Denoising of the CSM

J. Antoni, A. Dinsenmeyer and Q. Leclère
Laboratoire Vibrations Acoustique

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Context

Unwanted random noise :

- electronic, ambient, flow-induced,...
- short correlation lengths

Existing denoising methods :

- Physical removal : mic recession, porous treatment, ...
- Use of a background noise measurement
- Wavenumber filtering

CSM properties

$$\boldsymbol{S}_p = \frac{1}{N_s} \sum_i \boldsymbol{p}_i \boldsymbol{p}_i'$$

- Hermitian (conjugate symmetric)
- Positive semidefinite (nonnegative eigenvalues)

$$\underbrace{\boldsymbol{S}_p}_{\text{measured CSM}} = \underbrace{\boldsymbol{S}_a}_{\text{signal of interest}} + \underbrace{\boldsymbol{S}_n}_{\text{unwanted noise}}$$

- Signal CSM : one eigenvalue for one incoherent source
- Noise CSM : off-diagonal elements $\rightarrow 0$ with averaging

Denoising algorithms

Diagonal reconstruction

$$\text{maximize} \sum_i \sigma_{n_i} \quad \text{subject to} \quad \boldsymbol{S}_{pp} - \operatorname{diag}(\boldsymbol{\sigma}_n) \geq 0$$

Solved with CVX Matlab toolbox.

Robust Principal Component Analysis

$$\text{minimize } \|\boldsymbol{S}_a\|_* + \lambda \|\boldsymbol{S}_n\|_1 \quad \text{subject to} \quad \boldsymbol{S}_a + \boldsymbol{S}_n = \boldsymbol{S}_p$$

Solved with proximal gradient algorithm.

Probabilistic Factorial Analysis

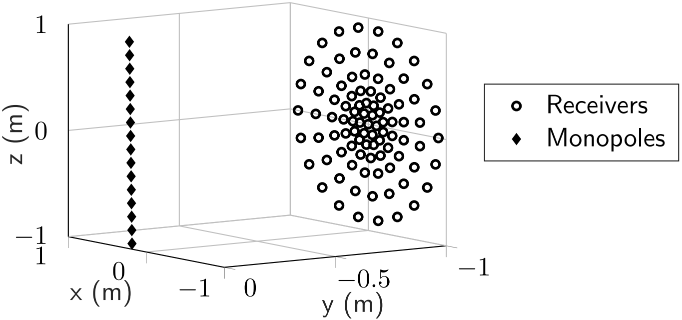
$$\boldsymbol{p} = \boldsymbol{L} \operatorname{diag}(\boldsymbol{\alpha}) \boldsymbol{C} + \operatorname{diag}(\boldsymbol{\sigma}) \boldsymbol{\epsilon}$$

$$\boldsymbol{L}, \boldsymbol{C}, \boldsymbol{\sigma} \sim \mathcal{N}(0, \boldsymbol{\Omega}^2_{L,C,\sigma}) \quad \text{and} \quad \boldsymbol{\alpha} \sim \mathcal{N}(\boldsymbol{\mu}_\alpha, \boldsymbol{\Omega}^2_\alpha)$$

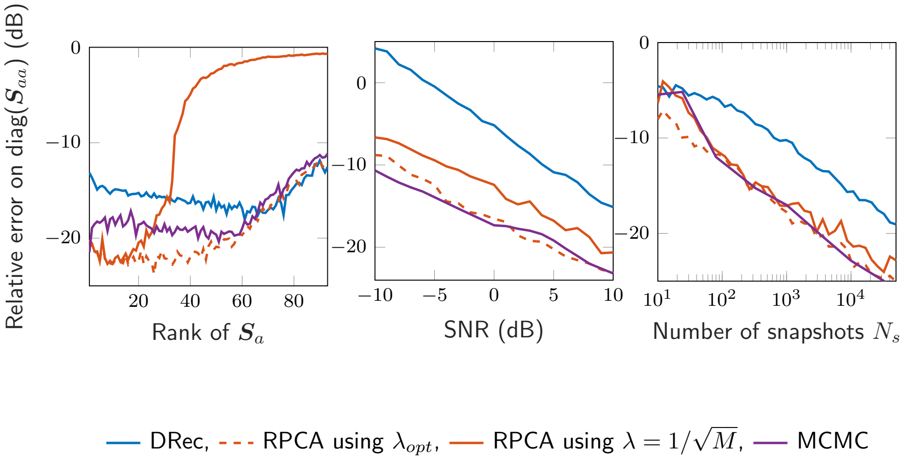
Solved with Gibbs sampling algorithm.

Test case

- frequency : 15 kHz
- 20 monopoles
- 93 receivers
- heterogeneous noise : SNR 10 dB lower on 10 random receivers
- SNR : 10 dB
- 10^4 snapshots



Results



Relative error on diag(S_{aa}) (dB)
0
−10
−20
20
40
60
80
Rank of S_a
0
−10
−20
−10
−5
0
5
10
SNR (dB)
0
−10
−20
10^1
10^2
10^3
10^4
Number of snapshots N_s
DRec, RPCA using λ_{opt}, RPCA using $\lambda = 1/\sqrt{M}$, MCMC