ASSET PRICING THEORY — Problem Set 3

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David With the Head of Goliath and two Soldiers (de Boulogne, 17th century). David outperformed his large enemy, but at a great risk.

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Exercise (*i*)

Rationale for the Two Adjustments

Adjustment for Returns (Delisting Returns)

When a stock is delisted, shareholders may still receive some return (e.g., from a merger or acquisition). This return is not captured in the `dlret` variable in CRSP. By adjusting the last recorded return (`ret`) to include the delisting return (`dlret`), we account for the final payoff to shareholders. The formula $(1 + R_{adj}) = (1 + R_{normal}) \times (1 + R_{delist})$ ensures that the adjusted return reflects the total return, including the delisting event. Overall, this adjustment ensures that the dataset accurately reflects the performance of all stocks, including those that are delisted, removing the potential downward bias in portfolio returns.

Adjustment for Survivor Bias (Two-Year Rule)

By excluding firms until they have appeared on COMPUSTAT for two years, we ensure that our dataset includes only firms that have survived long enough to be reliably tracked. This reduces the bias introduced by excluding firms that fail early and provides a more accurate representation of the population of firms.This adjustment ensures that the dataset is not biased towards firms that have already demonstrated some level of survivability, providing a more accurate and representative sample for analysis.

Share of Stocks in Each Portfolio over Time

Figure 1 shows the share of portfolios over time. The share of the portfolios is relatively stable over time.

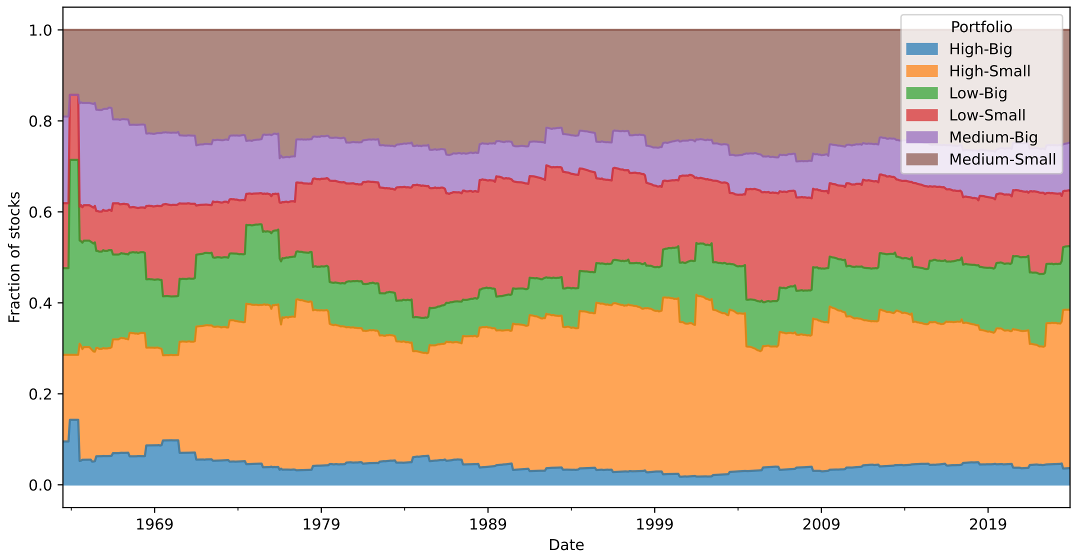


Figure 1: Share of the Six Portfolios Over Time.

Figure 2 shows the value-weighted returns of the six portfolios. They are generally fluctuating in the ±20 percent over time. The returns of the *Low Small* and *High Small* portfolios are more volatile than the other portfolios.

