36}}, \quad 2.6 + 2.575\frac{0.3}{\sqrt{36}}\right) = (2.47, \; 2.73)$$

➢**Example (3)**: An electric scale gives a reading equal to the true weight plus a random error that is normally distributed with mean 0 and standard deviation σ = 0.1 mg. Suppose that the results of five successive weightings of the same object are as follows : 3.142, 3.163, 3.155, 3.150, and 3.141.

(a) Determine a 95% confidence interval estimate of the true weight.

(b) Determine a 99% confidence interval estimate of the true weight.

(c) Determine the lower and upper 95% confidence interval estimates of μ.

Solution: $\sigma = 0.1$, $n = 5$, $\overline{X} = \sum_{i=1}^{n} x_i \backslash n = 3.502$

(a) A 95% confidence interval for the true weight μ, α = 0.05,

$$\left(\overline{X} - Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}, \quad \overline{X} + Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}\right)$$

$$Z_{\frac{\alpha}{2}} = Z_{0.025} = 1.96$$

Then , the 95 percent C.I. for μ is: $(3.0625,\ 3.2378)$

(b) A 99% confidence interval for μ, $\alpha = 0.01$

$$\left(\overline{X} - Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}, \quad \overline{X} + Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}\right)$$

$$Z_{\frac{\alpha}{2}} = Z_{0.005} = 2.58$$

Then , the 99 percent confidence interval for μ is ,

$$\left(3.1502 - 2.58\frac{0.1}{\sqrt{5}}, \quad 3.1502 + 2.58\frac{0.1}{\sqrt{5}}\right) = (3.0348,\ 3.2656)$$

(c) $\alpha=0.05$, and $Z_{\alpha} = Z_{0.05} = 1.645$

(i) the lower 95 percent confidence interval estimates of μ is

$$\left(\overline{X} - Z_{\alpha} \frac{\sigma}{\sqrt{n}}, \quad \infty\right) = (3.0766, \infty)$$

(ii) the upper 95 percent confidence interval estimates of μ is

$$\left(-\infty, \quad \overline{X} + Z_{\alpha} \frac{\sigma}{\sqrt{n}}\right) = (-\infty, 3.2238)$$

➢**Example (4)**: The standard deviation of test scores on a certain achievement test is 11.3. If a random sample of 81 students had a sample mean score of 74.6, find a 90 percent confidence interval estimate for the average score of all students.

Solution

$\sigma = 11.3$, $n = 81$, and $\overline{X} = 74.6$

A 90% C.I. for the average score of all students μ, $\alpha = 0.1$

$$\left(\overline{X} - Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}, \quad \overline{X} + Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}\right)$$

$$Z_{\frac{\alpha}{2}} = Z_{0.05} = 1.645$$

$$\left(74.6 - 1.645 \frac{11.3}{\sqrt{81}}, \quad 74.6 + 1.645 \frac{11.3}{\sqrt{81}}\right) = (72.5346, \quad 76.6654)$$

Sampling Error

- The required sample size can be found to reach a desired margin of error (e) with a specified level of confidence $(1 - \alpha)$

- The margin of error is also called sampling error
  - the amount of imprecision in the estimate of the population parameter
  - the amount added and subtracted to the point estimate to form the confidence interval

Determining Sample Size

Determining Sample Size

For the Mean

$$\bar{X} \pm Z\frac{\sigma}{\sqrt{n}}$$

Sampling error (margin of error)

$$e = Z\frac{\sigma}{\sqrt{n}}$$

Determining Sample Size

(continued)

Determining Sample Size

For the Mean

$$e = Z\frac{\sigma}{\sqrt{n}}$$

Now solve for n to get

$$\boxed{n = \frac{Z^2\,\sigma^2}{e^2}}$$

Determining Sample Size

(continued)

- To determine the required sample size for the mean, you must know:
  - The desired level of confidence ($1 - \alpha$), which determines the critical Z value
  - The acceptable sampling error (margin of error), e
  - The standard deviation, σ

Required Sample Size Example

➢ **Example (5):**

If σ = 45, what sample size is needed to estimate the mean within ± 5 with 90% confidence?

