Comparison of microphone array denoising techniques and application to flight test measurements

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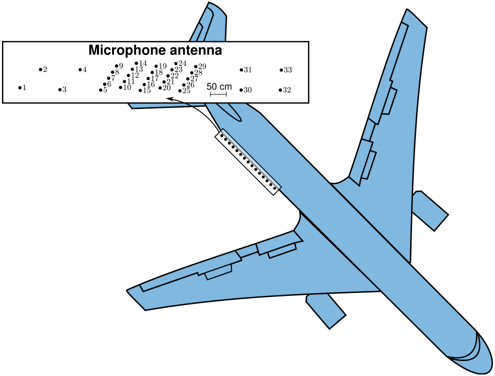
Context

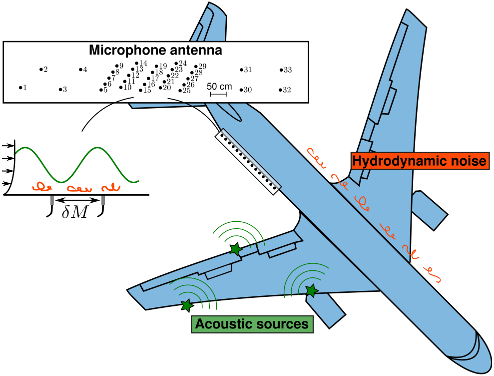
- **Unwanted noise** : electronic, ambient, flow-induced, . . .
- **Multi-channel acquisition** : inflight/wind tunnel tests for aircraft design



Context

- **Unwanted noise** : electronic, ambient, flow-induced, ...
- **Multi-channel acquisition** : inflight/wind tunnel tests for aircraft design
- 2 kinds of pressure excitation:
  - from the acoustic sources (signal)
  - from the turbulent boundary layer (noise)

  } very low SNR



Context

How to separate the acoustic sources from the noise ?

Existing methods:

- physical removal : windscreen, mic recession, porous treatment,...
- background subtraction $\rightarrow$ not always available or representative
- wavenumber filtering $\rightarrow$ requires high spatial sampling
- diagonal removal $\rightarrow$ underestimation of source level
- other post-processing (inverse problem) $\rightarrow$ for long records

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We propose a new method

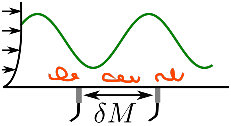
- Solve an inverse problem to recover the source signal

and compare it to an existing method

- Use noise-free channels as references

Outline

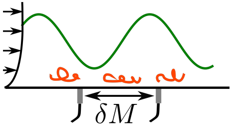
CSM properties



At one frequency and for $i = 1, \ldots, I$ snapshots

$$\underbrace{\boldsymbol{p}_i}_{\text{measured spectra}} = \underbrace{\boldsymbol{a}_i}_{\text{acoustic part}} + \underbrace{\boldsymbol{n}_i}_{\text{unwanted noise}}$$

CSM properties



$$\boldsymbol{S}_{xy} = \frac{1}{I}\sum_{i=1}^{I} \boldsymbol{x}_i \boldsymbol{y}_i^H$$

At one frequency, for averaged Cross-Spectral Matrix:

$$\underbrace{\boldsymbol{S}_{pp}}_{\text{measured CSM}} = \underbrace{\boldsymbol{S}_{aa}}_{\text{acoustic CSM}} + \underbrace{\boldsymbol{S}_{nn}}_{\text{unwanted noise}} + \underbrace{\boldsymbol{S}_{an} + \boldsymbol{S}_{na}}_{\text{cross-terms}}$$

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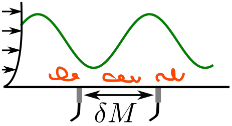

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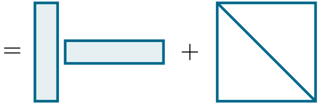

- TBL noise with **short** spatial correlation: **diagonal CSM**
- Acoustic field - with **high** spatial correlation
  - few equivalent monopoles: **low-rank CSM**

Probabilistic Factor Analysis (PFA)

- **Statistical model**

  At one frequency and for the ith snapshot:

vector of mic. pressure

$$\boldsymbol{p}_i = \boldsymbol{L} \ \boldsymbol{c}_i + \boldsymbol{n}_i$$

noise (+model errors)

mixing matrix

vector of latents factors

  - Capture dominant correlation with few factors (close to PCA)
    $\hookrightarrow$ low-rank acoustic CSM : $\boldsymbol{S}_{aa} = \boldsymbol{L}\boldsymbol{S}_{cc}\boldsymbol{L}^H$
  - Extract anisotropic noise

