Computer Assignment #7

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Part 1

In this part we want to solve the differential equation for a series RLC circuit.

1-1

first we write the differential equation:

$$R\frac{di(t)}{dt} + \mathrm{L}\,\frac{d^2 i(t)}{dt} + \frac{1}{c}\int_{-\infty}^{t} i(\tau)d\tau = v_{in}(t)$$

1-2

Now we apply the Laplace Transform to the equation and find $I(s)$:

$$RsI(s) + Ls^2 I(s) + \frac{1}{c}\,I(s) = sV\left(s\right)$$

$$\Rightarrow\; I(s)\left(\mathrm{L}s^2 + Rs + \frac{1}{c}\right) = sV\left(s\right)$$

$$\Longrightarrow I(s) = \frac{sV(s)}{Ls^2 + Rs + \frac{1}{c}}$$

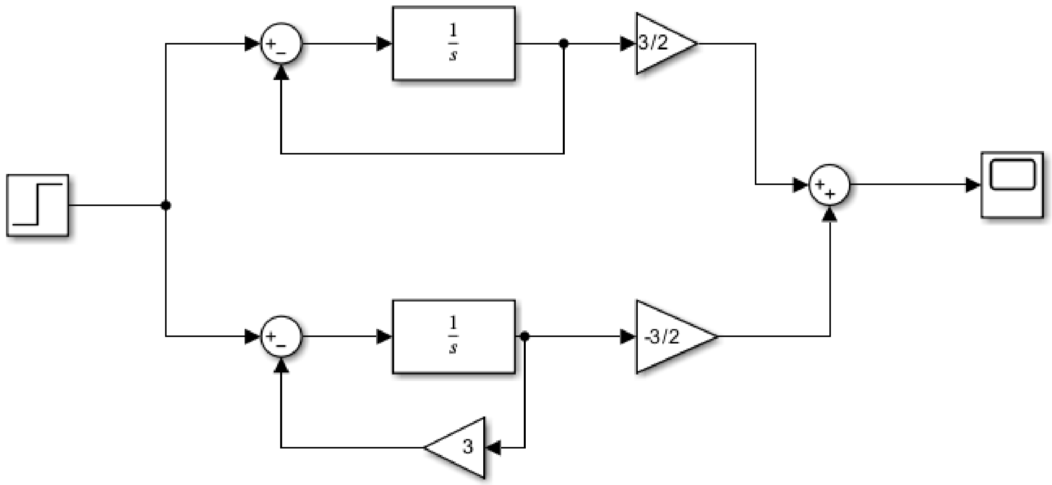
1-3

We know that $v_c(t) \;=\; \frac{1}{c}\int_{-\infty}^{t} i(\tau)d\tau$ so $V_c(s) = \frac{1}{\mathrm{c}s} I\left(s\right)$. Now if we assume that $x(t) = V_{in}(t)$ and $y(t) = V_c(t)$ this can be concluded:

$$Y(s) = \frac{1}{c}\frac{X(s)}{Ls^2 + Rs + \frac{1}{c}}$$

1-4

If we assume that $c = \frac{4}{3}, R = 1 \text{ and } L = 0.25$ the block diagram will be like this:



1-5

Now we want to calculate the step response of the system:

we know that the impulse response is as below:

$$H(s) = \frac{Y(s)}{X(s)} = \frac{3}{s^2 + 4s + 3} = \frac{3}{(s+1)(s+3)}$$

We know that The Laplace Transform of step function is $U(s) = \frac{1}{s}$ so:

