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**Section: BESE-13 A Date: 9th March 2024**

**Design and Analysis of Algorithm Assignment 01**

**Tools Used:**

* Visual Studio Code
* Jupyter Notebook

**Referances:**

* www.programiz.com
* Geeksforgeeks

Importing the Libraries

[39]:

**import time import random**

**import matplotlib.pyplot as plt**

Implementation of Bubble Sort

[26]:

**def** bubble\_sort(arr): n = len(arr)

**for** i **in** range(n):

*# Outer loop; Dictating number of passes*

**for** j **in** range(0, n-i-1): *# Inner Loop; Dictating number of swaps*

**if** arr[j] > arr[j+1]:

arr[j], arr[j+1] = arr[j+1], arr[j] *# Swapping of numbers*

print(f"Sorted Array for Size **{**n**}**: ", arr)

**return** arr

[40]:

Brief Overview of the Code:

Get the length of the array arr and store it in the variable n. The outer loop (for i in range(n)) runs n times, determining the number of passes needed to sort the array. The inner loop (for j in range(0, n-i-1)) iterates through the unsorted portion of the array, performing comparisons and swaps. In each iteration of the inner loop, adjacent elements are compared, and if they are in the wrong order, they are swapped. After completing one pass through the array, the largest unsorted element is moved to its correct position at the end of the array. The process repeats until the entire array is sorted.

Block for random value arrays of different size

array\_sizes = [10, 100, 1000, 10000] *# Array of different sized arrays*

execution\_times = []

**for** size **in** array\_sizes:

arr = [random.randint(0, 1000) **for** \_ **in** range(size)] start\_time = time.time()

sorted\_arr = bubble\_sort(arr) end\_time = time.time()

execution\_time = end\_time - start\_time execution\_times.append(execution\_time)

Sorted Array for Size 10: [223, 226, 325, 426, 478, 478, 479, 569, 659, 824]

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| 167, 192, 204, 205, 208, 225, 241, | | | | | | | 275, 282, 285, 290, 302, | | | | | 315, 321, 340, 344, | | | |
| 354, 364, 367, 371, 374, 374, 376, | | | | | | | 386, 388, 396, 396, 400, | | | | | 400, 401, 405, 406, | | | |
| 422, 443, 443, 449, 482, 493, 497, | | | | | | | 497, 503, 509, 517, 517, | | | | | 532, 542, 544, 545, | | | |
| 555, 559, 560, 603, 613, 618, 629, | | | | | | | 644, 648, 657, 661, 675, | | | | | 695, 699, 704, 704, | | | |
| 718, 725, 728, 733, 742, 766, 773, | | | | | | | 778, 786, 803, 826, 828, | | | | | 840, 863, 867, 907, | | | |
| 928, 937, 975, 977, 979, 990, 991, 995]  Sorted Array for Size 1000: [0, 0, 1, 1, 2, 3, 4, 4, 5, 6, | | | | | | | | | | | | 7, 8, 10, 10, 11, | | | |
| 13, 13, 16, 16, 17, | | | | 17, 19, 20, 21, 22, | | | | 22, 23, 23, 24, 24, | | | | 25, 25, 28, 29, 29, | | | |
| 30, 30, 31, 32, 32, | | | | 32, 34, 35, 35, 36, | | | | 36, 37, 38, 39, 40, | | | | 40, 40, 41, 41, 42, | | | |
| 43, 43, 44, 45, 47, | | | | 47, 48, 48, 49, 49, | | | | 49, 51, 51, 52, 53, | | | | 54, 54, 56, 57, 58, | | | |
| 59, 59, 60, 61, 61, | | | | 62, 63, 70, 72, 73, | | | | 74, 76, 78, 79, 80, | | | | 80, 83, 84, 84, 85, | | | |
| 86, 89, 89, 90, 90, | | | | 90, 92, 92, 92, 95, | | | | 95, 95, 97, 97, 97, | | | | 98, 98, 98, 99, 100, | | | |
| 101, | 101, | 102, | 103, | 103, | 105, | 106, | 112, | 113, | 114, | 115, | 115, | 116, | 117, | 118, | 119, |
| 121, | 122, | 125, | 125, | 127, | 127, | 128, | 128, | 130, | 131, | 135, | 135, | 135, | 136, | 136, | 138, |
| 139, | 142, | 148, | 149, | 150, | 151, | 153, | 154, | 154, | 154, | 155, | 155, | 157, | 159, | 160, | 160, |
| 161, | 162, | 165, | 167, | 167, | 169, | 171, | 172, | 173, | 173, | 175, | 176, | 177, | 177, | 180, | 180, |
| 181, | 181, | 182, | 183, | 187, | 187, | 189, | 190, | 190, | 192, | 192, | 193, | 193, | 193, | 196, | 198, |
| 198, | 198, | 199, | 199, | 200, | 204, | 205, | 207, | 208, | 209, | 211, | 211, | 212, | 212, | 212, | 212, |
| 213, | 215, | 217, | 218, | 218, | 220, | 221, | 221, | 221, | 221, | 224, | 224, | 225, | 225, | 228, | 229, |
| 231, | 231, | 232, | 232, | 232, | 233, | 234, | 235, | 235, | 237, | 239, | 240, | 241, | 242, | 244, | 247, |
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| 321, | 323, | 324, | 325, | 326, | 326, | 330, | 331, | 332, | 332, | 333, | 334, | 336, | 337, | 337, | 339, |
| 340, | 340, | 341, | 341, | 342, | 343, | 344, | 344, | 345, | 347, | 349, | 350, | 352, | 356, | 356, | 357, |
| 360, | 361, | 361, | 362, | 362, | 364, | 364, | 365, | 366, | 366, | 366, | 367, | 368, | 369, | 370, | 372, |
| 373, | 375, | 375, | 377, | 377, | 379, | 380, | 380, | 383, | 384, | 384, | 384, | 385, | 386, | 386, | 388, |
| 389, | 391, | 391, | 392, | 393, | 394, | 394, | 394, | 395, | 396, | 396, | 396, | 397, | 398, | 399, | 401, |
| 402, | 402, | 403, | 405, | 405, | 405, | 406, | 407, | 409, | 410, | 411, | 411, | 412, | 413, | 413, | 414, |
| 415, | 417, | 418, | 422, | 426, | 426, | 427, | 428, | 428, | 429, | 430, | 430, | 431, | 431, | 432, | 433, |
| 433, | 433, | 434, | 436, | 436, | 437, | 437, | 440, | 443, | 443, | 444, | 447, | 449, | 450, | 451, | 451, |
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| 487, | 487, | 488, | 490, | 492, | 492, | 493, | 494, | 495, | 495, | 496, | 497, | 498, | 501, | 502, | 504, |
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| 521, | 521, | 522, | 523, | 523, | 524, | 525, | 525, | 527, | 528, | 530, | 530, | 533, | 534, | 535, | 535, |
| 536, | 536, | 538, | 538, | 539, | 541, | 541, | 541, | 544, | 546, | 549, | 549, | 550, | 550, | 551, | 552, |
| 555, | 557, | 558, | 559, | 559, | 560, | 560, | 561, | 561, | 562, | 562, | 563, | 563, | 565, | 567, | 567, |
| 569, | 571, | 572, | 572, | 573, | 573, | 574, | 575, | 575, | 575, | 576, | 577, | 577, | 578, | 579, | 580, |
| 580, | 582, | 582, | 583, | 588, | 588, | 589, | 592, | 595, | 595, | 595, | 597, | 597, | 600, | 601, | 601, |
| 601, | 603, | 603, | 603, | 604, | 605, | 605, | 605, | 607, | 607, | 607, | 608, | 610, | 610, | 610, | 610, |
| 611, | 612, | 613, | 614, | 614, | 615, | 621, | 622, | 622, | 623, | 624, | 625, | 626, | 627, | 627, | 628, |
| 628, | 632, | 633, | 633, | 635, | 636, | 637, | 637, | 638, | 638, | 639, | 644, | 644, | 647, | 648, | 648, |

Sorted Array for Size 100: [11, 18, 22, 28, 41, 44, 56, 86, 93, 100, 156, 159,

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| 649, | 654, | 658, | 660, | 662, | 664, | 666, | 666, | 666, | 667, | 667, | 667, | 667, | 668, | 668, | 670, |
| 670, | 673, | 673, | 674, | 676, | 678, | 678, | 679, | 680, | 680, | 683, | 684, | 685, | 685, | 687, | 688, |
| 688, | 688, | 689, | 689, | 690, | 691, | 692, | 692, | 697, | 698, | 699, | 701, | 701, | 702, | 704, | 705, |
| 705, | 706, | 706, | 708, | 709, | 709, | 712, | 712, | 713, | 713, | 713, | 715, | 715, | 716, | 716, | 720, |
| 720, | 720, | 721, | 721, | 722, | 723, | 723, | 723, | 725, | 726, | 728, | 730, | 730, | 734, | 736, | 736, |
| 737, | 738, | 739, | 741, | 744, | 745, | 746, | 747, | 747, | 749, | 750, | 750, | 751, | 751, | 752, | 752, |
| 754, | 754, | 755, | 756, | 757, | 758, | 758, | 759, | 760, | 760, | 760, | 761, | 762, | 763, | 764, | 765, |
| 766, | 769, | 771, | 772, | 772, | 773, | 773, | 774, | 774, | 776, | 776, | 777, | 778, | 781, | 782, | 782, |
| 783, | 783, | 784, | 784, | 784, | 785, | 786, | 786, | 786, | 786, | 788, | 788, | 789, | 792, | 792, | 797, |
| 797, | 798, | 800, | 801, | 802, | 802, | 802, | 803, | 804, | 806, | 806, | 806, | 809, | 811, | 813, | 814, |
| 817, | 818, | 820, | 824, | 826, | 826, | 827, | 828, | 829, | 830, | 831, | 832, | 832, | 834, | 834, | 834, |
| 835, | 836, | 837, | 838, | 839, | 839, | 840, | 842, | 843, | 844, | 846, | 847, | 847, | 853, | 853, | 853, |
| 854, | 854, | 855, | 857, | 859, | 860, | 861, | 862, | 862, | 862, | 862, | 868, | 868, | 869, | 869, | 870, |
| 871, | 872, | 872, | 873, | 873, | 874, | 874, | 876, | 879, | 880, | 881, | 882, | 882, | 883, | 884, | 884, |
| 887, | 887, | 888, | 889, | 890, | 890, | 893, | 895, | 895, | 895, | 895, | 896, | 898, | 898, | 899, | 899, |
| 900, | 901, | 903, | 903, | 904, | 906, | 907, | 910, | 910, | 910, | 912, | 913, | 917, | 918, | 918, | 919, |
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| 937, | 940, | 941, | 942, | 942, | 944, | 944, | 944, | 945, | 945, | 945, | 946, | 946, | 951, | 951, | 951, |
| 952, | 953, | 953, | 954, | 954, | 956, | 957, | 960, | 960, | 961, | 962, | 965, | 965, | 967, | 967, | 967, |
| 968, | 968, | 969, | 970, | 971, | 972, | 973, | 974, | 976, | 977, | 978, | 979, | 980, | 981, | 982, | 982, |
| 983, | 983, | 984, | 985, | 985, | 985, | 986, | 987, | 988, | 988, | 989, | 989, | 989, | 990, | 993, | 994, |
| 994, | 994, | 995, | 996, | 1000] |  |  |  |  |  |  |  |  |  |  |  |
| Sorted Array for Size 10000: | | | | | | [0, 0, 0, | | 0, 0, 0, 0, 0, | | | 0, 0, 0, | | 0, 1, | 1, 1, 1, | |
| 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, | | | | | | 2, 2, 3, 3, 3, | | | 3, 3, 3, 3, 3, | | | 4, 4, 4, 4, 4, | | | 4, 4, |
| 4, 4, 4, 4, 4, 5, 5, 5, 5, 5, | | | | | | 5, 5, 5, 5, 5, | | | 6, 6, 6, 6, 6, | | | 6, 7, 7, 7, 7, | | | 7, 7, |
| 7, 7, 7, 7, 8, 8, 8, 8, 8, 8, | | | | | | 8, 8, 8, 8, 8, | | | 8, 8, 8, 9, 9, | | | 9, 9, 9, 9, 9, | | | 9, 9, |

9, 9, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 11, 11, 11, 11, 11, 11, 11, 11,

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[23]:

1000, 1000, 1000, 1000]

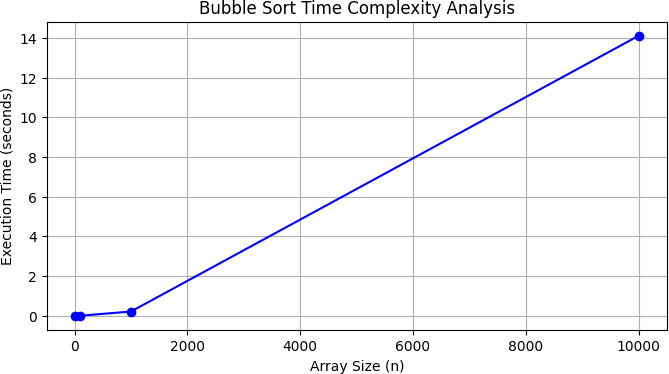
Block for Graph Visualization Using MatPlotLib

plt.figure(figsize=(8, 4))

plt.plot(array\_sizes, execution\_times, marker='o', color='b', linestyle='-') plt.title('Bubble Sort Time Complexity Analysis')

plt.xlabel('Array Size (n)') plt.ylabel('Execution Time (seconds)') plt.grid(**True**)

plt.show()



Explanation:

Bubble sort is a relatively slow sorting algorithm, especially for large datasets. Time Complexity:

The time complexity of bubble sort is O(n^2), which means that the time it takes to sort an array

[16]:

grows quadratically with the size of the array. This is because bubble sort iterates through the array multiple times, comparing adjacent elements and swapping them if they are in the wrong order. In the worst case scenario, bubble sort will need to make n-1 passes through the array, and in each pass, it will need to compare n-i elements, where i is the current pass number. This results in a total of O(n^2) comparisons.

Space Complexity:

The space complexity of bubble sort is O(1), which means that it only uses a constant amount of additional memory regardless of the size of the input. This is because bubble sort only uses a few temporary variables to store the elements being compared and swapped.

