On the Denoising of Cross-Spectral Matrices for (Aero)Acoustic Applications

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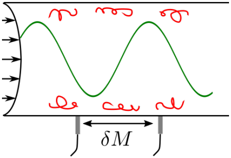
March 5, 2018 – 7th BeBeC

Context



- Unwanted random noise:
  - electronic, ambient, flow-induced,...
  - short correlation lengths

- Existing denoising methods:
  - Physical removal : mic recession, porous treatment, vibrating structure filtering. . .
  - Use a background noise measurement, → not always available or representative
  - Wavenumber filtering
  - Diagonal removal → underestimation of of source level

- Multichannel system → use spatial properties to separate signal from noise

Context – CSM properties

$$\underbrace{\boldsymbol{p}}_{\text{measured spectra}} = \underbrace{\boldsymbol{a}}_{\text{source spectrum}} + \underbrace{\boldsymbol{n}}_{\text{Gaussian noise}}$$

Averaging over N_s snapshots $\rightarrow$ Cross-spectral matrix (covariance of Fourier component):

$$\boldsymbol{S}_{pp} = \frac{1}{N_s} \sum_i \boldsymbol{p}_i \boldsymbol{p}_i^H$$

- Hermitian (conjugate symmetric)
- Positive semidefinite (nonnegative eigenvalues)

$$\underbrace{\boldsymbol{S}_{pp}}_{\text{measured CSM}} = \underbrace{\boldsymbol{S}_{aa}}_{\text{signal of interest}} + \underbrace{\boldsymbol{S}_{nn}}_{\text{unwanted noise}} + \underbrace{\boldsymbol{S}_{an} + \boldsymbol{S}_{na}}_{\text{cross-terms}}$$

- Signal CSM : Rank given by the number of incoherent sources (*ie* number of uncorrelated sources)

For $N_s \rightarrow \inf$

- Short correlation length : off-diagonal elements of $S_{nn} \rightarrow 0$
- Independent signal/noise : cross-terms $\rightarrow 0$

$$\boldsymbol{S}_{pp} \approx \boldsymbol{S}_{aa} + \operatorname{diag}\left(\sigma^2\right)$$

Diagonal Reconstruction

"Remove as much noise as possible as long as denoised CSM remains positive"

Convex optimization (Hald, 2017)

$$\text{maximize } \|\boldsymbol{\sigma}_n^2\|_1 \ \text{ subject to } \ \boldsymbol{S}_{pp} - \mathrm{diag}\left(\boldsymbol{\sigma}_n^2\right) \geq 0$$

Problem solved with CVX Matlab toolbox.

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Linear optimization (Dougherty, 2016)

$$\text{maximize } \|\boldsymbol{\sigma}_n^2\|_1 \text{ subject to } \boldsymbol{V}_{(k-1)}^H \left(\boldsymbol{S}_{pp} - \operatorname{diag}\left(\boldsymbol{\sigma}_n^2\right)_{(k)}\right) \boldsymbol{V}_{(k-1)} \geq 0$$

$\boldsymbol{V}_{(k-1)}$: eigenvectors of $\boldsymbol{S}_{pp} - \operatorname{diag}\left(\boldsymbol{\sigma}_n^2\right)_{(1,\ldots,k-1)}$

Solved with *linprog* Matlab function .

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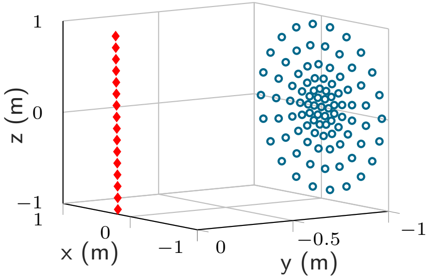
Alternating Projections (Leclère et al., 2015)

$$\boldsymbol{S}_{pp_{(k+1)}} := \bar{\boldsymbol{S}}_{pp_{(0)}} + \boldsymbol{V}_{(k)}^H \boldsymbol{s}_{(k)}^{+} \boldsymbol{V}_{(k)}$$

$\boldsymbol{V}_{(k)}^H$ and $\boldsymbol{s}_{(k)}$: eigenvectors/values of $\boldsymbol{S}_{pp_{(k)}}$.

