Kinematics of the Stewart Platform

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Introduction

A Stewart platform is a type of parallel manipulator that has six prismatic actuators, commonly hydraulic jacks or electric linear actuators, attached in pairs to three positions on the platform's base-plate, crossing over to three mounting points on a top plate. All 12 connections are made via universal joints. Devices placed on the top plate can be moved in the six degrees of freedom in which it is possible for a freely-suspended body to move: three linear movements x, y, z (lateral, longitudinal, and vertical), and the three rotations (pitch, roll, and yaw). In flight simulation, it is known as platform base.

Project Objectives

This project concerns a two-dimensional version of the Stewart platform. It will model a manipulator composed of a triangular platform in a fixed plane controlled by three struts (Refer to Fig 1). The primary objective of this project is to find the position of the platform, given the three strut lengths. It is called the forward kinematics problem for this manipulator. The secondary objective is to find the strut lengths given the position of the platform. Finally, we are required to mention about the best method for solving this problem. Below activities indicate the flow of the project work in a sequential manner.

- Activity 1: Defining a function
- Activity 2: Visualizing the function
- Activity 3: Solving the forward kinematics problem
- Activity 4: Resolving the problem for various parameters

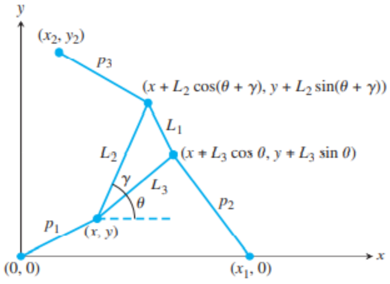

Fig 1: Schematic of planar Stewart platform

Kinematics of the Stewart Platform

Project Activities

Activity 1: Defining a function

As you see in Fig 1, the position of the Stewart platform can be expressed in x, y and θ. The function is defined as $f(\theta) = N_1^2 + N_2^2 - P_1^2 D^2$ and

$x = N_1/D$

$y = N_2/D$, where D=!0 and

$N_1 = B_3(p_2^2 - p_1^2 - A_2^2 - B_2^2) - B_2(p_3^2 - p_1^2 - A_3^2 - B_3^2)$

$N_2 = -A_3(p_2^2 - p_1^2 - A_2^2 - B_2^2) + A_2(p_3^2 - p_1^2 - A_3^2 - B_3^2)$

$D = 2(A_2B_3 - B_2A_3)$

$p_1^2 = x^2 + y^2$

$p_2^2 = (x + A_2)^2 + (y + B_2)^2$

$p_3^2 = (x + A_3)^2 + (y + B_3)^2$

$A_2 = L_3\cos\theta - x_1$

$B_2 = L_3\sin\theta$

$A_3 = L_2\cos(\theta + \gamma) - x_2$

$B_3 = L_2\sin(\theta + \gamma)$

After defining the function, f(θ) is calculated for θ= -π/4 and +π/4

The parameters L1, L2, L3,γ, x1, x2, y2 are fixed constants, and the strut lengths p1, p2, p3 will be known for a given pose. By passing these parameters as L1 = 2, L2 = L3 = √ 2, γ = π/2, p1 = p2 = p3 = √ 5, below values are obtained.

	x	y	theta	f_of_theta
0	1.0	2.0	-0.785398	-0.0
1	2.0	1.0	0.785398	-0.0

As f(θ) is zero, it clearly evident that -π/4 and +π/4 are the roots of this function

Activity 2: Visualizing the function

By plotting the graph of θ versus f(θ) for various values of θ between -π and +π, it is clear that f(θ) intersects x-axis at -π/4 and +π/4. Refer to Fig 2.

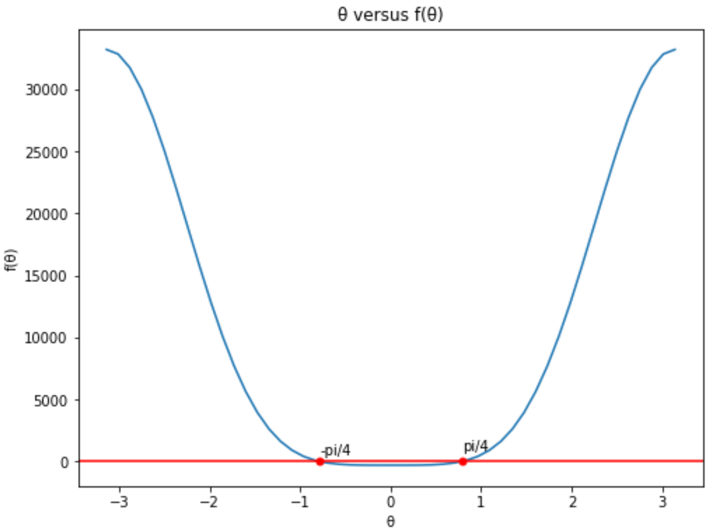

Fig 2: Graph of θ versus f(θ)

For the above values, we have drawn below schematics for the two poses to see the variations in the inner red triangle (Stewart platform).