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- Strong data compression

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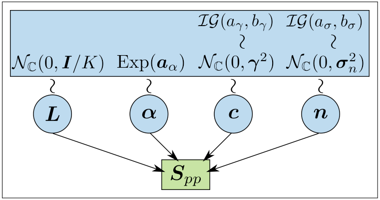
- **Bayesian approach :** See parameters as random variables

$\boldsymbol{L} \sim \mathcal{N}_{\mathbb{C}}(0, \lceil \frac{\mathbf{1}}{K} \rfloor)$	$\boldsymbol{c}_i \sim \mathcal{N}_{\mathbb{C}}(0, \lceil \boldsymbol{\gamma}^2 \rfloor)$	$\boldsymbol{n}_i \sim \mathcal{N}_{\mathbb{C}}(0, \lceil \boldsymbol{\sigma}_n^2 \rfloor)$	$\boldsymbol{\alpha} \sim \mathcal{E}(a_\alpha)$

$+$ hyperparameters : $\boldsymbol{\gamma}^2, \boldsymbol{\sigma}^2 \sim \mathcal{IG}(\boldsymbol{a}_{\gamma,\sigma}, \boldsymbol{b}_{\gamma,\sigma})$

Probabilistic Factor Analysis (PFA) – Optimization



$\longrightarrow$ **Parametric model**: $\mathcal{M}(\boldsymbol{\theta})$
with $\boldsymbol{\theta} = \{\boldsymbol{L},\ \boldsymbol{\alpha},\ \boldsymbol{c},\ \boldsymbol{n},\ \boldsymbol{a}_{\gamma,\alpha,\sigma},\ \boldsymbol{b}_{\gamma,\sigma}\}$

Optimization step:

$$\boxed{\boldsymbol{\theta} = \underset{\boldsymbol{\theta}}{\mathrm{argmax}} \quad \underbrace{\mathsf{P}\left(\boldsymbol{\theta} \mid \boldsymbol{S}_{pp}\right)}_{\text{objective function}}}$$

- The objective function is the joint posterior probability
  $\hookrightarrow$ no closed-form $\rightarrow$ approximated with numerical methods

Probabilistic Factor Analysis (PFA) – Optimization

Maximizing the posterior probability distribution

$\hookrightarrow$ *Find the optimal parameter set that best fits the data*

- Numerical method: the Gibbs sampler
- MCMC algorithm
- Global optimization process

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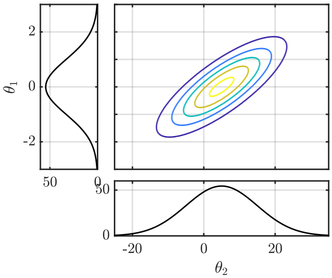

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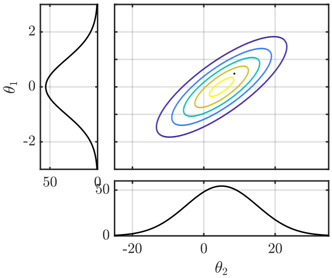

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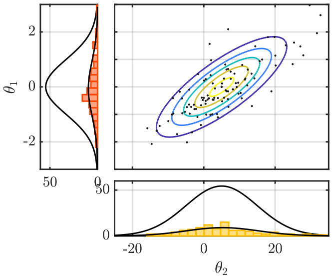

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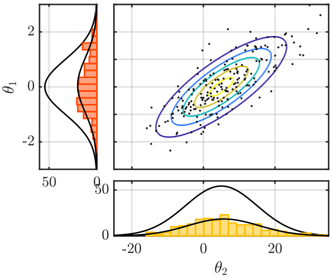

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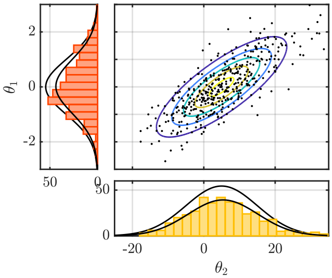

