

Autonomous vehicular overtaking maneuver: A survey and taxonomy

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ABSTRACT

Autonomous vehicles (AVs) are the next-generation driver-less vehicular entities with advanced technologies. Overtaking is an important and challenging maneuver that needs to be frequently performed in diverse environments during the run. The maneuvering of AVs, in present scenarios, improves driving precision, fuel efficiency, and travel time and minimizes road accidents caused by human error, waiting time, carbon emission, etc. Despite all these benefits, AVs face several critical issues, such as traffic safety, market issues, environmental aspects, technical compatibility, market introduction, etc. In traffic safety, the overtaking scenario is considered one of the most complex driving maneuvers out of several driving maneuvers, such as lane changing, lane following, tailgating, and many more. Despite this, to the best of authors' knowledge, an extensive study on autonomous vehicular overtaking maneuvers still needs to be explored. Therefore, this survey aims to visualize the current methods used to enhance traffic safety in an intelligent transport system by handling the most complex driving maneuver in autonomous driving (overtaking). In this survey, we present the taxonomy, simulators, and methods used for AVs overtaking maneuver, and the state-of-the-art methods are further categorized under theoretical and AI-based techniques. Furthermore, the theoretical and AI-based techniques are classified based on the applications of the respective technique in overtaking maneuvers. While designing this survey, several theoretical and practical studies are taken into consideration. As the outcomes of this study, several research gaps, challenges, future research directions, and open research problems in overtaking maneuvers are identified. The outcomes of this study would be helpful for the researchers to carry out their research in the domain of AVs overtaking maneuvers.

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1. Introduction

Autonomous vehicles (AVs) are the future of new-age automobile industries and the foundation of next-generation intelligent vehicles. As the AVs move on the roads, traffic efficiency will undoubtedly be increased [1]. The AVs are expected to lead in the growth of several sectors such as Electric Vehicles (EVs), clean energy industries, i.e., Batteries, Compresses Natural Gas (CNG), solar energy, IT sectors, and the sectors related to electronics components such as telecommunication, industrial electronics, and consumer electronics [2]. AVs are eco-friendly as most of the vehicles are shifted from traditional petrol or diesel engines to clean energy fuel consumption, such as batteries and CNGs. Several giant industries which are jumped to AVs and EVs include Tesla, Nvidia, Ford,

and BMW, and their interest in AVs grabs the attention of other sectors such as commercial and public transport, delivery services, healthcare, cyber-security, and many more [3], [4].

Despite the several applications and advantages of AVs in the industrial sector, the AVs industry faces several issues [5], including legal issues [6], market introduction [7], traffic safety [8], environmental aspects [9], technical aspects [10], market issues i.e., competing with existing vehicles [11], cybersecurity [12], dynamic weather condition [13], and many more. Based on state-of-the-art survey studies [14], [15], it is observed that traffic safety is one of the significant issues [16] for AVs. In particular, during overtaking, vehicles perform various maneuvers like lane changing, lane merging, lane following, accelerations, and other related movements. Performing these maneuvers is tricky for AVs as some actions involve combining two to three movements. Overtaking maneuver [17] is one of the most challenging and complex AVs driving scenarios that majorly includes the combination of lane following [18], lane changing [19], and acceleration [20] operations as shown in Fig. 1. In an actual traffic scenario, most traffic rule violations and accidents occur due to overtaking maneuvers in

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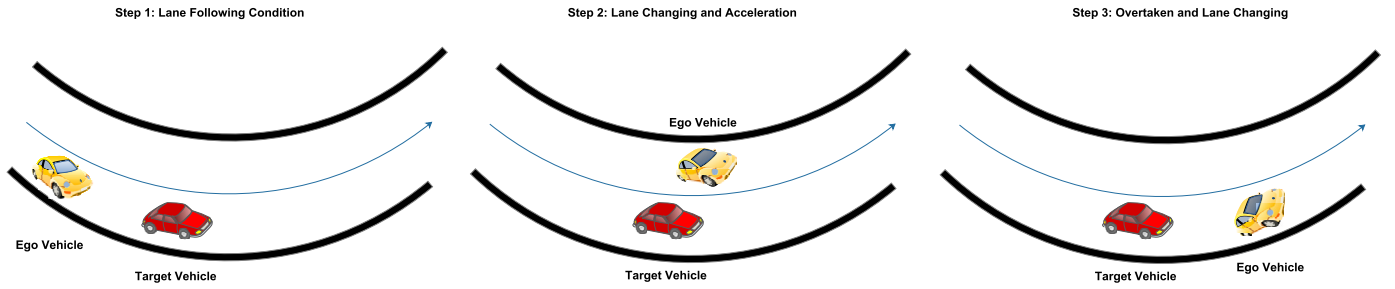


Fig. 1. Overtaking scenario in AVs driving systems.

an in-disciplined way without following the standard traffic rules. Approximately 1.3 million people die each year from road traffic crashes. Road traffic crashes cost most countries 3% of their gross domestic product. Road traffic injuries are the leading cause of death for children and young adults aged 5-29 years [21].

In manual control, overtaking is one of the most challenging maneuvers that the driver performs. In the case of autonomous driving, it becomes more complex. In current scenarios, AVs are yet to arrive on actual roads. However, many simulations and real-life application-based research efforts are being made considering several aspects of AV driving. This survey addresses the overtaking issue, the most complex driving maneuver. Several approaches have been proposed to solve the overtaking issue. All these approaches are categorized under two categories: the theoretical-based methods and the AI (Artificial Intelligence) based methods. As per our findings, both of these approaches are evaluated under simulation. However, in some cases, only few of these studies are done in real-time scenarios. To conduct this study, state-of-the-art methods are thoroughly explored for solving the overtaking issue in AVs. In particular, IEEE Xplore, Elsevier, and Springer are the key publishers explored to collect research articles. All the definitions, techniques, terms, and formulas applied to solve the overtaking issue are referenced from the standard books and research papers, which are properly cited in the contents. This survey used several keywords to explore the related contents, which include “overtaking autonomous vehicles”, “overtaking autonomous vehicle using ML”, “overtaking autonomous vehicle using DL”, “overtaking autonomous vehicles using DRL”, and “overtaking autonomous vehicles using RL”. This study is helpful for researchers in the AV overtaking domain.

1.1. Motivation

In AV driving maneuvers, few surveys have been published related to a single movement of the vehicles, such as lane changing, lane following, lane merging, and acceleration profile. However, considering its significant components, an extensive survey study on overtaking still needs to be included in the state-of-the-art. The motivation factors to write this survey paper are itemized as follows:

1. The detailed classification of approaches to solving the overtaking issues has yet to be explored extensively in existing survey papers.
2. The existing surveys only focus on the single movement of AVs, and the overtaking issue requires more than one movement, which needs to be explored.
3. A comparative study of different approaches for solving the overtaking problem considering several metrics has not yet been explored in existing surveys.

This motivated us to conduct an extensive study on AVs one of the most complex driving issue i.e., “overtaking”.

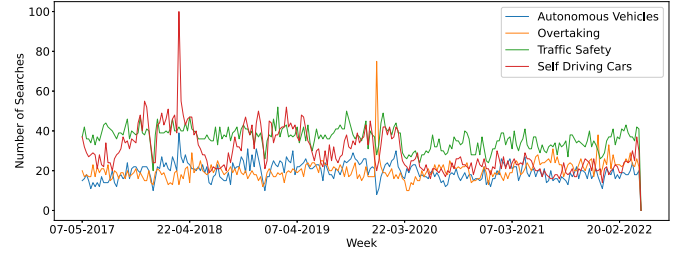


Fig. 2. Google trends for specified keywords.

1.2. Google trends and publications

Currently, the research in AV design and development is on high trend, and this domain is not in the automobile industry but is also being rapidly explored in academia. With the introduction of Tesla electric cars with a glimpse of self-driving features, all the big players in the automobile industry focused their research and development on AVs [22]. In Fig. 2 and Fig. 3, the Google trend graph is plotted to show people's interest in the research and development of AVs. Four search keywords are chosen: i) Autonomous vehicles, ii) Overtaking, iii) Traffic safety, and iv) Self-driving cars.

Apart from the Google trends over the internet, a comparative bar graph of the number of publications is plotted and represented in Fig. 4 to show the interest of researchers in solving the overtaking issues of AVs in intelligent transport systems. Majorly, IEEE Explore, Elsevier, and Springer are the key publishers explored for this study. From Fig. 4, it is observed that in early's 2016, less effort was made to handle the overtaking maneuvers in AV driving. After three to four years, a significant increase in research articles was recorded. In the last five years, the research on AV overtaking has become a prime concern in AV driving.

1.3. Contribution

Table 1 presents the existing surveys on AVs driving and overtaking maneuvers. With this study, it is observed that most of the surveys focus on individual-related technology for solving AV overtaking issues and challenges. In contrast, this work presents the progress related to the overtaking issues in AV driving through a comprehensive survey with detailed information on AVs driving maneuvers. The overtaking methods are categorized according to the techniques applied for AVs driving maneuvers. Furthermore, to make intelligent transportation systems more user-friendly, this survey presents significant issues, advantages and disadvantages, tools, techniques, and environments required to handle overtaking issues. The outcomes of this survey would assist the researchers and industrialists in understanding fundamentals, various strategies, and their classification related to overtaking maneuvers.

However, the major contributions of this survey are enlisted as follows:

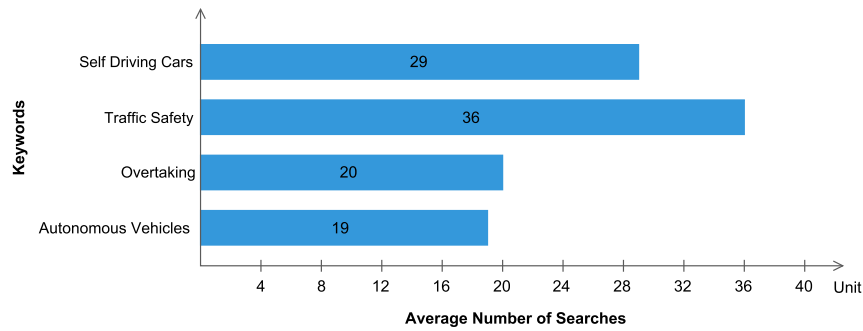


Fig. 3. Average Count for specified keyword related to overtaking in autonomous driving.

Table 1

A summary of related state-of-the-art surveys.

Technique	Ref. & Yr.	Contribution	Overt.	AI.	Theo.
MPC and related concepts	[23], 2016	This survey focuses on safe automated highway driving using several control approaches	×	×	✓
	[24], 2021	This survey focuses on the MPCs applied on Advance driving assistance system (ADAS)	×	×	✓
	[25], 2021	This survey focuses on the MPC application to multiple autonomous ground vehicle	✓	×	✓
DRL and related concepts	[26], 2021	This survey provides taxonomy for automated driving task using DRL in real world	×	✓	×
	[27], 2021	This survey provides overview of transport engineering for AVs using IL and DRL	×	✓	×
	[28], 2021	This survey summarizes the RL techniques for motion planning and control of AVs	✓	✓	×
	[29], 2022	This paper reviews the IL approaches for end-to-end AVs decision and planning approaches	✓	✓	×
ML and related concepts	[30], 2018	This survey analyzes the suitable ML techniques applicable in AVs ADAS	×	✓	×
	[31], 2019	This paper surveys current state-of-the-art on DL technologies for AVs driving	×	✓	×
	[32], 2020	This survey focuses on the AVs control system using DL approaches	✓	✓	×
	[33], 2021	This surveys explores ML & DL techniques for AVs in motion planning, vehicle localization, pedestrian detection, traffic sign detection, road-making detection, automated parking, vehicle cybersecurity and fault diagnosis	✓	✓	×
	[34], 2021	This survey concentrates on DL application for object and scene perception in AVs	×	✓	×
	[35], 2021	This study elaborates the importance of DL-Lidar strategies to formulate AVs driving research challenges	×	✓	×
	[36], 2022	This literature discusses the DL techniques for AVs system from the past decades for real-life implementation	✓	✓	×
Planning related concepts	[37], 2018	This survey reviews the trajectory planning and tracking of AVs overtaking systems	✓	×	✓
	[38], 2019	This survey focuses on AVs behavior and motion planning for highway overtaking, trajectory generation and many more	✓	×	×
	[39], 2021	This surveys focuses on road safety aspects of ADAS by exploring the crash avoidance and overtaking advice subsystems	✓	×	×
Hybrid overtaking approach	The proposed survey	This survey combines the theoretically based concepts, i.e., MPCs and control system related, and AI-based concepts i.e., DRL and ML-related concepts for overtaking maneuver in AVs	✓	✓	✓

Overt.: Overtaking included; AI.: AI-based methods; Theo.: Theoretical-based methods.

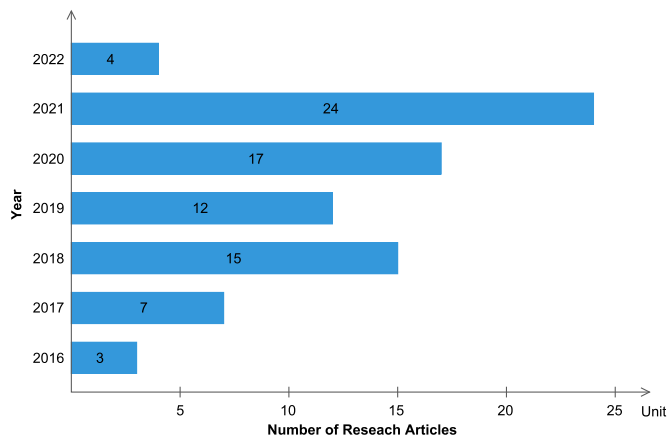


Fig. 4. Number of publications year-wise in key publishers.

2. This study presents an in-depth comparative analysis of the methods used to solve AVs overtaking maneuvers considering several overtaking scenarios. A new categorization of the AVs overtaking maneuver approaches has been proposed accounting theoretical and AI-based approaches.
3. It identifies most of the critical issues, challenges, methods, tools, and techniques for overtaking maneuvers in AV driving systems, and it also highlights their significant impacts.
4. In the end, taken as a whole, this survey aims to answer the following questions: i) What are the significant scenarios considered in overtaking maneuver study? ii) What broadly used simulators are available for handling AVs overtaking maneuvers? iii) Are there any techniques and models that analyze real-life and simulation data for handling overtaking scenarios? iv) What are the mathematical, ML, DL, RL, and DRL-based techniques used in AVs overtaking maneuvers? v) What are the gaps, current challenges, future research directions, and open problems in the AV overtaking maneuver?

The road map of the survey has been reported in pictorial form in Fig. 5. To enhance the readability of this survey, a short notation

1. It offers a taxonomy for recent state-of-the-art approaches used to solve the overtaking problem in autonomous driving.

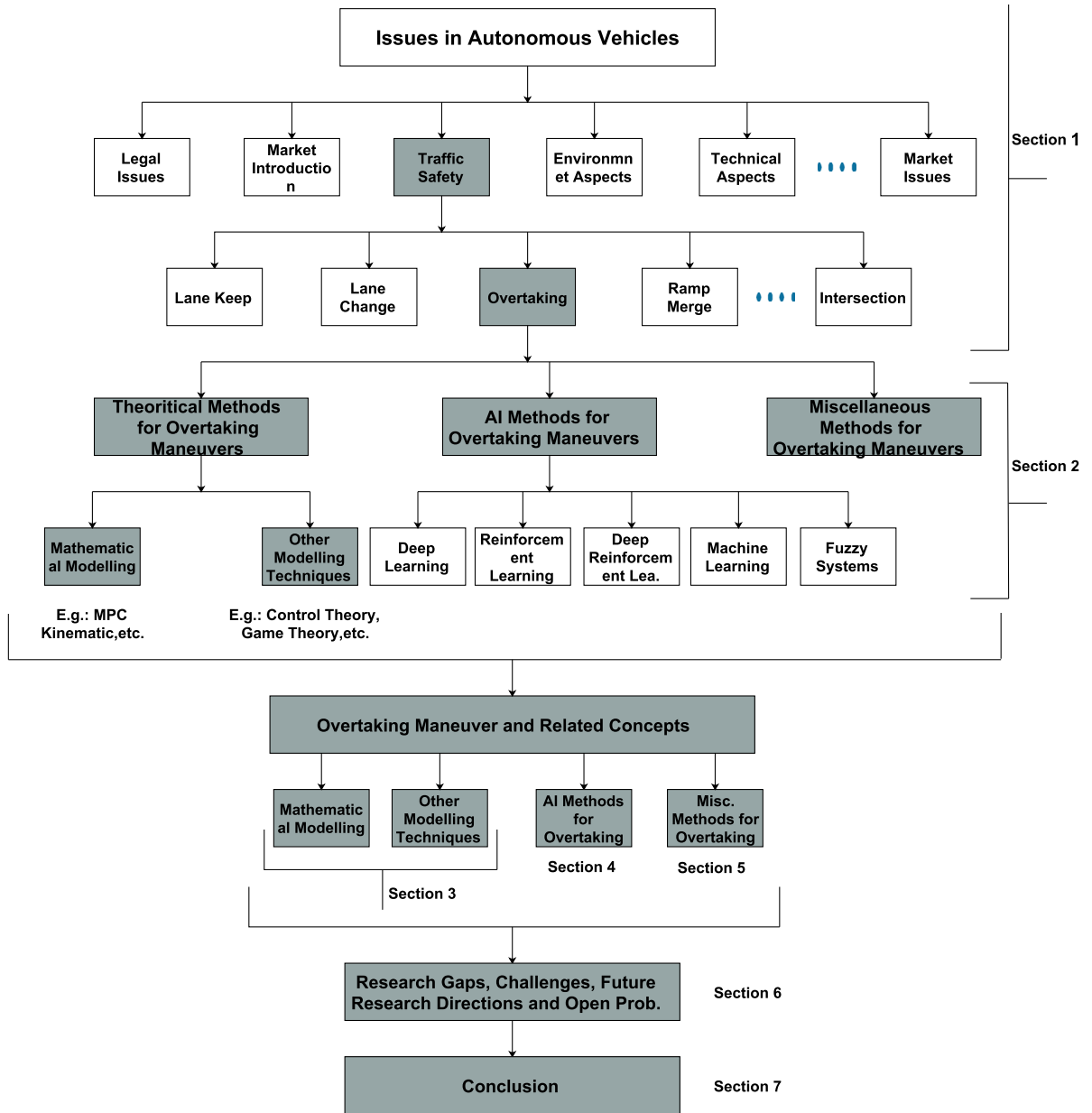


Fig. 5. Road map of the paper.

and abbreviation are presented in Table 2. An overtaking scenario in AV driving systems and their respective solutions approaches are provided in Section 2. The solution approach related to theoretical methods for overtaking maneuvers is given in Section 3. The AI methods for overtaking and its subcategories are discussed in Section 4. Some miscellaneous methods related to AV overtaking maneuvers are elaborated in Section 5. The research gaps, challenges, future research direction, and open problems are described in Section 6. Section 7 finally concluded the survey.