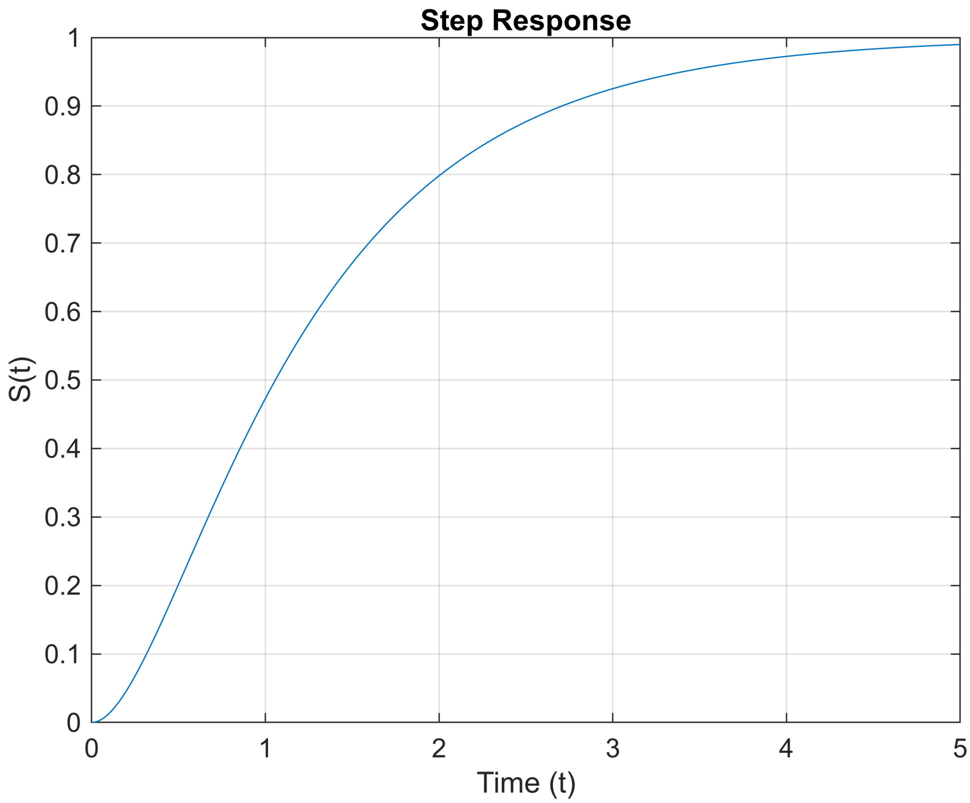
$$\Rightarrow H(s)U(s) = \frac{3}{s(s+1)(s+3)} = \frac{-\frac{3}{2}}{(s+1)} + \frac{\frac{1}{2}}{(s+3)} + \frac{1}{s}$$

Now if we calculate the inverse Laplace Transform of it we'll have the step response of the system:

$$S(t) \ = \ -\frac{3}{2}e^{-t}u(t) + \frac{1}{2}e^{-3t}u(t) + u\left(t\right)$$

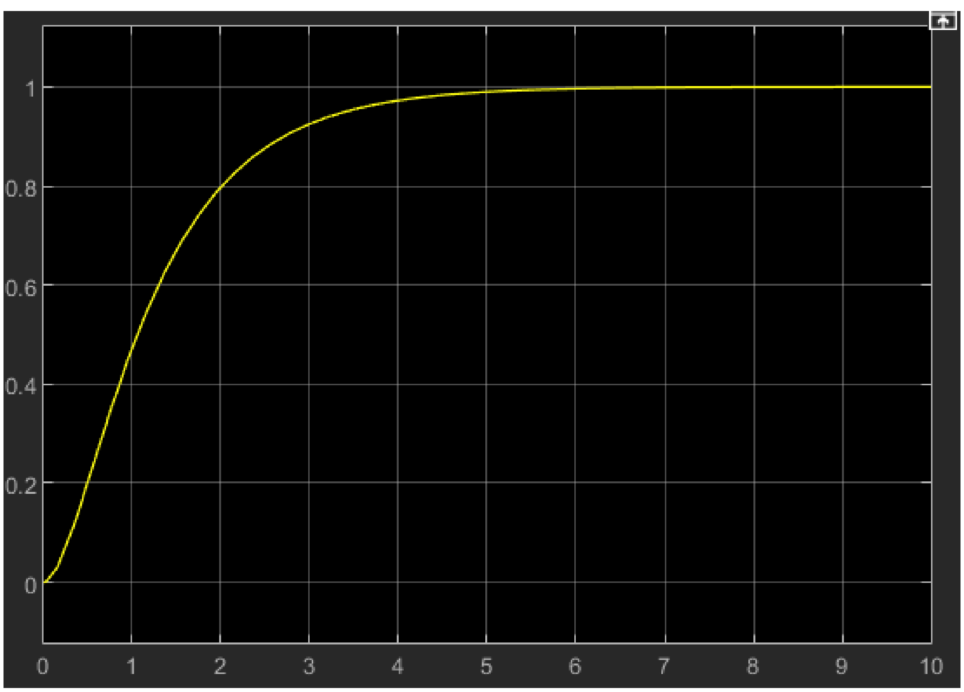
If we plot this function in MATLAB:

```
t = 0 : 1/1000 : 5;
u = heaviside(t);

y = -3/2 * exp(-t) .* u + 1/2 * exp(-3*t) .* u + u;

plot(t, y);
title('Step Response');
xlabel('Time (t)');
ylabel('S(t)');
grid on;
```



