شركة تدريب هندسي

E.CAMP

الطريق الدائري بجوار المدرسة المعمارية

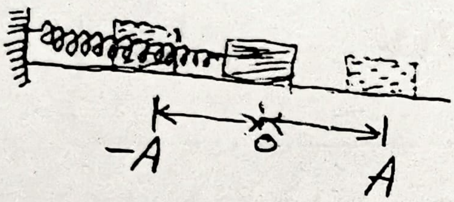
01064763583

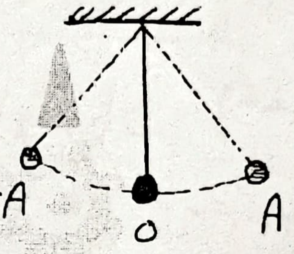
PHYSICS 1

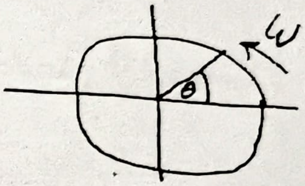
2021 - 2022 No.12

"CH. 5: Simple Harmonic Motion" SHM

* الحركة التوافقية البسيطة مثل حركة البندول البسيط وحركة الزنبرك.







* **Periodic time (T):** (الزمن الدوري)

هو الزمن اللازم لعمل دورة كاملة (sec)

* **Frequency (F = ν):** (التردد)

هو عدد الإهتزازات في الثانية الواحدة (HZ)

* **angular velocity (ω):** السرعة الزاوية

$$\boxed{\omega = 2\pi F} \ \text{rad/sec}$$

* **Phase angle (φ):** زاوية الطور

تكون لها قيمة إذا الجسم لم يبدأ من أقصى إزاحة

($\phi \rightarrow$ rad)

1 S.H.M equations:

(a) Displacement: الإزاحة

$$\boxed{X(t) = A\cos(\omega t + \phi)} \quad m$$

$A \equiv X_m$ ---- amplitude (أقصى إزاحة (السعة))

(b) Velocity: السرعة

$$V(t) = \frac{dX(t)}{dt}$$

$$\therefore \boxed{V(t) = -A\omega \sin(\omega t + \phi)} \quad m/s$$

$$\boxed{V_{max} = A\omega}$$

(c) acceleration: العجلة

$$a(t) = \frac{dV(t)}{dt}$$

$$\therefore \boxed{a(t) = -A\omega^2 \cos(\omega t + \phi)} \quad m/s^2$$

$$\boxed{a_{max} = A\omega^2} \quad , \quad \boxed{a = -\omega^2 X}$$

→ Sketch $X(t)$, $v(t)$ and $a(t)$?