2. Overtaking in autonomous vehicles

AVs driving maneuvers [40] are very complicated movements that require special techniques for control mechanisms, route planning, obstacle handling, overtaking, etc. Overtaking is one of the maneuvers of AVs that constitute multiple driving movements like a lane change, vehicle acceleration, braking, and lane following. A lot of research has been conducted to perform all these maneuvers independently. However, overtaking issue employs all these

movements simultaneously. Commonly, the ahead-moving vehicle is overtaken based on conditions such as the ahead vehicle moving slowly, other lanes being empty, and no vehicle coming from the opposite lane, etc. If all such conditions are satisfied, then a maneuver like the first lane change is performed. Next, the vehicle is accelerated to surpass the ahead-moving vehicle. At last, the vehicle moves to its original lane, which was being driven earlier. This is the simplest overall overtaking scenario. Other conditions might affect the overtaking maneuver, including braking, vehicle following, obstacles, etc. The overtaking maneuver, in real-life scenarios, requires essential information that includes the speed of target vehicles, the number of vehicles in overtaking lane, and the acceleration necessary for overtaking. It seems to be simple in a typical simulation environment compared to a real scenario. In general, the longitudinal and lateral controllers are beneficial in performing overtaking tasks. Lane changing [41] for the AV itself is a complex task. Different techniques, including mathematical modeling, DL, ML, and many more, have been utilized to carry out this overtaking maneuver. In acceleration or de-acceleration, various longitudinal

Table 2
List of abbreviations.

Abbreviations	Technique
MPC	Model Predictive Control
V2V	Vehicle to Vehicle Communication
V2X	Vehicle to moving Parts in Traffic Communication
FSM	Finite State Machine
MIP	Mixed Integer Programming
MDP	Markov Decision Process
MIQP	Mixed Integer Quadratic Programming
SQP	Sequential Quadratic Programming
CAM	Computer Aided Manufacturing
PID	Proportional Integral Derivative
GPR	Gaussian Process Regression
PPO	Proximal Policy Optimization
ML	Machine Learning
DL	Deep Learning
DRL	Deep Reinforcement Learning
RL	Reinforcement Learning
DQN	Deep Q Network
DDPG	Deep Deterministic Policy Gradients
IRL	Inverse Reinforcement Learning
NAC	Natural Actor Critics
HMM	Hidden Markov Model
CNN	Convolution Neural Network
RNN	Recurrent Neural Network
GAN	Generative Adversarial Network

Table 3
List of scenarios and environments.

Scenarios	Type	Condition	References
Highway Roads	Two lanes	Two-way	[44], [45]
	Five Lanes	One-way	[46]
	Two Lanes	One-way	[47]
	Three lanes	One-way	[48], [49], [50]
Urban Roads	Five Lanes	One-way	[51]
	Three lane	Two-way	[52], [53]
	Three lane	One-way	[54]
	Two Lanes	Two-way	[55], [56], [57], [58]
	Two Lanes	One-way	[59], [60], [61], [62], [63], [64], [65]
Race Tracks	Two Lanes	One-way	[66]
	Three lane	One-way	[52]

controller tasks are used to increase or decrease the vehicle's speed [42]. In vehicle following, a mix of vehicle speed and steering control is considered for a comfortable vehicle following maneuvers [43]. Most of the accidents, in real-life scenarios, happened during overtaking tasks. The lack of traffic safety rules, human errors, control over various dynamics of vehicles, and visibility at night cause loss of human life and liabilities.

2.1. Scenarios

Various approaches are used to solve the overtaking issues considering different scenarios to validate the methods in simulations and real-life scenarios. This subsection presents widely used environments/scenarios for experimental testing and training of overtaking maneuvers in Table 3 with their specifications and constraints. From Table 3, it is observed that there are three scenarios for experimental testing, i.e., highway roads, urban roads, and race tracks. In highway roads, two lanes are used for one-way and two-way traffic. Three and five lanes are used for only one-way traffic. Similarly, three and five lanes are used for both one-way and two-way traffic for urban roads, respectively. Five lanes are used for one-way traffic. However, race tracks are considered for one-way traffic with two and three lanes. The studied approaches, in this survey, are majorly tested in these scenarios.

2.2. Techniques applied for overtaking issues

As discussed, three maneuvers participate in the overtaking scenario; lane changing, lane following, and acceleration or deceleration. This sub-section briefly describes possible techniques and procedures used to tackle the three above mentioned overtaking maneuvers. The techniques used to make the overtaking maneuvers more efficient and reliable in the simulation and real-life scenarios. Broadly, these techniques are classified into two types; 1) theoretical-based techniques and 2) AI-based techniques. A taxonomy for AVs overtaking is shown in Fig. 6. The detailed description of theoretical based techniques and AI-based techniques is elaborated in upcoming Sections 3 and 4, respectively. However, a background of these techniques is summarized in the following sub-section.

2.2.1. Theoretical techniques

Theoretical-based techniques consist of methods dependent on mathematical modeling or conversion of any scenario into mathematical models. For AV overtaking issues, vehicular dynamics like steering, yaw angle, tires, and acceleration, basically all the components that participate in the longitudinal and lateral movement of vehicles, are taken into account. All the components that participate in the vehicular movement are formulated into the mathematical model and transformed accordingly into the optimization problem based on the desired output. In this survey, theoretical-based techniques are further categorized into two subcategories; i) MPC-based approach and ii) other mathematical modeling-based approaches. Both approaches are briefly introduced in this subsection.

Model Predictive Control MPC is an optimized control-dependent approach for selecting control inputs by minimizing an objective function. The objective function is assessed using an explicit model to anticipate future process outputs, and it is specified in terms of predicting system variables. A nonlinear, deterministic process model is described in discrete time by [67]:

$$x_{k+1} = f(x_k, u_k) \quad y_k = g(x_k) \quad (1)$$

Here, y_k , u_k , x_k , and x_{k+1} represent the output, control, set of states, and future set of states respectively. All the variables are represented as vectors.

The MPC control problem [68] is defined as: with the proper knowledge of the current output y_k , a control is chosen that minimizes the objective function J as:

$$J = \phi(y_k + N|k) + \sum_{j=0}^{N-1} L(y_{k+j|k}, u_{k+j|k}, \nabla u_{k+j|k}) \quad (2)$$

The controls, outputs, and states are doubly indexed, and $x_{k+j|k}$, $y_{k+j|k}$, and $u_{k+j|k}$ indicate values at time $k+j$ having the information up to time k . Here, L denotes the objective function stage cost. The function L is chosen based on the type of objective required by the user. The first control sequence is implemented out of the total N -moves control sequences, which helps minimize the above objective function. Whenever a new measurement becomes available, the problem's parameters are updated, and a new optimization problem is generated, the solution of which determines the next control. One of the main distinguishing elements of the MPC is the recurrent optimization utilizing an objective function that is updated by the process feedback. As the MPC requires prediction, the double subscript carries the information of prediction when the first index is greater than the second. Singly indexed variables are used to represent outputs, states, and variables that have been computed up to time k . An asterisk ($u_{k+j|k}^*$) is frequently used to

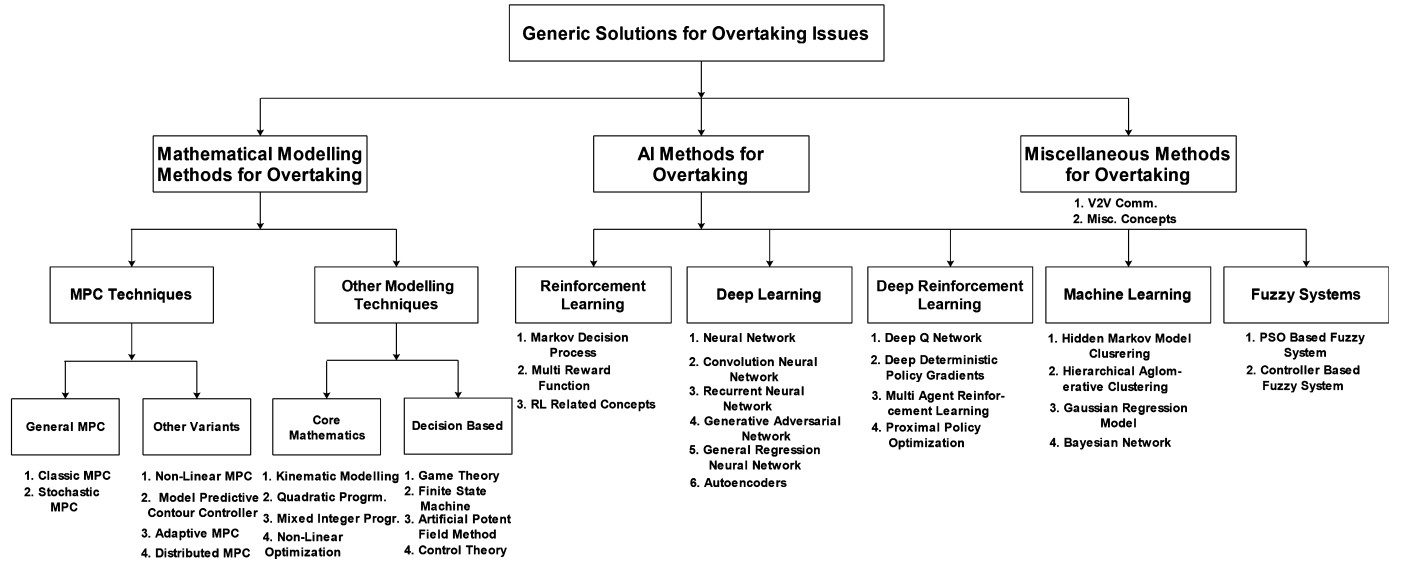


Fig. 6. Various techniques categorization for solving overtaking issues.

denote an ideal control sequence. The benefit of using the MPC in AV driving is that it can work efficiently with the constraints of the actual scenario. Further, for complex systems, a simpler control policy can be constructed. The main disadvantage is that the complex MPC algorithm requires a longer time than standard controllers, which causes an increase in the computational load that is not an acceptable solution for AVs.

Other mathematical modeling based approaches In other mathematical modeling-based approaches, techniques include kinematic modeling, game theory, MIP, control theory, FSM, and other miscellaneous concepts. These methods are merged or sometimes used standalone to make an efficient overtaking maneuver. For example, game theory concepts incorporated with some MPC-based concepts [49] lead to more efficient overtaking maneuvers. Likewise, when V2V communication is combined with probability theory [69] will give a better expectation of decision framework during overtaking decision making.

Quadratic programming minimizes or maximizes a quadratic objective function equipped with problem-specific constraints. The approach is widely utilized in operations research and statistics studies. Generally, the quadratic programming [70] problem is mathematically represented as:

$$\text{Maximize : } z = C'X + \frac{1}{2}X'HX \quad (3)$$

$$\text{subject to : } A'X \leq b, N'X = e, X \geq 0 \quad (4)$$

where, (C, X) , b , and e represent n -dimensional constant vector, m -dimensional constant vector, and p -dimensional constant vector respectively. The H , A , and N are $n \times n$ constant Hessian matrix, $n \times m$ constant matrix, and $n \times m$ constant matrix respectively. It is a simple nonlinear programming method that can efficiently model any actual scenario problem, such as AV driving. The formulated problem is a straightforward optimization since the objective function belongs to the convex optimization problem [71]. The higher computational complexity is the major drawback of this formulation.

A *non-linear optimization problem* is a problem having general form:

$$P : \text{minimize}_x f(x) \quad \text{s.t.} \quad x \in \mathbf{F} \quad x \in R^n \quad (5)$$

Here, $x = (x_1, \dots, x_n)^T$ are decision variable [72]. The function $f(\cdot) : R^n \rightarrow R$ is an objective function that is further optimized. The statement $x \in \mathbf{F}$ indicates that the decision variable x must lie in another specified set \mathbf{F} . If the final output is maximization instead of minimization, then $f(\cdot)$ is replaced by $-f(\cdot)$, or minimize is replaced by maximize. A non-linear optimization problem handles real-life problems which are non-linear. The major drawback of the non-linear optimization technique is high computational complexity while computing the global optimum minima or maxima for the respective problem.

Mixed Integer Programming usually maximizes or minimizes the linear objective functions having one or more constraints with an additional condition that at least one of the variables is integer [73]. In operations research, this approach is widely used. A typical mathematical representation of the MIP problem is:

$$\text{Maximize (or minimize) : } z = \sum_{i=1}^n c_i x_i \quad (6)$$

$$\text{subject to : } \sum_{i=1}^n a_{ji} x_i = b_j, x_i \geq 0 \quad (7)$$

$$(j = 1, 2, 3, \dots, m), (i = 1, 2, 3, \dots, n) \quad (8)$$

where, z is an objective function, c is $1 \times n$ matrix having n constraints, b is $1 \times m$ matrix having m constraints, and a is $m \times n$ constant matrix.

Some x_i are restricted to integer values. The x_i is the decision variable, where $1 \leq i \leq n$, Eq. (6) is termed as an objective function, and the m inequalities are termed as constraints. The constraint bounds, the b_j 's, are often called right-hand sides (RHS). For handling complex actual scenarios related to AV driving, MIP has more modeling power to handle non-linear problems. Since the modeling power increases, the non-linear model becomes complex, increasing computational complexity.

A *kinematic model* is a mathematical representation of a robot's motion that overlooks the forces that affect motion and focuses on the geometrical relationship among components. A link connects two or more manipulator joints in a robot's kinematic model, where a joint combines two or more links. A tree structure can be used to depict this kinematic model [74]. The tree depicts the kinematic chain, the connection of robotic linkages to joints, and their inter-dependencies. The underlying geometric informa-

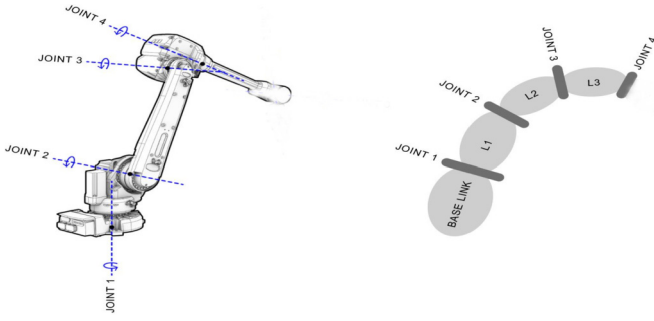


Fig. 7. An industrial robot's links and joints, and the according tree structure describing the kinematic model.

tion and the tree structures are expressed in the Unified Robot Description Format (URDF), which can be used to describe any robot, as shown in Fig. 7. These linkages and joints can also be added if the robot is positioned on external axes. In AVs, kinematic modeling converts vehicle motion into a mathematical representation. It is a powerful mathematical technique to analyze the motion of AVs. The disadvantage is that it cannot fully convert the movement of AVs, such as engine movement, tire pressure, and many more, into a mathematical representation.

2.2.2. AI techniques

AI-based techniques include methods that are related to end-to-end input and output procedures. AI-based methods like DL, ML, RL, DRL, and Fuzzy systems are used to execute overtaking maneuvers successfully. In this paper, the AI-based techniques are categorized into five sub-categories: DL-based approach, DRL-based approach, RL-based approach, ML-based approach, and other fuzzy system-based approaches. In ML approaches, HMMs are the prominent techniques utilized for autonomous driving, along with DL neural networks are used. In RL, MDP techniques play a crucial role in handling overtaking maneuvers, and Deep Q-Learning in DRL helps precisely control the vehicle movement. In further sub-sections, these prominent techniques are explained briefly with their importance in AV driving maneuvers.

Hidden Markov Model An HMM is a graphical model frequently used to represent temporal data. In several ML problems, the states of the system were only partially recognizable. For example, the states are not independent in the time sequence model. HMMs [75], unlike regular Markov models, claim that the data is produced by the underlying hidden states rather than the model's actual state. HMMs grow according to two rules:

1. The first rule states that the model progresses from one state to the next and might be in the same state based on a probability distribution that depends on the current state.

$$p(s_t | s_{t-1}) = p(s_t | s_{t-1}, \dots, s_0) \quad (9)$$

where, s_t represents the transition states, and $p(\cdot)$ represents transition probability. This is called the Markov property. Intuitively, this rule indicates that the system evolves regardless of previous states relying solely on the current state.

2. According to the second rule, the model generates an observation whose distributions depend only on the current state after each transition.

$$p(o_t | s_t) = p(o_t | s_t, o_{t-1}, s_{t-1}, \dots, o_0, s_0) \quad (10)$$

where o_t represents the observation state. The states producing the observations are termed as hidden states since the model only emits observations and the states having the observations are hidden from the observer.

In AVs, HMM plays a very critical role in driving. It has efficient learning algorithms that work directly on the raw sequential data, which benefits AVs. It can also handle the variable length input in the sequential form. The HMM can accumulate many unstructured parameters that are not desired. Further, no dependencies among the hidden states result in poor performance.

Neural network A neural network is a set of algorithms that detect hidden patterns in a data set using a similar method to how the human brain works [76]. Forward propagation is the process of transmitting input through a neural network, and the forward propagation executed in a perceptron is explained below:

1. Multiply the input value x_i with weight w_i , where $i \in (1, n)$ and add all the multiplied values for each input. Here, z represents the intermediate output value.

$$z = (x_1 \times w_1) + (x_2 \times w_2) + \dots + (x_n \times w_n) \quad (11)$$

$$z = \sum_i^n x_i \cdot w_i \quad (12)$$

2. Add bias b .

$$z = x \cdot w + b \quad (13)$$

3. Pass the value of z to some non-linear activation function.

$$\hat{y} = \sigma(z) = \frac{1}{1 + \exp^{-z}} \quad (14)$$

where, σ is sigmoid function [77], and \hat{y} is predicted output/activation value.

As discussed in previous sections, neural networks are more efficient than other learning techniques. It has a feature of continuous learning where it keeps improvising the desired result. Also, it can perform multitasking which is beneficial for AV driving. Apart from these benefits, the major disadvantage of the neural network is that it requires a vast amount of hardware and data.