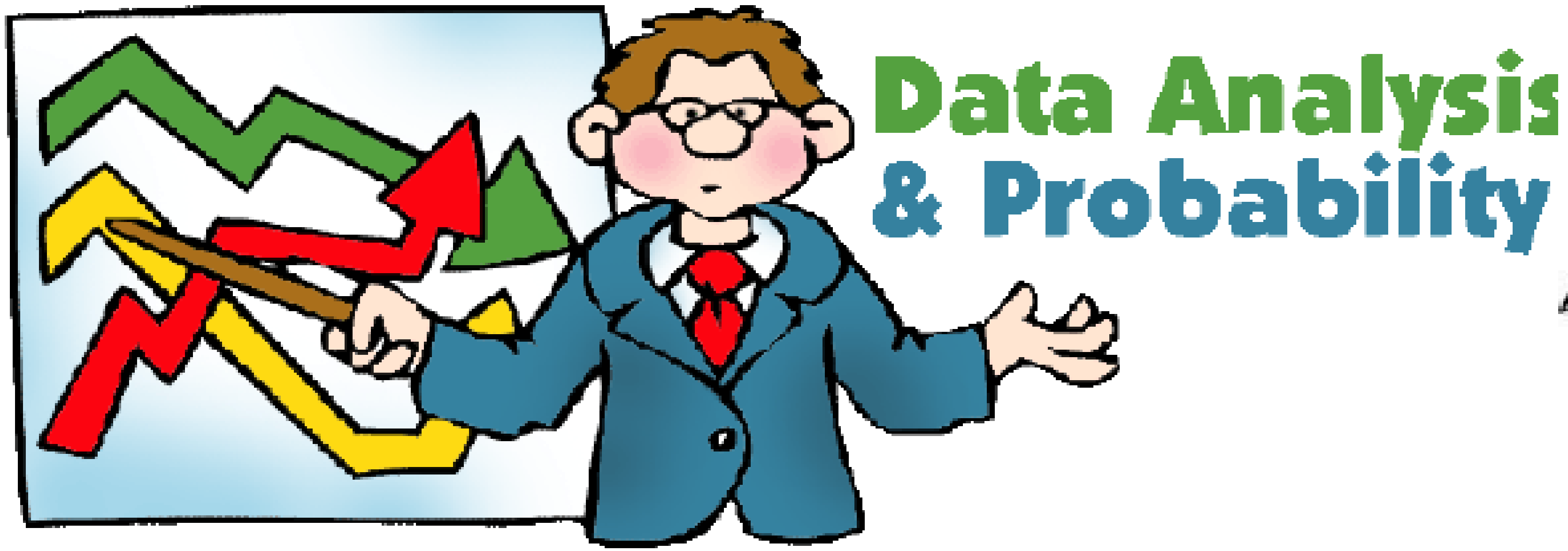
$$n = \frac{Z^2 \sigma^2}{e^2} = \frac{(1.645)^2 (45)^2}{5^2} = 219.19$$

So the required sample size is **n = 220**

(Always round up)

Hypothesis Testing

- The purpose of hypothesis testing is to help the researcher or administrator in reaching a decision concerning a population by examining a sample from that population



What is a Hypothesis?

- A hypothesis is a claim (assumption) about a population parameter:
  - For a population mean

Example: **The mean monthly cell phone bill of this city is $\mu = \$42$**

The Null Hypothesis, H_0

- States the assumption (numerical) to be tested

Example: **The average number of TV sets in U.S. homes is equal to three** $(H_0: \ \mu = 3)$

- Is always about a population parameter, not about a sample statistic

$$H_0 : \mu = 3 \qquad \cancel{H_0 : \bar{X} = 3}$$

•Begin with the assumption that the null hypothesis is true

Similar to the notion of innocent until proven guilty!

•Always contains "=" , "≤" or "≥" sign

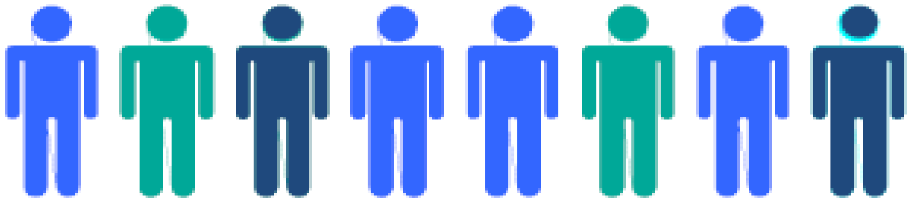
•May or may not be rejected

The Alternative Hypothesis, H_1

- Is the opposite of the null hypothesis
  - e.g., The average number of TV sets in U.S. homes is not equal to 3 (H_1: $\mu \neq 3$)
- Challenges the above status
- Never contains the "=" , "≤" or "≥" sign
- May or may not be proven
- Is generally the hypothesis that the researcher is trying to prove

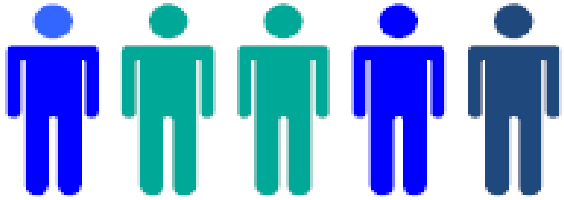
Hypothesis Testing Process

Claim: the population mean age is 50. (Null Hypothesis: $H_0: \mu = 50$)



Population

Now select a random sample



Sample

Suppose the sample mean age is 20: $\bar{X} = 20$

Is $\bar{X} = 20$ likely if $\mu = 50$?

