Definition of the Laplace Transform

Let f be a function defined for $t \geq 0$. Then the integral

$$\boxed{L\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt}$$

is said to be the Laplace Transform of f, provided the integral converages

Evaluate $L\{1\}$

from laplace transform

$$L\{1\} = \int_0^\infty e^{-st}(1)dt = \lim_{b\to\infty}\int_0^b e^{-st}dt$$

$$= \lim_{b\to\infty}\frac{-e^{-st}}{s}\Big|_0^b = \lim_{b\to\infty}\frac{-e^{-st}+1}{s} = \frac{1}{s}$$

provided $s > 0$

In order words,

when $s > 0$ the exponent $-sb$ is negative

and $e^{-st} \to 0$ as $b \to \infty$.

The integral diverges for $s < 0$

Evaluate $L\{\mathrm{t}\}$

from laplace transform

$$L\{t\} = \int_0^{\infty} e^{-st} t dt = \lim_{b\to\infty} \int_0^{b} e^{-st} t dt$$

integrating ..

$$\lim_{t\to\infty} te^{-st} = 0, s > 0$$

$$L\{t\} = \frac{-te^{-st}}{s} \Big|_0^{\infty} + \frac{1}{s}\int_0^{\infty} e^{-st} dt = \frac{1}{s} L\{1\} = \frac{1}{s}\left(\frac{1}{s}\right) = \frac{1}{s^2}$$

$$\int f(x)g(x)\,dx = f(x)\int g(x)\,dx - \int \left(\int g(x)\,dx\right)\, d(f(x))$$

Evaluate $L\{e^{-3t}\}$

from laplace transform

$$L\{e^{-3t}\} = \int_0^{\infty} e^{-st} e^{-3t} dt \ = \int_0^{\infty} e^{-(s+3)t} dt$$

$$= \frac{-e^{-(s+3)t}}{s+3} \Big|_0^{\infty} = \frac{1}{s+3}, s > -3$$

Evaluate $L\{s\text{in}2\text{t}\}$

from laplace transform

$$L\{s\text{in}2\text{t}\} = \int_0^\infty e^{-st} s\text{in}2\text{t}dt \; = \frac{-e^{-st}\sin 2t}{s}\Big|_0^\infty + \frac{2}{s}\int_0^\infty e^{-st}\cos 2\text{t}dt$$

$$= \frac{2}{s}\int_0^\infty e^{-st}\cos 2\text{t}dt \quad , s > 0$$

$$\Rightarrow \frac{2}{s}\left[\frac{-e^{-st}\cos 2\text{t}dt}{s}\Big|_0^\infty - \frac{2}{s}\int_0^\infty e^{-st} s\text{in}2\text{t}dt\right]$$

$$= \frac{2}{s^2} - \frac{4}{s^2} L\{s\text{in}2\text{t}\}$$

$$\therefore L\{s\text{in}2\text{t}\} = \frac{2}{s^2+4}$$

Laplace Transforms of Some Basic Functions

$$(\mathrm{a})\ L\{1\} = \frac{1}{s}$$

$$(\mathrm{b})\ L\{t^n\} = \frac{n!}{s^{n+1}}, n = 1,2,3\ldots \qquad (c) L\{e^{at}\} = \frac{1}{s-a}$$

$$(d) L\{\sin kt\} = \frac{k}{s^2+k^2} \qquad (e) L\{\cos kt\} = \frac{s}{s^2+k^2}$$

$$(f) L\{\sinh kt\} = \frac{k}{s^2-k^2} \qquad (g) L\{\cosh kt\} = \frac{s}{s^2-k^2}$$

Laplace Transforms of Some Basic Functions

	$f(t)$	$\mathcal{L}(f)$
1	1	$1/s$
2	t	$1/s^2$
3	t^2	$2!/s^3$
4	t^n $(n = 0, 1, \cdots)$	$\dfrac{n!}{s^{n+1}}$
5	t^a (a positive)	$\dfrac{\Gamma(a+1)}{s^{a+1}}$
6	e^{at}	$\dfrac{1}{s-a}$

	$f(t)$	$\mathcal{L}(f)$
7	$\cos \omega t$	$\dfrac{s}{s^2+\omega^2}$
8	$\sin \omega t$	$\dfrac{\omega}{s^2+\omega^2}$
9	$\cosh at$	$\dfrac{s}{s^2-a^2}$
10	$\sinh at$	$\dfrac{a}{s^2-a^2}$
11	$e^{at}\cos \omega t$	$\dfrac{s-a}{(s-a)^2+\omega^2}$
12	$e^{at}\sin \omega t$	$\dfrac{\omega}{(s-a)^2+\omega^2}$

