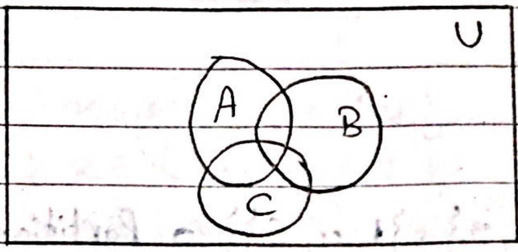
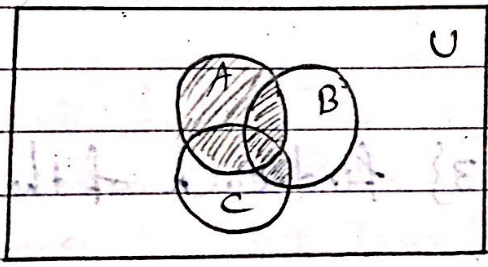
* Exercise Set 5.2 :

21. Consider the venn diagram below.

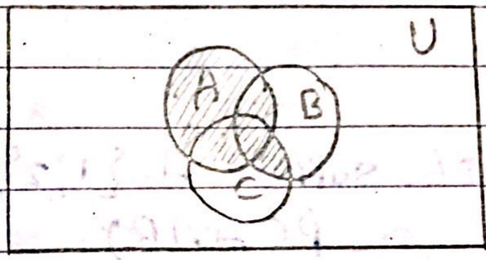


a. illustrate one of the distributive Laws by shading in the region Corresponding to $A \cup (B \cap C)$ on one Copy of the diagram and $(A \cup B) \cap (A \cup C)$ on another.

1

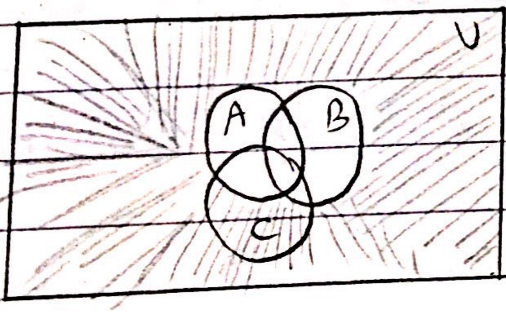


2

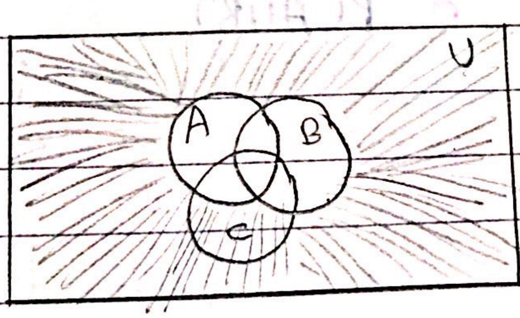


c. illustrate one of Demorgan's law by shading in the region Corresponding to $(A \cup B)^c$ on one Copy of the diagram and $A^c \cap B^c$ on the other

1



2



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* for each 5 _ 17 Prove each statement that true and find a Counter example for each statement that is false. assume all sets are subsets of a universal set U

9 - for all sets A, B and C, if $A \subseteq C$ and $B \subseteq C$ then $A \cup B \subseteq C$

$\rightarrow$ let $x \in A \cup B$

$\therefore x \in A$ or $x \in B$

$\rightarrow$ if $x \in A$:

$\therefore A \subseteq C \rightarrow \therefore x \in C \rightarrow$ ①

$\rightarrow$ if $x \in B$:

$\therefore B \subseteq C \rightarrow \therefore x \in C \rightarrow$ ②

from ① and ② $\rightarrow x \in C$

$\therefore x \in A \cup B$, $x \in C$

$\therefore A \cup B \subseteq C$

$\therefore$ The statement is true.

14 - for all sets A and B, $P(A \cup B) \subseteq P(A) \cup P(B)$.