Markov Decision Process A MDP model [78] consists of; a set of feasible world states S , a set of feasible actions A , a real-valued reward function $R(S, A)$, and a description T of every action's effects in every state. The deterministic actions are represented by:

$$T : S \times A \rightarrow S \quad (15)$$

For each action and state, a new state has been verified. Stochastic actions are represented by:

$$T : S \times A \rightarrow \text{Prob}(S) \quad (16)$$

For each action and state, a transition probability distribution (Prob) is specified over the next states. Distribution is represented by:

$$P(S' | S, A) \quad (17)$$

where, A is the action, S is the corresponding state, and P represents the transition probability distribution.

A policy π is a mapping from state S to action A . There are three steps for following a policy π .

1. Initially determine the current state S .
2. Execute the corresponding action.
3. And then Goto Step 1.

In AVs, an action is a driving reaction, states are the next position of the car, and rewards are the collection of corresponding action values. MDP forms sequential decision-making, each affecting the corresponding reward and the following state action. Policy formation is the major drawback of this process for AVs because of the uncertain and dynamic behavior of the environment around AVs.

Deep Q-Learning A Deep Q-Learning is a non-policy approach that does not get trained using a policy function [79]. An action-value function is used. This method creates a function that calculates a policy's maximum predicted future reward after performing an action in a given state. In general, it considers a terminal state time T , time from start to terminal state $t \in [0, T]$, state s , action a , reward r , training units (s_t, a_t, r_t, s_{t+1}) , discount factor $\gamma \in (0, 1)$, future discounted reward $R = \sum_{t=0}^{t=T} \gamma r_t$, policy function $\pi(s_t) = a_t : \max E[R]$, and the expected value (E).

$$(Action - Value)/(Q - Function Value) : Q(s, a) \quad (18)$$

$$Q(s, a) = \max E[R|s_t = s, a_t = a, \pi] \quad (19)$$

The expected future reward R is maximized using Q function value for taking action a at state s . Further Bellman Equation is applied to Q .

$$Q_{i+1}(s_t, a_t) = E[r_t + \gamma \max_{a_{t+1}} Q_i(s_{t+1}, a_{t+1}) | s_t, a_t] \quad (20)$$

where, Q_i gives output as the action values at a particular state s_{t+1} , the maximum value associated with action is chosen, Q_{i+1} depicts the future state values. The current reward, the discount factor, and the product of chosen maximum value are used to estimate the total reward for the current action and state. If the value of action and space is minimal, Bellman Equation is used to optimize the value of Q ; otherwise, the total reward of that particular action and space is reduced drastically. Considering neural networks weights θ , Q value (v), the neural network $Q(s; \theta)$ can be defined as:

$$Q(s = (... , images, params, ...); \theta) \rightarrow v_i \text{ for } i \text{ actions} \quad (21)$$

$$Loss : L_i = (r_t + \gamma \max_{a_{t+1}} Q(s_{t+1}; \theta_{i-1}) - Q_i(s_t; \theta_i))^2 \quad (22)$$

$$Loss : L_i = (Q_{i+1}(s_t; \theta_{i-1}) - Q_i(s_t; \theta_i))^2 \quad (23)$$

Compared to MDP, DQN resolves the convergence problem using value function instead of policy. DQN takes less time for training and requires a small amount of data for training. The major problem is the overestimation of Q -values using the same network for training and testing.

Fuzzy logic is a type of computation that relies on the degree of truth. To create a specific output, a fuzzy logic system combines the degree of truth, and linguistic factors in the input [80]. The condition of this input determines the type of output. The fuzzy logic architecture contains four components, as shown in Fig. 8: i) Rule Base contains all rules, ii) Fuzzifier converts raw input into fuzzy sets, iii) Inference engine sets ideal rules for a specific input, and iv) Defuzzifier converts fuzzy sets into explicit output. In AVs, fuzzy logic is helpful because of its flexibility. This concept can handle multiple inputs at a time and can make a precise decision. The space and data requirement is significantly less. The decision-making formulation is less complex, and it resembles human reasoning. The main disadvantage of fuzzy logic is that it depends on human intelligence and expertise. These systems are not able to recognize neural network platforms.

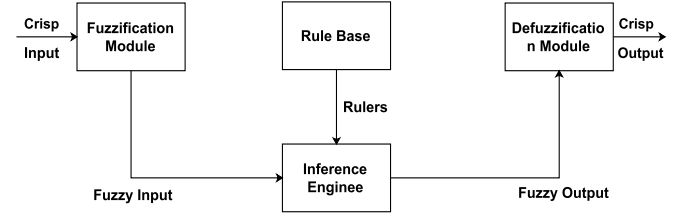


Fig. 8. Fuzzy logic architecture.

2.3. Simulators and tools

Numerous tools and simulators are used to test and train AVs for issues like lane changing, lane following, speed control, sudden braking, etc. Generally, before going to the field, an agent is created to test in the simulated environment; if it performs well, the final model is deployed in a real-life environment.

In this sub-section, all the major simulators, tools, and prototypes used to solve the overtaking issue for the AVs are presented in Table 4. In AV driving and navigation, each simulator is utilized according to the user's need. TORCS simulator and Logitech G27 racing wheel are used for AVs racing on competition tracks. Similarly, Network Simulator (NS) V3, SUMO, and IPG Carmaker simulators are utilized for handling the AVs driving behavior through communication. Further, Simulink, SCANer Studio, PreScan, and CARLA simulators tackle the overall driving maneuver. Apart from AV driving, mobility can also be addressed by considering traffic control. SimMobility, SUMO, CarSim, and Traffic Simulator (TS) on Pygame are the simulators used for traffic simulations.

3. Theoretical methods for overtaking maneuver

Theoretical-based techniques consist of modeling several scenarios and formulating the moving parts of vehicles into mathematical models. The overtaking issue usually incorporates these modeling techniques to complete the maneuvers. This section illustrates two variants of modeling approaches to achieve the overtaking maneuver: i) MPC-based approach and ii) Other mathematical modeling-based approaches. Both approaches are elaborated in upcoming subsections.

3.1. Model Predictive Control based approach

MPC is a multi-variable discrete-time control framework. The MPC controller utilizes the internal model to anticipate future plant behavior at each control interval. The controller calculates the best control actions depending on this prediction. The MPC estimates the controller state and sometimes predicts future plant outputs using noise models, disturbance, and linear plant [67]. In Fig. 9, the MPC technique and its variants are represented to show their accountability in solving the overtaking issue in AVs.

3.1.1. General Model Predictive Control

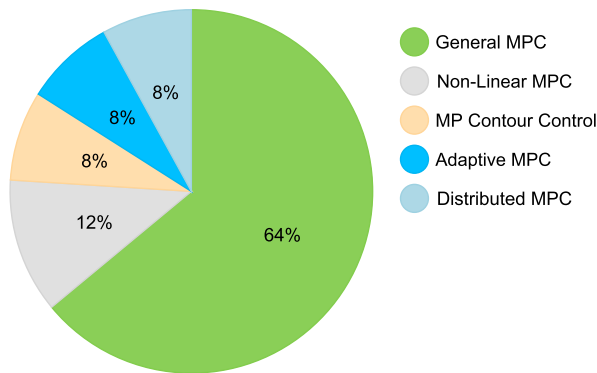
This subsection covers most of the research based on trivial MPC techniques. In Table 5, a comparative analysis of the classic MPC is illustrated that includes the tools, techniques, advantages, disadvantages, and other parameters. An appropriate overtaking motion-generating method using the MPC for AVs, considering real-time scenarios, is developed in [81]. Since overtaking motion generation is considered a non-linear and non-convex problem [82], their motion planner generates primitive control variables for autonomous cars. Non-convex systems and non-holonomic models are used to deal with the real-world environment [83]. The non-convex optimal control problem typically arises when using the MPC strategy to solve the overtaking problem [84]. In [85], the

Table 4

Tools and simulators.

Year	Simulators/Tools	Coding Language	OS Requirement	Application	Category
1984	Simulink	MATLAB	Windows	Control systems, AVs modeling	MM
1990	SCANE Studio	MATLAB, C, C++	Windows and Linux	Complete AVs dynamic simulation	MM
1997	TORCS	Python	Linux, FreeBSD, OpenSolaris, MacOSX and Windows	AVs car racing	A
1999	IPG CarMaker	C, C++	Windows	AVs driving assistance, V2X communication	A, M
2000	Traffic Simulator on Pygame	Python	Windows, Linux	AVs traffic simulation	A, M
2001	SUMO	Python	Windows, Linux, macOS	Traffic flow, AVs communication	A, M
2003	FORCES Pro	MATLAB	Windows and Linux	Control systems, mechatronics and robotics	
2005	CarSim	MATLAB, C, C++	Windows and Linux	Control and real-time system	MM, A, M
2008	Network Simulator V3	C++, Python	Linux, FreeBSD and MacOSX	AVs communication	M
2009	PreScan	MATLAB	Windows	Control and real time systems, AVs modeling	A
2010	RovisLab AMTU	Python	Actual hardware	AVs Forest road navigation	A, M
2010	Logitech G27	Python	Actual hardware	AVs and manual racing	A, M
2016	SimMobility	C++	Linux	AVs mobility	MM, A, M
2017	CARLA	Python	Windows and Linux	AVs driving systems	A

MM: Mathematical modeling methods; A: AI-based approaches; M: Miscellaneous methods.

**Fig. 9.** MPC techniques participation in overtaking maneuver.

problem of convexification has been investigated and compared to the non-convex basic formulation. It was initially demonstrated that, under constant longitudinal velocity, the corresponding optimization problems could be equivalent to convex quadratic programming by translating their objective function weights.

Nguyen et al. in [88] utilizes the stochastic MPC to offer a control strategy for AVs overtaking. The method ensures that the prediction has a high probability. More precisely, depending on their previous data, the authors forecast the location of the nearby vehicles at every prediction step. Using the MPC, Molinari and Fabio in [86] tackle the challenge of AVs overtaking. A MIP technique is proposed to obtain the goal, in which nearby automobiles are considered moving obstacles. Using MIP, the authors proposed three approaches for autonomous overtaking [96]. As a result, the second one [97], [98] was demonstrated to reduce the amount of unknown binary parameters and speed up online computing time. Moser et al. in [87] proposes control formulations that incorporate safety indications such as headway time as well as time to collision. Considering the benefits and drawbacks of both techniques i.e., MPC and MIP, a hybrid strategy is recommended. To account for the switching nature of this problem, the authors proposed the MPC framework using a MIP problem without explicitly specifying a reference vehicle. Chen et al. in [90] suggested a dynamic trajectory planning-based overtaking obstacles algorithm for autonomous driving, primarily consisting of trajectory tracking control and dynamic trajectory planning. The complex and dynamic trajectory planning method designs a lane change trajectory based on a polynomial function. A trajectory tracking controller depending on the MPC algorithm is meant to monitor the planned predicted route safely and securely to overcome the barrier. Dixit et al. in [89] proposed a framework for autonomous overtaking, including tactical awareness and trajectory planning. The authors offer a

mathematical framework comprising potential field-like functions and the MPC for developing an autonomous high-speed overtaking maneuver.

Zhou et al. in [91] presented a new method for evaluating the performance of autonomous overtaking approaches. The authors identified three gaps as safety indications rather than collision rate, which demands an exact model of human drivers' response to cut-in operations. These gaps are the time-to-collision gap, the time-headway gap of the rear vehicle, and the longitudinal inter-car distance gap of the lane-change maneuver. Nemeth et al. in [92] proposed the MPC-based technique for AVs overtaking maneuvers. The method employs a graph-based optimization approach considering the likelihood of colliding with nearby vehicles. The trajectory and velocity profiles were created using the MPC-based control techniques that considered specific limitations, such as acceleration and jerk limits. The discussion in [95] includes a control system capable of managing more difficult circumstances, such as rural road traffic with incoming automobiles. The overtaking algorithm is evaluated for several country road traffic scenarios in a stochastic traffic setting. An intelligent overtaking system was developed in [94], incorporating an overtaking possibility strategy (OPS) and an autonomous overtaking control scheme before successfully applying to AVs. An autonomous overtaking control system employing sequential linearization-based MPC was designed to determine the optimal throttle, brake, and steering angle actuators for an automobile. Finally, the suggested approach was integrated into an electric golf car and tested realistically. Weckx et al. in [93] provided a robust tracking and path planning solution that dynamically avoided an unexpected barrier by performing an overtaking maneuver to be adaptable to a variety of application areas. The method is based on MPC, which consists of multi-domain objectives that can be applied to various vehicle models and adapted multiple shooting strategy that ensures constraint satisfaction across the time domain [93].

The study in [53] presented a revolutionary integrated threat assessment method for the decision-making system. To analyze the possible threat, the motion of the adjacent vehicle is first predicted probabilistically using the interaction of multiple models. The authors then develop an integrated threat assessment function that synthesizes the current time-to-collision, time-headway [99], [100], and the initially intended time-to-front to assess the threat in each condition. The dynamic MPC solves action-state sequences and introduces a decision-making mechanism based on the MDP. Andersen et al. in [55] offered a method for autonomously overtaking unexpected objects in a complex urban environment using a trajectory-generating mechanism. Safe routes are developed by solving a non-linear constraint optimization, expressed as a receding horizon planner, in real-time to optimize the visibility of the

Table 5
Comparative study of the overtaking issue solved by the MPC approaches.

Authors	Language	Simulator	Issues	Approach	Environment	Outcome	Limitations	Multiple vehicle
Karlsson et al. (2016) [84]	Matlab	Simulink	Overtaking	MPC, Convex control problem	—	<ul style="list-style-type: none"> Two different approached i.e. convexification and non-convexification are compared for overtaking 	<ul style="list-style-type: none"> Not suitable for dynamic environment 	×
Obayash et al. (2016) [81]	Matlab, C++	CarSim & fmicon function	Overtaking, acceleration & steering change ratio	MPC	<ul style="list-style-type: none"> Single road with two surrounding vehicle and one ego vehicle 	<ul style="list-style-type: none"> Used techniques that mimic real world scenarios 	<ul style="list-style-type: none"> No consideration for other aspects like tyres forces, vehicle dynamics and many more 	×
Molinari et al. (2017) [86]	Matlab, C++	IPG CarMaker, Simulink	Moving obstacle avoidance	MPC	<ul style="list-style-type: none"> Two surrounding vehicle with one ego vehicle in a two lane straight road 	<ul style="list-style-type: none"> Overtaking computation time is reduced 	<ul style="list-style-type: none"> Not able to predict future events in heavy traffic 	×
Moser et al. (2017) [87]	Matlab, C++	IPG CarMaker	Overtaking, riding comfort and fuel efficiency	MPC	<ul style="list-style-type: none"> Two surrounding vehicle with one ego vehicle in a three lane straight road 	<ul style="list-style-type: none"> Combined approach of headway and collision helps in traffic efficiency 	<ul style="list-style-type: none"> No scope for testbed implementation 	×
Nguyen et al. (2017) [88]	Matlab, C++	IPG CarMaker	Overtaking	Stochastic MPC	<ul style="list-style-type: none"> Three vehicles in a two lane one way road 	<ul style="list-style-type: none"> High probability of prediction for overtaking 	<ul style="list-style-type: none"> No scope for testbed implementation 	✓
Dixit et al. (2018) [89]	Matlab	MPT3 Toolbox	Overtaking	MPC based controller	<ul style="list-style-type: none"> Two lane one way straight road 	<ul style="list-style-type: none"> Safety consideration for high speed overtaking in highways and urban roads 	<ul style="list-style-type: none"> Not tested in crowded scenarios 	×
Chen et al. (2018) [90]	Matlab, C++	Simulink, CarSim	Overtaking Obstacles, dynamic trajectory planning	MPC	<ul style="list-style-type: none"> Two lane one way road with multiple obstacle 	<ul style="list-style-type: none"> Minimizing the overtaking energy consumption Real time system Dynamic overtaking advantage 	<ul style="list-style-type: none"> No consideration for multiple vehicle handling 	×
Zhou et al. (2019) [91]	Matlab	CarMaker	Performance assessment of overtaking function	MPC, longitudinal inter-vehicle distance gap	<ul style="list-style-type: none"> Two lane road with one way traffic with multiple vehicles 	<ul style="list-style-type: none"> Instead of collision rate other adjacent techniques are utilized for overtaking scenarios 	<ul style="list-style-type: none"> No safety is considered while AVs maneuvers 	✓
Nemeth et al. (2019) [92]	C, C++	CarMaker	Overtaking	MPC, Graph based route planning, Velocity search algorithm	<ul style="list-style-type: none"> Two lane highway with one way traffic with multi-vehicle (Three) scenarios 	<ul style="list-style-type: none"> Motion of the surrounding vehicle can be incorporated 	<ul style="list-style-type: none"> Algorithm is not tested for safety maneuvers around multiple vehicles 	✓
Dixit et al. (2020) [45]	Matlab	IPG CarMaker, Simulink	High speed overtaking, Risk zone detection, Safe target identification, Trajectory generation	MPC, ROS framework	<ul style="list-style-type: none"> High speed structured environment with two lanes road 	<ul style="list-style-type: none"> Better technique for collision avoidance Safety consideration Safe speed overtaking 	<ul style="list-style-type: none"> Cannot handle multiple traffic and more complex road network overtaking scenario 	×
Andersen et al. (2020) [55]	Matlab	Simulink, One north in Singapore (Real)	Overtaking of an unexpected obstacle, Behavior planning	MPC, ROS framework	<ul style="list-style-type: none"> Urban environment with two way street 	<ul style="list-style-type: none"> Detection of blind-spot Analyze traffic situations Behavior planning of vehicle Handle unexpected scenarios 	<ul style="list-style-type: none"> No testing for commercialize vehicle and scenario 	✓
Xu et al. (2020) [53]	Matlab	Carsim, Simulink	Overtaking, Cut-in scenario	MPC	<ul style="list-style-type: none"> Three surrounding vehicles with three-lanes 	<ul style="list-style-type: none"> Handles Collision risk in multi-lane traffic Lane keeping Lane changing Double lane changing Safe trajectory 	<ul style="list-style-type: none"> No steering angle and velocity optimization for overtaking maneuvers 	×
Weckx et al. (2020) [93]	—	—	Overtaking obstacle, Path planning and tracking	MPC	<ul style="list-style-type: none"> Straight road with static obstacle in between 	<ul style="list-style-type: none"> Can follow predefined path as close as possible Can detect and avoid unforeseen obstacles 	<ul style="list-style-type: none"> No moving obstacle considered and need for position tracker 	×
Huan et al. (2020) [94]	—	Real Scenario	Lane following, Overtaking	Linearization based MPC, Time to lane change (TLC)	<ul style="list-style-type: none"> Long single lane with T junction with two-lane road 	<ul style="list-style-type: none"> Effective driving safety Decline in unnecessary acceleration and urgent steering 	<ul style="list-style-type: none"> Not tested on two way traffic with safety consideration 	✓
Sulejmani et al. (2020) [95]	Matlab, C++	IPG CarMaker	Overtaking	MPC	<ul style="list-style-type: none"> Single-lane country road segment 	<ul style="list-style-type: none"> Outperform human one with faster and smoother reaction 	<ul style="list-style-type: none"> Not testing on dynamic environment No consideration for gear shifting 	×
Jeon et al. (2022) [61]	Simulink	Matlab	Overtaking, Trajectory planner, High level decision maker	MPC	<ul style="list-style-type: none"> Two-lane straight road with one way traffic 	<ul style="list-style-type: none"> Guarantees safety Trip efficiency Passenger comfort Optimal overtaking trajectory 	<ul style="list-style-type: none"> No real scenario testing No consideration for two way traffic 	✓

AVs. Umberto et al. in [45] presented a unique framework for trajectory planning and situational awareness for autonomous overtaking in high-speed structured environments like highways and motorways. Simulink and IPG CarMaker simulator tool is utilized for experimental purposes in a high-speed structured environment like a highway and motorway in a two-lane scenario.

