Interband conductivity of graphene (Fermi's Golden Rule), $A = A_0 e^{-i\omega t}$, $E = -\frac{\partial A}{\partial t} = i\omega A = E_0 e^{-i\omega t}$

$H_0 = \hbar v_F \vec{\sigma}.\vec{k}$, $H_{int} = \frac{e v_F}{i\omega} \sigma . E_0 e^{-i\omega t}$

Fermi Golden Rule: $\frac{dw_{i \to f}}{dt} = \frac{2\pi}{\hbar} |\langle i | H_{int} | f \rangle|^2 \times$

$f_i (1 - f_f)\, \delta(\hbar\omega_{if} - \hbar\omega) \Rightarrow$

Total Power absorbed: $\sum_i \hbar\omega \frac{dw_{i \to f}}{dt}$

$P_a = 2 R_e \int \sigma(\omega) E(\omega) E^*(\omega)\, d^2 r = 2 [R\sigma(\omega)] |E(\omega)|^2 L^2$

$= 2 R_e \sigma(\omega) |E_0|^2 L^2 \Rightarrow$

$R\sigma(\omega) = \frac{\hbar\omega}{2|E_0|^2 L^2} \sum_i \frac{2\pi}{\hbar} |\langle i | H_{int} | f \rangle|^2 f_i (1 - f_f) \times$

$\delta(\hbar\omega_{if} - \hbar\omega)$, let $E_0 = E_0 \hat{e_x} \Rightarrow$

$\left(\frac{e v_F}{\omega}\right)^2 |\langle f | \sigma_x | i \rangle|^2 \Rightarrow$ mat element is $\frac{1}{2} \delta_{k,k'}$ $($

$n e^{-i\theta_k} + n' e^{i\theta_k}) \Rightarrow$

$$\frac{1}{2} \left(\frac{e^2 v_F^2}{\omega^2}\right) \frac{4}{(2\pi)^2} \int k\, dk\, d\theta \int \frac{2\pi}{\hbar} \frac{1}{2} (1 + n n' \cos 2\theta_k) \times$$

$$\delta(n' \hbar v_F k - n \hbar v_F k - \hbar\omega) f_{nk} (1 - f_{n'k})$$

Only looking at interband contribution gives us:

$$\mathrm{Re}\, G(\omega) = \frac{e^2}{4\hbar} f(-\hbar\omega/2)\left(1 - f(\hbar\omega/2)\right)$$

at $T=0$, $-\hbar\omega/2 < \mu$, $\hbar\omega/2 > \mu$ $\rightarrow$

$-\hbar\omega < 2\mu$, $\hbar\omega > 2\mu$ $\rightarrow$ $-\hbar\omega < 2\mu < \hbar\omega$

$\Rightarrow$ $\Theta(\hbar\omega - 2\mu)$, $\Theta(2\mu + \hbar\omega)$

For $\hbar\omega < 2\mu$, No dissipation due to Pauli Blocking:

$|a|^2 = 2.3\%$ (absorption coefficient)

$$\mathrm{Im}\, G(\omega) = -\frac{2\omega}{\pi} \int \frac{\mathrm{Re}\, G(\omega')}{\omega'^2 - \omega^2} d\omega' =$$

Radiative transfer between two graphene sheets:

Current-Current response function:

$$H_{int} = \sum_q e^{iq\cdot r} e v_F\, \sigma \cdot A^{ext}(q,t) =$$

$$-L^2 \sum_q j_q^{\dagger} \cdot A(q,t) = -\int e^{-i\omega t} d\omega\, L^2 \sum_q j \cdot A(q,\omega)$$

$$\psi_{nk}(r) = \frac{1}{L\sqrt{2}} \begin{pmatrix} n \\ e^{i\theta(k)} \end{pmatrix} e^{ik\cdot r} \Rightarrow$$

two possibilities: $j \sim \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} e^{iqx}$,

$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} e^{iqx} \Rightarrow$

First possibility $\Rightarrow \frac{1}{2}\frac{1}{L^2}\int d^2r\, e^{iq\cdot r} e^{ik\cdot r} e^{-ik'\cdot r}$

$\left(n e^{-i\theta(k')} + n' e^{i\theta(k)}\right) = \frac{1}{2}\int d^2r\, e^{i(k+q-k')\cdot r} n'$

