Write the negation of each of the following propositions without using any form of the word "not":

Today is Thursday

Today is Monday or Tuesday or Wednesday or Friday or Saturday or Sunday

$2 + 1 = 3$

$2+1 < 3$ or $2+1 > 3$

There is no wind in Illinois

Illinois is windy.

The summer in Champaign is hot and muggy

The summer in Champaign is cold or dry

Let *p*, *q*, and *r*, be the propositions:

p: You get an A on the final exam.

q: You do every exercise in this book.

r: You get an A in this class.

Write these propositions using *p*, *q*, and *r*, and logical connectives.

You get an A in this class, but you do not do every exercise in this book.

$r \wedge \sim q$

You get an A on the final, you do every exercise in this book,and you get an A in this class.

$p \wedge q \wedge r$

To get an A in this class, it is necessary for you to get an A on the final

$p \rightarrow r$

You get an A on the final, but you don't do every exercise in this book;nevertheless, you get an A in this class

$p \wedge \sim q \wedge r$

Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class

$(p \wedge q) \rightarrow r$

You will get an A in this class if and only if you either do every exercise in this book, or you get an A on the final

$(r \leftrightarrow (p \vee q))$

- Restate each proposition in the form (if p then q) of a conditional proposition.

Joey will pass the discrete mathematics exam if he studies hard

If Joey studies hard then he will pass the discrete math. exam

A sufficient condition for Katrina to take the algorithms course is that she pass discrete mathematics

If Katrina passes discrete math. then she will to take the algorithms course

- **Write the converse of first proposition**

If Joey passes the discrete math.exam then he studied hard

- **Refer to the propositions p, q and r; p is true, q is false and r's status is unknown at this time. Tell whether each proposition is true, is false or has unknown status at this time**

$p \vee r$ TRUE $(p \wedge r) \leftrightarrow r$ TRUE

$q \rightarrow r$ TRUE $(q \vee r) \leftrightarrow r$ TRUE

Represent the given statement symbolically by letting

if 4 < 2 then 7 < 10 $p \rightarrow q$

7<10 if and only if (4 < 2 and 6 is not less than 6) $q \leftrightarrow (p \wedge \sim r)$

Formulate the symbolic expression in word using

p: Today is Monday

q: It is raining

r: It is hot

$p \rightarrow q$ If today is Monday then it is raining

$\sim(p \vee q) \leftrightarrow r$ It is not the case that today is Monday or it is raining if and only if it is hot

Use a truth table to determine whether the following is a tautology: $(\sim p \wedge (p \rightarrow q)) \rightarrow \sim q$

It is not a tautology

p	q	$\sim p$	$p \rightarrow q$	$\sim p \wedge (p \rightarrow q)$	$\sim q$	$(\sim p \wedge (p \rightarrow q)) \rightarrow \sim q$
T	T	F	T	F	F	T
T	F	F	F	F	T	T
F	T	T	T	T	F	F
F	F	T	T	T	T	T

Find a counterexample, if possible, to these universally quantified statements, where the universe of discourse for all variables consists of all integers

$\forall x\ \exists y\ (x = 1/y)$

Let $x = 0$. There is no integer y for which $0 = 1/y$

$\forall x\ \exists y\ (y^2 - x < 100)$

Let x be an integer < -100

$\forall x\ \forall y\ (x^2 \neq y^3)$

Let $(x,y) = (0,0)$ or $(1,1)$

Write the negation of each of the following:

Only students eat pizza

Someone who is not a student eats pizza

All students eat pizza

There is a student who doesn't eat pizza

Some students eat only pizza

No student eats only pizza. All students don't eat only pizza. All students eat something other than pizza

Prove that if n is a positive integer,
then n is even if and only if 7n + 4 is even

First assume that n is even, so that n=2k for some integer k. Then 7n+4 = 14k+4 = 2(7k+2) which is even. To prove the converse, suppose (indirectly) that n=2k+1 for some integer k. Then 7n+4 = 14k+7+4 = 14k+10+1 = 2(7k+5)+1 which is odd. Hence n is odd if and only if 7n+4 is odd

If Sherlock Holmes is successful, the Professor Albert will be apprehended. If Dr. Watson doesn't slip up, then Sherlock Holmes will locate the missing clue. If Sherlock Holmes locates the missing clue, then he is successful. Dr. Watson doesn't slip up. Therefore Professor Albert will be apprehended

p: Sherlock Holmes is successful
q: Professor Albert is apprehended
r: Dr. Watson slips up
s: Sherlock Holmes locates the missing clue

The hypotheses are:

$p \rightarrow q$
$\sim r \rightarrow s$
$s \rightarrow p$
$\sim r$

The conclusion is: q

Determine the truth value of each statement. The domain of discourse is the set of real number.

For every x, $x^2>x$ **False** a counterexample is $x=1/2$

For some x, if $x>1$, then $x^2>x$ **True**

For every x, for every y, $x^2<y+1$ **False** a counterexample is $x=2$ $y=0$

For some x, for some y, $x^2<y+1$ **True** $x=0$ $y=0$

For every x, for every y, $x^2+y^2=9$ **False** $x=y=2$

For some x, for some y, $x^2+y^2=9$ **True** $x=1$ $y=\sqrt{8}$

For some x, for every y, $x^2+y^2>=0$ **True** $x=0$ for all y