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ON THE DENOISING OF CROSS-SPECTRAL MATRICES FOR (AERO)ACOUSTIC APPLICATIONS

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Abstract

Array systems and multichannel pressure measurements are widely used for source localization and quantification. Measurement noise such as calibration, electronic or ambient noise affects the performance of acoustic imaging algorithms. In aeroacoustic applications, acoustic pressure measurements can be highly disturbed by the presence of flow-induced noise [4]. However, signals are supposed to be stationary and performing averaging of cross-spectral quantities over several time snapshots will concentrate uncorrelated noise along the cross-spectral matrix (CSM) diagonal. A common practice is thus to set the CSM diagonal to zero, which is known to improve the dynamic range of the source localization maps, yet this also leads to underestimated source levels [3]. More advanced techniques have been recently developed to avoid such problems by preserving or reconstructing source information that lies in the CSM diagonal [2, 7].

Several existing approaches for CSM denoising are investigated in this paper and some new ones are proposed as well. We consider an unknown number of uncorrelated sources and no reference background noise. New methods are proposed based on the decomposition of the CSM into a low-rank part and a residual diagonal part attached to the unwanted noise; the corresponding inference problem is set up within a probabilistic framework which is ideally suited to take the non-deterministic nature of the estimated CSM into account. This is then solved by computing the maximum aposteriori estimates of the decomposition by using an expectation-maximization algorithm or by estimating the full aposteriori probability distribution by running a Markov chain Monte Carlo. For each method, reconstruction errors and convergence are compared in the frame of various numerical experiments, for different acoustic signals and noise structures.

1 Numerical experiments

1.1 Simulation of CSM

Source spectra are generated randomly :

$$\mathbf{c} \sim \mathcal{N}(0, \frac{c_{rms}}{\sqrt{2}}) \tag{1}$$

where c_{rms} stands for the root mean squared value of the spectra chosen equal to one for all the monopoles.

The propagator $\mathbf{H}$ is the Green functions for free field monopoles :

$$\mathbf{H} = \frac{e^{jk\mathbf{r}}}{4\pi\mathbf{r}} \tag{2}$$

k being the wavenumber and $\mathbf{r}$ distances between sources and receivers.

The acoustic signal is obtained from source propagation :

$$\mathbf{a} = \mathbf{Hc} \tag{3}$$

and a Gaussian noise is added :

$$\mathbf{p} = \mathbf{a} + \mathbf{n} \tag{4}$$

with

$$\mathbf{n} \sim \mathcal{N}(0, \frac{n_{rms}}{\sqrt{2}}) \tag{5}$$

where $n_{rms} = \langle \mathbf{a} \rangle_M \times 10^{-SNR/20}$ and $\langle \bullet \rangle$ is the average value over the M receivers.

Finally, cross-spectra are calculated :

$$\mathbf{S}_{aa} = \frac{1}{N_s}\mathbf{a}\mathbf{a}' \quad \text{used as the refence signal for denoising,} \tag{6}$$

$$\mathbf{S}_{pp} = \frac{1}{N_s}\mathbf{p}\mathbf{p}' \quad \text{which is the measurement spectrum to be denoised.} \tag{7}$$

1.2 Configuration for the simulated acoustical propagation

The acoustical field produced by a line of K uncorrelated monopoles is measured by $M = 93$ receivers distributed as a spiral. Default values for each parameter are given by the table 1.

Parameter	Notation	Default value
Frequency	f	15 kHz
Number of monopoles	K	20
Number of receivers	M	93
SNR	SNR	10 dB
Number of snapshots	N_s	10^4

Table 1: Default values for the numerical simulations.

