Deep Learning and Temporal Data Processing

0 - Gradient Descent

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Agenda

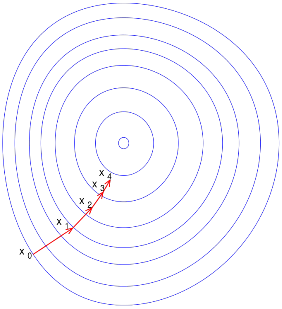
Gradient Descent

Credits

References

Gradient Descent

Gradient descent is an iterative optimization algorithm for finding the minimum of a function. How? Take step proportional to the negative of the gradient of the function at the current point.



Gradient descent on a series of level sets

Gradient Descent Update

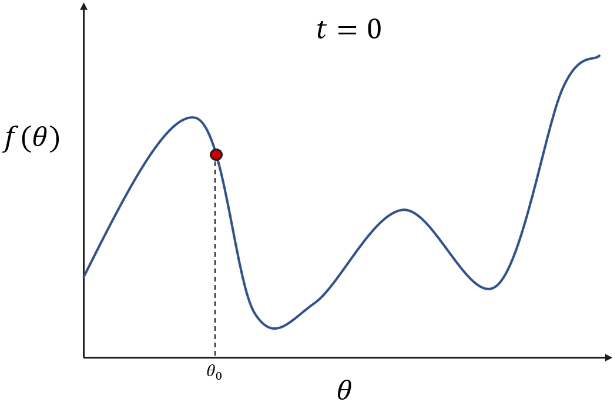
If we consider a function $f(\boldsymbol{\theta})$, the **gradient descent update** can be expressed as:

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} f(\boldsymbol{\theta}) \tag{1}$$

for each parameter θ_j.

The size of the step is controlled by **learning rate** α.

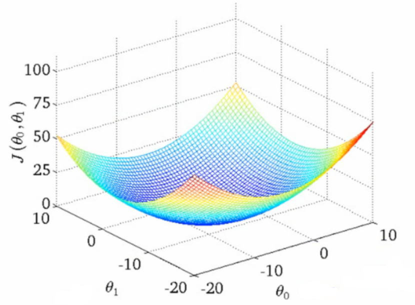
Gradient Descent for 1-d function $f(\theta)$.



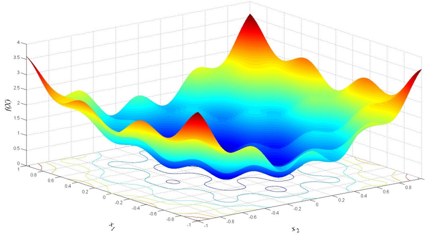
$$\frac{df}{d\theta} < 0$$

$$\theta_{new} = \theta_{old} - \alpha \frac{df}{d\theta}$$

Turns out that if the function is **convex** gradient descent will converge to the **global minimum**. For **non-convex** functions, it may converge to **local minima**.



Convex Function



Non-Convex Function

Gradient Descent

Gradient descent is often used in machine learning to **minimize a cost function**, usually also called *objective* or *loss* function and denoted $L(\cdot)$ or $J(\cdot)$.

The cost function depends on the model's parameters and is a proxy to evaluate model's performance. Generally speaking, in this framework minimizing the cost equals to maximizing the effectiveness of the model.

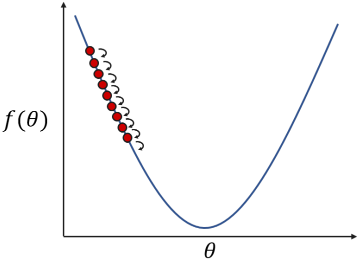
Stochastic Gradient Descent

In principle, to perform a single update step you should run through all your training examples. This is known as **batch gradient descent**.

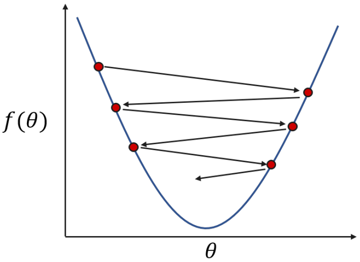
A different strategy is the one of **minibatch stochastic gradient descent**. In this case, only a small subset of the training dataset is considered at each update step.

In the extreme case in which only a random example of the training set is considered to perform the update step, we talk of **stochastic gradient descent**.

Choosing the the right **learning rate** α is essential to correctly proceed towards the minimum. A step *too small* could lead to an extremely *slow* convergence. If the step is *too big* the optimizer could *overshoot* the minimum or even *diverge*.



Learning Rate too small



Learning Rate too big

In practice, it's quite rare to see the procedure described above (so called **vanilla SGD**) used for optimization in the real-world.

Conversely, a number of cutting-edge optimizers [2, 1, 3] are commonly used. However, these advanced optimization techniques are out of the scope of this short overview.

Credits

These slides heavily borrow from a number of awesome sources. I'm really grateful to all the people who take the time to share their knowledge on this subject with others.

In particular:

- Stanford CS231n Convolutional Neural Networks for Visual Recognition
  http://cs231n.stanford.edu/
- Deep Learning Book (GoodFellow, Bengio, Courville)
  http://www.deeplearningbook.org/
- Convolution arithmetic animations
  https://github.com/vdumoulin/conv_arithmetic

- Andrej Karphathy personal blog
  http://karpathy.github.io/
- WildML blog on AI, DL and NLP
  http://www.wildml.com/
- Michael Nielsen Deep Learning online book
  http://neuralnetworksanddeeplearning.com/

References

References i

[1] J. Duchi, E. Hazan, and Y. Singer.
Adaptive subgradient methods for online learning and stochastic optimization.
Journal of Machine Learning Research, 12(Jul):2121–2159, 2011.

[2] D. Kingma and J. Ba.
Adam: A method for stochastic optimization.
arXiv preprint arXiv:1412.6980, 2014.

[3] M. D. Zeiler.
Adadelta: an adaptive learning rate method.
arXiv preprint arXiv:1212.5701, 2012.