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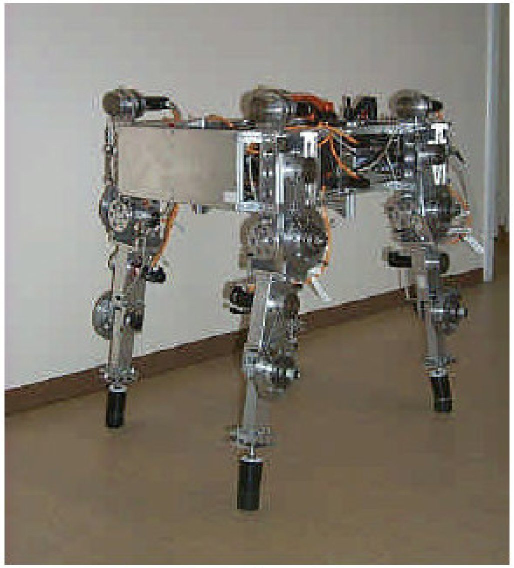
Locomotion



- **Locomotion** is the process of causing an autonomous robot to move
    - In order to produce motion, forces must be applied to the vehicle

Wheeled Mobile Robots (WMR)

Yamabico

MagellanPro

Sojourner

ATRV-2

Hilare 2-Bis

Koy

Wheeled Mobile Robots

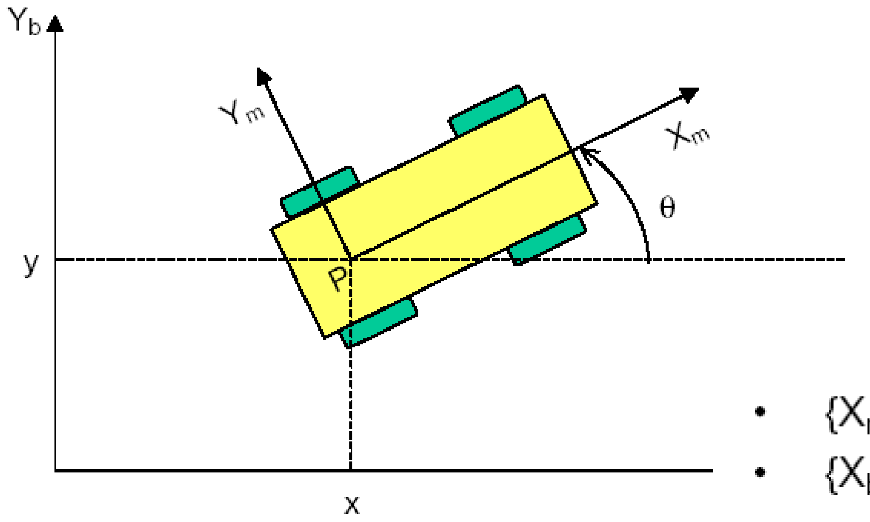
- **Combination of various physical (hardware) and computational (software) components**

- **A collection of subsystems:**
  - **Locomotion:** how the robot moves through its environment
  - **Sensing:** how the robot measures properties of itself and its environment
  - **Control:** how the robot generate physical actions
  - **Reasoning:** how the robot maps measurements into actions
  - **Communication:** how the robots communicate with each other or with an outside operator

Wheeled Mobile Robots

- **Locomotion —** the process of causing an robot to move.
  - In order to produce motion, forces must be applied to the robot
  - Motor output, payload
- **Kinematics** – study of the mathematics of motion without considering the forces that affect the motion.
  - Deals with the geometric relationships that govern the system
  - Deals with the relationship between control parameters and the behavior of a system.
- **Dynamics** – study of motion in which these forces are modeled
  - Deals with the relationship between force and motions.

Notation



Posture: position(x, y) and orientation θ

- $\{X_m, Y_m\}$ – moving frame
- $\{X_b, Y_b\}$ – base frame

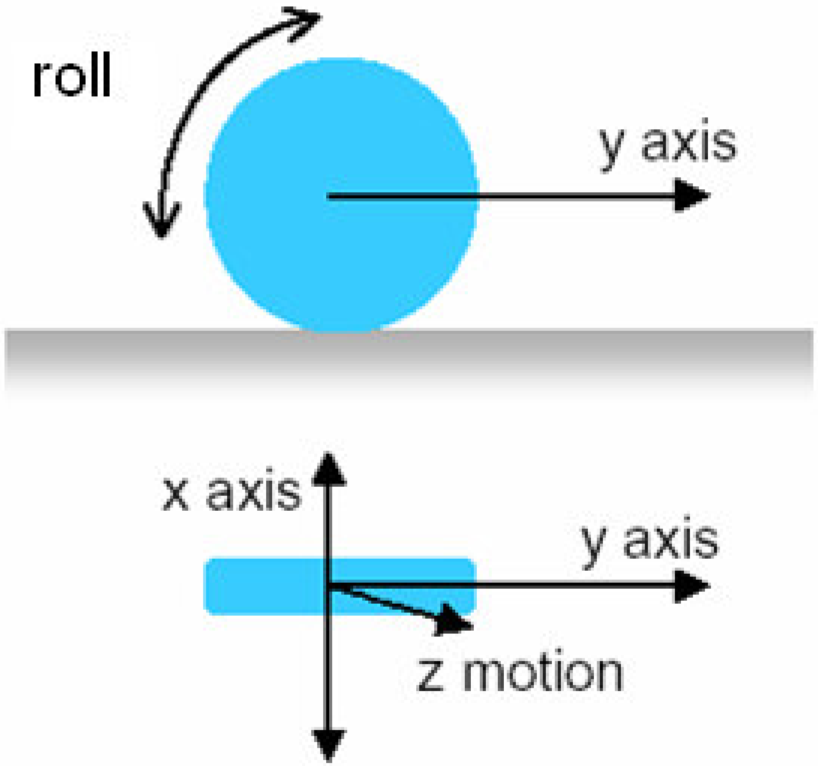
$$q = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

robot posture in base frame

$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotation matrix expressing the orientation of the base frame with respect to the moving frame

Wheels

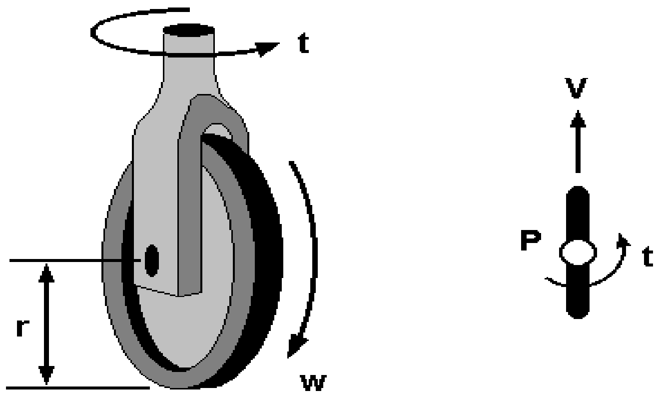


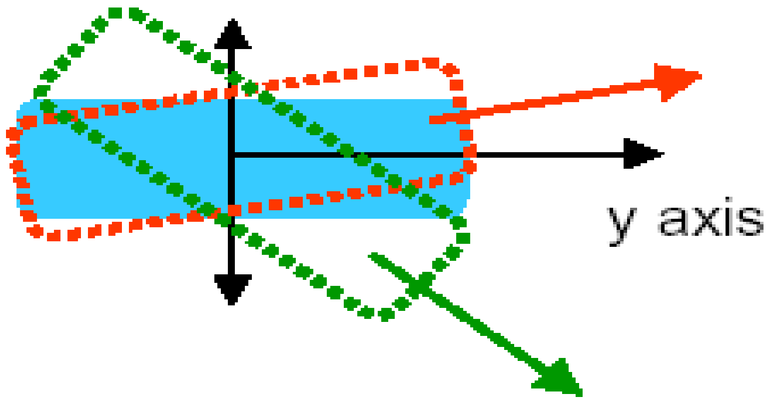
Rolling motion

Lateral slip

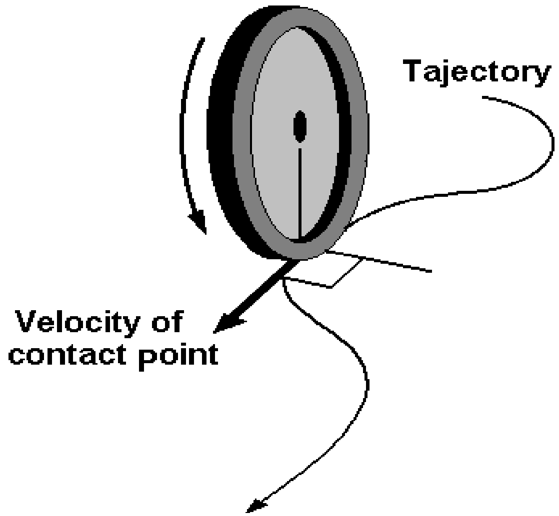
Steered Wheel

- **Steered wheel**
  - The orientation of the rotation axis can be controlled





Idealized Rolling Wheel



Non-slipping and pure rolling

- **Assumptions**

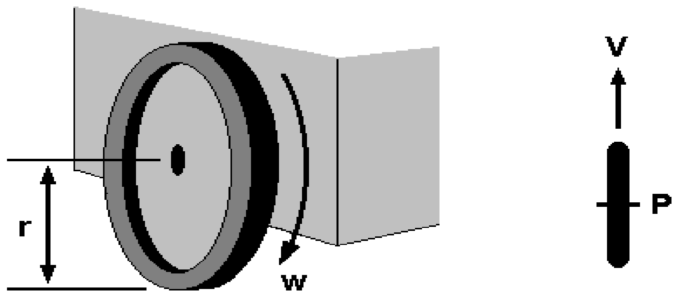
1. The robot is built from rigid mechanisms.
2. No slip occurs in the orthogonal direction of rolling (non-slipping).
3. No translational slip occurs between the wheel and the floor (pure rolling).
4. The robot contains at most one steering link per wheel.
5. All steering axes are perpendicular to the floor.