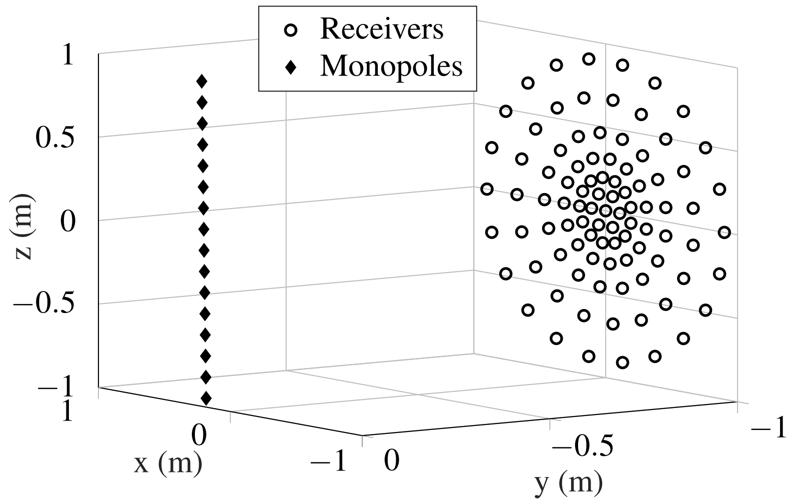

Figure 1: Receiver and source positions used for acoustic field simulation (source line is tilted of 1 degree from the vertical to break antenna symmetry).

2 Diagonal reconstruction

In this section, three methods for diagonal reconstruction are presented. The idea of these methods is to reduce as much as possible the self-induced noise concentrated on the diagonal elements of the CSM. The diagonal is minimized as long as the denoised CSM stays positive semidefinite. This problem can be written as :

$$\text{maximize} \sum_i D_{ii} \text{ subject to } \mathbf{S}_{pp} - \mathbf{D} \geq 0 \tag{8}$$

D being a diagonal matrix.

2.1 Convex optimization

Hald use convex programming with the CVX toolbox [5, 6] to solve this problem [7].

2.2 Linear optimization

Dougherty express this problem as a linear programming problem :

$$\text{maximize } \sum_i D_{ii} \ \text{ subject to } \ \mathbf{V}^{i-1}\left(\mathbf{S}_{pp} - \mathbf{D}^i\right)\mathbf{V}^{i-1} \geq 0 \tag{9}$$

where $\mathbf{V}^{i-1}$ are the eigenvectors of $\mathbf{S}_{pp} - \mathbf{D}^{i-1}$.

2.3 Alternating projections

Alternating projections method can also be used. The solution is the intersection between two sets : the first set is the positive semi-definite matrices and the second is the set of diagonal matrices for the noise matrix :

$$\begin{aligned}
&\bar{\mathbf{S}}^0_{pp} = \mathbf{S}_{pp} - \text{diagonal}(\mathbf{S}_{pp}) \\
&\textbf{for } i \\
&\quad \mathbf{s}^i = \text{eigenvalues}(\mathbf{S}^i_{pp}) \\
&\quad \mathbf{V}^i = \text{eigenvectors}(\mathbf{S}^i_{pp}) \\
&\quad \mathbf{s}^i(s \leq 0) = 0 \\
&\quad \bar{\mathbf{S}}^{i+1}_{pp} = \bar{\mathbf{S}}^0_{pp} + \mathbf{V}^{i\prime}\mathbf{s}^i\mathbf{V}^i \\
&\textbf{endfor}
\end{aligned} \tag{10}$$

This algorithm stops when all the eigenvalues of the denoised CSM are nonnegative.

2.4 Comparison of diagonal reconstruction methods

For each of these methods, we study the relative error on the estimated signal matrix $\hat{\mathbf{S}}_{aa}$ defined as :

$$err = \frac{\|\,\text{diag}(\mathbf{S}_{aa}) - \text{diag}(\hat{\mathbf{S}}_{aa})\|_2}{\|\,\text{diag}(\mathbf{S}_{aa})\|_2} \tag{11}$$

For each method, three parameters are successively changed :

- rank of signal matrix $\mathbf{S}_{aa}$ by increasing the number of monopoles from 1 to M,
- SNR from -10 to 10 dB,
- number of snapshots N_s.

When one parameter is changed, the others are constant, given by the table 1.

