EE243: Advanced Computer Vision
Assignment #1

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1 Problem 1

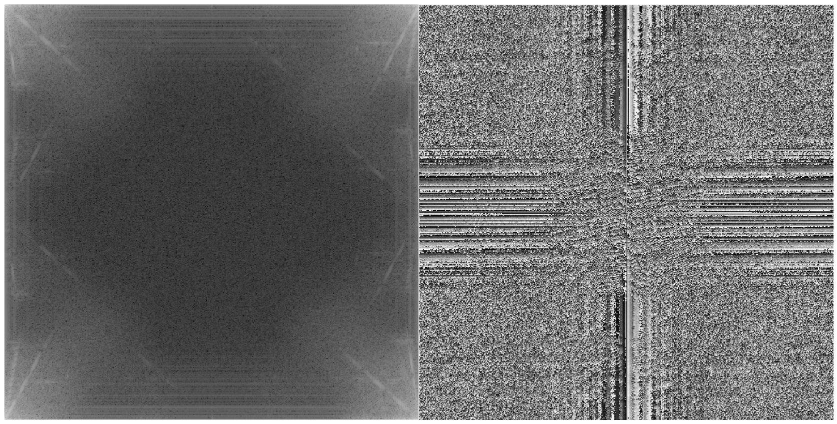
1.1 Part a

This problem asks for computing the 2D Discrete Fourier transform of *house.tif* image and displaying it.

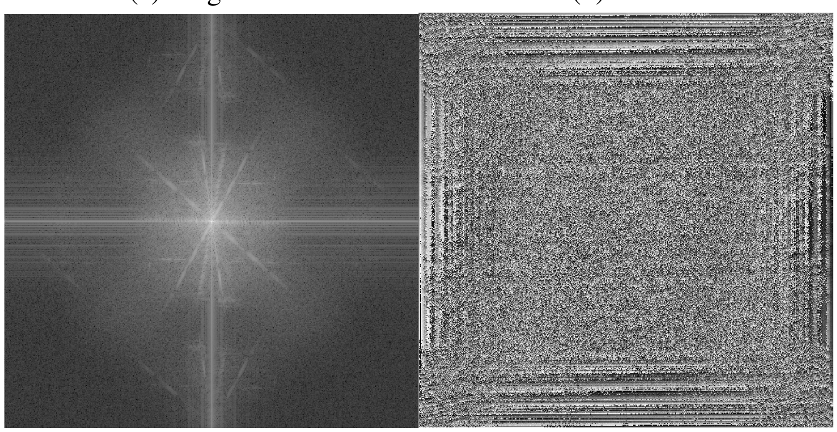
Figure 1: *house.tif* original image.

Figure 1. shows the original image. We applied *fft2* function to the image. The elements of the result matrix are complex numbers which corresponds to $phase$ and $magnitude$ of that element. In order to show the magnitude of the transformed image, we scaled the results by *log* function to obtain a more understandable view

of the magnitudes of the transformed image components, shown in Figure 2a. Figure 2b. also demonstrates the phase of the transformed image.



(a) Magnitude (b) Phase



(c) Shifted magnitude (d) Shifted phase

Figure 2: DFT results for *house.tif*.

Figure 2c. and Figure 2d. show *magnitude* and *phase* of the shifted transformed image using *fftshift* function, respectively. *fftshit* function rearranges the outputs of *fft2* by moving the zero-frequency component to the center of the array. It is useful for visualizing a Fourier transform with the zero-frequency component in the middle of the spectrum to make it visually clearer.

1.2 Part b

In this part, we apply Gaussian smoothing to the *house.tif* image shown in Figure 1. We sample the smoothed image, and try to reconstruct the original image (of the same size) from the samples. Then, we repeat for different choices of the Gaussian filter width and number of samples. Finally, we explain the relationship between the reconstruction error, smoothing filter width, and sampling window. First, we draw these parameters relationship conclusions visually. However, visual images can not help us to realized the exact relationship of these two parameters. Thus, we use MSE metric to see how these parameters behave.