Implementation of Selection Sort

**def** selection\_sort(arr): n = len(arr)

**for** i **in** range(n): *# Outer loop for number of passes*

key = i *# For setting minimum element*

**for** j **in** range(i+1, n):

**if** arr[j] < arr[key]: *# Checks for minimum element across the array*

key = j

arr[i], arr[key] = arr[key], arr[i] *# Places the min element at the*␣

↪*start array at appropriate positon*

print(f"Sorted Array for Size **{**n**}**: ", arr)

**return** arr

[ ]:

Brief Overview of the Code:

Get the length of the array arr and store it in the variable n. The outer loop (for i in range(n)) iterates through each element of the array, representing the number of passes. Within each pass, key is set to the current index i, indicating the position where the minimum element will be placed. The inner loop (for j in range(i+1, n)) iterates through the unsorted portion of the array, searching for the minimum element from arr[i+1] to arr[n-1]. If a smaller element than arr[key] is found at index j, key is updated to j, indicating the new index of the minimum element found so far. After completing the inner loop, the minimum element found (arr[key]) is swapped with the element at index i, effectively placing the minimum element at its correct position in the sorted portion of the array. The process continues until the entire array is sorted.

Block for random value arrays of different size

execution\_times\_selection\_sort = []

**for** size **in** array\_sizes:

arr = [random.randint(0, 1000) **for** \_ **in** range(size)] start\_time = time.time()

sorted\_arr = selection\_sort(arr) end\_time = time.time()

execution\_time = end\_time - start\_time execution\_times\_selection\_sort.append(execution\_time)

Block for Graph Visualization Using MatPlotLib

[27]:

plt.figure(figsize=(8, 4))

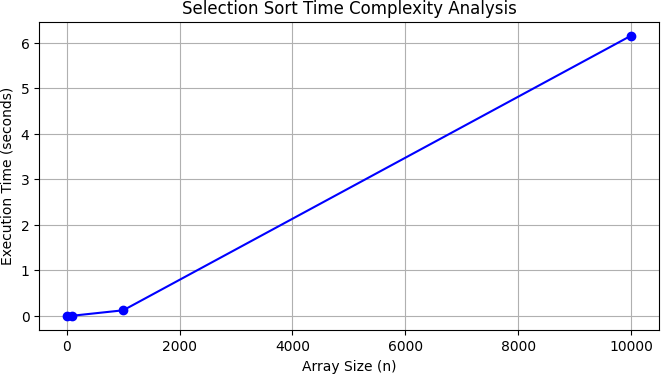
plt.plot(array\_sizes, execution\_times\_selection\_sort, marker='o', color='b',␣

↪linestyle='-')

plt.title('Selection Sort Time Complexity Analysis') plt.xlabel('Array Size (n)')

plt.ylabel('Execution Time (seconds)') plt.grid(**True**)

plt.show()



Explanation:

The sudden increase in execution time for selection sort can be attributed to the O(n^2) time complexity of the algorithm. This means that as the size of the input (n) increases, the execution time grows quadratically.

Time Complexity:

The time complexity of selection sort is O(n^2), which means that the time it takes to sort an array grows quadratically with the size of the array. This is because selection sort iterates through the array multiple times, searching for the minimum element in each pass. In the worst case scenario, selection sort will need to make n-1 passes through the array, and in each pass, it will need to compare n-i elements, where i is the current pass number. This results in a total of O(n^2) comparisons..

Space Complexity:

The space complexity of bubble sort is O(1), which means that it only uses a constant amount of additional memory regardless of the size of the input. This is because bubble sort only uses a few temporary variables to store the elements being compared and swapped.

Implementation of Insertion Sort

[32]:

**def** insertion\_sort(arr): n = len(arr)

**for** i **in** range(1, n): *# Outer loop for number of passes*

key = arr[i] *# First element is alwas sorted*

j = i-1

**while**(j>=0 **and** arr[j] > key): *# Inner loop checks the sorted list*␣

↪*with curr element of unsorted*

arr[j+1] = arr[j]

j = j-1 *# Moves backward in the sorted array*

arr[j+1] = key *# Places the unsorted elem at right place in sorted*␣

↪*array*

print(f"Sorted Array for Size **{**n**}**: ", arr)

**return** arr

[33]:

Brief Overview of the Code:

Get the length of the array arr and store it in the variable n. The outer loop (for i in range(1, n)) iterates through each element of the array starting from the second element (index 1), representing the number of passes. Within each pass, key is set to the current element arr[i], representing the element to be inserted into the sorted portion of the array. The inner loop (while loop) iterates backward from the current element arr[i] towards the beginning of the array, comparing key with each element in the sorted portion of the array. If an element arr[j] is greater than key, it shifts arr[j] one position to the right to make space for key. The process continues until either key is greater than the element being compared or until the beginning of the array is reached. After finding the correct position for key, it is placed in the sorted portion of the array. The process continues until the entire array is sorted.

Block for random value arrays of different size

execution\_times\_insertion\_sort = []

**for** size **in** array\_sizes:

arr = [random.randint(0, 1000) **for** \_ **in** range(size)] start\_time = time.time()

sorted\_arr = insertion\_sort(arr) end\_time = time.time()

execution\_time = end\_time - start\_time execution\_times\_insertion\_sort.append(execution\_time)

Sorted Array for Size 10: [33, 46, 238, 255, 364, 385, 842, 861, 892, 962]

Sorted Array for Size 100: [7, 11, 23, 25, 46, 57, 71, 115, 120, 129, 140, 145,

153, 154, 167, 181, 198, 219, 226, 229, 238, 240, 241, 241, 242, 250, 252, 299,

308, 316, 323, 325, 326, 328, 334, 362, 363, 370, 371, 376, 376, 400, 407, 449,

453, 467, 474, 497, 504, 507, 514, 526, 542, 545, 546, 551, 552, 554, 578, 581,

588, 599, 604, 625, 641, 644, 645, 653, 654, 667, 671, 682, 702, 703, 709, 744,

748, 758, 774, 775, 790, 800, 802, 807, 819, 834, 844, 853, 892, 904, 912, 913,

915, 927, 944, 952, 977, 981, 998, 998]

Sorted Array for Size 1000: [0, 2, 2, 3, 3, 4, 5, 5, 6, 6, 7, 10, 11, 14, 14,

18, 18, 18, 19, 19, 19, 20, 20, 22, 22, 23, 23, 25, 25, 25, 26, 26, 27, 27, 28,

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[34]:

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Block for Graph Visualization Using MatPlotLib

plt.figure(figsize=(8, 4))

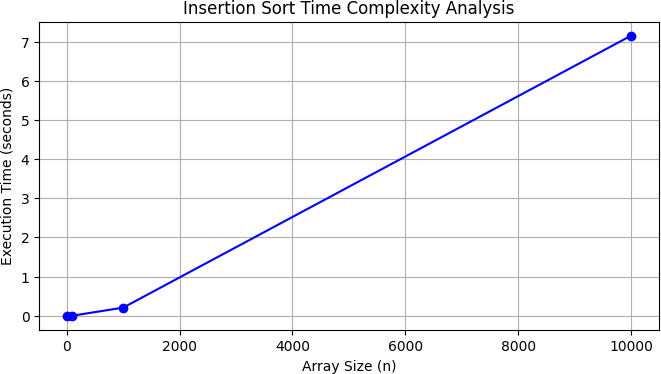
plt.plot(array\_sizes, execution\_times\_insertion\_sort, marker='o', color='b',␣

↪linestyle='-')

plt.title('Insertion Sort Time Complexity Analysis') plt.xlabel('Array Size (n)')

plt.ylabel('Execution Time (seconds)') plt.grid(**True**)

plt.show()



Explanation:

This sharp rise is a direct consequence of insertion sort’s time complexity, which is O(n^2) in the worst case. The sudden increase in execution time occurs because the number of comparisons and shifts performed by insertion sort grows quadratically with the input size. Let’s break down the factors contributing to this

Time Complexity:

Worst Case: This occurs when the input array is sorted in descending order. In each iteration, insertion sort needs to compare the current element with all previously sorted elements to find its correct position. This requires n-1 comparisons in the first pass, n-2 comparisons in the second pass, and so on, leading to a total of n(n-1)/2 comparisons, which is roughly equal to O(n^2).

Average Ca) ): Although insertion sort might perform better than the worst case for random inputs, the average number of comparisons and shifts still grows quadratically with the input size, making

[46]:

it O(n^2) on averag

.

Best ( n)): This scenario occurs when the input array is already sorted in ascending order. In this case, insertion sort only needs to perform n-1 comparisons (one for each element except the last) to confirm that the array is sorted, resulting in a linear time complexity of O(n).</ </ <

Space Complexity:

Insertion sort is considered space-efficient because it only uses a constant amount of additional space regardless of the input size. This space is typically used for temporary variables like the element being inserted and its current position during comparisons.<

Implementation of Merge Sort

**def** merge\_sort(arr): n = len(arr)

**if** len(arr) > 1:

mid = len(arr) // 2 *# Floor Division for finding mid index* left\_arr = arr[:mid] *# Slicing array from start to mid* right\_arr = arr[mid:] *# Slicing array from mid to end*

merge\_sort(left\_arr) *# recursive call on left array*

merge\_sort(right\_arr) *# recursive call on right array*

i = j = k = 0

**while** i < len(left\_arr) **and** j < len(right\_arr): *# This block compares*␣

↪*elemements of two subarrays and merge them into main array*

**if** left\_arr[i] < right\_arr[j]: arr[k] = left\_arr[i]

i += 1

**else**:

arr[k] = right\_arr[j] j += 1

k += 1

**while** i < len(left\_arr): *# This block copies remaining elements of*␣

↪*left subarray*

arr[k] = left\_arr[i] i += 1

k += 1

**while** j < len(right\_arr): *# This block copies remaining elements of*␣

↪*right subarray*

arr[k] = right\_arr[j] j += 1

k += 1

**return** arr

**def** printList(arr): *# Function for printing arrays with size*

n = len(arr)

print(f"Sorted Array for Size **{**n**}**: ", arr)

[28]:

Brief Overview of the Code:

The merge\_sort function takes an array arr as input. It first checks if the length of the array is greater than 1, indicating that it needs to be sorted. If the array has more than one element, it calculates the midpoint of the array and divides the array into two halves: left\_arr and right\_arr. It then recursively calls itself on the left and right halves of the array. After the recursive calls return, it merges the sorted left and right halves into a single sorted array. This merging process is done by comparing elements from the left and right halves and placing them in the correct order in the original array. Finally, it returns the sorted array. Printing Function (printList):

The printList function is used to print the sorted array along with its size. It takes an array arr as input and prints the size of the array along with its elements.

Block for random value arrays of different size

execution\_times\_merge\_sort = []

**for** size **in** array\_sizes:

arr = [random.randint(0, 1000) **for** \_ **in** range(size)] start\_time = time.time()

sorted\_arr = merge\_sort(arr) printList(sorted\_arr) end\_time = time.time()

execution\_time = end\_time - start\_time execution\_times\_merge\_sort.append(execution\_time)

Sorted Array for Size 10: [61, 156, 349, 368, 477, 510, 517, 574, 876, 893]

Sorted Array for Size 100: [10, 21, 27, 35, 40, 53, 54, 55, 57, 60, 60, 77, 95,

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| 113, 125, 128, 128, 134, 138, 143, | | | 159, 167, 179, 180, 181, | | | 202, 216, 217, | 220, |
| 233, 254, 262, 290, 292, 295, 300, | | | 300, 311, 315, 317, 322, | | | 329, 333, 339, | 361, |
| 365, 367, 384, 393, 402, 430, 463, | | | 504, 507, 510, 511, 515, | | | 529, 547, 554, | 574, |
| 591, 603, 632, 648, 657, 665, 666, | | | 669, 672, 702, 702, 705, | | | 709, 736, 739, | 744, |
| 761, 763, 767, 771, 783, 796, 810, | | | 814, 836, 847, 869, 871, | | | 901, 904, 915, | 919, |
| 928, 929, 932, 941, 948, 950, 971] | | |  | | |  |  |
| Sorted Array for Size 1000: [0, 0, 1, 1, 2, 5, 6, 6, 7, 8, | | | | | | 9, 10, 11, 12, | 12, |
| 13, 15, 16, 16, 17, | | 18, 18, 19, 19, 20, | | 21, 24, 25, 26, 27, | | 29, 29, 30, 31, 32, | |
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| 4, 5, 5, 5, 5, 5, 5, 5, 5, 5, | 5, 5, 5, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, |
| 7, 7, 7, 7, 7, 7, 7, 8, 8, 8, 8, 8, 8, 8, 8, 8, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9,  9, 10, 10, 10, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 12, 12, 12, 12, 12, 12, | |

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[41]:

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Block for Graph Visualization Using MatPlotLib

plt.figure(figsize=(8, 4))

plt.plot(array\_sizes, execution\_times\_merge\_sort, marker='o', color='b',␣

↪linestyle='-')

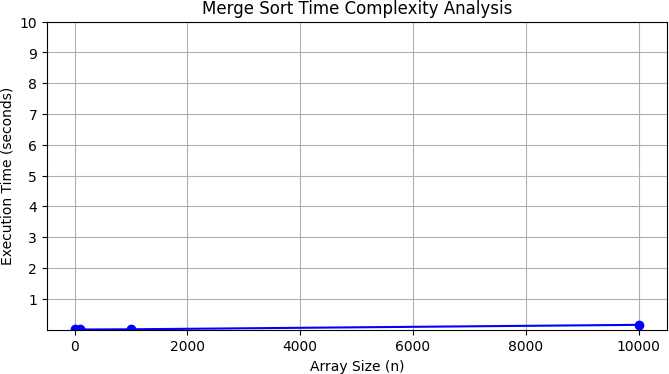
plt.title('Merge Sort Time Complexity Analysis') plt.xlabel('Array Size (n)')

plt.ylabel('Execution Time (seconds)') plt.grid(**True**)

*# Set y-axis ticks to range from 1 to 10*

plt.yticks(range(1, 11))

plt.show()



Explanation:

Merge Sort is a efficient sorting algorithm that is often used for large arrays. However, it does require extra space to store the sublists during the sorting process. Merge Sort is also a stable sorting algorithm, which means that it does not change the relative order of equal elements.

Time Complexity:

Merge Sort follows a divide-and-conquer approach. It breaks down the original list into halves until single-element sublists (already sorted) are obtained. The divide operation takes constant time. Sorting the sublists (recursive calls) contributes a factor of n log n (logarithmic due to halving the size in each step). Merging the sorted sublists back together takes linear time (proportional to the size of the sublists being merged). Since the merging process dominates the overall complexity, and it happens n times (once for each level of the recursion tree), the total time complexity becomes O(n log n).

Space Complexity:

The additional space is needed for temporary sublists created during the divide and merge steps. In the worst case, the depth of the recursion tree can be logarithmic in the size of the input (log n). However, at each level, the space used to store sublists scales linearly with the original list size

[48]:

(n). Therefore, the overall space complexity is O(n).ped.

to a total of n(n-1)/2 comparisons, which is roughly equal to O(n^2).

Average Ca) ): Although insertion sort might perform better than the worst case for random inputs, the average number of comparisons and shifts still grows quadratically with the input size, making it O(n^2) on averag

.