3.1.2. Other variants of MPC

Apart from the classical MPC approaches, several other variants of the MPC are introduced to handle the issue of overtaking in AVs. The nonlinear MPC, Model predictive contour control, adaptive MPC, distributed MPC, and many more are a few variants illustrated in the following subsections. In Table 6, a comparative study of other variants of the MPC approach is presented to solve the overtaking problem effectively.

Non-linear Model Predictive Control Non-linear MPC (NMPC) is a model in which non-linear system models are used to make predictions. The iterative solutions of optimum control issues on a limited prediction horizon are required in NMPC, similar to the linear MPC. While these issues are convex in classic MPC, they are not always convex in NMPC. Due to this, both the NMPC stability theory as well as the numerical solution face the issue of convergence of the optimization problem [67], [110]. Direct optimum control techniques employ Newton-type optimization procedures in direct collocation, direct single shooting, or direct multi-shooting methods, commonly used to numerically solve NMPC optimal control problems [67]. Path-following techniques never try to iterate any optimal control problem for convergence but instead take a few more iterations toward finding the solution to the most recent NMPC problem [111].

The cooperative optimum trajectory creation for autonomous driving is addressed in [104]. The cooperative route planning problem was solved using a decentralized NMPC [112]. The suggested solution does not require a different path-tracking since it immediately obtains the best control commands for the actuator using a limited non-linear optimization procedure. The collaborative design of a lane-changing controller employing an NMPC and a Sliding Mode Controller (SMC) is discussed in [106]. The objective is to achieve the performance and efficiency required for the overtaking operation. Nossier et al. in [108] proposed a local path planning strategy for AVs that avoids dynamic and static multi-obstacles. The suggested method can change the established waypoints from global path planning when obstacles are discovered. In this method, NMPC is a trajectory-tracking system that considers the vehicle's dynamics.

Palatti et al. in [107] offered a trajectory and behavior planning strategy for safer autonomous overtaking. The suggested method optimizes the trajectory by ensuring safety and limiting intrusion into the next lane. Furthermore, the technology allows the AVs to abandon the overtaking operation and merge back into the lane if safety is compromised due to opposing traffic emerging during the maneuver execution. NMPC is then used to plan feasible and collision-free paths.

Model Predictive Contour Controller The control objective of the bi-axial contouring system is to maximize correctness while decreasing traversal time. In contouring applications, the main aim is to minimize the distance between the current location and the desired path, referred to as contouring error. The control inputs for contouring systems are determined by optimizing a cost function that captures the trade-off between these conflicting issues, subject to the actuator and state restrictions, as suggested in [113]. A linear time-varying technique is presented to ease real-time implementation [113], and stability is ensured by introducing an additional contraction constraint. Andersen et al. in [101] offered a

technique for autonomously overcoming static barriers in a variable urban environment using a trajectory-generating mechanism. Schwarting et al. in [102] formulated a Model Predictive Contour Controller (MPCC) problem to trace the middle road while avoiding obstructions. The benefits of these techniques are maximizing visibility, prioritizing safety, and adhering to the road's borders while performing the maneuver. By learning spatial information, the research in [109] offered an algorithm that enhances the possibility of a safe overtaking move. The approach is used in an autonomous racing environment where cars must identify and operate within dynamic handling constraints. A Switched Model Predictive Contouring Controller (SMPCC) incorporates the policy learning technique into the path planning and control setup.

Adaptive Model Predictive Control MPC is a sophisticated approach that uses a linear-time-invariant (LTI) dynamic model to forecast future behavior. These predictions are inaccurate thus, it is a good strategy for making MPC robust for prediction errors [114]. Adaptive MPC is a control system that employs a fixed model structure. However, it enables the model parameters to develop over time to address this deterioration by modifying the prediction model for changing operating conditions. Franco et al. in [105] introduced a new short-term path planning technique for self-driving cars that navigates multiple moving obstacles. The primary goal of this technique is to develop a motion planning and execution system that allows ATLASCAR2 to exist with other approaching vehicles by minimizing collisions and overtaking them when required. Based on the adaptive MPC [115], an enhanced technique for short-term motion planning of AVs is offered. The core component of this study was an obstacle detection system that uses steering angle and throttle to maneuver the car around moving impediments in the lane.

Distributed Model Predictive Control Distributed MPC is a collection of predictive control architecture in which local controllers manipulate the subset of inputs to control a subset of outputs. Each controller uses the MPC method to regulate its system. It considers objectives, dynamics, disturbances of the subsystem, constraints, and the relationships between the systems. Every local controller handles the MPC issue using local data and can also exchange the data with another controller to increase overall performance [116]. The issue of cooperative optimal trajectory design for autonomous driving with a human driver vehicle (HDV) in traffic is addressed in [103]. The hierarchical MPC architecture with mixed-integer quadratic programming (MIQP) at the planning layer was used to tackle the challenge [117]. The authors suggest this [118] strategy is ideal for achieving on-road autonomous driving since MIQP performed well in overtaking scenarios. This study's anticipated HDV errors are constants throughout the trajectory prediction. The Simulink tool uses three vehicles on a three-lane one-way road to conduct experimental research.

3.2. Other mathematical modeling-based approaches

In the previous sub-section, MPC-based mathematical modeling techniques were surveyed, and this sub-section explores other mathematical models (excluding subsection 3.1) used for autonomous driving maneuvers. This sub-section also explores other mathematical modeling techniques for autonomous, safe, and secure driving maneuvers in different scenarios. This subsection is further categorized into two categories: i) Core mathematical, ii) Decision-based to improve autonomous vehicles' driving skills and to handle overtaking-related issues. In Fig. 10, several other mathematical modeling techniques are represented, and their participation in solving the overtaking issue in AVs is presented.

Table 6
Comparative study of the overtaking issue solved by the variants of MPC approaches.

Authors	Language	Simulator	Issues	Approach	Environment	Outcome	Limitations	Obstacle Movement
Andersen et al. (2017) [101]	Matlab, C++	ROS framework, FORCES Pro	Overtaking static obstacle, trajectory generation	MPC, Model Predictive Contour Control	<ul style="list-style-type: none"> Two way street road in urban environment 	<ul style="list-style-type: none"> Minimizes blind spot while overtaking 	<ul style="list-style-type: none"> Not considered for testbed implementation 	×
Schwarting et al. (2017) [102]	Matlab	FORCES Pro	static and moving obstacle avoidance	Model Predictive Contouring Control	<ul style="list-style-type: none"> Two lane with one way road 	<ul style="list-style-type: none"> Safety is insured with minimal deviation from human inputs Can be applied to on road autonomous driving in overtaking scenarios 	<ul style="list-style-type: none"> Involvement of more traffic with real time implementation Certain assumptions are made in human drive machine which are different from real scenarios 	✓
Viana et al. (2018) [103]	Matlab	Simulink	Static obstacle avoidance, cooperative path planning architecture	Hierarchical MPC	<ul style="list-style-type: none"> Three vehicles in a three lane one way road 	<ul style="list-style-type: none"> Provides planned trajectories with collision avoidance 	<ul style="list-style-type: none"> Involvement of human driver 	×
Viana et al. (2019) [104]	Matlab, C++	CarSim	Cooperative Overtaking, Trajectory generation	Non-linear MPC	<ul style="list-style-type: none"> Two vehicle in a three-lane road one-way road 	<ul style="list-style-type: none"> Can apply brake while obstacle avoidance to avoid collision 	<ul style="list-style-type: none"> No lane following feature integrated 	✓
Franco et al. (2019) [105]	Real Scenario	–	Moving Obstacle Avoidance, Motion planning	Adaptive MPC	<ul style="list-style-type: none"> Two lane highway with two way traffic 	<ul style="list-style-type: none"> New alternative for lateral and longitudinal control of AVs in highway roads 	<ul style="list-style-type: none"> No traffic consideration for urban roads 	×
Santana et al. (2020) [106]	Matlab	Simulink	Overtake	Non-linear MPC, Sliding mode controller	<ul style="list-style-type: none"> Straight road with two lane 	<ul style="list-style-type: none"> Overtaking can be aborted if safety compromised Can handle driving behaviors, Handle multiple objectives 	<ul style="list-style-type: none"> No testing on urban and highway roads 	✓
Palatti et al. (2021) [107]	Matlab	Simulink	Overtaking & Abort Overtaking	Non-linear MPC	<ul style="list-style-type: none"> Closed loop simulation 	<ul style="list-style-type: none"> Can avoid both static and moving obstacles Deal with both single and multi-vehicle 	<ul style="list-style-type: none"> Not suitable for heavy traffic scenarios 	✓
Nossier et al. (2021) [108]	Matlab	Simulink	Overtaking obstacle, Optimized path	Genetic algorithm, Non-linear MPC, Sigmoid function	<ul style="list-style-type: none"> Five-lane road having maximum three obstacle 	<ul style="list-style-type: none"> Effective overtaking strategy based on position advantage Successful in learning region of high probabilities on tracks 	<ul style="list-style-type: none"> Only suitable for high speed vehicle with limited vehicle in one way 	✓
Bhargav et al. (2022) [109]	–	Real Scenario	Overtaking	Switched Model Predictive Contouring Controller	<ul style="list-style-type: none"> Two race tracks 			✓

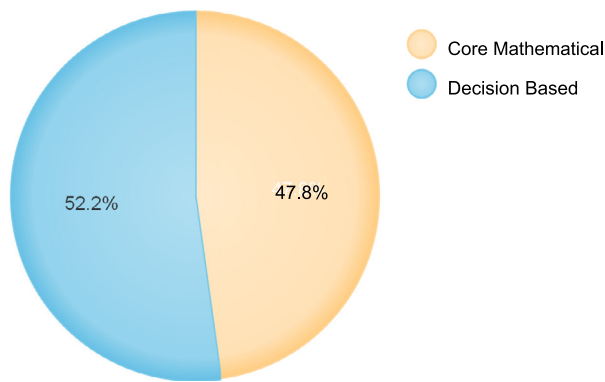


Fig. 10. Other mathematical modeling techniques participation in overtaking maneuver.

3.2.1. Core mathematical modeling based concepts

Core mathematical modeling-based techniques include kinematic modeling, quadratic programming, MIP, non-linear optimization, etc. A summarized and comparative study is illustrated in Table 7 to tackle the overtaking issue in AVs.

Kinematic modeling Kinematics is a branch of physics that explains the movement of objects, points, and groups of stuff without considering the issues that act them to move. It evolved from classical mechanics. Kinematics is sometimes described as the “geometry of motion” and is sometimes considered a part of mathematics. The location, speed, and acceleration of any unidentified portions of the system are calculated using geometry arguments [74]. Petrov et al. offered an adaptive controller and mathematical model for a self-driving car overtaking maneuver in [59]. They investigated the concept of autonomous overtaking without road infrastructure and created a feedback controller, which requires data on the present mutual orientation and inter-vehicle position. The overtaking vehicle is equipped with an adaptive nonlinear controller that allows it to track desirable trajectories even when the overtaken vehicle's velocity is unknown. The work in [121] aimed to provide a feasible strategy for designing real-time overtaking trajectories. The fundamental idea is to use a probabilistic Kalman-filtering approach to forecast all traffic participants on the road in order to obtain prohibited zones across the prediction horizon. Once the behavior module decides that overtaking is desirable, an initial trajectory is modeled using the path-tracking control scheme for lane change and a traditional PID controller to reach the desired speed. The controls are set up so that the simulated path conforms to kinematic restrictions.

The work in [93] offered a flexible tracking and path planning system that dynamically avoids an unanticipated barrier by performing an overtaking maneuver, intending to be adaptable to various application areas. This work uses ultra-wideband (UWB) technology to locate the Automated Guided Vehicle (AGV). Two UWB tags are deployed on the AGV to gather heading information and improve range data reception. To eliminate UWB range data, an extended Kalman filter is used with an outlier filter for fusion. In [123], simulation software is utilized to model the vehicle's dynamics, with the steering angle, coordinates, distance, and other factors specified to discover the ideal overtaking path. This study uses simulation software as a foundation, creates a radar model in the program, and uses the Kalman filter to track and recognize vehicles in simulation. This study employs a Kalman filter to ensure a safe gap between the two automobiles while simulating the car's overtaking maneuver to produce a better overtaking path.

Quadratic programming Quadratic programming is a technique for optimizing a multivariate quadratic function that is either linearly

constrained or not [124]. In quadratic programming, numerical optimization techniques are employed to minimize an objective function. The objective function comprises a set of choice variables that are either bounded or not [70]. The optimal resource issue of the Connected Autonomous Vehicle (CAV) communications is examined in [125]. In an overtaking situation, the goal is to investigate ideal velocity profiles from the viewpoint of balancing the cost of opportunity and the cost of energy. The optimization issue is handled by iterating the solution until an optimal solution is achieved using MATLAB's `fmincon` function, which is built on the Sequential Quadratic Programming approach [125]. The study in [120] aims to find the best way to safely drive an AV to overtake a slower leading vehicle. An investigation of the similarities between a convex relaxation and a conventional MIQP is presented [126]. Both techniques are examined in variable longitudinal velocity and lateral position, and the solution accuracy and computing effort are analyzed and compared. In the context of varied but known longitudinal speeds, the study in [56] investigated the topic of efficient overtaking of a slow-moving vehicle. The overtaking problem is stated by sampling the relative distance to the vehicle ahead, substituting the velocity state with its inverse, and applying a nonlinear change of control variables in a computationally efficient modeling technique. These three processes result in a nonlinear control problem that is computationally efficient and can be addressed using SQP. Designing a human-compatible control algorithm is a fundamental problem in autonomous driving. Coskun et al. in [47] investigated the challenge of autonomous overtaking with dynamic surrounding vehicles under both environment and state limitations. The answer is defined using quadratic programming and solved using the receding horizon method.

Mixed integer programming In mathematics, a linear objective function is maximized (or minimized) using linear programming when one or more restrictions are applied to the available parameters [73]. When one of the available parameters accepts an integer value, it is termed as MIP. Autonomous overtaking is addressed in [86] by utilizing MPC. As a result, a MIP technique was proposed. In this work, the nearby automobiles were treated as moving obstacles. Unlike current MIP methods, this study suggested more efficient formulations by decreasing the number of binary variables, allowing for online computing acceleration. The MIP approach for static obstacle avoidance was examined in [96]. However, this study states that the approach is used for moving obstacles. In the context of autonomous overtaking, the study in [87] proposed control formulations incorporating safety indications such as time headway and time to collision. Indeed, setting a maximum risk level for all surrounding vehicles, which is defined as the inverse of its lowest safety indication, is beneficial in ensuring collision prevention. The suggested control formulations are verified using the IPG CarMaker high-fidelity vehicle simulator.

Non-linear optimization Non-linear programming (NLP) is a mathematical term that refers to the method of solving an optimization problem in which most of the objective functions or constraints are non-linear [127]. The optimization problem is the computation of the extrema of an objective function over the collection of unknown variables and constraints. It is a branch of mathematical optimization that deals with non-linear issues [72]. The work in [121] aimed to provide a feasible strategy for designing real-time overtaking trajectories and can be accomplished in three steps. First, predict all traffic participants, then find all the free spaces, and finally, an optimization problem for finding the optimal trajectory is proposed. The method was implemented in C++, and the non-linear optimization issue was tackled using WORHP's SQP method [128]. The study in [122] offered a motion planning technique based on the vehicle kinetics model prediction to tackle

Table 7
Comparative study of the overtaking issue solved by core mathematical modeling concepts.