If not likely,

REJECT Null Hypothesis

Level of Significance, α

- **Defines the unlikely values of the sample statistic if the null hypothesis is true**
  - Defines rejection region of the sampling distribution
- Is designated by α , (level of significance)
  - Typical values are .01, .05, or .10
- Is selected by the researcher at the beginning
- Provides the critical value(s) of the test

Hypothesis-Testing Common Phrases

Is greater than	Is less than	Is equal to	Is not equal to
Is above	Is below	Is the same as	Is different from
Is higher than	Is lower than	Has not changed from	Has changed from
Is longer than	Is shorter than	Is the same as	Is not the same as
Is bigger than	Is smaller than	$=$	$\neq$
Is increased	Is decreased or reduced from		
$>$	$<$		

Level of Significance and the Rejection Region

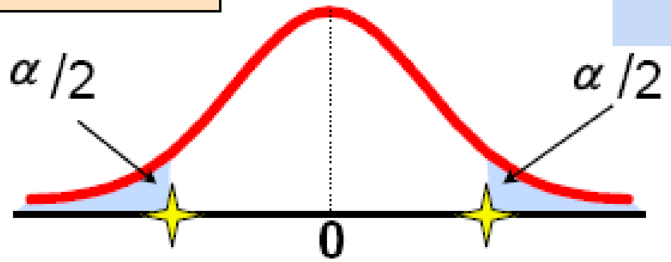
Level of significance = α

Represents critical value

Rejection region is shaded

$H_0: \mu = 3$

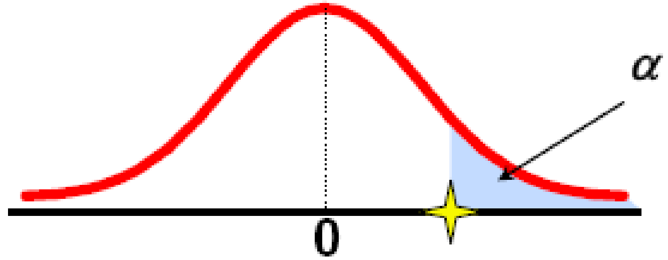
$H_1: \mu \neq 3$



Two-tailed test

$H_0: \mu \leq 3$

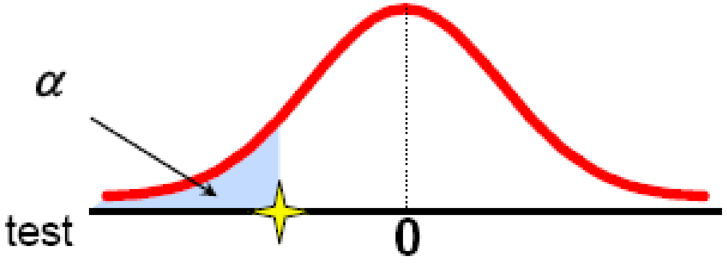
$H_1: \mu > 3$



Upper-tailed test

$H_0: \mu \geq 3$

$H_1: \mu < 3$



Lower-tailed test

Errors in Making Decisions

- **Type I Error**
  - Rejecting the null hypothesis when it's true.
  - Considered a serious type of error

The probability of Type I Error is α

  - Called level of significance of the test
  - Set by researcher in advance

- **Type II Error**
  - Accepting the null hypothesis when it's false.

probability of Type II Error is β

Type I & II Error Relationship

If Type I error probability (α) $\Uparrow$, then

Type II error probability (β) $\Downarrow$

Outcomes and Probabilities

Possible Hypothesis Test Outcomes

		Actual Situation	
		H_0 is true	H_0 is false
Decision	accept H_0	correct decision probability = 1- α = confidence level	incorrect decision (type II error) probability = β
	reject H_0	incorrect decision (type I error) probability = α = level of significance	correct decision probability = 1- β = power of the test

Z Test of Hypothesis for the Mean (σ Known)