Comparison on a test case

- Default parameters:
  - 20 uncorrelated free field monopoles: ♦
  - 93 receivers: ○
  - SNR: 10 dB
  - 10^4 snapshots
  - frequency: 15 kHz

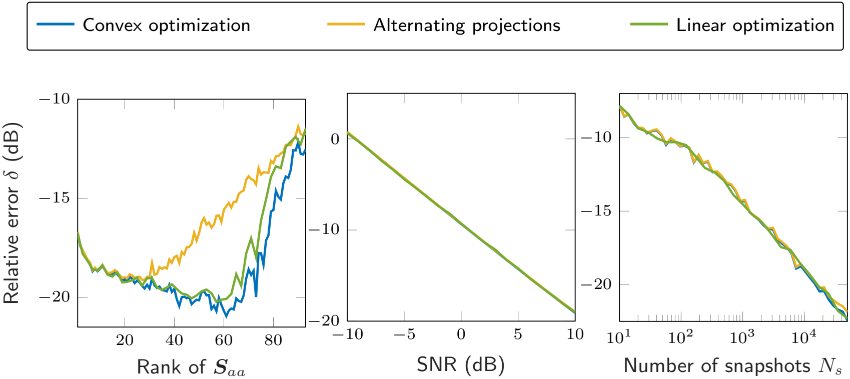


- Varying parameters:
  - number of ♦ (rank of $\boldsymbol{S}_{aa}$) : from 1 to 93
  - SNR from -10 to 10 dB
  - Number of snapshots (level of extra-diagonal terms): from 10 to 5.10^4

- Error on the signal CSM:

$$\delta = \frac{\| \operatorname{diag}(\boldsymbol{S}_{aa}) - \operatorname{diag}\left(\hat{\boldsymbol{S}}_{aa}\right) \|_2}{\| \operatorname{diag}(\boldsymbol{S}_{aa}) \|_2}$$

Comparison on a test case



Select Convex Optimization (DRec) for further comparison

✓ Fast, simple code
✓ Better performance

✗ Local optimization
✗ Modifies only auto-spectra

RPCA

"Search $\boldsymbol{S}_{aa}$ as a low rank matrix and $\boldsymbol{S}_{nn}$ as a sparse matrix"

$$\boxed{\text{minimize } \|\boldsymbol{S}_{aa}\|_* + \lambda\|\boldsymbol{S}_{nn}\|_1 \quad \text{subject to} \quad \boldsymbol{S}_{aa} + \boldsymbol{S}_{nn} = \boldsymbol{S}_{pp}}$$

$\|\cdot\|_*$: nuclear norm (related to rank)
$\|\cdot\|_1$: ℓ_1-norm (related to sparsity)

Solved with a proximal gradient algorithm.

RPCA (Wright et al., 2009)

✓ Modifies the whole CSM

✗ Local optimization
✗ Choose regularization parameter:
- L-curve criterion,
- Generalized cross validation method,
- Bayesian criterion, ...

$\hookrightarrow$ For comparison :
- optimal λ (unknown on real case)
- "universal" constant parameter $\lambda = M^{-\frac{1}{2}} = 0.1$

Probabilistic Factor Analysis

- Gibbs sampling in the Bayesian hierarchical model :

$$\boldsymbol{p} = \boldsymbol{L}\boldsymbol{c} + \boldsymbol{n}$$

Loading factors
$\boldsymbol{L} \sim \mathcal{N}_{\mathbb{C}}(0, \boldsymbol{\gamma}^2)$

Latent variables
$\boldsymbol{c} \sim \mathcal{N}_{\mathbb{C}}(0, \boldsymbol{I}\boldsymbol{\alpha}^2)$

Uncorrelated noise
$\boldsymbol{n} \sim \mathcal{N}_{\mathbb{C}}(0, \boldsymbol{I}\boldsymbol{\sigma}^2)$

- Hyperparameters:

$$\boldsymbol{\gamma}^2 \sim \mathcal{IG}(a_\gamma, b_\gamma) \qquad \boldsymbol{\alpha}^2 \sim \mathcal{IG}(a_\alpha, b_\alpha) \qquad \boldsymbol{\sigma}^2 \sim \mathcal{IG}(a_\sigma, b_\sigma)$$