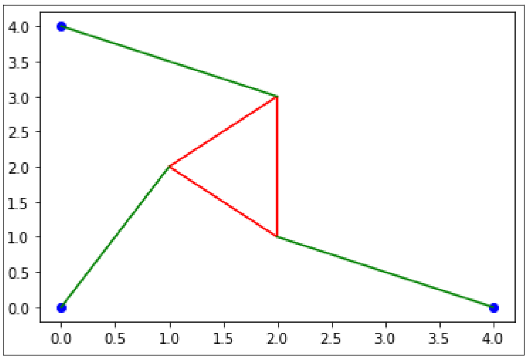

Fig 3: Stewart platform at $\theta = -\pi/4$

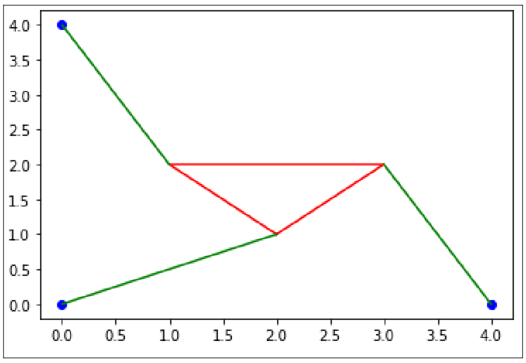

Fig 4: Stewart platform at $\theta = +\pi/4$

Activity 3: Solving the forward kinematics problem

Using the input parameters $x1 = 5$, $(x2, y2) = (0,6)$, $L1 = L3 = 3$, $L2 = 3\sqrt{2}$, $\gamma = \pi/4$, $p1 = p2 = 5$, $p3 = 3$, $f(\theta)$ is again calculated and below graph is plotted. As a result, it can be concluded that the function has 4 roots as the curve cuts x-axis at four different positions.