Best ( n)): This scenario occurs when the input array is already sorted in ascending order. In this case, insertion sort only needs to perform n-1 comparisons (one for each element except the last) to confirm that the array is sorted, resulting in a linear time complexity of O(n).</ </ <

Space Complexity:

Insertion sort is considered space-efficient because it only uses a constant amount of additional space regardless of the input size. This space is typically used for temporary variables like the element being inserted and its current position during comparisons.<

Implementation of Quick Sort

**def** partition(arr, start, end): pivot = arr[end]

p\_index = start

**for** i **in** range(start, end):

**if** arr[i] <= pivot:

arr[i], arr[p\_index] = arr[p\_index], arr[i] p\_index += 1

arr[p\_index], arr[end] = arr[end], arr[p\_index]

**return** p\_index

**def** quick\_sort(arr, start, end):

**if** start < end:

p\_index = partition(arr, start, end) quick\_sort(arr, start, p\_index - 1) quick\_sort(arr, p\_index + 1, end)

**return** arr

Brief Overview of the Code:

Partition Function (partition):

This function takes an array arr, a start index, and an end index as input. It selects the last element of the array (arr[end]) as the pivot. It iterates through the array from start to end - 1, and for each element that is less than or equal to the pivot, it swaps that element with the element at the p\_index. After the iteration, it swaps the pivot with the element at the p\_index, ensuring that all elements less than the pivot are on the left, and all elements greater than the pivot are on the right. Finally, it returns the p\_index, which represents the position of the pivot after partition i

ng. Quick Sort Function (quick\_s

[49]:

rt):

This function takes an array arr, a start index, and an end index as input. It first checks if start is less than end, indicating that there is more than one element in the subarray to be sorted. If so, it calls the partition function to partition the array around a pivot element. It then recursively calls itself on the left and right partitions, i.e., the elements to the left and right of the pivot respectively, until the entire array is sorted.

Block for random value arrays of different size

execution\_times\_quick\_sort = []

**for** size **in** array\_sizes:

arr = [random.randint(0, 1000) **for** \_ **in** range(size)] start\_time = time.time()

sorted\_arr = quick\_sort(arr, 0, len(arr) - 1) printList(sorted\_arr)

end\_time = time.time()

execution\_time = end\_time - start\_time execution\_times\_quick\_sort.append(execution\_time)

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| --- | --- | --- | --- |
| Sorted Array for Size 10: [50, 59, 84, 287, 354, 598, 660, 798, 803, 978]  Sorted Array for Size 100: [19, 21, 22, 41, 52, 69, 70, 96, 98, 115, 116, | | | 117, |
| 117, 160, 160, 180, 191, 198, 200, | 230, 251, 270, 290, | 296, 304, 312, 326, | 334, |
| 353, 371, 381, 402, 421, 428, 428, | 446, 451, 461, 494, | 503, 508, 510, 515, | 523, |
| 524, 543, 546, 553, 554, 574, 577, | 601, 626, 630, 641, | 643, 662, 664, 673, | 676, |
| 686, 698, 700, 703, 710, 711, 716, | 723, 725, 729, 749, | 761, 771, 773, 782, | 791, |
| 810, 810, 818, 818, 818, 831, 848, | 850, 851, 864, 878, | 892, 912, 917, 917, | 917, |
| 925, 931, 976, 980, 990, 991, 994, | 998] |  |  |
| Sorted Array for Size 1000: [0, 2, 8, 11, 13, 14, 14, | | 15, 15, 15, 18, 19, | 19, |

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| 20, 20, 22, 23, 23, | | | | 24, 25, 28, 29, 31, | | | | 31, 32, 35, 35, 35, | | | | 36, 36, 36, 36, 36, | | | |
| 40, 40, 40, 42, 42, | | | | 43, 43, 45, 45, 47, | | | | 48, 52, 52, 53, 55, | | | | 55, 59, 60, 63, 63, | | | |
| 65, 65, 66, 67, 69, | | | | 70, 70, 73, 75, 75, | | | | 77, 80, 81, 84, 85, | | | | 86, 86, 88, 88, 89, | | | |
| 89, 90, 91, 93, 93, | | | | 95, 97, 99, 99, 99, | | | | 102, | 102, | 103, | 104, | 107, | 107, | 107, | 108, |
| 109, | 110, | 111, | 112, | 112, | 113, | 114, | 116, | 118, | 119, | 119, | 120, | 122, | 122, | 122, | 123, |
| 123, | 124, | 124, | 125, | 126, | 126, | 126, | 129, | 129, | 129, | 130, | 130, | 132, | 132, | 133, | 134, |
| 135, | 140, | 141, | 142, | 146, | 147, | 147, | 149, | 150, | 150, | 150, | 152, | 153, | 153, | 157, | 158, |
| 158, | 160, | 163, | 164, | 166, | 168, | 168, | 168, | 168, | 169, | 169, | 177, | 177, | 180, | 181, | 183, |
| 184, | 184, | 185, | 186, | 186, | 186, | 187, | 187, | 187, | 188, | 189, | 190, | 191, | 191, | 191, | 192, |
| 193, | 194, | 196, | 197, | 197, | 200, | 200, | 201, | 203, | 203, | 204, | 205, | 206, | 207, | 207, | 209, |
| 209, | 211, | 211, | 213, | 213, | 213, | 214, | 216, | 217, | 220, | 221, | 222, | 222, | 225, | 225, | 225, |
| 225, | 226, | 228, | 229, | 230, | 230, | 230, | 230, | 231, | 231, | 231, | 231, | 231, | 234, | 235, | 236, |
| 236, | 238, | 238, | 239, | 239, | 239, | 241, | 241, | 242, | 243, | 244, | 245, | 245, | 246, | 246, | 248, |
| 249, | 249, | 250, | 252, | 252, | 252, | 255, | 255, | 257, | 258, | 258, | 258, | 260, | 260, | 261, | 263, |
| 264, | 264, | 265, | 268, | 270, | 270, | 271, | 271, | 271, | 271, | 276, | 276, | 276, | 278, | 279, | 280, |
| 283, | 284, | 287, | 287, | 287, | 287, | 288, | 289, | 289, | 290, | 292, | 295, | 295, | 296, | 296, | 296, |
| 297, | 298, | 298, | 299, | 300, | 301, | 301, | 302, | 303, | 307, | 308, | 309, | 309, | 310, | 311, | 315, |
| 315, | 316, | 317, | 317, | 318, | 319, | 319, | 320, | 320, | 323, | 325, | 328, | 328, | 329, | 330, | 334, |
| 335, | 335, | 336, | 339, | 339, | 340, | 342, | 344, | 346, | 347, | 348, | 351, | 351, | 351, | 354, | 354, |
| 354, | 356, | 356, | 356, | 357, | 358, | 359, | 360, | 361, | 361, | 361, | 363, | 364, | 365, | 365, | 366, |

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| 366, | 369, | 370, | 372, | 373, | 373, | 373, | 374, | 375, | 376, | 377, | 377, | 377, | 381, | 382, | 383, |
| 383, | 384, | 386, | 387, | 387, | 389, | 389, | 390, | 391, | 392, | 394, | 394, | 394, | 396, | 396, | 397, |
| 399, | 401, | 401, | 402, | 404, | 404, | 404, | 405, | 406, | 406, | 407, | 408, | 409, | 409, | 410, | 412, |
| 412, | 413, | 415, | 416, | 417, | 418, | 419, | 420, | 421, | 425, | 426, | 426, | 426, | 427, | 427, | 428, |
| 430, | 430, | 431, | 432, | 432, | 435, | 436, | 436, | 437, | 437, | 437, | 437, | 437, | 438, | 438, | 438, |
| 439, | 440, | 440, | 440, | 441, | 443, | 443, | 443, | 443, | 443, | 443, | 444, | 448, | 450, | 451, | 452, |
| 453, | 453, | 454, | 455, | 456, | 456, | 457, | 457, | 457, | 461, | 462, | 464, | 465, | 465, | 465, | 465, |
| 465, | 468, | 469, | 469, | 470, | 471, | 471, | 480, | 480, | 482, | 482, | 483, | 484, | 485, | 485, | 487, |
| 490, | 490, | 493, | 494, | 495, | 495, | 496, | 497, | 498, | 500, | 501, | 505, | 506, | 506, | 507, | 508, |
| 508, | 509, | 509, | 510, | 510, | 511, | 512, | 513, | 516, | 518, | 520, | 521, | 522, | 522, | 523, | 524, |
| 527, | 528, | 528, | 528, | 529, | 530, | 531, | 531, | 532, | 533, | 534, | 534, | 535, | 535, | 537, | 537, |
| 537, | 538, | 539, | 540, | 540, | 540, | 541, | 543, | 543, | 544, | 544, | 544, | 545, | 545, | 546, | 547, |
| 547, | 547, | 548, | 549, | 549, | 552, | 554, | 557, | 558, | 559, | 562, | 564, | 564, | 564, | 567, | 570, |
| 570, | 571, | 572, | 573, | 574, | 576, | 578, | 578, | 579, | 580, | 582, | 582, | 584, | 585, | 587, | 587, |
| 587, | 587, | 589, | 591, | 591, | 591, | 591, | 593, | 593, | 593, | 594, | 594, | 595, | 596, | 597, | 598, |
| 599, | 601, | 602, | 603, | 603, | 603, | 604, | 606, | 607, | 608, | 608, | 608, | 609, | 612, | 616, | 618, |
| 619, | 619, | 619, | 620, | 620, | 620, | 621, | 621, | 622, | 623, | 623, | 624, | 624, | 625, | 626, | 627, |
| 628, | 628, | 629, | 630, | 630, | 631, | 631, | 632, | 633, | 634, | 634, | 634, | 635, | 637, | 638, | 638, |
| 639, | 641, | 642, | 648, | 649, | 650, | 652, | 655, | 656, | 657, | 657, | 658, | 660, | 662, | 662, | 662, |
| 662, | 665, | 666, | 666, | 668, | 670, | 671, | 671, | 672, | 673, | 674, | 676, | 676, | 677, | 682, | 683, |
| 685, | 685, | 687, | 687, | 689, | 693, | 694, | 695, | 698, | 699, | 699, | 700, | 702, | 703, | 704, | 704, |
| 705, | 705, | 705, | 705, | 708, | 708, | 708, | 712, | 713, | 714, | 714, | 714, | 715, | 716, | 720, | 722, |
| 723, | 724, | 725, | 725, | 725, | 725, | 726, | 726, | 727, | 728, | 731, | 732, | 734, | 734, | 735, | 737, |
| 737, | 738, | 744, | 745, | 747, | 747, | 747, | 748, | 750, | 750, | 750, | 752, | 752, | 754, | 754, | 756, |
| 756, | 760, | 761, | 762, | 764, | 765, | 765, | 765, | 766, | 767, | 767, | 768, | 770, | 771, | 771, | 772, |
| 774, | 775, | 776, | 777, | 778, | 778, | 778, | 779, | 780, | 780, | 782, | 782, | 783, | 783, | 783, | 784, |
| 784, | 784, | 786, | 787, | 788, | 789, | 791, | 793, | 793, | 795, | 796, | 796, | 797, | 799, | 799, | 799, |
| 800, | 801, | 803, | 803, | 803, | 803, | 804, | 805, | 808, | 808, | 809, | 809, | 810, | 811, | 811, | 813, |
| 813, | 813, | 814, | 815, | 815, | 816, | 817, | 817, | 818, | 818, | 819, | 819, | 820, | 821, | 822, | 822, |
| 824, | 825, | 825, | 825, | 826, | 826, | 827, | 828, | 830, | 830, | 831, | 833, | 835, | 836, | 836, | 836, |
| 838, | 838, | 840, | 840, | 840, | 841, | 842, | 843, | 843, | 843, | 844, | 845, | 845, | 846, | 846, | 846, |
| 846, | 847, | 848, | 849, | 849, | 850, | 851, | 852, | 853, | 854, | 855, | 856, | 857, | 858, | 858, | 858, |
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| 876, | 876, | 876, | 878, | 879, | 880, | 881, | 881, | 884, | 885, | 885, | 885, | 887, | 888, | 888, | 889, |
| 891, | 892, | 893, | 897, | 898, | 898, | 899, | 900, | 901, | 901, | 903, | 905, | 907, | 907, | 911, | 911, |
| 912, | 913, | 914, | 915, | 916, | 916, | 917, | 918, | 918, | 919, | 920, | 920, | 921, | 922, | 922, | 923, |
| 923, | 924, | 925, | 926, | 931, | 931, | 932, | 933, | 935, | 936, | 936, | 937, | 939, | 940, | 943, | 944, |
| 944, | 945, | 945, | 947, | 948, | 948, | 949, | 949, | 950, | 950, | 951, | 952, | 952, | 953, | 954, | 956, |
| 957, | 958, | 959, | 961, | 964, | 964, | 965, | 966, | 967, | 967, | 967, | 968, | 971, | 971, | 971, | 972, |
| 974, | 977, | 977, | 978, | 978, | 979, | 979, | 981, | 981, | 987, | 988, | 989, | 990, | 990, | 990, | 990, |
| 991, | 992, | 993, | 993, | 993, | 995, | 995, | 996, | 996, | 996, | 997, | 1000, 1000] | | | | |
| Sorted Array for Size 10000: | | | | | | [0, 0, 0, | | 0, 0, 0, 0, 0, | | | 0, 0, 0, 0, 1, | | | 1, 1, 1, | |
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| 3, 3, 4, 4, 4, 4, 4, 5, 5, 5, | | | | | | 5, 5, 5, 5, 5, | | | 5, 5, 5, 5, 6, | | | 6, 6, 6, 6, 6, | | | 6, 6, |

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| 6, 7, 7, 7, 7, 7, 8, 8, 8, 8, | 8, 8, 8, 8, 8, | 9, 9, 9, 9, 10, 10, 10, 10, 10, 10, | |
| 10, 10, 10, 10, 10, 10, 10, 10, 11, 11, 11, 11, 11, 11, 11, | | | 11, 11, 12, 12, 12, |
| 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 13, 13, 13, 13, 13, | | | 13, 13, 13, 13, 13, |
| 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 15, 15, | | | 15, 15, 15, 15, 15, |

