شركة تدريب هندسي

# E.CAMP

الطريق الدائري بجوار المدرسة المعمارية

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## PHYSICS 1

2021 - 2022 No.11

# "surface - tension"

التوتر
السطحى

* هى ظاهرة تتمكن من خلالها السوائل من حمل أوزان صغيرة مثل حشرة أو إبرة.

## * Definitions:

### (1) coefficient of surface tension ($\gamma$):

"معامل التوتر السطحى"

→ Is the force per unit length acting normal to a line on the surface of a liquid.

$$\boxed{\gamma = \frac{F}{L}}$$ N/m or J/m²

* لاحظ:
عند حل مسائل التوتر السطحى تكون صورة القانون

$$\boxed{\gamma = \frac{F}{2L}}$$

لأن قوة التوتر السطحى تؤثر على جانبى الجسم المحمول فوق السائل.

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### (2) Cohesion force: قوى التماسك بين جزيئات السائل

→ Is the attractive force between like molecules.

## (3) Adhesion force:

قوى التلاصق بين السائل وجدار الأنبوبة

→ Is the attractive force between unlike molecules.

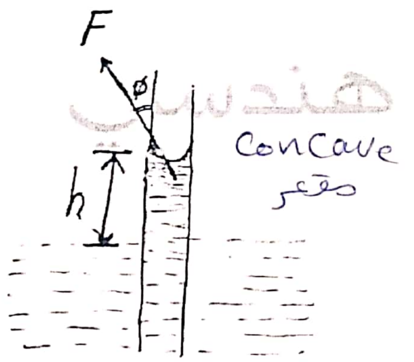
## (4) The angle of contact (∅):

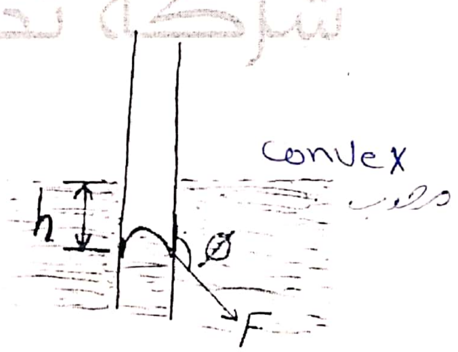
زاوية التلامس

*هى الزاوية بين المماس لسطح السائل و جدار الأنبوبة عند نقطة التقاطع.

* Is the angle between the line tangent to the surface of the liquid and the solid surface at the intersection point.






Water (∅ < 90°)

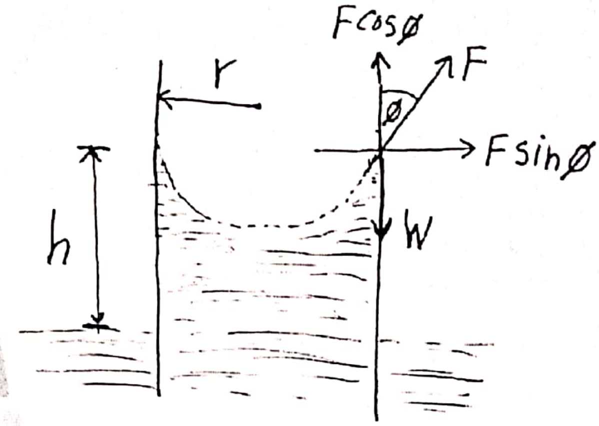
mercury (∅ > 90°)

Cohesion < adhesion

Cohesion > adhesion

## 2 Capillarity action:

→ at equilibrium




$$W = F\cos\phi$$

$$\therefore mg = \gamma L \cos\phi$$

$$\therefore \rho V g = \gamma (2\pi r)\cos\phi$$

"$L \equiv$ المحيط"

$$\therefore \rho(\pi r^2 h) g = \gamma(2\pi r)\cos\phi$$

$$\therefore h = \frac{\gamma(2\pi r)\cos\phi}{\rho \pi r^2 g} = \boxed{\frac{2\gamma\cos\phi}{\rho r g}}$$

حفظ

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EX: Find the hieght for which the water will rise in a capillary tube of radius $r = 5\times10^{-5}$ m, assum that the angle of contact is small enough to be considered as zero and $\gamma = 0.073$ N/m

answer

$$h = \frac{2\gamma\cos\phi}{\rho g r} = \frac{2\times0.073\cos(0)}{1000\times9.8\times(5\times10^{-5})} = \boxed{0.30\,m}$$

EX.2: suppose ethyl alcohol rises 0.25m in a thin tube. الكحول الإيثلي

→ Estimate the radius of the tube if the contact angle approximately zero.

