

# Introduction to ROBOTICS

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## Kinematics of Robot Manipulator

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# Outline

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- Review
- Robot Manipulators
  - Robot Configuration
  - Robot Specification
    - Number of Axes, DOF
    - Precision, Repeatability
- Kinematics
  - Preliminary
    - World frame, joint frame, end-effector frame
    - Rotation Matrix, composite rotation matrix
    - Homogeneous Matrix
  - Direct kinematics
    - Denavit-Hartenberg Representation
    - Examples
  - Inverse kinematics



# Review

- What is a robot?
  - By general agreement a robot is:
    - A programmable machine that imitates the actions or appearance of an intelligent creature—usually a human.
  - To qualify as a robot, a machine must be able to:
    - 1) Sensing and perception: get information from its surroundings
    - 2) Carry out different tasks: Locomotion or manipulation, do something physical—such as move or manipulate objects
    - 3) Re-programmable: can do different things
    - 4) Function autonomously and/or interact with human beings
- Why use robots?
  - Perform 4A tasks in 4D environments
    - 4A: Automation, Augmentation, Assistance, Autonomous
    - 4D: Dangerous, Dirty, Dull, Difficult



# Manipulators

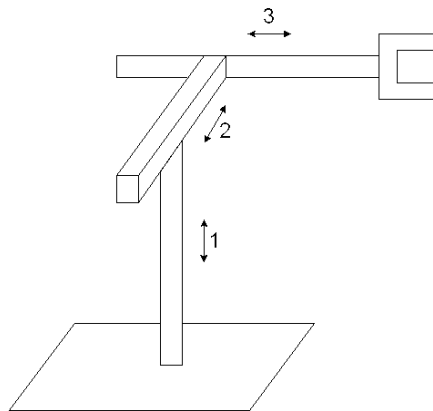
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- Robot arms, industrial robot
  - Rigid bodies (links) connected by joints
  - Joints: revolute or prismatic
  - Drive: electric or hydraulic
  - End-effector (tool) mounted on a flange or plate secured to the wrist joint of robot

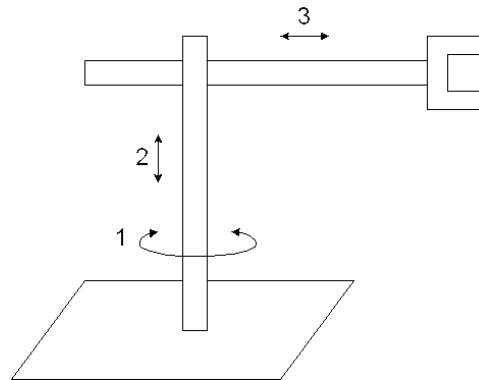


# Manipulators

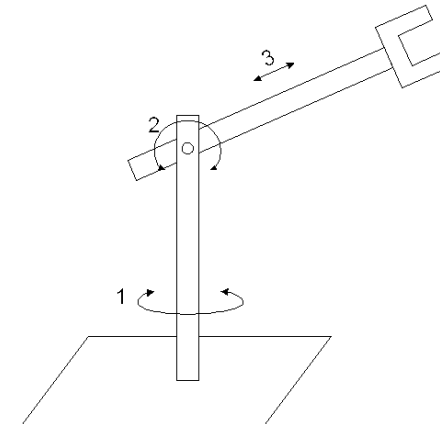
- Robot Configuration:



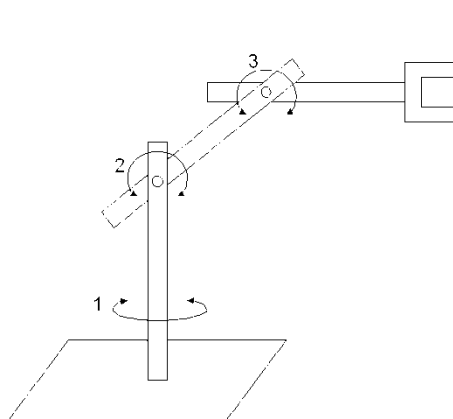
Cartesian: PPP



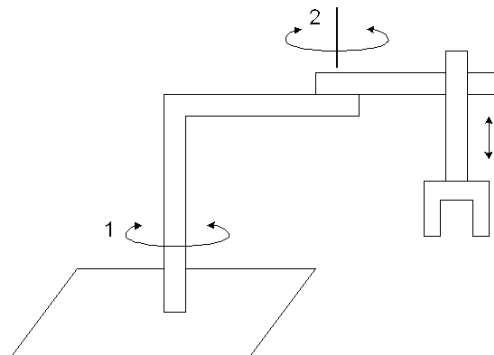
Cylindrical: RPP



Spherical: RRP

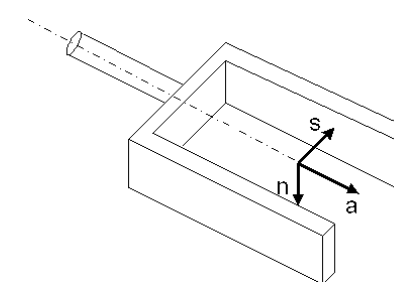


Articulated: RRR



SCARA: RRP

(Selective Compliance Assembly Robot Arm)



Hand coordinate:

**n**: normal vector; **s**: sliding vector;

**a**: approach vector, normal to the

tool mounting plate



# Manipulators

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- Motion Control Methods
  - Point to point control
    - a sequence of discrete points
    - spot welding, pick-and-place, loading & unloading
  - Continuous path control
    - follow a prescribed path, controlled-path motion
    - Spray painting, Arc welding, Gluing



# Manipulators

- Robot Specifications

- Number of Axes

- Major axes, (1-3) => Position the wrist
    - Minor axes, (4-6) => Orient the tool
    - Redundant, (7-n) => reaching around obstacles, avoiding undesirable configuration

- Degree of Freedom (DOF)

- Workspace

- Payload (load capacity)

- Precision v.s. Repeatability

Which one is more important?

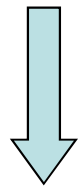


# What is Kinematics

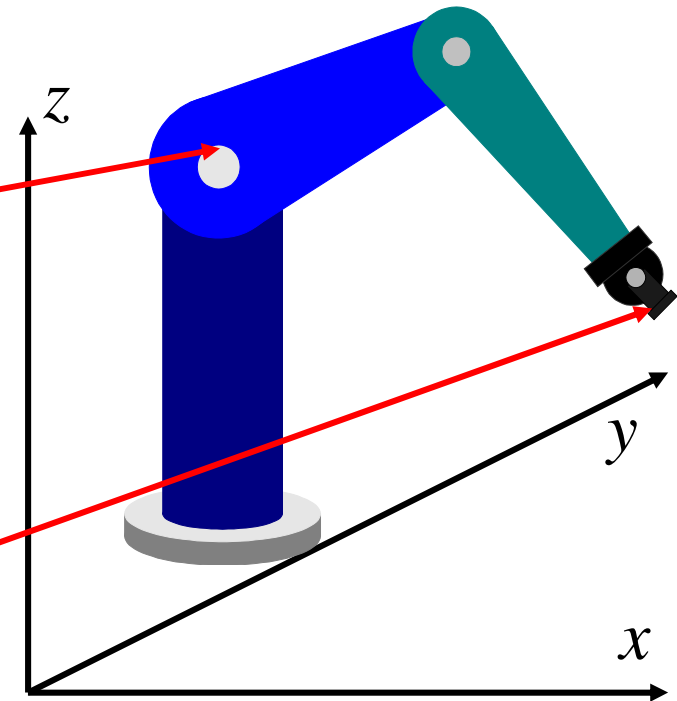
- Forward kinematics

Given joint variables

$$q = (q_1, q_2, q_3, q_4, q_5, q_6, \dots, q_n)$$



$$Y = (x, y, z, O, A, T)$$



End-effector position and orientation, -Formula?