$e^{i\theta(k)}\big(\underbrace{1 + nn' e^{-i\theta_k - i\theta_{k'}}}\big) \Rightarrow$

$$\left|\frac{1+nn' e^{i(\theta_k+\theta_{k'})}}{2}\right|^2 = \frac{1+1+2nn'\cos(\theta_k+\theta_{k'})}{4}$$

$= \frac{1+\cos(\theta_k+\theta_{k'})}{2}$ ✓ (Note that $\frac{1}{L^2}\int e^{iq\cdot r + ik\cdot r - ik'\cdot r} d^2r = \delta_{q+k,k'}$)

For transverse polarization,

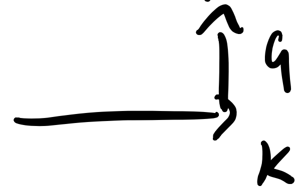
$(n' \;\; e^{-i\theta_{k'}}) \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} n \\ e^{i\theta_k} \end{pmatrix} \Rightarrow -n' e^{i\theta_k} + n e^{-i\theta_{k'}}$

$= n e^{-i\theta_{k'}}\left(1 - nn' e^{i(\theta_k+\theta_{k'})}\right) \Rightarrow$

$$\frac{1}{4}\left(2 - 2\cos(\theta_k+\theta_{k'})\right) = \boxed{\frac{1-\cos(\theta_k+\theta_{k'})}{2}}$$

(Note that if instead of $\sigma_x \to \sigma_y$ we take

$q_x \to q_y$, we obtain



Two plasmon absorption (Jabbar/Cheng 2018)

$$H_p = -\frac{1}{2} e \varphi_p \sum e^{iq\cdot r_j} e^{-i\omega t} + c.c$$

$$\frac{e^2}{4\pi\varepsilon_0 r} \sim \frac{\hbar c \alpha}{r} \sim \frac{\text{eV s meter}}{\text{s} \times \text{meter}} \to \text{eV}$$

$$\gamma_p = \frac{P}{W}, \quad \boxed{W = \varepsilon_r \varepsilon_0 |E_p|^2 \frac{A}{q}}, \quad Q = \omega/\gamma_p$$

$$P = \sum_n 2\hbar\omega e^4 \frac{2\pi}{\hbar} \left| \sum_m \frac{\langle n| e^{iq\cdot r} |m\rangle\langle m| e^{iq\cdot r} |p\rangle}{E_m - E_0 - \hbar\omega} \right|^2 \delta(E_n - E_0 - 2\hbar\omega)$$

$$= \sum_{n_1, n_2} 4\pi e^4 \omega \left| \sum_m \frac{\langle n_2, k+2q | m, k+q\rangle\langle m, k+q | n_1, k\rangle}{\varepsilon_{m,k+q} - \varepsilon_{n,k} - \hbar\omega} \right|^2 \delta(\varepsilon_{m,k+q} - \varepsilon_{n,k} - \hbar\omega)$$

$$\gamma_2 = \frac{P}{W} \Rightarrow \gamma_2 = \omega F^{(2)}(\omega) \left|\frac{E_p}{E_e}\right|^2 = \frac{P}{W} =$$

$$\frac{2q}{\varepsilon_r \varepsilon_0 |E_p|^2 A} \sum 4\pi e^4 \omega \left| \sum_m \frac{\langle n_2, k+2q | m, k+q\rangle\langle m, k+q | n, k\rangle}{\varepsilon_{m,k+q} - \varepsilon_{n,k} - \hbar\omega} \right|^2 \delta(\,)$$

$$\Rightarrow F^2(\omega) = \frac{2|E_e|^2 q}{\varepsilon_r \varepsilon_0 |E_p|^4 A} \sum 4\pi e^4 \omega \left| \quad \right|$$

$\delta(\qquad)$

$$E_F = \hbar v_F k_F \rightarrow \frac{E_F^3}{k_F^2} = \frac{\hbar^3 v_F^3 k_F^3}{k_F^2} = \boxed{\hbar^3 v_F^3 k_F}$$

$$\Rightarrow \frac{\omega^6}{v_F^6 q^6} \frac{E_F^3}{k_F^2} = \frac{\omega^6}{v_F^6 q^6} \hbar^3 v_F^3 k_F = \frac{\hbar^3 \omega^6 k_F}{v_F^3 q^6} =$$

$$\left(\frac{\omega}{\varepsilon q}\right)^6 k_F \rightarrow$$

$$F^2(\omega) \equiv \frac{1}{\text{angstrom}} \frac{1}{\text{angstrom}^2} \frac{1}{eV^3}$$

$$\frac{eV^3}{\quad} \frac{1}{eV^3} \Rightarrow F^{(2)}(\omega) \text{ is unitless:}$$