1-6

Now we test this system with step input in Simulink:



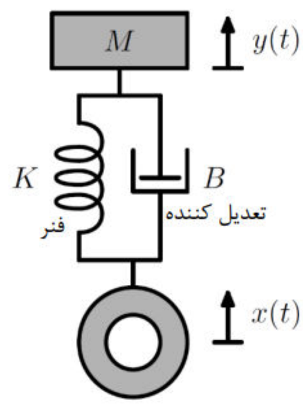
As you see the result is the same function that we calculated in the previous part.

Part 2

In a car, the body is connected to its wheels via suspension system. When the car hits a bump, this system tries to cancel the sudden movement of the wheels so the car body doesn't move that much. it uses a spring and a damper which both compresses when the car hits a bump. damper creates a force to keep the height of the car body and the damper makes a fraction force.

Now we want to model this in a mathematical equation:

- M is the mass of the car.
- $x(t)$ is the vertical displacement of the wheels.
- $y(t)$ is the vertical displacement of the M which basically shows the movements of the car body after it hits a bump.



Using dynamics we can write this equation between our variables:

$$K\left(x(t)-y(t)\right)+B\left(\frac{dx(t)}{dt}-\frac{dy(t)}{dt}\right)=M\frac{d^2y(t)}{dt^2}$$

2-1

First we assume that $M=1$ and $K=1$ and then rewrite the equation in a second-order ODE format:

$$\frac{d^2y(t)}{dt^2}+B\frac{dy(t)}{dt}+y(t)=x(t)+B\frac{dx(t)}{dt}$$

2-2

Then we apply Lapalce Transform to the equation and find the function that converts $Y(s)$ to $X(s)$:

$$s^2Y(s)+BsY(s)+Y(s)=X(s)+BsX(s)$$

$$\Rightarrow Y(s)(s^2+Bs+1)=X(s)(1+Bs)$$

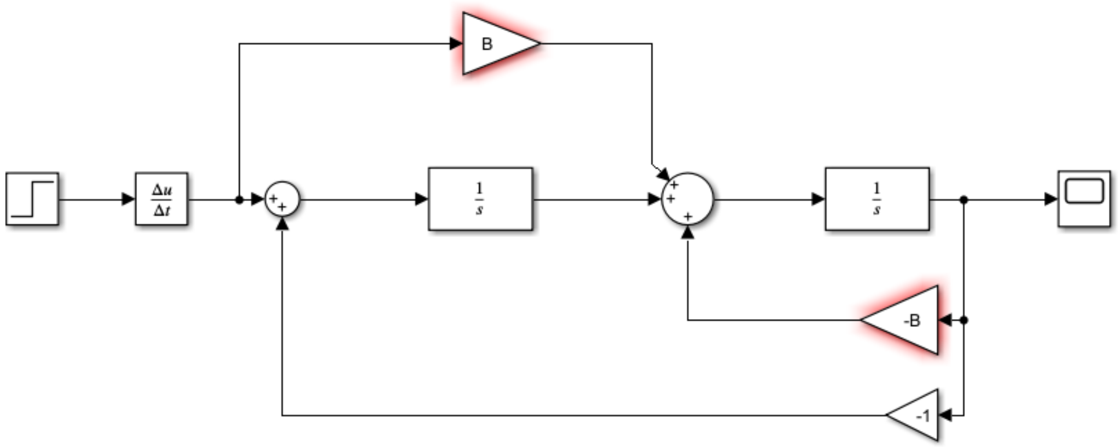
$$\Rightarrow H(s)=\frac{1+Bs}{s^2+Bs+1}$$