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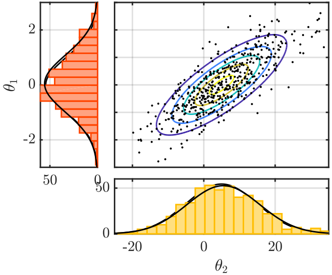

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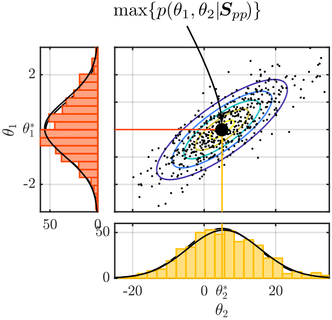

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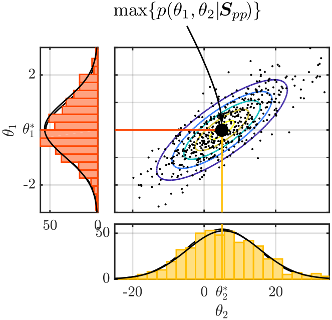
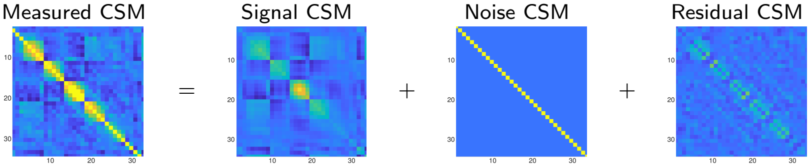


Example of inflight measurements



Probabilistic Factor Analysis (PFA)

$\oplus$

- The bayesian approach:
  - prior knowledges are part of the model
  - gives credible interval

- Probabilistic Factor Analysis :
  - preserves the CSM properties
  - reduces data dimension
  - no input parameter to set
  - blind: no hypothesis on the source

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- Sensitive to prior choices esp. for ill-posed problem
- Computationally expensive

Denoising with reference channels

Hypothesis : the TBL noise does not affect the sensors inside the cabin

$$\left.\begin{array}{ll} \boldsymbol{p} & : \text{noisy measurements} \\ \boldsymbol{r} & : \text{noise-free reference signals} \end{array}\right\} \text{synchronous acquisitions}$$

$\boldsymbol{a}$: denoised signals

$$\boxed{\boldsymbol{S}_{aa} = \boldsymbol{S}_{pr}\boldsymbol{S}_{rr}^{-1}\boldsymbol{S}_{rp}} \quad \rightarrow$$

Generalization of the coherent spectrum (Bendat and Piersol, 1980)

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Generalization of the coherent spectrum (Bendat and Piersol, 1980)

+

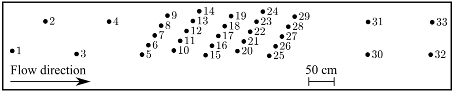
- Simple to implement
- Low computational cost

-

- Require extra measurements
- Reference channels have to be noise-free (or independant from TBL)

Application to flight test measurements

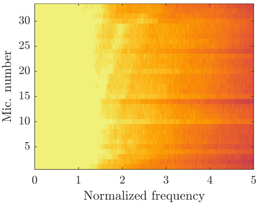
- Cruise flight condition: Mach 0.85
- High engine speed + 1 background meas. (idle speed)
- 33 microphones flushmounted on the aft fuselage
- 9 microphones + 6 accelerometers in the cabin for reference only
- CSM:
  - 4 Hz resolution
  - 60 sec record length
  - Hanning window
  - overlap : 70% (500 snapshots)



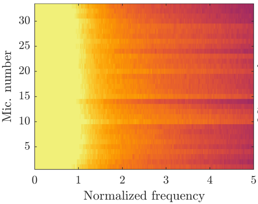
Outer microphone antenna

Application to flight test measurements

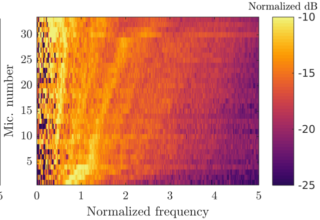
Autospectra of the outer mic. in the MF frequency range



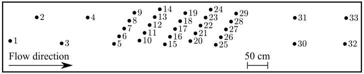
Raw (high engine speed)

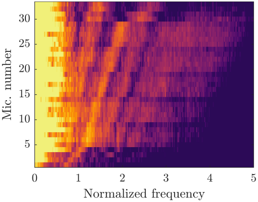


Background noise (idle)