- Convert sample statistic ($\overline{X}$) to a Z **test statistic**

Hypothesis Tests for μ

σ Known | σ Unknown

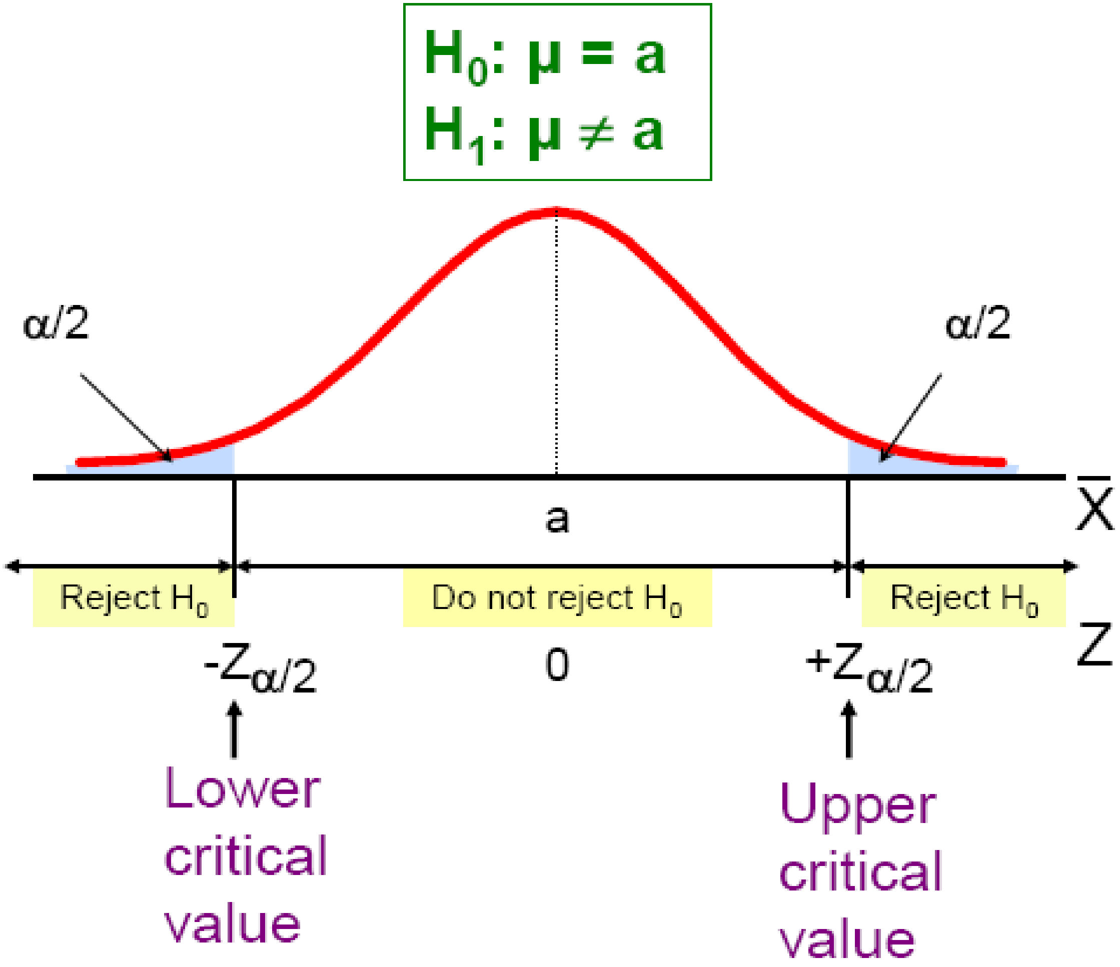
The test statistic is:

$$Z = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$\text{Test value} = \frac{(\text{observed value}) - (\text{expected value})}{\text{standard error}}$$

Two-Tailed Tests

- There are two cutoff values (critical values), defining the regions of rejection



One-Tailed Tests

- In many cases, the alternative hypothesis focuses on a particular direction

$H_0: \mu \geq 3$

$H_1: \mu < 3$

→ This is a lower-tailed test since the alternative hypothesis is focused on the lower tail below the mean of 3

