On the Denoising of Cross-Spectral Matrices for (Aero)Acoustic Applications

A. Dinsenmeyer[1,2], J. Antoni[1], Q. Leclère[1] and A. Pereira[2]

[1] Laboratoire Vibrations Acoustique
[2] Laboratoire de Mécanique des Fluides et d'Acoustique
Lyon, France

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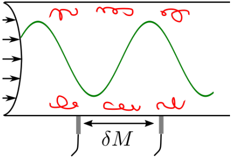
Context

- Unwanted random noise:
  - electronic, ambient, flow-induced,...
  - short correlation lengths



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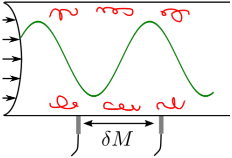

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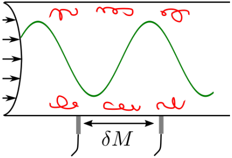

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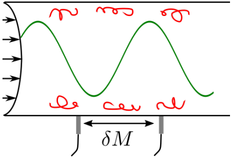

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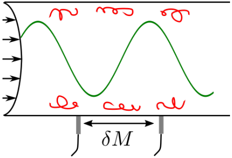

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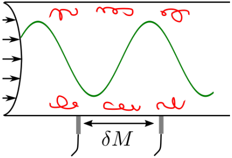

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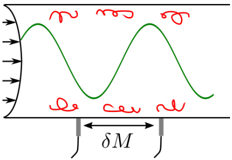

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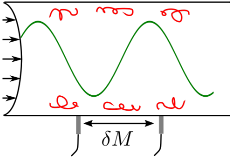

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- Rank of $\boldsymbol{S}_{aa}$ = number of uncorrelated monopoles

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For $N_s \to \infty$

- Short correlation length : off-diagonal elements of $S_{nn} \to 0$
- Independent signal/noise : cross-terms $\to 0$

Problématique

How to separate signal part from noise ?
Studied existing methods:

- 3 diagonal reconstruction methods
- Robust Principal Component Analysis (RPCA)

Proposed method:

- PFA

What are there performance when noise level, Ns or the number of sources vary ?

Diagonal Reconstruction

"Remove as much noise as possible as long as denoised CSM remains positive"

Convex optimization (Hald, 2017)

$$\text{maximize } \|\boldsymbol{\sigma}_n^2\|_1 \;\text{ subject to }\; \boldsymbol{S}_{pp} - \mathrm{diag}\left(\boldsymbol{\sigma}_n^2\right) \geq 0$$

Problem solved with CVX Matlab toolbox.

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Linear optimization (Dougherty, 2016)

$$\text{maximize } \|\boldsymbol{\sigma}_n^2\|_1 \;\; \text{subject to } \;\; \boldsymbol{V}_{(k-1)}^H \left(\boldsymbol{S}_{pp} - \mathrm{diag}\left(\boldsymbol{\sigma}_n^2\right)_{(k)}\right) \boldsymbol{V}_{(k-1)} \geq 0$$

$\boldsymbol{V}_{(k-1)}$: eigenvectors of $\boldsymbol{S}_{pp} - \mathrm{diag}\left(\boldsymbol{\sigma}_n^2\right)_{(1,\ldots,k-1)}$

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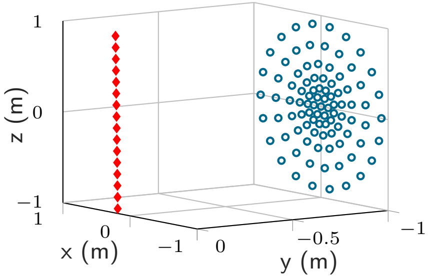
Alternating Projections (Leclère et al., 2015)

$$\boldsymbol{S}_{pp_{(k+1)}} := \bar{\boldsymbol{S}}_{pp_{(0)}} + \boldsymbol{V}_{(k)}^H \boldsymbol{s}_{(k)}^{+} \boldsymbol{V}_{(k)}$$

$\boldsymbol{V}_{(k)}^H$ and $\boldsymbol{s}_{(k)}$: eigenvectors/values of $\boldsymbol{S}_{pp_{(k)}}$.