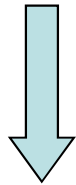




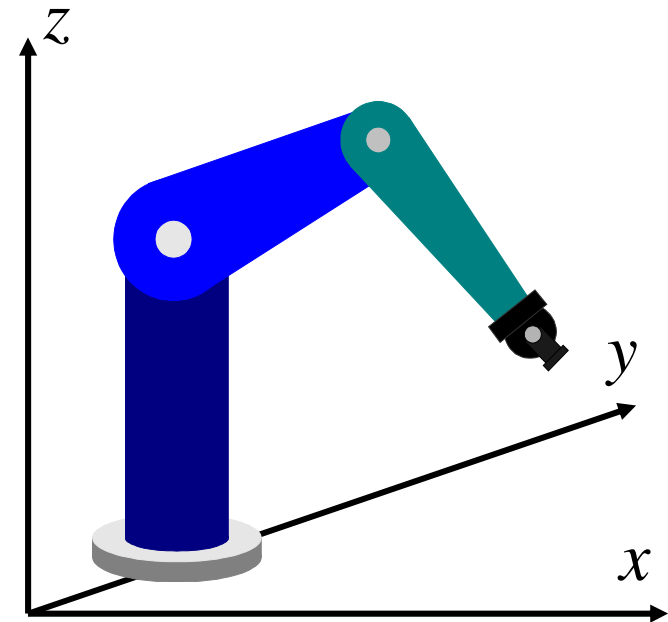
# What is Kinematics

- Inverse kinematics  
End effector position  
and orientation

$(x, y, z, O, A, T)$



$q = (q_1, q_2, q_3, q_4, q_5, q_6, \dots, q_n)$



Joint variables -Formula?



# Example 1

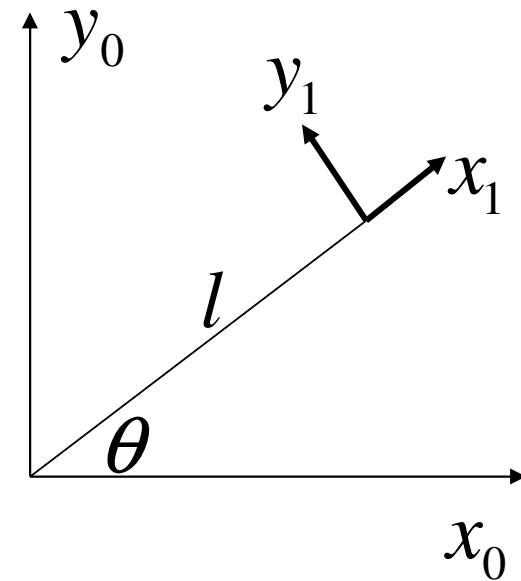
Forward kinematics

$$x_0 = l \cos \theta$$

$$y_0 = l \sin \theta$$

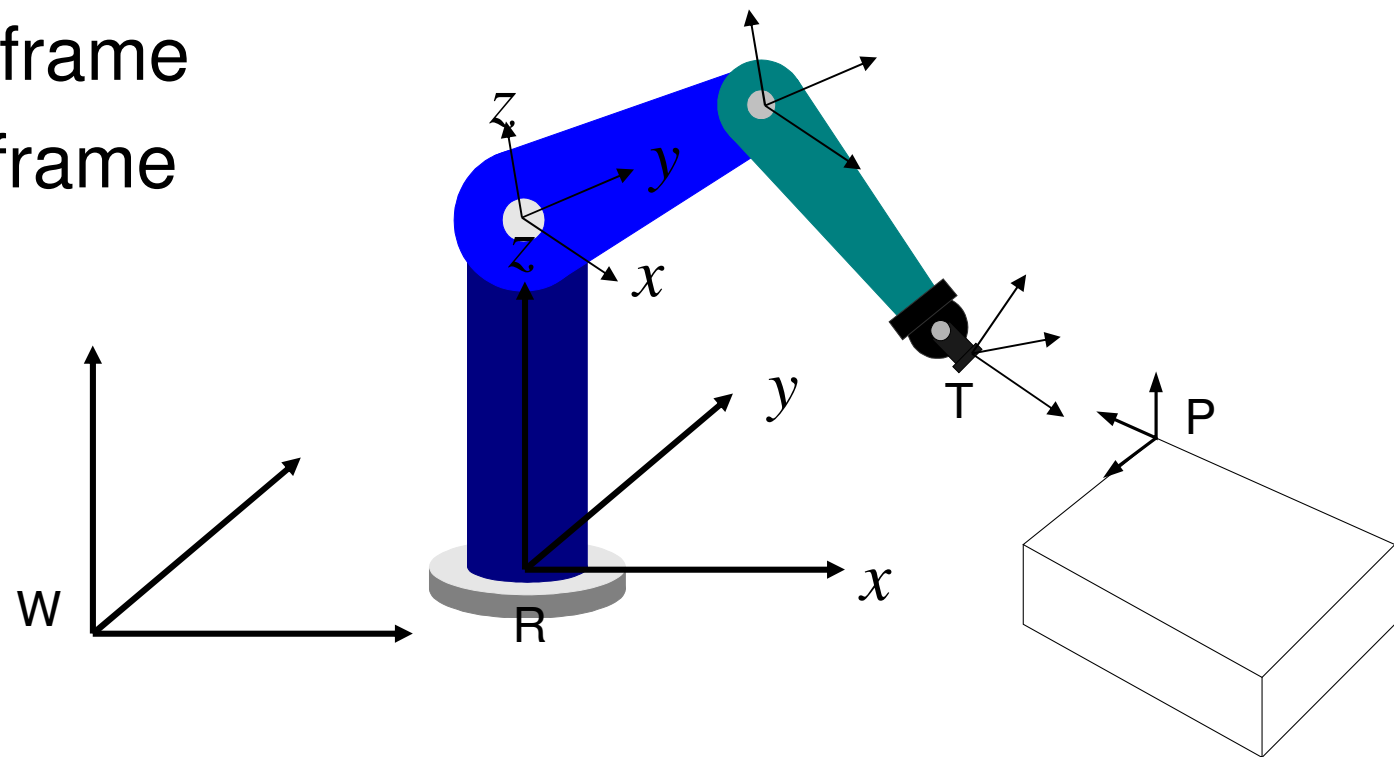
Inverse kinematics

$$\theta = \cos^{-1}(x_0 / l)$$



# Preliminary

- Robot Reference Frames
  - World frame
  - Joint frame
  - Tool frame



# Preliminary

- Coordinate Transformation
  - Reference coordinate frame OXYZ
  - Body-attached frame O'uvw

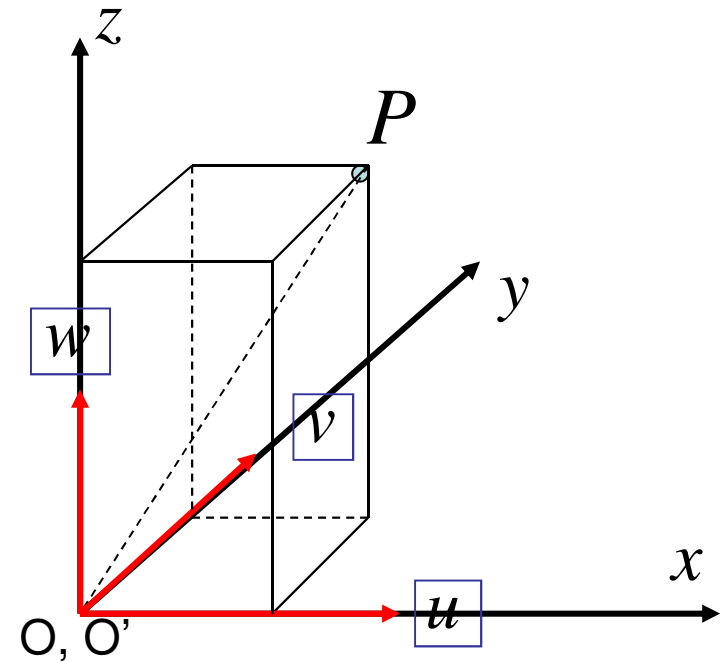
Point represented in OXYZ:

$$P_{xyz} = [p_x, p_y, p_z]^T$$

$$\vec{P}_{xyz} = p_x \mathbf{i}_x + p_y \mathbf{j}_y + p_z \mathbf{k}_z$$

Point represented in O'uvw:

$$\vec{P}_{uvw} = p_u \mathbf{i}_u + p_v \mathbf{j}_v + p_w \mathbf{k}_w$$



Two frames coincide  $\implies p_u = p_x \quad p_v = p_y \quad p_w = p_z$



# Preliminary

## Properties: Dot Product

Let  $x$  and  $y$  be arbitrary vectors in  $R^3$  and  $\theta$  be the angle from  $x$  to  $y$ , then