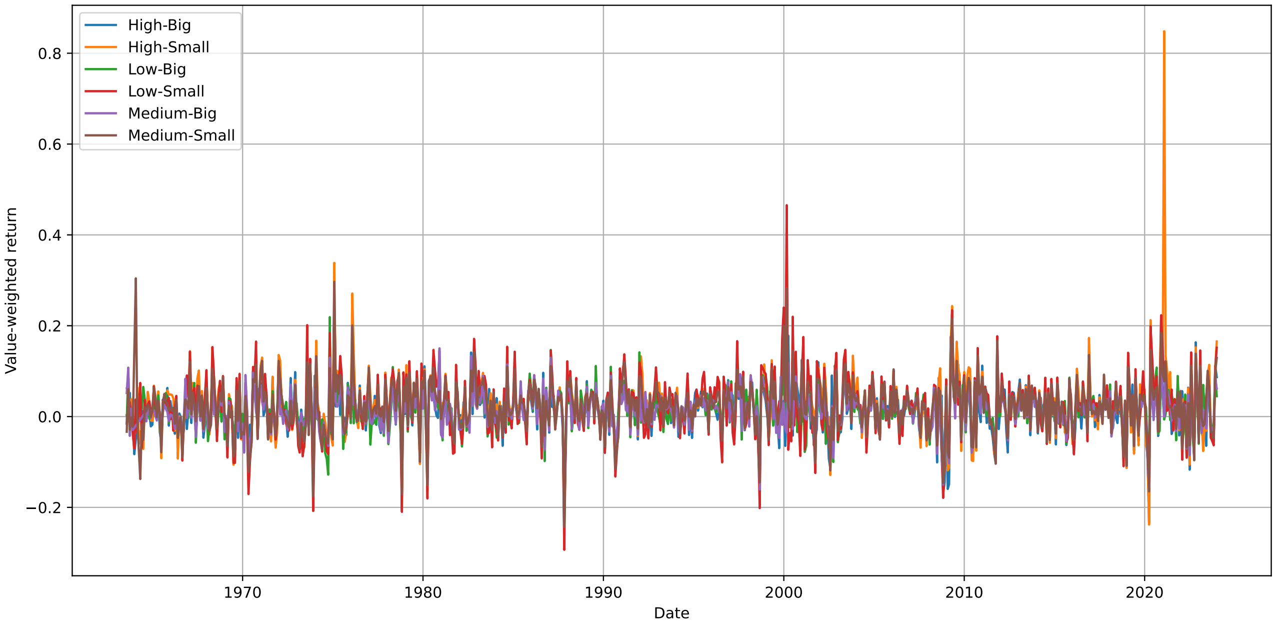


Figure 2: Value Weighted Returns of the Six Portfolios.

Rationale for the Construction of Book-to-Market Portfolios

Look-ahead bias occurs when future information (not available at the time of portfolio formation) is inadvertently used in the analysis. Fama and French (1992)'s methodology avoids this bias in the following ways:

1. The book-to-market ratio is calculated using the lagged data (e.g., December 1979 data is used to form portfolios in June 1980). This ensures that the information used to construct portfolios is available to investors at the time of portfolio formation.
2. Portfolios are formed in June and held until the following June. This ensures that the returns are calculated using only information available at the time of portfolio formation.
3. The methodology does not rely on any future data (e.g., future returns or financials) to construct portfolios, ensuring that the results are free from look-ahead bias.

Bowles et al. (2024) show that the conventional method of portfolio rebalancing used in financial research leads to anomaly portfolios that rely on information that can be severely outdated. Hence, it can be argued that the June rebalancing approach by Fama and French (1992), while avoiding look-ahead bias, introduces staleness bias.

Discrepancy with Fama French (Importing other data)

We see from Figure 3 that the time series of our return differences and French's, while being arguably similar, still are far away from being perfect matches. In particular, from Figure 4, we see that our portfolios have consistently higher cumulative returns than French's. One major reason as to why our results differ is that Fama and French (1992) exclude financial firms from their portfolio constructions, which we have not. Indeed, if financial firms then overperformed during our sample period, this would explain part of the difference. For instance, we see that in the times of crises, i.e., the IT bubble, the Great recession, and during the start of the Covid-19 pandemic, some of our portfolios have far greater returns than their French counterparts, which could be an artifact of the different portfolio compositions. Ceteris paribus, this could then explain the differences in cumulative returns.