Robot wheel parameters

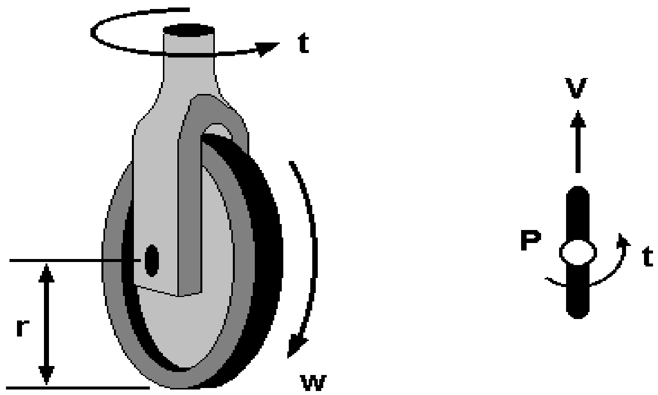
- For low velocities, rolling is a reasonable wheel model.
  - This is the model that will be considered in the kinematics models of WMR
- Wheel parameters:
  - r = wheel radius
  - v = wheel linear velocity
  - w = wheel angular velocity
  - t = steering velocity

Wheel Types

Fixed wheel

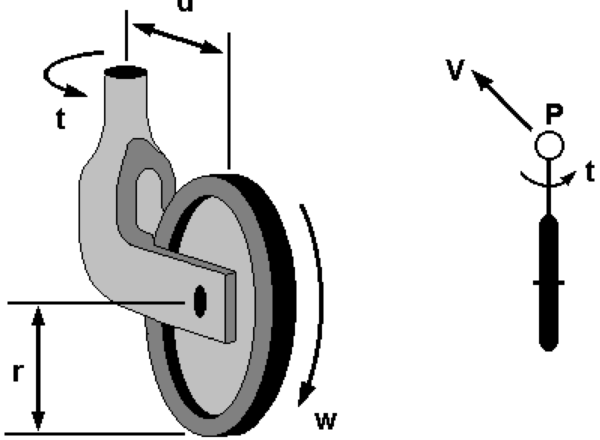


Centered orientable wheel

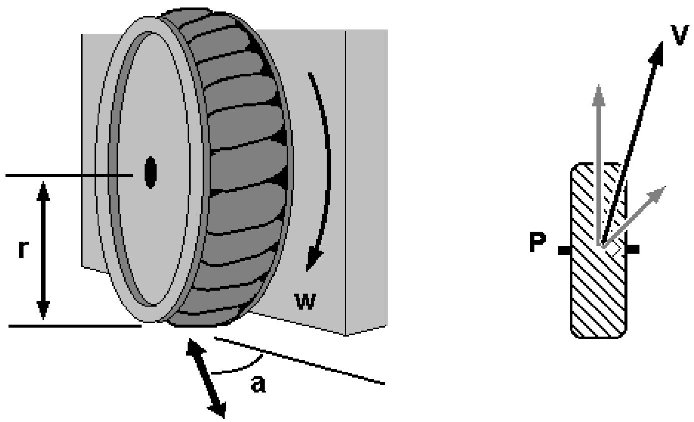


Off-centered orientable wheel

(Castor wheel)



Swedish wheel:omnidirectional property



Fixed wheel

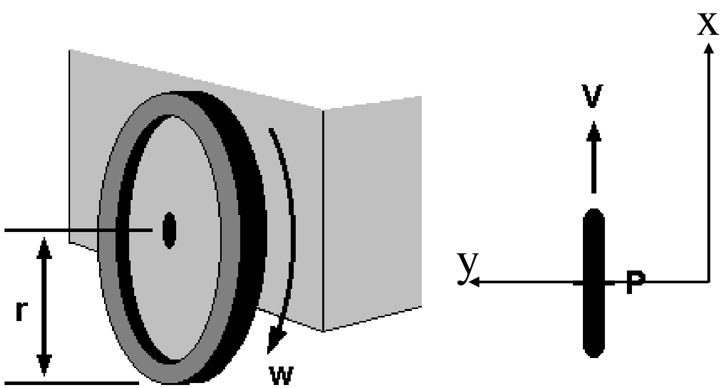
– Velocity of point **P**

$$V = (r \times w)a_x$$

where, $\mathbf{a_x}$: A unit vector to X axis

– Restriction to the robot mobility

Point **P** cannot move to the direction perpendicular to plane of the wheel.