Some Inverse Transforms

$$\text{(a)}\ 1 = L^{-1}\left\{\frac{1}{s}\right\}$$

$$\text{(b)}\ t^n = L^{-1}\left\{\frac{n!}{s^{n+1}}\right\}, n = 1,2,3\ldots \qquad (c)\ e^{at} = L^{-1}\left\{\frac{1}{s-a}\right\}$$

$$(d)\ \sin kt = L^{-1}\left\{\frac{k}{s^2+k^2}\right\} \qquad (e)\ \cos kt = L^{-1}\left\{\frac{s}{s^2+k^2}\right\}$$

$$(f)\ \sinh kt = L^{-1}\left\{\frac{k}{s^2-k^2}\right\} \qquad (g)\ \cosh kt = L^{-1}\left\{\frac{s}{s^2-k^2}\right\}$$

Both L and L^{-1} are linear

Examples

(a) Evaluate $L^{-1}\{\frac{1}{s^5}\}$

$$L^{-1}\{\frac{1}{s^5}\} = \frac{1}{4!}L^{-1}\left\{\frac{4!}{s^5}\right\} = \frac{1}{24}t^4$$

(b) Evaluate $L^{-1}\{\frac{1}{s^2+7}\}$

$$L^{-1}\{\frac{1}{s^2+7}\} = \frac{1}{\sqrt{7}}L^{-1}\left\{\frac{\sqrt{7}}{s^2+7}\right\} = \frac{1}{\sqrt{7}}\sin\sqrt{7}t$$

Termwise Division and Linearity

Evaluate $L^{-1}\left\{\dfrac{-2s+6}{s^2+4}\right\}$

$$L^{-1}\left\{\frac{-2s+6}{s^2+4}\right\} = L^{-1}\left\{\frac{-2s}{s^2+4}+\frac{6}{s^2+4}\right\} = -2L^{-1}\left\{\frac{s}{s^2+4}\right\}+\frac{6}{2}L^{-1}\left\{\frac{2}{s^2+4}\right\}$$

$$= -2\cos 2t + 3\sin 2t$$

Partial Fraction Expansions

$$\frac{s+1}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$$

- Expand into a term for each factor in the denominator.
- Recombine RHS

$$\frac{s+1}{(s+2)(s+3)} = \frac{A(s+3) + B(s+2)}{(s+2)(s+3)}$$

- Equate terms in s and constant terms. Solve.
- Each term is in a form so that inverse Laplace transforms can be applied.

$$A+B=1 \qquad 3A+2B=1$$

$$\frac{s+1}{(s+2)(s+3)} = \frac{-1}{s+2} + \frac{2}{s+3}$$

Partial Fractions and Linearity

Evaluate $L^{-1}\left\{\dfrac{s^2+6s+9}{(s-1)(s-2)(s+4)}\right\}$

$$\frac{s^2+6s+9}{(s-1)(s-2)(s+4)} = \frac{A}{s-1}+\frac{B}{s-2}+\frac{C}{s+4}$$

$$=\frac{A(s-2)(s+4)+B(s-1)(s+4)+C(s-1)(s-2)}{(s-1)(s-2)(s+4)}$$

$$\to s^2+6s+9 = A(s-2)(s+4)+B(s-1)(s+4)+C(s-1)(s-2)$$

set s = 1,2,-4

$$16 = A(-1)(5),\ 25 = B(1)(6),\ 1 = C(-5)(-6)$$

$$..A=-\frac{16}{5}, B=\frac{25}{6}, C=\frac{1}{30}$$

$$L^{-1}\left\{\frac{s^2+6s+9}{(s-1)(s-2)(s+4)}\right\} = L^{-1}\left\{\frac{-\frac{16}{5}}{s-1}+\frac{\frac{25}{6}}{s-2}+\frac{\frac{1}{30}}{s+4}\right\}$$

$$=-\frac{16}{5}e^{t}+\frac{25}{6}e^{2t}+\frac{1}{30}e^{-4t}$$

Transform of a Derivative

If $f, f', \ldots f^{(n-1)}$ are continuous on $[0,\infty)$ and are of exponential order and if $f^{(n)}(t)$ is piecewise continuous on $[0,\infty)$, then

$$L\{f^{(n)}(t)\} = s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \ldots - f^{(n-1)}(0)$$

$$where\ F(s) = L\{f(t)\}$$

Laplace Transform of Derivatives

The transforms of the first and second derivatives of f(t) satisfy

$$\mathcal{L}(f') = s\mathcal{L}(f) - f(0)$$

$$\mathcal{L}(f'') = s^2\mathcal{L}(f) - sf(0) - f'(0).$$

Laplace Transform of the Derivative $f^{(n)}$ of Any Order

Let $f, f', \ldots, f^{(n-1)}$ be continuous for all $t \geq 0$ and satisfy the growth restriction.