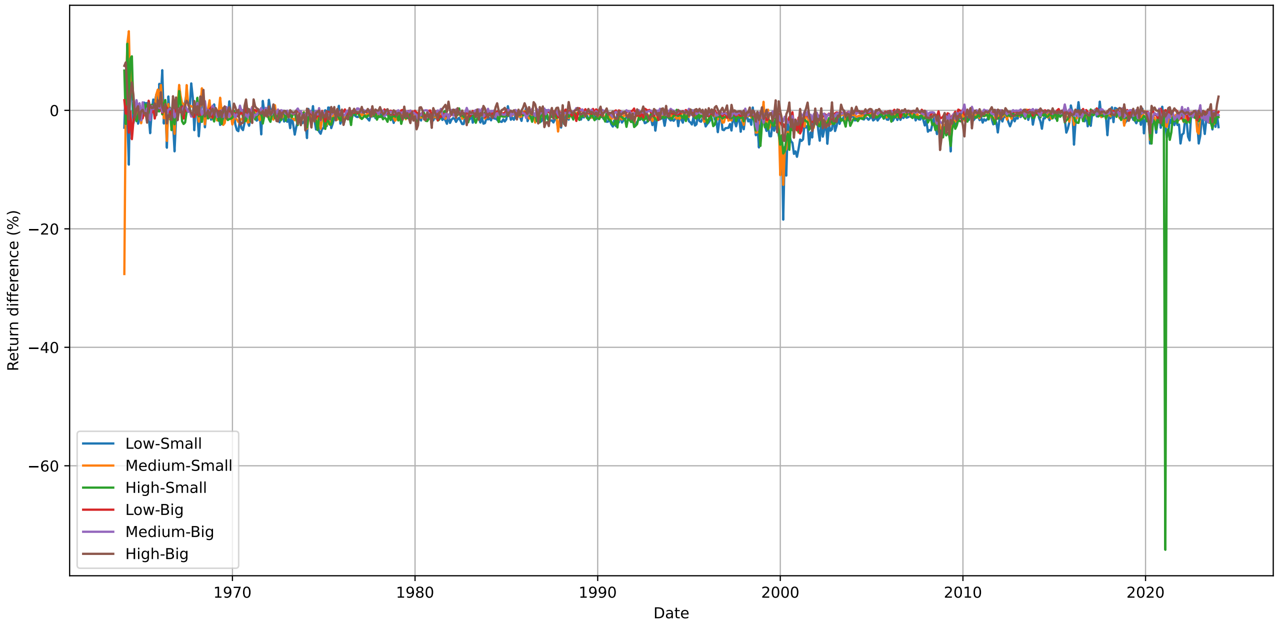


Figure 3: Return Differences Between French and Value-Weighted Portfolios.

