On the Denoising of Cross-Spectral Matrices for (Aero)Acoustic Applications

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Context

Unwanted random noise:

- electronic, ambient, flow-induced,...
- short correlation lengths

Existing denoising methods:

- Physical removal : mic recession, porous treatment, ...
- Use of a background noise measurement,
- Wavenumber filtering.

CSM properties

$$\boldsymbol{S}_p = \frac{1}{N_s} \sum_i \boldsymbol{p}_i \boldsymbol{p}_i'$$

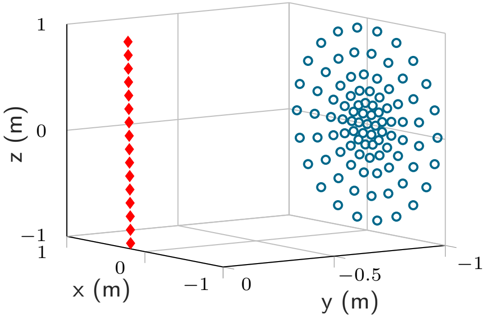
- Hermitian (conjugate symmetric)
- Positive semidefinite (nonnegative eigenvalues)

$$\underbrace{\boldsymbol{S}_p}_{\text{measured CSM}} = \underbrace{\boldsymbol{S}_a}_{\text{signal of interest}} + \underbrace{\boldsymbol{S}_n}_{\text{unwanted noise}}$$

- Signal CSM : one eigenvalue for one incoherent source
- Noise CSM : off-diagonal elements $\rightarrow 0$ with averaging

Test case

- frequency: 15 kHz
- 20 uncorrelated monopoles: ♦
- 93 receiver: ○
- SNR: 10 dB
- 10^4 snapshots



Error on the signal CSM:

$$\delta = \frac{\| \operatorname{diag}(\boldsymbol{S}_{aa}) - \operatorname{diag}\left(\hat{\boldsymbol{S}}_{aa}\right) \|_2}{\| \operatorname{diag}(\boldsymbol{S}_{aa}) \|_2}$$

Diagonal Reconstruction

Convex optimization

$$\text{maximize } \|\boldsymbol{\sigma}_n^2\|_1 \quad \text{subject to} \quad \boldsymbol{S}_{pp} - \mathrm{diag}\left(\boldsymbol{\sigma}_n^2\right) \geq 0$$

Problem solved with CVX Matlab toolbox (Hald, 2017).

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Linear optimization

$$\text{maximize } \|\boldsymbol{\sigma}_n^2\|_1 \;\; \text{subject to } \;\; \boldsymbol{V}_{(k-1)}^H \left(\boldsymbol{S}_{pp} - \mathrm{diag}\left(\boldsymbol{\sigma}_n^2\right)_{(k)}\right) \boldsymbol{V}_{(k-1)} \geq 0$$

Solved with *linprog* Matlab function (Dougherty, 2016).

Diagonal Reconstruction

Convex optimization

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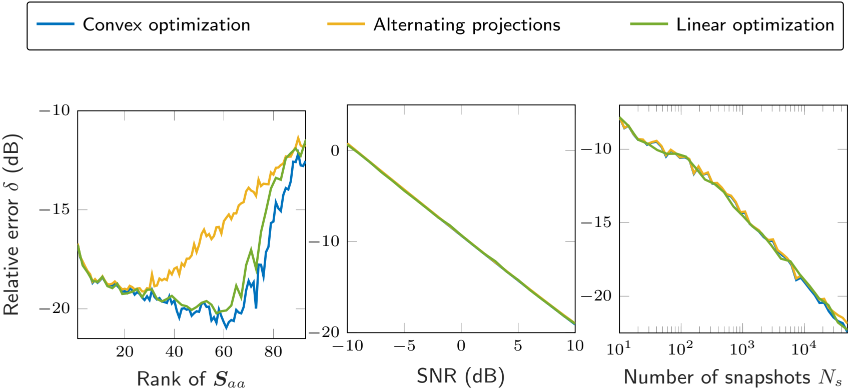
Alternating Projections

$$\boldsymbol{S}_{pp_{(k+1)}} := \bar{\boldsymbol{S}}_{pp_{(0)}} + \boldsymbol{V}_{(k)}^H \boldsymbol{s}_{(k)}^{+} \boldsymbol{V}_{(k)}$$

with $\boldsymbol{V}_{(k)}^H$ and $\boldsymbol{s}_{(k)}$ eigenvector/values of $\boldsymbol{S}_{pp_{(k)}}$.
Algorithm from Leclère et al. (2015)