Furthermore, let $f^{(n)}$ be piecewise continuous on every finite interval on the semi-axis $t \geq 0$. Then the transform of $f^{(n)}$ satisfies

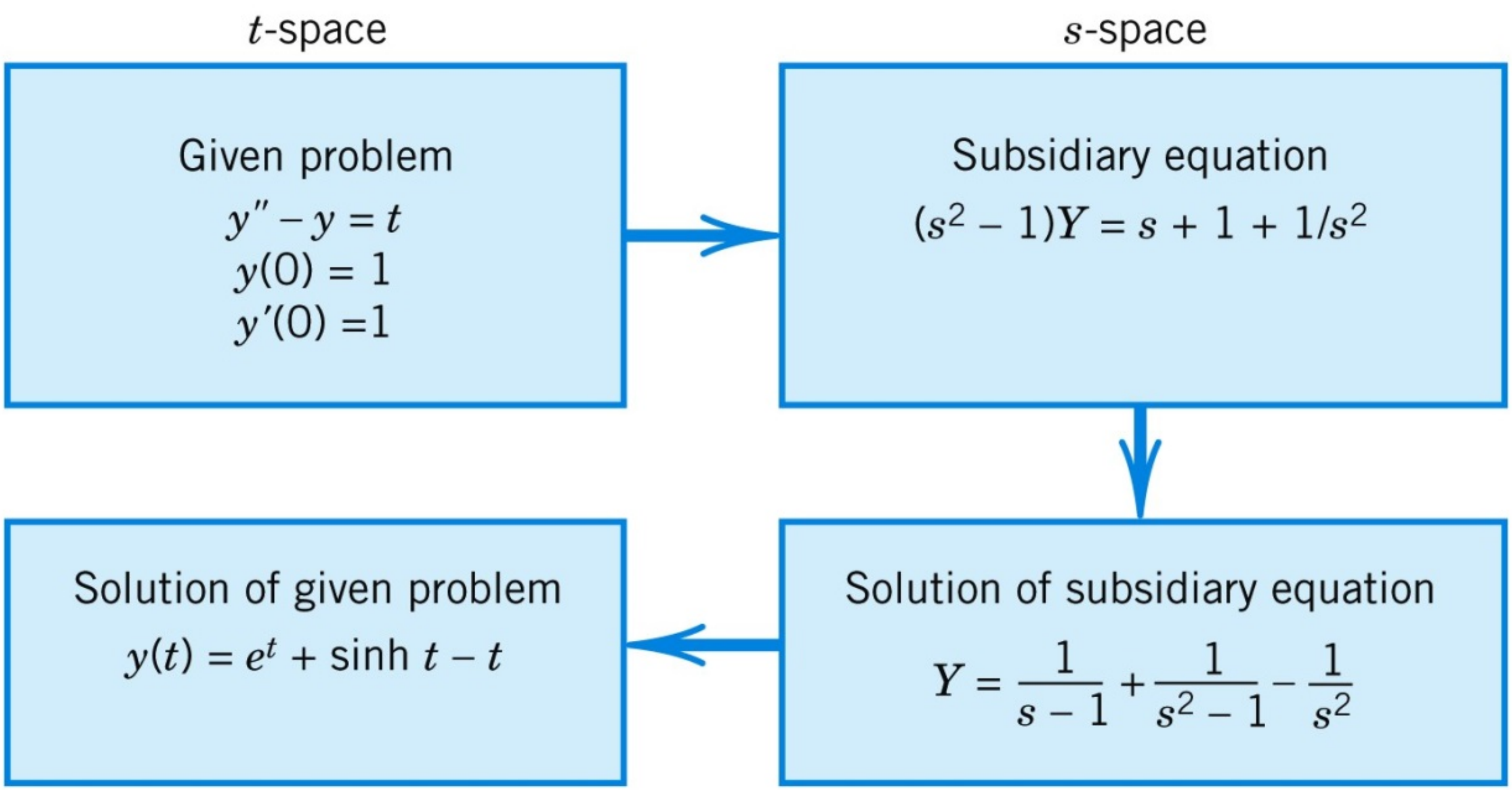
$$\mathcal{L}(f^{(n)}) = s^n \mathcal{L}(f) - s^{n-1} f(0) - s^{n-2} f'(0) - \ldots - f^{(n-1)}(0).$$