→ assume that $\rho = 0.806 \times 10^{3}\ kg/m^{3}$

$\gamma = 0.022\ N/m$

"answer"

$$\therefore h = \frac{2\gamma \cos\phi}{\rho g r}$$

$$\therefore r = \frac{2\gamma \cos\phi}{\rho g h} = \frac{2 \times 0.022 \times \cos 0^{\circ}}{(0.806 \times 10^{3}) \times 9.8 \times 0.25}$$

$$= \boxed{2.23 \times 10^{-5}\ m}$$

"summary"

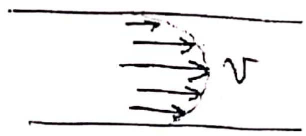
(1) $\gamma = \frac{F}{L}$ or $\gamma = \frac{F}{2L}$

(2) $h = \frac{2\gamma \cos\phi}{\rho g r}$ "capillarity action"

# "Viscosity" اللزوجة

**1** **Viscosity:** is the internal friction of a fluid. هي الإحتكاك داخل الموائع

$$\boxed{F = \eta \frac{A v}{d}}$$



F --- Friction force
$\eta$ -- viscosity coefficient
A --- area of the fluid layer طبقة
$v$ -- velocity of layer
d --- distance between layers.

هام ⇒ **Units of $\eta$:**

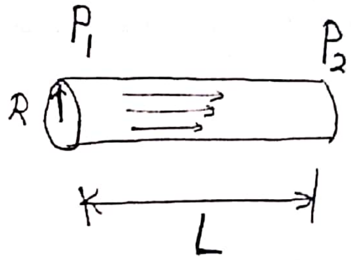
* in SI $N \cdot s/m^2$
* in French 1 Poise = 10 $N \cdot s/m^2$
* For small values centipoise (1cp) = $10^{-2}$ P

**2** **Poiseuille's law:** قانون بوازيل

← قانون يصف العلاقة بين المعدل الحجمي لتدفق سائل خلال أنبوبة ومعامل اللزوجة.

$$\boxed{\text{Rate of flow} = \frac{\Delta V}{\Delta t} = \frac{\pi R^4 (P_1 - P_2)}{8 \eta L}}$$

$\frac{\Delta V}{\Delta t}$ -- Volumetric flow rate




يستقبل

<u>EX</u>: A Patient receives a blood through a needle (سرنجة) of radius 0.2 mm and length 2 cm, the density of the blood is 1050 $kg/m^3$, the bottle supplying blood is 0.5 m above the patient's arm, What is the rate of flow through the needle? $\eta = 2.7 \times 10^{-3}$ $N \cdot s/m^2$



<u>answer</u>-

$$P_1 - P_2 = \rho g h = (1050) \times (9.8) \times (0.5)$$
$$= 5.15 \times 10^3 \; Pa$$

$$* \; \frac{\Delta V}{\Delta t} = \frac{\pi R^4 (P_1 - P_2)}{8 \eta L} = \frac{\pi \times (0.2 \times 10^{-3})^4 \times (5.15 \times 10^3)}{8 \times (2.7 \times 10^{-3}) \times (2 \times 10^{-2})}$$

$$= \boxed{6 \times 10^{-8} \; m^3/s}$$

EX.2: A pipe carrying water from a tank 20 m tall must cross $3\times10^{2}$ km to reach a town, find the radius of the pipe so that the volumetric flow rate at least $0.05\ m^3/s$

$\eta_{water} = 1\times10^{-3}\ N.s/m^2$

answer

$$P_1 - P_2 = \rho g h = 1000\times9.8\times20 = 19.6\times10^{4}\ Pa$$

$$\therefore \frac{\Delta V}{\Delta t} = \frac{\pi R^4 (P_1 - P_2)}{8\eta L}$$

$$\therefore 0.05 = \frac{\pi R^4 \times (19.6\times10^{4})}{8\times(1\times10^{-3})\times(3\times10^{2})}$$

$$\therefore \boxed{R = 0.118\ m}$$

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## 3 Reynolds number: رقم رينولد