- Signal CSM :

$$\hat{\boldsymbol{S}}_{aa} = \frac{1}{N_s}\boldsymbol{L}\left(\sum_{i=1}^{N_s} \boldsymbol{c}_i\boldsymbol{c}_i^H\right)\boldsymbol{L}^H$$

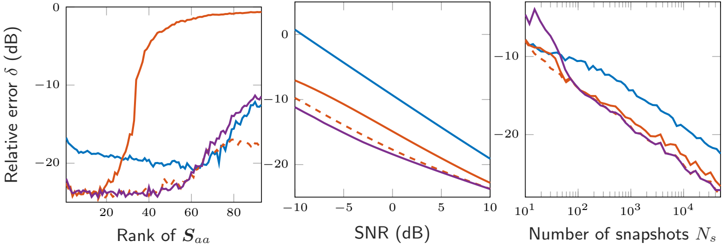
PFA

✓ Global optimization
✗ Computationally expensive
✗ Here, model for uncorrelated noise → ✓ but flexible

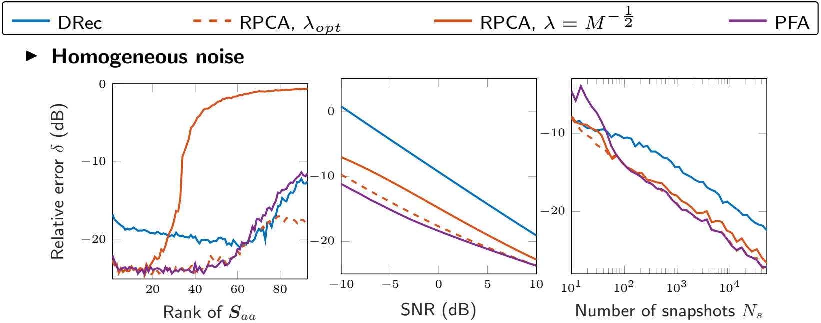
How to fix number of factor κ ? Enforce sparsity on factors → will set useless factors will be set to zero

Comparison

- **Homogeneous noise**



Comparison

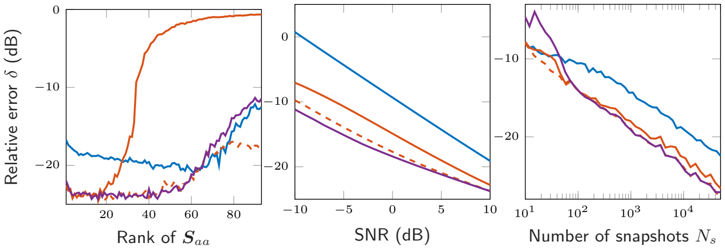


- **Homogeneous noise**

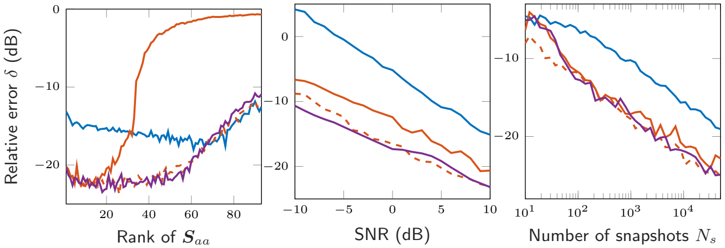
↪ Error linearly decreases with increasing SNR
↪ Error linearly decreases with logarithmically increasing N_s
↪ For $N_{src} \geq 0.75M$: denoising problem becomes poorly conditioned

Comparison

- **Homogeneous noise**



- **Heterogeneous noise**: SNR 10 dB lower on 10 random receivers



Conclusion

- Common to all the methods :
  - Error linearly decreases with increasing SNR
  - Error linearly decreases with logarithmically increasing N_s
  - Error steady with rank below $0.75M$
  - For $N_{src} \geq 0.75M$: denoising problem becomes poorly conditioned
- DRec: fast and simple but error at least 5 dB higher in all configurations
- PFA performance similar to RPCA using λ_{opt}
- PFA and RPCA more robust to heterogeneous noise
- PFA: flexible model $\rightarrow$ to be adapted for correlated noise

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