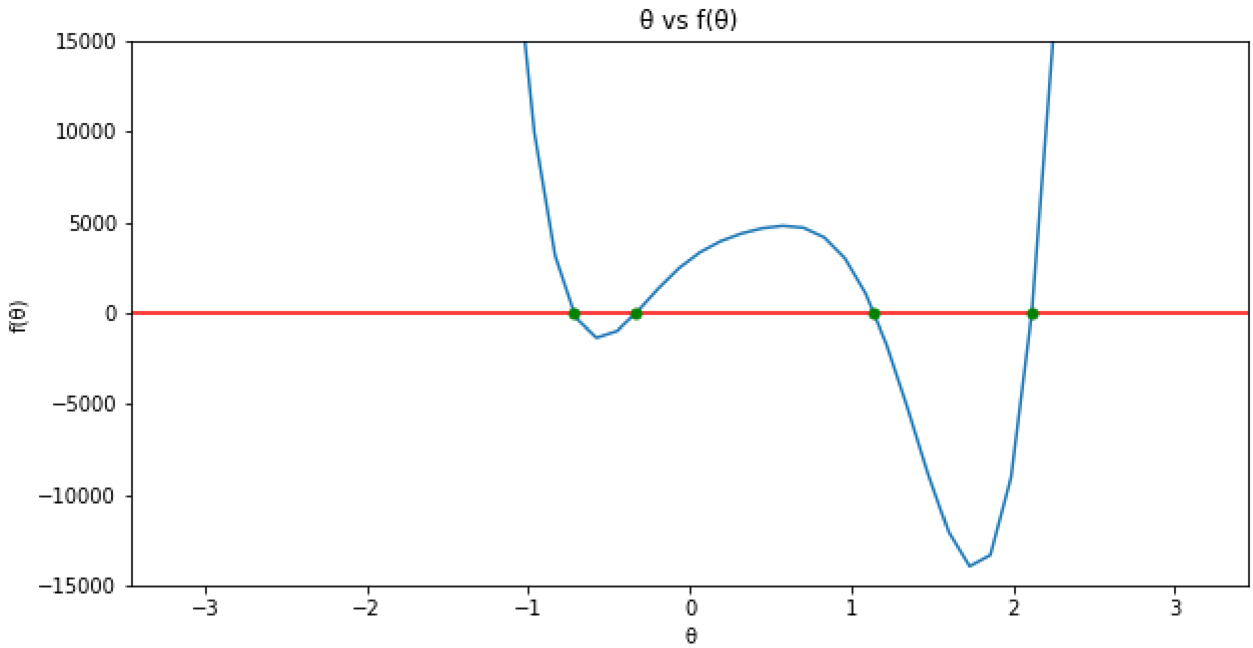

Fig 5: Graph of θ versus f(θ) for activity 3

In the next step, Bisection method and Newton's methods are used to find the roots, computational times and the number of iterations. Below is the comparison of the time taken in seconds by the both the methods to find the roots.

	Newton roots	Bisection roots	Newton_Times	Bisection_Times	Newton_Itr	Bisection_Itr	Newton_time-Bisection_time	Newton_Itr-Bisection_Itr
Root 1	-0.720852	-0.720840	0.000293	0.000311	15	15	-0.000018	0
Root 2	-0.331005	-0.331009	0.000072	0.000320	4	15	-0.000248	-11
Root 3	1.143687	1.143692	0.000088	0.000297	5	15	-0.000209	-10
Root 4	2.115910	2.115907	0.000106	0.000331	6	16	-0.000225	-10

From the above results it can be concluded that

1. The roots obtained in Newtons and Bisection method are almost similar.
2. Except for root 1, the computational time taken by Bisection method is more than that of the Newton's method.
3. Except for root 1, the number of iterations taken by Bisection method are approximately thrice to that of Newton's method.

As a result, Newton’s method is computationally less expensive and superior to Bisection method.

The below four poses are plotted as shown below to see variations in the position of Stewart platform.

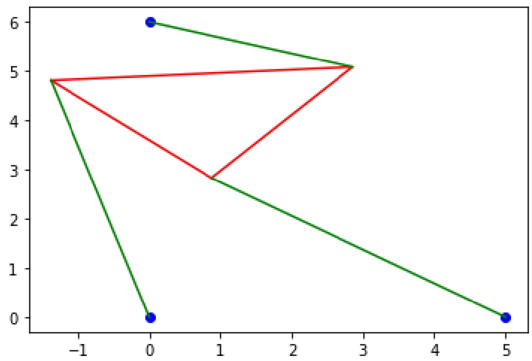

Fig 6: Pose 1 at root1 = -0.7208

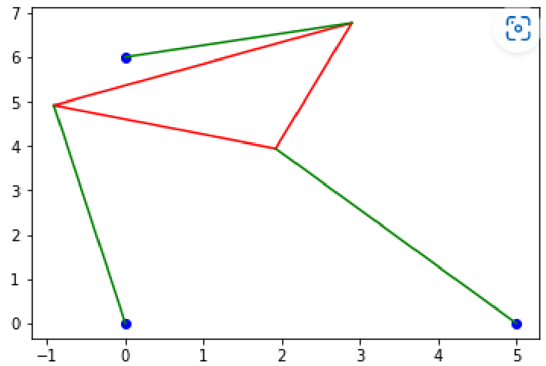

Fig 7: Pose 2 at root2 = -0.3310

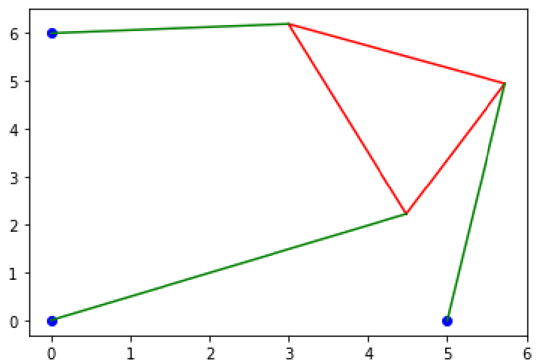

Fig 8: Pose 3 at root 3 = 1.143

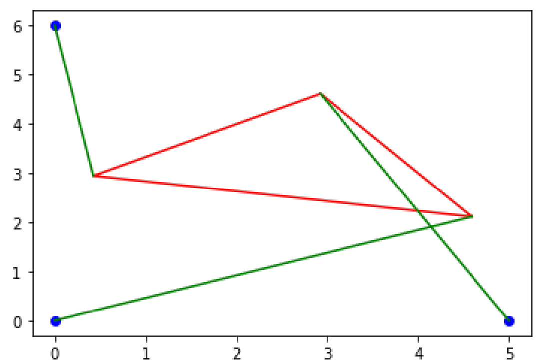

Fig 9: Pose 4 at root 4 = 2.1159

Activity 4 : Re-solving the problem for different values of parameters

Just by changing the value of p2 = 7, it is observed that the function has six roots. Refer to below plot of f(θ).