In order to easily draw the block diagram of the equation, we rewrite the equation as below:

$$Y(s) + \frac{BY(s)}{s} + \frac{Y(s)}{s^2} = \frac{X(s)}{s^2} + \frac{BX(s)}{s}$$

$$\Rightarrow Y(s) = \frac{1}{s^2}(X(s) - Y(s)) + \frac{1}{s}(BX(s) - BY(s))$$

Now we draw the block diagram in Simulink:

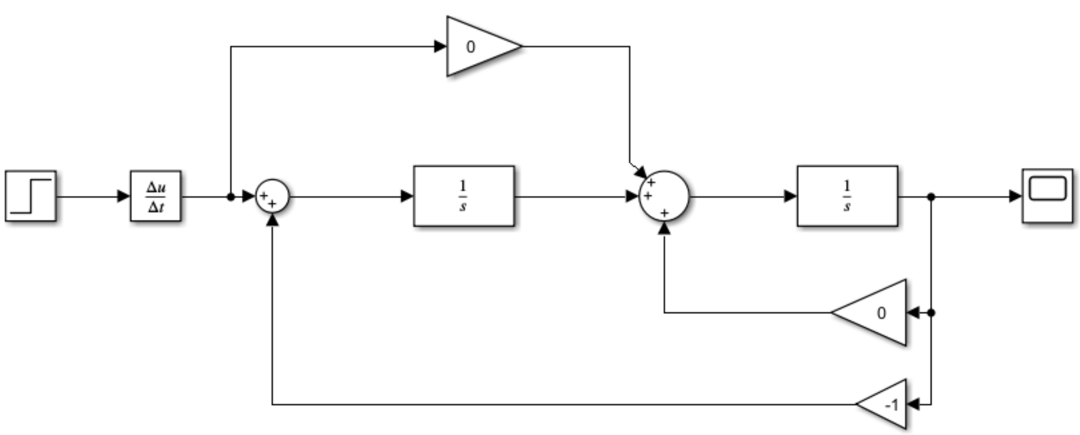


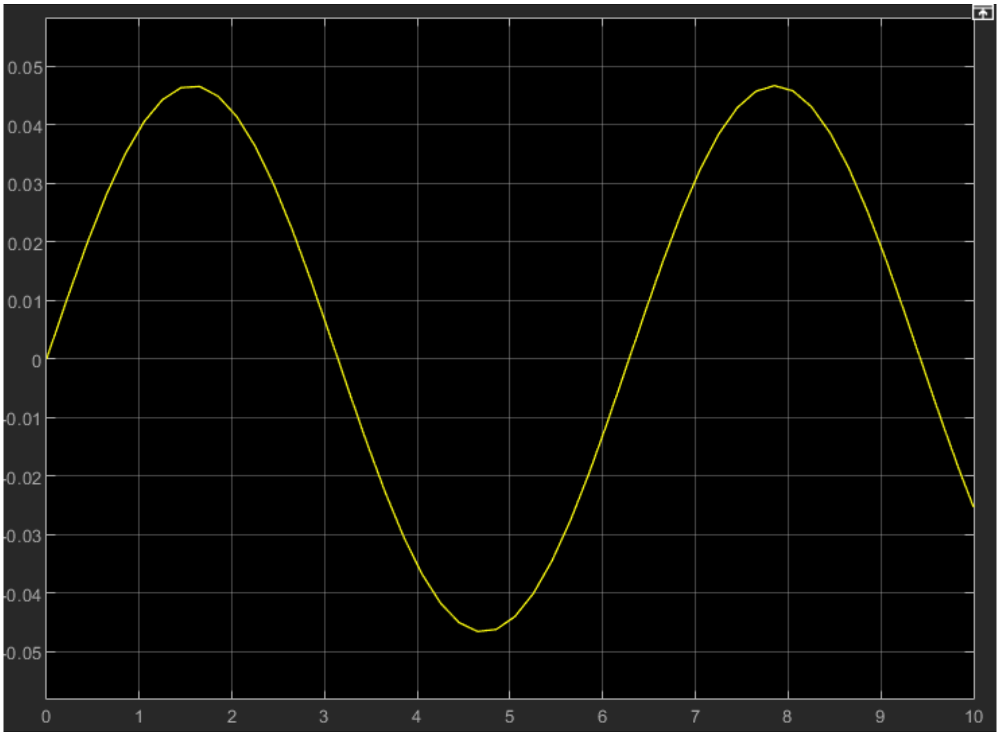
2-3

In this section we assume that $B = 0$ which means the damper don't generate any force. In this situation we can write the impulse respnose as below:

$$H(s) = \frac{1}{s^2 + 1} \longrightarrow h(t) = \sin\left(t\right)u(t)$$

We can test it in simulink and as we see the result is a sinus function:





This means that if we don't have damper in suspension system, the car body will go up and down repeatedly after it hits a bump!

2-4

The poles of $H(s)$ are the answers of $s^2 + Bs + 1 = 0$ so if we want the poles to be Real, the answers to that equation should be Real:

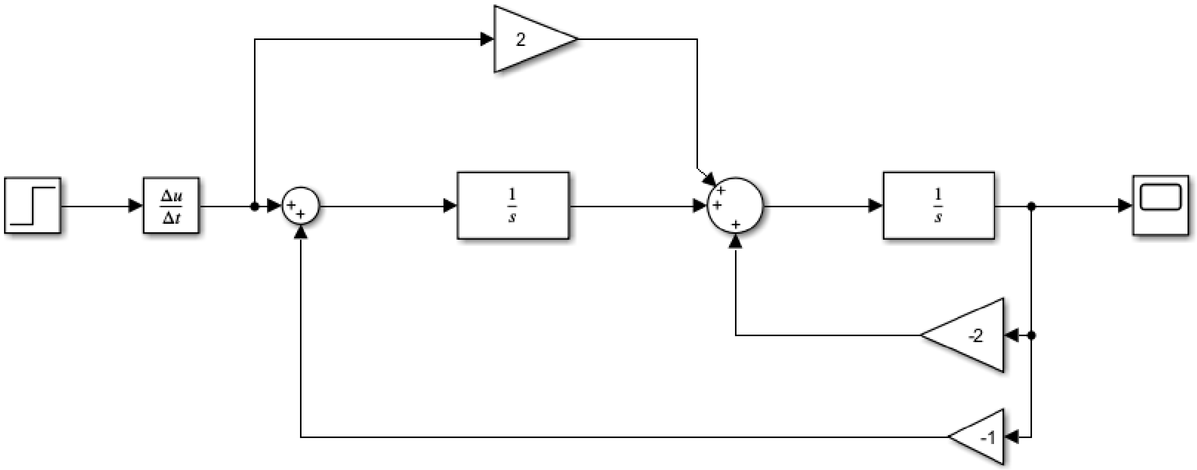
$$\Delta = B^2 - 4 \geq 0 \ \Rightarrow \ |B| \ \geq \ 2$$