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| 795, | 795, | 795, | 795, | 795, | 795, | 795, | 796, | 796, | 796, | 796, | 796, | 796, | 796, | 796, | 796, |
| 796, | 796, | 796, | 797, | 797, | 797, | 798, | 798, | 798, | 798, | 798, | 798, | 798, | 798, | 799, | 799, |
| 799, | 799, | 799, | 799, | 799, | 799, | 799, | 799, | 800, | 800, | 800, | 800, | 800, | 800, | 800, | 800, |
| 800, | 801, | 801, | 801, | 801, | 801, | 801, | 801, | 801, | 801, | 801, | 801, | 801, | 801, | 801, | 801, |
| 802, | 802, | 802, | 802, | 802, | 802, | 802, | 802, | 802, | 802, | 802, | 803, | 803, | 803, | 803, | 803, |
| 803, | 803, | 803, | 803, | 803, | 803, | 803, | 803, | 803, | 804, | 804, | 804, | 804, | 804, | 804, | 804, |
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| 819, | 819, | 819, | 819, | 820, | 820, | 820, | 820, | 820, | 820, | 820, | 820, | 820, | 821, | 821, | 821, |
| 821, | 821, | 821, | 821, | 822, | 822, | 822, | 822, | 822, | 822, | 822, | 822, | 822, | 822, | 822, | 822, |
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| 884, | 884, | 884, | 884, | 885, | 885, | 885, | 885, | 885, | 885, | 885, | 885, | 885, | 885, | 885, | 885, |
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[50]:

plt.figure(figsize=(8, 4))

plt.plot(array\_sizes, execution\_times\_quick\_sort, marker='o', color='b',␣

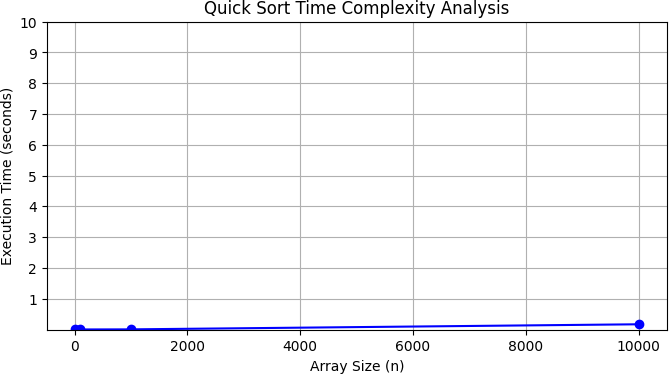
↪linestyle='-')

plt.title('Quick Sort Time Complexity Analysis') plt.xlabel('Array Size (n)') plt.ylabel('Execution Time (seconds)') plt.grid(**True**)

*# Set y-axis ticks to range from 1 to 10*

plt.yticks(range(1, 11))

plt.show()



[61]:

Explanation:

The graph illustrates that the execution time of Merge Sort increases logarithmically with the size of the array. In simpler terms, as the array size doubles, the execution time only increases by a constant factor.Iever, it’s important to note that this merging process is still linear in the size of the sub-lists being merged. This characteristic contributes to the overall time complexity of Merge Sort being O(n log n), where n represents the size of the input list.

Time Comp O(n log n)lexity:</

Applicable in all cases (best, average, and worst). Merge Sort efficiently divides the problem into subproblems and merges them back in a relatively quick linear time.isons..

</li>

<li>

<h4>Space Co O(n)mplexity:Merge Sort typically uses extra space to store sub-lists during

</li>

Implementation of Counting Sort

**def** counting\_sort(arr): size = len(arr) output = [0] \* size

*# Find the maximum element in the array*

max\_element = max(arr)

*# Initialize count array with size max\_element + 1*

count = [0] \* (max\_element + 1)

*# Store the count of each element in the count array*

**for** i **in** range(0, size): count[arr[i]] += 1

*# Store the cumulative count*

**for** i **in** range(1, max\_element + 1): count[i] += count[i - 1]

*# Find the index of each element of the original array in the count array # Place the elements in the output array*

i = size - 1

**while** i >= 0:

output[count[arr[i]] - 1] = arr[i] count[arr[i]] -= 1

i -= 1

*# Copy the sorted elements into the original array*

**for** i **in** range(0, size): arr[i] = output[i]

print(f"Sorted Array for Size **{**size**}**: ", arr)

**return** arr

Brief Overview of the Code:

The provided code implements the Counting Sort algorithm. It begins by initializing necessary variables, including the size of the input array and an output array initialized with zeros. Counting occurrences of each element in the input array is facilitated by a count array. After computing the cumulative count, the algorithm sorts the array by iterating through the input array in reverse order, placing elements into the output array according to their count. Finally, it copies the sorted elements from the output array back to the original array. The sorted array is then printed along with its size, and the function returns the sorted array. Counting Sort is efficient for sorting integer arrays, especially when the range of elements is not significantly larger than the number of elements in the array. It achieves a time complexity of O(n + k), where n represents the number of elements in the input array and k denotes the range of input values.The provided code implements the Counting Sort algorithm. It begins by initializing necessary variables, including the size of the input array and an output array initialized with zeros. Counting occurrences of each element in the input array is facilitated by a count array. After computing the cumulative count, the algorithm sorts the array by iterating through the input array in reverse order, placing elements into the output array according to their count. Finally, it copies the sorted elements from the output array back to the original array. The sorted array is then printed along with its size, and the function returns the sorted array. Counting Sort is efficient for sorting integer arrays, especially when the range of elements is not significantly larger than the number of elements in the array. It achieves a time complexity of O(n + k), where n represents the number of elements in the input array and k denotes the range of input values.

Block for random value arrays of different size

[62]:

execution\_times\_counting\_sort = []

**for** size **in** array\_sizes:

arr = [random.randint(0, 1000) **for** \_ **in** range(size)] start\_time = time.time()

counting\_sort(arr) end\_time = time.time()

execution\_time = end\_time - start\_time execution\_times\_counting\_sort.append(execution\_time)

Sorted Array for Size 10: [31, 231, 336, 607, 762, 813, 873, 895, 965, 996]

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| Sorted Array for Size 100: [0, 22, 24, 34, 38, 49, 56, 62, | | | | | | | | | | | | 92, 104, 119, 121, | | | |
| 125, 134, 140, 144, 150, 152, 155, | | | | | | | 165, 185, 188, 195, 204, | | | | | 212, 224, 224, | | | 265, |
| 266, 294, 306, 310, 313, 317, 320, | | | | | | | 322, 322, 343, 350, 359, | | | | | 363, 366, 369, | | | 372, |
| 386, 393, 408, 413, 436, 466, 484, | | | | | | | 495, 500, 509, 512, 518, | | | | | 532, 543, 545, | | | 557, |
| 567, 568, 587, 588, 598, 634, 659, | | | | | | | 674, 674, 697, 712, 713, | | | | | 765, 769, 771, | | | 774, |
| 778, 788, 790, 798, 806, 809, 814, | | | | | | | 843, 871, 885, 895, 903, | | | | | 908, 910, 921, | | | 925, |
| 928, 935, 937, 945, 958, 962, 965, | | | | | | | 983] | | | | |  | | |  |
| Sorted Array for Size 1000: [0, 1, 2, 2, 4, 5, 5, 6, 7, 8, | | | | | | | | | | | | 8, 8, 9, 9, 9, | | | 9, 9, |
| 10, 11, 11, 13, 14, | | | | 15, 16, 16, 16, 16, | | | | 16, 19, 19, 20, 20, | | | | 20, 21, 21, 22, 23, | | | |
| 23, 25, 28, 28, 31, | | | | 31, 31, 32, 33, 33, | | | | 35, 35, 37, 39, 40, | | | | 41, 41, 44, 45, 46, | | | |
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| 66, 67, 67, 71, 77, | | | | 77, 80, 80, 80, 81, | | | | 83, 83, 83, 83, 85, | | | | 85, 87, 87, 87, 88, | | | |
| 89, 90, 90, 91, 91, | | | | 92, 93, 95, 96, 97, | | | | 97, 98, 99, 101, 101, 101, 101, 103, | | | | | | | |
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| 597, | 598, | | 600, | | 601, | 602, | | 603, | | 603, | | 603, | 603, | 604, | | 605, | | 607, | 607, | 608, | | 610, | | 611, |
| 611, | 612, | | 612, | | 614, | 614, | | 615, | | 616, | | 616, | 616, | 620, | | 622, | | 623, | 624, | 625, | | 626, | | 627, |
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| 648, | 649, | | 649, | | 650, | 652, | | 653, | | 653, | | 654, | 654, | 655, | | 655, | | 658, | 658, | 659, | | 659, | | 663, |
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| 676, | 678, | | 681, | | 681, | 681, | | 682, | | 682, | | 683, | 683, | 683, | | 685, | | 687, | 688, | 688, | | 689, | | 689, |
| 689, | 692, | | 694, | | 695, | 695, | | 695, | | 696, | | 697, | 697, | 698, | | 700, | | 701, | 701, | 702, | | 703, | | 705, |
| 706, | 708, | | 709, | | 710, | 711, | | 711, | | 714, | | 714, | 716, | 717, | | 718, | | 719, | 720, | 721, | | 724, | | 724, |
| 726, | 726, | | 727, | | 728, | 729, | | 731, | | 732, | | 733, | 737, | 737, | | 738, | | 740, | 742, | 743, | | 743, | | 744, |
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| 778, | 778, | | 779, | | 784, | 784, | | 784, | | 787, | | 788, | 788, | 788, | | 793, | | 793, | 793, | 794, | | 794, | | 795, |
| 795, | 797, | | 799, | | 800, | 801, | | 802, | | 803, | | 804, | 804, | 806, | | 806, | | 808, | 808, | 811, | | 811, | | 811, |
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| 837, | 837, | | 841, | | 841, | 841, | | 842, | | 842, | | 842, | 842, | 844, | | 847, | | 847, | 847, | 849, | | 849, | | 850, |
| 852, | 852, | | 852, | | 858, | 858, | | 859, | | 859, | | 860, | 861, | 864, | | 866, | | 867, | 869, | 873, | | 873, | | 873, |
| 873, | 873, | | 873, | | 875, | 875, | | 876, | | 877, | | 877, | 878, | 880, | | 880, | | 881, | 883, | 884, | | 886, | | 888, |
| 890, | 890, | | 890, | | 892, | 892, | | 894, | | 894, | | 894, | 894, | 894, | | 894, | | 895, | 895, | 896, | | 899, | | 900, |
| 901, | 902, | | 903, | | 904, | 904, | | 904, | | 904, | | 904, | 905, | 906, | | 906, | | 909, | 909, | 910, | | 910, | | 912, |
| 913, | 914, | | 914, | | 915, | 915, | | 921, | | 922, | | 922, | 923, | 923, | | 924, | | 925, | 925, | 925, | | 925, | | 926, |
| 927, | 929, | | 931, | | 932, | 933, | | 934, | | 934, | | 935, | 936, | 937, | | 937, | | 940, | 941, | 942, | | 942, | | 943, |
| 943, | 944, | | 944, | | 944, | 945, | | 945, | | 949, | | 949, | 949, | 949, | | 950, | | 950, | 952, | 953, | | 954, | | 954, |
| 954, | 956, | | 957, | | 957, | 959, | | 961, | | 961, | | 962, | 963, | 963, | | 963, | | 965, | 965, | 965, | | 966, | | 966, |
| 966, | 968, | | 972, | | 973, | 974, | | 974, | | 975, | | 976, | 977, | 978, | | 980, | | 981, | 981, | 982, | | 982, | | 982, |
| 985, | 985, | | 986, | | 987, | 987, | | 987, | | 988, | | 991, | 991, | 991, | | 991, | | 993, | 994, | 994, | | 996, | | 997, |
| 998, | 999, | | 999, | | 999, | 1000] | |  | |  | |  |  |  | |  | |  |  |  | |  | |  |
| Sorted Array for Size 10000: | | | | | | | | | | [0, 0, 0, | | | 0, 0, 0, 0, 0, | | | | | 1, 1, 1, | | 1, 1, | | 1, 1, 1, | | |
| 1, 1, 2, 2, | | | | 2, | 2, 2, 2, | | 2, 2, | | | 2, 2, | | 3, 3, 3, | | 3, | 3, 3, 3, 3, | | | | 3, 3, 3, 3, | | | | 3, | 4, 4, |
| 4, 4, 4, 4, | | | | 4, | 4, 4, 4, | | 4, 4, | | | 4, 5, | | 5, 5, 5, | | 5, | 5, 5, 5, 5, | | | | 5, 5, 6, 6, | | | | 6, | 6, 6, |
| 6, 6, 6, 6, | | | | 6, | 6, 7, 7, | | 7, 7, | | | 7, 7, | | 7, 7, 7, | | 7, | 7, 7, 7, 7, | | | | 8, 8, 8, 8, | | | | 8, | 8, 8, |
| 8, 8, 8, 8, | | | | 8, | 8, 8, 9, | | 9, 9, | | | 9, 9, | | 9, 9, 9, | | 9, | 9, 9, 9, 10, 10, 10, 10, 10, 10, | | | | | | | | | |
| 10, | 11, | 11, | | 11, | 11, | 11, | 11, | | 11, | | 12, | 12, | 12, | 12, | 12, | | 12, | 12, | 12, | 12, | 12, | | 13, | 13, |
| 13, | 13, | 13, | | 13, | 13, | 13, | 13, | | 14, | | 14, | 14, | 14, | 14, | 14, | | 14, | 14, | 14, | 14, | 14, | | 14, | 14, |
| 14, | 15, | 15, | | 15, | 15, | 15, | 15, | | 15, | | 15, | 15, | 15, | 16, | 16, | | 16, | 16, | 16, | 16, | 16, | | 17, | 17, |
| 17, | 17, | 17, | | 17, | 17, | 17, | 17, | | 17, | | 17, | 17, | 18, | 18, | 18, | | 18, | 18, | 18, | 18, | 18, | | 18, | 18, |
| 18, | 18, | 19, | | 19, | 19, | 19, | 19, | | 19, | | 19, | 19, | 19, | 19, | 19, | | 19, | 19, | 19, | 20, | 20, | | 20, | 20, |
| 20, | 21, | 21, | | 21, | 21, | 21, | 21, | | 21, | | 22, | 22, | 22, | 22, | 22, | | 22, | 22, | 22, | 23, | 23, | | 23, | 23, |
| 23, | 23, | 23, | | 23, | 24, | 24, | 24, | | 24, | | 24, | 24, | 24, | 24, | 25, | | 25, | 25, | 25, | 25, | 25, | | 25, | 25, |
| 25, | 26, | 26, | | 26, | 26, | 26, | 26, | | 26, | | 26, | 26, | 26, | 26, | 26, | | 26, | 27, | 27, | 27, | 27, | | 27, | 27, |
| 27, | 27, | 28, | | 28, | 28, | 28, | 28, | | 28, | | 29, | 29, | 29, | 29, | 29, | | 29, | 29, | 29, | 29, | 29, | | 29, | 29, |
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| 31, | 32, | 32, | | 32, | 32, | 32, | 32, | | 32, | | 32, | 32, | 32, | 32, | 32, | | 32, | 33, | 33, | 33, | 33, | | 33, | 33, |