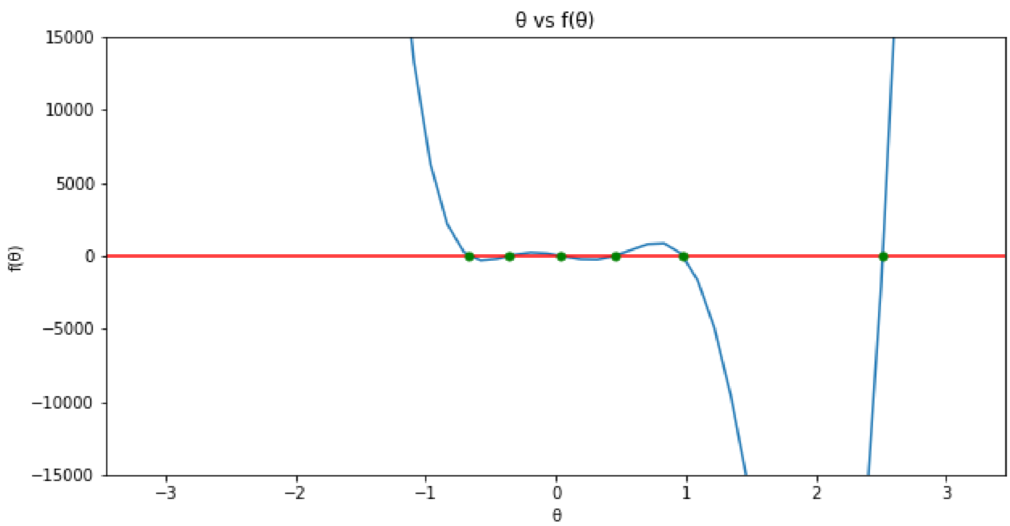

Fig 10: Graph of θ versus f(θ) for activity 4

The above graph is magnified below to clearly see the six points of intersection between the curve and the x-axis.

Then, Newton's method is used to find these 6 roots as -0.673, -0.3547, 0.0377, 0.4588, 0.9776, 7.260

In the next step, we have solved this problem as an Inverse kinematics problem to find the strut length p2 by using x, y and θ and other given parameters. By using following code snippet, we have obtained **p2 as 5** for the two poses given in the problem.

```
def p2_values(x,y,theta):
    L1 = 3; L3 = 3
    L2 = 3*math.sqrt(2)
    p1 = 5; p3 = 3
    gamma = math.pi/4
    x1 = 5
    theta = -0.7208
    A2 = L3*math.cos(theta)-x1
    B2 = L3*math.sin(theta)
    p2 = math.sqrt((x+A2)**2+(y+B2)**2)

    return p2
```

In the last activity of this project, our target is to find the intervals in p2 for various poses such as 0, 2, 4, 6, 8 and 10. To arrive at the solution, we have plotted θ versus f(θ) for various values of p2 from 0 to 10 in 20 steps. After observing all these 20 plots, below results are obtained.

p2 intervals	Number of poses
0 - 3.6	0
3.6 - 4.8	2
4.8 - 6.3	4
6.3 - 7.3	6
7.3 - 8	4
8 - 9.2	2
9.2 -10	0

Hence, it can be concluded that number of poses will increase as p2 increases up to a certain point. If p2 is further increased, then the number of poses will start to decrease.

Conclusion

As our task is to find the position of the planar Stewart platform, we have defined a function in terms of θ and found the roots of this function using Bisection method and Newton's method. We have observed that though the roots obtained by both of these methods are same, Newton's method has performed better than Bisection method in terms of computation time and the number of iterations. In addition to this, divergence problem has not been noticed while applying Newton's method. Hence, we prefer to use Newton's method in this case. In this project, we have also calculated poses for various values of strut length p2 and identified that number of poses will increase with increase in p2 value up to a certain point and then will decrease with further increase in p2 value.

References

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