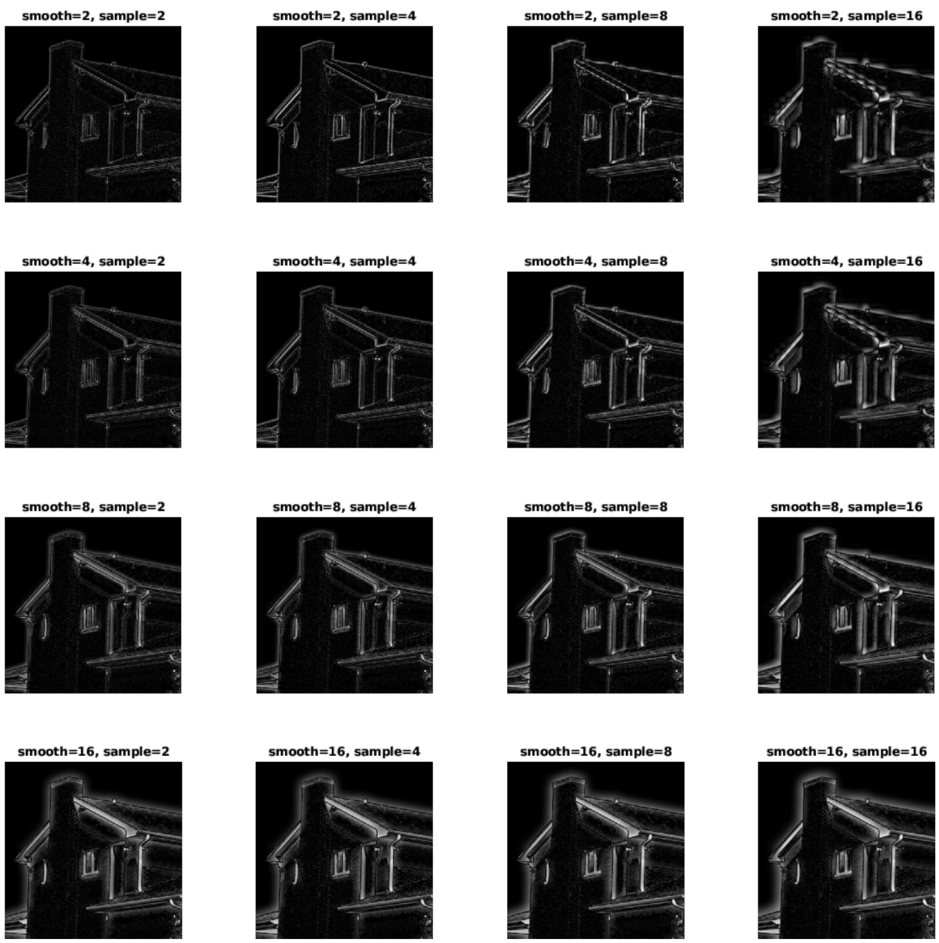


Figure 3: The difference of *house.tif* from the reconstructed one with different filter width and sampling window values.

The aforementioned procedure was performed on the image with Gaussian smoothing filter width of [2,4,8,16], and sampling window on [2,4,8,16]. Then, the sampled images was reconstructed. Figure 3. shows the absolute difference of the original image and the reconstructed one for different filter width and sampling window.

Visual conclusion: As we can see in the difference images of Figure 3, from top-left toward top-right, as the sampling window increases, the difference becomes higher and uneven around the image edges. By increasing the smoothing filter width, from top to bottom, as the sampling window increases, the difference around the edges becomes more smooth, even and regular. However, the difference is still increasing by increasing the sampling window.

MSE-based conclusion: Figure 4. shows the MSE of the orignal image and the reconstructed one with difference filter width and sampling windows. We can see that by increasing either of smoothing filter width or and sampling window size, reconstruction error increases. But, the rate by which the reconstruction error increases is different for each of them. According to Figure 4, for smoothing filter width of size 2 as an example, as the sampling window increases, the reconstruction error increases quicker than larger smoothing filter widths. Although larger smoothing filter width has larger MSE for the same sampling window, the rate of MSE getting larger by increasing sampling window size is smaller in larger smoothing filter widths.

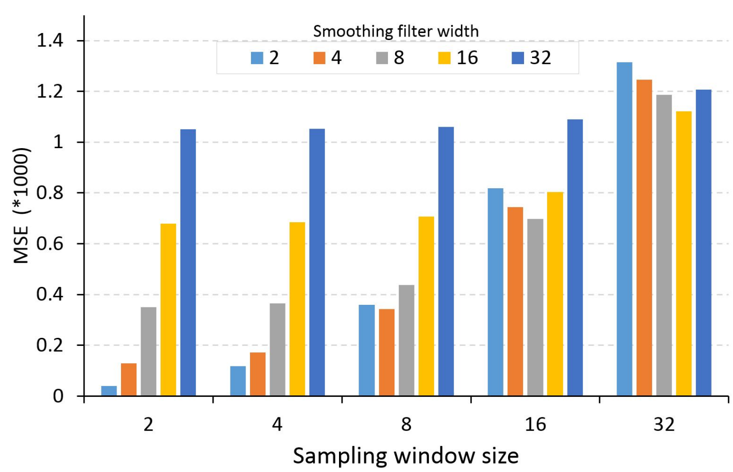
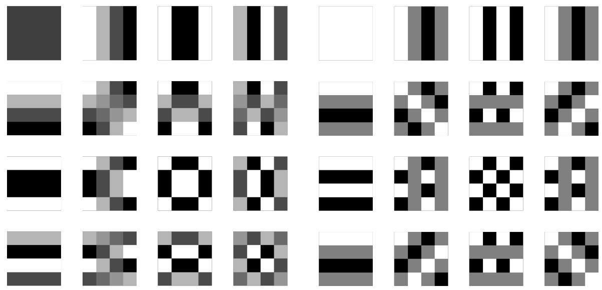