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| 37, | 37, | 38, | | 38, | 38, | 38, | 38, | 38, | | 38, | 38, | 38, | 38, | 39, | | 39, | 39, | 39, | 39, | 39, | | 39, | 39, |
| 39, | 39, | 39, | | 40, | 40, | 40, | 40, | 40, | | 40, | 40, | 40, | 40, | 40, | | 40, | 40, | 40, | 40, | 41, | | 41, | 41, |
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| 68, | 68, | 68, | | 68, | 69, | 69, | 69, | 69, | | 69, | 69, | 69, | 69, | 69, | | 69, | 70, | 70, | 70, | 70, | | 70, | 70, |
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| 74, | 74, | 74, | | 74, | 74, | 74, | 75, | 75, | | 75, | 75, | 75, | 75, | 75, | | 75, | 75, | 75, | 75, | 75, | | 75, | 75, |
| 75, | 75, | 75, | | 76, | 76, | 76, | 76, | 76, | | 76, | 76, | 76, | 76, | 76, | | 76, | 77, | 77, | 77, | 77, | | 77, | 78, |
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| 86, | 86, | 86, | | 86, | 86, | 86, | 86, | 86, | | 86, | 87, | 87, | 87, | 87, | | 87, | 87, | 87, | 87, | 88, | | 88, | 88, |
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| 95, | 95, | 95, | | 95, | 95, | 95, | 96, | 96, | | 96, | 96, | 96, | 96, | 96, | | 96, | 96, | 97, | 97, | 97, | | 97, | 97, |
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[64]:

plt.figure(figsize=(8, 4))

plt.plot(array\_sizes, execution\_times\_counting\_sort, marker='o', color='b',␣

↪linestyle='-')

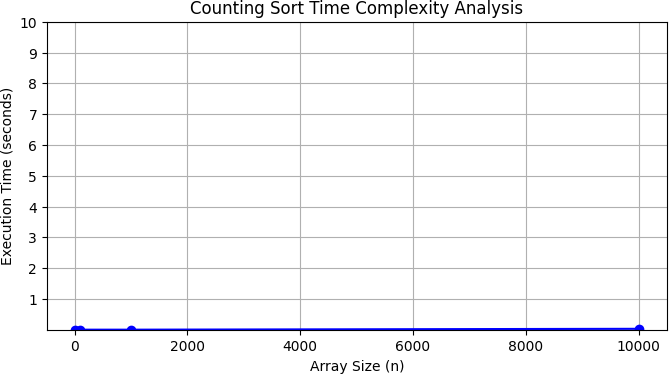
plt.title('Counting Sort Time Complexity Analysis') plt.xlabel('Array Size (n)')

plt.ylabel('Execution Time (seconds)') plt.grid(**True**)

*# Set y-axis ticks to range from 1 to 10*

plt.yticks(range(1, 11))

plt.show()



Explanation:

Counting Sort’s time complexity is dominated by two factors:

Counting the occurrences of each element in the input array (proportional to n and k). Populating the sorted output array based on the count array (proportional to n). Since both operations are linear in n and k, the overall time complexity remains O(n + k) in all cases (best, average, and worst). This is because Counting Sort doesn’t involve comparisons between elements, making it efficient for scenarios with a limited value range (k is small compared to n).

Time Complexity: O(n log n)

Average Case & Best Case: O(n + k) n represents the number of elements in the input array. k represents the range of possible values in the input array (i.e., the largest value - the smallest value

+ 1). Worst Case: O(n + k) (similar to average and best case)ime. i>

Space Complexity O(n + k)

Similar to time complexity, Counting Sort requires extra space for the count array, which scales with the range of values (k) and the number of elements (n) in the worst case.

n

Counting Sort’s complexity doesn’t directly translate to a single curve on a graph like Merge Sort. This is because the time complexity depends on both n (number of elements) and k (range of values). However, we can conceptually understand that the execution time will increase linearly with both n and k, but k is typically much smaller than n for Counting Sort to be efficient.ase.

Implementation of Radix Sort

[74]:

**def** counting\_sort(arr, place): *# Using counting sort to sort the elements in*␣

↪*the basis of significant places*

size = len(arr) max\_elem = max(arr) output = [0] \* size

count = [0] \* (max\_elem + 1)

**for** i **in** range(0, size): *# Calculate count of elements*

index = arr[i] // place count[index % 10] += 1

**for** i **in** range(1, 10): *# Calculate cumulative count*

count[i] += count[i - 1] i = size - 1

**while** i >= 0:

index = arr[i] // place output[count[index % 10] - 1] = arr[i] count[index % 10] -= 1

i -= 1

**for** i **in** range(0, size): arr[i] = output[i]

**def** radix\_sort(arr): *# Main function to implement radix sort*

max\_element = max(arr) n = len(arr)

place = 1 *# Apply counting sort to sort elements based on place value.*

**while** max\_element // place > 0: counting\_sort(arr, place) place \*= 10

print(f"Sorted Array for Size **{**n**}**: ", arr)

**return** arr

Brief Overview of the Code:

The radix\_sort function serves as the main function to implement Radix Sort. It initializes the maximum element in the array and sets the initial value of place to 1. It iterates through each significant place of the elements in the array, applying the counting\_sort function to sort the elements based on each place. After sorting all significant places, it prints the sorted array along with its size and returns the sorted array. Radix Sort achieves its sorting by sorting digits from the least significant digit (rightmost) to the most significant digit (leftmost) using a stable sorting algorithm such as Counting Sort. It is particularly efficient for sorting integers, especially when the range of integers is known and not very large. Radix Sort has a time complexity of O(d \* (n + k)), where d is the number of digits in the maximum element, n is the number of elements in the array, and k is the range of input.

Block for random value arrays of different size

[76]:

execution\_times\_radix\_sort = []

**for** size **in** array\_sizes:

arr = [random.randint(0, 1000) **for** \_ **in** range(size)] start\_time = time.time()

radix\_sort(arr) end\_time = time.time()

execution\_time = end\_time - start\_time execution\_times\_radix\_sort.append(execution\_time)

Sorted Array for Size 10: [311, 350, 479, 497, 506, 632, 741, 861, 900, 908]

Sorted Array for Size 100: [6, 8, 13, 26, 36, 47, 56, 61, 75, 79, 87, 116, 135,

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| 298, 300, 303, 328, 352, 364, 376, | | | | | | | 403, 416, 417, 420, 449, | | | | | 458, 467, 477, 482, | | | |
| 485, 489, 492, 504, 506, 512, 514, | | | | | | | 532, 534, 534, 556, 557, | | | | | 564, 568, 572, 577, | | | |
| 579, 597, 599, 618, 621, 632, 644, | | | | | | | 650, 675, 681, 683, 683, | | | | | 707, 717, 722, 724, | | | |
| 736, 752, 766, 777, 791, 796, 801, | | | | | | | 818, 826, 843, 854, 857, | | | | | 891, 909, 913, 929, | | | |
| 933, 933, 938, 953, 977, 983, 986]  Sorted Array for Size 1000: [0, 0, 1, 2, 2, 5, 5, 5, 6, 8, | | | | | | | | | | | | 8, 9, 14, 15, 16, | | | |
| 17, 17, 18, 18, 19, | | | | 20, 20, 21, 22, 22, | | | | 22, 23, 25, 26, 27, | | | | 28, 33, 34, 35, 36, | | | |
| 39, 41, 43, 46, 49, | | | | 49, 50, 50, 52, 53, | | | | 53, 54, 54, 55, 55, | | | | 56, 56, 56, 57, 60, | | | |
| 60, 61, 61, 63, 63, | | | | 63, 64, 65, 65, 66, | | | | 68, 70, 71, 71, 73, | | | | 74, 74, 76, 78, 78, | | | |
| 79, 79, 84, 85, 86, | | | | 87, 87, 87, 88, 90, | | | | 91, 92, 92, 93, 95, | | | | 97, 98, 98, 98, 100, | | | |
| 102, | 102, | 105, | 105, | 106, | 108, | 109, | 110, | 110, | 111, | 111, | 111, | 113, | 114, | 115, | 115, |
| 116, | 118, | 120, | 120, | 121, | 121, | 123, | 123, | 124, | 126, | 127, | 127, | 127, | 127, | 129, | 131, |
| 133, | 133, | 134, | 135, | 136, | 138, | 139, | 140, | 140, | 142, | 143, | 145, | 148, | 150, | 151, | 151, |
| 151, | 151, | 157, | 157, | 160, | 160, | 162, | 163, | 163, | 163, | 166, | 167, | 167, | 167, | 167, | 168, |
| 169, | 170, | 171, | 171, | 176, | 176, | 179, | 179, | 180, | 181, | 183, | 185, | 187, | 187, | 188, | 190, |
| 190, | 191, | 192, | 192, | 194, | 194, | 194, | 196, | 197, | 197, | 200, | 201, | 202, | 203, | 204, | 205, |
| 205, | 206, | 206, | 206, | 208, | 211, | 211, | 212, | 212, | 213, | 214, | 214, | 215, | 216, | 216, | 217, |
| 217, | 217, | 218, | 219, | 220, | 220, | 220, | 220, | 221, | 222, | 223, | 224, | 225, | 225, | 226, | 227, |
| 227, | 228, | 230, | 232, | 232, | 233, | 233, | 234, | 235, | 236, | 237, | 239, | 239, | 240, | 242, | 242, |
| 243, | 243, | 245, | 246, | 246, | 247, | 247, | 248, | 248, | 249, | 249, | 250, | 251, | 254, | 256, | 256, |
| 256, | 257, | 257, | 258, | 258, | 259, | 260, | 260, | 260, | 261, | 261, | 266, | 267, | 269, | 270, | 270, |
| 273, | 274, | 274, | 274, | 274, | 275, | 275, | 277, | 278, | 278, | 281, | 284, | 285, | 288, | 289, | 289, |
| 291, | 291, | 292, | 292, | 292, | 294, | 296, | 297, | 297, | 299, | 299, | 301, | 304, | 305, | 307, | 307, |
| 308, | 309, | 309, | 310, | 311, | 312, | 312, | 313, | 313, | 316, | 320, | 322, | 323, | 323, | 325, | 325, |
| 332, | 333, | 333, | 333, | 335, | 337, | 338, | 338, | 339, | 341, | 342, | 342, | 345, | 345, | 345, | 347, |
| 348, | 348, | 349, | 350, | 351, | 355, | 356, | 356, | 358, | 358, | 361, | 361, | 363, | 365, | 366, | 368, |
| 368, | 369, | 369, | 370, | 371, | 371, | 371, | 372, | 373, | 373, | 376, | 379, | 379, | 380, | 382, | 382, |
| 384, | 384, | 384, | 385, | 386, | 386, | 387, | 390, | 392, | 392, | 392, | 396, | 396, | 398, | 398, | 400, |
| 402, | 402, | 402, | 402, | 403, | 404, | 405, | 406, | 406, | 407, | 407, | 407, | 410, | 410, | 412, | 413, |
| 414, | 415, | 418, | 418, | 419, | 422, | 423, | 424, | 424, | 425, | 425, | 426, | 426, | 426, | 427, | 428, |
| 428, | 429, | 431, | 433, | 433, | 437, | 437, | 439, | 440, | 441, | 444, | 444, | 445, | 445, | 448, | 448, |
| 449, | 449, | 449, | 449, | 450, | 450, | 453, | 454, | 458, | 459, | 460, | 460, | 460, | 461, | 461, | 462, |
| 464, | 464, | 466, | 468, | 469, | 469, | 469, | 470, | 471, | 471, | 472, | 473, | 474, | 475, | 476, | 476, |
| 478, | 478, | 479, | 482, | 482, | 484, | 484, | 484, | 486, | 487, | 488, | 490, | 491, | 491, | 491, | 492, |
| 492, | 493, | 494, | 497, | 498, | 499, | 499, | 503, | 504, | 505, | 505, | 506, | 507, | 509, | 511, | 512, |