Laplace Transform of Integral

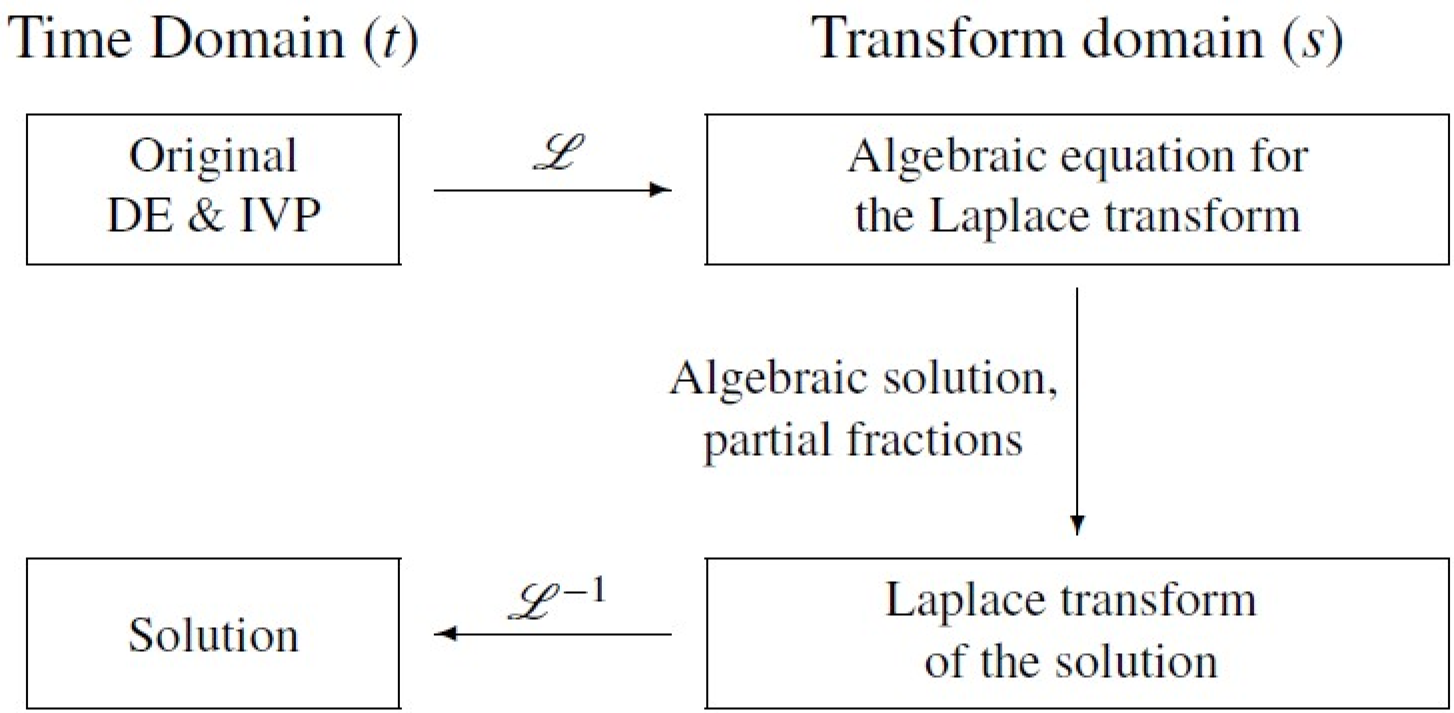
- *Let $F(s)$ denote the transform of a function $f(t)$ which is piecewise continuous for $t \geq 0$ and satisfies a growth restriction* (2), *Sec.* 6.1. *Then, for $s > 0$, $s > k$, and $t > 0$,*

$$\mathcal{L}\left\{\int_0^t f(\tau)\,d\tau\right\} = \frac{1}{s}F(s), \quad \text{thus} \quad \int_0^t f(\tau)\,d\tau = \mathcal{L}^{-1}\left\{\frac{1}{s}F(s)\right\}.$$