Figure 4: MSE of *house.tif* and the reconstructed one with different filter width and sampling window values.

2 Problem 2

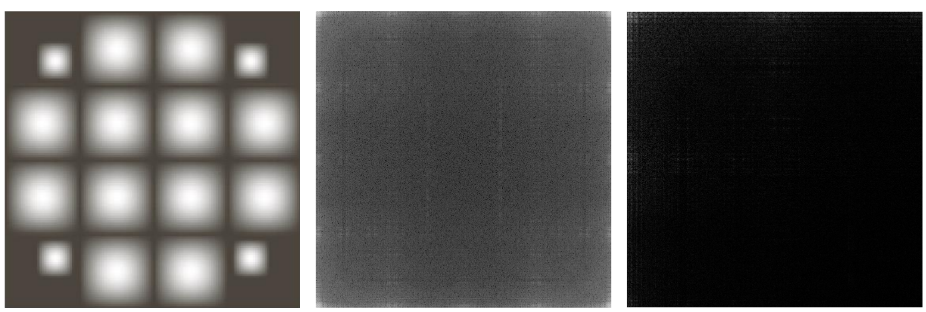
In this problem, we are asked to plot the 4×4 DFT and DCT basis images. Figure 5a. shows DCT and Figure 5b. shows DFT basis images.



(a) DCT. (b) DFT.

Figure 5: 4×4 DCT and DFT basis images.

Then we are asked to compute the DCT and DFT transformed image of Figure 6a. and display. Figure 6c. and Figure 6b. show the DCT and DFT of the image, respectively.



(a) Original image. (b) DFT. (c) DCT.

Figure 6: *GonzalezWoods* image and its DCT and DFT.

As we know, Fourier transform generates complex numbers, which consist of real and imaginary components. On the other hard, Cosine transform generates only real numbers. Thus, in order to store a given image with DFT we have to store two components. However, for DCT we have to store one component for the same image which is half of the DFT of same image and we still have same quality. Hence, DCT works better for image compressing.

3 Problem 3

In this part we are asked to prove the 2D Fourier Transform (Equation 1) properties related to Linearity, Convolution and Energy Conservation.

$$\begin{aligned} X(\omega_1, \omega_2) &\triangleq \mathfrak{F}[x(m,n)] \\ &\triangleq \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x(m,n) exp[-j\ (m\omega_1 + n\omega_2)] \\ &- \pi \leq \omega_1, \omega_2 < \pi \end{aligned} \tag{1}$$

In the rest of this report, we used Equation 2. in order to be brief in the writing.

$$ex^{m,n} \leftrightarrow exp[-j\ (m\omega_1 + n\omega_2)] \tag{2}$$

3.1 Linearity

Equation 3 is the mathematical representation of the linearity property of 2D Fourier transform, which is asked to be proved in this question.

$$\mathfrak{F}[ax_1(m,n) + bx_2(m,n)] \triangleq aX_1(\omega_1, \omega_2) + bX_2(\omega_1 + \omega_2) \tag{3}$$

Equation 4. proves that the Fourier transform of a sum of functions, is the sum of the Fourier transforms of the functions.

$$\begin{aligned} \mathfrak{F}[x_1(m,n) + x_2(m,n)] &\triangleq \sum\sum [x_1(m,n) + x_2(m,n)] ex^{m,n} \\ &\triangleq \sum\sum [x_1(m,n) ex^{m,n} + x_2(m,n) ex^{m,n}] \\ &\triangleq \sum\sum x_1(m,n) ex^{m,n} + \sum\sum x_2(m,n) ex^{m,n} \\ &\triangleq \mathfrak{F}[x_1(m,n)] + \mathfrak{F}[x_2(m,n)] \\ &\triangleq X_1(\omega_1, \omega_2) + X_2(\omega_1, \omega_2) \end{aligned} \tag{4}$$

