Assignment 1
Due April 19, 2019 at 11.59 PM via ilearn Assignment-1 Submission folder

IMAGES. All images may be found in the TEST IMAGES folder on iLearn or in the folder uploaded specific to this assignment.

Independent Reading. Please go through the uploaded file under Independent Reading-Assignment1.

Problem 1. [2 pts]

(a) On the `house.tif` image, compute the 2D Discrete Fourier transform and display.
(b) Apply Gaussian smoothing to the image, sample the smoothed image, and try to reconstruct the original image (of the same size) from the samples. Repeat for different choices of the Gaussian filter width and number of samples. What do you observe in terms of the relationship between the reconstruction error and the number of samples?

Write MATLAB scripts for your solutions.

Problem 2. [2 pts]

Plot the 4×4 DFT and DCT basis images. Compute the DCT and DFT transformed image of `gonazalezwoods725.png` and display. Write a MATLAB script for your solution. Which transform will work better for image compression ? Explain briefly.

Problem 3. [3 pts]

Prove the 2D Fourier Transform properties related to Linearity, Convolution and Energy Conservation in the image below. Please read independent reading materials mentioned above for help.

Problem 4. [3 pts]

`lena_gray_256_noisy.png` is a noisy version of the image `lena_gray_256.tif`. De-noise the image by applying a suitable image filtering technique to obtain an image as close as possible to the original one. You are allowed to hand-pick the filter parameters for this problem, but explain clearly the reason behind the choice of filter and its parameters. Write a MATLAB script for your solution.

Submission Protocol. All codes should be written in MATLAB. Inbuilt functions can be used for problem 1, 2 and 4. You should submit codes as well as explanations to each problem (if required). You should add comments to your codes to make them reader friendly. All coding related problems should be in separate scripts (.m files) named after the problem number. For e.g. Problem 1 has two parts (a) and (b). The codes corresponding to them should be in Problem1a.m and Problem1b.m. If you may require to call functions for a problem, you may do so, but include them in your submission. Keep all the images necessary to run a code in the same folder as the code, while you are submitting. You MUST also include a report written electronically (using the likes of LaTeXor MS Word). It should contain explanations, images, etc (as required).

Each student must do the assignment independently, although you may discuss prior to that. While discussion is allowed, we will be particularly careful about any plagiarism, whether from each other or from other sources.

TABLE 2.4 Properties and Examples of Fourier Transform of Two-Dimensional Sequences

Property	Sequence	Transform
	$x(m, n), y(m, n), h(m, n), \cdots$	$X(\omega_1, \omega_2), Y(\omega_1, \omega_2), H(\omega_1, \omega_2), \cdots$
Linearity	$a_1 x_1(m, n) + a_2 x_2(m, n)$	$a_1 X_1(\omega_1, \omega_2) + a_2 X_2(\omega_1, \omega_2)$
Conjugation	$x^*(m, n)$	$X^*(-\omega_1, -\omega_2)$
Separability	$x_1(m) x_2(n)$	$X_1(\omega_1) X_2(\omega_2)$
Shifting	$x(m \pm m_0, n \pm n_0)$	$\exp[\pm j(m_0 \omega_1 + n_0 \omega_2)] X(\omega_1, \omega_2)$
Modulation	$\exp[\pm j(\omega_{01} m + \omega_{02} n)] x(m, n)$	$X(\omega_1 \mp \omega_{01}, \omega_2 \mp \omega_{02})$
Convolution	$y(m, n) = h(m, n) \circledast x(m, n)$	$Y(\omega_1, \omega_2) = H(\omega_1, \omega_2) X(\omega_1, \omega_2)$
Multiplication	$h(m, n) x(m, n)$	$\left(\frac{1}{4\pi^2}\right) H(\omega_1, \omega_2) \circledast X(\omega_1, \omega_2)$
Spatial correlation	$c(m, n) = h(m, n) \star x(m, n)$	$C(\omega_1, \omega_2) = H(-\omega_1, -\omega_2) X(\omega_1, \omega_2)$
Inner product	$I = \sum\sum_{m, n=-\infty}^{\infty} x(m, n) y^*(m, n)$	$I = \frac{1}{4\pi^2} \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} X(\omega_1, \omega_2) Y^*(\omega_1, \omega_2)\, d\omega_1\, d\omega_2$
Energy conservation	$\mathscr{E} = \sum\sum_{m, n=-\infty}^{\infty} \lvert x(m, n)\rvert^2$	$\mathscr{E} = \frac{1}{4\pi^2} \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} \lvert X(\omega_1, \omega_2)\rvert^2\, d\omega_1\, d\omega_2$
	$\sum\sum_{m, n=-\infty}^{\infty} \exp[j(m\omega_{01} + n\omega_{02})]$	$4\pi^2 \delta(\omega_1 - \omega_{01}, \omega_2 - \omega_{02})$
	$\delta(m, n)$	$\frac{1}{4\pi^2} \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} \exp[-j(\omega_1 m + \omega_2 n)]\, d\omega_1\, d\omega_2$