$$x \cdot y = |x||y| \cos \theta$$

## Properties of orthonormal coordinate frame

- Mutually perpendicular
- Unit vectors

$$\vec{i} \cdot \vec{j} = 0$$

$$|\vec{i}| = 1$$

$$\vec{i} \cdot \vec{k} = 0$$

$$|\vec{j}| = 1$$

$$\vec{k} \cdot \vec{j} = 0$$

$$|\vec{k}| = 1$$



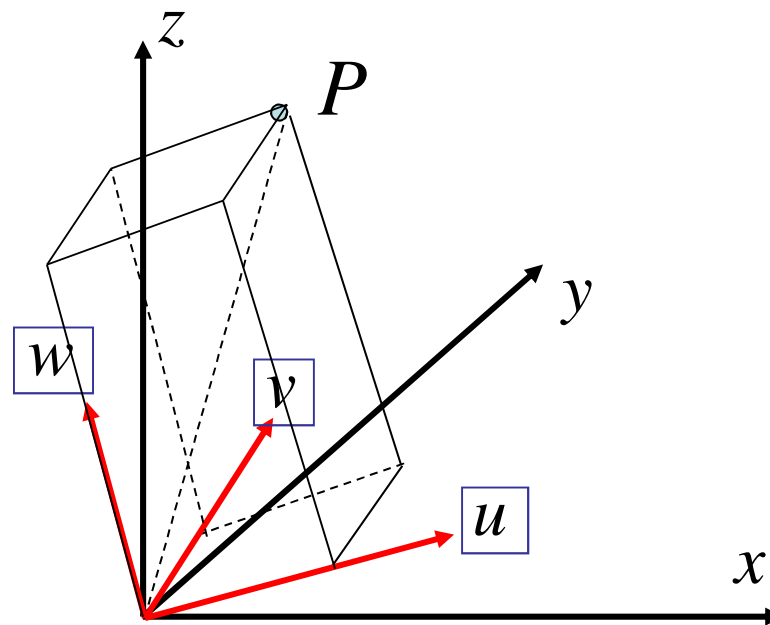
# Preliminary

- Coordinate Transformation
  - Rotation only

$$\vec{P}_{xyz} = p_x \mathbf{i}_x + p_y \mathbf{j}_y + p_z \mathbf{k}_z$$

$$\vec{P}_{uvw} = p_u \mathbf{i}_u + p_v \mathbf{j}_v + p_w \mathbf{k}_w$$

$$P_{xyz} = RP_{uvw}$$



How to relate the coordinate in these two frames?



# Preliminary

- Basic Rotation

- $p_x$ ,  $p_y$ , and  $p_z$  represent the projections of  $P$  onto OX, OY, OZ axes, respectively

- Since  $P = p_u \mathbf{i}_u + p_v \mathbf{j}_v + p_w \mathbf{k}_w$

$$p_x = \mathbf{i}_x \cdot P = \mathbf{i}_x \cdot \mathbf{i}_u p_u + \mathbf{i}_x \cdot \mathbf{j}_v p_v + \mathbf{i}_x \cdot \mathbf{k}_w p_w$$

$$p_y = \mathbf{j}_y \cdot P = \mathbf{j}_y \cdot \mathbf{i}_u p_u + \mathbf{j}_y \cdot \mathbf{j}_v p_v + \mathbf{j}_y \cdot \mathbf{k}_w p_w$$

$$p_z = \mathbf{k}_z \cdot P = \mathbf{k}_z \cdot \mathbf{i}_u p_u + \mathbf{k}_z \cdot \mathbf{j}_v p_v + \mathbf{k}_z \cdot \mathbf{k}_w p_w$$



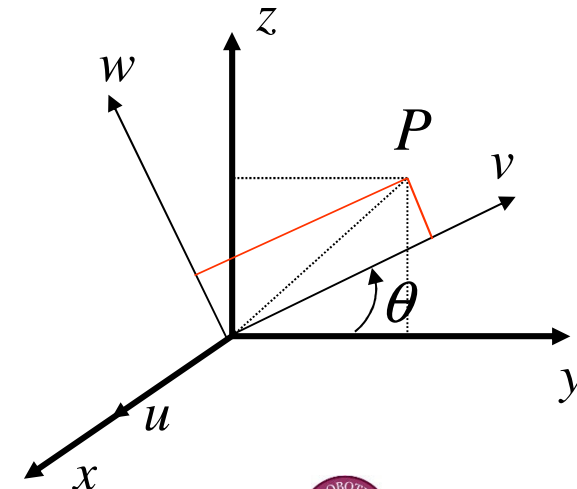
# Preliminary

- Basic Rotation Matrix

$$\begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} \mathbf{i}_x \cdot \mathbf{i}_u & \mathbf{i}_x \cdot \mathbf{j}_v & \mathbf{i}_x \cdot \mathbf{k}_w \\ \mathbf{j}_y \cdot \mathbf{i}_u & \mathbf{j}_y \cdot \mathbf{j}_v & \mathbf{j}_y \cdot \mathbf{k}_w \\ \mathbf{k}_z \cdot \mathbf{i}_u & \mathbf{k}_z \cdot \mathbf{j}_v & \mathbf{k}_z \cdot \mathbf{k}_w \end{bmatrix} \begin{bmatrix} p_u \\ p_v \\ p_w \end{bmatrix}$$

– Rotation about x-axis with  $\theta$

$$Rot(x, \theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\theta & -S\theta \\ 0 & S\theta & C\theta \end{bmatrix}$$





# Preliminary

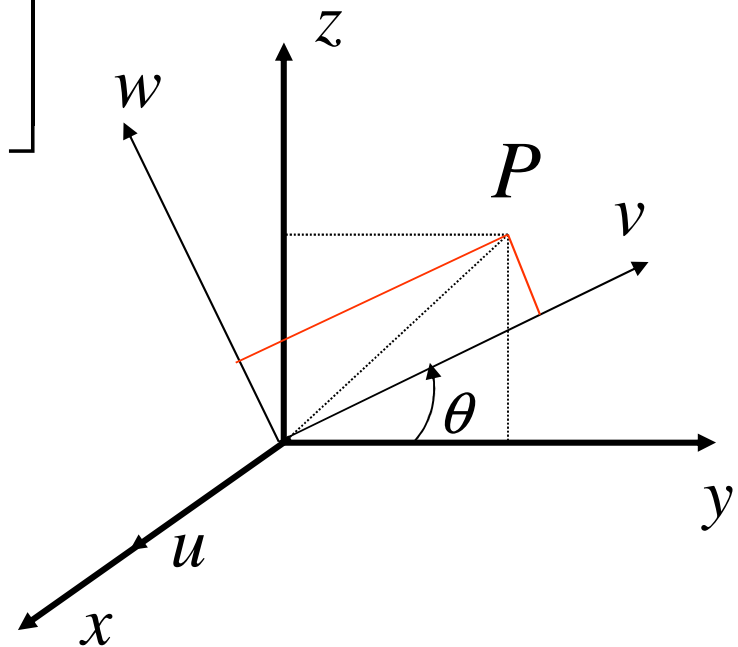
- Is it True?
  - Rotation about x axis with  $\theta$

$$\begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} p_u \\ p_v \\ p_w \end{bmatrix}$$

$$p_x = p_u$$

$$p_y = p_v \cos \theta - p_w \sin \theta$$

$$p_z = p_v \sin \theta + p_w \cos \theta$$



# Basic Rotation Matrices

- Rotation about x-axis with  $\theta$

$$Rot(x, \theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\theta & -S\theta \\ 0 & S\theta & C\theta \end{bmatrix}$$

- Rotation about y-axis with  $\theta$

$$Rot(y, \theta) = \begin{bmatrix} C\theta & 0 & S\theta \\ 0 & 1 & 0 \\ -S\theta & 0 & C\theta \end{bmatrix}$$

- Rotation about z-axis with  $\theta$

$$P_{xyz} = RP_{uvw} \quad Rot(z, \theta) = \begin{bmatrix} C\theta & -S\theta & 0 \\ S\theta & C\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



# Preliminary

- Basic Rotation Matrix

$$R = \begin{bmatrix} \mathbf{i}_x \cdot \mathbf{i}_u & \mathbf{i}_x \cdot \mathbf{j}_v & \mathbf{i}_x \cdot \mathbf{k}_w \\ \mathbf{j}_y \cdot \mathbf{i}_u & \mathbf{j}_y \cdot \mathbf{j}_v & \mathbf{j}_y \cdot \mathbf{k}_w \\ \mathbf{k}_z \cdot \mathbf{i}_u & \mathbf{k}_z \cdot \mathbf{j}_v & \mathbf{k}_z \cdot \mathbf{k}_w \end{bmatrix} \quad P_{xyz} = RP_{uvw}$$

- Obtain the coordinate of  $P_{uvw}$  from the coordinate of  $P_{xyz}$  Dot products are commutative!