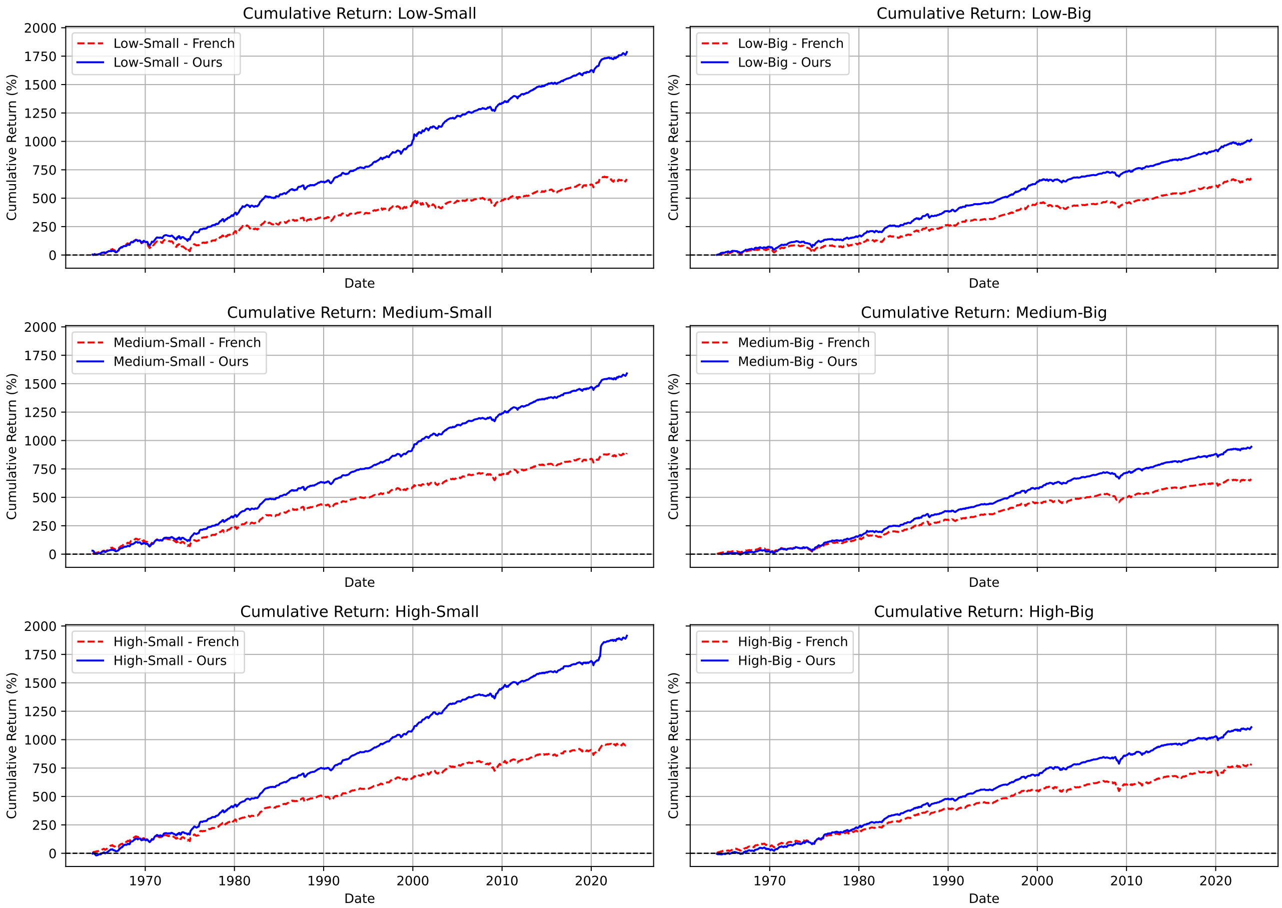


Figure 4: Cumulative Returns of the French and Value-Weighted Portfolios.

Exercise (*ii*)

The vector of sample mean excess returns is illustrated in Table 1. It illustrates that the small firms have higher mean excess returns than the big firms in our data. However, these excess returns for the small firms come at a cost of higher volatility as displayed in Figure 5.

Table 1: Sample Mean Excess Returns of the Six Portfolios.

Portfolio	Mean Excess Return (%)
Low Small	2.12
Medium Small	1.85
High Small	2.30
Low Big	1.05
Medium Big	0.96
High Big	1.18