هو رقم يحدد طبيعة السريان

$$\boxed{RN = \frac{\rho v d}{\eta}}$$

$\rho \rightarrow$ density
$v \rightarrow$ Velocity
$d \rightarrow$ Diameter

$$RN = \frac{\text{Inertia Force}}{\text{Viscosity Force}}$$

Inertia Force ← قوة القصور الذاتي

Viscosity Force ← قوة اللزوجة

$$= \frac{\text{Dynamic Pressure}}{\text{shear stress}} = \frac{\rho V^2}{\eta V/d}$$

$$RN = \boxed{\frac{\rho V d}{\eta}}$$

Dimensionless.

ليس له أبعاد

Note: (1) $RN < 2000$ streamline flow

(2) $2000 < RN < 3000$ Unstable flow

(3) $R_N > 3000$ turbulent flow

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EX: Determine the speed at which blood flowing through artery (شريان) of diameter 0.2 cm will become turbulent, if

$RN = 3000$, $\eta = 2.7 \times 10^{-3}$, $\rho = 1.05 \times 10^{3}\ kg/m^3$

answer

$$RN = \frac{\rho V d}{\eta}$$

$$\therefore V = \frac{(RN)\eta}{\rho d} = \frac{3000 \times (2.7 \times 10^{-3})}{1.05 \times 10^{3} \times (0.2 \times 10^{-2})} = \boxed{3.9\ m/s}$$

## $\boxed{4}$ Stokes's Law: قانون ستوكس

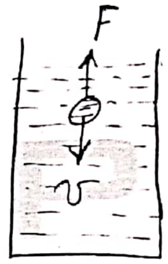
$$\boxed{F = 6\pi\eta v r}$$

Viscosity force قوة اللزوجة

**EX:** Using dimensional analysis derive the relation between viscous force acting on a solid sphere moving through a liquid with Velocity (r).

**answer**

$$F \propto \eta^{x} v^{y} r^{z}$$




$$\therefore F = K \eta^{x} v^{y} r^{z}$$

$$MLT^{-2} = (ML^{-1}T^{-1})^{x} (LT^{-1})^{y} (L)^{z}$$

$$\therefore M^{1} L^{1} T^{-2} = M^{x} L^{-x+y+z} T^{-x-y}$$

$$\therefore \boxed{x = 1}$$

$$-2 = -x - y \Rightarrow y = -x + 2 = -1 + 2 = \boxed{1}$$

$$1 = -x + y + z \Rightarrow \boxed{z = 1}$$

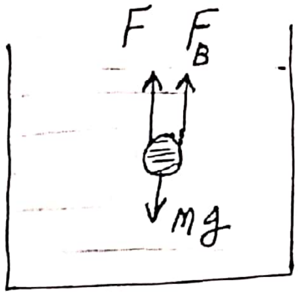
$$\therefore F = K\eta v r \Rightarrow \boxed{F = 6\pi\eta v r}$$

## 5 Terminal Velocity:

* Derive an expression for the terminal velocity ($V_t$) for a soild sphere will density ($\rho$) and radius ($r$), moving through a viscous liquid of density ($\rho_F$).

**answer**

→ Because the sphere has constant velocity




$$F + F_B = mg$$

$$6\pi \eta V_t r + \rho_F V g = (\rho V) g$$

$$\therefore V_t = \frac{\rho V g - \rho_F V g}{6 \pi \eta r} \qquad \because V = \frac{4}{3}\pi r^3$$

$$\therefore V_t = \frac{(\rho - \rho_F)\frac{4}{3}\pi r^3 g}{6 \pi \eta r}$$

$$\therefore \boxed{V_t = \frac{2(\rho - \rho_F) r^2 g}{9 \eta}} \quad m/s$$

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**Example (4.4):**

A large storage tank, open at the top and filled with water, develops a small hole in its side at a point **16 m** below the water level. If the rate of flow from the leak is equal to **2.5 × 10$^3$ m$^3$/min**, determine **(a)** the speed at which the water leaves the hole and **(b)** the diameter of the hole.