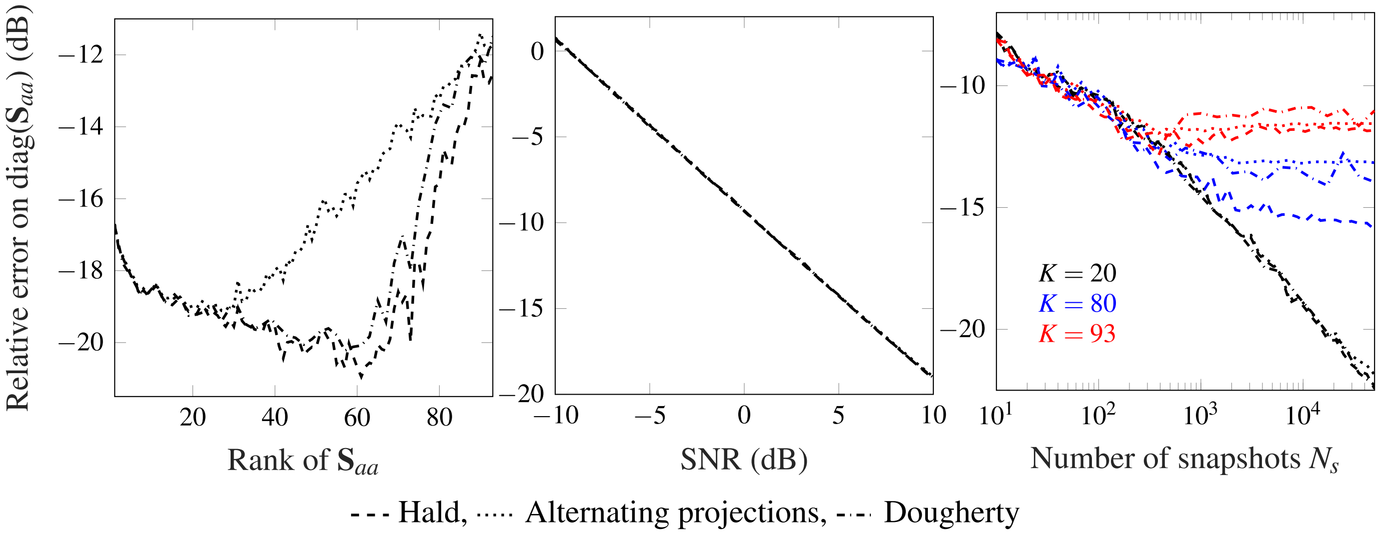

Figure 2: Error on the reconstructed diagonal, for different simulation parameters. On the right, error is plotted for 20, 80 and 93 sources.

3 Robust principal component analysis (RPCA)

The optimisation problem :

$$\text{minimize } \|\hat{\mathbf{S}}_{aa}\|_* + \lambda \|\hat{\mathbf{S}}_{nn}\|_1 \quad \text{subject to} \quad \hat{\mathbf{S}}_{aa} + \hat{\mathbf{S}}_{nn} = \mathbf{S}_{pp} \tag{12}$$

where $\| \bullet \|_*$ is the nuclear norm (sum of the eigenvalues), is solved using an accelerated proximal gradient algorithm, developed by Wright *et al.* [8].

3.1 Choosing the regularization parameter

According to [8**?**], a constant parameter equal to $1/\sqrt{M}$ can be chosen as far as the rank of the signal matrix is reasonably low. As shown by [1], this parameter is not very accurate but far easier to implement than a trade-off curve analysis. The reason is that the trade-off curve is very fluctuating, as shown on figure 3.

Figure **??** shows relative error for different regularization parameters and as a function of the number of sources. The parameter called "optimal" is the one that gives the smallest relative error (unknown on non-synthetic case).
Détailler optimal et minimisation reconstruction

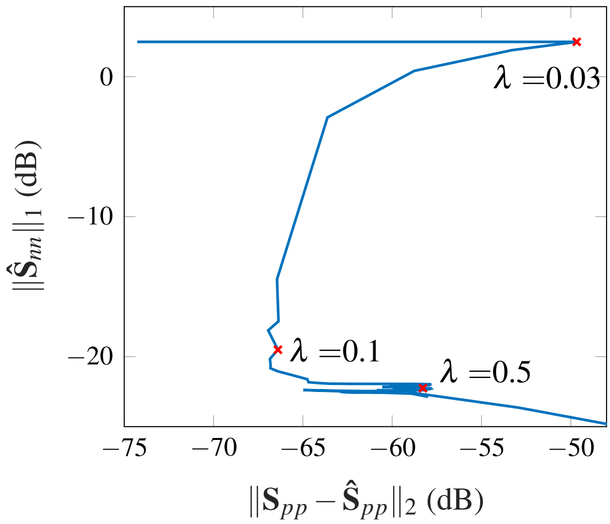

Figure 3: Trade-off curve as a function of λ (for default values from table 1).