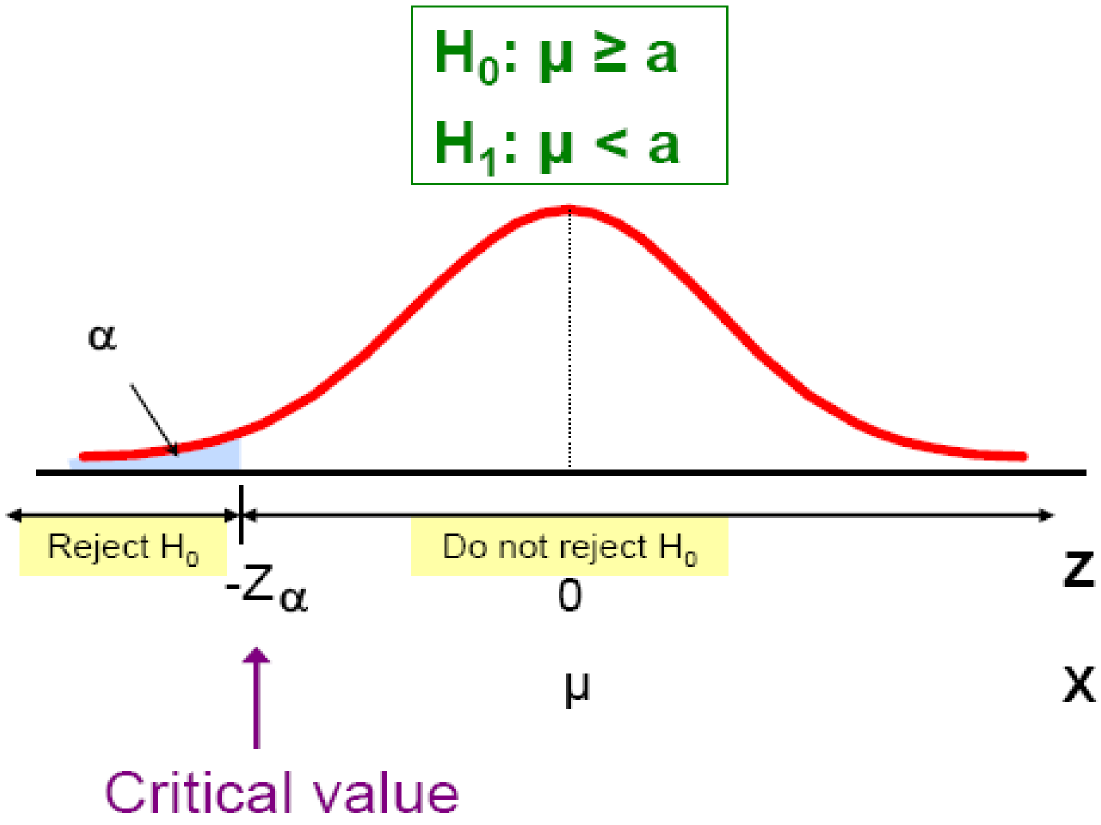
$H_0: \mu \leq 3$

$H_1: \mu > 3$

→ This is an upper-tailed test since the alternative hypothesis is focused on the upper tail above the mean of 3

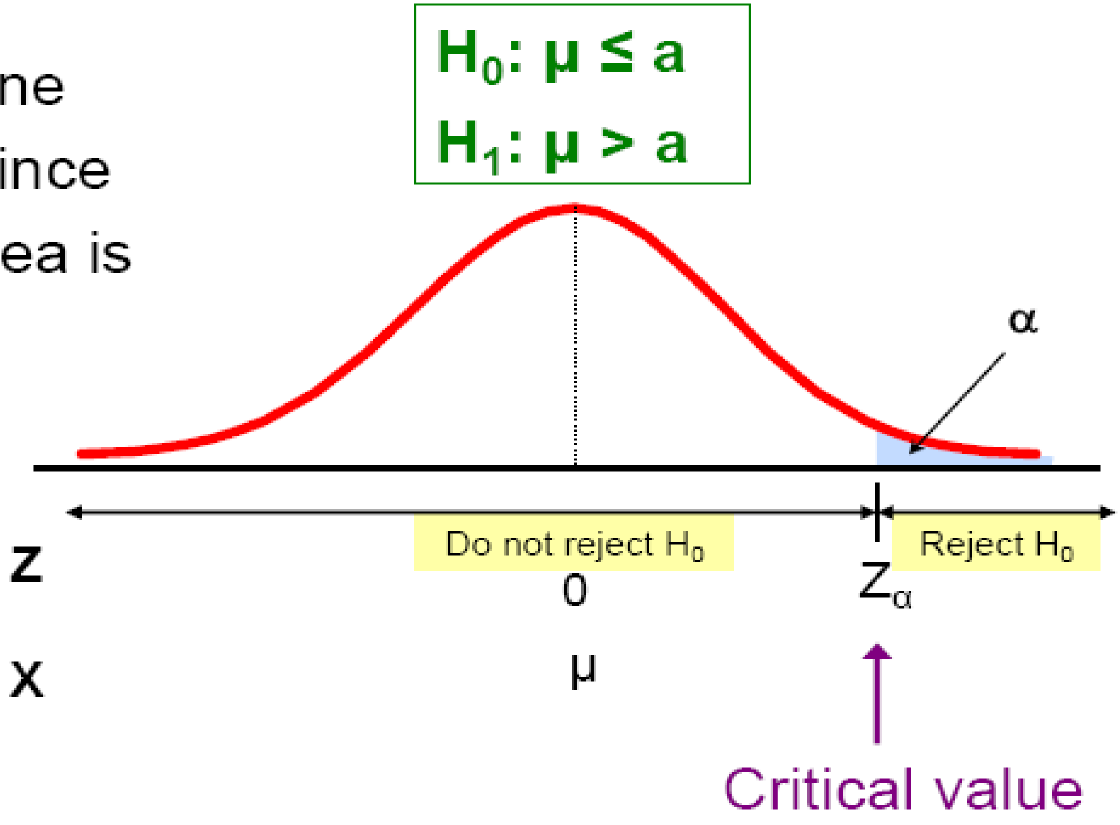
Lower-Tailed Tests

- There is only one critical value, since the rejection area is in only one tail



Upper-Tailed Tests

- There is only one critical value, since the rejection area is in only one tail



Procedure Table

Solving Hypothesis-Testing Problems (Traditional Method)

Step 1 State the hypotheses and identify the claim.

Step 2 Find the critical value(s) from the appropriate table.

Step 3 Compute the test statistic value.