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| 513, | 513, | | 514, | | 516, | 517, | 517, | | 518, | | 519, | 520, | 520, | | 523, | | 524, | 524, | 524, | | 525, | | 527, |
| 527, | 527, | | 527, | | 528, | 528, | 529, | | 530, | | 531, | 531, | 532, | | 534, | | 536, | 537, | 539, | | 540, | | 540, |
| 541, | 541, | | 542, | | 545, | 545, | 546, | | 547, | | 547, | 547, | 550, | | 550, | | 551, | 552, | 552, | | 553, | | 553, |
| 553, | 555, | | 556, | | 559, | 559, | 560, | | 560, | | 561, | 563, | 563, | | 565, | | 565, | 567, | 569, | | 569, | | 569, |
| 571, | 572, | | 573, | | 573, | 573, | 575, | | 575, | | 575, | 575, | 576, | | 577, | | 577, | 577, | 578, | | 579, | | 580, |
| 581, | 582, | | 582, | | 583, | 583, | 583, | | 586, | | 586, | 586, | 587, | | 587, | | 587, | 588, | 588, | | 589, | | 590, |
| 591, | 591, | | 592, | | 592, | 596, | 596, | | 599, | | 601, | 601, | 601, | | 604, | | 605, | 609, | 610, | | 611, | | 613, |
| 614, | 615, | | 615, | | 617, | 619, | 624, | | 625, | | 627, | 627, | 629, | | 629, | | 630, | 635, | 637, | | 637, | | 638, |
| 638, | 638, | | 639, | | 642, | 642, | 643, | | 643, | | 643, | 644, | 646, | | 648, | | 648, | 649, | 649, | | 650, | | 651, |
| 651, | 652, | | 652, | | 653, | 653, | 653, | | 654, | | 655, | 656, | 656, | | 658, | | 659, | 659, | 660, | | 661, | | 662, |
| 662, | 662, | | 664, | | 664, | 667, | 668, | | 669, | | 669, | 671, | 672, | | 672, | | 673, | 674, | 679, | | 680, | | 683, |
| 684, | 685, | | 685, | | 687, | 687, | 687, | | 688, | | 688, | 689, | 690, | | 691, | | 692, | 693, | 694, | | 694, | | 694, |
| 695, | 695, | | 696, | | 697, | 697, | 699, | | 700, | | 701, | 704, | 705, | | 706, | | 707, | 707, | 707, | | 710, | | 710, |
| 710, | 712, | | 713, | | 713, | 714, | 715, | | 716, | | 716, | 717, | 717, | | 718, | | 719, | 720, | 720, | | 721, | | 723, |
| 724, | 726, | | 726, | | 728, | 728, | 728, | | 729, | | 730, | 733, | 733, | | 735, | | 736, | 736, | 738, | | 739, | | 743, |
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| 764, | 764, | | 767, | | 769, | 769, | 771, | | 772, | | 772, | 773, | 774, | | 775, | | 778, | 783, | 784, | | 784, | | 784, |
| 784, | 785, | | 788, | | 789, | 789, | 789, | | 790, | | 791, | 791, | 792, | | 793, | | 796, | 797, | 797, | | 797, | | 799, |
| 800, | 800, | | 802, | | 802, | 802, | 802, | | 803, | | 803, | 803, | 805, | | 805, | | 805, | 806, | 806, | | 806, | | 807, |
| 807, | 808, | | 809, | | 812, | 812, | 812, | | 813, | | 816, | 817, | 819, | | 819, | | 819, | 821, | 825, | | 826, | | 826, |
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| 842, | 842, | | 845, | | 845, | 848, | 849, | | 850, | | 851, | 851, | 852, | | 852, | | 855, | 855, | 855, | | 857, | | 857, |
| 860, | 861, | | 861, | | 862, | 862, | 862, | | 863, | | 863, | 865, | 866, | | 866, | | 867, | 869, | 875, | | 875, | | 877, |
| 877, | 877, | | 877, | | 879, | 879, | 880, | | 882, | | 882, | 882, | 882, | | 883, | | 883, | 884, | 884, | | 884, | | 888, |
| 891, | 891, | | 891, | | 892, | 893, | 893, | | 893, | | 893, | 896, | 896, | | 897, | | 897, | 898, | 899, | | 901, | | 902, |
| 903, | 905, | | 906, | | 906, | 908, | 908, | | 909, | | 910, | 910, | 910, | | 911, | | 913, | 915, | 917, | | 918, | | 921, |
| 922, | 923, | | 923, | | 923, | 924, | 924, | | 926, | | 927, | 929, | 930, | | 931, | | 931, | 934, | 938, | | 939, | | 939, |
| 942, | 942, | | 943, | | 943, | 949, | 950, | | 950, | | 950, | 951, | 951, | | 953, | | 954, | 956, | 957, | | 960, | | 961, |
| 963, | 963, | | 966, | | 966, | 969, | 971, | | 971, | | 973, | 974, | 975, | | 976, | | 977, | 978, | 979, | | 979, | | 980, |
| 981, | 981, | | 985, | | 987, | 987, | 989, | | 989, | | 990, | 991, | 991, | | 992, | | 993, | 993, | 993, | | 993, | | 993, |
| 993, | 994, | | 995, | | 995, | 996, | 997, | | 997, | | 998, | 999] |  | |  | |  |  |  | |  | |  |
| Sorted Array for Size 10000: | | | | | | | | | [0, 0, 0, | | | 0, 0, 0, 0, 0, | | | | | 0, 1, 1, | | 1, 1, | | 1, 2, 2, | | |
| 2, 2, 2, 2, | | | | 2, | 2, 3, 3, | | 3, 3, | | 3, 3, | | 3, 3, 3, | | 4, | 4, 4, 4, 4, | | | | 4, 4, 4, 5, | | | | 5, | 5, 5, |
| 5, 5, 5, 5, | | | | 5, | 5, 5, 6, | | 6, 6, | | 6, 6, | | 6, 6, 6, | | 6, | 6, 6, 6, 7, | | | | 7, 7, 7, 7, | | | | 8, | 8, 8, |
| 8, 8, 8, 8, | | | | 8, | 8, 8, 9, | | 9, 9, | | 9, 9, | | 9, 9, 9, | | 9, | 9, 9, 9, 10, 10, 10, 10, 10, 10, | | | | | | | | | |
| 10, | 10, | 10, | | 10, | 10, | 11, | 11, | 11, | | 11, | 11, | 11, | 11, | 11, | | 11, | 11, | 11, | 12, | 12, | | 12, | 12, |
| 12, | 12, | 13, | | 13, | 13, | 13, | 13, | 13, | | 13, | 13, | 13, | 13, | 14, | | 14, | 14, | 14, | 14, | 14, | | 14, | 14, |
| 15, | 15, | 15, | | 15, | 15, | 15, | 15, | 16, | | 16, | 16, | 16, | 16, | 16, | | 16, | 16, | 16, | 16, | 16, | | 16, | 17, |
| 17, | 17, | 17, | | 17, | 17, | 17, | 17, | 18, | | 18, | 18, | 18, | 18, | 18, | | 18, | 18, | 18, | 18, | 18, | | 18, | 18, |
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| 22, | 22, | 22, | | 22, | 22, | 22, | 23, | 23, | | 23, | 23, | 23, | 23, | 23, | | 23, | 23, | 23, | 23, | 23, | | 23, | 23, |
| 24, | 24, | 24, | | 24, | 24, | 24, | 25, | 25, | | 25, | 25, | 25, | 26, | 26, | | 26, | 26, | 26, | 26, | 26, | | 26, | 26, |
| 26, | 26, | 27, | | 27, | 27, | 27, | 27, | 27, | | 27, | 27, | 27, | 27, | 27, | | 27, | 27, | 27, | 27, | 27, | | 27, | 27, |
| 28, | 28, | 28, | | 28, | 28, | 28, | 28, | 28, | | 28, | 28, | 29, | 29, | 29, | | 29, | 29, | 30, | 30, | 30, | | 30, | 30, |
| 31, | 31, | 31, | | 31, | 31, | 31, | 31, | 31, | | 31, | 31, | 32, | 32, | 32, | | 32, | 32, | 32, | 32, | 32, | | 32, | 32, |
| 33, | 33, | 33, | | 33, | 33, | 33, | 33, | 33, | | 33, | 34, | 34, | 34, | 34, | | 34, | 34, | 34, | 34, | 34, | | 34, | 35, |

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| 897, | 897, | 897, | 897, | 898, | 898, | 898, | 898, | 898, | 898, | 898, | 898, | 898, | 898, | 898, | 898, |
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| 900, | 900, | 900, | 901, | 901, | 901, | 901, | 901, | 901, | 901, | 901, | 901, | 902, | 902, | 902, | 902, |
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| 978, | 978, | 978, | 978, | 978, | 978, | 979, | 979, | 979, | 979, | 979, | 979, | 979, | 980, | 980, | 980, |
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| 981, | 981, | 982, | 982, | 982, | 982, | 982, | 982, | 982, | 982, | 982, | 983, | 983, | 983, | 983, | 983, |
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| 984, | 985, | 985, | 985, | 985, | 985, | 985, | 986, | 986, | 986, | 986, | 986, | 986, | 986, | 986, | 986, |
| 986, | 986, | 986, | 986, | 986, | 987, | 987, | 987, | 987, | 987, | 987, | 987, | 987, | 987, | 987, | 987, |
| 987, | 987, | 988, | 988, | 988, | 988, | 988, | 988, | 988, | 988, | 988, | 988, | 988, | 988, | 988, | 988, |
| 989, | 989, | 989, | 989, | 989, | 989, | 989, | 989, | 989, | 989, | 990, | 990, | 990, | 990, | 990, | 990, |
| 990, | 990, | 990, | 990, | 990, | 991, | 991, | 991, | 991, | 991, | 991, | 991, | 991, | 991, | 992, | 992, |
| 992, | 992, | 992, | 992, | 992, | 992, | 993, | 993, | 993, | 993, | 993, | 993, | 993, | 993, | 993, | 993, |
| 993, | 994, | 994, | 994, | 994, | 994, | 994, | 994, | 994, | 994, | 995, | 995, | 995, | 995, | 995, | 995, |
| 995, | 995, | 996, | 996, | 996, | 996, | 996, | 996, | 996, | 996, | 996, | 996, | 997, | 997, | 997, | 997, |
| 997, | 997, | 997, | 997, | 997, | 997, | 997, | 997, | 998, | 998, | 998, | 998, | 998, | 998, | 998, | 998, |
| 998, | 998, | 998, | 998, | 998, | 999, | 999, | 999, | 999, | 999, | 999, | 999, | 999, | 999, | 999, | 999, |
| 999, | 999, | 999, | 999, | 1000, 1000, 1000, 1000, 1000] | | | | | | | | | | | |

[77]:

plt.figure(figsize=(8, 4))

plt.plot(array\_sizes, execution\_times\_radix\_sort, marker='o', color='b',␣

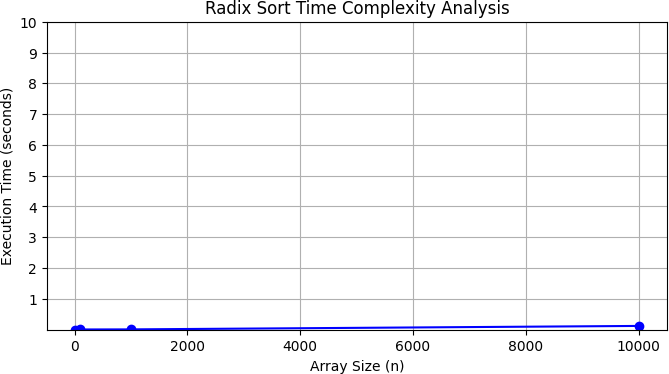
↪linestyle='-')

plt.title('Radix Sort Time Complexity Analysis') plt.xlabel('Array Size (n)') plt.ylabel('Execution Time (seconds)') plt.grid(**True**)

*# Set y-axis ticks to range from 1 to 10*

plt.yticks(range(1, 11))

plt.show()



Explanation:

Radix sort iterates through each digit position (place value) in the input array. In each iteration, it calls the counting\_sort function, which has a time complexity of O(n + k) due to counting element occurrences and populating the sorted output based on the count array. However, for radix sort, k (number of digits) is typically much smaller than n (number of elements) for most practical inputs.

Since radix sort performs multiple passes (one for each digit position), the total time complexity becomes O(nk). This is because the number of passes is bounded by the number of digits (k), and the counting sort within each pass dominates the time with its linear dependence on n ankTime Complexity

Since radix sort performs multiple passes (one for each digit position), the total time complexity becomes O(nk).

Space Comple

The count array used in the counting\_sort function. This array has a size of (max\_element + 1) to accommodate the range of possible digit values. In the worst case, this could be as large as the original input array (n) if all elements have unique digits. The temporary output array used to store the sorted elements during each pass of the counting sort within radix sort. This has a size of n to hold the elements being sorted. Therefore, the overall space complexity is O(n + k), where k is typically much smaller than n for most practical inputs.

s.

Implementation of Bucket Sort

[82]:

**def** bucket\_sort(arr): max\_element = max(arr)

bucket = [[] **for** \_ **in** range(len(arr))]

*# Create empty buckets*

**for** j **in** arr: *# Insert elements into their respective buckets* index\_b = int((len(arr) - 1) \* (j / max\_element)) bucket[index\_b].append(j)

**for** i **in** range(len(arr)): bucket[i] = sorted(bucket[i])

k = 0

**for** i **in** range(len(arr)):

**for** j **in** range(len(bucket[i])): arr[k] = bucket[i][j]

k += 1

print(f"Sorted Array for Size **{**len(arr)**}**: ", arr)

**return** arr

[83]:

Brief Overview of the Code:

The provided code implements Bucket Sort, a sorting algorithm that divides an array into a finite number of buckets, each capable of holding a range of values. Elements from the input array are distributed into these buckets based on their value range. Subsequently, the elements within each bucket are sorted individually, and the sorted elements are concatenated to form the final sorted array. Bucket Sort is particularly efficient when the input data is uniformly distributed across a range. However, its performance may degrade if the elements are not uniformly distributed or if the number of elements in each bucket becomes too large.

Block for random value arrays of different size

execution\_times\_bucket\_sort = []

**for** size **in** array\_sizes:

arr = [random.randint(0, 1000) **for** \_ **in** range(size)] start\_time = time.time()

bucket\_sort(arr) end\_time = time.time()

execution\_time = end\_time - start\_time execution\_times\_bucket\_sort.append(execution\_time)

Sorted Array for Size 10: [320, 350, 466, 496, 670, 719, 759, 762, 893, 911]

Sorted Array for Size 100: [1, 5, 9, 24, 38, 46, 54, 96, 97, 100, 112, 133,

138, 146, 159, 185, 191, 208, 225, 229, 234, 234, 237, 241, 241, 287, 314, 319,

320, 327, 352, 361, 372, 379, 386, 396, 401, 402, 406, 424, 441, 445, 447, 461,

502, 509, 524, 533, 539, 568, 579, 584, 609, 610, 611, 618, 645, 646, 652, 674,

681, 684, 684, 685, 685, 700, 704, 719, 720, 727, 728, 734, 738, 755, 763, 764,

779, 792, 802, 805, 813, 834, 849, 852, 866, 883, 883, 895, 899, 902, 911, 914,

919, 926, 934, 948, 956, 958, 961, 970]

Sorted Array for Size 1000: [0, 1, 1, 1, 2, 6, 8, 8, 8, 9, 10, 10, 11, 12, 12,

|  |  |  |  |
| --- | --- | --- | --- |
| 13, 13, 13, 14, 19, | 20, 21, 21, 22, 25, | 26, 28, 28, 28, 29, | 29, 30, 31, 31, 31, |
| 35, 35, 35, 39, 40, | 41, 41, 42, 42, 43, | 43, 43, 44, 45, 46, | 46, 47, 48, 49, 50, |
| 51, 51, 51, 55, 55, | 58, 59, 60, 60, 62, | 63, 64, 65, 66, 67, | 69, 70, 70, 70, 72, |
| 72, 72, 72, 73, 73, | 73, 73, 74, 75, 75, | 77, 78, 78, 78, 78, | 78, 79, 80, 80, 81, |
| 82, 83, 83, 83, 86, | 87, 88, 88, 89, 91, | 92, 92, 93, 93, 94, | 97, 98, 98, 99, 101, |