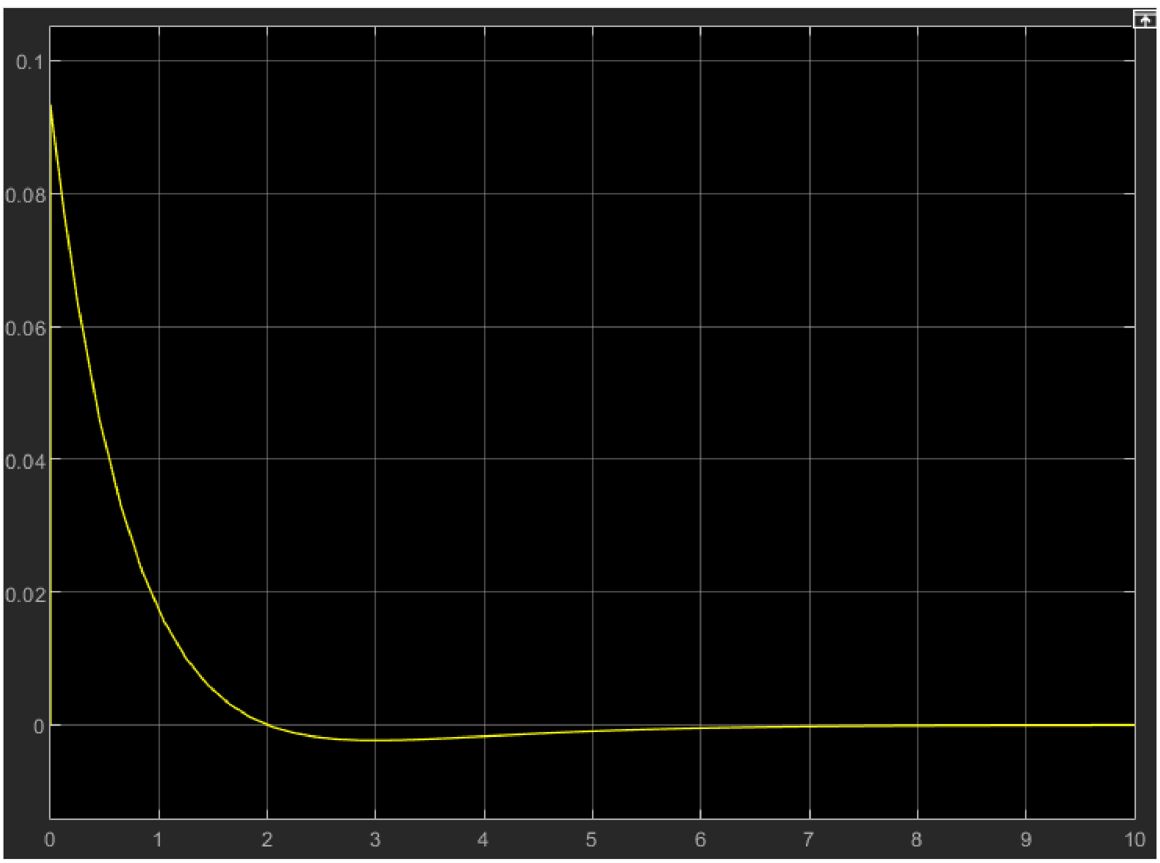
So the least positive value for B is 2.

In this situation we find the impulse response of the equation as below:

$$H(s) = \frac{1 + 2s}{s^2 + 2s + 1} \ \longrightarrow h(t) \ = \ -\mathrm{e}^{-t}\left(t - 2\right)u(t)$$

We test this situation in Simulink:





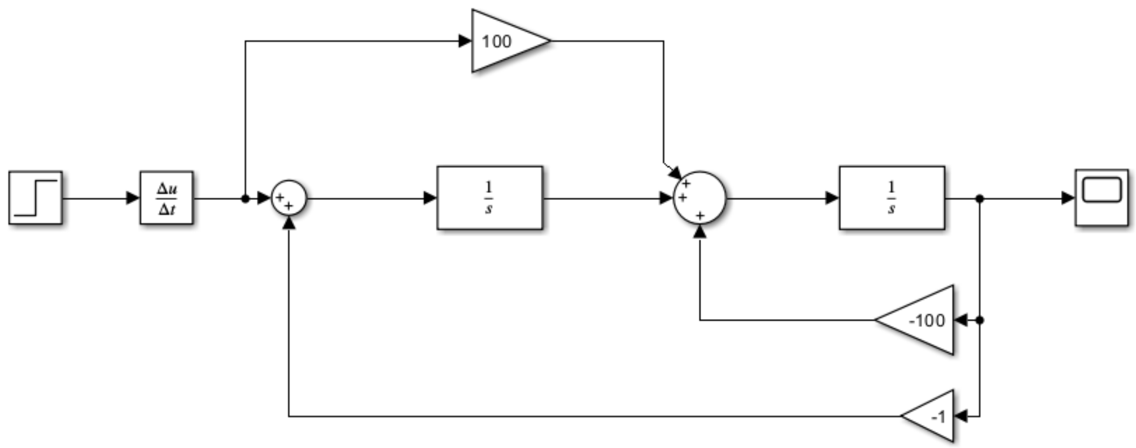
In this situation, suddenly the car body goes up but then goes down in a smooth way and then it becomes stable and we don't have that sinus shape that we had in the previous part.

