Exercices supplémentaires : Calcul intégral

Exercice 1: Primitives

Donner les primitives des fonctions suivantes :

a)
$$f(x) = 3x^2 + 2x + 1$$

b)
$$f(x) = \sin(x)$$

c)
$$f(x) = \frac{1}{x}$$

d)
$$f(x) = x - \frac{1}{x^2}$$

e)
$$f(x) = -x^2 + x$$

$$f) f(x) = \frac{1}{x^3}$$

g)
$$f(x) = \frac{x^4 + 1}{x^2}$$

$$f(x) = 3\sin(x) + 2\cos(x)$$

i)
$$f(x) = 2(2x+1)^3$$

j)
$$f(x) = (3x+1)^{-5}$$

k)
$$f(x) = (-2x+1)^5$$

l)
$$f(x) = \frac{2x+1}{(x^2+x+1)^4}$$

$$m) f(x) = \sin(x) \cos^3(x)$$

$$f(x) = \frac{\ln^2(x)}{x}$$

$$o) f(x) = \frac{1}{\sqrt{x+1}}$$

p)
$$f(x) = \frac{3x}{\sqrt{x^2 + 1}}$$

q)
$$f(x) = \frac{1}{x^2 \sqrt{1 + \frac{1}{x}}}$$

$$f(x) = 3\sin(3x + \frac{\pi}{2})$$

$$f(x) = x\cos(x^2 + \pi)$$

t)
$$f(x) = \frac{\sin(\sqrt{x})}{\sqrt{x}}$$

u)
$$f(x) = \frac{2x^2 + 3x + 5}{x}$$

v)
$$f(x) = \frac{\ln(x)}{x}$$

$$\mathbf{w}) f(x) = \frac{\mathbf{e}^x}{\mathbf{e}^x + 1}$$

$$x) f(x) = \frac{1}{e^{2x}}$$

$$y) f(x) = \frac{\sin(x)}{2 + \cos(x)}$$

z)
$$f(x) = \frac{x^3}{1+x^2}$$

Exercice 2: Calcul d'intégrales

Calculer les intégrales suivantes :

a)
$$I = \int_0^3 (x+4) dx$$

b)
$$I = \int_{-1}^{1} (2t^2 - 1) dt$$

c)
$$I = \int_{1}^{2} \frac{3}{\sqrt{t}} dt$$

d)
$$I = \int_0^{\pi} \sin(t) dt$$

e)
$$I = \int_0^1 (2x+3)(x^2+3x-5) dx$$

f)
$$I = \int_{-1}^{1} \frac{2t+1}{(t^2+t+1)^2} dt$$

g)
$$I = \int_0^{\pi} \sin^2(t) dt$$

h)
$$I = \int_{-1}^{0} \frac{2t}{t^2 + 1} dt$$

i)
$$I = \int_{1}^{2} x \ln x dx$$

j)
$$I = \int_0^1 (2x+1)e^x dx$$

$$k) I = \int_1^e \frac{\ln x}{x^2} dx$$

$$I) \quad I = \int_{1}^{x} \ln t \, \mathrm{d}t$$

Exercice 3: Primitives

Donner les primitives des fonctions suivantes :

a)
$$f(x) = \frac{1}{x^4 - x}$$

b)
$$f(x) = \frac{3x+1}{x^2-1}$$

Exercice 4: Changement de variables

Calculer les intégrales suivantes :

a)
$$I = \int_0^1 \sqrt{1 - x^2} dx$$

b)
$$I = \int_0^{\frac{\pi}{4}} \frac{\tan x}{\cos x (\cos x + \sin x)} dx$$