$$\begin{bmatrix} p_u \\ p_v \\ p_w \end{bmatrix} = \begin{bmatrix} \mathbf{i}_u \cdot \mathbf{i}_x & \mathbf{i}_u \cdot \mathbf{j}_y & \mathbf{i}_u \cdot \mathbf{k}_z \\ \mathbf{j}_v \cdot \mathbf{i}_x & \mathbf{j}_v \cdot \mathbf{j}_y & \mathbf{j}_v \cdot \mathbf{k}_z \\ \mathbf{k}_w \cdot \mathbf{i}_x & \mathbf{k}_w \cdot \mathbf{j}_y & \mathbf{k}_w \cdot \mathbf{k}_z \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} \quad P_{uvw} = QP_{xyz}$$

$$Q = R^{-1} = R^T$$

$$QR = R^T R = R^{-1} R = I_3 \quad \Leftarrow \text{3X3 identity matrix}$$



# Example 2

- A point  $a_{uvw} = (4,3,2)$  is attached to a rotating frame, the frame rotates 60 degree about the OZ axis of the reference frame. Find the coordinates of the point relative to the reference frame after the rotation.

$$\begin{aligned} a_{xyz} &= Rot(z, 60) a_{uvw} \\ &= \begin{bmatrix} 0.5 & -0.866 & 0 \\ 0.866 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} -0.598 \\ 4.964 \\ 2 \end{bmatrix} \end{aligned}$$



# Example 3

- A point  $a_{xyz} = (4,3,2)$  is the coordinate w.r.t. the reference coordinate system, find the corresponding point  $a_{uvw}$  w.r.t. the rotated OU-V-W coordinate system if it has been rotated 60 degree about OZ axis.

$$\begin{aligned} a_{uvw} &= Rot(z, 60)^T a_{xyz} \\ &= \begin{bmatrix} 0.5 & 0.866 & 0 \\ -0.866 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 4.598 \\ -1.964 \\ 2 \end{bmatrix} \end{aligned}$$



# Composite Rotation Matrix

- A sequence of finite rotations
  - matrix multiplications do not commute
  - rules:
    - if rotating coordinate O-U-V-W is rotating about principal axis of OXYZ frame, then **Pre-multiply** the previous (resultant) rotation matrix with an appropriate basic rotation matrix
    - if rotating coordinate OUVW is rotating about its own principal axes, then **post-multiply** the previous (resultant) rotation matrix with an appropriate basic rotation matrix



# Example 4

- Find the rotation matrix for the following operations:

Rotation  $\phi$  about OY axis

Rotation  $\theta$  about OW axis

Rotation  $\alpha$  about OU axis

Answer...

$$R = Rot(y, \phi) I_3 Rot(w, \theta) Rot(u, \alpha)$$

$$= \begin{bmatrix} C\phi & 0 & S\phi \\ 0 & 1 & 0 \\ -S\phi & 0 & C\phi \end{bmatrix} \begin{bmatrix} C\theta & -S\theta & 0 \\ S\theta & C\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\alpha & -S\alpha \\ 0 & S\alpha & C\alpha \end{bmatrix}$$

$$= \begin{bmatrix} C\phi C\theta & S\phi S\alpha - C\phi S\theta C\alpha & C\phi S\theta S\alpha + S\phi C\alpha \\ S\theta & C\theta C\alpha & -C\theta S\alpha \\ -S\phi C\theta & S\phi S\theta C\alpha + C\phi S\alpha & C\phi C\alpha - S\phi S\theta S\alpha \end{bmatrix}$$

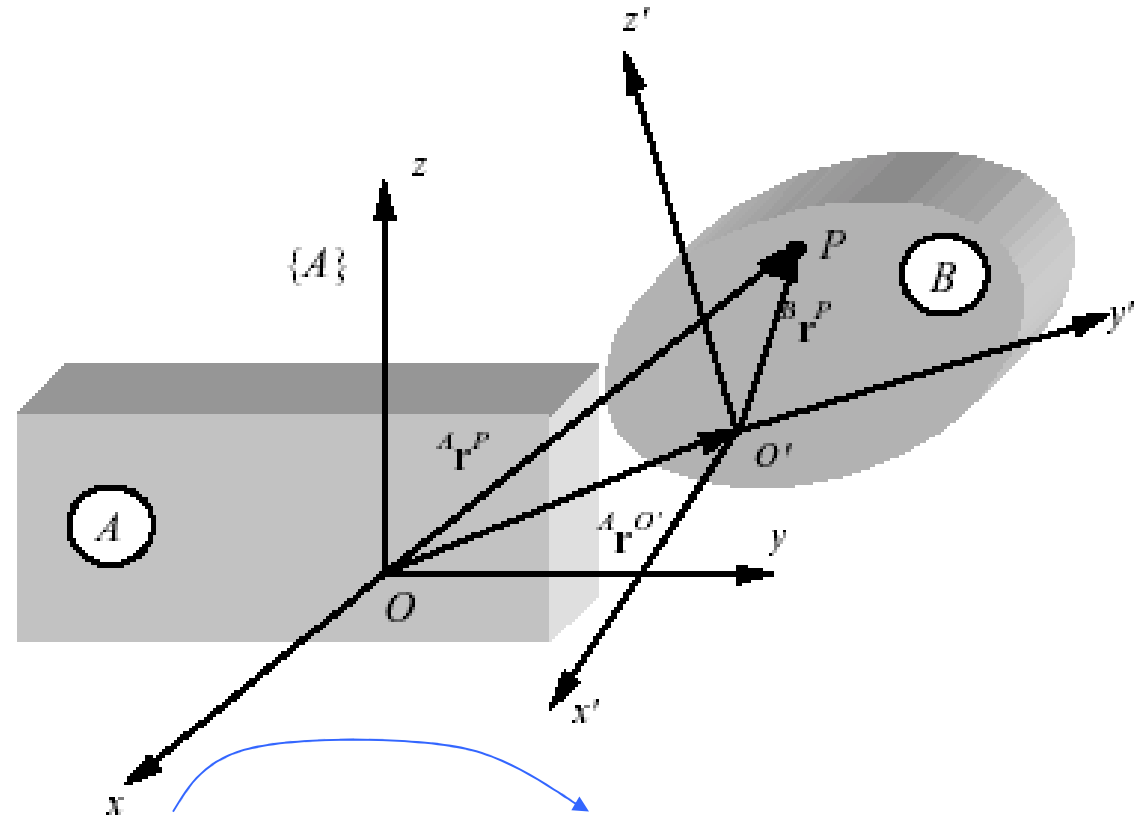
Pre-multiply if rotate about the OXYZ axes

Post-multiply if rotate about the OUVW axes



# Coordinate Transformations

- position vector of  $P$  in  $\{B\}$  is transformed to position vector of  $P$  in  $\{A\}$
- description of  $\{B\}$  as seen from an observer in  $\{A\}$



Rotation of  $\{B\}$  with respect to  $\{A\}$

$${}^A \mathbf{r}^P = {}^A \mathbf{R}_B {}^B \mathbf{r}^P + {}^A \mathbf{r}^{O'}$$