Diagonal Reconstruction

- Default parameters:
  - 20 uncorrelated free field monopoles: ♦
  - 93 receivers: ○
  - SNR: 10 dB
  - 10^4 snapshots
  - frequency: 15 kHz



- Varying parameters:
  - number of ♦ (rank of $\boldsymbol{S}_{aa}$) : from 1 to 93
  - SNR from -10 to 10 dB
  - Number of snapshots (level of extra-diagonal terms): from 10 to 5.10^4

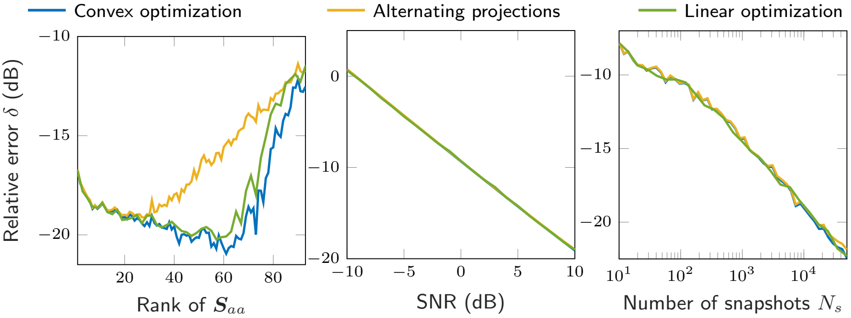
- Error on the signal CSM:

$$\delta = \frac{\| \operatorname{diag}\left(\boldsymbol{S}_{aa}\right) - \operatorname{diag}\left(\hat{\boldsymbol{S}}_{aa}\right) \|_2}{\| \operatorname{diag}\left(\boldsymbol{S}_{aa}\right) \|_2}$$

Diagonal Reconstruction

Default values:

20 sources	93 receivers	SNR: 10 dB	10^4 snapshots	frequency: 15 kHz



Select Convex Optimization (DRec) for further comparison

✓ Fast, simple code
✓ Better performance

✗ Local optimization
✗ Denoises only diagonal

RPCA

"Search $\boldsymbol{S}_{aa}$ as a low rank matrix and $\boldsymbol{S}_{nn}$ as a sparse matrix"

$$\boxed{\text{minimize } \|\boldsymbol{S}_{aa}\|_* + \lambda\|\boldsymbol{S}_{nn}\|_1 \quad \text{subject to} \quad \boldsymbol{S}_{aa} + \boldsymbol{S}_{nn} = \boldsymbol{S}_{pp}}$$

$\|\cdot\|_*$: nuclear norm (related to rank)
$\|\cdot\|_1$: ℓ_1-norm (related to sparsity)

Solved with a proximal gradient algorithm.

RPCA (Wright et al., 2009)

✓ Modifies the whole CSM

✗ Local optimization

✗ Choose regularization parameter:
- L-curve criterion,
- Generalized cross validation method,
- Bayesian criterion, ...

$\hookrightarrow$ For comparison :
- optimal λ (unknown on real case)
- "universal" constant parameter $\lambda = M^{-\frac{1}{2}} = 0.1$

Diapo d'intro aux méthodes bayésiennes ?

Probabilistic Factor Analysis

- Gibbs sampling in the Bayesian hierarchical model :

$$\boldsymbol{p} = \boldsymbol{L}\boldsymbol{c} + \boldsymbol{n}$$

Loading factors
$\boldsymbol{L} \sim \mathcal{N}_{\mathbb{C}}(0, \boldsymbol{\gamma}^2)$

Latent variables
$\boldsymbol{c} \sim \mathcal{N}_{\mathbb{C}}(0, \boldsymbol{I}\boldsymbol{\alpha}^2)$

Uncorrelated noise
$\boldsymbol{n} \sim \mathcal{N}_{\mathbb{C}}(0, \boldsymbol{I}\boldsymbol{\sigma}^2)$

- Hyperparameters:

$$\boldsymbol{\gamma}^2 \sim \mathcal{IG}(a_\gamma, b_\gamma) \qquad \boldsymbol{\alpha}^2 \sim \mathcal{IG}(a_\alpha, b_\alpha) \qquad \boldsymbol{\sigma}^2 \sim \mathcal{IG}(a_\sigma, b_\sigma)$$

- Signal CSM :