Initial Value Problem: The Basic Laplace Steps



How Laplace Transforms Turn Initial Value Problems Into Algebraic Equations



$solve$ the IVP) $\dfrac{dy}{dt} + 3y = 13\sin 2t,\ \ y(0) = 6$

$$L\left\{\frac{dy}{dt}\right\} + 3L\{y\} = 13L\{\sin 2t\}$$

$$L\left\{\frac{dy}{dt}\right\} = sY(s) - y(0) = sY(s) - 6$$

Since $\quad L\{\sin 2t\} = 2/(s^2+4)$

$$sY(s) - 6 + 3Y(s) = \frac{26}{s^2+4}, (s+3)Y(s) = 6 + \frac{26}{s^2+4}$$

$$Y(s) = \frac{6}{s+3} + \frac{26}{(s+3)(s^2+4)} = \frac{6s^2+50}{(s+3)(s^2+4)}$$

$$\frac{6s^2+50}{(s+3)(s^2+4)} = \frac{A}{s+3} + \frac{Bs+C}{s^2+4}$$

set s = -3, A = 8

$$6 = A + B, 0 = 3B + C, \rightarrow B = -2, C = 6$$

$$\frac{6s^2+50}{(s+3)(s^2+4)} = \frac{8}{s+3} + \frac{-2s+6}{s^2+4}$$

$$y(t) = 8L^{-1}\left\{\frac{1}{s+3}\right\} - 2L^{-1}\left\{\frac{s}{s^2+4}\right\} + 3L^{-1}\left\{\frac{2}{s^2+4}\right\}$$

$$= 8e^{-3t} - 2\cos 2t + 3\sin 2t$$

How Laplace Transforms Turn Initial Value Problems Into Algebraic Equations

1. The first key property of the Laplace transform is the way derivatives are transformed.
   1.1 $\mathscr{L}\{y\}(s) =: Y(s)$ (This is just notation.)
   1.2 $\mathscr{L}\{y'\}(s) = sY(s) - y(0)$
   1.3 $\mathscr{L}\{y''\}(s) = s^2Y(s) - sy(0) - y'(0)$
   1.4 $\mathscr{L}\left\{y^{(n)}(t)\right\}(s) = s^nY(s) - s^{n-1}y(0) - s^{n-2}y'(0) - \cdots - y^{(n-1)}(0)$
2. The right sides above do not involve derivatives of whatever Y is.
3. The other key property is that constants and sums “factor through” the Laplace transform:
   $\mathscr{L}\{f+g\} = \mathscr{L}\{f\} + \mathscr{L}\{g\}$ and $\mathscr{L}\{af\} = a\mathscr{L}\{f\}$.
   (That is, the Laplace transform is linear.)

Solve the Initial Value Problem

$y'' + 7y' + 12y = 0,\ y(0) = 1,\ y'(0) = 2$

Finding the Laplace transform of the solution.

$$
\begin{aligned}
y'' + 7y' + 12y &= 0,\ y(0) = 1,\ y'(0) = 2 \\
s^2Y - s - 2 + 7sY - 7 + 12Y &= 0 \\
\left(s^2 + 7s + 12\right)Y &= s + 9 \\
Y &= \frac{s+9}{s^2 + 7s + 12} \\
&= \frac{s+9}{(s+3)(s+4)}
\end{aligned}
$$

Solve the Initial Value Problem

$y'' + 7y' + 12y = 0,\ y(0) = 1,\ y'(0) = 2$

Partial fraction decomposition.

$$
\begin{aligned}
Y = \frac{s+9}{(s+3)(s+4)} &= \frac{A}{s+3} + \frac{B}{s+4} \\
\frac{s+9}{(s+3)(s+4)} &= \frac{A(s+4)+B(s+3)}{(s+3)(s+4)} \\
s+9 &= A(s+4)+B(s+3) \\
& \quad \text{Heaviside's Method :} \\
s=-3 & \quad A=6 \\
s=-4 & \quad B=-5 \\
Y &= \frac{6}{s+3} - \frac{5}{s+4}
\end{aligned}
$$

Solve the Initial Value Problem

$y'' + 7y' + 12y = 0,\ y(0) = 1,\ y'(0) = 2$

Inverting the Laplace Transform.

$$Y = \frac{6}{s+3} - \frac{5}{s+4}$$

Use the transform table.