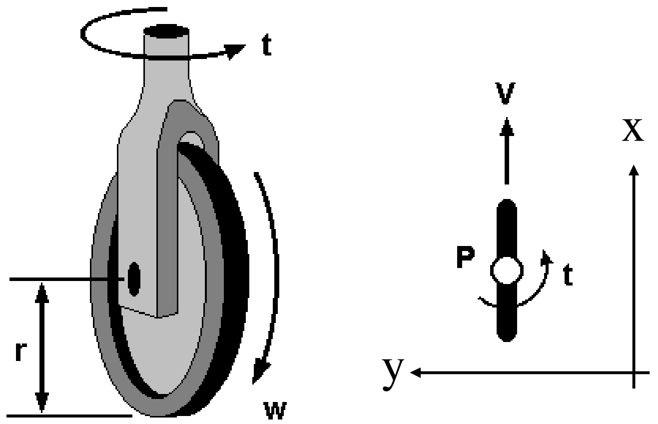
Centered orientable wheels

- Velocity of point **P**

$$V = (r \times w)a_x$$

where, a_x : A unit vector of x axis
a_y : A unit vector of y axis

- Restriction to the robot mobility



Off-Centered Orientable Wheels

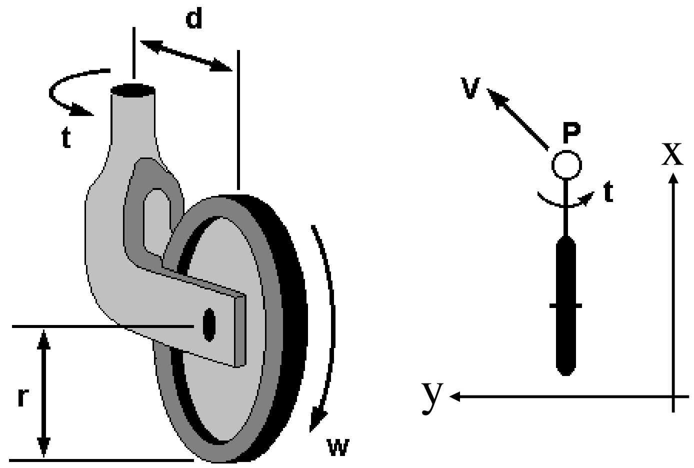
– Velocity of point **P**

$$v = (r \times w)a_x + (d \times t)a_y$$

where, a_x : A unit vector of x axis

a_y : A unit vector of y axis

– Restriction to the robot mobility



Swedish wheel

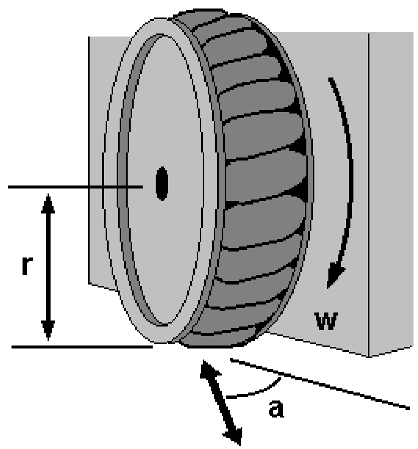
– Velocity of point **P**

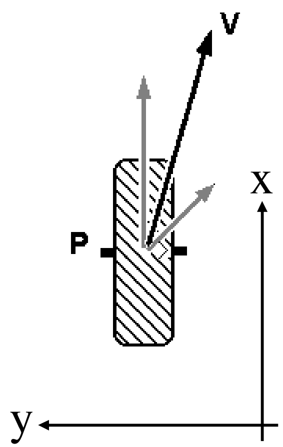
$$v = (r \times w)a_x + Ua_s$$

where, a_x : A unit vector of x axis

a_s : A unit vector to the motion of roller

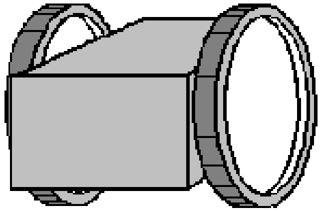
– Omnidirectional property





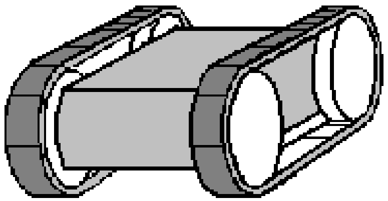
Examples of WMR

Example



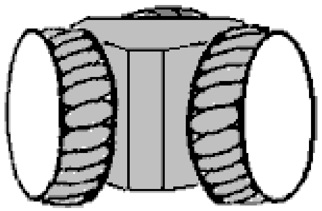
Bi-wheel type robot

- Smooth motion
- Risk of slipping
- Some times use roller-ball to make balance



Caterpillar type robot

- Exact straight motion
- Robust to slipping
- Inexact modeling of turning

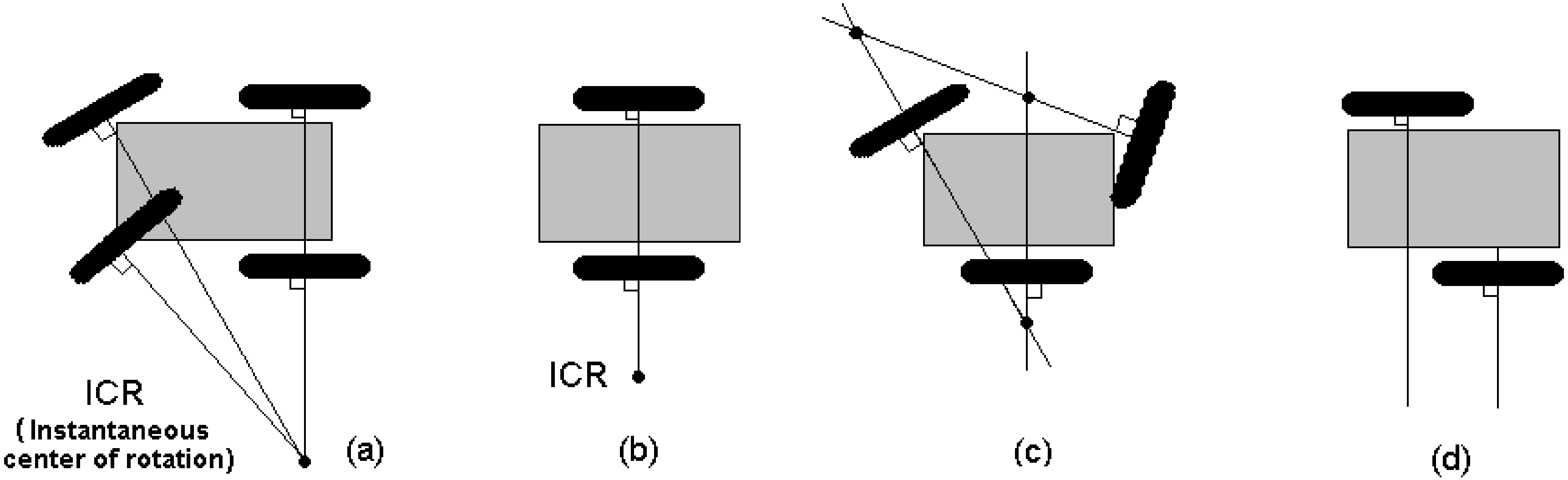


Omnidirectional robot

- Free motion
- Complex structure
- Weakness of the frame

Mobile Robot Locomotion

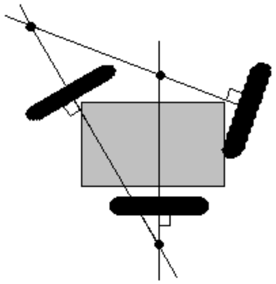
- Instantaneous center of rotation (ICR) or Instantaneous center of curvature (ICC)
  - A cross point of all axes of the wheels



Degree of Mobility

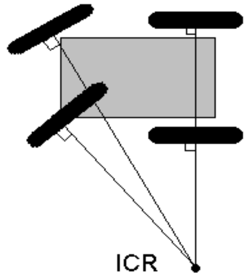
- **Degree of mobility**

  The degree of freedom of the robot motion



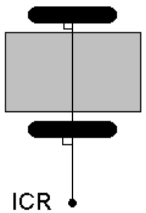
Cannot move anywhere (No ICR)

- Degree of mobility : 0



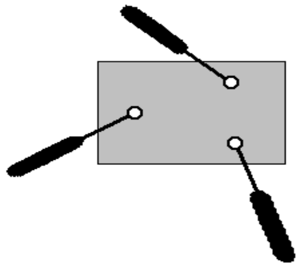
Fixed arc motion (Only one ICR)

- Degree of mobility : 1



Variable arc motion (line of ICRs)

- Degree of mobility : 2



Fully free motion

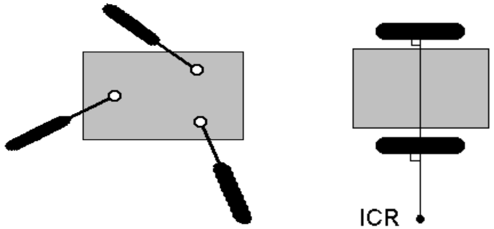
(ICR can be located at any position)

- Degree of mobility : 3

Degree of Steerability

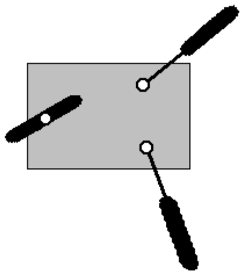
- **Degree of steerability**

The number of centered orientable wheels that can be steered independently in order to steer the robot

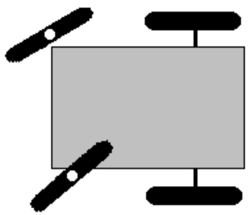


No centered orientable wheels

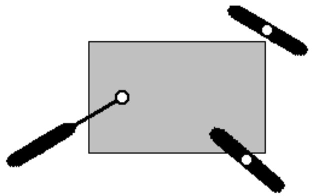
- Degree of steerability : 0



One centered orientable wheel



Two mutually dependent centered orientable wheels



Two mutually independent centered orientable wheels

- Degree of steerability : 1
- Degree of steerability : 2

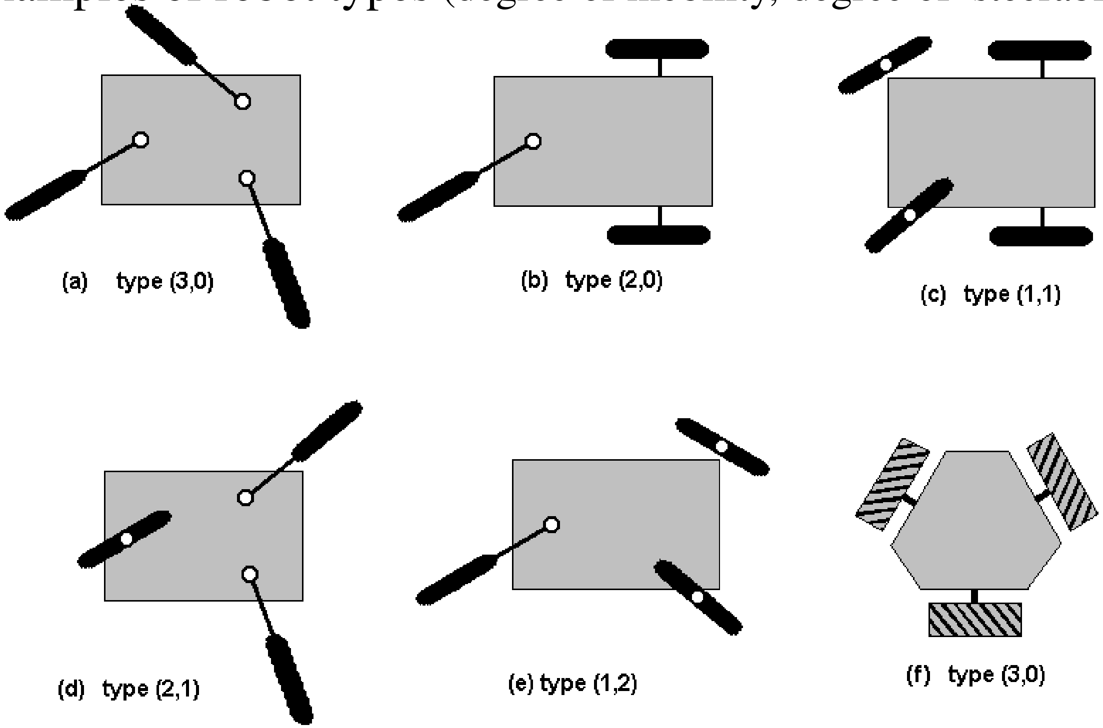
Degree of Maneuverability

- The overall degrees of freedom that a robot can manipulate:

$$\delta_M = \delta_m + \delta_s$$

Degree of Mobility	3	2	2	1	1
Degree of Steerability	0	0	1	1	2

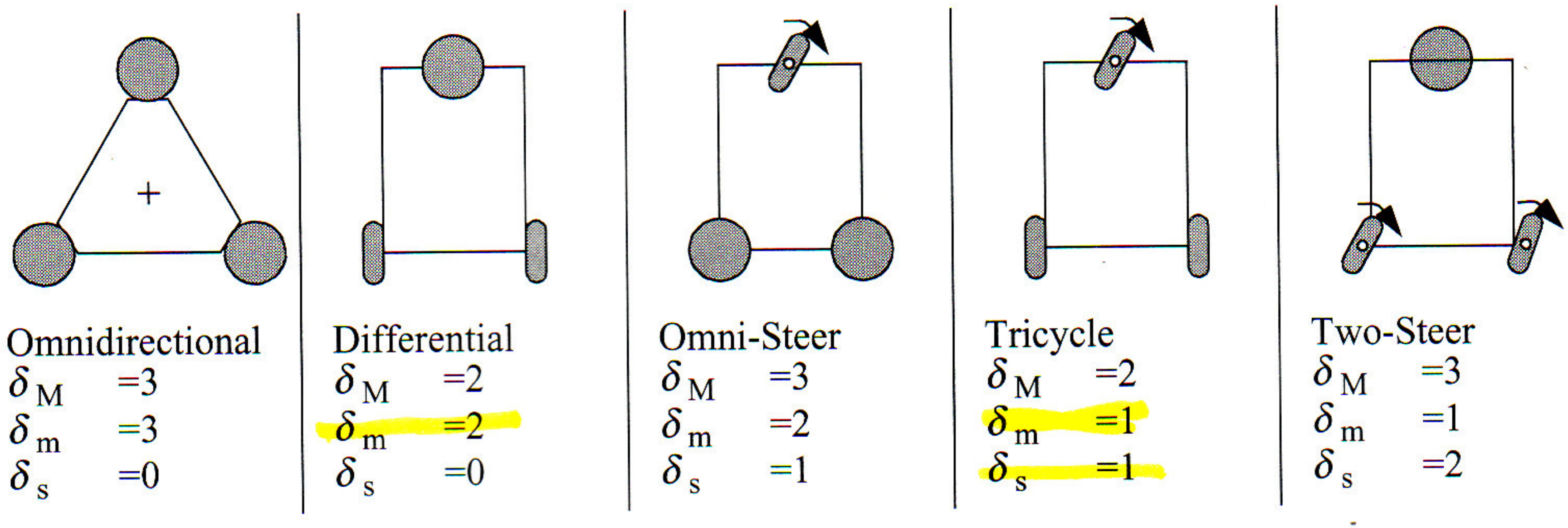
- Examples of robot types (degree of mobility, degree of steerability)



(a) type (3,0)
(b) type (2,0)
(c) type (1,1)
(d) type (2,1)
(e) type (1,2)
(f) type (3,0)