Step 4 Make the decision to reject or not reject the null hypothesis.

Step 5 Summarize the results.

➢Example(1): **Two-tailed Z test (σ Known)**

At a 5% significance level, test the claim that the true mean # of TV sets in U.S. homes is equal to 3. Consider a sample of size 100 with mean 2.84 (Assume σ = 0.8)

1- State the appropriate null and alternative hypotheses

- H_0: $\mu = 3$ H_1: $\mu \neq 3$ (This is a two-tailed test)

2- Specify the desired level of significance and set up the critical values

- α = .05 is chosen for this test
- For α = .05, the critical Z values are ± 1.96

Hypothesis Testing Example

(continued)

3- Compute the test statistic

- The sample results are

  $n = 100$, $\overline{X} = 2.84$ ($\sigma = 0.8$ is assumed known)

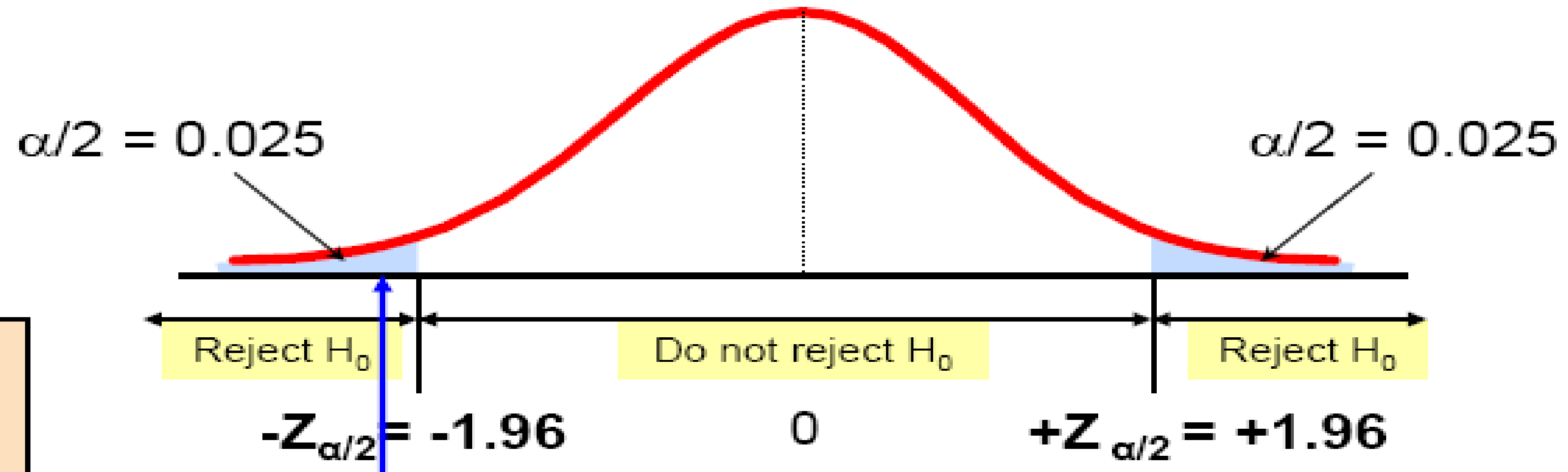
So the test statistic is:

$$Z_c = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{2.84 - 3}{\frac{0.8}{\sqrt{100}}} = \frac{-.16}{.08} = -2.0$$

Hypothesis Testing Example

(continued)

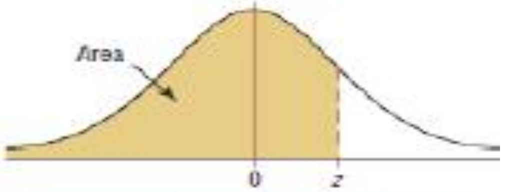
4- Is the test statistic in the rejection region?