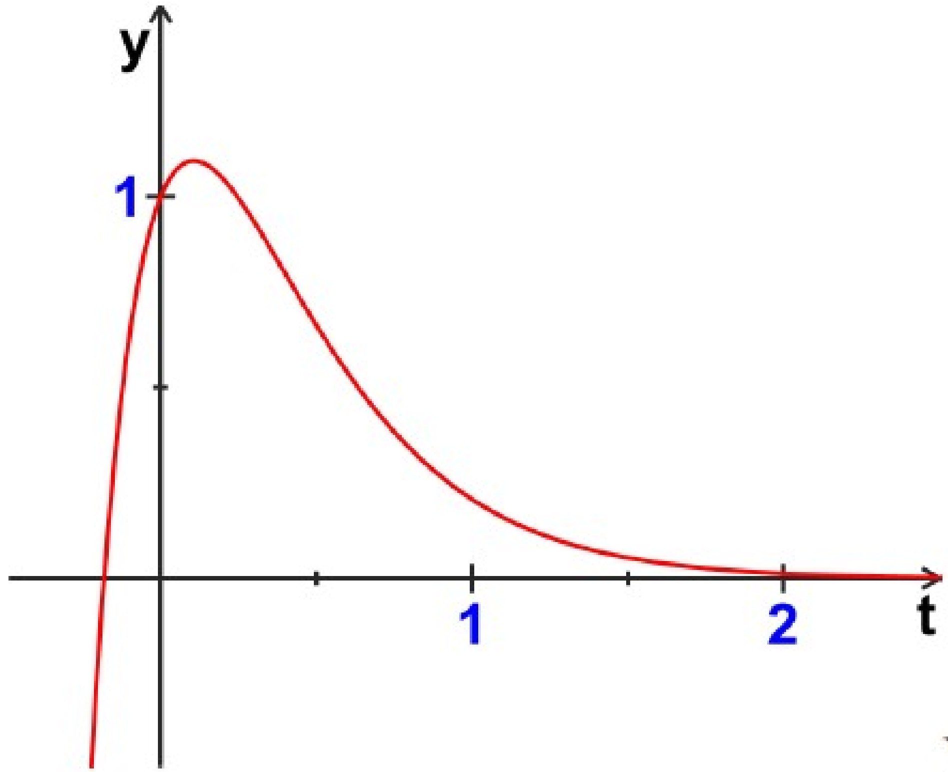
$$\mathscr{L}\left\{e^{at}\right\}(s) = \frac{1}{s-a}$$

$$= 6\frac{1}{s-(-3)} - 5\frac{1}{s-(-4)}$$

$$y = 6e^{-3t} - 5e^{-4t}$$

Solve the Initial Value Problem

$y'' + 7y' + 12y = 0,\ y(0) = 1,\ y'(0) = 2$



$$y = 6e^{-3t} - 5e^{-4t}$$

Does $y = 6e^{-3t} - 5e^{-4t}$ Really Solve the Initial Value Problem $y'' + 7y' + 12y = 0$, $y(0) = 1$, $y'(0) = 2$?

Checking the differential equation.

$$
\begin{aligned}
y'' + 7y' + 12y &\stackrel{?}{=} 0 \\
\left(54e^{-3t} - 80e^{-4t}\right) + 7\left(-18e^{-3t} + 20e^{-4t}\right) + 12\left(6e^{-3t} - 5e^{-4t}\right) &\stackrel{?}{=} 0 \\
(54 - 126 + 72)e^{-3t} + (-80 + 140 - 60)e^{-4t} &\stackrel{?}{=} 0 \\
0 &\stackrel{\surd}{=} 0
\end{aligned}
$$

Does $y = 6e^{-3t} - 5e^{-4t}$ Really Solve the Initial Value Problem $y'' + 7y' + 12y = 0$, $y(0) = 1$, $y'(0) = 2$?

Checking the initial values.

$$
\begin{aligned}
y &= 6e^{-3t} - 5e^{-4t} \\
y(0) &= 6 - 5 = 1 \quad \surd \\
y' &= -18e^{-3t} + 20e^{-4t} \\
y'(0) &= -18 + 20 = 2 \quad \surd
\end{aligned}
$$

Laplace Transform: General Formulas

Formula	Name, Comments
$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt$ $f(t) = \mathcal{L}^{-1}\{F(s)\}$	Definition of Transform Inverse Transform
$\mathcal{L}\{af(t) + bg(t)\} = a\mathcal{L}\{f(t)\} + b\mathcal{L}\{g(t)\}$	Linearity
$\mathcal{L}\{e^{at}f(t)\} = F(s-a)$ $\mathcal{L}^{-1}\{F(s-a)\} = e^{at}f(t)$	s-Shifting (First Shifting Theorem)