Degree of Maneuverability

$$\delta_M = \delta_m + \delta_s$$

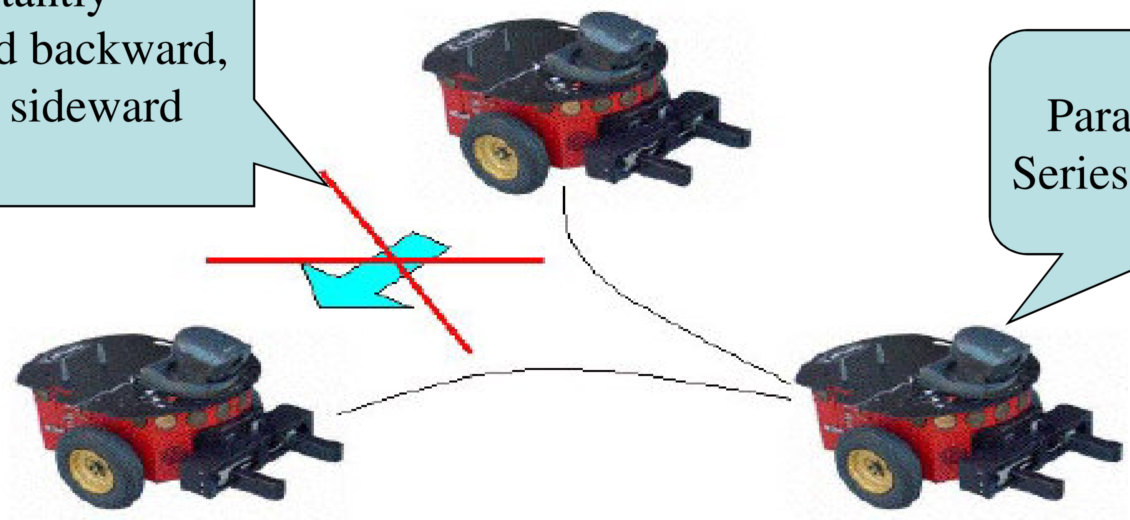


Non-holonomic constraint

A non-holonomic constraint is a constraint on the feasible **velocities** of a body

So what does that mean?

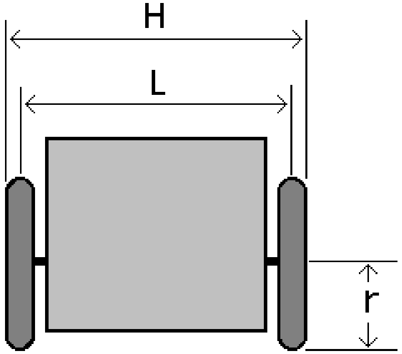
Your robot can move in some directions (forward and backward), but not others (sideward).

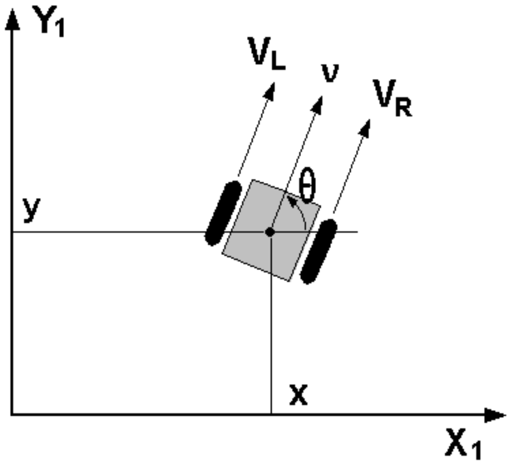


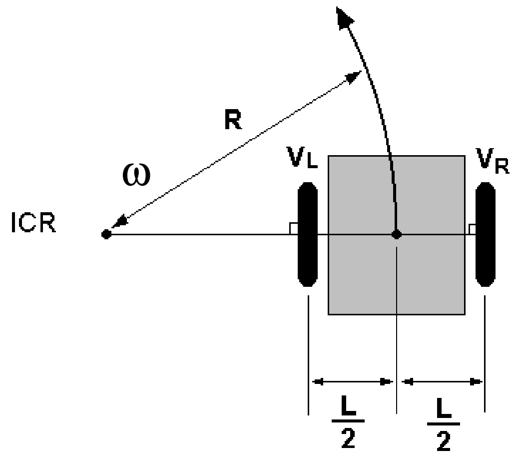
Mobile Robot Locomotion

- Differential Drive
  - two driving wheels (plus roller-ball for balance)
  - simplest drive mechanism
  - sensitive to the relative velocity of the two wheels (small error result in different trajectories, not just speed)
- Steered wheels (tricycle, bicycles, wagon)
  - Steering wheel + rear wheels
  - cannot turn ±90º
  - limited radius of curvature
- Synchronous Drive
- Omni-directional
- Car Drive (Ackerman Steering)

Differential Drive







- Posture of the robot

$$P = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix}$$

(x,y) : Position of the robot

θ : Orientation of the robot

- Control input

$$U = \begin{pmatrix} v \\ w \end{pmatrix}$$

v : Linear velocity of the **robot**

w : Angular velocity of the **robot**

(notice: not for each wheel)

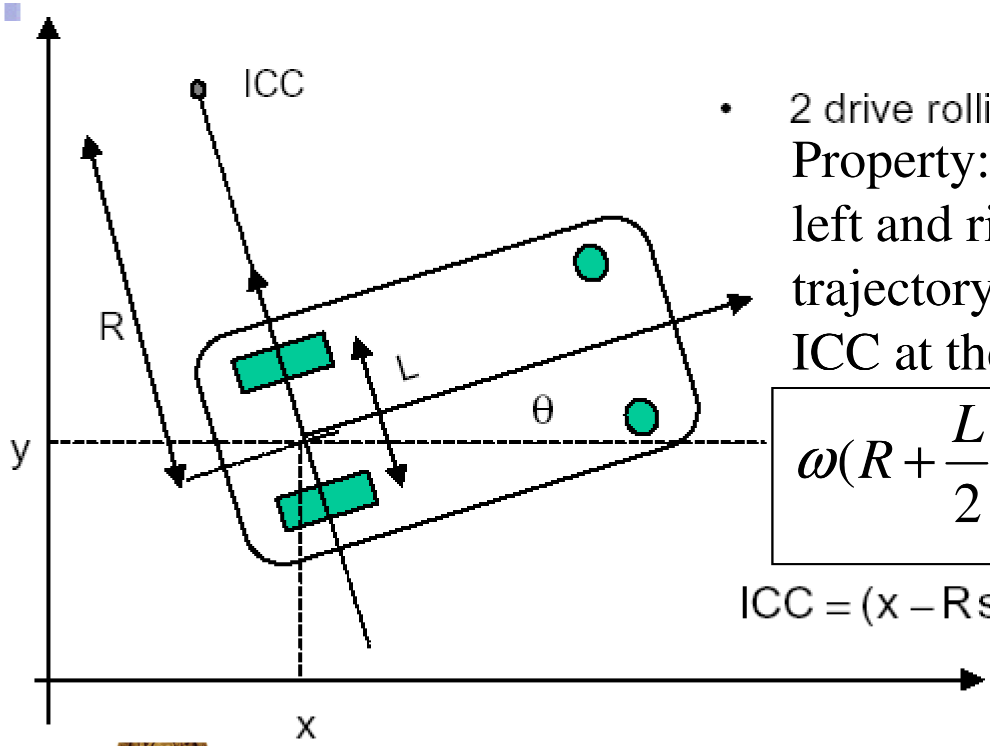
Differential Drive

$V_R(t)$ – linear velocity of right wheel

$V_L(t)$ – linear velocity of left wheel

r – nominal radius of each wheel

R – instantaneous curvature radius of the robot trajectory (distance from ICC to the midpoint between the two wheels).



- 2 drive rolling wheels

Property: At each time instant, the left and right wheels must follow a trajectory that moves around the ICC at the same angular rate ω, i.e.,

$$\omega(R+\frac{L}{2})=V_R \qquad \omega(R-\frac{L}{2})=V_L$$

$$ICC=(x-R\sin\theta,\ y+R\cos\theta)$$

Differential Drive

Posture Kinematics Model: Kinematics model in world frame

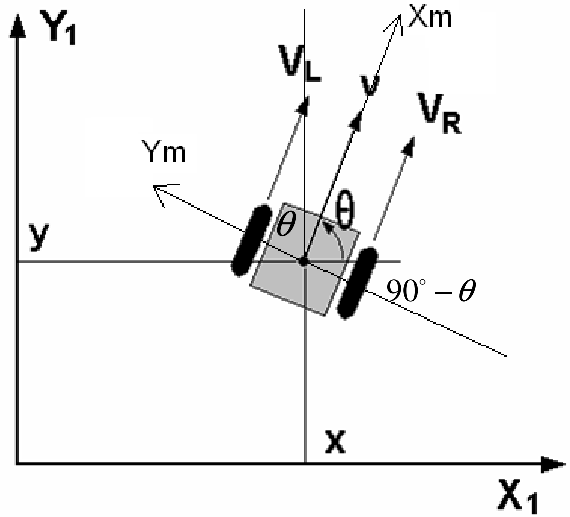
- Relation between the control input and speed of wheels

$$V_L = r\,\omega_L \qquad V_R = r\,\omega_R$$

$$\omega = \frac{V_R - V_L}{L} \qquad v = \frac{V_R + V_L}{2}$$

- Kinematic equation

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix}$$



- Nonholonomic Constraint

$$\begin{bmatrix} \sin\theta & -\cos\theta \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \dot{x}\sin\theta - \dot{y}\cos\theta = 0$$

Physical Meaning?

