

Char	Freq	Prob.	Range	CumFreq
		Total	CumFreq=	10
S	5	$5/10 = 0.5$	$[0.5, 1.0)$	5
W	1	$1/10 = 0.1$	$[0.4, 0.5)$	4
I	2	$2/10 = 0.2$	$[0.2, 0.4)$	2
M	1	$1/10 = 0.1$	$[0.1, 0.2)$	1
␣	1	$1/10 = 0.1$	$[0.0, 0.1)$	0

Table 2.46: Frequencies and Probabilities of Five Symbols.

Char.		The calculation of low and high
S	L	$0.0 + (1.0 - 0.0) \times 0.5 = 0.5$
	H	$0.0 + (1.0 - 0.0) \times 1.0 = 1.0$
W	L	$0.5 + (1.0 - 0.5) \times 0.4 = 0.70$
	H	$0.5 + (1.0 - 0.5) \times 0.5 = 0.75$
I	L	$0.7 + (0.75 - 0.70) \times 0.2 = 0.71$
	H	$0.7 + (0.75 - 0.70) \times 0.4 = 0.72$
S	L	$0.71 + (0.72 - 0.71) \times 0.5 = 0.715$
	H	$0.71 + (0.72 - 0.71) \times 1.0 = 0.72$
S	L	$0.715 + (0.72 - 0.715) \times 0.5 = 0.7175$
	H	$0.715 + (0.72 - 0.715) \times 1.0 = 0.72$
␣	L	$0.7175 + (0.72 - 0.7175) \times 0.0 = 0.7175$
	H	$0.7175 + (0.72 - 0.7175) \times 0.1 = 0.71775$
M	L	$0.7175 + (0.71775 - 0.7175) \times 0.1 = 0.717525$
	H	$0.7175 + (0.71775 - 0.7175) \times 0.2 = 0.717550$
I	L	$0.717525 + (0.71755 - 0.717525) \times 0.2 = 0.717530$
	H	$0.717525 + (0.71755 - 0.717525) \times 0.4 = 0.717535$
S	L	$0.717530 + (0.717535 - 0.717530) \times 0.5 = 0.7175325$
	H	$0.717530 + (0.717535 - 0.717530) \times 1.0 = 0.717535$
S	L	$0.7175325 + (0.717535 - 0.7175325) \times 0.5 = 0.71753375$
	H	$0.7175325 + (0.717535 - 0.7175325) \times 1.0 = 0.717535$

Table 2.47: The Process of Arithmetic Encoding.

Char.	Code-low	Range
S	$0.71753375 - 0.5 = 0.21753375$	$/0.5 = 0.4350675$
W	$0.4350675 - 0.4 = 0.0350675$	$/0.1 = 0.350675$
I	$0.350675 - 0.2 = 0.150675$	$/0.2 = 0.753375$
S	$0.753375 - 0.5 = 0.253375$	$/0.5 = 0.50675$
S	$0.50675 - 0.5 = 0.00675$	$/0.5 = 0.0135$
□	$0.0135 - 0 = 0.0135$	$/0.1 = 0.135$
M	$0.135 - 0.1 = 0.035$	$/0.1 = 0.35$
I	$0.35 - 0.2 = 0.15$	$/0.2 = 0.75$
S	$0.75 - 0.5 = 0.25$	$/0.5 = 0.5$
S	$0.5 - 0.5 = 0$	$/0.5 = 0$

Table 2.49: The Process of Arithmetic Decoding.

Char	Prob.	Range	Char	Prob.	Range
$a_1$	0.001838	[0.998162, 1.0)	eof	0.000001	[0.999999, 1.0)
$a_2$	0.975	[0.023162, 0.998162)	$a_1$	0.001837	[0.998162, 0.999999)
$a_3$	0.023162	[0.0, 0.023162)	$a_2$	0.975	[0.023162, 0.998162)
			$a_3$	0.023162	[0.0, 0.023162)

(a) (b)

Table 2.50: (Skewed) Probabilities of Three Symbols.

$a_2$	$0.0 + (1.0 - 0.0) \times 0.023162 = 0.023162$
	$0.0 + (1.0 - 0.0) \times 0.998162 = 0.998162$
$a_2$	$0.023162 + .975 \times 0.023162 = 0.04574495$
	$0.023162 + .975 \times 0.998162 = 0.99636995$
$a_1$	$0.04574495 + 0.950625 \times 0.998162 = 0.99462270125$
	$0.04574495 + 0.950625 \times 1.0 = 0.99636995$
$a_3$	$0.99462270125 + 0.00174724875 \times 0.0 = 0.99462270125$
	$0.99462270125 + 0.00174724875 \times 0.023162 = 0.994663171025547$
$a_3$	$0.99462270125 + 0.00004046977554749998 \times 0.0 = 0.99462270125$
	$0.99462270125 + 0.00004046977554749998 \times 0.023162 = 0.994623638610941$

Table 2.51: Encoding the String  $a_2a_2a_1a_3a_3$ .