Laplace Transform: General Formulas

$\mathcal{L}(f') = s\mathcal{L}(f) - f(0)$ $\mathcal{L}(f'') = s^2\mathcal{L}(f) - sf(0) - f'(0)$ $\mathcal{L}(f^{(n)}) = s^n\mathcal{L}(f) - s^{(n-1)}f(0) - \cdots$ $\cdots - f^{(n-1)}(0)$	Differentiation of Function
$\mathcal{L}\left\{\int_0^t f(\tau)\,d\tau\right\} = \frac{1}{s}\mathcal{L}(f)$	Integration of Function
$(f * g)(t) = \int_0^t f(\tau)g(t-\tau)\,d\tau$ $= \int_0^t f(t-\tau)g(\tau)\,d\tau$ $\mathcal{L}(f * g) = \mathcal{L}(f)\mathcal{L}(g)$	Convolution

Laplace Transform: General Formulas

$\mathcal{L}\{f(t-a)u(t-a)\} = e^{-as}F(s)$ $\mathcal{L}^{-1}\{e^{-as}F(s)\} = f(t-a)u(t-a)$	t-Shifting (Second Shifting Theorem)
$\mathcal{L}\{tf(t)\} = -F'(s)$ $\mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(\widetilde{s})\,d\widetilde{s}$	Differentiation of Transform Integration of Transform
$\mathcal{L}(f) = \frac{1}{1-e^{-ps}}\int_0^p e^{-st}f(t)\,dt$	f Periodic with Period p

Laplace Transform: General Formulas

$\mathscr{L}(f') = s\mathscr{L}(f) - f(0)$ $\mathscr{L}(f'') = s^2\mathscr{L}(f) - sf(0) - f'(0)$ $\mathscr{L}(f^{(n)}) = s^n\mathscr{L}(f) - s^{(n-1)}f(0) - \cdots$ $\cdots - f^{(n-1)}(0)$	Differentiation of Function
$\mathscr{L}\left\{\int_0^t f(\tau)\,d\tau\right\} = \frac{1}{s}\mathscr{L}(f)$	Integration of Function
$(f * g)(t) = \int_0^t f(\tau)g(t-\tau)\,d\tau$ $= \int_0^t f(t-\tau)g(\tau)\,d\tau$ $\mathscr{L}(f * g) = \mathscr{L}(f)\mathscr{L}(g)$	Convolution

Laplace Transform: General Formulas

$\mathcal{L}\{f(t-a)u(t-a)\} = e^{-as}F(s)$ $\mathcal{L}^{-1}\{e^{-as}F(s)\} = f(t-a)u(t-a)$	t-Shifting (Second Shifting Theorem)
$\mathcal{L}\{tf(t)\} = -F'(s)$ $\mathcal{L}\left\{\dfrac{f(t)}{t}\right\} = \displaystyle\int_s^{\infty} F(\tilde{s})\,d\tilde{s}$	Differentiation of Transform Integration of Transform
$\mathcal{L}(f) = \dfrac{1}{1-e^{-ps}}\displaystyle\int_0^p e^{-st}f(t)\,dt$	f Periodic with Period p

Table of Laplace Transforms

	$F(s) = \mathcal{L}\{f(t)\}$	$f(t)$
1	$1/s$	1
2	$1/s^2$	t
3	$1/s^n \quad (n = 1, 2, \cdots)$	$t^{n-1}/(n-1)!$
4	$1/\sqrt{s}$	$1/\sqrt{\pi t}$
5	$1/s^{3/2}$	$2\sqrt{t/\pi}$
6	$1/s^a \quad (a > 0)$	$t^{a-1}/\Gamma(a)$
7	$\dfrac{1}{s-a}$	e^{at}
8	$\dfrac{1}{(s-a)^2}$	te^{at}
9	$\dfrac{1}{(s-a)^n} \quad (n = 1, 2, \cdots)$	$\dfrac{1}{(n-1)!}t^{n-1}e^{at}$
10	$\dfrac{1}{(s-a)^k} \quad (k > 0)$	$\dfrac{1}{\Gamma(k)}t^{k-1}e^{at}$

Table of Laplace Transforms

11	$\dfrac{1}{(s-a)(s-b)} \qquad (a \neq b)$	$\dfrac{1}{a-b}(e^{at}-e^{bt})$
12	$\dfrac{s}{(s-a)(s-b)} \qquad (a \neq b)$	$\dfrac{1}{a-b}(ae^{at}-be^{bt})$
13	$\dfrac{1}{s^2+\omega^2}$	$\dfrac{1}{\omega}\sin \omega t$
14	$\dfrac{s}{s^2+\omega^2}$	$\cos \omega t$
15	$\dfrac{1}{s^2-a^2}$	$\dfrac{1}{a}\sinh at$
16	$\dfrac{s}{s^2-a^2}$	$\cosh at$
17	$\dfrac{1}{(s-a)^2+\omega^2}$	$\dfrac{1}{\omega}e^{at}\sinh \omega t$
18	$\dfrac{s-a}{(s-a)^2+\omega^2}$	$e^{at}\cos \omega t$