Authors	Language	Simulator	Issues	Approach	Environment	Outcome	Limitations	Multiple Vehicles
Petrov et al. (2014) [59]	Matlab	Simulink	Overtaking	Kinematic modeling	• Two vehicle in two-lane one-way road	• Overtaking without use of V2V communication or road info	• This strategy works only for one vehicle avoidance	×
Moser et al. (2017) [87]	Matlab, C++	IPG CarMaker	Overtaking, riding comfort and fuel efficiency	Mixed integer programming	• Two surrounding vehicle with one ego vehicle in a three-lane straight road	• Combined approach of headway and collision helps in traffic efficiency	• Not tested in real time scenario	×
Molinari et al. (2017) [86]	Matlab, C++	IPG CarMaker, Simulink	Moving obstacle avoidance	Mixed Integer Programming	• Two surrounding vehicle with one ego vehicle in a two lane straight road	• Overtaking computation time is reduced	• Not able to predict surrounding vehicle in heavy traffic	×
Strykowski et al. (2018) [125]	Matlab	Simulink-fmicon function	Overtaking, cruising, optimal velocity profile	Sequential quadratic problem	–	• Considers optimal energy aspects	• Does not consider energy efficiency for entire traffic	✓
Karlsson et al. (2018) [120]	Matlab	Yalmip	Trajectory generation for overtaking	Second order cone problem (SOCP), Mixed integer quadratic program (MIQP)	• Two-lane road with one-way traffic with multiple vehicles	• SOCP are more computationally efficient than MIQP	• Both strategies are computationally expensive	✓
Graf et al. (2019) [121]	C++	WORHP, Real Scenario	Overtaking, lane changing	Non-linear optimization, behavior planning, Kalman filtering method	• Two-lane highway with two-way traffic	• Can be practical for planning overtaking in real scenario	• Not tested on crowded urban traffic	✓
Weckx et al. (2020) [93]	–	–	Overtake obstacle, path planning and tracking	Kalman filtering, Kinematic Modeling	Straight road with static obstacle in between	• Can follow predefined path as close as possible • Can detect and avoid unforeseen obstacles	• No modeling considered for tyre pressure and engine dynamics	×
Karlsson et al. (2020) [56]	–	–	Optimal overtaking of slow moving leading vehicle	Sequential quadratic programming	• Straight two lane roads with oncoming traffic	• New technique of overtaking named inverse speed as a state	• Not able to work on curvy roads with variable speed	✓
Yang et al. (2020) [122]	Matlab, C++	Simulink, CarSim	Overtake	Non-linear optimization solver	• Two-way two-lane straight road	• Takes care of the slip angle and roll of the vehicle a leading vehicle	• Security issues while planning trajectories for overtaking maneuver	✓
Coskun et al. (2021) [47]	Matlab	Simulink	Overtaking, trajectory generation	Quadratic programming, non-linear programming	• Two-lane highway with multiple vehicles • One way traffic	• Highlights the effect of traffic behavior in trajectory planning	• No testing in high traffic urban scenario	✓
Lin et al. (2021) [123]	–	–	Overtaking	Kalman filtering, kinematic modeling	• Two cars in a two-lane road	• Efficient overtaking path than usual one without Kalman filtering method	• No safety on overtaking maneuvers and no real time testing	×

an optimization problem incorporating vehicle dynamics restrictions and safety and comfort considerations. The author opted for a two-degree-of-freedom vehicle dynamics model, established slip and roll restrictions, and solved the numerical optimization issue using a non-linear optimization solver. The suggested technique can plan a smooth and safe overtaking route according to simulation findings.

3.2.2. Decision based modeling concepts

Decision-based techniques include game theory, FSM, artificial potent field method, control theory, etc. A comparative study of the decision-based modeling technique is summarized in Table 8 with all details, including tools, techniques, advantages, environments, etc., in compact form.

Game theory Game theory is a branch of mathematics that studies mathematical models of rational actors' strategic interactions. It is used in logic, systems science, computer science, and all branches of social science. It was designed to deal with two-person zero-sum events, in which the other players perfectly balance each player's earnings or losses. Game theory is currently an umbrella name for the science of rational decision-making in people, animals, and computers, and it relates to various behavioral connections in the twenty-first century. The notion of mixed-strategy equilibria of two-person zero-sum games and John von Neumann's demonstration gave rise to modern game theory [132]. In [66], the authors presented a nonlinear receding horizon game-theoretic planner for autonomous automobiles competing with other cars. This work uses the game theory idea of Nash equilibria. The algorithm was designed for a multi-car racing game in which each car focuses on overtaking the ahead car until the race track ends. The vehicles share similar collision avoidance constraints to formalize the notion that all automobiles aim to avoid crashes. A high-fidelity simulator is used on the race track for experimental purposes. The study published in [119] presented a decision-making model for Connected AVs (CAV) interactions. It describes and validates methods for selecting control references strategically for cruising, platooning, and overtaking. The method is based on a cost analysis of energy usage versus time.

Finite state machine An FSM is a computational mathematical model. It is a machine that can only be in one of a finite set of states at any moment. In response to some inputs, the FSM can transition from one state to another [133]. A list of its states describes an FSM, and the inputs are the basis for each transition. There are two forms of FSM: deterministic and non-deterministic FSM [134]. Any non-deterministic FSM can be built as a deterministic FSM [135]. To enhance journey efficiency and riding pleasure while ensuring safety, the study in [61] proposed an overtaking algorithm that uses an FSM as a high-level decision-maker and probability-constrained MPC as a trajectory planner. By sub-manuevers of overtaking driving and correct transition conditions, the Conditional State Machine creates the proper reference and safety limitations in overtaking planning. Using an FSM, Palatti et al. in [107] offered an autonomous overtaking approach that employs rules to determine an acceptable behavior (maneuver) at each moment. An FSM is utilized to pick an acceptable maneuver at each moment and iteratively build intermediate reference targets depending on the current maneuver using a mix of safe and reachable sets. The simulation study shows that combining intermediate reference generation with MPC can manage many actions in a single framework, such as trailing a lead vehicle, overtaking, and aborting the overtake. The Simulink tool is utilized in a closed-loop environment for the simulation study.

Artificial potent field method An artificial potential field algorithm is commonly used in robot path planning using magnetic force to achieve the target point. The algorithm uses attracting forces to reach the goal point and repulsive forces to avoid an obstacle in the dynamic environment. The system is basic yet successful in avoiding robot collisions with obstacles since it enables real-time course planning [136]. The study in [89] proposes a framework for AVs overtaking movements, including trajectory planning and situational awareness. Initially, a potential field-like function is used to determine safe zones on a route, and then vehicle reachability settings are updated. The surrounding vehicle's region is mapped using an artificial potential field based on the objects' location, relative velocity, and orientation. The target generation block evaluates the reference state set point to be monitored at each sampling instant by identifying the safest road location consistent with the surrounding vehicles' dynamics. To accomplish this, the target generation blocks integrate the potential field's safe zones with the surrounding vehicle dynamics. Xie et al. in [131] presented a distributed motion planning methodology depending on the artificial potential field method for AVs escape overtaking in dynamic situations. According to MATLAB and Unreal Engine simulation results, the suggested strategy was accurate and successful for autonomous overtaking in a complicated dynamic environment with human-driven vehicles.

Control theory Control theory is a domain of applied mathematics that uses feedback to affect a system's behavior to achieve an objective. The regulation of system dynamics in engineered operations and machinery is the subject of control theory. The goal is to create a model or algorithm that governs the system inputs to guide the system to an ideal state while reducing delay and assuring control stability; this is generally done to achieve optimality [137]. In [88], Nguyen et al. described a control algorithm that employs a relaxed prediction. They estimated the location of the adjacent vehicles at every prediction phase based on the gathered data. The technique is based on accurate predictions of the surrounding vehicles' longitudinal and lateral speeds. As a result, these data provide adequate dynamics constraints for the presented control algorithm, which calculates the necessity for overtaking action while avoiding obstacles by tracking an appropriate longitudinal speed and lateral position. In [65] work, Raghavan et al. explored the problem of a classical vehicle overtaking. The stochastic control framework of this issue and the control technique, based on the probability of collision computation, are the main contributions of this study. A control method is proposed that minimizes the likelihood of a collision. The effect of computational noise and collision avoidance decision-making is investigated using simulation.

4. AI methods for overtaking maneuvers

In this section, AI-based AVs overtaking solutions are presented. AI-based techniques are more robust than theoretical-based techniques in handling real-life and simulation-based scenarios. In Fig. 11, each AI-based technique is presented to show their participation in solving the overtaking issue in AVs. AI-based methods include DL, DRL, RL, Fuzzy, and several ML approaches used to solve the overtaking problem in AVs. Forthcoming sub-sections detail the same.

4.1. Machine Learning

"ML is the study of computer algorithms that can improve automatically through experience, and by the use of data [138]." In AVs, ML is already applied to solve various issues in advanced driving assistance systems such as safety, security, obstacle detection, privacy,

Table 8
Comparative study of the overtaking issue solved by decision based modeling concepts.

Authors	Language	Simulator	Issues	Approach	Environment	Outcome	Limitations	Obstacle Movement
Naranjo et al. (2008) [60]	–	–	Overtaking, path tracking, lane changing	Fuzzy logic	<ul style="list-style-type: none"> Two-lane straight road with one-way traffic 	<ul style="list-style-type: none"> No trajectory is needed to carry out experiments 	<ul style="list-style-type: none"> GPS information is required for more accurate position of the surrounding vehicles 	✓
Milanes et al. (2012) [64]	–	Real Scenario	Overtaking	Fuzzy logic	<ul style="list-style-type: none"> Two-lane straight one-way road 	<ul style="list-style-type: none"> Tested in real environment with good results Width of surrounding vehicle can be calculated accurately 	<ul style="list-style-type: none"> Vehicle detection range is less and overtaking is performed at slower speed 	✓
Zhang et al. (2018) [129]	–	–	Overtaking (conservative and aggressive)	Fuzzy logic	<ul style="list-style-type: none"> Two-lane road with one-way traffic 	<ul style="list-style-type: none"> Handling overtaking problem in non-collaborative way 	<ul style="list-style-type: none"> No consideration for multiple vehicle and collaborative overtaking 	×
Nguyen et al. (2017) [88]	Matlab, C++	IPG CarMaker	Overtaking	Control algorithm	<ul style="list-style-type: none"> Three vehicles in a two-lane one-way road 	<ul style="list-style-type: none"> High probability of prediction for overtaking 	<ul style="list-style-type: none"> No testing performance in actual scenario 	✓
Dixit et al. (2018) [89]	Matlab	MPT3 toolbox	Overtaking	Artificial potential field algorithm, target generation block	<ul style="list-style-type: none"> Two-lane one-way straight road 	<ul style="list-style-type: none"> Safety consideration for high speed overtaking 	<ul style="list-style-type: none"> No safety is insured in case of heavy traffic 	×
Raghavan et al. (2018) [65]	–	–	Overtaking	Stochastic control approach, Control, probability theory	<ul style="list-style-type: none"> Three cars two lane one way straight road 	<ul style="list-style-type: none"> Irrespective of multiple agents outcome depend upon the main agent probability space 	<ul style="list-style-type: none"> Not able to return to original after overtaking 	×
Chai et al. (2021) [130]	–	–	Multi-objective optimal overtaking trajectory	Particle swarm optimization (PSO), multiobjective PSO, fuzzy adaptive rules	–	<ul style="list-style-type: none"> Less overtaking time Trajectory smoothness Vehicle visibility 	<ul style="list-style-type: none"> Very complex overtaking scenario and not tested in actual scenario 	✓
Wang et al. (2021) [66]	–	High fidelity simulator	Overtaking and blocking	Nash equilibrium, game theory	<ul style="list-style-type: none"> Race tracks 	<ul style="list-style-type: none"> Suitable for online planning Exhibits complex competitive behaviors in racing scenarios with multiple cars 	<ul style="list-style-type: none"> Not suitable for general traffic scenario having lane change, lane merger, intersection and many more 	×
Strykowski et al. (2021) [119]	C++	–	Cruising, platooning, overtaking, energy efficiency with time	Cooperative game theory, Nash equilibrium, connected autonomous vehicles	<ul style="list-style-type: none"> Cars and trucks are considered 	<ul style="list-style-type: none"> Road transport energy efficiency 	<ul style="list-style-type: none"> The cost function used for minimization is computationally expensive 	✓
Palatti et al. (2021) [107]	Matlab	Simulink	Overtaking & abort overtaking	Finite state machine	<ul style="list-style-type: none"> Closed loop simulation 	<ul style="list-style-type: none"> Overtaking can be aborted if safety compromised 	<ul style="list-style-type: none"> Cannot work in real complex urban and highway scenario 	✓
Xie et al. (2021) [131]	Matlab, C++	Unreal Engine, Simulink	Obstacle avoidance or overtaking in dynamic environment	Artificial potential field method, target tracking control protocol	<ul style="list-style-type: none"> Highway scenario with four lanes Three autonomous cars with two human driven vehicle 	<ul style="list-style-type: none"> Proposed method was accurate and effective for automatic overtaking in complex dynamic environment with human driven vehicles 	<ul style="list-style-type: none"> Slight stability issues while performing overtaking maneuver 	✓
Jeon et al. (2022) [61]	Matlab	Simulink	Overtaking	Finite state machine	<ul style="list-style-type: none"> Two-lane straight road with one-way traffic 	<ul style="list-style-type: none"> Guarantees safety Trip efficiency Passenger comfort Optimal overtaking trajectory 	<ul style="list-style-type: none"> Simpler simulation environment with no actual scenario testing 	✓

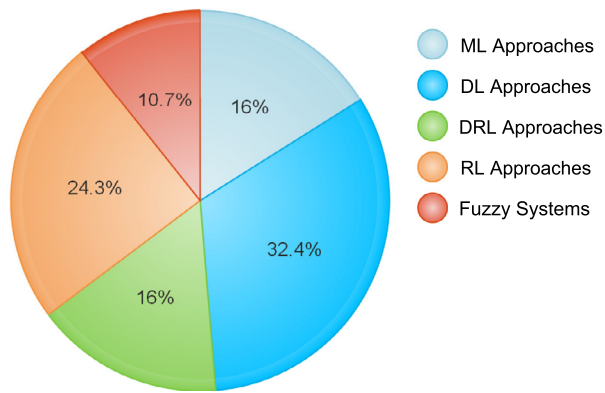


Fig. 11. Application based techniques participation in overtaking maneuver.

and many more [139]. This section explores research in AVs overtaking scenarios using ML approaches, and the summarization of the study is presented in Table 9.

4.1.1. Hidden Markov Model

An HMM is a network in which a node represents the probability distribution across labels and edges represent the probability distribution of movement from one node to another. This can be used to calculate the likelihood of the label sequence for a given input sequence [75]. Martinsson et al. in [141] proposed a combination of several HMMs to detect the clusters in trajectories, which might help speed up the labeling process of the raw dataset from the sensors. To cluster maneuver trajectory data, a probabilistic model-based clustering approach is used. The method employs a combination of several variants of HMMs to locate natural groups in the trajectory data. The HMM is a versatile model that can learn noisy and asynchronous behaviors and is not limited to sequences of the same length. This makes it an excellent choice for these unlabeled types of datasets. Soft assignment of motion trajectories can also be done using a combination of HMMs. Several variants of HMMs could be utilized on highways and country roads to detect standard sequences in the relative motion trajectory of drivers' behaviors.

4.1.2. Agglomerative hierarchical clustering

Agglomerative clustering is the most frequent class of hierarchical clustering, used to put items in clusters based on similarities. In this approach, each item is first treated as a singleton cluster. Following this, pairs of clusters are merged one by one until all the clusters have been combined into a single large cluster holding all items. The output is a dendrogram, a tree-based presentation of the items [145]. The approach in [50] visualizes the importance of combining collision data with participants' feedback to see the effect on the roads while performing automated overtaking maneuvers. Two crucial goals are addressed in particular. The first is identifying situational elements associated with overtaking crashes to highlight relevant risk scenarios. The second purpose is to examine the link between perceived risk and objective risk in overtaking, especially if situational parameters related to objective risk in overtaking impact perceived risk during autonomous maneuver execution. For the analysis, several algorithms were studied [146], out of which a hierarchical agglomerative approach is chosen as the best approach for AVs movement [147]. The cluster analysis was done with R statistical software, and the dissimilarity matrix was calculated using the cluster package's daisy function. The function derived from the "stats" package was used to conduct hierarchical clustering. Road Accident In-depth Studies (RAIDS) and the incorporated legacy study On The Spot (OTS) contain on-scene investigations.

4.1.3. Gaussian Process Regression

GPR is a highly powerful family of ML algorithms that can act on a few parameters to conduct predictive analysis. As the GPR has significantly fewer parameters, it is used to handle a wide range of supervised learning algorithms, even if there is limited data. The kernel parameterization of the regression can be thought of as kernelized Bayesian linear regression, where the choice of kernel function and the data to generate predictions determine the kernel parameterization [148]. A collaborative collision avoidance method for AVs performing lane changing and overtaking maneuvers (CCAV-OLC) is suggested in [144]. The CCAV-OLC scheme's performance is limited by the capacity of IRL to adapt to a high-dimensional AV environment. To address this, the IRL in CCAV-OLC employs a Gaussian Process regression model (IRL-GP), which allows data-efficient Bayesian predictions even with a small number of samples.

A Bagging GPR approach is suggested in [142] to emulate the driver's behavior for dynamic, complicated driving tasks such as overtaking, which combines Bagging and GPR. Human driver data is acquired using the simulation software, i.e., Prescan, and the data is used to train and validate various driver behavior models. A Bagging GPR technique for driving behavior modeling, which integrates GPR with Ensemble Learning, is presented to increase the performance of driving behavior modeling. The GPR considers the random interference components in driver behavior data.

4.1.4. Bayesian networks

A Bayesian network is a probabilistic graphical model that uses a directed acyclic graph to describe a set of variables and their conditional relationships [149]. Bayesian networks are perfect for forecasting an event that has already occurred and predicting the probability that the contributing factor is among several known causes. Influence diagrams are the generalizations of Bayesian networks that can solve and express decision problems under uncertainty [150]. The issue of maneuver estimation is explored in [140], which focuses on the situations involving the interaction among the traffic participants. Collective maneuvers are defined by Schulz et al. based on trajectory homotopy, which describes the relative motion of several vehicles in a scenario. The probability distribution across the various movements is produced using a Bayesian statistic and an interactive multiple-model estimator, which is a basis for the AVs decision-making during overtaking or lane changing. Bellingard et al. in [143] suggested a novel technique called Distance Awareness for Adaptive Velocity Profile to ensure the safety of a dynamic movement. This concept is based on the authors' prior work in [151], where they offered a Multi-level Bayesian decision-making approach [152] using a Two-Sequential Level Decision Network. This method works well while dealing with various scenarios, configurations, and many decision criteria considering ambiguity in mind.

4.2. Deep Learning

"DL is a class of ML algorithms that uses multiple layers to progressively extract higher-level features from the raw input [153]." DL is the critical component of self-driving automobiles, allowing them to detect a stop sign or differentiate between pedestrians and lampposts. DL models can attain accuracy, even surpassing human performance in some cases. Models are trained to utilize a considerable amount of labeled data and multi-layer neural network architectures [154]. The comparative study of several types of research involving the DL approach is summarized in Table 10.

4.2.1. Neural network

A neural network is a computational learning system that uses a network of functions to understand and translate a data input of one form

Table 9
Comparative study of the overtaking issue solved by the ML approaches.