Differential Drive

Kinematics model in robot frame

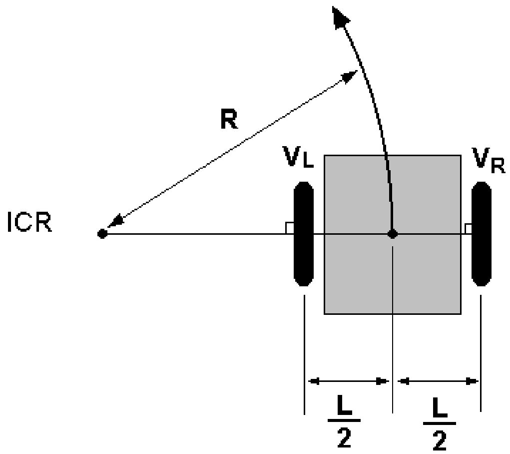
---configuration kinematics model

$$\begin{bmatrix} v_x(t) \\ v_y(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} r/2 & r/2 \\ 0 & 0 \\ -r/L & r/L \end{bmatrix} \begin{bmatrix} w_l(t) \\ w_r(t) \end{bmatrix}$$

- $w_r(t)$ – angular velocity of right wheel
- $w_l(t)$ – angular velocity of left wheel

Basic Motion Control

- Instantaneous center of rotation



$$(V_R - V_L)/\ L = V_R\ /(R + \frac{L}{2})$$

$$R = \frac{L}{2}\ \frac{V_R + V_L}{V_R - V_L}$$

R : Radius of rotation

- Straight motion

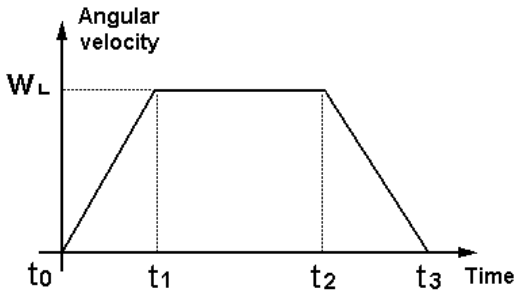
  R = Infinity → $V_R = V_L$

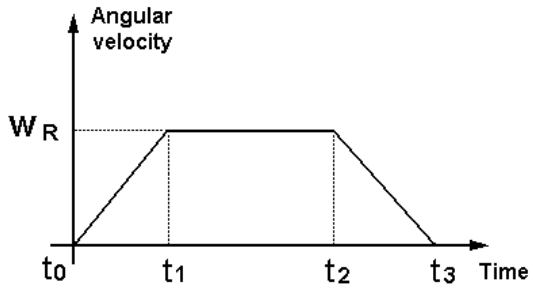
- Rotational motion

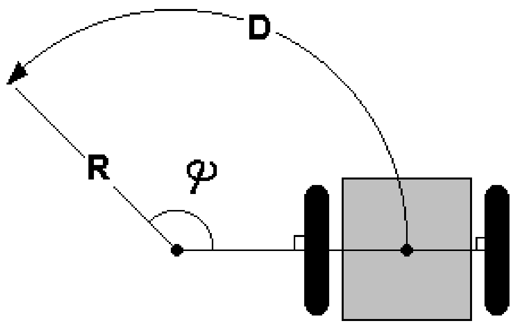
  R = 0 → $V_R = -V_L$

Basic Motion Control

- Velocity Profile







$$R = \frac{L}{2}\frac{V_R + V_L}{V_R - V_L} = \frac{L}{2}\frac{W_R + W_L}{W_R - W_L}$$

$$D = \int \frac{V_L + V_R}{2}\, dt = \frac{1}{2} r \frac{W_L + W_R}{2}(t_3 - t_0 + t_2 - t_1)$$

$$\varphi = \frac{D}{R} = \frac{r}{2L}(W_R - W_L)(t_3 - t_0 + t_2 - t_1)$$

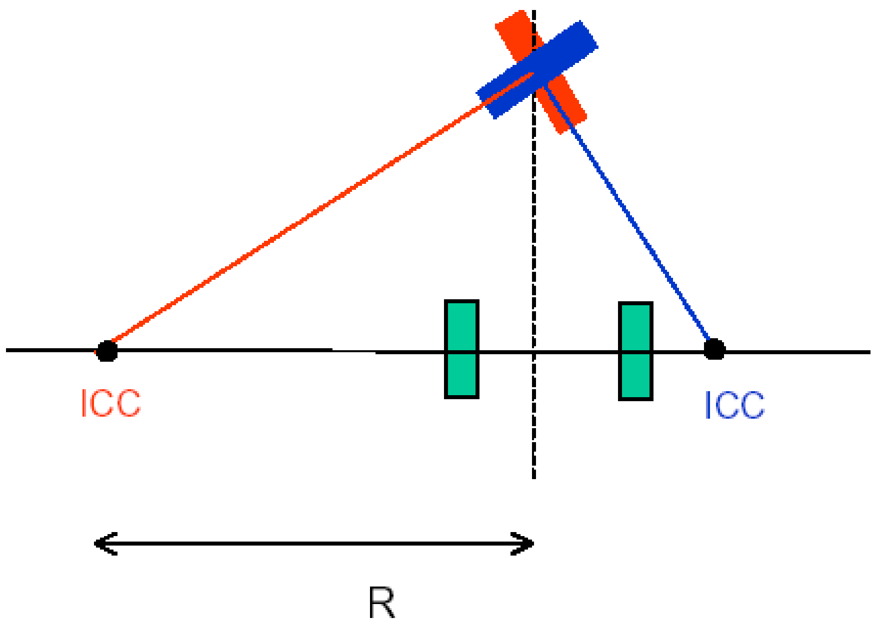
R : Radius of rotation

D : Length of path

φ : Angle of rotation

Tricycle

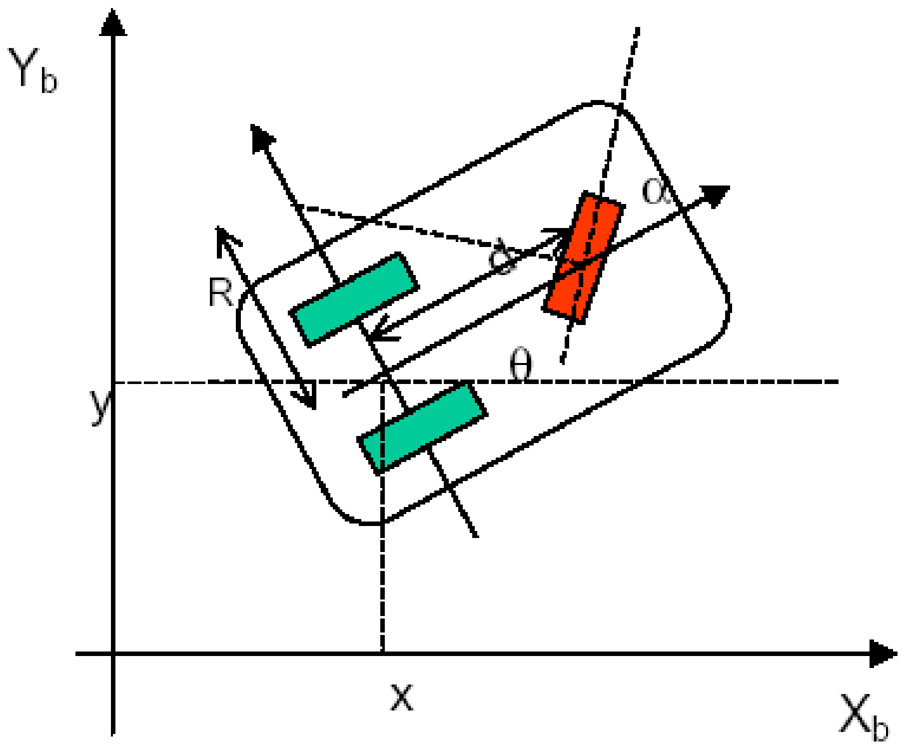
- Three wheels and odometers on the two rear wheels
- Steering and power are provided through the front wheel
- control variables:
  - steering direction $\alpha(t)$
  - angular velocity of steering wheel $w_s(t)$



The ICC must lie on the line that passes through, and is perpendicular to, the fixed rear wheels

Tricycle

- If the steering wheel is set to an angle α(t) from the straight-line direction, the tricycle will rotate with angular velocity ω(t) about ICC lying a distance R along the line perpendicular to and passing through the rear wheels.



Tricycle

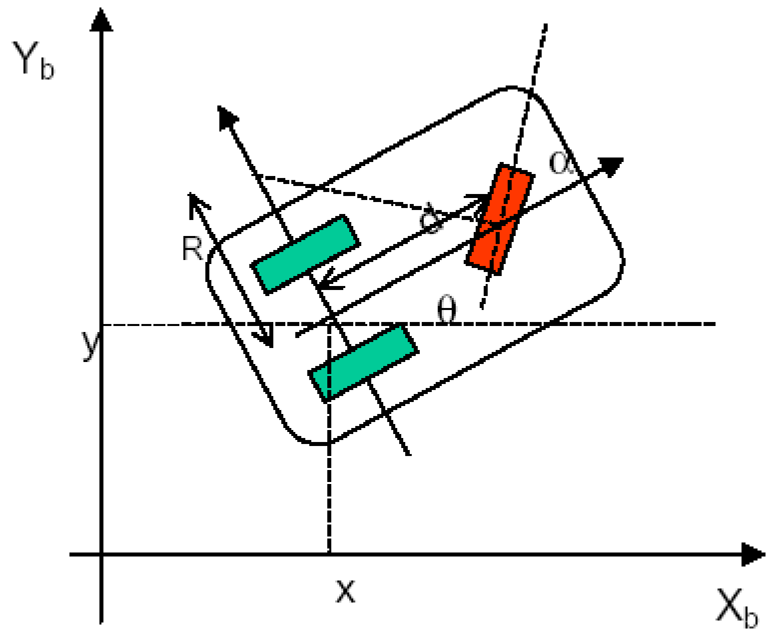
r = steering wheel radius

$$v_s(t) = w_s(t)\, r$$ linear velocity of steering wheel

$$R(t) = d\, tg\left(\pi/2 - \alpha(t)\right)$$

$$w(t) = \frac{w_s(t)\, r}{\sqrt{d^2 + R(t)^2}}$$ angular velocity of the moving relative to the base frame