Table of Laplace Transforms

19	$\dfrac{1}{s(s^2 + \omega^2)}$	$\dfrac{1}{\omega^2}(1 - \cos \omega t)$
20	$\dfrac{1}{s^2(s^2 + \omega^2)}$	$\dfrac{1}{\omega^3}(\omega t - \sin \omega t)$
21	$\dfrac{1}{(s^2 + \omega^2)^2}$	$\dfrac{1}{2\omega^3}(\sin \omega t - \omega t \cos \omega t)$
22	$\dfrac{s}{(s^2 + \omega^2)^2}$	$\dfrac{t}{2\omega} \sin \omega t$
23	$\dfrac{s^2}{(s^2 + \omega^2)^2}$	$\dfrac{1}{2\omega}(\sin \omega t + \omega t \cos \omega t)$
24	$\dfrac{s}{(s^2 + a^2)(s^2 + b^2)} \quad (a^2 \neq b^2)$	$\dfrac{1}{b^2 - a^2}(\cos at - \cos bt)$

Table of Laplace Transforms

25	$\dfrac{1}{s^4 + 4k^4}$	$\dfrac{1}{4k^3}(\sin kt \cos kt - \cos kt \sinh kt)$
26	$\dfrac{s}{s^4 + 4k^4}$	$\dfrac{1}{2k^2} \sin kt \sinh kt$
27	$\dfrac{1}{s^4 - k^4}$	$\dfrac{1}{2k^3}(\sinh kt - \sin kt)$
28	$\dfrac{s}{s^4 - k^4}$	$\dfrac{1}{2k^2}(\cosh kt - \cos kt)$
29	$\sqrt{s-a} - \sqrt{s-b}$	$\dfrac{1}{2\sqrt{\pi t^3}}(e^{bt} - e^{at})$
30	$\dfrac{1}{\sqrt{s+a}\sqrt{s+b}}$	$e^{-(a+b)t/2} I_0\left(\dfrac{a-b}{2}t\right)$
31	$\dfrac{1}{\sqrt{s^2 + a^2}}$	$J_0(at)$

Table of Laplace Transforms

32	$\dfrac{s}{(s-a)^{3/2}}$	$\dfrac{1}{\sqrt{\pi t}}e^{at}(1+2at)$
33	$\dfrac{1}{(s^2-a^2)^k}$ $(k>0)$	$\dfrac{\sqrt{\pi}}{\Gamma(k)}\left(\dfrac{t}{2a}\right)^{k-1/2} I_{k-1/2}(at)$
34	e^{-as}/s	$u(t-a)$
35	e^{-as}	$\delta(t-a)$
36	$\dfrac{1}{s}e^{-k/s}$	$J_0(2\sqrt{kt})$
37	$\dfrac{1}{\sqrt{s}}e^{-k/s}$	$\dfrac{1}{\sqrt{\pi t}}\cos 2\sqrt{kt}$
38	$\dfrac{1}{s^{3/2}}e^{k/s}$	$\dfrac{1}{\sqrt{\pi k}}\sinh 2\sqrt{kt}$
39	$e^{-k\sqrt{s}}$ $(k>0)$	$\dfrac{k}{2\sqrt{\pi t^3}}e^{-k^2/4t}$

Table of Laplace Transforms

	$F(s) = \mathcal{L}\{f(t)\}$	$f(t)$
40	$\frac{1}{s}\ln s$	$-\ln t - \gamma \quad (\gamma \approx 0.5772)$
41	$\ln \frac{s-a}{s-b}$	$\frac{1}{t}(e^{bt} - e^{at})$
42	$\ln \frac{s^2+\omega^2}{s^2}$	$\frac{2}{t}(1 - \cos \omega t)$
43	$\ln \frac{s^2-a^2}{s^2}$	$\frac{2}{t}(1 - \cosh at)$
44	$\arctan \frac{\omega}{s}$	$\frac{1}{t}\sin \omega t$
45	$\frac{1}{s}\operatorname{arccot} s$	$\mathrm{Si}(t)$