Authors	Language	Simulator	Issues	Approach	Environment	Outcome	Limitations	Obstacle Movement
Schulz et al. (2017) [140]	Matlab, C++	–	Overtaking scenarios with oncoming traffic, maneuver estimation, merging scenarios at intersection	Bayesian Collective maneuver estimation framework, Interacting Multi-model (IMM), Kalman filter, multi-agent planning	• Two lane and three lane road with oncoming and single way traffic	• Can handle relative motion of multiple vehicles • Works on intersection and merging lanes	• No consideration of human behavior and actual scenario testing	✓
Martinsson et al. (2018) [141]	Python	Python pomegranate library	Cut-in, overtaking, lane keeping	Hidden Markov models, clustering	• Highway roads, track roads, country roads, basically two lane one way	• Useful in finding typical patterns in relative motion trajectories of driver maneuvers on highways and country roads	• Not able to handle the complex driving behavior having multiple vehicles	×
Chen et al. (2019) [142]	Matlab	PreScan, Logitech G29	Fast overtaking, obstacle avoidance	Bagging, Gaussian Process regression (GPR)	• Two lane road with left as overtaking lane	• Improving the performance of driving behavior learning	• Only tested on simpler roads and no actual vehicle testing	×
Sourelli et al. (2021) [50]	Python, Matlab, C, R	SCANer Studio	Overtaking	Hierarchical agglomerative clustering, unsupervised learning	• Three-lane motorway in single direction	• Identification of various factors related to overtaking, works in night conditions	• No user preference overtaking maneuver	✓
Bellingard et al. (2021) [143]	Matlab	Simulink	Overtaking	Multilevel Bayesian decision making approach, Time to collision	• Two lane one way highway	• Better safety and feasibility • Proper prediction of safety distance from other vehicle	• Not feasible for multiple obstacle and actual scenarios	✓
Prathiba et al. (2022) [144]	Python, C++	SUMO, Network Simulator V3	Overtaking, lane changing	Gaussian Process regression model, 6 th Gen Vehicle 2 Vehicle communication	• Random routes provided by SUMO simulator	• Cooperative communication minimize the risk of collision • IRL-GP accurately extract the reward function	• No testbed implementation with increase in latency of the network	✓

Table 10
Comparative study of the overtaking issue solved by the DL approaches.

Authors	Language	Simulator	Issues	Approach	Environment	Outcome	Limitations	Multiple Vehicles
Yu et al. (2017) [158]	–	Real Scenario	Overtaking	Deep Stacked Autoencoders	<ul style="list-style-type: none"> Three-lane one-way straight road Testing on two-lane road 	<ul style="list-style-type: none"> Works better at higher speed and urban real scenario 	<ul style="list-style-type: none"> Not tested on highway roads with adjacent vehicle moving at high speed 	✓
Verma et al. (2018) [159]	SimMobility Tool & Real Scenario	Python, C++	Overtaking, Stop and Go at Intersection, Turning, Lane Changing	YOLO, Kalman Filters (Vehicle Tracking)	<ul style="list-style-type: none"> Real Driving Scenarios 	<ul style="list-style-type: none"> Vehicle detection from camera image Classify the behavior of tracked vehicles in different lane scenarios 	<ul style="list-style-type: none"> Limited route testing with few surrounding vehicles 	✓
C. Lu et al. (2018) [52]	Python	PreScan, Logitech G27	Overtaking	General Regression Neural Network (GRNN)	<ul style="list-style-type: none"> Three lane urban road with three vehicles 	<ul style="list-style-type: none"> Very low learning error for all the drivers selected 	<ul style="list-style-type: none"> Traffic environment chosen is different from real scenario for testing 	×
Cha et al. (2018) [160]	–	Real Scenarios, RC platforms	Collision avoidance, Overtaking on straight road & corners	3D-CNN, 2D-CNN, LRCN, LSTM	<ul style="list-style-type: none"> Two lane one way road with two vehicles 	<ul style="list-style-type: none"> Real time collision avoidance algorithm Behavior prediction 	<ul style="list-style-type: none"> More training data is required 	×
Liu et al. (2019) [161]	Matlab	NGSIM, Simulink	Longitudinal and lateral motion of vehicles, freeway overtaking	LSTM	<ul style="list-style-type: none"> Two lane one way road 	<ul style="list-style-type: none"> Applicable to real world scenario Better performance in collision condition and computation time 	<ul style="list-style-type: none"> Training on limited real world data with specific area 	✓
Hegedus et al. (2020) [63]	C, C++	CarMaker	Overtaking, Multiobjective trajectory design	Neural Network, Search Algorithm, Complex Computation	<ul style="list-style-type: none"> One way two lane road with three vehicle 	<ul style="list-style-type: none"> Can handle the motion of surrounding vehicle Projected trajectory is comfortable with smooth profile and low lateral acceleration 	<ul style="list-style-type: none"> Can be more complex while integrating algorithm in real world scenario 	×
Markolf et al. (2020) [156]	–	–	Trajectory planning & control, Overtaking	Supervised Learning, Non-linear vehicle models, Approximate dynamic programming	<ul style="list-style-type: none"> Three vehicle in two lane road in a two way traffic 	<ul style="list-style-type: none"> Trajectory planning with non-linear dynamics 	<ul style="list-style-type: none"> Usage of more sophisticated network structures 	✓
Nemeth et al. (2020) [157]	C, C++	CarMaker	Trajectory design, control design, overtaking	Supervised learning technique, graph based algorithm	<ul style="list-style-type: none"> Two lane straight one way road with two vehicles 	<ul style="list-style-type: none"> Collision avoidance with surrounding vehicle 	<ul style="list-style-type: none"> No safety is insured while overtaking maneuver 	×
Ginerica et al. (2021) [162]	Python	CARLA	Overtaking, side-cutoff, opposite direction & rear pass vehicle handling	Vision based prediction based control	<ul style="list-style-type: none"> Two lane road with two way traffic having multiple vehicle 	<ul style="list-style-type: none"> Avoid collision between ego vehicle Better performance in aggressive driving contexts 	<ul style="list-style-type: none"> Not tested on actual roads 	✓
Liu et al. (2021) [51]	Python	TORCS	Overtaking, longest distance between infraction	Generative Adversarial Network (GAN)	<ul style="list-style-type: none"> Five tracks with diverse routes and scenarios 	<ul style="list-style-type: none"> Conditional GAN estimate human gaze map accurately Imitation network with gaze info have lower model uncertainty and better data accuracy 	<ul style="list-style-type: none"> No automatic selection of driving maneuvers 	✓
Ginerica et al. (2021) [163]	Python	CARLA, RovisLab	Overtaking, side cutoff, opposite direction, rear pass, multi-vehicle	CNN, RNN	<ul style="list-style-type: none"> 40 CARLA routes for testing 	<ul style="list-style-type: none"> Improves path tracking and obstacles avoidance accuracy 	<ul style="list-style-type: none"> No testbed implementation of the proposed strategy 	✓
Li et al. (2021) [164]	–	AMTU PanoSim	Object detection and state estimation, overtaking	CNN, RNN	<ul style="list-style-type: none"> Four obstacle vehicle with one ego vehicle More than four lanes 	<ul style="list-style-type: none"> Sensors failure does not affect the accuracy of the approach 	<ul style="list-style-type: none"> Applied only on obstacle avoidance not on overtaking 	×

into the desired output, usually in another form [155]. Neural networks, in this context, refer to a set of neurons that could be artificial. Several layers of linked nodes make up a neural network. Each node is a perceptron, which works similarly to multiple linear regression. The perceptron converts the signal from a multiple linear regression into a non-linear activation function [76]. Hegedus et al. [63] suggested a trajectory design technique for overtaking maneuvers built on a simple mathematical representation of possible trajectories utilizing the potential field approach. Here, neural networks are used to approximate the solution of the optimization's complex calculations. A unique strategy for designing reference trajectories for collaborative AVs using non-linear dynamics is presented in [156]. Using a supervised learning technique, the approach suggested in this research seeks for approximate solution: First, training data is created by solving optimal control problems, and then a neural network is trained with it. Three cars are chosen for the testing on a two-lane road with two-way traffic. The study in [157] aimed to look at trajectory design and control design methodologies for ML-based overtaking approaches that can ensure collision avoidance scenarios. The suggested neural network-based trajectory design approach successfully generated a suitable path. Simulations using CarMaker were used to demonstrate the method's performance. The technique is based on a robust control framework and is unaffected by the ML-based agent's structure.

4.2.2. Convolution Neural Network

CNN is an artificial neural network. It is used in computer vision and image recognition. A mathematical function generated by integrating two separate functions is referred to as "convolutional." Convolution is a mathematical term that describes how the structure of one function is affected by the shape of another. The CNN algorithm is the fundamental method to detect and classify different areas of the road and make suitable judgments during the AVs movement on the roads [165].

Verma et al. [159] introduced a sensor fusion system that combines Lidar and vision to reliably track and detect vehicles in complicated urban environments. Authors used DL algorithms to recognize vehicles from camera images and enhance their location estimation by combining Lidar data. They tested the method by following a leading vehicle engaged in common urban driving behaviors such as lane keeping, stop-and-go at junctions, lane changes, overtaking, and turning. Cha et al. [160] offered a revolutionary overtaking-based collision avoidance framework for AVs. The suggested Overtaking Procedure for Collision Avoidance Systems uses behavioral cloning and employs low-cost monocular camera images. Experiments were conducted to determine the optimal architecture for predicting steering and throttle values for a given image. The 3D-CNN model showed the most potential in reliably predicting throttle and steering values for overtaking movements and lane-keeping maneuvers. Ginerica et al. [163] employed ObserveNet to forecast the ego vehicle's future sensory observations. It acts as input for a local route planner. The route planner's skills were evaluated in aggressive driving circumstances (e.g., overtaking maneuvers, side cut-off situations, lane changing, and many more) by comparing it to a traditional path planner-controller [166] in the first round of trials in the CARLA simulator. To accomplish object identification and state estimation, Li et al. [164] suggests a two-stage fusion approach. Objects are recognized using radar and camera data in the first step, and the point cloud from LiDAR is turned into a 2D point map using bird's-eye projection. Ginerica et al. [162] proposed ObserveNet Control, a vision-based dynamic approach for AVs maneuvers control. The technique consists of a deep neural network that can accurately predict future sensory data across a time horizon of up to ten and a temporal planner that uses the projected sensory data to compute a safe vehicle state trajectory for overtaking scenarios.

4.2.3. Recurrent Neural Network

"RNN are a type of neural networks where the output from the previous step is fed as input to the current step [167]." RNN is a kind of artificial neural network that works with time series or sequential data. To learn, RNNs use training data. RNN is characterized by its "memory," which allows impacting on current output and input by using knowledge from previous inputs. Unlike standard deep neural networks, which believe that outputs and inputs are independent of one another, RNN output relies on the sequence's preceding components. While future events help in deciding the output of a sequence, unidirectional RNNs cannot account for them in their predictions [168].

The Overtaking Procedure for Collision Avoidance System (OP-CAS) is proposed in [160]. LSTM units are used in the long-term recurrent convolutional network design to retain prior action data as a memory for the subsequent controller output. The study in [161] aimed to integrate RL and DL techniques to develop a predictive overtaking strategy for AVs on freeways. The next Generation SIMulation (NGSIM) dataset extracts real-world driving information. To forecast the lateral and longitudinal motion of vehicles, the LSTM model is used. A two-stage fusion approach is suggested in [164] study to accomplish object identification and state estimation. Because of the uncertainty in the sensor and detection results, an Extended Kalman filter is used to estimate the object's best state. The approach in [162] tries to learn the kinematics of the visible driving environment in a self-supervised way without manually defining training labels using the vehicle's past state and sensory data in the form of Lidar point clouds. The abilities of ObserveNet Control in aggressive driving conditions, such as overtaking movements or side cut-off situations, are assessed by the simulation process.

4.2.4. Generative Adversarial Network

GAN is a DL-based approach to generative modeling. In ML, generative modeling is an unsupervised approach that automatically detects and learns patterns in input data [169]. GAN is an intelligent technique of training a generative model by framing the task as a supervised learning technique with two sub-models: the generator model, which produces new instances, and the discriminator model, which is used to categorize examples as real or fake [170]. Liu et al. [51] used a Conditional GAN to successfully predict human gaze mappings while running in both seen and unseen contexts. The authors quantified performance on single images and closed-loop testing, revealing that gaze-modulated dropout has the lowest prediction error, the best success rate in overtaking automobiles, the longest distance between breaches, and enhanced data efficiency.

4.2.5. General Regression Neural Network

A memory-based network that approximates continuous variables and converges to the non-linear or linear regression platform is termed as GRNN [76]. This GRNN has a highly parallel architecture and a one-pass method of learning. The approach enables smooth transitions from one observed value to another, even with small datasets in multi-dimensional measurement space. Chao et al. [52] developed a combined learning framework based on GRNN and NAC learning. Based on historical data, GRNN can be trained offline. As an outcome, the offline module's generic behavior can be reused and altered by the online module to capture the driver's particular behavior. A complicated overtaking scenario defines the learning issue and tests the situation. The authors created a learning system that can address the problem of driver-specific behavior adaption and learning during overtaking maneuvers for both longitudinal and lateral behavior.

4.2.6. Autoencoders

Autoencoder is an unsupervised artificial neural network that can compress and encode data effectively before reconstructing it to a representation similar to the original input. By definition, an autoencoder reduces the data dimensionality by excluding the noise present in the data [171]. Encoder, Bottleneck, Decoder, and Reconstruction Loss are the four essential components of autoencoders [172]. Backpropagation is then used in training to reduce the network's reconstruction loss. Yu et al. [158] proposed an autonomous overtaking decision-making approach based on a deep Q-learning network, which uses a deep neural network to learn the Q function from chosen action to a state transition. The authors used a deep-stacked autoencoder (SAE) [173] neural network to estimate the Q function. The states are used as inputs, while the Q values for each action are used as outputs. The deep SAE neural networks are built based on autoencoders. Experimental testing is conducted in a real scenario with a three-lane straight road with one-way traffic.

4.3. Deep Reinforcement Learning

"DRL is an ML technique that integrates RL with DL. The challenge of computational agent learning is to make a decision through trial and error [174]." DRL is a technique that includes DL, allowing agents to make choices based on unorganized input data without manually designing the state space. Table 11 presents a comparative summarized study of the most recent same-line studies.

4.3.1. Deep Q Network

A neural network is used to approximate the Q-value function in Deep Q-learning. The state is provided as an input, and the output is the Q-value of potential actions. DQN [79] are neural networks that use deep Q learning to create models such as intelligent video gameplay simulations. DQN consists of Convolutional Neural Networks and other structures that learn about various processes using specialized approaches. In [158], an autonomous overtaking decision-making approach based on a Deep Q-learning network is suggested, in which a deep neural network is used to learn the Q function from chosen action to a state transition. An actual scenario is selected for experimental study considering a three-lane one-way straight road. Mo et al. [176] suggested an intelligent AVs decision-making strategy based on RL for an upcoming overtaking scenario. To learn policies for both lane change decisions and longitudinal speed, a Double DQN agent was utilized. Prioritized Experience Replay [179] was utilized to accelerate policy convergence.

4.3.2. Deep Deterministic Policy Gradient

DDPG is an algorithm that learns both a Q-function and a policy simultaneously [180]. It learns the Q-function using the Bellman equation [181] and off-policy data and then utilizes the Q-function to understand the policy. It combines DQN (Deterministic Quality Network) and DPG (Deterministic Policy Gradient) principles. It is built on DPG, which operates across continuous action spaces. The DDPG approach is used in [175] to study overtaking strategies for a car in a simulated highway environment with several other vehicles. The uniqueness comes from the training technique, which involved teaching the agent to drive in a similar way as human beings learn to drive. This method learns smooth, collision-free movements in which the agent overtakes all other vehicles. In [177], a DRL technique based on a DDPG is suggested to train AVs overtaking actions. A framework for reward function design was proposed and applied to the overtaking maneuver for autonomous overtaking. By changing the reward function's parameters and constraints, this framework can be readily expanded to additional

Table 11
Comparative study of the overtaking issue solved by the DRL approaches.

Authors	Language	Simulator	Issues	Approach	Environment	Outcome	Limitations	Multiple Vehicles
Yu et al. (2017) [158]	-	Real Scenario	Overtaking	Deep Q Network	• Three-lane one-way straight road • Testing on two-lane road	• Works better at higher speed • Works in urban real scenario • Overtaking at higher speeds	• Not tested in real highway roads with crowded traffic	✓
Kaushik et al. (2018) [175]	Python	TORCS	Overtaking	Deep deterministic policy gradients, curriculum learning	• Highway scenario with single track having four vehicles		• Use of RNN strategy might increase the efficiency of the proposed methodology	✓
Mo et al. (2019) [176]	Python	SUMO	Overtaking	Double Deep Q Learning	• Two-way traffic with three oncoming vehicles	• Works better than DQN in SUMO simulator testing	• Use of DDQN strategy might increase the chances of accuracy of overtaking maneuvers	✓
C. Yu et al. (2020) [44]	-	Real Scenario	Overtaking, following vehicles, lane changing	Multi agent RL, Markov Decision process, Coordinated Graphs	• Different Highway Scenario	• This approach helps in learning of coordinated maneuvers for group of vehicles, safety features, rich pattern of driving	• No consideration of traffic intersection and other road issues	✓
Yi et al. (2020) [177]	-	-	Overtaking	Deep Deterministic Policy Gradients	-	• Low energy cost for overtaking • Performs better than fuzzy system and DQN	• Not tested on multivehicle scenario	×
Chen et al. (2021) [178]	Python	Highway-Env	Overtaking	Proximal policy optimization with self attention, intelligent driving model	• Two-way urban roads with multiple lanes	• Increases the efficiency & safety of overtaking decision making	• No consideration of traffic lights, pedestrian and traffic signals	✓

aspects of autonomous driving, such as merging, platooning, and formation.

4.3.3. Multi Agent Reinforcement Learning

Multi-agent RL (MARL) is the application of RL to regulate several agents [182]. It is similar to single-agent RL in that each agent tries to learn its policy to maximize its reward. It is feasible to apply a single policy for all agents, but this would need numerous agents to connect with a central server to calculate their actions, which is impractical in most real-world circumstances. Hence decentralized multi-agent RL is utilized instead [183]. Yu et al. [44] proposed a framework to use MARL to overcome the communication problem in autonomous driving. The emphasis is on the high-level strategic decision-making for overtaking a group of AVs on highways. This issue is considered as lane changing and overtaking movements are the two essential methods for autonomous driving. In dynamic situations, the authors developed a Dynamic Coordination Graph formulation for a distributed coordination issue with constantly changing dependencies.