$$w(t) = \frac{v_s(t)}{d} \sin\alpha(t)$$



d: distance from the front wheel to the rear axle

Tricycle

Kinematics model in the robot frame
---configuration kinematics model

$$v_x(t) = v_s(t)\cos\alpha(t)$$

$$v_y(t) = 0$$

with no splippage

$$\dot{\theta}(t) = \frac{v_s(t)}{d}\sin\alpha(t)$$

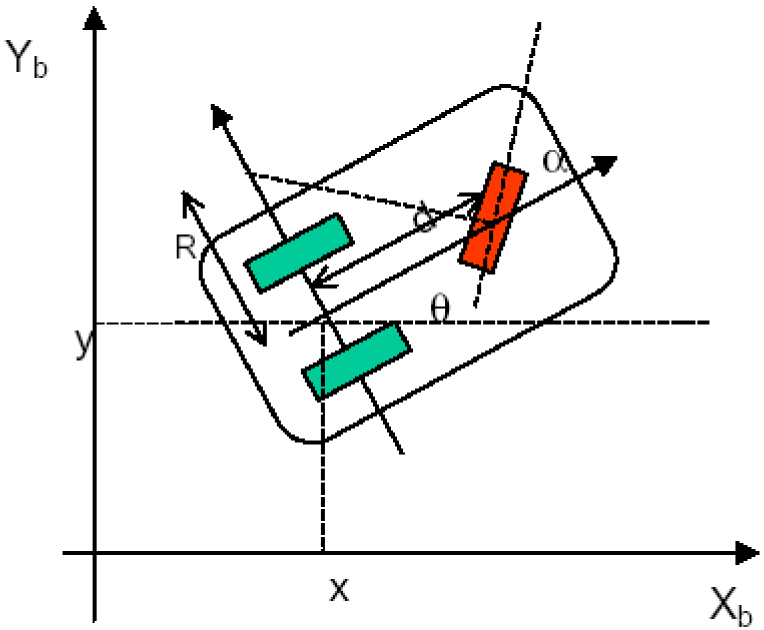
Tricycle

Kinematics model in the world frame
---Posture kinematics model

$$\dot{x}(t) = v_s(t)\cos\alpha(t)\cos\theta(t)$$
$$\dot{y}(t) = v_s(t)\cos\alpha(t)\sin\theta(t)$$
$$\dot{\theta}(t) = \frac{v_s(t)}{d}\sin\alpha(t)$$

$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} \cos\theta(t) & 0 \\ \sin\theta(t) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v(t) \\ w(t) \end{bmatrix}$$

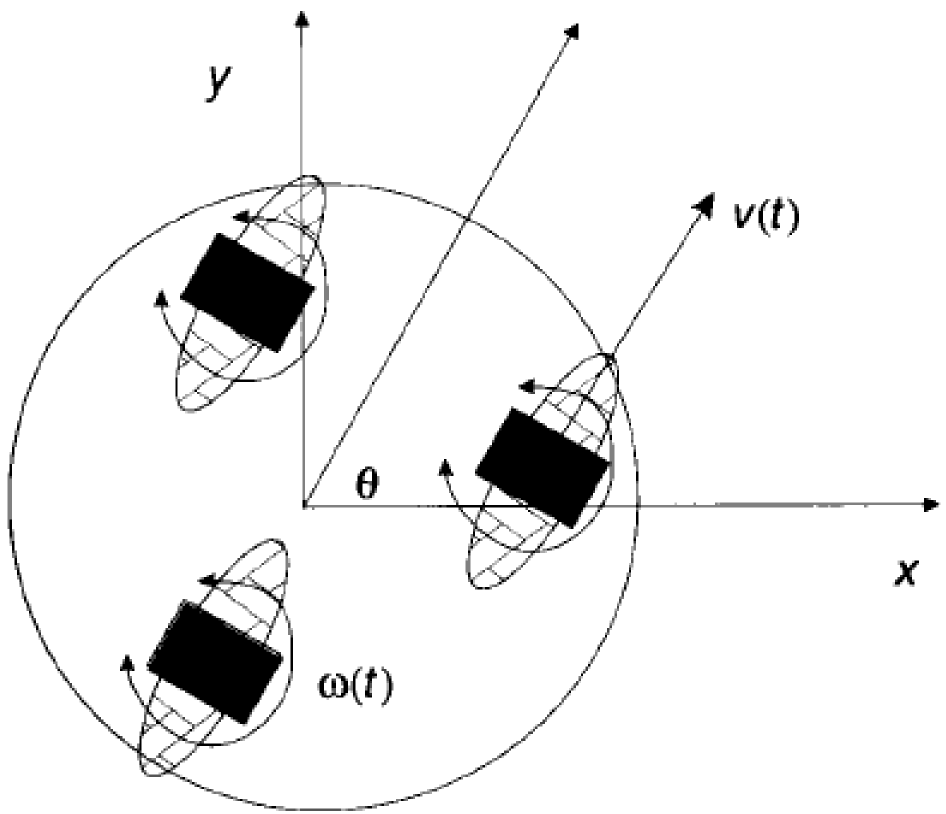
$$v(t) = v_s(t)\cos\alpha(t)$$
$$w(t) = \frac{v_s(t)}{d}\sin\alpha(t)$$



Synchronous Drive

- In a synchronous drive robot (synchronous drive) each wheel is capable of being driven and steered.
- Typical configurations
  - Three steered wheels arranged as vertices of an equilateral
  - triangle often surmounted by a cylindrical platform
  - All the wheels turn and drive in unison
- This leads to a holonomic behavior

Synchronous Drive

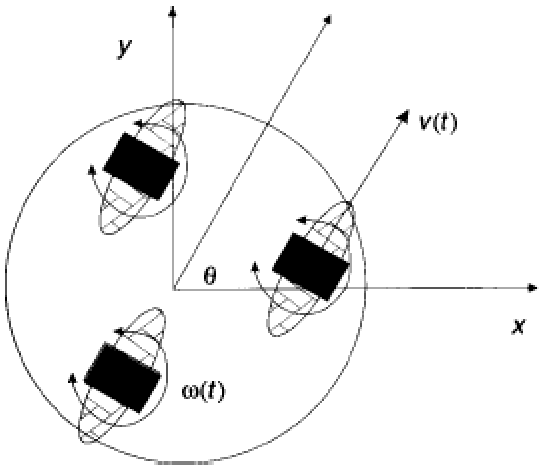


Synchronous Drive

- All the wheels turn in unison
- All of the three wheels point in the same direction and turn at the same rate
  - This is typically achieved through the use of a complex collection of belts that physically link the wheels together
  - Two independent motors, one rolls all wheels forward, one rotate them for turning
- The vehicle controls the direction in which the wheels point and the rate at which they roll
- Because all the wheels remain parallel the synchro drive always rotate about the center of the robot
- The synchro drive robot has the ability to control the orientation θ of their pose directly.

Synchronous Drive

- Control variables (independent)
  - v(t), ω(t)



$$x(t) = \int_0^t v(\sigma)\cos(\theta(\sigma))d\sigma$$

$$y(t) = \int_0^t v(\sigma)\sin(\theta(\sigma))d\sigma$$

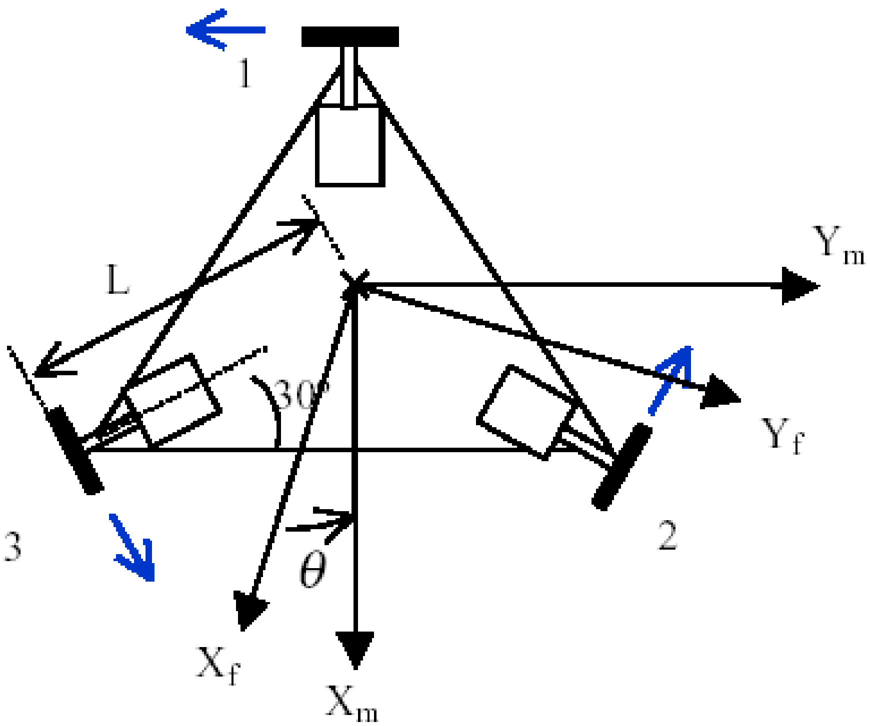
$$\theta(t) = \int_0^t w(\sigma)d\sigma$$

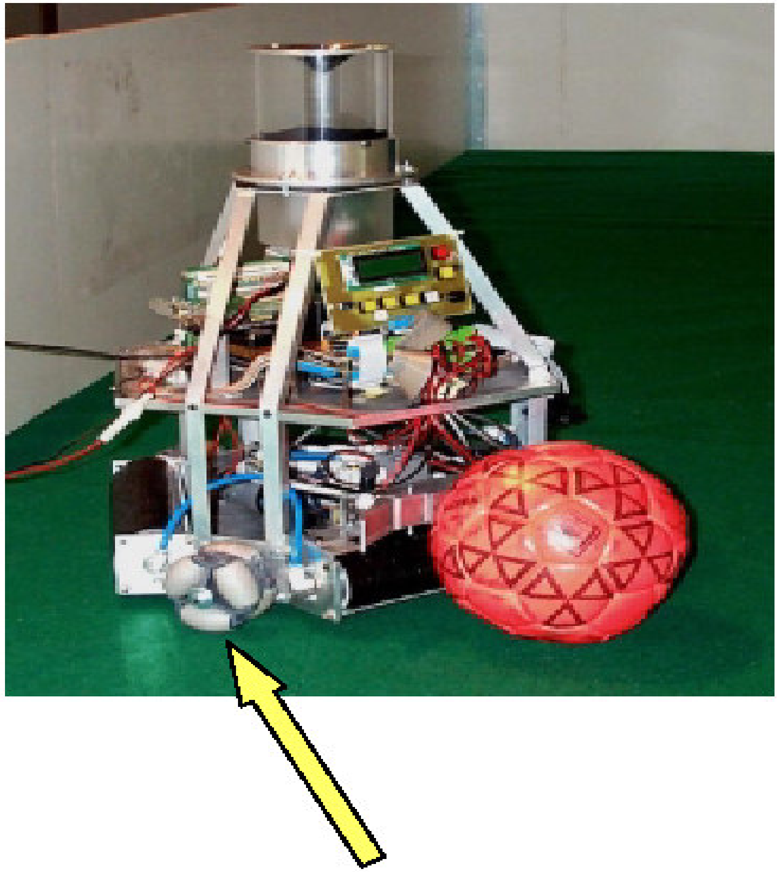
- The ICC is always at infinity
- Changing the orientation of the wheels manipulates the direction of ICC

Synchronous Drive

- Particular cases:
  - v(t)=0, w(t)=w during a time interval Δt, The robot rotates in place by an amount w Δt .
  - v(t)=v, w(t)=0 during a time interval Δt , the robot moves in the direction its pointing a distance v Δt.