Check: $\left(\frac{\hbar\omega}{\hbar v_F q}\right)^6 \frac{\hbar^3 v_F^3 k_F}{A} =$

$$\frac{(\hbar\omega)^6}{\hbar^3 v_F^3 q^6} \frac{k_E}{A}$$

$$F^{(2)}(\omega) = \frac{|E_e|^2}{|E_p|^2} \frac{1}{\omega} \frac{2q}{\varepsilon_r \varepsilon_0 |E_p|^2 A} \sum \frac{2\hbar\omega\, 2\pi}{\hbar} \left(\qquad\right)$$

$$= \frac{|E_e|^2}{|E_p|^4} \frac{2q}{\varepsilon_r \varepsilon_0 A\omega} \sum 4\omega\pi \left(\qquad\right) =$$

$$\frac{|E_e|^2}{|E_p|^4} \frac{8\pi q}{\varepsilon_r \varepsilon_0 A} \left(\qquad\right) \sim$$

$$\frac{1}{\text{Efield}^2}\ \frac{8\pi}{\text{Volume}}\ \frac{1}{\varepsilon_0}\left(\quad\right) \sim \frac{1}{eV}\left(\qquad\right)$$

parentheses has units of $\frac{eV^4}{eV^2 \cdot eV} = eV \Rightarrow$

$F^2(\omega)$ is unitless as before.

$$\frac{|F_e|^2}{|F_p|^4}\ \frac{8\pi q}{\varepsilon_r\varepsilon_0 A} = \frac{|F_e|^2}{q^3 \cancel{\varphi^4}}\ \frac{8\pi}{\varepsilon_r\varepsilon_0 A}\left(\frac{\cancel{\varphi^4}}{16}\qquad\right)$$

$$\cancel{= \frac{e^2}{4\pi(\varepsilon_r\varepsilon_0)^2}\ \frac{1}{\varphi_e^2}\ \frac{\pi}{2A}\ \frac{1}{q^3}} =$$

$$W = \frac{1}{2}\varepsilon_r\varepsilon_0\frac{A|E_p|^2}{q} \rightarrow$$

$$\frac{\gamma^{(2)}}{\omega} = \frac{P}{W}\frac{1}{\omega} = \frac{2q}{\varepsilon_r\varepsilon_0 A|F_p|^2}\frac{1}{\omega}\sum_i 2\hbar\omega\frac{2\pi}{\cancel{\hbar}}\left(\quad\right)$$

$$= \frac{8\pi q}{\varepsilon_r\varepsilon_0 A|E_p|^2}\sum_i \frac{e^4}{16}\varphi_p^4\left(\qquad\right)$$

Final Expression for $F^{(2)}(\omega) =$

$$\left(\frac{\omega}{v_F q}\right)^6 \frac{E_F^3}{k_F^2}\ \frac{\pi^2}{(2\pi)^2}\int d^2k\ \Big|\langle n_3, k+q| \ n_2, k+q$$

$$\left.\frac{\langle n_2, k+q \rangle n_1, k\rangle|}{\varepsilon_{n_2, k+q} - \varepsilon_{n_1,k} - \hbar\omega}\right| \quad \Rightarrow$$

Numerically, what we do is

$$\left(\frac{\omega^b}{v_F q}\right)^b \frac{E_F^3}{k_F^2} \left(\frac{1}{\sqrt{2} N_k}\right) \sum_k \Big| \langle n_3, k+2q | n_2, k+q \rangle \langle n_2, k+q | n_1, k\rangle \Big|^2 \delta(\varepsilon_{n_3, k+2q} - \varepsilon_{n_1,k} - 2\hbar\omega)\, f_{n_1 k} (1 - f_{n_3 k+2q})$$