4.3.4. Proximal Policy Optimization

PPO is a policy gradient approach that can be utilized in continuous or discrete action space contexts [79]. It uses on-policy training to train a stochastic policy. It also employs the actor-critic approach. The critic predicts the agent's rewards for the given observation, whereas the actor maps the observation to action. First, it samples from the latest edition of the stochastic policy to gather a collection of paths for each epoch. Then, the rewards-to-go and advantage estimations are computed to fit the value function and update the policy. A stochastic gradient ascent optimizer is used to update the policy, while a gradient descent technique fits the value function. The overtaking behavior of AVs is suggested in [178] using a DRL algorithm. First, a two-way urban road is constructed, with the AVs attempting to reach their goal safely and efficiently while taking into account many traffic participants. When the AVs does an overtaking maneuver, new responses from other vehicles are received. Then, a hierarchical control structure is provided to manage cars on the road, this monitors vehicle behaviors at the higher layer and regulates mobility at the lower layer. The high-level decision-making principles are derived using the DRL approach known as PPO. In order to increase the algorithm's performance, a self-attention mechanism is included [184]. Finally, the ego vehicle's overtaking tactics in various training time steps are examined.

4.4. Reinforcement Learning

"RL is a feedback-based ML technique in which an agent learns to behave in an environment by performing the actions and seeing the results of actions. For each suitable action, the agent gets positive feedback; for each bad action, the agent gets negative or a penalty. An RL agent senses and comprehends its surroundings and generally learns via trial and error [185]". Developers establish a system of rewards, desired actions, and penalizing negative behaviors in RL. This strategy motivates the agent by providing positive values to desired acts and negative values to undesirable behaviors. The agents are trained to behave ideally in every scenario. A brief description of RL-based methods used for overtaking scenarios is illustrated in Table 12.

4.4.1. Markov Decision Process

In RL, the MDP is a mathematical framework for describing an environment [78]. A Markov decision process consists of a set of states, models, possible actions, a real-valued reward function, and a policy to find the optimized solution.

Zhao et al. [53] presented a comprehensive threat assessment function that objectively estimates the collision risk in multi-lane traffic. The suggested function estimates the collision probability of any given strategy, such as lane-keeping, lane-changing, or overtaking. In this case, the decision-making architecture is based on the MDP, which can determine a safe and reliable path in complicated and frequently changing traffic scenarios using the optimum search approach. Xu et al. [48] proposed an RL strategy for autonomous decision-making of intelligent automobiles on roads. The sequential decision-making issue for lane change and overtaking was tackled using an MDP. A multi-objective approximate policy iteration approach was used to develop an optimum autonomous decision-making policy. A real highway scenario and a custom highway simulator are used for data collection and experimental testing considering a two-lane highway scenario.

A probabilistic MDP model to characterize vehicle behaviors was proposed in [46]. This MDP model accounts for road geometry and can simulate a more comprehensive driving behavior range. A new idea called the "dynamic cell" allows changing the traffic condition dynamically based on the surrounding vehicle's size, driver signals, and vehicle speeds [186]. Bezier curves [187] were utilized to plan smooth lane change pathways. Designing the driver's reward function successfully demonstrated several common driving behaviors, such as tailgating and overtaking.

4.4.2. Multi reward function

Instead of a single reward function, a multi-reward function is used for autonomous overtaking maneuvers in a Q-Network strategy [188]. An AVs overtaking framework was proposed in [129] based on relative position information such as distance, velocity, and angle. The Q-learning framework was used to suggest various overtaking methods. The study in [161] aims to integrate RL and DL techniques to develop a proactive overtaking technique for AVs on highways. First, the Next Generation SIMulation (NGSIM) dataset was used to extract real-world driving data. RL is used to lead the target vehicle through the scenario as quickly as possible based on the expected driving paths of the surrounding vehicles. Yuan et al. [189] proposed a Multi-Reward Architecture based RL method for highway driving policies. To better present multi-dimensional driving strategies, a single reward function was splatted into many reward functions. The total reward was divided into three-dimensional rewards: the reward for a lane change, the reward for overtaking, the reward for speed, and a significant penalty for the accident. For each reward, a branch of the Q-network is trained for the appropriate domain knowledge.

4.4.3. RL related concepts

IRL is an ML framework that solves the inverse issue of RL [190]. In essence, IRL is about learning from other people. IRL studies an agent's goals, values, and rewards based on its actions. A Cooperative Collision Avoidance Scheme for Autonomous Vehicles during Overtaking and Lane Changing Maneuvers (CCAV-OLC) is proposed in [144]. In the CCAV-OLC system, IRL automatically acquires the reward function from expert demonstrations by simulating actual human driving tactics and judgments. The SUMO Simulator was used to conduct experimental testing in randomly generated routes.

The actor updates in NAC are obtained via stochastic policy gradients utilizing Amari's natural gradient technique [191]. In contrast, the critic derives both the natural policy gradient and extra parameters of a value function using linear regression [192]. In [52], Lu et al. built a combined learning framework based on NAC learning and GRNN. GRNN can be trained offline using previous data, and NAC must be done online. As a result, the offline module's generic behavior was reused and changed by the online module to catch driver-specific behavior. To define the learning is-

Table 12
Comparative study of the overtaking issue solved by the RL approaches.

Authors	Language	Simulator	Issues	Approach	Environment	Outcome	Limitations	Multiple Vehicles
Lu et al. (2018) [52]	Python	PreScan & Logitech G27	Overtaking	Natural Actor Critics	<ul style="list-style-type: none"> Three lane urban road with three vehicles 	<ul style="list-style-type: none"> Very low learning error for all the drivers selected vehicle 	<ul style="list-style-type: none"> No consideration of real traffic scenario while performing overtaking maneuvers 	×
Zhang et al. (2018) [129]	–	–	Conservative & Aggressive Overtaking	Q-Learning	<ul style="list-style-type: none"> Two-lane road with one-way traffic 	<ul style="list-style-type: none"> Handles overtaking issues in non-collaborative way 	<ul style="list-style-type: none"> No collaboration among the ego vehicles while overtaking maneuver 	×
Liu et al. (2019) [161]	Matlab	NGSIM & Simulink	Longitudinal & lateral motion of vehicles, freeway overtaking	Speedy Q-Learning	<ul style="list-style-type: none"> Two-lane one-way road 	<ul style="list-style-type: none"> Applicable to real world scenario Better performance in collision condition & computation time 	<ul style="list-style-type: none"> Training on limited real world data with specific area 	✓
Yuan et al. (2019) [189]	Python	Unity & Unity ML-Agent	Speed change, Overtake, Lane Change	Multi-Reward Function	<ul style="list-style-type: none"> Five-lane one-way highway road 	<ul style="list-style-type: none"> Outperforms the previous state-of-art DQN method 	<ul style="list-style-type: none"> No consideration of other scenarios with heavy traffic 	✓
Xu et al. (2020) [53]	Matlab	CarSim, Simulink	Overtaking & Cut-in-Scenario	Markov Decision Process	<ul style="list-style-type: none"> Three-lane with three surrounding vehicles 	<ul style="list-style-type: none"> Safer trajectory while lane keeping, lane changing, double lane changing, collision avoidance in multi-lane traffic 	<ul style="list-style-type: none"> No consideration of steering angle and dynamic velocity while overtaking maneuver 	×
X. Xu et al. (2020) [48]	Python	In-house traffic simulator on Pygame	Trajectory Planning & control, Highway Overtaking, lane switching	Bezier Curve & Markov Decision Process	<ul style="list-style-type: none"> Highway scenarios with five-lane road 	<ul style="list-style-type: none"> More diverse driving in curvatures Smooth driving style Generates smooth path 	<ul style="list-style-type: none"> No tracking control Complex traffic scenario and optimized trajectory generation 	×
You et al. (2020) [46]	–	Real highway, Custom highway Simulator	Overtaking, lane changing	Value function approximation, feature learning, Markov Decision Process	<ul style="list-style-type: none"> Highway scenario with two-lane highway 	<ul style="list-style-type: none"> Better learning efficiency due to feature learning 	<ul style="list-style-type: none"> No inclusion of pedestrian control Traffic signals and road intersections 	✓
Du et al. (2021) [62]	–	–	Overtaking	Heuristic RL	<ul style="list-style-type: none"> Five vehicle each in driving & overtaking lane 	<ul style="list-style-type: none"> Improves the optimization effect of RL Better performance than Q learning Tested on various driving conditions 	<ul style="list-style-type: none"> No testbed implementation for urban and highway roads 	✓
Prathiba et al. (2022) [144]	Python, C++	SUMO, Network Simulator V3	Overtaking, Lane Changing	Inverse RL	<ul style="list-style-type: none"> Random routes by SUMO simulator 	<ul style="list-style-type: none"> Accurately extract the reward function than standard one 	<ul style="list-style-type: none"> No consideration of the real traffic scenario in simulation 	✓

sues and test situations, complicated overtaking behavior is used. The research in [62] provides an intelligent overtaking decision based upon that heuristic RL approach called Hey-Dyna for an autonomous vehicle. The suggested overtaking control concentrate on autonomous vehicle driving safety and efficiency. The overtaking problem is first modeled, and the adaptive safe driving region is built. Then, a heuristic RL approach called Heu-Dyna is created to derive the ideal overtaking choice.

4.5. Fuzzy systems

A fuzzy control system is a unique control system that relies on fuzzy logic: a mathematical framework that evaluates analog input data in terms of logical factors that take consistent values between 0 and 1 [80]. The word “fuzzy” acknowledges the fact that the theory involved deals with ideas that are “partially true” rather than “true” or “false.” The main advantage of fuzzy logic is that human experts understand the problem’s solution when expressed in phrases. It allows their expertise to be implemented in controller design [193]. Table 13 presents the fuzzy system techniques for overtaking AV maneuvers.

Naranjo et al. [60] proposed an overtaking strategy for AVs, including lane change and path tracking capabilities. During overtaking movements, the system employs fuzzy controllers that simulate human behavior and reflexes in real-time. It can drive an autonomous car and pass another vehicle in the same road lane. Two fuzzy driving controllers are used at the low level: path tracking and lane changing. Only two linguistic parameters and four fuzzy rules describe each controller. The authors presented a technique for automating one of the riskiest AVs operations, overtaking, in [64] research. The method relies on a stereo vision technology that detects any previous vehicle and initiates the self-driving overtaking maneuver. To achieve this goal, a controller based on fuzzy logic was created to mimic how people overtake it. It receives data from a vision-based system and a positioning-based system that includes an inertial measurement unit and a differential global positioning system. The necessary output is sent to the vehicle’s controllers – the steering wheel, throttle, and brake pedals – through a fuzzy logic-based controller.

In this study, [129], Zhang et al. attempted to accomplish overtaking in a non-collaborative situation using relative position information such as distance, angle, and velocity between vehicles. A fuzzy inference system is used to assess the driving behavior of the previous vehicle based on relative location information to minimize the complexity of maneuvers. A unique swarm intelligence-based method suggests a solution for the multi-objective optimum overtaking path problem of AVs in [130]. The presented technique optimizes the maneuver time length, trajectory smoothness, and vehicle visibility while considering various mission-dependent limitations by solving multiple objective optimal control problems. The vehicle’s multiple objective optimal overtaking maneuver was created using the FAMOPSO (Fuzzy Adaptive Multi-Objective Particle Swarm Optimization) algorithm, which considered several physical restrictions.

5. Miscellaneous methods for overtaking maneuvers

Miscellaneous techniques related to AVs driving maneuvers include V2V communication, PID controllers, degree of freedom, lateral and longitudinal controller, etc., which are presented in Table 14 to understand their significance in solving AV overtaking problems. The table summarizes the tools and techniques, advantages, disadvantages, scenarios, and environment related to solving the overtaking issue using several concepts related to AVs maneuvers.

Table 13
Comparative study of the overtaking issue solved by fuzzy system concepts.

Authors	Language	Simulator	Issues	Approach	Environment	Outcome	Limitations	Obstacle Movement
Naranjo et al. (2008) [60]	-	-	Overtaking, path tracking, lane changing	Fuzzy logic	Two-lane straight road with one-way traffic	<ul style="list-style-type: none"> No trajectory is needed to carry out experiments 	<ul style="list-style-type: none"> GPS information is required for more accurate position of the surrounding vehicles 	✓
Milanes et al. (2012) [64]	-	Real Scenario	Overtaking	Fuzzy logic	Two-lane straight one-way road	<ul style="list-style-type: none"> Tested in real environment with good results Width of surrounding vehicle can be calculated accurately 	<ul style="list-style-type: none"> Vehicle detection range is less and overtaking performed at slower speed 	✓
Zhang et al. (2018) [129]	-	-	Overtaking (conservative and aggressive)	Fuzzy logic	Two-lane road with one-way traffic	<ul style="list-style-type: none"> Handling overtaking problem in non-collaborative way 	<ul style="list-style-type: none"> No consideration of multi-vehicles and collaborative overtaking 	×
Chai et al. (2021) [130]	-	-	Multi-objective optimal overtaking trajectory	Particle swarm optimization (PSO), multiobjective PSO, fuzzy adaptive rules	-	<ul style="list-style-type: none"> Less overtaking time Trajectory smoothness Vehicle visibility 	<ul style="list-style-type: none"> Very complex overtaking scenario and not tested in actual scenario 	✓

Table 14

Comparative study of the overtaking issue solved by miscellaneous techniques related to AVs driving maneuvers.

Authors	Language	Simulator	Issues	Approach	Environment	Outcome	Limitations	Obstacle Movement
Alia et al. (2017) [194]	Matlab	Simulink	Overtaking static obstacle, trajectory generation	Clothoid tentacles method	<ul style="list-style-type: none"> Two-lane straight one-way road with vehicle and static obstacle 	<ul style="list-style-type: none"> Overtaking is performed with road rules and security measurements 	<ul style="list-style-type: none"> Not suitable for high speed overtaking 	×
Deng et al. (2018) [195]	–	–	Overtaking, collision avoidance, lane changing	V2V communication for overtaking	<ul style="list-style-type: none"> Two-lane road with two-way traffic having multiple vehicle 	<ul style="list-style-type: none"> The theoretical and simulation result matches Can avoid collision efficiently 	<ul style="list-style-type: none"> No connection among the ego vehicles is considered 	✓
Messaoud et al. (2018) [196]	Matlab	Simulink	Obstacle or vehicle avoidance or creating a safe path	Sigmoid function, rolling horizon method	<ul style="list-style-type: none"> Real environment maps with obstacle placed in between 	<ul style="list-style-type: none"> Safe path planning instead of initial trajectory 	<ul style="list-style-type: none"> No multi-obstacle avoidance and their movement is not considered 	×
Monot et al. (2018) [197]	–	–	Design lateral controller of an autonomous vehicle	PID Design, single PID, weighted multi-PID	<ul style="list-style-type: none"> Highway scenario 	<ul style="list-style-type: none"> PID enables the stability at all speeds 	<ul style="list-style-type: none"> No consideration of road friction while PID designing 	×
Deng et al. (2019) [69]	–	–	Overtaking, collision avoidance	V2X communication based overtaking, Poisson point process	<ul style="list-style-type: none"> Two-lane random straight road 	<ul style="list-style-type: none"> Collision avoided efficiently Better accuracy in detecting the vehicle with largest collision probability 	<ul style="list-style-type: none"> High latency due to cellular network Less accuracy due to absence of V2V communication 	✓
Ortega et al. (2020) [54]	Matlab	Prescan, Simulink	Overtaking	Adaptive cruise control, Technology independent sensors	<ul style="list-style-type: none"> urban, one way three lane highway 	<ul style="list-style-type: none"> Increase understanding of overtaking maneuver, safety and reliability while overtaking Can be used in real time 	<ul style="list-style-type: none"> Dynamic scenarios are not considered and sensor dependent traffic rules are followed 	✓
Bolufe et al. (2021) [198]	Matlab	Simulink	Unsafe overtaking	V2V communication based overtaking	<ul style="list-style-type: none"> Three AV's with two lane straight road 	<ul style="list-style-type: none"> Stable CAM transmission rate Best incident detection rate (IDR) in different condition 	<ul style="list-style-type: none"> Increased IDR at low SNR (signal to noise ratio) 	✓
Abdelkader et al. (2021) [199]	–	Theoretical Concept	Overtake	V2V communication, cooperative perception	<ul style="list-style-type: none"> Two lane highway with two way traffic 	<ul style="list-style-type: none"> Informed decision concept for a safe and efficient passing 	<ul style="list-style-type: none"> Not tested with multiple vehicles with platooning concept 	×
Zhang et al. (2021) [200]	–	Six degree of freedom simulator, Real Scenario	Overtaking	Behavior analysis, Time To Collision	<ul style="list-style-type: none"> Highway scenario in two lane 	<ul style="list-style-type: none"> Behavior analysis based on different overtaking scenarios 	<ul style="list-style-type: none"> No simulation or actual scenario testing performed 	×

Vehicle to Vehicle Communication V2V is a communication technique that aids collision avoidance [201]. The data comprises speed, location, braking, stability, travel direction among the vehicles, and other things. V2V communication allows vehicles to receive, send, and re-transmit signals through a wireless mesh network [202]. A collaborative autonomous driving system in which a vehicle overtakes the vehicle ahead of it based on shared perception is presented in [195]. The authors presented a V2X-based collaborative collision avoidance strategy to avoid vehicle accidents in the opposite lane. The overtaking vehicle uses V2V communications to calculate its distance from its neighbors and determines whether or not to overtake. Deng et al. [69] investigated a V2X-based collision prevention strategy in the overtaking scenario. In this approach, vehicles evaluated their distance from each other using V2X communication, accounting for independent and collaborative detection scenarios. In the overtaking situation, the authors created a V2X-based collision avoidance technique (V2X-CA). The overtaking vehicle uses the Received signal strength to determine the distance between itself and the vehicles with the highest collision probability in the proposed scheme. The capacity and interference features of the V2X-based autonomous driving system are analyzed using a Poisson point process.

Abdelkader et al. [199] presented a lane overtaking decision algorithm for connected cars built on cooperative sensing to convey real-time critical safety data about their state, including location, speed, and signal direction. The first scenario features an incoming vehicle approaching from another direction, requiring another vehicle's comprehensive investigation to make a safe overtaking. The second scenario features a more critical circumstance with high uncertainty traffic conditions due to the leading vehicle's low vision. The effectiveness of relevant awareness control techniques, such as IVTRC [203], POSACC [204], and many more to enable the V2V-based overtaking application in autonomous driving, is evaluated in [198]. The key contribution of this research is to assess the influence of the targeted awareness control techniques on forecasting critical overtaking movements while accounting for motion state sensor faults and packet drop due to channel fading. The capability of POSACC (POSITION-ACCURACY) to respond to changes in vehicle dynamics and attain a steady CAM transmission rate, which enhances its likelihood of detecting risky overtaking movements, is demonstrated via simulation study [198].