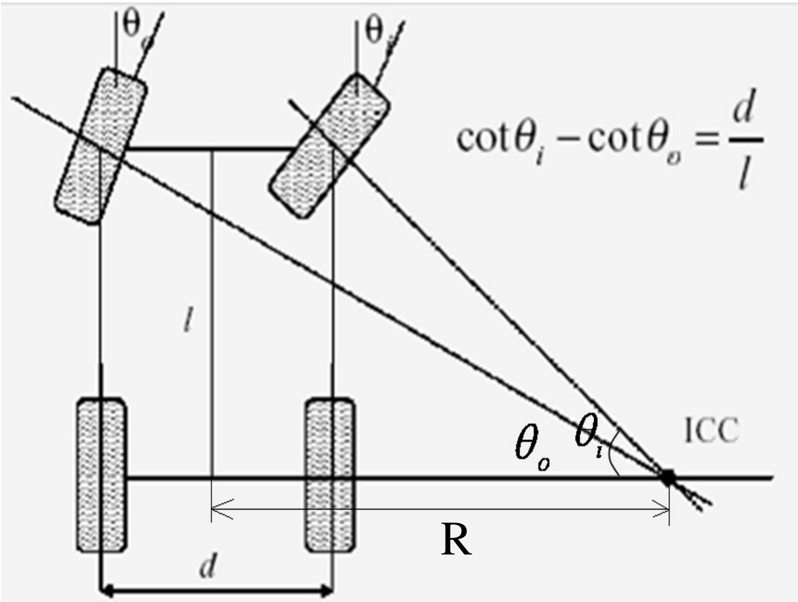
Omidirectional





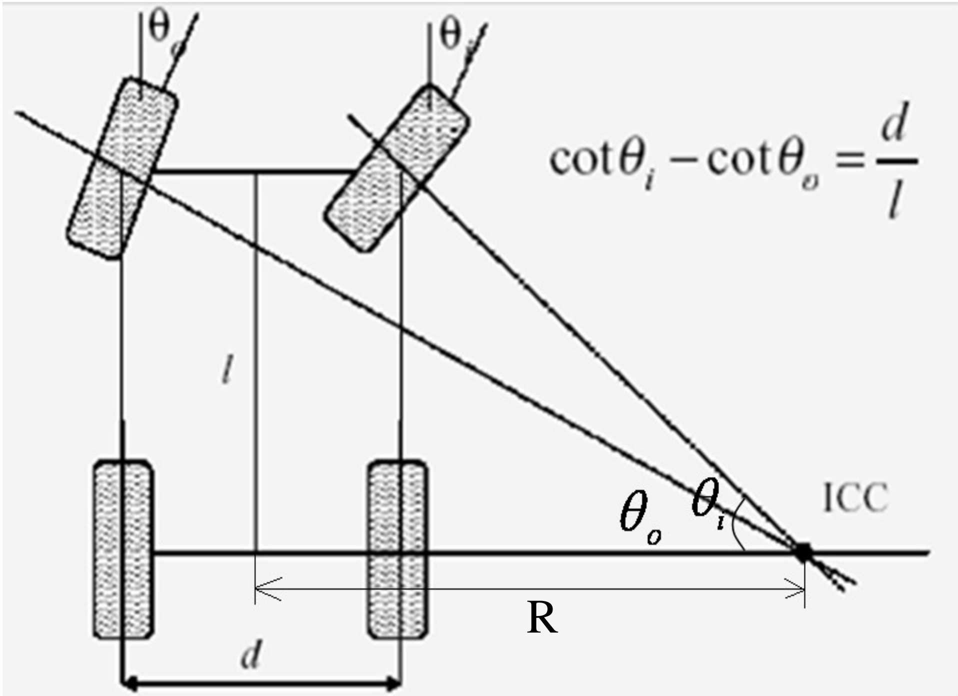
Swedish Wheel

Car Drive (Ackerman Steering)



- Used in motor vehicles, the inside front wheel is rotated slightly sharper than the outside wheel (reduces tire slippage).
- Ackerman steering provides a fairly accurate dead-reckoning solution while supporting traction and ground clearance.
- Generally the method of choice for outdoor autonomous vehicles.

Ackerman Steering



where

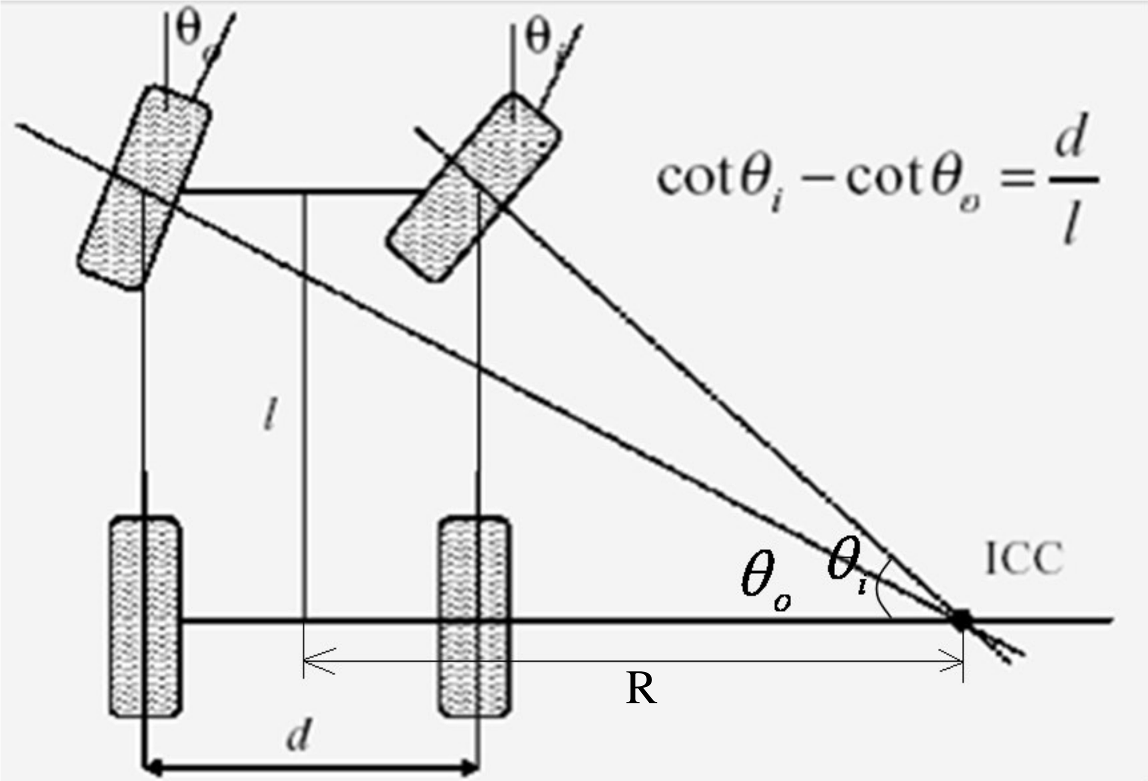
d = lateral wheel separation
l = longitudinal wheel separation
θi = relative steering angle of inside wheel
θo = relative steering angle of outside wheel
R=distance between ICC to centerline of the vehicle

Ackerman Steering

- The Ackerman Steering equation:
  - :

$$\cot\theta_i - \cot\theta_o = \frac{d}{l}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$



