

## Advanced Statistics

### Homework-6

*Due: After April 14, 2025 and on or before April 18, 2025*

**Question 1:** Scientists and engineers frequently wish to compare two different techniques for measuring or determining the value of a variable. In such situations, interest centers on testing whether the mean difference in measurements is zero. The article “*Evaluation of the Deuterium Dilution Technique Against the Test Weighing Procedure for the Determination of Breast Milk Intake*” (Amer. J. of Clinical Nutr., 1983: 996–1003) reports the accompanying data on amount of milk ingested by each of 14 randomly selected infants.

**Table 1: Infants 1–5**

| Infant     | 1    | 2    | 3    | 4    | 5    |
|------------|------|------|------|------|------|
| DD method  | 1509 | 1418 | 1561 | 1556 | 2169 |
| TW method  | 1498 | 1254 | 1336 | 1565 | 2000 |
| Difference | 11   | 164  | 225  | −9   | 169  |

**Table 2: Infants 6–10**

| Infant     | 6    | 7    | 8    | 9    | 10   |
|------------|------|------|------|------|------|
| DD method  | 1760 | 1098 | 1198 | 1479 | 1281 |
| TW method  | 1318 | 1410 | 1129 | 1342 | 1124 |
| Difference | 442  | −312 | 69   | 137  | 157  |

**Table 3: Infants 11–14**

| Infant     | 11   | 12   | 13   | 14   |
|------------|------|------|------|------|
| DD method  | 1414 | 1954 | 2174 | 2058 |
| TW method  | 1468 | 1604 | 1722 | 1518 |
| Difference | −54  | 350  | 452  | 540  |

- (a) Is it plausible that the population distribution of differences is normal?
- (b) Does it appear that the true average difference between intake values measured by the two methods is something other than zero? Determine the  $P$ -value of the test, and use it to reach a conclusion at significance level 0.05.

**Question 2:** The article “*Slender High-Strength RC Columns Under Eccentric Compression*” (*Magazine of Concrete Res.*, 2005: 361–370) gave the accompanying data on cylinder strength (MPa) for various types of columns cured under both moist conditions and laboratory drying conditions.

|     | Type |      |      |      |      |      |
|-----|------|------|------|------|------|------|
|     | 1    | 2    | 3    | 4    | 5    | 6    |
| M:  | 82.6 | 87.1 | 89.5 | 88.8 | 94.3 | 80.0 |
| LD: | 86.9 | 87.3 | 92.0 | 89.3 | 91.4 | 85.9 |

  

|     | 7    | 8    | 9    | 10   | 11   | 12   |
|-----|------|------|------|------|------|------|
| M:  | 86.7 | 92.5 | 97.8 | 90.4 | 94.6 | 91.6 |
| LD: | 89.4 | 91.8 | 94.3 | 92.0 | 93.1 | 91.3 |

a. Estimate the difference in true average strength under the two drying conditions in a way that conveys information about reliability and precision, and interpret the estimate. What does the estimate suggest about how true average strength under moist drying conditions compares to that under laboratory drying conditions?

b. Check the plausibility of any assumptions that underlie your analysis of (a).

**Question 3:** In an experiment to compare the tensile strengths of  $I = 5$  different types of copper wire,  $J = 4$  samples of each type were used. The between-samples and within-samples estimates of  $\sigma^2$  were computed as  $MSTr = 2673.3$  and  $MSE = 1094.2$ , respectively. Use the F test at level .05 to test  $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$  versus  $H_a$ : at least two  $\mu_i$ 's are unequal.

**Question 4:** The lumen output was determined for each of  $I = 3$  different brands of lightbulbs having the same wattage, with  $J = 8$  bulbs of each brand tested. The sums of squares were computed as  $SSE = 4773.3$  and  $SSTr = 591.2$ . State the hypotheses of interest (including word definitions of parameters), and use the F test of ANOVA ( $\alpha = .05$ ) to decide whether there are any differences in true average lumen outputs among the three brands for this type of bulb by obtaining as much information as possible about the P-value.

**Question 5:** It is common practice in many countries to destroy (shred) refrigerators at the end of their useful lives. During this process, material from insulating foam may be released into the atmosphere.

The article “*Release of Fluorocarbons from Insulation Foam in Home Appliances During Shredding*” (*J. of the Air and Waste Mgmt. Assoc.*, 2007: 1452–1460) provided the following data on foam density (g/L) for each of two refrigerators produced by four different manufacturers:

1. 30.4, 29.2
2. 27.7, 27.1
3. 27.1, 24.8
4. 25.5, 28.8

Does it appear that the true average foam density is not the same for all these manufacturers? Carry out an appropriate hypothesis test by obtaining as much information as possible about the P-value, and summarize your analysis in an ANOVA table.

**Question 6:** Consider the following summary data on the modulus of elasticity ( $\times 10^6$  psi) for lumber of three different grades [in close agreement with values in the article “*Bending Strength and Stiffness of Second-Growth Douglas-Fir Dimension Lumber*” (*Forest Products J.*, 1991: 35–43)], except that the sample sizes there were larger:

| Grade | $J$ | $\bar{x}_i$ | $s_i$ |
|-------|-----|-------------|-------|
| 1     | 10  | 1.63        | 0.27  |
| 2     | 10  | 1.56        | 0.24  |
| 3     | 10  | 1.42        | 0.26  |

Use this data and a significance level of 0.01 to test the null hypothesis of no difference in mean modulus of elasticity for the three grades.