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| 101, | 101, | 102, | 104, | 104, | 104, | 105, | 105, | 107, | 107, | 108, | 108, | 109, | 110, | 111, | 111, |
| 112, | 113, | 114, | 114, | 115, | 115, | 116, | 116, | 116, | 116, | 117, | 117, | 118, | 118, | 119, | 123, |
| 123, | 124, | 126, | 126, | 128, | 129, | 137, | 137, | 138, | 138, | 139, | 140, | 141, | 141, | 142, | 144, |
| 144, | 144, | 145, | 145, | 145, | 145, | 146, | 148, | 149, | 149, | 150, | 150, | 152, | 153, | 154, | 156, |
| 158, | 159, | 160, | 162, | 163, | 163, | 164, | 165, | 165, | 165, | 166, | 166, | 167, | 167, | 168, | 168, |
| 169, | 169, | 170, | 172, | 172, | 173, | 173, | 174, | 176, | 176, | 178, | 180, | 182, | 183, | 184, | 184, |
| 186, | 188, | 189, | 189, | 190, | 190, | 194, | 195, | 195, | 196, | 196, | 196, | 196, | 197, | 199, | 200, |
| 201, | 202, | 202, | 204, | 205, | 207, | 208, | 209, | 209, | 210, | 211, | 212, | 212, | 213, | 213, | 213, |
| 214, | 218, | 218, | 218, | 219, | 220, | 221, | 222, | 223, | 224, | 225, | 226, | 226, | 227, | 227, | 228, |
| 229, | 230, | 230, | 232, | 232, | 233, | 234, | 235, | 235, | 236, | 238, | 238, | 239, | 243, | 246, | 246, |
| 246, | 250, | 250, | 251, | 252, | 252, | 252, | 253, | 253, | 256, | 256, | 258, | 260, | 261, | 262, | 263, |
| 263, | 265, | 265, | 266, | 267, | 270, | 270, | 271, | 272, | 275, | 275, | 275, | 278, | 278, | 278, | 279, |
| 280, | 285, | 286, | 287, | 288, | 289, | 289, | 289, | 291, | 291, | 292, | 292, | 293, | 293, | 297, | 299, |
| 300, | 301, | 301, | 302, | 302, | 303, | 304, | 305, | 305, | 306, | 307, | 311, | 312, | 312, | 313, | 314, |
| 315, | 316, | 317, | 318, | 319, | 319, | 320, | 321, | 321, | 324, | 325, | 325, | 326, | 328, | 329, | 330, |
| 330, | 331, | 331, | 332, | 333, | 333, | 333, | 335, | 335, | 338, | 338, | 340, | 342, | 342, | 344, | 345, |
| 349, | 349, | 352, | 353, | 353, | 355, | 356, | 357, | 357, | 358, | 360, | 360, | 360, | 362, | 362, | 363, |
| 363, | 364, | 364, | 364, | 364, | 366, | 366, | 366, | 368, | 368, | 369, | 369, | 369, | 369, | 370, | 370, |
| 371, | 371, | 374, | 378, | 379, | 379, | 379, | 381, | 384, | 385, | 388, | 389, | 389, | 390, | 391, | 392, |
| 393, | 394, | 396, | 396, | 398, | 399, | 399, | 401, | 401, | 402, | 402, | 404, | 408, | 411, | 412, | 412, |
| 413, | 416, | 416, | 418, | 419, | 420, | 422, | 424, | 426, | 427, | 428, | 428, | 430, | 430, | 431, | 432, |
| 432, | 433, | 434, | 434, | 434, | 435, | 439, | 440, | 441, | 441, | 441, | 442, | 442, | 442, | 442, | 443, |
| 445, | 447, | 448, | 451, | 451, | 452, | 453, | 453, | 455, | 457, | 458, | 459, | 459, | 461, | 461, | 461, |
| 462, | 464, | 464, | 465, | 466, | 466, | 466, | 466, | 468, | 469, | 469, | 470, | 471, | 473, | 474, | 474, |
| 474, | 474, | 475, | 475, | 475, | 476, | 476, | 477, | 477, | 482, | 482, | 483, | 483, | 483, | 484, | 484, |
| 485, | 485, | 485, | 486, | 487, | 487, | 487, | 488, | 491, | 493, | 494, | 494, | 495, | 496, | 497, | 498, |
| 498, | 502, | 505, | 506, | 507, | 508, | 508, | 510, | 514, | 514, | 515, | 516, | 517, | 517, | 518, | 518, |
| 518, | 518, | 519, | 519, | 520, | 523, | 523, | 524, | 525, | 526, | 526, | 528, | 530, | 532, | 534, | 535, |
| 535, | 535, | 539, | 540, | 540, | 541, | 541, | 541, | 541, | 542, | 543, | 544, | 546, | 548, | 549, | 549, |
| 550, | 552, | 553, | 555, | 557, | 559, | 564, | 565, | 566, | 568, | 569, | 569, | 571, | 574, | 575, | 576, |
| 578, | 579, | 579, | 579, | 580, | 586, | 588, | 588, | 588, | 588, | 589, | 591, | 593, | 593, | 593, | 593, |
| 594, | 595, | 595, | 596, | 597, | 599, | 599, | 604, | 606, | 607, | 610, | 610, | 611, | 611, | 612, | 612, |
| 614, | 614, | 615, | 616, | 617, | 618, | 618, | 619, | 621, | 623, | 623, | 623, | 625, | 626, | 626, | 627, |
| 631, | 631, | 633, | 636, | 637, | 638, | 638, | 639, | 639, | 641, | 641, | 641, | 641, | 642, | 643, | 643, |
| 644, | 644, | 645, | 645, | 646, | 646, | 646, | 647, | 647, | 649, | 650, | 650, | 651, | 657, | 657, | 658, |
| 658, | 658, | 658, | 658, | 658, | 659, | 660, | 660, | 661, | 663, | 663, | 663, | 664, | 665, | 666, | 667, |
| 667, | 668, | 669, | 669, | 671, | 673, | 674, | 675, | 677, | 679, | 680, | 681, | 682, | 682, | 682, | 682, |
| 683, | 684, | 684, | 684, | 687, | 688, | 689, | 690, | 691, | 692, | 693, | 694, | 695, | 695, | 695, | 697, |
| 697, | 697, | 697, | 698, | 701, | 701, | 703, | 703, | 704, | 706, | 707, | 708, | 709, | 710, | 711, | 712, |
| 713, | 714, | 714, | 717, | 718, | 718, | 720, | 721, | 721, | 721, | 721, | 724, | 724, | 725, | 726, | 726, |
| 727, | 730, | 733, | 733, | 736, | 738, | 738, | 739, | 739, | 740, | 741, | 741, | 742, | 742, | 743, | 743, |
| 743, | 743, | 743, | 746, | 747, | 748, | 748, | 748, | 748, | 749, | 750, | 750, | 751, | 752, | 753, | 753, |

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| 754, | 754, | | 754, | | 755, | 756, | 758, | | 762, | | 765, | 765, | 765, | | 770, | | 770, | 771, | 772, | | 773, | | 774, |
| 776, | 777, | | 777, | | 778, | 778, | 781, | | 784, | | 784, | 785, | 789, | | 789, | | 792, | 793, | 793, | | 798, | | 798, |
| 799, | 800, | | 801, | | 801, | 802, | 802, | | 805, | | 806, | 807, | 807, | | 808, | | 809, | 809, | 809, | | 811, | | 811, |
| 812, | 812, | | 813, | | 814, | 815, | 815, | | 816, | | 816, | 816, | 817, | | 818, | | 818, | 819, | 819, | | 824, | | 824, |
| 825, | 831, | | 832, | | 833, | 834, | 836, | | 837, | | 837, | 837, | 841, | | 842, | | 843, | 844, | 845, | | 846, | | 846, |
| 846, | 847, | | 848, | | 849, | 849, | 849, | | 850, | | 850, | 854, | 854, | | 856, | | 857, | 857, | 858, | | 859, | | 860, |
| 860, | 864, | | 867, | | 870, | 871, | 872, | | 872, | | 874, | 874, | 877, | | 878, | | 879, | 880, | 880, | | 881, | | 882, |
| 883, | 886, | | 888, | | 888, | 890, | 891, | | 892, | | 898, | 898, | 898, | | 898, | | 899, | 901, | 902, | | 902, | | 902, |
| 902, | 905, | | 907, | | 908, | 908, | 909, | | 910, | | 912, | 913, | 915, | | 917, | | 918, | 918, | 919, | | 920, | | 920, |
| 920, | 923, | | 924, | | 929, | 930, | 931, | | 931, | | 933, | 934, | 934, | | 935, | | 936, | 937, | 937, | | 937, | | 940, |
| 943, | 943, | | 943, | | 944, | 945, | 946, | | 946, | | 949, | 949, | 949, | | 951, | | 952, | 952, | 952, | | 954, | | 956, |
| 956, | 956, | | 957, | | 957, | 958, | 958, | | 961, | | 962, | 963, | 966, | | 967, | | 968, | 970, | 970, | | 970, | | 971, |
| 972, | 975, | | 976, | | 979, | 979, | 980, | | 980, | | 981, | 983, | 984, | | 985, | | 987, | 989, | 991, | | 991, | | 993, |
| 993, | 994, | | 995, | | 996, | 996] |  | |  | |  |  |  | |  | |  |  |  | |  | |  |
| Sorted Array for Size 10000: | | | | | | | | | [0, 0, 0, | | | 0, 0, 0, 0, 0, | | | | | 0, 0, 1, | | 1, 1, | | 1, 1, 1, | | |
| 1, 2, 2, 2, | | | | 2, | 2, 2, 2, | | 2, 2, | | 2, 2, | | 2, 3, 3, | | 3, | 3, 3, 3, 3, | | | | 3, 3, 3, 3, 4, | | | | | 4, 4, |
| 4, 4, 4, 4, | | | | 4, | 5, 5, 6, | | 6, 6, | | 6, 6, | | 6, 6, 7, | | 7, | 7, 7, 7, 7, | | | | 7, 7, 7, 7, 7, | | | | | 7, 7, |
| 8, 8, 8, 8, | | | | 8, | 8, 8, 8, | | 9, 9, | | 9, 9, | | 9, 9, 9, | | 9, | 9, 9, 9, 9, | | | | 9, 10, 10, 10, | | | | | 10, |
| 11, | 11, | 11, | | 11, | 11, | 11, | 11, | 11, | | 11, | 11, | 11, | 12, | 12, | | 12, | 12, | 12, | 12, | 12, | | 12, | 12, |
| 12, | 13, | 13, | | 13, | 13, | 13, | 13, | 13, | | 13, | 13, | 13, | 13, | 13, | | 13, | 13, | 13, | 13, | 14, | | 14, | 14, |
| 14, | 14, | 14, | | 14, | 14, | 14, | 14, | 15, | | 15, | 15, | 15, | 15, | 16, | | 16, | 16, | 16, | 16, | 16, | | 17, | 17, |
| 17, | 17, | 17, | | 17, | 17, | 17, | 17, | 17, | | 18, | 18, | 18, | 18, | 18, | | 18, | 18, | 19, | 19, | 19, | | 19, | 19, |
| 19, | 19, | 19, | | 19, | 19, | 20, | 20, | 20, | | 20, | 20, | 20, | 20, | 20, | | 21, | 21, | 21, | 21, | 21, | | 21, | 22, |
| 22, | 22, | 22, | | 22, | 22, | 22, | 22, | 22, | | 22, | 22, | 22, | 22, | 22, | | 22, | 22, | 23, | 23, | 23, | | 23, | 23, |
| 23, | 23, | 23, | | 23, | 23, | 23, | 24, | 24, | | 24, | 24, | 24, | 24, | 24, | | 24, | 24, | 25, | 25, | 25, | | 25, | 25, |
| 25, | 25, | 25, | | 25, | 25, | 25, | 26, | 26, | | 26, | 26, | 26, | 26, | 26, | | 27, | 27, | 27, | 27, | 27, | | 27, | 27, |
| 27, | 27, | 28, | | 28, | 28, | 28, | 28, | 29, | | 29, | 29, | 29, | 29, | 29, | | 29, | 29, | 29, | 29, | 29, | | 29, | 29, |
| 29, | 29, | 30, | | 30, | 30, | 30, | 30, | 31, | | 31, | 31, | 31, | 31, | 31, | | 31, | 31, | 31, | 31, | 31, | | 32, | 32, |
| 32, | 32, | 32, | | 32, | 32, | 32, | 32, | 32, | | 32, | 32, | 33, | 33, | 33, | | 33, | 33, | 34, | 34, | 34, | | 34, | 34, |
| 35, | 35, | 35, | | 35, | 35, | 35, | 35, | 36, | | 36, | 36, | 36, | 36, | 36, | | 36, | 36, | 36, | 36, | 36, | | 36, | 36, |
| 36, | 36, | 36, | | 36, | 36, | 37, | 37, | 37, | | 37, | 37, | 37, | 37, | 37, | | 37, | 37, | 37, | 37, | 37, | | 37, | 37, |
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plt.figure(figsize=(8, 4))

plt.plot(array\_sizes, execution\_times\_bucket\_sort, marker='o', color='b',␣

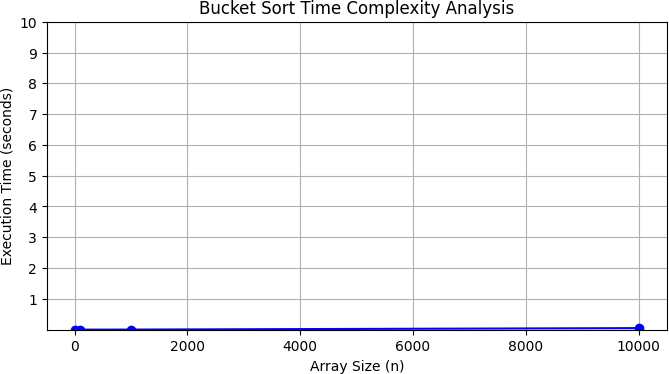
↪linestyle='-')

plt.title('Bucket Sort Time Complexity Analysis') plt.xlabel('Array Size (n)') plt.ylabel('Execution Time (seconds)') plt.grid(**True**)

*# Set y-axis ticks to range from 1 to 10*

plt.yticks(range(1, 11))

plt.show()



Explanation:

Bucket Sort is a compelling choice for sorting data with a limited and well-distributed value range, offering a time complexity of O(n + k) on average. However, the worst-case complexity and space complexity considerations require careful evaluation when choosing this algorithm for specific applications.

Time Complexity:

This approach leads to an attractive time complexity of O(n + k) in the average and best cases. Here, n represents the number of elements in the input array and k represents the number of elements in each bucket (ideally, a small constant value). The efficiency stems from avoiding

[101]:

expensive comparisons between elements and leveraging a simple sorting algorithm within buckets. However, the time complexity can degrade to O(n^2) in the worst case, which occurs when all elements have similar values and end up in the same bucket, causing the inner sorting operation on that single bucket to dominate the overall time.

Space Complexity:

The space complexity of Bucket Sort is also O(n + k). This is because it requires extra space for the array of buckets itself, which scales with the number of buckets. In the worst case, each bucket could potentially hold all elements, making the space complexity approach O(n) for the buckets. However, by choosing a small constant number of buckets (k), the space complexity remains dominated by the size of the input array (n), resulting in the overall O(n + k) complexity.

Implementation of Heap Sort

**def** heapify(arr, n, i): largest = i

l = 2 \* i + 1 r = 2 \* i + 2

**if** l < n **and** arr[l] > arr[largest]: largest = l

**if** r < n **and** arr[r] > arr[largest]: largest = r

**if** largest != i:

arr[i], arr[largest] = arr[largest], arr[i] heapify(arr, n, largest)

**def** heap\_sort(arr): n = len(arr)

*# Build max heap*

**for** i **in** range(n // 2 - 1, -1, -1): heapify(arr, n, i)

*# Extract elements one by one*

**for** i **in** range(n - 1, 0, -1):

arr[i], arr[0] = arr[0], arr[i] *# Swap*

heapify(arr, i, 0)

print(f"Sorted Array for Size **{**len(arr)**}**: ", arr)

**return** arr

Brief Overview of the Code:

The heapify function is crucial for maintaining the heap property within the array. It compares a node with its left and right child nodes, ensuring that the largest element resides at the root. If necessary, elements are swapped to maintain the heap property, and the process is recursively

[102]:

applied to child nodes.