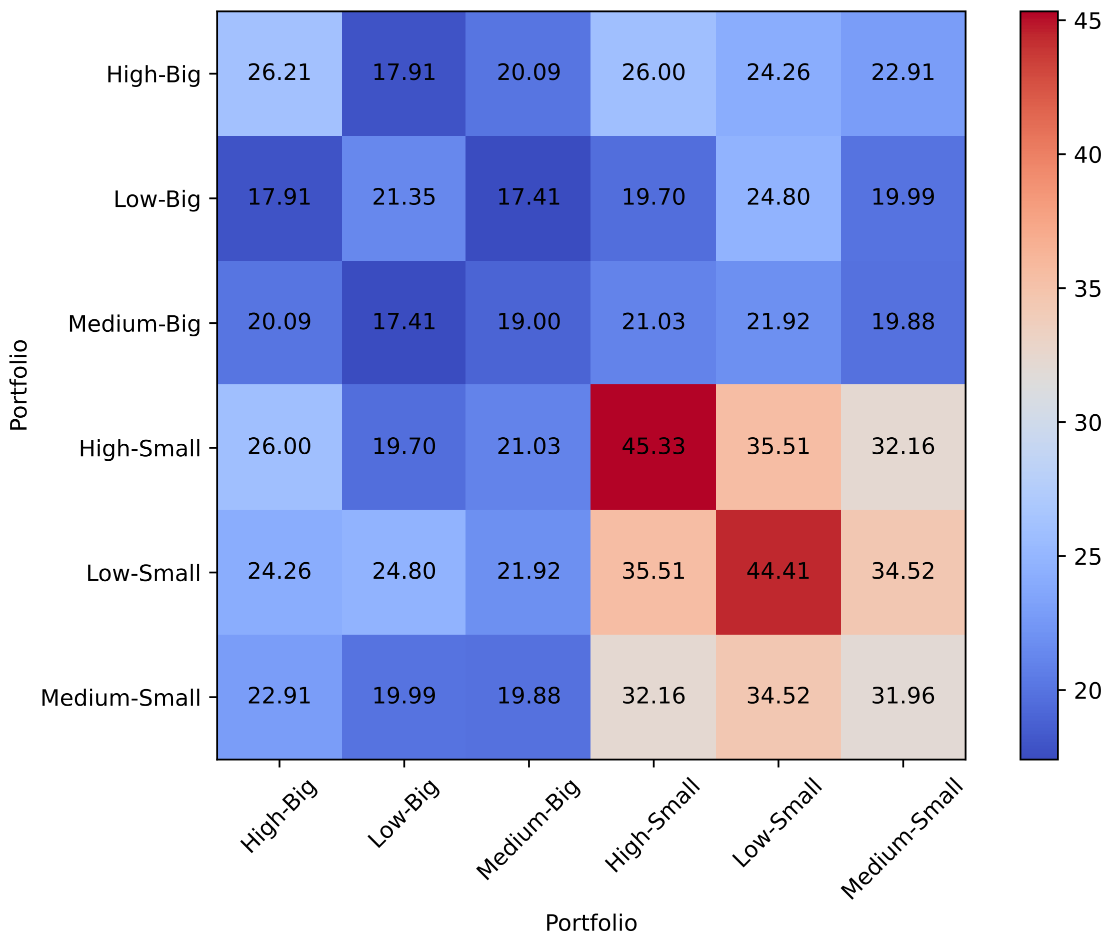
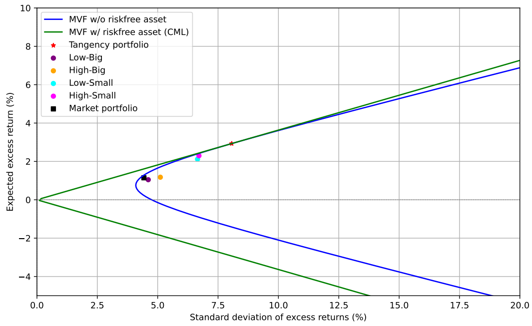
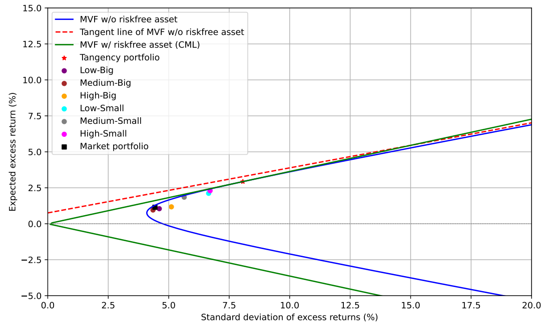


Figure 5: Covariance Matrix of the Value-Weighted Portfolios.

In our data, the Sharpe ratio of the tangency portfolio and the market portfolio is approximately 0.36 and 0.26, respectively. Hence, our value-weighted market portfolio is not mean-variance efficient ex post. This suggests that, according to the mean variance framework, our proxy for the market portfolio does not achieve the optimal risk-adjusted return, potentially due to market frictions, data limitations, or deviations from CAPM. However, we observe from Figure 6 that the market portfolio is not mean-variance dominated by any of the four Fama-French portfolios. In that sense, there is still no free lunch; to obtain a higher return, one must also accept a higher risk.



(a) Mean Variance Frontiers with Four Portfolios.



(b) Mean Variance Frontiers with Six Portfolios.

Figure 6: Ex-Post Mean Variance Frontiers.