Char.	Code-low	Range
$a_2$	$0.99462270125 - 0.023162 = 0.97146170125$	$/0.975 = 0.99636995$
$a_2$	$0.99636995 - 0.023162 = 0.97320795$	$/0.975 = 0.998162$
$a_1$	$0.998162 - 0.998162 = 0.0$	$/0.00138 = 0.0$
$a_3$	$0.0 - 0.0 = 0.0$	$/0.023162 = 0.0$
$a_3$	$0.0 - 0.0 = 0.0$	$/0.023162 = 0.0$

Table 2.52: Decoding the String  $a_2a_2a_1a_3a_3$ .

$a_3$	$0.0 + (1.0 - 0.0) \times 0.0 = 0.0$
	$0.0 + (1.0 - 0.0) \times 0.023162 = 0.023162$
$a_3$	$0.0 + .023162 \times 0.0 = 0.0$
	$0.0 + .023162 \times 0.023162 = 0.000536478244$
$a_3$	$0.0 + 0.000536478244 \times 0.0 = 0.0$
	$0.0 + 0.000536478244 \times 0.023162 = 0.000012425909087528$
$a_3$	$0.0 + 0.000012425909087528 \times 0.0 = 0.0$
	$0.0 + 0.000012425909087528 \times 0.023162 = 0.0000002878089062853235$
eof	$0.0 + 0.0000002878089062853235 \times 0.999999 = 0.0000002878086184764172$
	$0.0 + 0.0000002878089062853235 \times 1.0 = 0.0000002878089062853235$

Table 2.53: Encoding the String  $a_3a_3a_3a_3$ eof.

Char.	Code-low	Range
$a_3$	$0.0000002878086184764172-0 = 0.0000002878086184764172$	$/0.023162=0.00001242589666161891247$
$a_3$	$0.00001242589666161891247-0=0.00001242589666161891247$	$/0.023162=0.000536477707521756$
$a_3$	$0.000536477707521756-0 = 0.000536477707521756$	$/0.023162=0.023161976838$
$a_3$	$0.023161976838-0.0 = 0.023161976838$	$/0.023162=0.999999$
eof	$0.999999-0.999999 = 0.0$	$/0.000001=0.0$

Table 2.54: Decoding the String  $a_3a_3a_3a_3$ eof.

1	2	3	4	5
S	$L = 0 + (1 - 0) \times 0.5 = 0.5$	5000		5000
	$H = 0 + (1 - 0) \times 1.0 = 1.0$	9999		9999
W	$L = 0.5 + (1 - .5) \times 0.4 = 0.7$	7000	7	0000
	$H = 0.5 + (1 - .5) \times 0.5 = 0.75$	7499	7	4999
I	$L = 0 + (0.5 - 0) \times 0.2 = 0.1$	1000	1	0000
	$H = 0 + (0.5 - 0) \times 0.4 = 0.2$	1999	1	9999
S	$L = 0 + (1 - 0) \times 0.5 = 0.5$	5000		5000
	$H = 0 + (1 - 0) \times 1.0 = 1.0$	9999		9999
S	$L = 0.5 + (1 - 0.5) \times 0.5 = 0.75$	7500		7500
	$H = 0.5 + (1 - 0.5) \times 1.0 = 1.0$	9999		9999
□	$L = .75 + (1 - .75) \times 0.0 = 0.75$	7500	7	5000
	$H = .75 + (1 - .75) \times 0.1 = .775$	7749	7	7499
M	$L = 0.5 + (.75 - .5) \times 0.1 = .525$	5250	5	2500
	$H = 0.5 + (.75 - .5) \times 0.2 = 0.55$	5499	5	4999
I	$L = .25 + (.5 - .25) \times 0.2 = 0.3$	3000	3	0000
	$H = .25 + (.5 - .25) \times 0.4 = .35$	3499	3	4999
S	$L = 0.0 + (0.5 - 0) \times 0.5 = .25$	2500		2500
	$H = 0.0 + (0.5 - 0) \times 1.0 = 0.5$	4999		4999
S	$L = .25 + (.5 - .25) \times 0.5 = .375$	3750	3750	
	$H = .25 + (.5 - .25) \times 1.0 = 0.5$	4999		4999

Table 2.55: Encoding "SWISS<sub>□</sub>MISS" by Shifting.

- Here are all the decoding steps for our example:
0. Initialize Low=0000, High=9999, and Code=7175.
  1.  $\text{index} = [(7175 - 0 + 1) \times 10 - 1] / (9999 - 0 + 1) = 7.1759 \rightarrow 7$ . Symbol "S" is selected.