let $\phi = \text{zero}$

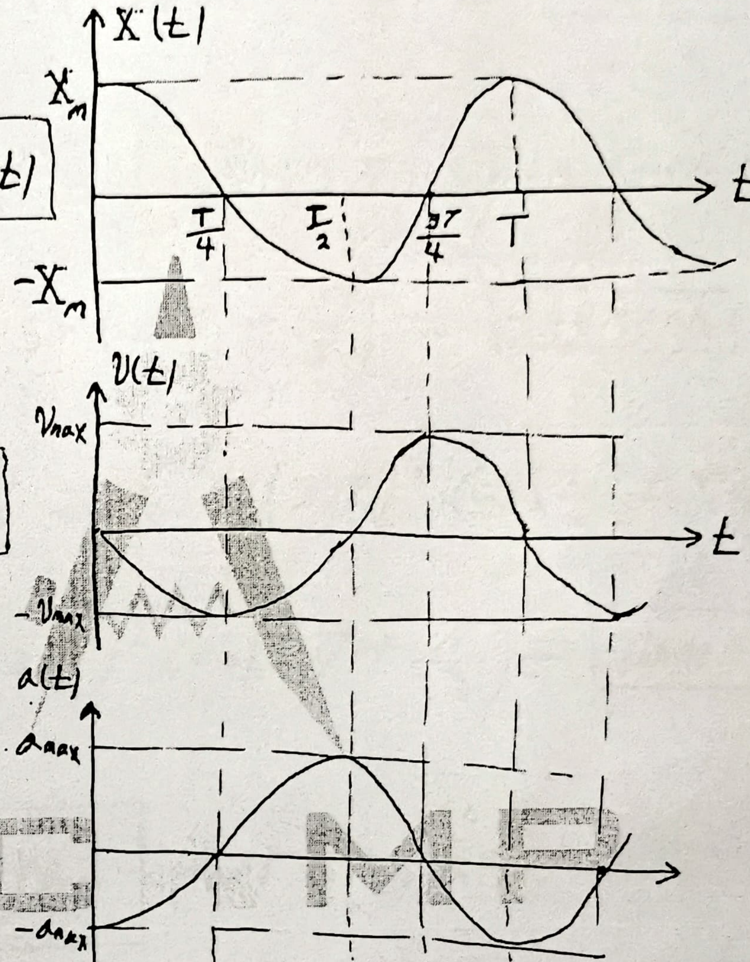
$$\boxed{X(t) = X_m \cos(\omega t)}$$

$$\boxed{v(t) = -v_{max} \sin(\omega t)}$$

$v_{max} = \omega X_m$

$$\boxed{a(t) = -a_m \cos(\omega t)}$$

$a_{max} = \omega^2 X_m$



هام جداً

EX: a body oscillates with SHM according equation

$$X(t) = 6 \cos\left(3\pi t + \frac{\pi}{3}\right) m \text{, Find}$$

(a) displacement at $t = 2$ sec.

(b) velocity at $t = 2$ sec

(c) acceleration at $t = 2$ sec

(d) the phase (e) Frequency

(f) periodic time

(a) $X(t) = 6\cos\left(3\pi t + \frac{\pi}{3}\right)$

Shift + Mode + 4 الآلة الحاسبة rad

$$X(2) = 6\cos\left(3\pi \times 2 + \frac{\pi}{3}\right) = \boxed{3\ m}$$

(b) $v(t) = \frac{dX(t)}{dt} = (-6)(3\pi)\sin\left(3\pi t + \frac{\pi}{3}\right)$

$$v(2) = -18\pi\sin\left(6\pi + \frac{\pi}{3}\right) = \boxed{-48.97\ m/s}$$

(c) $a(t) = \frac{dv(t)}{dt} = (-6)(3\pi)^2\cos\left(3\pi t + \frac{\pi}{3}\right)$

$$= (-6)(3\pi)^2\cos\left(6\pi + \frac{\pi}{3}\right) = \boxed{-266.48\ m^2/s}$$

(d) $\boxed{\phi = \frac{\pi}{3}\ rad}$ or 60°

(e) $\omega = 3\pi$

$$2\pi F = 3\pi$$

$$F = \frac{3\cancel{\pi}}{2\cancel{\pi}} = \boxed{1.5\ Hz}$$

(F) $T = \frac{1}{F} = \frac{1}{1.5} = \boxed{0.667\ sec}$

⇒ <u>Note</u>: $X_{max} = 6$ (amplitude)

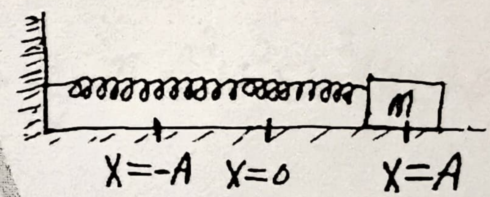
3 S.H.M examples

- (a) mass-spring system
- (b) simple pendulum
- (c) Physical Pendulum
- (d) Torsional Pendulum

(a) mass-spring system:

$\because F \propto X$

$\because F = -KX$ Hook's law



$\because F = ma$

$\therefore ma = -KX$

$\therefore m(-W^2X) = -KX$ "$a = -W^2X$"

$\therefore mW^2 = K$

$\therefore \boxed{W = \sqrt{\frac{K}{m}}}$ rad/s

$\therefore \boxed{f = \frac{1}{2\pi}\sqrt{\frac{K}{m}}}$ Hz "$W = 2\pi f$"

$\boxed{T = 2\pi\sqrt{\frac{m}{K}}}$ s "$T = \frac{1}{f}$"

⇨ Total Mechanical energy:

الطاقة الميكانيكية الكلية فى النظام

$$E = \text{Potential energy} + \text{Kinetic energy}$$

$$E = U + K$$

$$E = \frac{1}{2}KX^2 + \frac{1}{2}mV^2$$

$$E = \frac{1}{2}KA^2\cos^2(\omega t+\phi) + \frac{1}{2}mA^2\omega^2\sin^2(\omega t+\phi)$$

$$\because \omega = \sqrt{\frac{K}{m}} \Rightarrow \omega^2 = \frac{K}{m} \Rightarrow \omega^2 m = K$$

$$\therefore E = \frac{1}{2}KA^2\cos^2(\omega t+\phi) + \frac{1}{2}KA^2\sin^2(\omega t+\phi)$$

$$\therefore E = \frac{1}{2}KA^2\left[\underbrace{\cos^2(\omega t+\phi) + \sin^2(\omega t+\phi)}_{1}\right]$$

$$\therefore \boxed{E = \frac{1}{2}KA^2}$$

$$\Rightarrow \therefore \cancel{\frac{1}{2}}KA^2 = \cancel{\frac{1}{2}}KX^2 + \cancel{\frac{1}{2}}mV^2$$

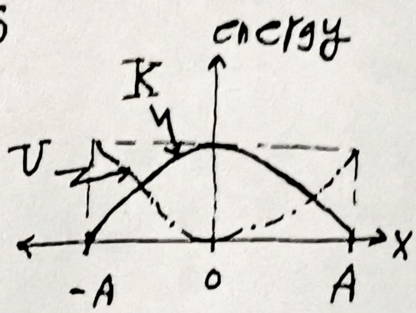
$$\therefore KA^2 - KX^2 = mV^2$$

$$K(A^2 - X^2) = mV^2$$

$$\therefore V^2 = \frac{K}{m}(A^2 - X^2)$$

$$\therefore V = \pm\sqrt{\frac{K}{m}}\sqrt{A^2 - X^2}$$

$$\therefore \boxed{V = \pm W\sqrt{A^2 - X^2}} \; m/s$$



*<u>Note</u>: $E = K + U = \frac{1}{2}KA^2$

*ملخص القوانين:

(1) $W = 2\pi f$; $f = \frac{1}{T}$

(2) $X(t) = A\cos(Wt + \phi)$

(3) $V(t) = -AW\sin(Wt + \phi)$; $V_{max} = AW$

(4) $a(t) = -AW^2\cos(Wt + \phi)$; $a_{max} = AW^2$

(5) $a = -W^2 X$

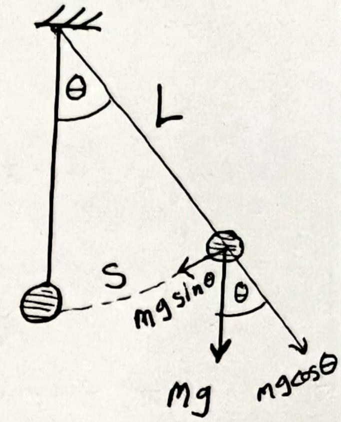
(6) $F = -KX$, $F = ma$

(7) $W = \sqrt{\frac{K}{m}}$, $f = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$, $T = 2\pi\sqrt{\frac{m}{K}}$

(8) $E = \frac{1}{2}KA^2$, $U = \frac{1}{2}KX^2$, $K = \frac{1}{2}mV^2$

(9) $V = \pm W\sqrt{A^2 - X^2}$

(b) Simple Pendulum:



$\because F = -mg\sin\theta$

$\because F = ma = -m\omega^2 S$

$\therefore \cancel{-m}\,\omega^2 S = \cancel{-m}\,g\sin\theta$

$\therefore \omega^2 S = g\sin\theta$ $\qquad \because \theta = \sin\theta$ θ very small

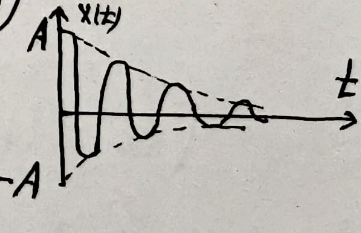
$\therefore \omega^2 \cancel{S} = g\left(\frac{\cancel{S}}{L}\right)$ $\qquad \because \theta = \frac{S}{L}$

$\therefore \boxed{\omega = \sqrt{g/L}}$, $\boxed{f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}}$, $\boxed{T = 2\pi\sqrt{L/g}}$