The statement is false

$\rightarrow$ let $A = \{1\}$ $B = \{2\}$

$A \cup B = \{1, 2\}$

$\rightarrow P(A \cup B) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$

$\rightarrow P(A) = \{\phi, \{1\}\}$

$\rightarrow P(B) = \{\phi, \{2\}\}$

$P(A) \cup P(B) = \{\phi, \{1\}, \{2\}\}$

$\therefore P(A \cup B) \nsubseteq P(A) \cup P(B)$

Exercise set 5.3

15- for all sets A and B, $P(A) \cup P(B) \subseteq P(A \cup B)$

let $x \in P(A) \cup P(B)$

$\therefore x \in P(A)$ or $x \in P(B)$

→ if $x \in P(A)$

$\therefore x \subseteq A \rightarrow \therefore x \subseteq A \cup B \rightarrow (1)$

→ if $x \in P(B)$

$\therefore x \subseteq B \rightarrow \therefore x \subseteq A \cup B \rightarrow (2)$

from (1) and (2)

$P(A) \cup P(B) \subseteq P(A \cup B)$

- The statement is true.

* use the Properties in theorem 5.2.2 to Construct an algebraic Proof for the given statement :-

27- for all sets A, B and C

$(A - B) - C = A - (B \cup C)$.

$\rightarrow (A - B) - C = (A \cap B^c) - C$ → set difference law

$= (A \cap B^c) \cap C^c$ → " "

$= A \cap (B^c \cap C^c)$ → associative law

$= A \cap (B \cup C)^c$ → De Morgan's law

$= A - (B \cup C)$ → set difference Law.

29- for all set A and B $((A^c \cup B^c) - A)^c = A$.

$\rightarrow ((A^c \cup B^c) - A)^c = ((A^c \cup B^c) \cap A^c)^c$ → set difference law

$= (A^c \cup B^c)^c \cup (A^c)^c$ → De Morgan's law.

$= ((A^c)^c \cap (B^c)^c) \cup (A^c)^c$ → " "

$= (A \cap B) \cup A$ → double Complement law

$= A \cup (A \cap B)$ → Commutative law.

$= A$ → absorption law

31- for all sets A and B, $A - (A \cap B) = A - B$

$\rightarrow A - (A \cap B) = A \cap (A \cap B)^c$ → set differenc law

$= A \cap (A^c \cup B^c)$ → De Morgan's law

$= (A \cap A^c) \cup (A \cap B^c)$ → distributive law.

$= \phi \cup (A \cap B^c)$ → Complement law

$= A \cap B^c$ → identity law

$= A - B$ → set differenc law.

* siplify the given expressions:-

35- $(A - (A \cap B)) \cap (B - (A \cap B))$

$= ((A - A) \cap (A - B)) \cap ((B - A) \cap (B - B))$ → distributive law

$= ((A \cap A^c) \cap (A \cap B^c)) \cap ((B \cap A^c) \cap (B \cap B^c))$ → set differenc law

$= (\phi \cap (A \cap B^c)) \cap ((B \cap A^c) \cap \phi)$ → Complement law.

$= \phi \cap \phi$ → universal bound law

$= \phi$ → universal bound law

39- let $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$ and $C = \{5, 6, 7, 8\}$ find each of the following sets:-

a- $A \Delta B$

$\rightarrow A \Delta B = (A - B) \cup (B - A)$

$= \{1, 2\} \cup \{5, 6\} = \{1, 2, 5, 6\}$

c- $(A \Delta B) \Delta C$

$\rightarrow (A \Delta B) \Delta C = \{1, 2, 5, 6\} \Delta \{5, 6, 7, 8\}$

$= ((A \Delta B) - C) \cup (C - A \Delta B))$

$= \{1, 2\} \cup \{7, 8\}$

$= \{1, 2, 7, 8\}$

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* refer to the definition of symmetric differenc given above Prove each of 40 - 45 assuming that A, B and C are all subsets of a universal set U

40- $A \Delta B = B \Delta A$

$A \Delta B = (A - B) \cup (B - A)$
$= (B - A) \cup (A - B)$
$= B \Delta A$ #

41- $A \Delta \phi = A$

$A \Delta \phi = (A - \phi) \cup (\phi - A)$
$= A \cup \phi = A$

* let $A = \{1, 2\}$, $B = \{2, 5, 6\}$ find:-

a) $A \times B$

$\{(1,2), (1,5), (1,6), (2,2), (2,5), (2,6)\}$

b) $A \times A$

$\{(1,1), (1,2), (2,1), (2,2)\}$

* let $A = \{1, 2\}$, $B = \{a, b\}$, $C = \{r, s, t\}$ find-

a) $A \times B \times C$

$\{(1,a,r), (1,a,s), (1,a,t), (1,b,r),$
$(1,b,s), (1,b,t), (2,a,r), (2,a,s),$
$(2,a,t), (2,b,r), (2,b,s), (2,b,t)\}$