Computing and Data Science

Simulations

Assignment no. 6 (Discrete Distributions)

3rd Year

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1. **Solve the last question for negative distribution that is required to reach the 3rd success.**

A variable X has a negative distribution with parameters n=1, p = 0.35, and q=0.65

1. Generate 7 observations of the variable.

2. Determine the observed mean and the standard deviation.
3. Compute the true mean and the standard deviation of the observations.

Using the following random numbers:
39 58 55 46 85 84 38 01 93 52 46 11 57 75 86 44 33 28 93 58 18 91 02 24 05 36 45 04 69 66 58 69 35 29 29 53 12 89 87 67 30

Answer:

p = 0.35 then the success RN 01 to 35

q = 0.65 then the failure RN 36 to 00

Generate 7 observations of the variable where I = 1,2,3,4,5,6,7

(1)Simulation	Number Random Number	Success
1	39	0
2	58	0
3	55	0
4	46	0
5	85	0
6	84	0
7	38	0
8	01	1
9	93	0
10	52	0
11	46	0
12	11	1
13	57	0
14	75	0
15	86	0
16	44	0
17	33	1

$x_1 = 17$

(2)Simulation	Number Random Number	Success
1	28	1
2	93	0
3	58	0
4	18	1
5	91	0
6	02	1

$x_2 = 6$

(3)Simulation	Number Random Number	Success
1	24	1
2	05	1
3	36	0
4	45	0
5	04	1

$x_3 = 5$

(4)Simulation	Number Random Number	Success
1	69	0
2	66	0
3	58	0
4	69	0
5	35	1
6	29	1
7	29	1

$x_4 = 7$

(5)Simulation	Number Random Number	Success
1	53	0
2	12	1
3	89	0
4	87	0
5	67	0
6	30	1
7	39	0
8	58	0
9	55	0
10	46	0
11	85	0
12	84	0
13	38	0
14	01	1

$X_5 = 14$

(6)Simulation	Number Random Number	Success
1	93	0
2	52	0
3	46	0
4	11	1
5	57	0
6	75	0
7	86	0
8	44	0
9	33	1
10	28	1

$X_6 = 10$

(1)Simulation	Number Random Number	Success
1	93	0
2	58	0
3	18	1
4	91	0
5	02	1
6	24	1

$X_7 = 6$

After this simulation, the 7 observation are obtained to be:

17 6 5 7 14 10 6

2. The observed mean of these observations = $\frac{17+6+5+7+14+10+6}{7} = 9.286$

   standard deviation = SD = $\sqrt{\frac{\sum(Xi-mean)^2}{N-1}}$ = 4.608

3. the true mean = $\frac{n}{P} = \frac{3}{0.35} = 8.571$

   the true standard deviation = $\sqrt{\frac{nq}{p^2}} = \sqrt{\frac{3*0.65}{0.35^2}} = 3.9898$

2. Suppose you are given a 3 question multiple-choice test. Each question has 4 responses and only one is correct. Suppose you want to find the probability that you can just guess at the answers and get 2 questions right. (Teachers do this all the time when they make up a multiple-choice test to see if students can still pass without studying. In most cases the students can't.) To help with the idea that you are going to guess, suppose the test is in Martian.

a. What is the random variable?

b. Is this a binomial experiment?

c. What is the probability of getting 2 questions right? (PDF)

d. What is the probability of getting zero right, one right, and all three right? (PDF)

Answer:

p = ¼ = 0.25 **q = 1- 0.25 = 0.75** **n = 3**

a. The random variable is the number of questions answered correctly out of three.

b. Yes, this is a binomial experiment because there are a fixed number of trials (three questions), two possible outcomes (correct or incorrect).

c. $P(X=2) = (nC2) * p^2 * (1-p)^{(n-2)}$

$= 3C2 * (0.25)^2 * (0.75)^1 = 0.140625$

d. $P(X=0) = (3C0) * (0.25)^0 * (0.75)^3 = 0.421875$

$P(X=1) = (3C1) * (0.25)^1 * (0.75)^2 = 0.421875$

$P(X=3) = (3C3) * (0.25)^3 * (0.75)^0 = 0.015625$

3. When looking at a person's eye color, it turns out that 1% of people in the world has green eyes ("What percentage of," 2013). Consider a group of 20 people.