4 Damped oscillation: الإهتزاز المخمد

* في حالة الإهتزاز في وسط تؤدي قوة الإحتكاك (Friction Force) ومقاومة الهواء إلى إخماد الإهتزاز وفي هذه الحالة تكون:

$\rightarrow X(t) = e^{\frac{-bt}{2m}} A\cos(\omega t + \phi)$



$\omega = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}$

where

$\omega_0 \rightarrow$ natural frequency التردد بدون إخماد

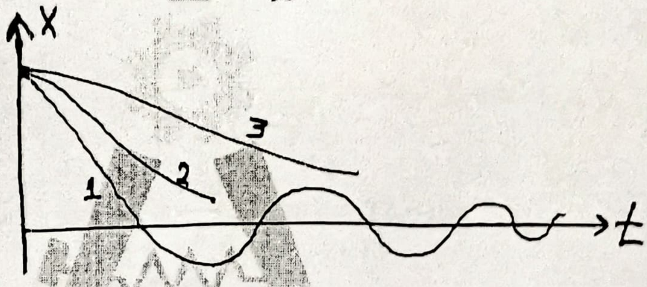
$b \rightarrow$ damping coefficient معامل الإخماد

* Types of damping: أنواع الإخماد

(1) Under-damped $\rightarrow \frac{b}{2m} < W_0$

(2) critical-damped $\rightarrow \frac{b}{2m} = W_0$

(3) over-damped $\rightarrow \frac{b}{2m} > W_0$



5 Forced oscillation:

* لأن الإخماد يؤدى إلى توقف الحركة فيجب أن نغذى النظام بقوة خارجية (driving force) لتستمر الحركة

$$F_{driving} = F_0 \sin(Wt)$$

$$A = \frac{F_0/m}{\sqrt{(W-W_0^2)^2 + \left(\frac{bW}{m}\right)^2}}$$

* **Resonance:** حالة الرنين $(W = W_0)$

$\rightarrow$ if the frequency of driving force equal the natural frequency, displacement is max.

Simple Harmonic Motion

Example (5.1):

An object oscillates with simple harmonic motion along the x axis. Its displacement from the origin varies with time according to the equation

$$X = (4.00\ m)\cos(\pi t + \frac{\pi}{4})$$

where t is in seconds and the angles in the parentheses are in radians. (a) Determine the amplitude, frequency, and period of the motion. (b) Determine the maximum speed and maximum acceleration of the object.

Solution:

$$(a)\ A = 4.00\ m,\ \omega = \pi \frac{rad}{s} \Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{(\pi \frac{rad}{s})} = 2\ s,\ f = \frac{1}{T} = \frac{1}{(2\ s)} = 0.5\ s$$

$$(b)\ v_{max} = \omega A = \left(\pi \frac{rad}{s}\right)(4\ m) = 12.6\ ms^{-1}$$

$$a_{max} = \omega^2 A = \left(\pi \frac{rad}{s}\right)^2 (4\ m) = 39.5\ ms^{-2}$$

Example (5.2):

A car with a mass of **1300 kg** is constructed so that its frame is supported by four springs. Each spring has a force constant of **20000 N/m**. If two people riding in the car have a combined mass of **160 kg**, find the frequency of vibration of the car after it is driven over a pothole in the road.

Solution:

We assume that the mass is evenly distributed. Thus, each spring supports one fourth of the load. The total mass is 1460 (1300+160) kg, and therefore each spring supports 365 kg.

Hence, the frequency of vibration

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{(2000\ Nm^{-1})}{(\frac{1300+160}{4})}} = 1.18\ s^{-1}$$

Example (5.3):

A block with a mass of **200 g** is connected to a light spring for which the force constant is **5 N/m** and is free to oscillate on a horizontal, frictionless surface. The block is displaced **5 cm** from equilibrium and released from rest (a) Find the period of its motion. (b) Determine the maximum speed of the block. (c) What is the maximum acceleration of the block?