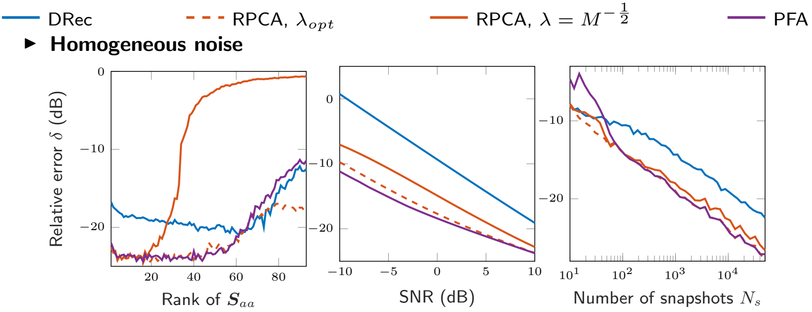
$$\hat{\boldsymbol{S}}_{aa} = \frac{1}{N_s}\boldsymbol{L}\left(\sum_{i=1}^{N_s}\boldsymbol{c}_i\boldsymbol{c}_i^H\right)\boldsymbol{L}^H$$

PFA

✓ Global optimization ✗ Computationally expensive
✗ Here, model for uncorrelated noise → ✓ but flexible

Comparison

- **Homogeneous noise**



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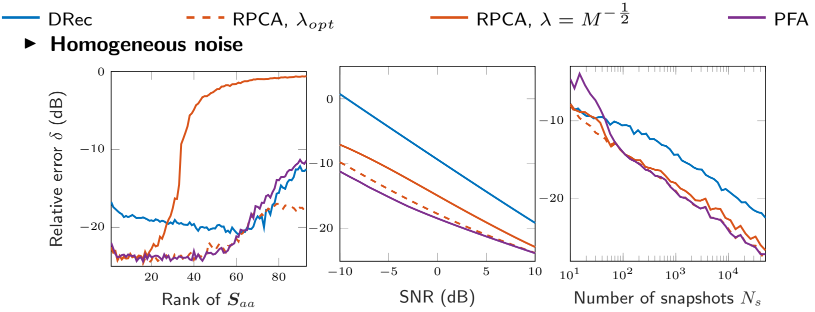


- **Homogeneous noise**

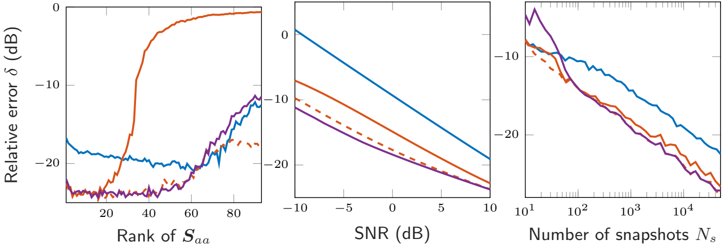
↪ Error linearly decreases with increasing SNR

↪ Error linearly decreases with logarithmically increasing N_s

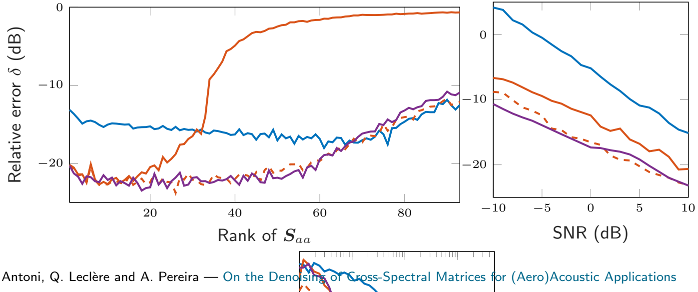
↪ For $N_{src} \geq 0.75M$: denoising problem becomes poorly conditioned

Comparison

- **Homogeneous noise**

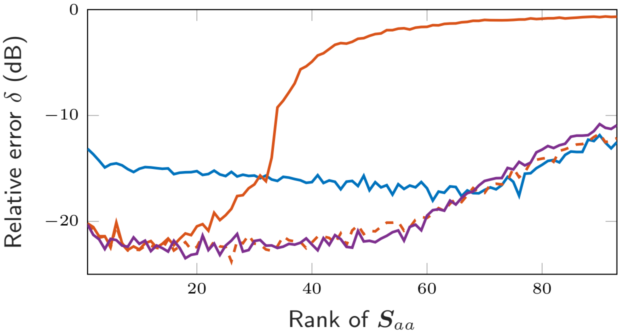


- **Heterogeneous noise**: SNR 10 dB lower on 10 random receivers



Conclusion

- DRec: fast and simple but error at least 5 dB higher in all configurations
- PFA
  - performance similar to RPCA using λ_{opt}
  - PFA and RPCA more robust to heterogeneous noise
  - flexible model → to be adapted for correlated noise



RPCA PFA DRec

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