Reject H_0 if $Z_c < -1.96$ or $Z_c > 1.96$; otherwise do not reject H_0

Here, $Z_c = -2.0 < -1.96$, so the test statistic is in the rejection region

The Cumulative Standardized Normal Table

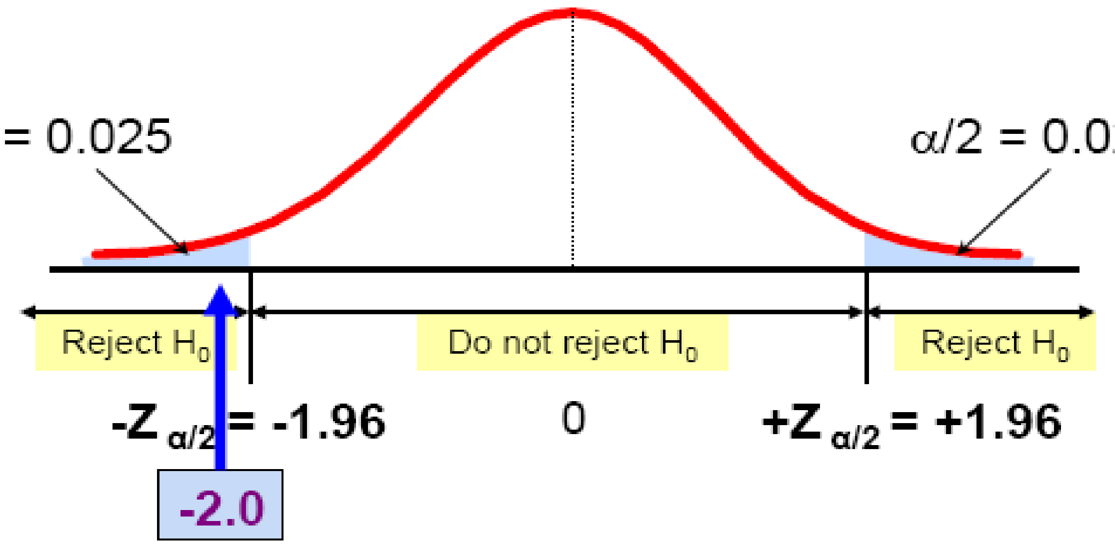


z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Hypothesis Testing Example

(continued)

5- Reach a decision and interpret the result



Since $Z_c = -2.0 < -1.96$, we <u>reject the null hypothesis</u> and conclude that there is sufficient evidence that the mean number of TVs in US homes is not equal to 3.

➢ Example (2): Upper-Tail Z Test for Mean (σ Known)

A phone industry manager thinks that customer monthly cell phone bill have increased, and now average over $52 per month. The company wishes to test this claim for a sample of 64 with mean 53.1. Use 0.10 significance level. (Assume $\sigma = 10$ is known)

1- Form of hypothesis test:

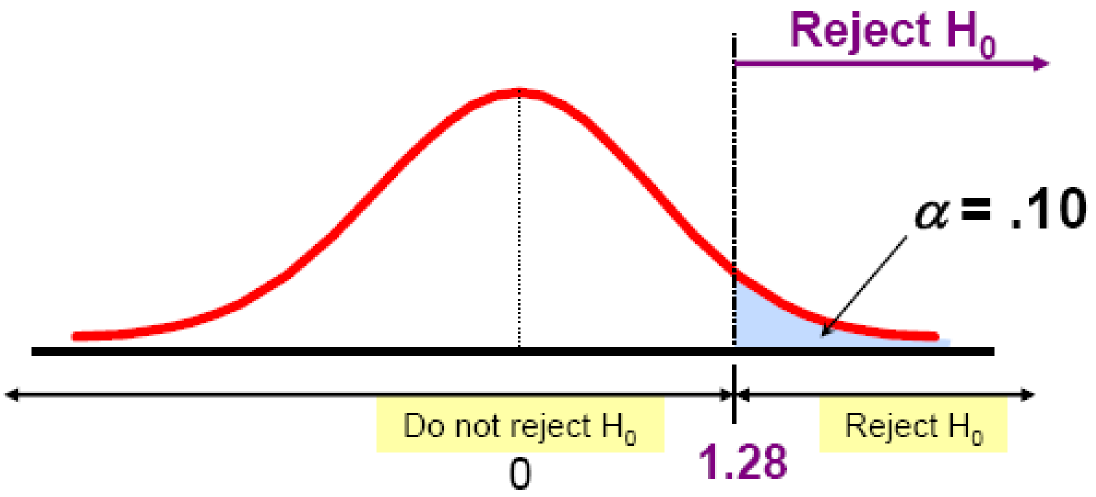
$H_0: \mu \le 52$	the average is not over $52 per month
$H_1: \mu > 52$	the average is greater than $52 per month (i.e., sufficient evidence exists to support the manager's claim)

Finding Rejection Region

(continued)

2- For the chosen $\alpha = .10$ to this test, the critical value is 1.28 (why?)