Now $\frac{1}{A}\sum_k = \frac{1}{(2\pi)^2}\int k\,dk\,d\theta =$

$$\frac{1}{(2\pi)^2}\frac{(\pi k_F^2)}{N_k N_\theta} \sum_{k,\theta} k(\quad)$$

Redo:

$$\frac{1}{A}\sum_{k_x,k_y} = \frac{1}{(2\pi)^2}\int dk_x\, dk_y\, f(k_x, k_y) =$$

$$\frac{1}{(2\pi)^2}\int k\,dk\,d\theta\, f(k,\theta) = \frac{1}{(2\pi)^2}\frac{(\pi k_F^2)}{N_k N_\theta}\sum_{k,\theta} k\, f(k,\theta)$$

$$\frac{1 \pm kq\cos\alpha}{|k+q|^{\varepsilon}}, \quad |k+2q| = \sqrt{(k\cos\alpha + 2q)^2 + k^2\sin^2\alpha}$$

$$= \sqrt{k^2 + 4q^2 + 4}$$

$$k + 2q = (k\cos\theta + 2q,\ k\sin\theta)$$

$$\frac{1}{2}\left(ss' e^{i(\varphi_k - \varphi_{k+q})} + 1 \right)$$

$$\rightarrow \frac{1}{2}\left(1 + ss' e^{i(\varphi_k - \varphi_{k+q})} \right)$$

Matrix elements change with doping

- Matrix elements in undoped case
- show bands don't move
- Losses in undoped case.

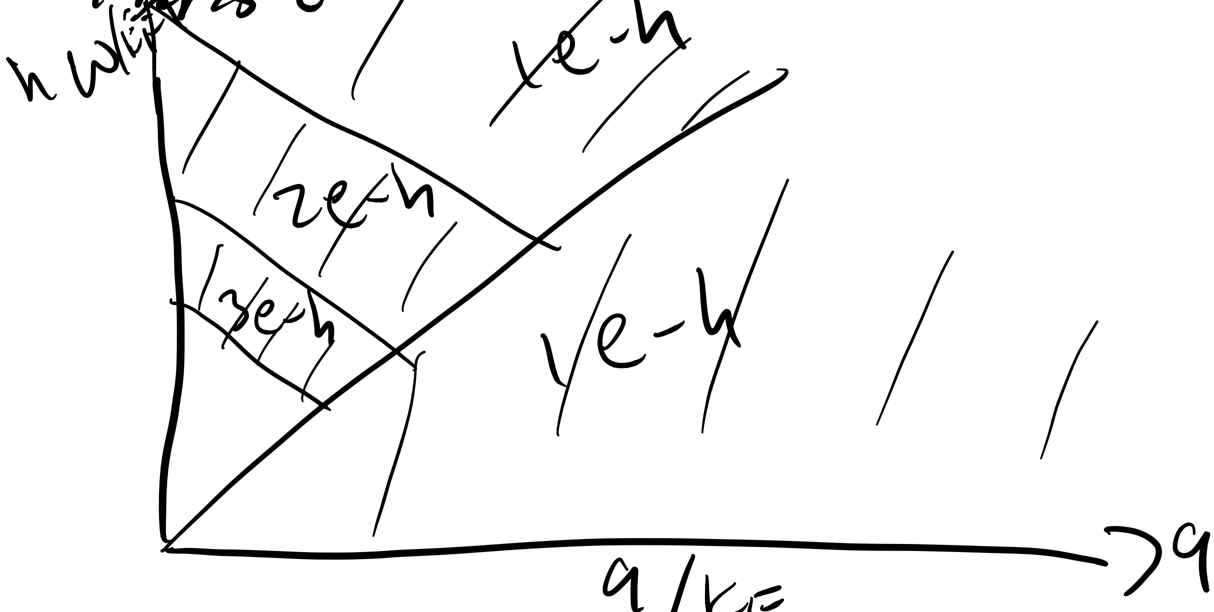
Typical values in graphene
(from 2 photon absorption paper by Mishchenko)

$n = 10^{12}\ \mathrm{cm}^{-2} \rightarrow n = \frac{N}{V}$, $V = \pi r_e^2 N \rightarrow$

$n = \frac{1}{\pi r_e^2} \rightarrow \boxed{r_e = \frac{1}{\sqrt{n\pi}}} \sim \underline{5.6\ \mathrm{nm}}$

$E_e = \frac{e}{4\pi \varepsilon_r \varepsilon_0 r_e^2}$

$= \frac{e n\pi}{4\pi \varepsilon_r \varepsilon_0 r_e^2} \approx 2\times10^{7}\ \mathrm{V/meter}$



<u>Loss mechanisms for plasmons in graphene</u> (shown above)

1e-h pair creation corresponds to

Lavender clumping.