## 2.14 Arithmetic Coding

117

- Low =  $0 + (9999 - 0 + 1) \times 5/10 = 5000$ . High =  $0 + (9999 - 0 + 1) \times 10/10 - 1 = 9999$ .
2.  $\text{index} = [(7175 - 5000 + 1) \times 10 - 1] / (9999 - 5000 + 1) = 4.3518 \rightarrow 4$ . Symbol "W" is selected.  
Low =  $5000 + (9999 - 5000 + 1) \times 4/10 = 7000$ . High =  $5000 + (9999 - 5000 + 1) \times 5/10 - 1 = 7499$ .  
After the 7 is shifted out, we have Low=0000, High=4999, and Code=1753.
  3.  $\text{index} = [(1753 - 0 + 1) \times 10 - 1] / (4999 - 0 + 1) = 3.5078 \rightarrow 3$ . Symbol "I" is selected.  
Low =  $0 + (4999 - 0 + 1) \times 2/10 = 1000$ . High =  $0 + (4999 - 0 + 1) \times 4/10 - 1 = 1999$ .  
After the 1 is shifted out, we have Low=0000, High=9999, and Code=7533.
  4.  $\text{index} = [(7533 - 0 + 1) \times 10 - 1] / (9999 - 0 + 1) = 7.5339 \rightarrow 7$ . Symbol "S" is selected.  
Low =  $0 + (9999 - 0 + 1) \times 5/10 = 5000$ . High =  $0 + (9999 - 0 + 1) \times 10/10 - 1 = 9999$ .
  5.  $\text{index} = [(7533 - 5000 + 1) \times 10 - 1] / (9999 - 5000 + 1) = 5.0678 \rightarrow 5$ . Symbol "S" is selected.  
Low =  $5000 + (9999 - 5000 + 1) \times 5/10 = 7500$ . High =  $5000 + (9999 - 5000 + 1) \times 10/10 - 1 = 9999$ .
  6.  $\text{index} = [(7533 - 7500 + 1) \times 10 - 1] / (9999 - 7500 + 1) = 0.1356 \rightarrow 0$ . Symbol "L" is selected.  
Low =  $7500 + (9999 - 7500 + 1) \times 0/10 = 7500$ . High =  $7500 + (9999 - 7500 + 1) \times 1/10 - 1 = 7749$ .  
After the 7 is shifted out, we have Low=5000, High=7499, and Code=5337.
  7.  $\text{index} = [(5337 - 5000 + 1) \times 10 - 1] / (7499 - 5000 + 1) = 1.3516 \rightarrow 1$ . Symbol "M" is selected.  
Low =  $5000 + (7499 - 5000 + 1) \times 1/10 = 5250$ . High =  $5000 + (7499 - 5000 + 1) \times 2/10 - 1 = 5499$ .  
After the 5 is shifted out we have Low=2500, High=4999, and Code=3375.

8.  $\text{index} = [(3375 - 2500 + 1) \times 10 - 1] / (4999 - 2500 + 1) = 3.5036 \rightarrow 3$ . Symbol “I” is selected.

$\text{Low} = 2500 + (4999 - 2500 + 1) \times 2/10 = 3000$ .  $\text{High} = 2500 + (4999 - 2500 + 1) \times 4/10 - 1 = 3499$ .

After the 3 is shifted out we have  $\text{Low}=0000$ ,  $\text{High}=4999$ , and  $\text{Code}=3750$ .

9.  $\text{index} = [(3750 - 0 + 1) \times 10 - 1] / (4999 - 0 + 1) = 7.5018 \rightarrow 7$ . Symbol “S” is selected.

$\text{Low} = 0 + (4999 - 0 + 1) \times 5/10 = 2500$ .  $\text{High} = 0 + (4999 - 0 + 1) \times 10/10 - 1 = 4999$ .

10.  $\text{index} = [(3750 - 2500 + 1) \times 10 - 1] / (4999 - 2500 + 1) = 5.0036 \rightarrow 5$ . Symbol “S” is selected.

$\text{Low} = 2500 + (4999 - 2500 + 1) \times 5/10 = 3750$ .  $\text{High} = 2500 + (4999 - 2500 + 1) \times 10/10 - 1 = 4999$ .

1	2	3	4	5
1 L=0+(1 - 0)×0.0 = 0.0	000000	0	000000	
H=0+(1 - 0)×0.023162= 0.023162	023162	0	231629	
2 L=0+(0.231629 - 0)×0.0 = 0.0	000000	0	000000	
H=0+(0.231629 - 0)×0.023162= 0.00536478244	005364	0	053649	
3 L=0+(0.053649 - 0)×0.0 = 0.0	000000	0	000000	
H=0+(0.053649 - 0)×0.023162= 0.00124261813	001242	0	012429	
4 L=0+(0.012429 - 0)×0.0 = 0.0	000000	0	000000	
H=0+(0.012429 - 0)×0.023162= 0.00028788049	000287	0	002879	
5 L=0+(0.002879 - 0)×0.0 = 0.0	000000	0	000000	
H=0+(0.002879 - 0)×0.023162= 0.00006668339	000066	0	000669	

Table 2.56: Encoding  $a_3a_3a_3a_3a_3$  by Shifting.