In the heap\_sort function, a max heap is initially constructed by applying heapify to each non-leaf node in reverse order. This ensures that the maximum element is placed at the root of the heap. The algorithm then repeatedly extracts the maximum element from the heap, swaps it with the last element of the array, and restores the heap property for the remaining elements. Through this process, the array is progressively sorted in ascending order. The sorted array is printed alongside its size, and the function returns the sorted array. Heap Sort exhibits an average and worst-case time complexity of O(n log n), rendering it efficient for sorting large datasets due to its ability to utilize the binary heap structure for optimized comparisons and swaps.

Block for random value arrays of different size

execution\_times\_heap\_sort = []

**for** size **in** array\_sizes:

arr = [random.randint(0, 1000) **for** \_ **in** range(size)] start\_time = time.time()

heap\_sort(arr) end\_time = time.time()

execution\_time = end\_time - start\_time execution\_times\_heap\_sort.append(execution\_time)

Sorted Array for Size 10: [126, 156, 273, 394, 659, 666, 824, 874, 906, 924]

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| Sorted Array for Size 100: [12, 17, 30, 38, 45, 47, 49, 50, 61, 62, 99, 105, | | | | | | | | | | | | | | | |
| 111, 117, 120, 121, 128, 142, 153, | | | | | | | 181, 209, 213, 214, 218, | | | | | 229, 248, 257, 267, | | | |
| 276, 283, 285, 299, 306, 319, 327, | | | | | | | 340, 342, 387, 389, 421, | | | | | 425, 429, 442, 456, | | | |
| 460, 473, 477, 491, 503, 507, 508, | | | | | | | 513, 520, 568, 589, 590, | | | | | 595, 610, 611, 616, | | | |
| 623, 634, 645, 646, 677, 701, 705, | | | | | | | 718, 722, 732, 736, 761, | | | | | 771, 783, 787, 791, | | | |
| 822, 838, 841, 842, 844, 849, 857, | | | | | | | 858, 861, 881, 915, 918, | | | | | 924, 931, 942, 953, | | | |
| 954, 954, 969, 970, 982, 991, 993, 993]  Sorted Array for Size 1000: [1, 4, 4, 5, 5, 6, 7, 8, 9, 9, | | | | | | | | | | | | 10, 10, 12, 12, 12, | | | |
| 14, 14, 15, 16, 17, | | | | 17, 17, 20, 21, 22, | | | | 24, 26, 27, 27, 28, | | | | 29, 29, 32, 33, 34, | | | |
| 35, 38, 41, 42, 44, | | | | 44, 46, 48, 48, 50, | | | | 54, 54, 55, 56, 58, | | | | 58, 58, 59, 59, 61, | | | |
| 62, 62, 62, 63, 63, | | | | 65, 66, 67, 70, 71, | | | | 71, 73, 73, 74, 74, | | | | 76, 81, 84, 84, 86, | | | |
| 87, 88, 88, 89, 90, | | | | 90, 92, 92, 93, 93, | | | | 94, 97, 97, 97, 99, | | | | 99, 100, 102, 104, | | | |
| 107, | 110, | 110, | 110, | 111, | 114, | 114, | 115, | 116, | 117, | 117, | 118, | 119, | 119, | 120, | 124, |
| 125, | 125, | 126, | 126, | 127, | 127, | 128, | 129, | 129, | 129, | 129, | 130, | 131, | 131, | 132, | 132, |
| 133, | 135, | 135, | 139, | 140, | 141, | 142, | 142, | 143, | 144, | 144, | 144, | 145, | 146, | 148, | 148, |
| 148, | 150, | 150, | 151, | 154, | 154, | 156, | 156, | 160, | 162, | 163, | 164, | 165, | 165, | 166, | 168, |
| 168, | 169, | 169, | 170, | 173, | 173, | 173, | 173, | 174, | 174, | 175, | 177, | 179, | 181, | 183, | 184, |
| 185, | 186, | 186, | 187, | 188, | 190, | 191, | 192, | 193, | 195, | 196, | 197, | 198, | 198, | 199, | 200, |
| 201, | 201, | 202, | 204, | 205, | 206, | 207, | 207, | 211, | 211, | 211, | 212, | 212, | 212, | 215, | 216, |
| 216, | 218, | 221, | 222, | 223, | 223, | 224, | 226, | 227, | 227, | 227, | 228, | 229, | 229, | 229, | 230, |
| 231, | 231, | 232, | 232, | 232, | 232, | 234, | 234, | 234, | 235, | 235, | 236, | 239, | 240, | 240, | 242, |
| 243, | 246, | 246, | 247, | 249, | 250, | 251, | 251, | 253, | 253, | 254, | 254, | 254, | 255, | 256, | 261, |
| 262, | 262, | 262, | 264, | 266, | 267, | 268, | 269, | 270, | 271, | 272, | 273, | 274, | 275, | 275, | 275, |
| 275, | 276, | 276, | 276, | 277, | 277, | 277, | 278, | 279, | 280, | 281, | 282, | 283, | 283, | 284, | 285, |
| 286, | 287, | 288, | 289, | 292, | 292, | 292, | 293, | 296, | 296, | 296, | 298, | 298, | 299, | 299, | 299, |
| 301, | 302, | 304, | 304, | 305, | 306, | 307, | 308, | 309, | 310, | 311, | 311, | 311, | 311, | 312, | 312, |

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| 316, | 317, | 318, | 319, | 319, | 319, | 320, | 322, | 324, | 325, | 327, | 328, | 329, | 330, | 330, | 330, |
| 330, | 331, | 331, | 333, | 333, | 334, | 335, | 335, | 337, | 338, | 339, | 340, | 340, | 340, | 340, | 341, |
| 343, | 345, | 345, | 346, | 347, | 348, | 350, | 351, | 353, | 358, | 358, | 360, | 360, | 362, | 362, | 363, |
| 364, | 364, | 366, | 367, | 368, | 369, | 370, | 370, | 370, | 371, | 373, | 374, | 374, | 374, | 375, | 375, |
| 375, | 380, | 382, | 383, | 383, | 386, | 386, | 388, | 388, | 388, | 388, | 389, | 389, | 389, | 390, | 391, |
| 392, | 392, | 392, | 395, | 395, | 395, | 396, | 398, | 398, | 398, | 399, | 399, | 400, | 401, | 401, | 401, |
| 401, | 403, | 404, | 405, | 407, | 408, | 411, | 411, | 411, | 413, | 414, | 414, | 415, | 415, | 416, | 416, |
| 418, | 419, | 422, | 422, | 424, | 424, | 426, | 427, | 427, | 427, | 428, | 429, | 430, | 431, | 431, | 433, |
| 433, | 434, | 434, | 434, | 437, | 438, | 438, | 438, | 441, | 443, | 448, | 448, | 448, | 449, | 451, | 453, |
| 453, | 453, | 454, | 455, | 455, | 456, | 457, | 457, | 457, | 459, | 461, | 461, | 462, | 462, | 462, | 462, |
| 463, | 463, | 465, | 467, | 468, | 468, | 469, | 472, | 474, | 476, | 478, | 479, | 480, | 481, | 481, | 482, |
| 483, | 484, | 484, | 484, | 485, | 487, | 487, | 487, | 488, | 488, | 490, | 491, | 494, | 495, | 495, | 497, |
| 497, | 498, | 500, | 501, | 501, | 502, | 503, | 503, | 504, | 504, | 506, | 506, | 506, | 507, | 511, | 511, |
| 511, | 513, | 513, | 515, | 517, | 517, | 519, | 520, | 522, | 523, | 524, | 525, | 525, | 526, | 526, | 527, |
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| 563, | 564, | 565, | 568, | 569, | 573, | 573, | 573, | 574, | 575, | 577, | 581, | 582, | 583, | 584, | 584, |
| 585, | 586, | 587, | 587, | 587, | 590, | 590, | 591, | 591, | 592, | 594, | 596, | 597, | 598, | 599, | 600, |
| 601, | 602, | 602, | 603, | 605, | 608, | 608, | 609, | 609, | 611, | 611, | 612, | 613, | 614, | 614, | 615, |
| 615, | 616, | 617, | 620, | 621, | 622, | 623, | 625, | 625, | 626, | 626, | 627, | 628, | 628, | 631, | 631, |
| 631, | 633, | 634, | 636, | 636, | 637, | 637, | 639, | 639, | 639, | 643, | 644, | 645, | 649, | 650, | 650, |
| 650, | 650, | 651, | 651, | 653, | 654, | 655, | 656, | 656, | 657, | 660, | 661, | 663, | 663, | 664, | 666, |
| 667, | 667, | 668, | 669, | 670, | 671, | 672, | 674, | 675, | 675, | 676, | 678, | 678, | 682, | 684, | 684, |
| 685, | 686, | 687, | 687, | 687, | 690, | 691, | 691, | 692, | 692, | 695, | 696, | 696, | 699, | 700, | 702, |
| 703, | 703, | 703, | 704, | 706, | 706, | 706, | 707, | 708, | 708, | 709, | 712, | 712, | 714, | 715, | 715, |
| 716, | 717, | 717, | 717, | 718, | 718, | 719, | 720, | 720, | 722, | 723, | 724, | 725, | 726, | 727, | 728, |
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| 740, | 740, | 741, | 741, | 742, | 743, | 744, | 747, | 747, | 747, | 750, | 750, | 752, | 752, | 754, | 755, |
| 755, | 756, | 761, | 764, | 764, | 766, | 766, | 768, | 768, | 769, | 770, | 771, | 773, | 773, | 774, | 775, |
| 777, | 778, | 779, | 779, | 780, | 780, | 780, | 781, | 781, | 781, | 782, | 785, | 786, | 786, | 789, | 791, |
| 791, | 793, | 793, | 794, | 795, | 798, | 798, | 799, | 801, | 803, | 803, | 804, | 804, | 805, | 806, | 807, |
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| 834, | 835, | 836, | 840, | 841, | 842, | 844, | 846, | 847, | 848, | 848, | 849, | 850, | 855, | 856, | 857, |
| 857, | 858, | 859, | 861, | 862, | 862, | 865, | 865, | 866, | 866, | 867, | 869, | 869, | 869, | 869, | 870, |
| 872, | 873, | 875, | 875, | 876, | 876, | 876, | 877, | 878, | 879, | 880, | 881, | 883, | 883, | 883, | 884, |
| 884, | 886, | 887, | 887, | 887, | 888, | 888, | 889, | 892, | 893, | 893, | 894, | 894, | 895, | 898, | 899, |
| 900, | 900, | 900, | 902, | 902, | 903, | 903, | 905, | 908, | 909, | 909, | 910, | 911, | 913, | 913, | 915, |
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| 932, | 933, | 933, | 933, | 936, | 937, | 939, | 940, | 942, | 942, | 943, | 944, | 947, | 948, | 948, | 951, |
| 952, | 953, | 954, | 955, | 956, | 957, | 963, | 963, | 965, | 965, | 967, | 967, | 970, | 973, | 973, | 973, |
| 975, | 975, | 977, | 979, | 979, | 979, | 980, | 981, | 981, | 982, | 983, | 983, | 983, | 983, | 983, | 984, |
| 986, | 987, | 992, | 993, | 994, | 994, | 995, | 997, | 999, | 999] |  |  |  |  |  |  |
| Sorted Array for Size 10000: | | | | | | [0, 0, 0, | | 0, 0, 0, 0, 0, | | | 1, 1, 1, 1, 1, | | | 1, 1, 1, | |
| 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, | | | | | | 2, 2, 3, 3, 3, | | | 3, 3, 4, 4, 4, | | | 4, 4, 4, 4, 4, | | | 4, 4, |
| 5, 5, 5, 5, 5, 5, 5, 5, 5, 6, | | | | | | 6, 6, 6, 6, 6, | | | 6, 6, 6, 6, 7, | | | 7, 7, 7, 7, 7, | | | 7, 7, |
| 7, 7, 7, 8, 8, 8, 8, 8, 8, 8, | | | | | | 8, 8, 8, 8, 8, | | | 9, 9, 9, 9, 9, | | | 9, 10, 10, 10, | | | 10, |

10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 11, 11, 11, 11, 11, 11, 11, 11, 12, 12,

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[103]:

plt.figure(figsize=(8, 4))

plt.plot(array\_sizes, execution\_times\_heap\_sort, marker='o', color='b',␣

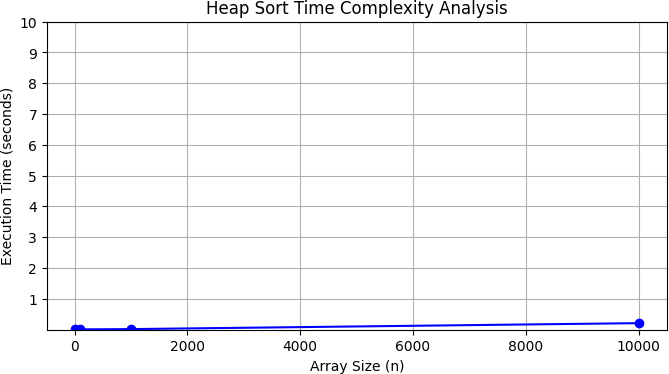
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plt.title('Heap Sort Time Complexity Analysis') plt.xlabel('Array Size (n)') plt.ylabel('Execution Time (seconds)') plt.grid(**True**)

*# Set y-axis ticks to range from 1 to 10*

plt.yticks(range(1, 11))

plt.show()



[ ]:

Explanation:

A heap is a specialized binary tree-based data structure that satisfies the heap property. In a max heap, for any given node, its value is greater than or equal to the values of its children. Conversely, in a min heap, the value of each node is less than or equal to the values of its children. Heaps are commonly implemented as arrays, where the parent-child relationships are determined by the indices of the array elements.

Time Complexity:

<pHeap operations maintain the heap property, allowing for efficient access to the maximum (or minimum) element, insertion of elements, and removal of the maximum (or minimum) element. Heapify, which takes O(log n) time complexity, restores the heap property after an element is added or removed. Building a heap from an array typically takes O(n) time complexity, achieved by heapifying the non-leaf nodes.

The most common application of heaps is in priority queues, where elements are organized by their priority. Priority queues built with heaps offer O(log n) time complexity for insertion and deletion, and O(1) time complexity for accessing the maximum (or minimum) element..

Space Complexity:

The space complexity of a heap is O(n) since it requires additional space proportional to the number of elements stored. Overall, heaps provide efficient solutions for managing priority-based data and are essential in various algorithms, including sorting algorithms like heap sort and graph algorithms like Dijkstra’s algorithm.ty.