Translation of the origin of  $\{B\}$  with respect to origin of  $\{A\}$  ↗





# Coordinate Transformations

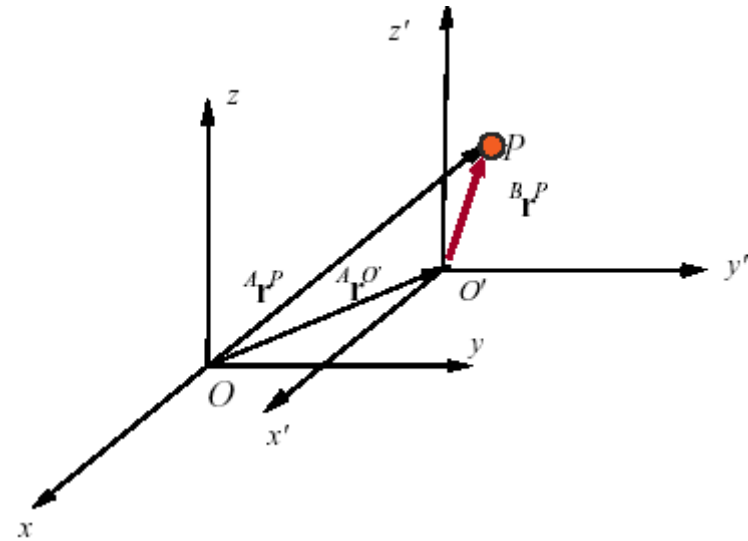
- Two Special Cases

$${}^A r^P = {}^A R_B {}^B r^P + {}^A r^{O'}$$

- Translation only

- Axes of  $\{B\}$  and  $\{A\}$  are parallel

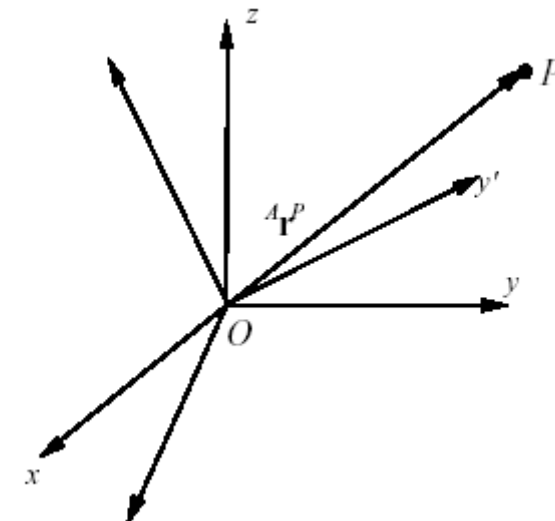
$${}^A R_B = 1$$



- Rotation only

- Origins of  $\{B\}$  and  $\{A\}$  are coincident

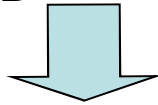
$${}^A r^{O'} = 0$$



# Homogeneous Representation

- Coordinate transformation from  $\{B\}$  to  $\{A\}$

$${}^A r^P = {}^A R_B {}^B r^P + {}^A r^{o'}$$



$$\begin{bmatrix} {}^A r^P \\ 1 \end{bmatrix} = \begin{bmatrix} {}^A R_B & {}^A r^{o'} \\ 0_{1 \times 3} & 1 \end{bmatrix} \begin{bmatrix} {}^B r^P \\ 1 \end{bmatrix}$$

- Homogeneous transformation matrix

$${}^A T_B = \begin{bmatrix} {}^A R_B & {}^A r^{o'} \\ 0_{1 \times 3} & 1 \end{bmatrix} = \begin{bmatrix} \boxed{R_{3 \times 3}} & \boxed{P_{3 \times 1}} \\ 0 & \boxed{1} \end{bmatrix}$$

Rotation matrix

Position vector

Scaling

*Diagram description: The equation shows the decomposition of the homogeneous transformation matrix. The top-left 3x3 block is labeled 'Rotation matrix' (blue box), the top-right 3x1 block is labeled 'Position vector' (yellow box), and the bottom-right 1x1 block is labeled 'Scaling' (red box). Arrows point from these labels to their respective blocks in the matrix. A curved arrow also points from the 'Position vector' label to the 'Rotation matrix' label.*



# Homogeneous Transformation

- Special cases

1. Translation

$${}^A T_B = \begin{bmatrix} I_{3 \times 3} & {}^A r^{o'} \\ 0_{1 \times 3} & 1 \end{bmatrix}$$

2. Rotation

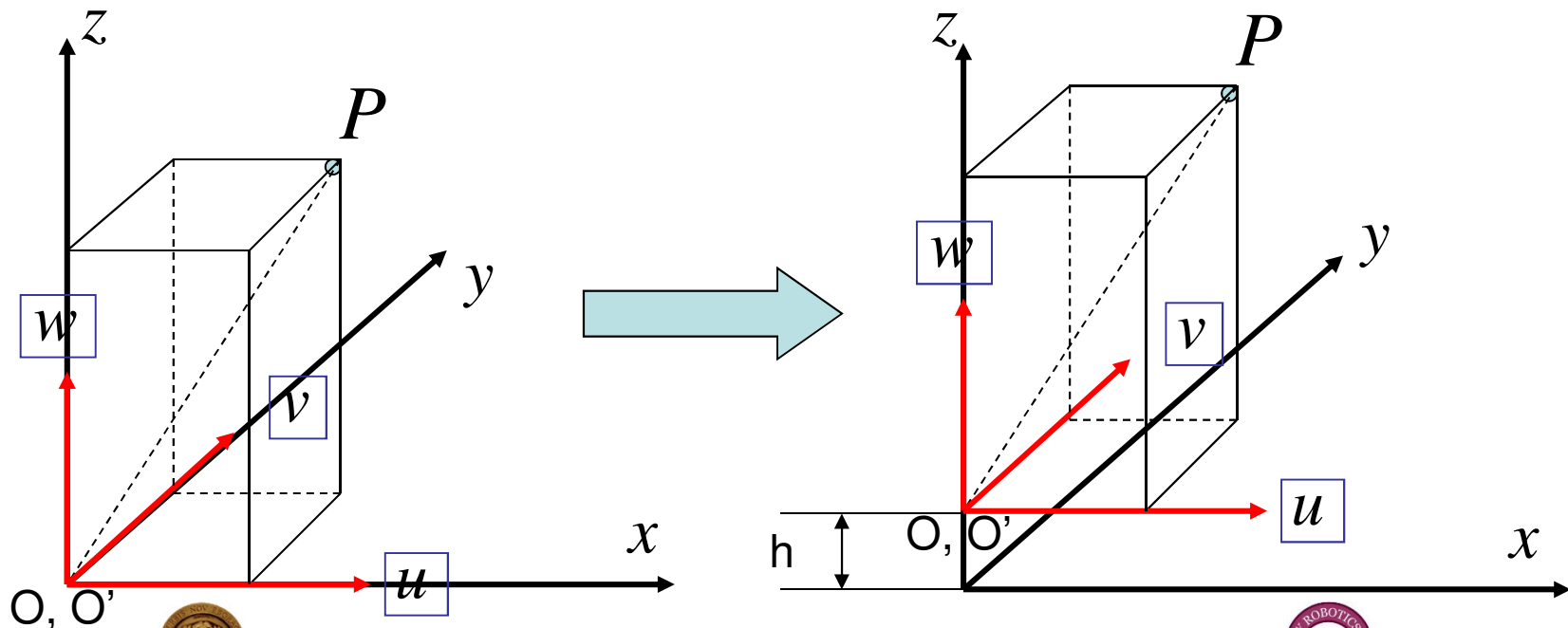
$${}^A T_B = \begin{bmatrix} {}^A R_B & 0_{3 \times 1} \\ 0_{1 \times 3} & 1 \end{bmatrix}$$



# Example 5

- Translation along Z-axis with  $h$ :

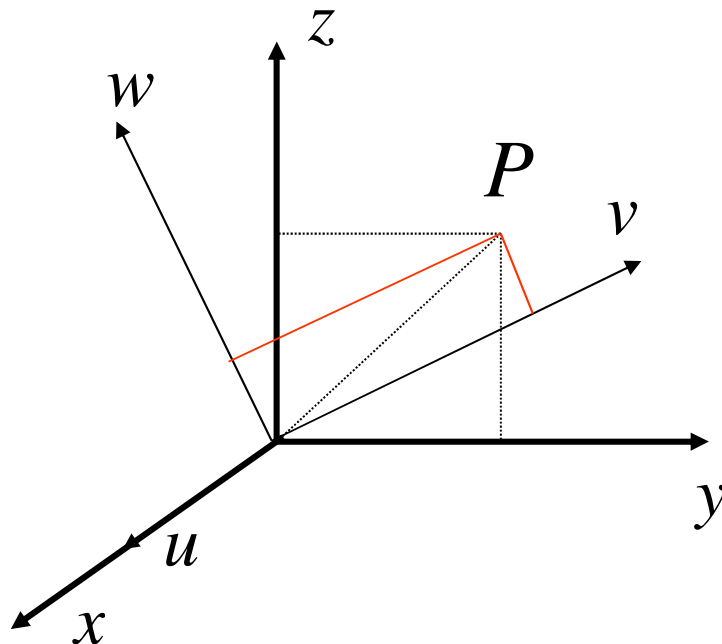
$$Trans(z, h) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & h \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & h \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_u \\ p_v \\ p_w \\ 1 \end{bmatrix} = \begin{bmatrix} p_u \\ p_v \\ p_w + h \\ 1 \end{bmatrix}$$



# Example 6

- Rotation about the X-axis by

$$Rot(x, \theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C\theta & -S\theta & 0 \\ 0 & S\theta & C\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C\theta & -S\theta & 0 \\ 0 & S\theta & C\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_u \\ p_v \\ p_w \\ 1 \end{bmatrix}$$



