

# Deep Learning and Temporal Data Processing

## 1 - Deep Neural Networks

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Andrea Palazzi

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University of Modena and Reggio Emilia

**Introduction**

**Linear classifiers**

**Neural Networks**

**Training a DNN**

# Introduction

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## Linear classifiers

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For the purpose of this lecture, we'll stick to the task of image classification.  
Let's assume we have a training dataset of  $N$  images

$$x_i \in \mathbb{R}^D, i = 1, \dots, N$$

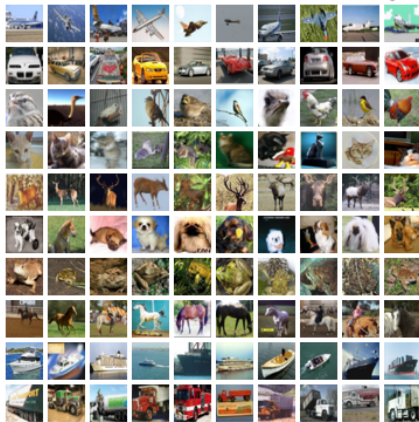
that we want to classify into  $K$  distinct classes.

Thus, training set is made by couples:

$$(x_i, y_i), \text{ where } y_i \in \{1, \dots, K\}$$

Our goal is to define a function  $f : \mathbb{R}^D \mapsto \mathbb{R}^K$  that maps images to class scores.

Making a real-world example: let's take the CIFAR-10 dataset, which consists of  $N = 60000$  32x32 RGB images belonging to 10 different classes.



Each image is  $32 \times 32 \times 3$ , thus it can be thought as a column vector  $x_i \in \mathbb{R}^{3072}$ .

Now we can define a **linear mapping**:

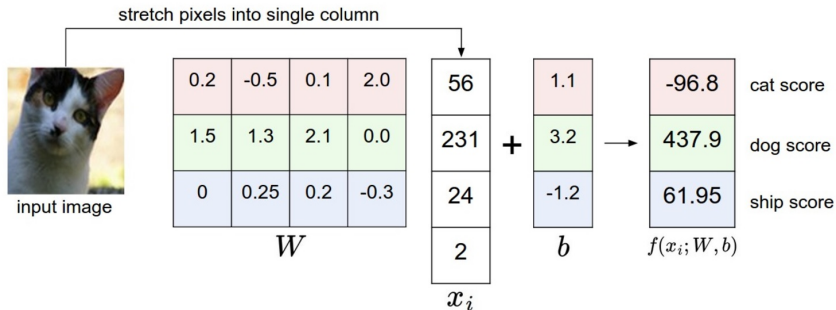
$$f(x_i, W, b) = Wx_i + b$$

where the parameters are:

- the weight matrix  $W \in \mathbb{R}^{10 \times 3072}$
- the bias vector  $b \in \mathbb{R}^{10}$ .

Intuitively, our goal is to learn the parameters from the training set *s.t.* when a new test image  $x_i^{test}$  is given as input, the score of the correct class is higher than the scores of other classes.





Example of mapping an image to a score. For the sake of visualization, here the image is assumed to have only 4 grayscale pixels.



First let's introduce the **softmax function**:

$$\text{softmax}_j(\mathbf{z}) = \frac{e^{z_j}}{\sum_k e^{z_k}}$$

It takes a vector of arbitrary real-valued scores  $\mathbf{z}$  and squashes it to a vector of values between zero and one that sum to one.

e.g.

$$\mathbf{z} = \begin{bmatrix} 1.2 \\ 5.1 \\ 2.7 \end{bmatrix} \quad \text{softmax}(\mathbf{z}) = \begin{bmatrix} 0.018 \\ 0.90 \\ 0.08 \end{bmatrix}$$

**Softmax Classifier** generalizes Logistic Regression classifier to multi-class classification.

In the Softmax classifier the scores of linear function mapping  $f(x_i, W) = Wx_i$  are interpreted as unnormalized log probabilities and we use the **cross-entropy loss**:

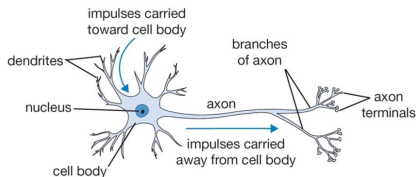
$$L_i = -\log \left( \frac{e^{f_{y_i}}}{\sum_j e^{f_j}} \right)$$

Prove yourself that this loss function makes sense.

# Neural Networks

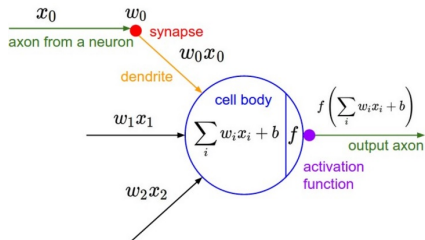
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Neural Networks are a mathematical model **coarsely** inspired by the way our own brain works.



Neurons are the basic computational unit of our brain. Approximately 86 billion neurons can be found in the human nervous system and they are connected with approximately  $10^{14}$  -  $10^{15}$  **synapses**. Each neuron receives input signals from its **dendrites** and produces output signals along its (single) **axon**. The axon eventually branches out and connects via synapses to dendrites of other neurons.

More formally, we can model a single **neuron** as follows:



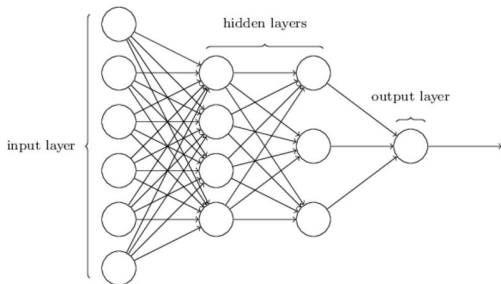
Each neuron can have multiple inputs. The neuron's output is the dot product between the inputs and its weights, plus the bias: then, a non-linearity is applied.

It's easy to see that a single neuron can be used to implement a binary classifier.

Indeed, when **cross-entropy loss** is applied to neuron's output, we can optimize a **binary Softmax classifier** (*a.k.a.* Logistic regression).



When we connect an ensemble of neurons in an acyclic graph is when the magic happens and we get an actual **neural network**.



Neural networks are arranged in **layers**, with one *input layer*, one *output layer* and  $N$  *hidden layers* in the middle.

*The network depicted here has a total of 47 learnable parameters. Does this make sense to you?*

From the computational point of view, we can express the 4-layer network previously depicted as follows.

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## Training a DNN

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Forward propagation is the process of computing the network output given its input.



[1] F. Yu and V. Koltun.

**Multi-scale context aggregation by dilated convolutions.**

*arXiv preprint arXiv:1511.07122*, 2015.