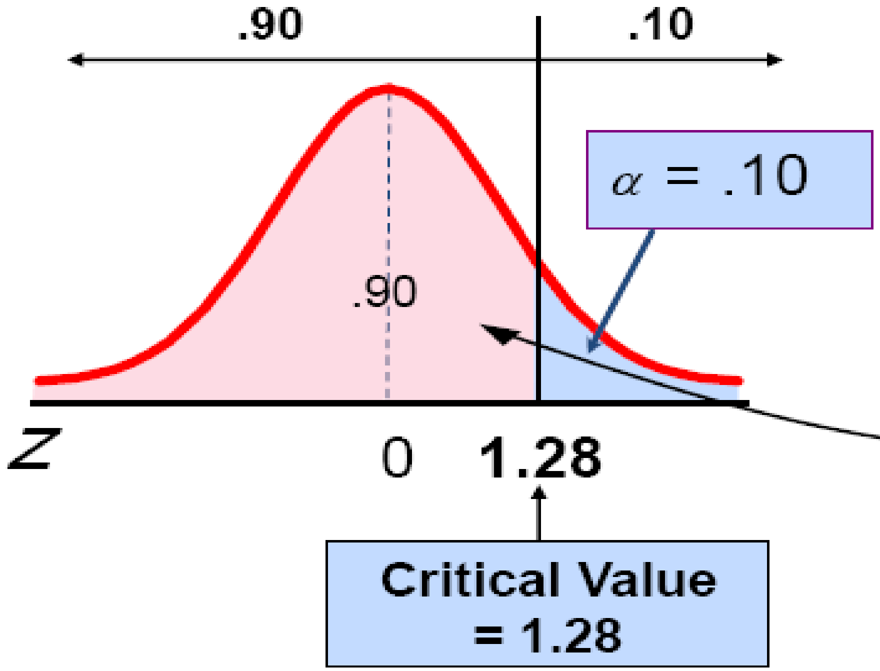
Find the rejection region:



Reject H_0 if $Z_c > 1.28$

Review:
One-Tail Critical Value

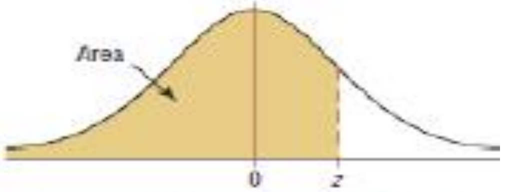
What is Z given $\alpha = 0.10$?



Standard Normal Distribution Table (Portion)

Z	.07	.08	.09
1.1	.8790	.8810	.8830
1.2	.8980	.8997	.9015
1.3	.9147	.9162	.9177

The Cumulative Standardized Normal Table



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
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2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Test Statistic

(continued)

3- Compute the test statistic with n = 64, $\overline{X}$ = 53.1, and σ = 10

– Then the test statistic is:

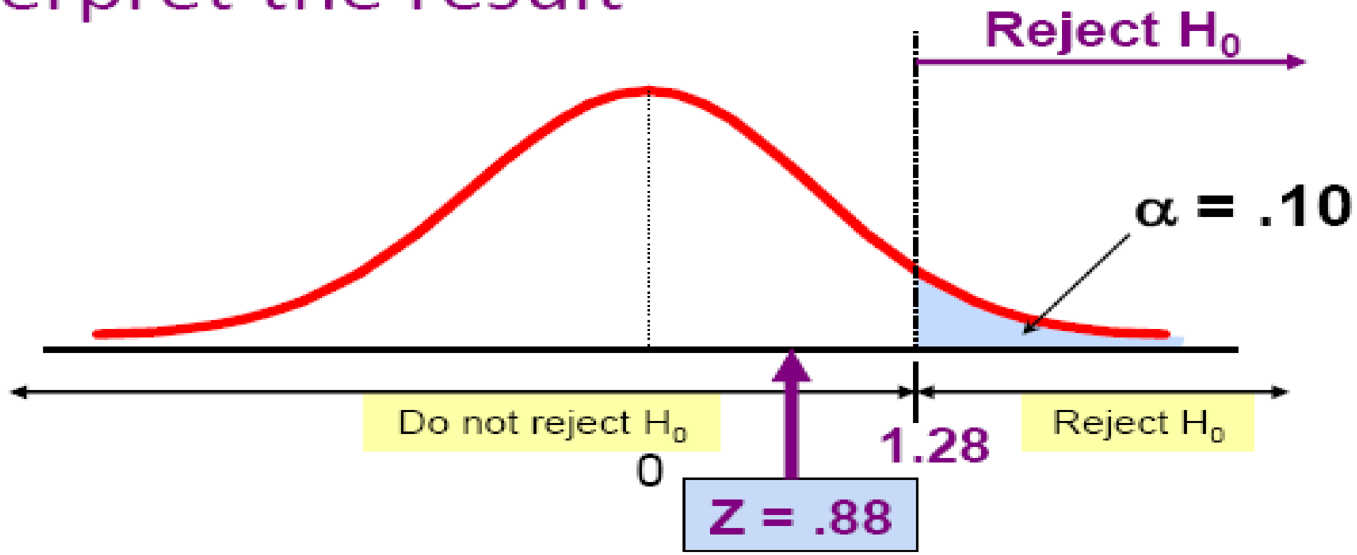
$$Z_c = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{53.1 - 52}{\frac{10}{\sqrt{64}}} = 0.88$$

Decision

(continued)

4- Reach a decision

5- interpret the result



Do not reject H_0 since $Z = 0.88 \le 1.28$

i.e.: there is not sufficient evidence that the mean bill is over $52

Questions!

"Success doesn't come to you, you go to it."

PRACTICE!

PRACTICE!

PRACTICE!

There is **NO** elevator to SUCCESS.

You have to take the STAIRS.