# Homogeneous Transformation

- Composite Homogeneous Transformation Matrix
- Rules:
  - Transformation (rotation/translation) w.r.t (X,Y,Z) (OLD FRAME), using pre-multiplication
  - Transformation (rotation/translation) w.r.t (U,V,W) (NEW FRAME), using post-multiplication



# Example 7

- Find the homogeneous transformation matrix (T) for the following operations:

Rotation  $\alpha$  about OX axis

$$T = R_{x,\alpha} I_{4 \times 4}$$

Translation of a along OX axis

Translation of d along OZ axis

Rotation of  $\theta$  about OZ axis

$$T = T_{z,\theta} T_{z,d} T_{x,a} T_{x,\alpha} I_{4 \times 4}$$

*Answer :*

$$= \begin{bmatrix} C\theta & -S\theta & 0 & 0 \\ S\theta & C\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C\alpha & -S\alpha & 0 \\ 0 & S\alpha & C\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

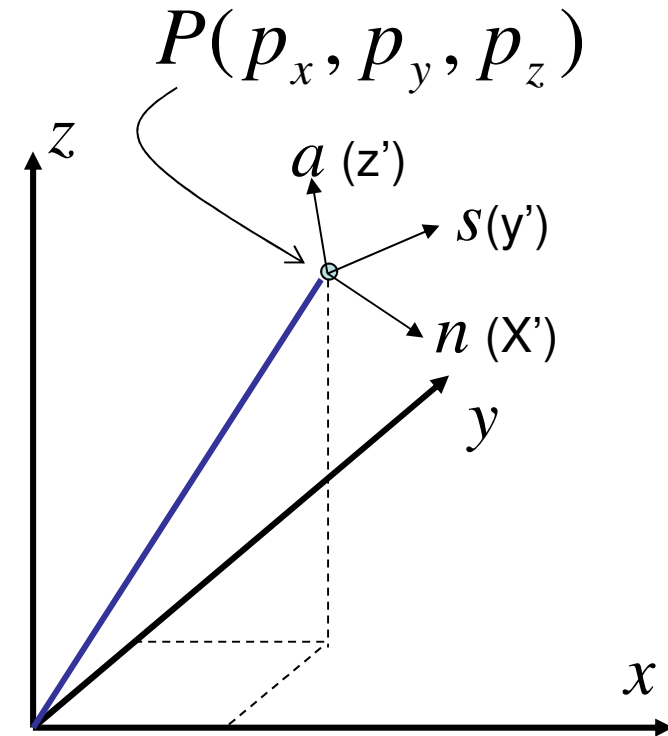


# Homogeneous Representation

- A frame in space (Geometric Interpretation)

$$F = \begin{bmatrix} R_{3 \times 3} & P_{3 \times 1} \\ 0 & 1 \end{bmatrix}$$

$$F = \begin{bmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Principal axis  $n$  w.r.t. the reference coordinate system



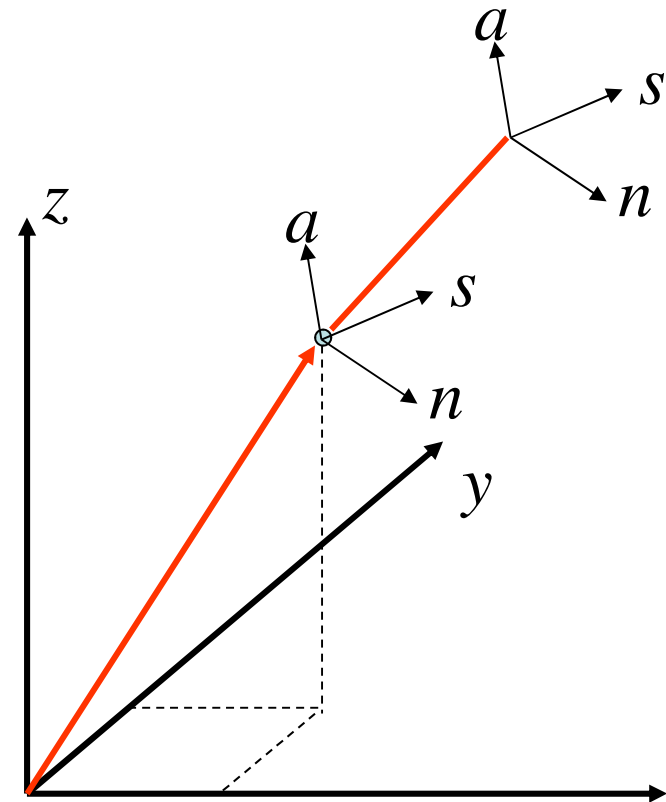


# Homogeneous Transformation

- Translation

$$F_{new} = \begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} n_x & s_x & a_x & p_x + d_x \\ n_y & s_y & a_y & p_y + d_y \\ n_z & s_z & a_z & p_z + d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