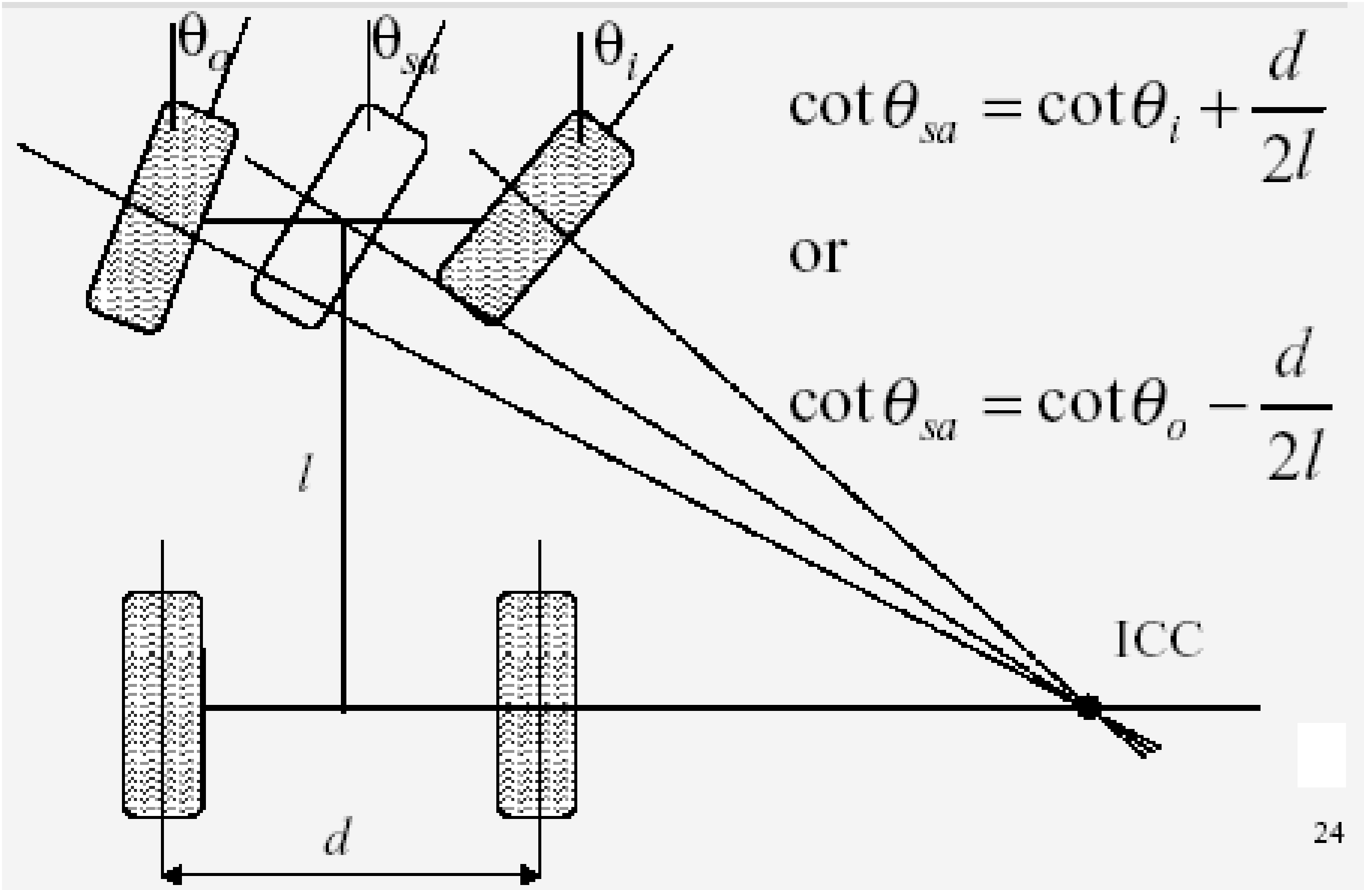
$$\cot\theta_i - \cot\theta_o$$

$$= \frac{R - d/2}{l} - \frac{R + d/2}{l}$$

$$= \frac{d}{l}$$

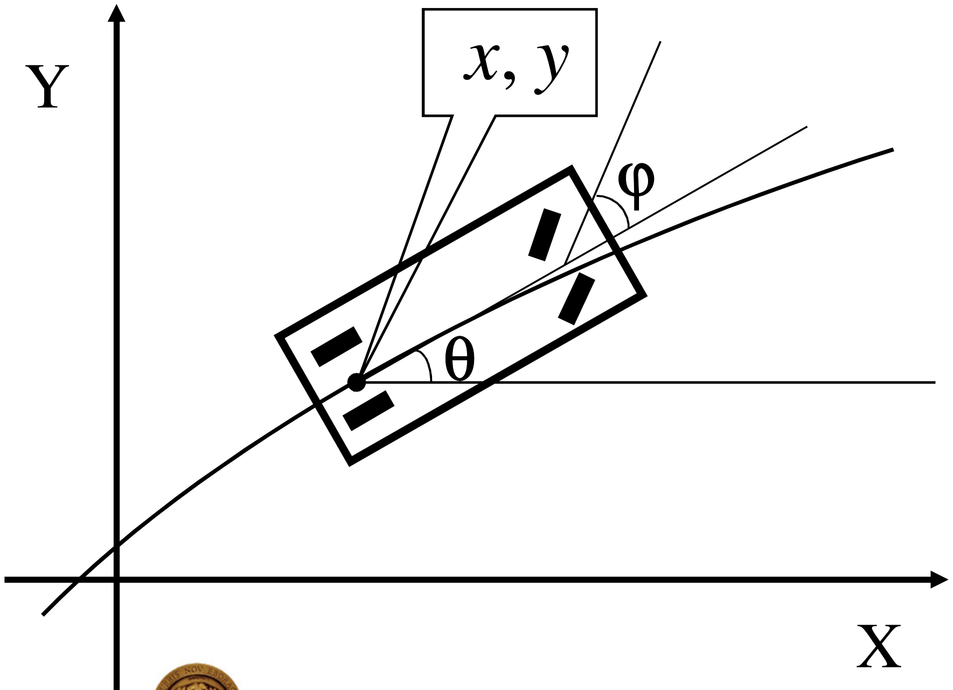
Ackerman Steering

Equivalent:



Kinematic model for car-like robot

- Control Input
- Driving type: Forward wheel drive



$$\{x, y, \theta, \varphi\}$$

$$\{u_1, u_2\}$$ u_1 : forward vel

u_2 : steering vel

$$\{\tau_1, \tau_2\}$$

Kinematic model for car-like robot

$$\dot{x} = u_1 \cos\theta$$

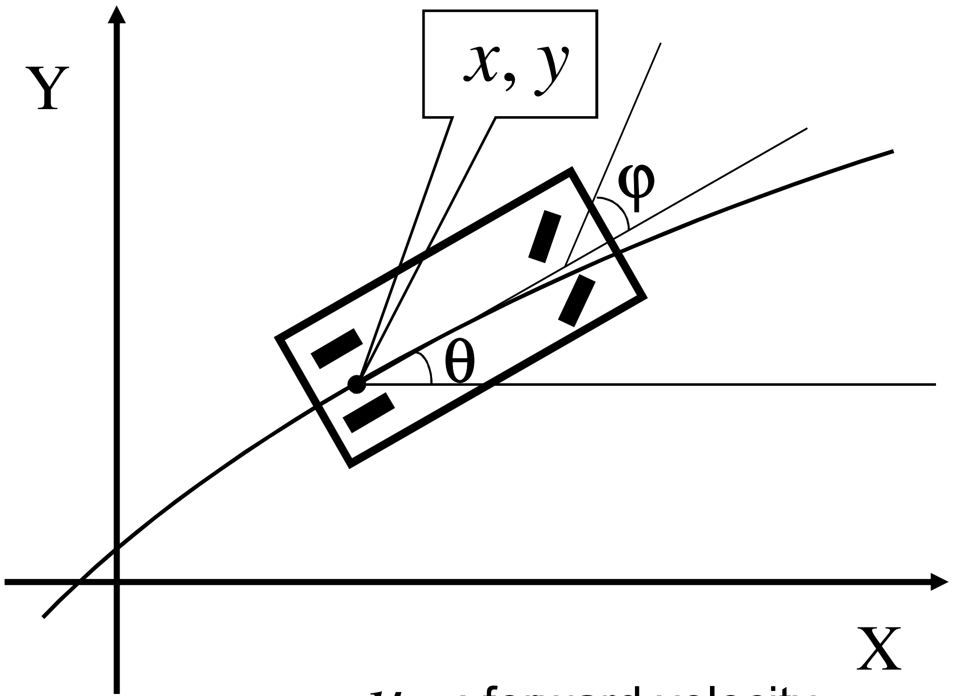
$$\dot{y} = u_1 \sin\theta$$

$$\dot{\theta} = \frac{u_1}{l} \tan\varphi$$

$$\dot{\varphi} = u_2$$

non-holonomic constraint:

$$\dot{x}\sin\theta - \dot{y}\cos\theta = 0$$

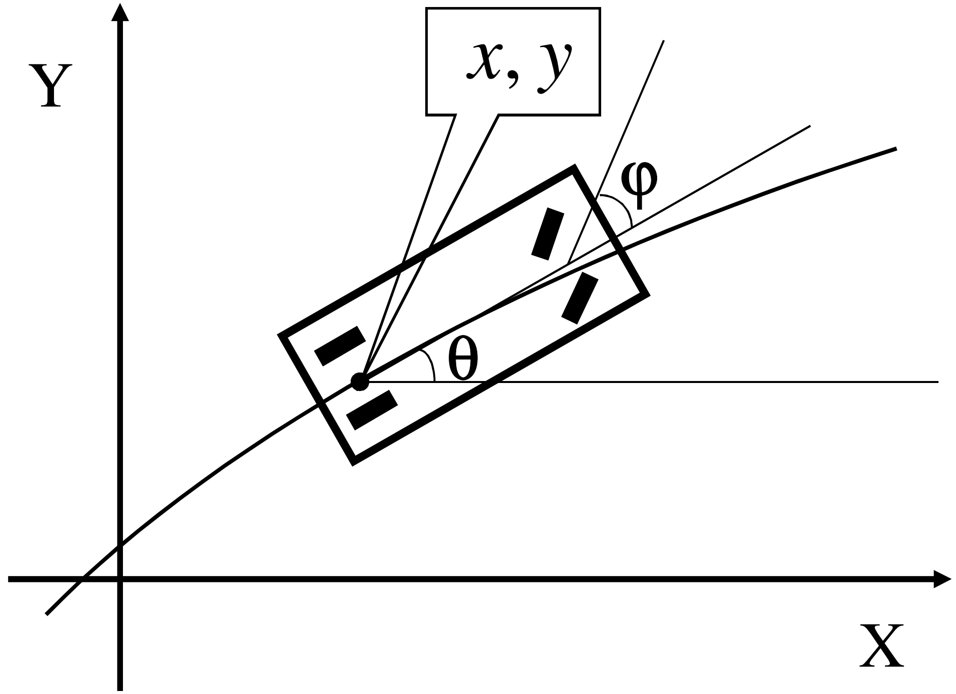


u_1 : forward velocity

u_2 : steering velocity

Dynamic Model

- Dynamic model



$$\begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I \end{pmatrix}\begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\theta} \end{pmatrix} = \begin{pmatrix} \sin\theta \\ \cos\theta \\ 0 \end{pmatrix}\lambda + \begin{pmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} f_1 \\ f_2 \end{pmatrix}$$

Summary

- Mobot: Mobile Robot
- Classification of wheels
  – Fixed wheel
  – Centered orientable wheel
  – Off-centered orientable wheel (Caster Wheel)
  – Swedish wheel
- Mobile Robot Locomotion
  – Degrees of mobility
  – 5 types of driving (steering) methods
- Kinematics of WMR
- Basic Control