P = 0.01 q = 0.99

a. State the random variable.

b. Argue that this is a binomial experiment

c. Find the probability that

I. None have green eyes (0 successes)

ii. Nine have green eyes (9 successes)

iii. At most three have green eyes. $P(X \le 3)$

iv. At most two have green eyes $P(X \le 2)$

v. At least four have green eyes. $P(X \ge 4) = 1 - P(X \le 3)$

d. In Europe, four people out of twenty have green eyes. Is this unusual? What does that tell you?

Answer:

a. The random variable is the number of people with green eyes

b. This is a binomial experiment because:

- There are a fixed number of trials (20 people).
- There are two possible outcomes for each trial (green eyes or not green eyes).
- The probability of success (getting a person with green eyes) is constant for each trial (0.01).

c. Find the probability that:

I. None have green eyes: x = 0

$20C0 * 0.01^{0} * 0.99^{20} = 0.818$

II. Nine have green eyes: x = 9

$20C9 * 0.01^{9} * 0.99^{11} = 0.15*10^{-12}$

III. At most three have green eyes : $P(X \le 3)$

$$\sum_{0}^{i=x} n\,C\,i * p^{i} * q^{n-i} = \sum_{0}^{i=3} 20\,C\,i * 0.01^{i} * 0.99^{20-i} = 0.99996$$

iv. At most two have green eyes: P(X≤2)

$$\sum_{0}^{i=x} n \, C \, i * p^{i} * q^{n-i} = \sum_{0}^{i=2} 20 \, C \, i * 0.01^{i} * 0.99^{20-i} = 0.999$$

v. At least four have green eyes. P(X≥4) = 1−P(X≤3)

1- 0.99996 = 0.00004

d. In Europe, four people out of twenty have green eyes, while only 1% of people in the world have green eyes. This is not unusual, as the frequency of green eyes is known to be higher in Europe than in other parts of the world.

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4. representative from the NFL Marketing division randomly selects people on a random street in Chicago loop, until he/she finds a person who attended the last home football game. Let p, the probability that she succeeds in finding such a person, is 0.2 and X denote the number of people asked until the first success.

a. What is the probability that the representative must select 4 people until he finds one who attended the last home game?

b. What is the probability that the representative must select more than 6 people before finding one who attended the last home game?

Answer:

This is geometric distribution with:

- P = 0.2
- q = 0.8
- x = success = 1
- x - 1= 4 - 1 = 3

a. p (x=4) = $q^{x-1} * p = (0.8)^{3} * (0.2)^{1} = 0.1024$

b. p (x > 6) =1- p (x ≤ 6) = 1- $[1 – (1- p)^{x}]$

= 1- $[1-(1 – 0.2)^{6}]$ = 0.262

5. A cloud computing system failure occurs according to a Poisson distribution with an average of 3 failures every 10 weeks. I.e., the failure within t week(s) is Poisson distributed with α = 0.3 t

a. Calculate the probability that there will not be more than one failure during a particular week

b. Calculate the probability that there will be at least one failure during a particular month (4 weeks)

<u>Answer:</u>

a. with λ = 0.3

$$p\ (x \leq 1) = \sum_{i=0}^{i=1} \frac{e^{-\lambda} * \lambda^{i}}{i!} = \sum_{i=0}^{i=1} \frac{e^{-0.3} * 0.3^{i}}{i!} = 0.963$$

b. with λ = 1.2

$$p\ (x \geq 1) = 1 - p(x = 0\ )$$

$$= 1 - \frac{e^{-\lambda} * \lambda^{x}}{x!} = 1 - \frac{e^{-1.2} * 1.2^{0}}{0!} = 0.6988$$