Diagonal Reconstruction



keep convex optimization :
+ fast, simple code
- local optimization

RPCA

citations

$$\text{minimize } \|\boldsymbol{S}_{aa}\|_* + \lambda \|\boldsymbol{S}_{nn}\|_1 \quad \text{subject to} \quad \boldsymbol{S}_{aa} + \boldsymbol{S}_{nn} = \boldsymbol{S}_{pp}$$

Solved with a proximal gradient algorithm.

$+$ joue théoriquement sur toute la CSM
- choose regularization parameter : l-curve, cross validation, ...
- local optimization ?

Probabilistic Factorial Analysis

$$\boldsymbol{p} = \boldsymbol{L}\boldsymbol{c} + \boldsymbol{n}$$

$$\boldsymbol{L} \sim \mathcal{N}_{\mathbb{C}}(0, \boldsymbol{\gamma}^2) \qquad \boldsymbol{c} \sim \mathcal{N}_{\mathbb{C}}(0, \boldsymbol{I}\boldsymbol{\alpha}^2) \qquad \boldsymbol{n} \sim \mathcal{N}_{\mathbb{C}}(0, \boldsymbol{I}\boldsymbol{\sigma}^2)$$

Hyperparameters:

$$\boldsymbol{\gamma}^2 \sim \mathcal{IG}(a_\gamma, b_\gamma) \qquad \boldsymbol{\alpha}^2 \sim \mathcal{IG}(a_\alpha, b_\alpha) \qquad \boldsymbol{\sigma}^2 \sim \mathcal{IG}(a_\sigma, b_\sigma)$$

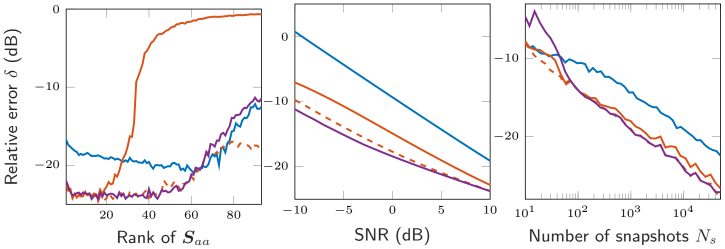
Inferred using Gibb's sampling (MCMC algorithm)
$\hat{\boldsymbol{S}}_{aa} = {N_s}^{-1} \boldsymbol{L} (\sum_{i=1}^{N_s} \boldsymbol{c}_i \boldsymbol{c}_i^H) \boldsymbol{L}^H$

+ convergence théorique assurée
- computationally expensive
+ flexible formulation + takes into account finite length record ?

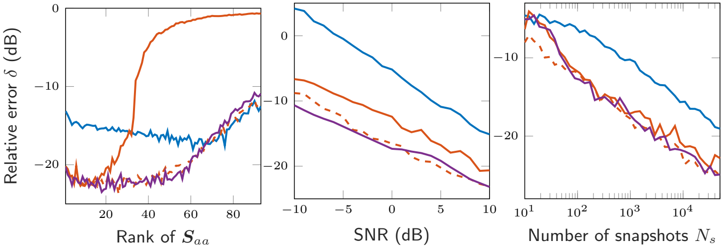
Comparison

DRec — RPCA, λ_{opt} — RPCA, $\lambda = M^{\frac{1}{2}}$ — PFA

- **Homogeneous noise**



- **Heterogeneous noise**: SNR 10 dB lower on 10 random receivers



Conclusion

contenu...

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