Solution:

$$(a)\ T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{(200\times10^{-3}\ kg)}{(5\ Nm^{-1})}} = 1.26\ s$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{(1.26\ s)} = 5\ rad/s$$

$$(b)\ v_{max} = \omega A = \left(5\frac{rad}{s}\right)(5\times10^{-2}\ m) = 0.25\ ms^{-1}$$

$$(c)\ a_{max} = \omega^2 A = \left(5\frac{rad}{s}\right)^2(5\times10^{-2}\ m) = 1.25\ ms^{-2}$$

Example (5.4):

Using a small pendulum of length **0.171 m**, a geophysicist counts **72** complete swings in a time of **60 s**. What is the value of g in this location?

Solution:

Calculate the period by dividing the total elapsed time by the number of complete oscillations:

$$T = \frac{total\ time}{total\ number\ of\ oscillations} = \frac{(60\ s)}{(72\ osscilations)} = 8.33\ s$$

$$T = 2\pi\sqrt{\frac{L}{g}} \Rightarrow g = \frac{4\pi^2 L}{T^2} = \frac{4\pi^2(0.171\ m)}{(8.33\ s)^2} = 9.711\ ms^{-2}$$

Example 5.5: What is the acceleration due to gravity in a region where a simple pendulum having a length 75.000 cm has a period of 1.7357 s?

Strategy

We are asked to find g given the period T and the length L of a pendulum. We can solve $T = 2\pi\sqrt{\frac{L}{g}}$ for g, assuming only that the angle of deflection is less than 15°.

Solution

1. Square $T = 2\pi\sqrt{\frac{L}{g}}$ and solve for g:

$$g = 4\pi^2\frac{L}{T^2}.$$

2. Substitute known values into the new equation:

$$g = 4\pi^2\frac{0.75000\ \text{m}}{(1.7357\ \text{s})^2}.$$

3. Calculate to find g:

$$g = 9.8281\ \text{m/s}^2.$$

Example 5.6: Suppose that a car is 900 kg and has a suspension system that has a force constant $k = 6.53\times10^4$ N/m. The car hits a bump and bounces with an amplitude of 0.100 m. What is its maximum vertical velocity if you assume no damping occurs?

Strategy

We can use the expression for v_{max} given in $v_{max} = \sqrt{\frac{k}{m}}X$ to determine the maximum vertical velocity. The variables m and k are given in the problem statement, and the maximum displacement X is 0.100 m.

Solution

1. Identify known.
2. Substitute known values into $v_{max} = \sqrt{\frac{k}{m}}X$:

$$v_{max} = \sqrt{\frac{6.53\times10^4\ \text{N/m}}{900\ \text{kg}}}(0.100\ \text{m}).$$

3. Calculate to find $v_{max} = 0.852$ m/s.

PROBLEMS H.W

1. A simple harmonic oscillator takes $12\ s$ to undergo five complete vibrations. Find (a) the period of its motion, (b) the frequency in hertz, and (c) the angular frequency in radians per second.

69

2. A $0.6\ kg$ block attached to a spring with force constant $130\ N/m$ is free to move on a frictionless, horizontal surface. The block is released from rest after the spring is stretched $0.13\ m$. At that instant, find (a) the force on the block and (b) its acceleration.

3. When a $4.25\ kg$ object is placed on top of a vertical spring, the spring compresses a distance of $2.62\ cm$. What is the force constant of the spring?

4. At an outdoor market, a bunch of bananas is set into oscillatory motion with amplitude of $20\ cm$ on a spring with a force constant of $16\ N/m$. It is observed that the maximum speed of the bunch of bananas is $40\ cm/s$. What is the weight of the bananas in Newton's?

5. The period of motion of an object spring system is $0.223\ s$ when a $35.4\ g$ object is attached to the spring. What is the force constant of the spring?

6. A man enters a tall tower, needing to know its height. He notes that a long pendulum extends from the ceiling almost to the floor and that its period is $15.5\ s$. (a) How tall is the tower? (b) If this pendulum is taken to the Moon, where the free-fall acceleration is $1.67\ m/s^2$, what is the period there?

7. A simple pendulum makes 120 complete oscillations in $3\ min$ at a location where $g = 9.8\ m/s^2$. Find (a) the period of the pendulum and (b) its length.

8. A "seconds" pendulum is one that moves through its equilibrium position once each second. (The period of the pendulum is $2\ s$.) The length of a seconds pendulum is $0.9927\ m$ at Tokyo and $0.9942\ m$ at Cambridge, England. What is the ratio of the free-fall accelerations at these two locations?