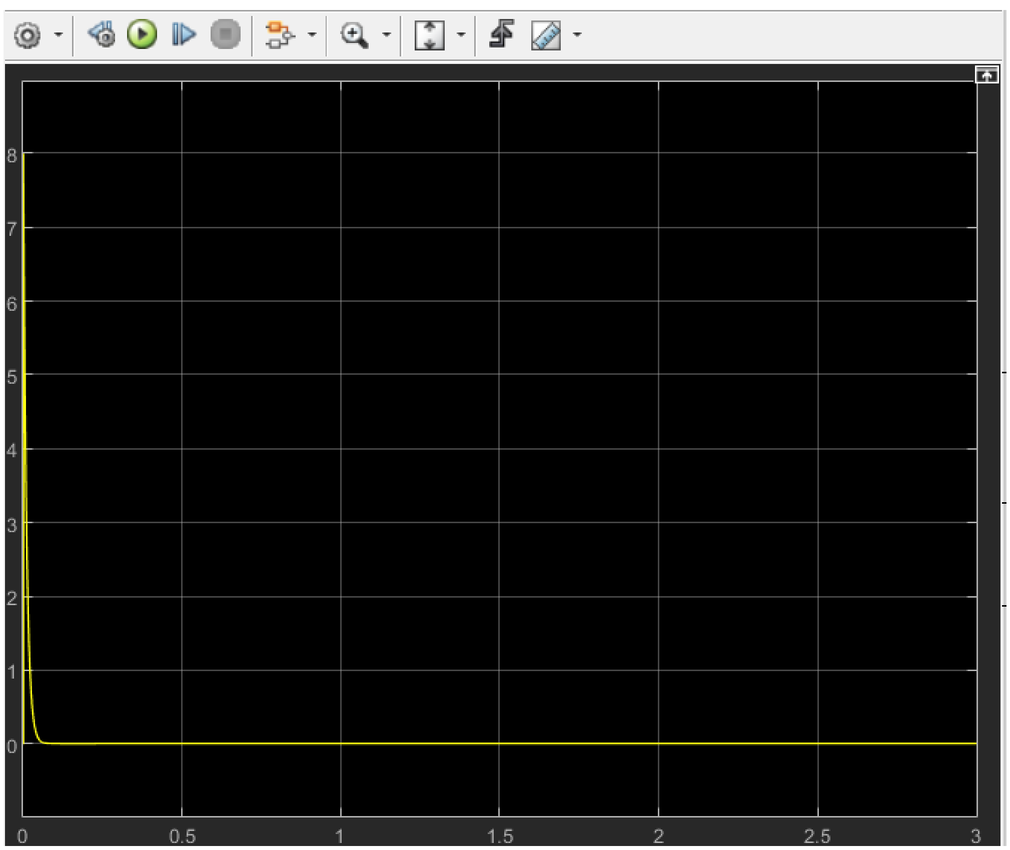
2-5

Now we assume that $B = 100$. In this situation the impulse respnse will be as below:

$$H(s) = \frac{1+100s}{s^2+100s+1} \approx \frac{1+100s}{(s+100)(s+0.01)} \longrightarrow h(t) = 100\,e^{-100t}u(t)$$

Now we test it in Simulink:





In this situation, it damps out oscillations more quickly but it's more suddent than B = 2.

Also the shape of the simulation result is similar to what we calculated in theory.

2-6

which value for B was better? *B = 2*

- B = 0: It was completely bad because we had no damping and after the car hits the bump, the car body will oscillate.
- B = 2: This case was ideal it damps out oscillations in a smooth way.
- B = 100: In this situation it damps out oscillations in a smooth way quicklier but the problem is its so sudden and when it hits the bump, the car body movement is so much higher than B = 2.

Part 3

If we don't have initial rest condition in our differential equation we can't use Laplace Transform to solve the equation instead we use Unilateral Laplace Transform

The equation is:

$$d^2\frac{\mathrm{y}(t)}{\mathrm{dt}^2}+3\,\frac{\mathrm{dy}(t)}{\mathrm{dt}}+2y(t)=x(t)$$

The initial conditions are:

$$y(0^-)=1,\, y'(0^-)$$

and $x(t)=5\,u(t)$.