**Solution:**

$$v_2 = \sqrt{2gh} = \sqrt{2(9.8\,ms^{-2})(16\,m)} = 17.7\,m/s$$

$$Flow\,rate = Av \Rightarrow \left(\frac{2.5\times10^3}{60}\,m^3/s\right) = \left(\pi\left(\frac{d}{2}\right)^2\right)(17.7\,m/s) \Rightarrow d = 1.73\times10^{-3}\,m$$

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**Example (4.5):**

Water at **20 ˚C** flows through a pipe of radius **1 cm**. if the maximum velocity **10 cm/s**. Find the pressure drop a long a **2 m** section of pipe due to viscosity. $\Delta P = \frac{8\eta L v}{R^2}$ , $v = \frac{v_{max}}{2}$

**Solution:**

$$\Delta P = \frac{4\eta L v_{max}}{R^2} = \frac{4\,(1.005\times10^{-2}\,kg\,m/s)(2\,m)(10\times10^{-2}\,m/s)}{(1\times10^{-2}\,m)^2} = 80.4\,Pa$$

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**Example (4.5):**

An aluminum ball of radius **1 cm** falls through water at **20 ˚C**. What is the terminal velocity, assuming laminar flow and including buoyancy?

**Solution:**

$$v_T = \frac{2}{9}\frac{r^2 g\,(\rho_s - \rho_L)}{\eta} = \frac{2}{9}\frac{(1\times10^{-2}\,m)^2(9.81\,m/s^2)\,(2700\,kg/m^3 - 1000\,kg/m^3)}{(10^{-3}\,P.s)} = 37.1\,m/s$$

H. W

# PROBLEMS

**1.** A garden hose pipe of inner radius **1 cm** carries water at **2 m/s**. the nozzle at the end has radius **0.2 cm**. how fast dose the water moves through the nozzle?

**2.** A horizontal segment of pipe tapers from a cross-section area **50 cm$^2$** to **0.5 cm$^2$**. The pressure at the larger end of the pipe is **1.2×10$^5$ Pa** and the speed is **0.04 m/s**. what is the pressure at the narrow end of the segment?

**3.** Water enters a house through a pipe **2 cm** in inside diameter, at an absolute pressure of **4×10$^5$ Pa**. The pipe leading to the second floor bathroom **5 m** above is **1 cm** in diameter. When the flow velocity at the inlet pipe is **4 m/s**, find the flow velocity and pressure in the bathroom.

**4.** A sniper fires rifle bullet into a gasoline tank, making a hole **50 m** below the surface of the gasoline. The tank was sealed and is under **3 atm** absolute pressures. The stored gasoline has a density of **660 kg/m$^3$**. At what speed does the gasoline begin to shoot out of the hole?

**5.** A nozzle is connected to a horizontal hose. The nozzle shoots out water moving at **25 m/s**. what is the gauge pressure of the water in the hose? Assuming the diameter of the nozzle is much smaller than the inner diameter of the hose.

**6.** What is the pressure difference required to make water flow through a tube of inner radius **2 mm** and length **0.2 m** at speed of **6 cm/s**? If the viscosity coefficient of water at **20 °C** is **1.005 cp**, calculate the total volume of water flow per unit time?

**7.** Oil at **20 °C** flows through a tube of inner radius **20 cm** with coefficient of viscosity $\eta$ = **9.86 poise**. if the pressure drop along a **4 m** section of pipe is **1200 Pa**. Find the velocity of the oil flow at radius **10 cm**.

**8.** A sphere of radius **1 cm** is dropped into a glass cylinder filled with a viscous liquid. The mass of the sphere is **12 g** and the density of the liquid is **1200 kg/m$^3$**. The sphere reaches a terminal speed of **0.15 m/s**. what is the viscosity of the liquid?

**9. (a)** With what terminal velocity will an air bubble **1 mm** in diameter rise in a liquid of viscosity **150 cp** and density **0.9 g/cm$^3$**. **(b)** What is the terminal velocity of the same bubble in water?