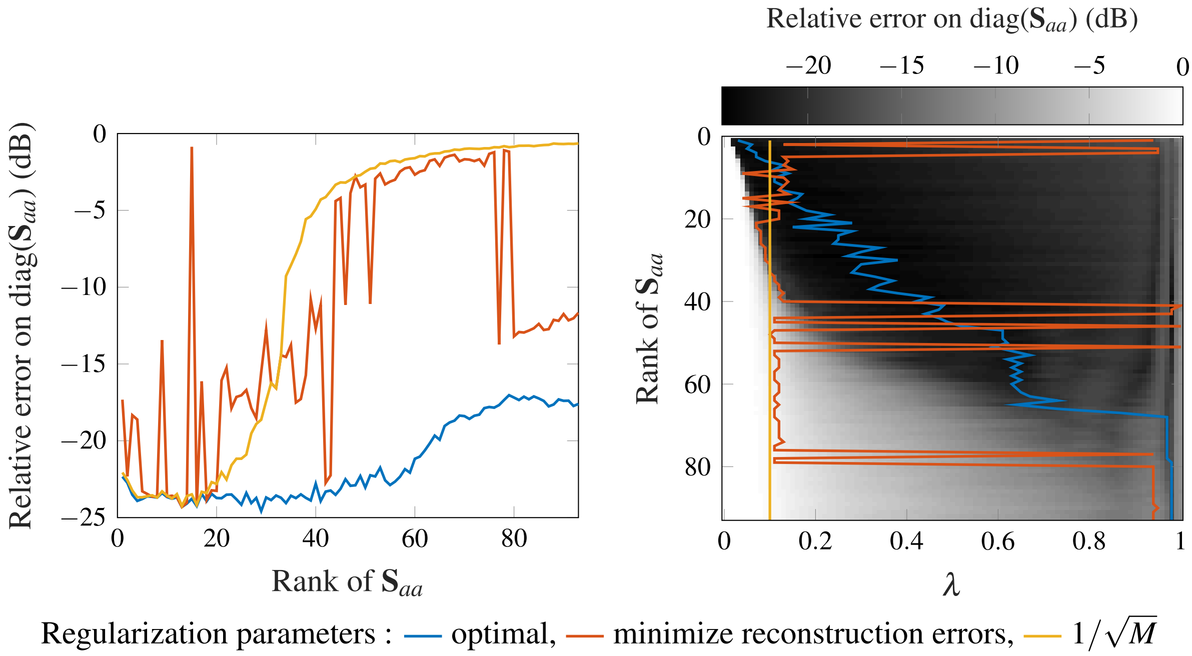

Figure 4: Error on the reconstructed diagonal solving RPCA with different regularization parameter, as a function of the rank of the signal matrix.

4 Probabilistic factorial analysis

Expectation-Maximisation algorithm: Noise CSM supposed to be a diagonal matrix.

Gibbs sampling: hyperparameters are chosen to be small (all equal to 1.1)
For both algorithms, the number of parameters κ is fixed by the user.

4.1 Choosing the number of factors

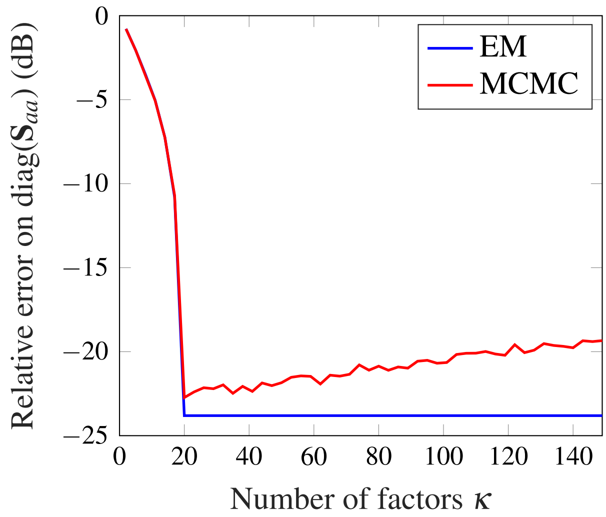

Figure 5: Error on the diagonal of the signal CSM for increasing number of factors. Error is minimal when the number of factors is equal to the number of uncorrelated sources (default value is 20).

As the number of sources is unknown, we choose $\kappa = M$.

5 Comparison of the different methods

5.1 Homogeneous noise

Previous methods are first compared when the noise is homogeneously added on the receivers. In the equation 5, , n_{rms} is the same for all the receivers, given by the SNR.

5.2 Heterogeneous noise

Ten receivers randomly chosen are now affected by a strong noise, for which SNR is 10 dB lower than for the other receivers.

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