Denoising of the CSM

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Context

CSM properties

$$\boldsymbol{S}_p = \frac{1}{N_s} \sum_i \boldsymbol{p}_i \boldsymbol{p}_i'$$

- Hermitian (conjugate symmetric)
- Positive semidefinite (nonnegative eigenvalues)

$$\underbrace{\boldsymbol{S}_p}_{\text{measured CSM}} = \underbrace{\boldsymbol{S}_a}_{\text{signal of interest}} + \underbrace{\boldsymbol{S}_n}_{\text{unwanted noise}}$$

- Signal CSM : one eigenvalue for one incoherent source
- Noise CSM : off-diagonal elements $\rightarrow 0$ with averaging

Denoising algorithms

Diagonal reconstruction

$$\text{maximize } \sum_i \sigma_{n_i} \text{ subject to } \boldsymbol{S}_{pp} - \mathrm{diag}(\boldsymbol{\sigma}_n) \geq 0$$

Solved with CVX Matlab toolbox

citation

Robust Principal Component Analysis

$$\text{minimize } \|\boldsymbol{S}_a\|_* + \lambda \|\boldsymbol{S}_n\|_1 \quad \text{subject to} \quad \boldsymbol{S}_{aa} + \boldsymbol{S}_{nn} = \boldsymbol{S}_{pp}$$

Solved with proximal gradient algorithm

citation

Probabilistic Factorial Analysis

$$\boldsymbol{p} = \boldsymbol{L}\,\mathrm{diag}(\boldsymbol{\alpha})\boldsymbol{C} + \mathrm{diag}(\boldsymbol{\sigma}^2)$$

$$\boldsymbol{L}, \boldsymbol{C}, \boldsymbol{\sigma}^2 \sim \mathcal{N}(0, \boldsymbol{\Omega}^2_{L,C,\sigma}) \quad \text{and} \quad \boldsymbol{\alpha} \sim \mathcal{N}(\boldsymbol{\mu}_\alpha, \boldsymbol{\Omega}^2_\alpha)$$

Solved with Gibbs sampling algorithm

Results