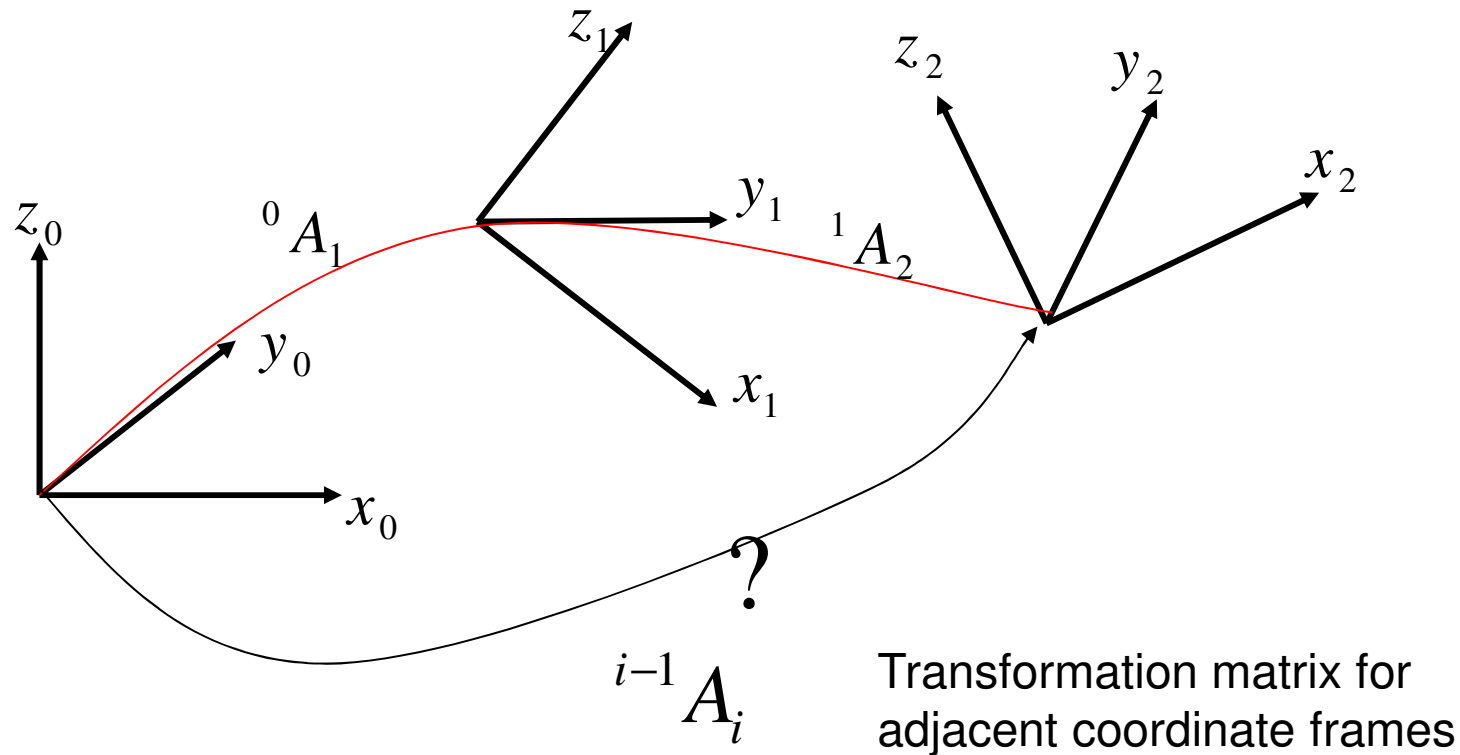


$$F_{new} = Trans(d_x, d_y, d_z) \times F_{old}$$



# Homogeneous Transformation

## Composite Homogeneous Transformation Matrix



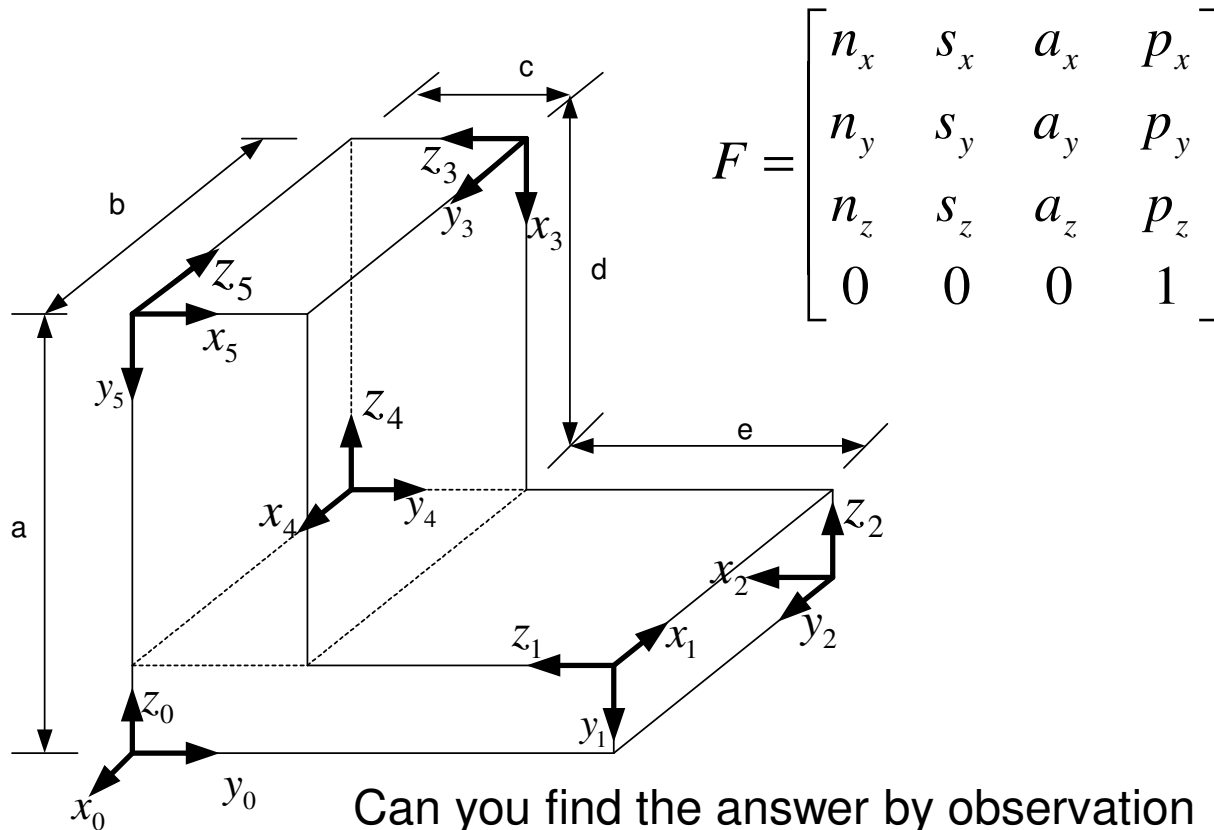
$${}^0A_2 = {}^0A_1 {}^1A_2$$

Chain product of successive coordinate transformation matrices



# Example 8

- For the figure shown below, find the 4x4 homogeneous transformation matrices  ${}^{i-1}A_i$  and  ${}^0A_i$  for  $i=1, 2, 3, 4, 5$



$${}^0A_1 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & e+c \\ 0 & -1 & 0 & a-d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1A_2 = \begin{bmatrix} 0 & -1 & 0 & b \\ 0 & 0 & -1 & a-d \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0A_2 = \begin{bmatrix} 0 & 1 & 0 & -b \\ -1 & 0 & 0 & e+c \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Can you find the answer by observation based on the geometric interpretation of homogeneous transformation matrix?



# Orientation Representation

$$F = \begin{bmatrix} R_{3 \times 3} & P_{3 \times 1} \\ 0 & 1 \end{bmatrix}$$

- Rotation matrix representation needs 9 elements to completely describe the orientation of a rotating rigid body.
- Any easy way?

Euler Angles Representation



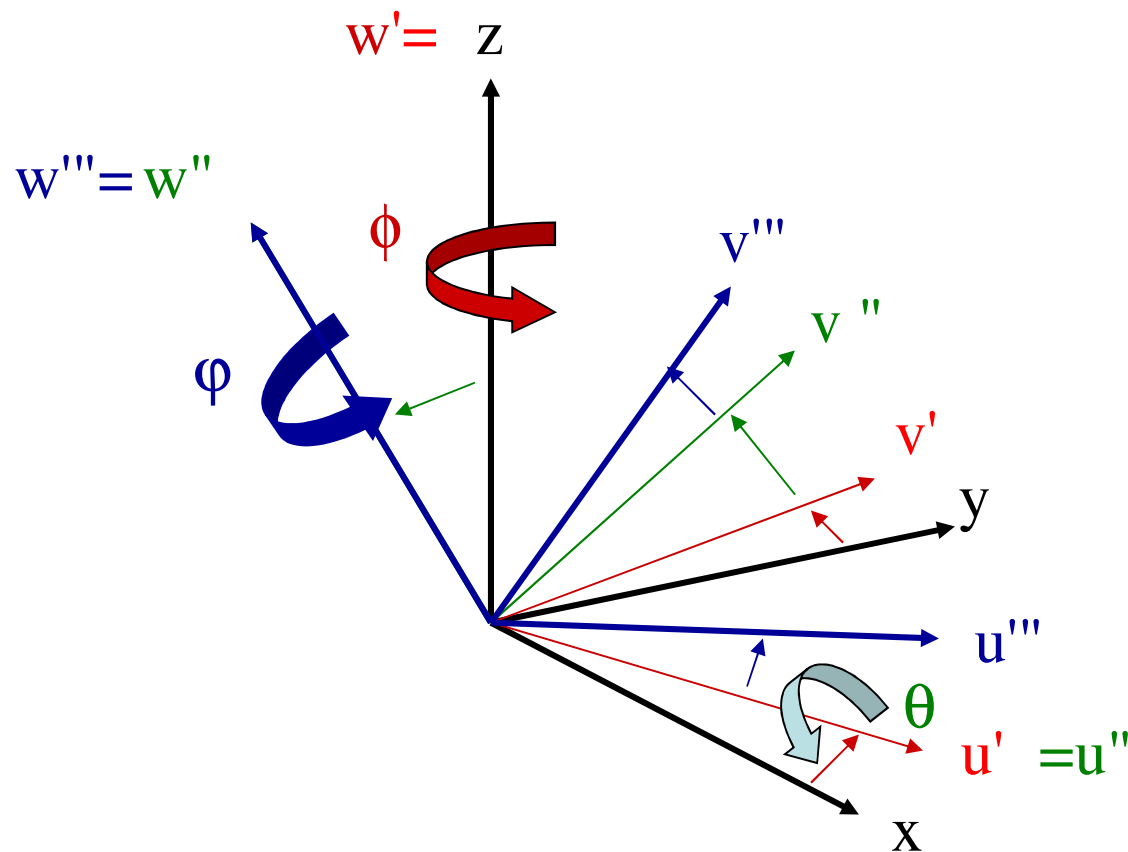
# Orientation Representation

- Euler Angles Representation (  $\phi, \theta, \psi$  )
  - Many different types
  - Description of Euler angle representations

	Euler Angle I	Euler Angle II	Roll-Pitch-Yaw
Sequence	$\phi$ about OZ axis	$\phi$ about OZ axis	$\psi$ about OX axis
of	$\theta$ about OU axis	$\theta$ about OV axis	$\theta$ about OY axis
Rotations	$\psi$ about OW axis	$\psi$ about OW axis	$\phi$ about OZ axis



# Euler Angle I, Animated



# Orientation Representation

- Euler Angle I

$$R_{z\phi} = \begin{pmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{pmatrix}, R_{u'\theta} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{pmatrix},$$

$$R_{w''\varphi} = \begin{pmatrix} \cos \varphi & -\sin \varphi & 0 \\ \sin \varphi & \cos \varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



# Euler Angle I

Resultant eulerian rotation matrix:

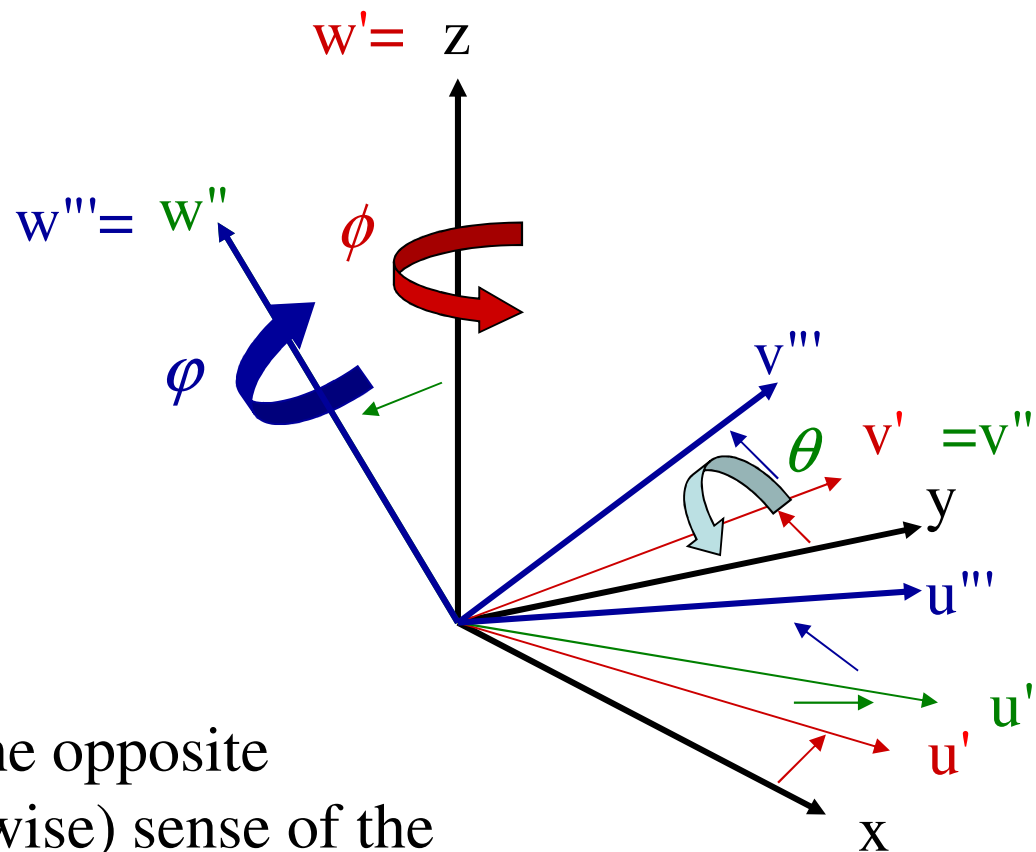
$$R = R_{z\phi} R_{u'\theta} R_{w''\varphi}$$

$$\begin{pmatrix} \cos \phi \cos \varphi & -\cos \phi \sin \varphi & \sin \varphi \sin \theta \\ -\sin \phi \sin \varphi \cos \theta & -\sin \phi \cos \varphi \cos \theta & \\ \sin \phi \cos \varphi & -\sin \phi \sin \varphi & -\cos \phi \sin \theta \\ +\cos \phi \sin \varphi \cos \theta & +\cos \phi \cos \varphi \cos \theta & \\ \sin \varphi \sin \theta & \cos \varphi \sin \theta & \cos \theta \end{pmatrix}$$





# Euler Angle II, Animated



Note the opposite  
(clockwise) sense of the  
third rotation,  $\phi$ .



# Orientation Representation

- Matrix with Euler Angle II

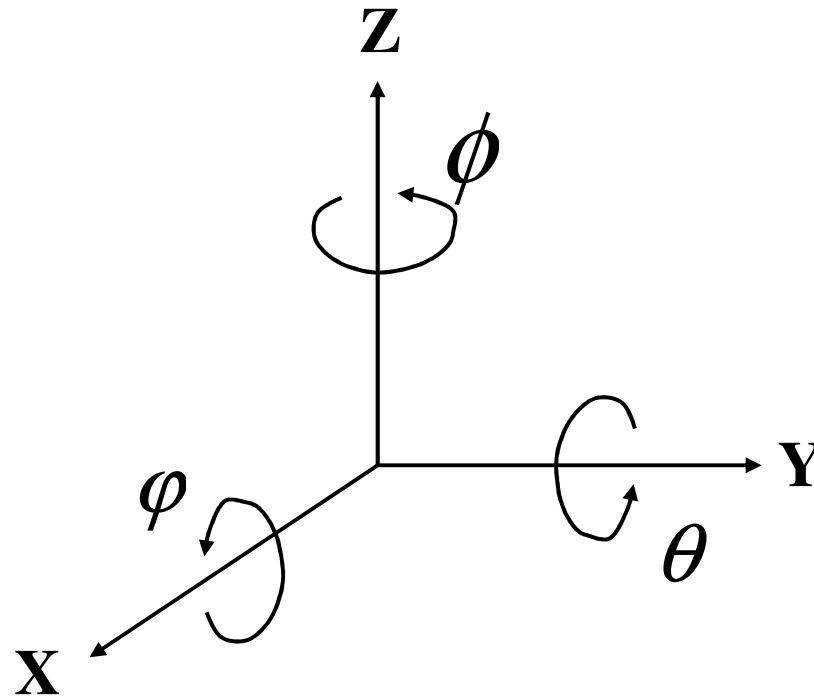
$$\begin{pmatrix} -\sin \phi \sin \varphi & -\sin \phi \cos \varphi & \cos \phi \sin \theta \\ +\cos \phi \cos \varphi \cos \theta & -\sin \phi \cos \varphi \cos \theta & \cos \phi \cos \theta \\ \cos \phi \sin \varphi & \cos \phi \cos \varphi & \sin \phi \sin \theta \\ +\sin \phi \cos \varphi \cos \theta & -\sin \phi \cos \varphi \cos \theta & \sin \phi \cos \theta \\ -\cos \phi \sin \theta & \sin \phi \sin \theta & \cos \theta \end{pmatrix}$$

Quiz: How to get this matrix ?



# Orientation Representation

- Description of Roll Pitch Yaw



Quiz: How to get rotation matrix ?

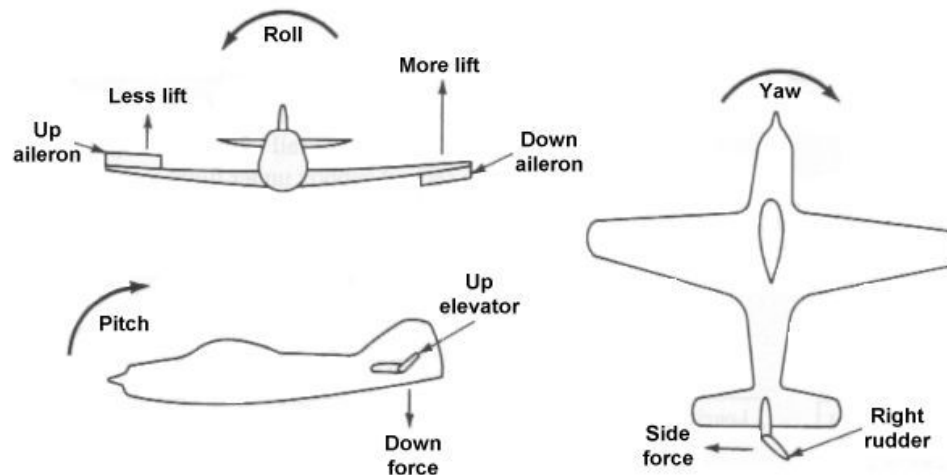
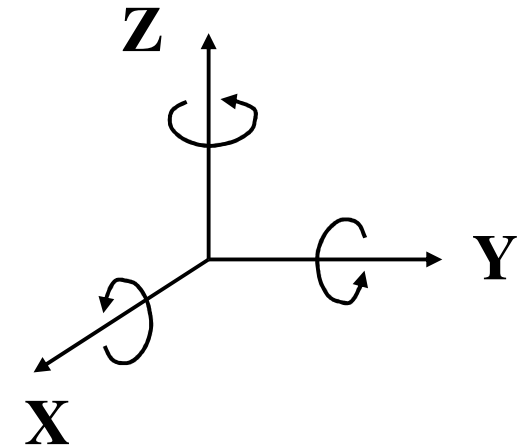


# Description of Roll Pitch Yaw

**X- Roll:** sağa-sola hareket

**Y- Pitch:** ileri-geri hareket

**Z- Yaw:** bulunduğu konumda sağa-sola döndürme



**Throttle:** deviri arttırmaya ve azaltmaya yarar