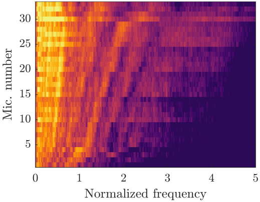


After background subtraction





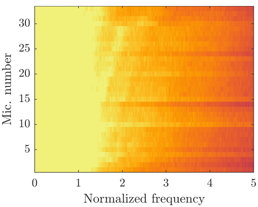
PFA denoising



Reference-based denoising

Application to flight test measurements

Autospectra of the outer mic. in the MF frequency range



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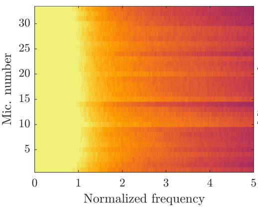

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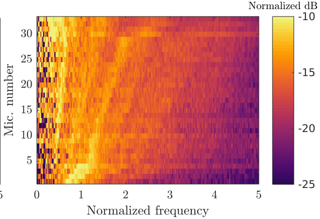

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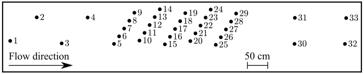
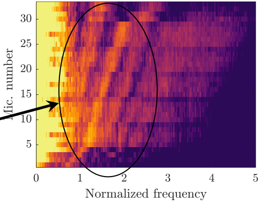


- Interference patterns: Regularly spaced monopoles in the jet (BBSAN)



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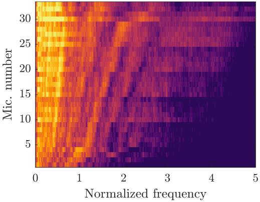
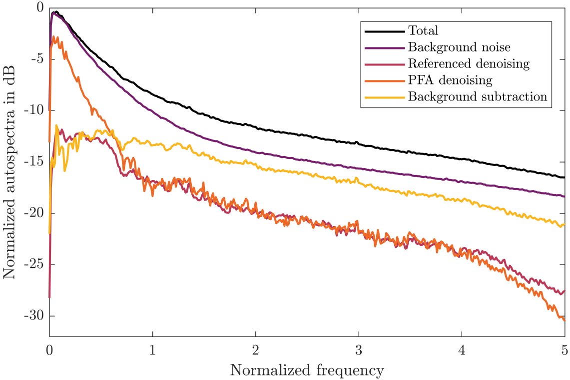


Reference-based denoising

Application to flight test measurements

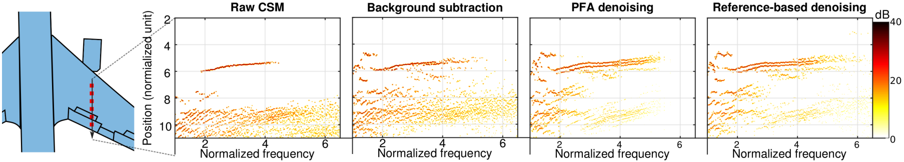
Autospectra averaged over the microphones



- Reduces the noise of 10-15 dB
- PFA : Few denoising at very low frequency $\rightarrow$ TBL noise is correlated
- PFA and reference-based denoising are in good agreement in the MF range

Application to flight test measurements – Imaging

Inverse method: Iterative Reweighted Least Squares, $p = 0$
and Bayesian regularization (Antoni et al., 2019)



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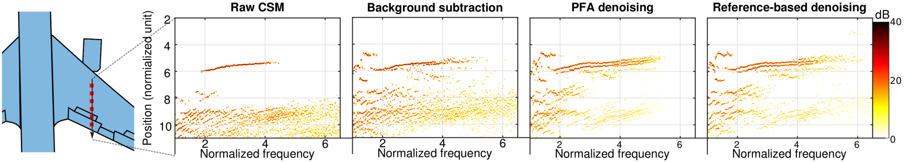

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↪ visible only with advanced denoising

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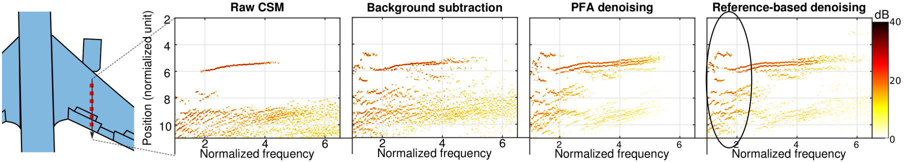