Exercise (*iii*) & (*iv*)

From Figure 7, we find that the small firms have expected excess returns well above what would be justified from their betas alone, and hence exhibit positive alpha. On the other hand, the large firms have have expected returns more in line with what would be expected from their betas, although the *Low Big* portfolio have somewhat of a negative alpha. These visual results are further strengthened by the regressions presented in the Table 2, which shows that the *Low Small* and *High Small* portfolios have positive alphas and the *Low Big* portfolio a negative alpha. All in all, our results suggest that there is a size-related risk premium that is not accounted for by the CAPM. In particular, small firms are sold at a discount (undervalued) versus what would be predicted by the CAPM. In addition, there is some evidence that large growth firms[1] are sold at a premium (overvalued) versus the large value firms[2].

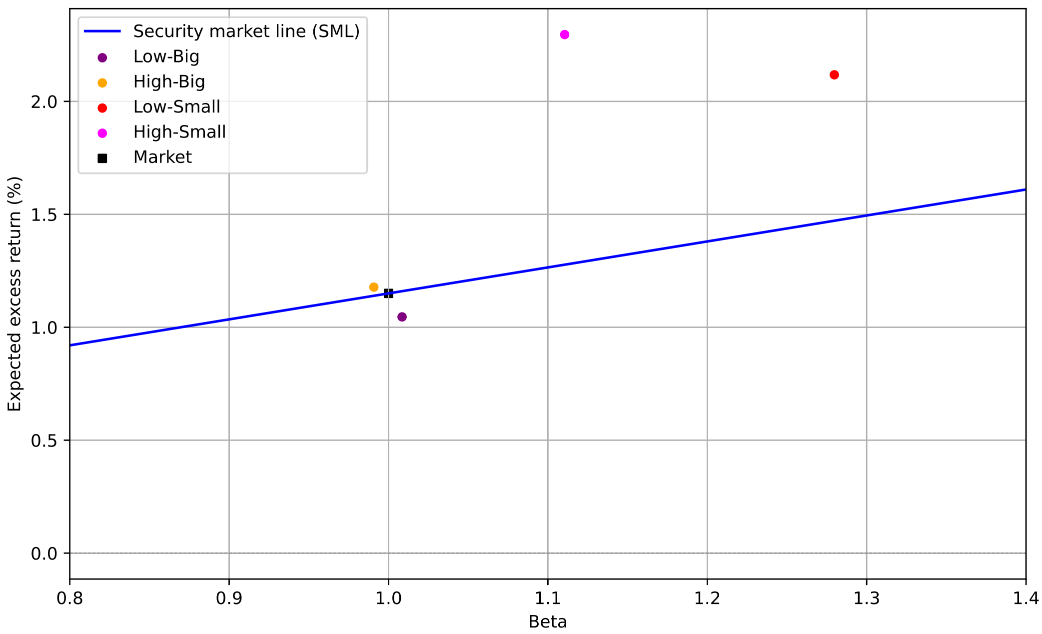


Figure 7: Security Market Line.

[1] *Low Big* firms.
[2] *High Big* firms.

Table 2: Regression Results for the Four Portfolios.

	Low-Big	High-Big	Low-Small	High-Small
β	1.01**	0.99**	1.28**	1.11**
	(0.01)	(0.02)	(0.03)	(0.04)
α	-0.11**	0.04	0.65**	1.02**
	(0.04)	(0.10)	(0.13)	(0.18)
Obs	720	720	720	720
R^2	0.94	0.74	0.73	0.54

Standard errors in parantheses
*** p<0.001, ** p<0.005, * p<0.01

References

B. Bowles, A. V. Reed, M. C. Ringgenberg, and J. R. Thornock. Anomaly time. *The Journal of Finance*, 79(5): 3543–3579, 2024.

V. de Boulogne. David with the head of goliath and two soldiers. https://commons.wikimedia.org/wiki/File:Valentin_de_Boulogne_-_David_with_the_Head_of_Goliath_and_Two_Soldiers_-_WGA24236.jpg, 17th century. Accessed: [Insert Date Here].

E. F. Fama and K. R. French. The cross-section of expected stock returns. *the Journal of Finance*, 47(2):427–465, 1992.