Miscellaneous techniques This sub-section describes some miscellaneous concepts, apart from the techniques discussed in previous sub-sections. Alia et al. [194] focused on the trajectory planning module and created a clothoid tentacles-based algorithm for local path planning. This work aimed to improve the tentacles technique by evaluating the overtaking maneuver and developing an appropriate trajectory for lane-changing operations while considering vehicle kinematics, traffic restrictions, and some security factors into account. Monot et al. [197] aimed to create a lateral controller for AVs that is resistant to large longitudinal speed changes. This study was used to construct a weighted PID controller as a function of longitudinal speed. The weights on the PIDs provide stability at all speeds, while a single PID cannot ensure this in all circumstances. The weights and intervals are calculated based on system dynamics. As an outcome, the proposed approach is used in handling lane-changing systems, overtaking systems, and many more. Ben-Messaoud et al. [196] proposed a smooth localized dynamic trajectory of a global path to avoid neighboring vehicles and obstacles. The suggested method can design trajectories for autonomous cars on a local level that is particularly useful for overtaking and collision avoidance maneuvers. The method constructs a local trajectory based on a global reference path in a relatively short execution time to avoid static and dynamic obstacles. Zhang et al. [200] gathered five testers to conduct a 2×3

in-test design experiment using a six-degrees-of-freedom simulation tool to investigate the features of driver overtaking behavior. The time-to-collision, the longitudinal distance between two cars, and the steering wheel angle at the start of the overtaking behavior are used as analytical indicators to define the driver's behavior features. The speed difference significantly affects the values of the following three indicators. Ortega et al. [54] presented a simulation of an overtaking maneuver scenario using AVs using PreScan simulation software, which integrates Technology Independent Sensors and an Adaptive Cruise Control to perform the overtaking move. The approach revealed that AVs could complete overtaking maneuvers in an urban setting, including conventional vehicles, by satisfying safety and reliability norms.

6. Research gaps, challenges, future research directions and open problems

In this section, an exploration of research gaps, challenges, future research directions, and open problems is carried out. This section is further divided into two categories: i) Research gaps and challenges and ii) Future research direction and open problems.

6.1. Research gaps and challenges

In this survey, we thoroughly investigated various overtaking maneuvers and explored several techniques to tackle the overtaking problem. Based on the above study, this sub-section presents the identified research gaps and challenges in overtaking AVs, as presented in Table 15, that can be useful for researchers in the same domain.

6.1.1. Overtaking in mixed autonomous vehicular traffic scenario

While performing the overtaking maneuver, all the tasks, including lane changing, acceleration, and lane following, are performed without human intervention in most state-of-the-art methods (Sections 3, 4, and 5). A driver can switch from automatic to manual driving while performing overtaking maneuvers in some cases that need human supervision. Combining both automatic and manual techniques makes the overtaking maneuver more user-friendly. Moreover, in a mixed traffic scenario, the human driver's random behavior is entirely missing in the state-of-the-art while making the overtaking decisions for AVs. As the AVs overtaking methods only consider the upcoming vehicles, tailgating vehicles, and the target vehicle. These methods missed the speed breakers, potholes, pedestrians, human drivers' random behavior, and other obstacles during the overtaking decisions, which need to be substantially explored.

6.1.2. Traffic light cooperative AVs driving in a smart city scenario

From the recent literature study [44], [48], [49], [45], it is observed that some research efforts have been made for AVs overtaking considering traffic rules. This led to safety concerns, as discussed in sections 2, 3, 4, and 5 during autonomous driving. Safety consideration is essential while performing overtaking maneuvers on the highway or urban roads. During autonomous driving in smart cities, there is a need to explore traffic light cooperative autonomous driving. This can be useful for minimizing travel time, overtaking efforts, fuel consumption, traffic congestion, and many more.

6.1.3. AVs overtaking with two-way traffic

In a real-life scenario, we find several situations where a vehicle is moving on the road having a two-way traffic scenario. To handle the two-way traffic for performing overtaking maneuvers, a few research efforts are carried out [178], [11]. However, this is a severe concern in developing countries such as India. Therefore,

Table 15
Research Gaps, Problems, Proposed solution, and Category.

Category	Research Gaps	Details in	Problems	Proposed Solutions
Traffic related	Mixed Vehicle Traffic	Sec:6.1.1	<ul style="list-style-type: none"> • Manual vehicle driving behavior • Potholes, speed breakers, pedestrians and many more 	<ul style="list-style-type: none"> • Advance advice system with AVs • Smooth switching between automatic and manual transmission
	Traffic light consideration	Sec:6.1.2	<ul style="list-style-type: none"> • Vehicle to traffic light connectivity • Traffic rules integration in AVs 	<ul style="list-style-type: none"> • Improved V2X connectivity • AVs driving Behavioral management
	Two way traffic consideration	Sec:6.1.3	<ul style="list-style-type: none"> • Oncoming traffic • Difficult lane change maneuver 	<ul style="list-style-type: none"> • Next lane vehicle detection system before lane change
Environment Related	Heterogeneous road infrastructure	Sec:6.1.4	<ul style="list-style-type: none"> • Unstructured road • Shattered road, road stops, potholes, etc. 	<ul style="list-style-type: none"> • Emergency action behavior in AVs • Early warning system
	Dynamic weather conditions	Sec:6.1.5	<ul style="list-style-type: none"> • Dynamic weather condition mainly focused on rainy weather 	<ul style="list-style-type: none"> • Improved traction control system in AVs • High quality lidar device for maximum visibility
	Unexpected Obstacle	Sec:6.1.6	<ul style="list-style-type: none"> • Random animals and person in middle of road • Natural calamity such as landslides, earthquake, and many more. 	<ul style="list-style-type: none"> • Equipped with advance tech like radar technology to detect obstacle from distance • Automatic switching to manual transmission with warning system in case of emergency
	Overtaking in variable environment	Sec:6.1.7	<ul style="list-style-type: none"> • Steep terrains • Twisted roads • Single lane roads • No lane markings 	<ul style="list-style-type: none"> • Considering the inclined movement of AVs (other than lateral and longitudinal) • Computing the mid lane using Lidar without lane marking
Behavior planning related and Others	Fuel efficiency consideration	Sec:6.1.8	<ul style="list-style-type: none"> • Rash driving behavior • Expected travel time • Driving comfort 	<ul style="list-style-type: none"> • Autonomous driving behavior management considering adequate throttle response
	Testbed/Field implementation	Sec:6.1.9	<ul style="list-style-type: none"> • Costly Hardware requirement • High reaction time • Computational complexity 	<ul style="list-style-type: none"> • Training in actual traffic • Used light weight models • Use V2V communication
	Heterogeneous Vehicle Dynamics	Sec:6.1.10	<ul style="list-style-type: none"> • More vehicles in real traffic compared to simulated environment • Types of vehicle in real traffic 	<ul style="list-style-type: none"> • Training in actual traffic • Use of average dynamics of all the general vehicles for common training procedures
	Dynamic trajectories consideration	Sec:6.1.11	<ul style="list-style-type: none"> • Dynamic trajectory prediction • Dynamic obstacle/surrounding vehicles 	<ul style="list-style-type: none"> • Quick reaction time • Use of light weight model for trajectory prediction

there is a need to explore the AV overtaking mechanism to tackle this most challenging scenario of overtaking.

6.1.4. AVs overtaking considering heterogeneous road infrastructure

In current road scenarios, structural damage, such as potholes, several speed bumps, shattered roads, road stoppers, etc., are constructed and generated at regular intervals. Such heterogeneous road infrastructure requires an intelligent mechanism to understand the kind of road while overtaking a vehicle, as the traditional overtaking approach cannot handle all these scenarios. This domain is entirely missing in state-of-the-art, which needs to be explored thoroughly.

6.1.5. AVs overtaking by accounting different weather conditions

While driving vehicles on roads, different weather conditions might affect driving behavior. While overtaking the vehicles, a scenario like a rainy season, snow-filled roads, and foggy climate creates a lot of hurdles during the overtaking maneuver. To the best of the authors' knowledge, research efforts still need to be carried out to handle all these situations to perform overtaking maneuvers safely. This needs to be explored for the sustainability of the AVs.

6.1.6. AVs overtaking considering dynamic random obstacles and unexpected anomalies

While performing the overtaking maneuver, there is a situation where a dynamic obstacle occurs. Dynamic obstacles include the sudden animals or objects in between roads, the sudden arrival of pedestrians, and road accidents that lead to an uncertain obstacle between roads. A few efforts have been made to handle these situations [142], [131]. However, more rigorous research is required to handle all possible scenarios while performing the overtaking maneuver by accounting for different localities in the cities/states or countries.

6.1.7. AVs overtaking performance analysis in variable environments

As listed in Table 3, only three types of environments are discussed in the literature, i.e., highway roads, urban roads, and race tracks. Apart from these, in backward areas, there exist several single-lane roads, twisted curvy roads in hilly areas, rolling terrains, steep terrains, and many more scenarios which have not been explored. Thus, there is a need to explore a framework for training and testing AVs in these variable environments to make the overtaking maneuver more robust and environment-friendly.

6.1.8. AVs overtaking considering fuel efficiency

As moderate and simple driving behavior will lead to better fuel economy if there is no rigid travel time requirement. On the unhand, fuel efficiency is not considered then aggressive driving behavior is used to minimize travel time for overtaking maneuvers. In the same line, some research efforts [52], [55], [160], [159], [142], [200] have been made to carry out the study of autonomous driving behavior. However, there is a need to explore AV overtaking strategies considering the travel time and fuel consumption trade-off that need to be included in the state-of-the-art study.

6.1.9. Field implementation of AVs overtaking methods

From the above study, it is observed that theoretical and AI-based approaches for solving overtaking problem lack infield implementation on actual AVs. As both strategies are tested under the simulated environment. Although some efforts have been performed in [158], [94] to mimic the behavior of an actual vehicle, such as AVs, to perform overtaking maneuvers in a real-life scenario. However, there is a need to analyze the performance of the overtaking methods in field implementation on actual AVs by accounting for real-time computational and response time requirements, communication and computational latency, and random traffic behaviors in several remote/urban scenarios.

6.1.10. AVs overtaking considering heterogeneous vehicular dynamics

According to the literature, majorly, experimental testing is carried out on either two-vehicle [160], [157], three-vehicle [103], [52], [88], and sometimes considering multi-vehicle scenario with four or five vehicles [164], [175], [131]. This case is entirely different from the actual scenario. As more than hundreds of vehicles run on a typical highway with heterogeneous vehicular dynamics, only a few efforts have been made considering the small number of vehicles on the roads [91], [120]. However, it is far too away from the actual traffic scenario. Hence, a rigorous effort is required to design the overtaking algorithms considering heterogeneous vehicular dynamics.

6.1.11. AVs driving considering dynamic trajectories for overtaking maneuvers

AVs first design the path trajectory for overtaking maneuvers, and then, the AVs follow the same path to perform overtaking maneuvers. Some researchers [57], [58], [123], [93] follow the same procedure to perform overtaking maneuvers. However, most of the existing trajectory design methods are static. There is an extreme need to explore dynamic trajectory design mechanisms for an effective AVs overtaking problem considering several actual field scenarios to meet the real-life application demands.

6.2. Future research directions and open research problems

Based on this survey study, this sub-section presents our findings in terms of future research directions and open research problems in the domain of AVs overtaking maneuvers. The corresponding proposed solutions and their respective reference are illustrated in Table 16. These can be very useful for researchers to carry out further research in the same domain.

6.2.1. AVs overtaking in hilly and sloppy areas

Based on this survey study, it is observed that most of the approaches have been designed for highway roads, urban roads, and race tracks, as shown in Table 3. Since hilly and sloppy areas require different driving maneuver profiles for handling AVs overtaking. Typically, in these areas, other parameters, such as road angle, friction etc., accounted for better overtaking decision-making apart from lane change, acceleration, and lane following. This domain needs to be explored explicitly.

6.2.2. Upgrade traditional overtaking approaches

In traditional data association techniques, generally, data are attached to moving objects, and location is computed by calculating the geometric average of the data. The computational complexity of the entire calculation is very high [222]. A more robust approach is required to calculate the moving object/vehicle's location efficiently. An extensive research effort is required to decrease the computational complexity of the traditional approach. Further, an accurate location estimation increases the efficiency of overtaking maneuvers, which also needs to be explored.

6.2.3. Robust model based approach for driving maneuver

Apart from theoretical based modeling approaches, a non-parametric approach that uses physical models of geometry and sensor models of objects can be used for vehicle movement. A physical and sensor model provides better accuracy for moving vehicles than traditional theoretical-based models. Hence, it allows exploring the physical and sensor-based model approaches to optimize the time for overtaking maneuvers.

6.2.4. Stereo vision based approach for vehicle movement and detection

A stereo vision-based technique can be used for object detection, depending on the color and depth information given by

Table 16
Open challenges, Problems, Proposed solution, and Recommended References.

Open Challenges	Problems	Proposed Solutions	References
Overtaking in hilly areas	<ul style="list-style-type: none"> • Road inclination • Sloppy roads • Sharp curves 	<ul style="list-style-type: none"> • Considered normal and lateral force in vehicle on inclined road • Satellite based navigation system is used for decision making 	[205], [206], [207].
Traditional overtaking approaches	<ul style="list-style-type: none"> • Computational complexity • Precise Location of objects/cars 	<ul style="list-style-type: none"> • Used light weight models • Considered less complex vehicle model 	[208], [209].
Physical model based approaches	<ul style="list-style-type: none"> • Computational complexity calculation 	<ul style="list-style-type: none"> • Compared with actual scenario testing for complexity calculation 	[210]
Stereo vision approach for overtaking	<ul style="list-style-type: none"> • Day and night effects • Weather effects 	<ul style="list-style-type: none"> • Improved Ladar sensor • Used CNN for better classification results 	[211], [212].
Grid map based overtaking approach	<ul style="list-style-type: none"> • Less range of sensors • High speed moving vehicles 	<ul style="list-style-type: none"> • Increased the range of sensors • Considered dynamic speed of AVs 	[213], [214].
V2V and V2X approach for overtaking	<ul style="list-style-type: none"> • Network Bandwidth • Distance connectivity • Privacy issues 	<ul style="list-style-type: none"> • Increased the number of base station of towers and their security services • Used real-time software-controlled re-configurable intelligent surface units 	[215], [216], [217].
Advanced ML, DL and DRL techniques	<ul style="list-style-type: none"> • Large Models • Vast amount of data 	<ul style="list-style-type: none"> • Used model compression techniques • Considered compact data storage 	[218], [219].
Hybridization approach for overtaking	<ul style="list-style-type: none"> • Increased computational complexity 	<ul style="list-style-type: none"> • Considered shared resources 	[220]
Overtaking using hybrid sensor data	<ul style="list-style-type: none"> • Difficult for real time systems • Higher cost 	<ul style="list-style-type: none"> • Used less complex sensor data 	[221]

stereo pairs of images. For vehicle movement, the color and depth information provides the accurate position of the vehicle as compared to AI-based techniques [223]. Hence, an in-depth study is required to develop a more accurate stereo-vision-based approach that helps in detecting vehicle movement.

6.2.5. Grid map based approach for vehicle movement in dynamic environment

For handling the dynamic environment, a grid-based approach can be developed and utilized to work efficiently in dynamic scenarios. In this approach, the first dynamic map is created, and then an object-level representation of the entire scene is created. A stereo camera is used for detecting and tracking moving objects/vehicles. This approach is new in the field of autonomous driving. Therefore, more focus is required to explore the drawbacks and advantages of using this approach in autonomous driving.

6.2.6. Surveying the V2V communication and V2X communication strategies

V2V communication can enhance the overtaking maneuver efficiently by gathering advanced vehicular dynamics for handling unexpected situations which might occur while overtaking the target vehicle. Also, V2X communication using 5G or 6G technology needs to be explored to enhance the overtaking decision-making in AVs, especially for highly accidental prone areas, i.e., hilly areas. Additionally, combining the above two strategies (V2V and V2X) will undoubtedly increase the efficiency of AVs, overtaking decision-making.

6.2.7. Emerging ML, DL and DRL models in AVs overtaking

In the current era, several emerging DL techniques, i.e., edge intelligence, multiple model integration, vision transformers, and other ML techniques such as AutoML, Unsupervised ML, and TinyML are, remained unexplored in autonomous driving and overtaking decision-making [224], [225]. These emerging approaches can be explored to solve the AVs overtaking issue and make the maneuver more efficient.

6.2.8. Hybridization of several approaches

In Sections 3, 4, and 5, the approaches proposed for solving overtaking issues are categorized under two categories: i) Theoretical and ii) AI-based approaches. An efficient hybridization enhances the efficiency of overtaking maneuvers since individual approaches focus only on one aspect at a time.

6.2.9. Usage of combined data received from several sensors in overtaking maneuver

A hybrid sensor fusion approach combines the data of all the available sensors in the AVs. Combined data enhances the decision-making in detecting the position and altering the vehicle's movement more efficiently than the single data from the individual sensor. The sensor fusion approach is beneficial for overtaking maneuvers, as it requires precise information about position and AVs movement. Thus, a thorough and rigorous study is required to analyze the impact of multiple sensors, different kinds of sensors, and their possible fusions to improve the overtaking efficiency in AVs.

7. Conclusion

This survey paper presented a comprehensive review of current strategies for solving the overtaking problem in the AVs driving systems. In this study, issues related to autonomous driving are thoroughly studied, and deep analysis is carried to figure out the most complex driving issues in autonomous driving. Overtaking maneuver is one of the most complex driving maneuvers, and it consists of a combination of three different maneuvers: i) Lane following, ii) Lane changing, and iii) Acceleration. Several categories of the proposed strategies are illustrated and compared to the previous state-of-the-art methods to solve the overtaking problem. Several informative tables and figures are presented to make a better comparative study among several approaches used for solving the overtaking problem. Based on this study, various research gaps and challenges are also identified, which can be helpful for the researchers in the domain of AVs overtaking maneuver. Furthermore, some crucial future research directions and open problems are explored to make the overtaking maneuver more accurate and robust considering real scenarios. We hope that this survey provides

fruitful knowledge to the researchers in the domain of overtaking maneuvers of autonomous driving systems.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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