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- 4 correlated sources in the 1-2 frequency range
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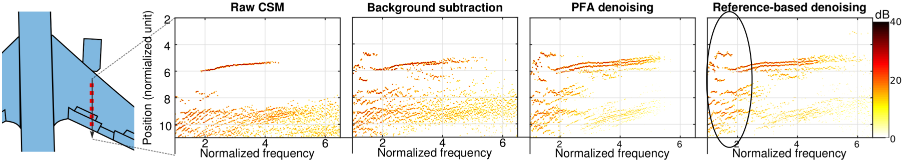
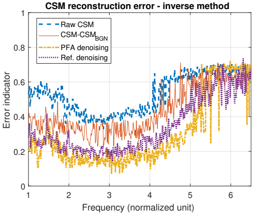


- 2 dominant wide-band sources
  ↪ visible only with advanced denoising
- 4 correlated sources in the 1-2 frequency range
  ↪ caused interference patterns
- PFA gives the lowest error

$$\text{Error} = \frac{\|\boldsymbol{S}_{pp}^{\text{denoised}} - \boldsymbol{G}\boldsymbol{S}_{qq}\boldsymbol{G}'\|_1}{\|\boldsymbol{S}_{pp}^{\text{denoised}}\|_1 + \|\boldsymbol{G}\boldsymbol{S}_{qq}\boldsymbol{G}'\|_1}$$



Conclusions

- Denoising with different requirements/hypothesis
  $\hookrightarrow$ but both give similar results

- Denoising increases of the imaging performance

- Possible improvements for PFA:
  - use a more efficient/robust solver
  - change the model to have better control on sparsity level
  - account for a correlated noise model for the low-frequency range

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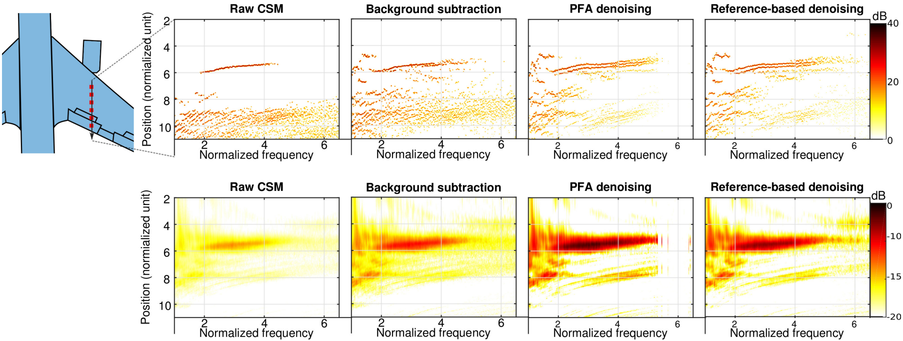
References

J. Antoni, T. Le Magueresse, Q. Leclère, and P. Simard. Sparse acoustical holography from iterated bayesian focusing. *Journal of Sound and Vibration*, 446:289–325, 2019.

J. Bendat and A. Piersol. *Engineering applications of correlation and spectral analysis*. Wiley-Interscience, New York, 1980.

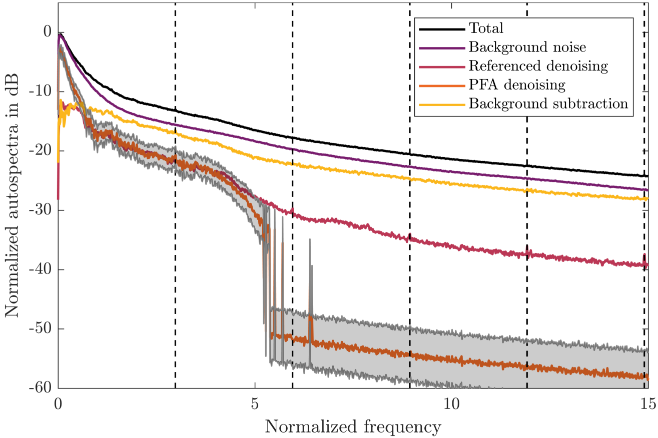
Appendix – Beamforming

Iterative Reweighted Least Squares, $p = 0$
and Bayesian regularization



Beamforming coherence

Appendix – Credible interval



In gray : 95% credible interval for PFA denoising