3-1

First we solve this equation using Unilateral Laplace Transform:

$$s\,(sy\,(s)+\,y\,(0^-))-\,y'\,(0^-)+3sy\,(s)-\,3y'\,(0^-)+2y\,(s)=\,X(s)$$

$$\Rightarrow s^2y\,(s)-\,s-1+3sy\,(s)-\,3+2y\,(s)=\,X\,(s)$$

$$\Longrightarrow s^2y\,(s)+\,(2+3s)y\,(s)=\,X\,(s)+\,s+4$$

since $X(s)=\dfrac{5}{s}$ then:

$$(s^2+3s+2)Y\,(s)=\frac{5}{s}+s+4$$

$$\Rightarrow Y\,(s)=\frac{\frac{5}{s}+s+4}{s^2+3s+2}=\frac{s^2+4s+5}{s(s+1)(s+2)}$$

then if we apply inverse Laplace Transform we can find y(t):

$$y\,(t)=\frac{e^{-2t}}{2}-2\,e^{-t}+\frac{5}{2}$$

3-2

In this section we solve the same equation using MATLAB functions:

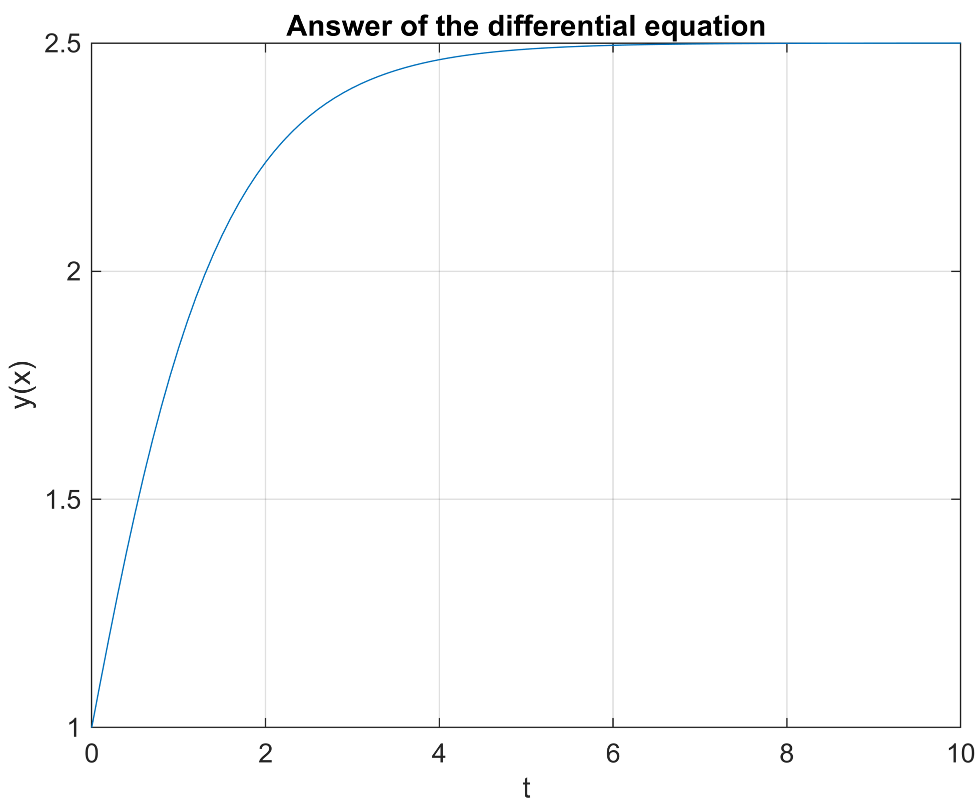
```
syms y(t)
```

```
sys = tf(1, 1);
Dy = diff(y);

ode = diff(y,t,2) + 3 * diff(y, t, 1) + 2*y == 5 * step(sys);
cond1 = y(0) == 1;
cond2 = Dy(0) == 1;
conds = [cond1 cond2];
ySol(t) = dsolve(ode,conds);
ySol = simplify(ySol)
```

ySol(t) =

$$\frac{e^{-2\,t}}{2} - 2\,e^{-t} + \frac{5}{2}$$

```
figure;
ts = 0 : 0.1 : 10;
plot(ts, ySol(ts));
title('Answer of the differential equation');
xlabel('t');
ylabel('y(x)');
grid on;
```



As you can see the answer is the same function that we calculated in the previous part.