According to Equation 5, any coefficients (in this case a and b) can be taken out out the summation, which means if we multiply a function by a constant, the

Fourier Transform is multiplied by the same constant.

$$
\begin{aligned}
\mathfrak{F}[ax_1(m,n)] &\triangleq \sum\sum ax_1(m,n)ex^{m,n} \\
&\triangleq a * [\sum\sum x_1(m,n)ex^{m,n}] \\
&\triangleq a * \mathfrak{F}[x_1(m,n)] \\
&\triangleq aX_1(\omega_1,\omega_2)
\end{aligned}
\tag{5}
$$

By combining Equation 4. and Equation 5. the linearity of 2D Fourier transform can be proved, i.e. Equation 3.

3.2 Convolution

Equation 3 is the mathematical representation of the convolution property of 2D Fourier transform, which is asked to be proved in this question.

$$
\mathfrak{F}[h(m,n) \circledast x(m,n)] \triangleq H(\omega_1,\omega_2)X(\omega_1+\omega_2) \tag{6}
$$

If we expand the right hand side of Equation 6, we will have Equation 7.

$$
\begin{aligned}
\mathfrak{F}[h(m,n) \circledast x(m,n)] &\triangleq \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}[\sum_{u=-\infty}^{\infty}\sum_{v=-\infty}^{\infty} h(m,n)x(u-m,v-n)ex^{u,v}] \\
&\triangleq \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}[h(m,n)\sum_{u=-\infty}^{\infty}\sum_{v=-\infty}^{\infty} x(u-m,v-n)ex^{u,v}]
\end{aligned}
\tag{7}
$$

Then, we use shift property of Fourier transform, which is stated in Equation 8, and prove the convolution property of 2D Fourier transform as shown in Equation 9.

$$
\sum_{u=-\infty}^{\infty}\sum_{v=-\infty}^{\infty} x(u-m,v-n)ex^{u,v} \xrightarrow{shiftproperty} X(\omega_1,\omega_2)ex^{m,n} \tag{8}
$$

$$
\begin{aligned}
&Using\ (8)\ in\ (7) \Rightarrow \\
&\qquad \triangleq \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}[h(m,n)\sum_{u=-\infty}^{\infty}\sum_{v=-\infty}^{\infty}[x(u-m,v-n)]ex^{u,v}] \\
&\qquad \triangleq \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} h(m,n)X(\omega_1,\omega_2)ex^{m,n} \\
&\qquad \triangleq X(\omega_1,\omega_2)[\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} h(m,n)ex^{m,n}] \\
&\qquad \triangleq H(\omega_1,\omega_2)X(\omega_1,\omega_2)
\end{aligned}
\tag{9}
$$

3.3 Energy Conservation

Equation 10 is the mathematical representation of the energy conservation (parseval) property of 2D Fourier transform, which is asked to be proved in this question.

$$
\begin{aligned}
\xi &= \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}|x(m,n)|^2 \qquad (sequence) \\
\mathfrak{F}[\xi] &\triangleq \frac{1}{4\pi^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}|X(\omega_1,\omega_2)|^2 d\omega_1 d\omega_2 \qquad (transform)
\end{aligned}
\tag{10}
$$

We know that Equation 11. holds, know is Parseval formula.

$$
f(m,n)f^*(m,n) = |f(m,n)|^2 \tag{11}
$$

Starting from left hand side of Equation 11. and using Parseval formula, we get to Equation 12. At the one line before the last line, we used *shifting property* and replaced all θs by ω.

$$
\begin{aligned}
\xi &= \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} |x(m,n)|^2 \\
&= \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x(m,n)x^*(m,n) \\
&= \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} [\frac{1}{4\pi^2} \int\int X(\omega_1,\omega_2) ex^{m,n} d\omega_1\omega_2 [\frac{1}{4\pi^2} \int\int X(\theta_1,\theta_2) ex^{m,n} d\theta_1 d\theta_2]^*] \\
&= \frac{1}{4\pi^2} \int\int \frac{1}{4\pi^2} \int\int X(\omega_1,\omega_2) X^*(\theta_1,\theta_2) [\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{j(m\omega_1+n\omega_2)} e^{-j(m\theta_1+n\theta_2)}] \\
&\quad d\omega_1 d\omega_2 d\theta_1 d\theta_2 \\
&= \frac{1}{4\pi^2} \int\int \frac{1}{4\pi^2} \int\int X(\omega_1,\omega_2) X^*(\theta_1,\theta_2) [4\pi^2 \delta(\theta-\omega)] d\omega_1 d\omega_2 d\theta_1 d\theta_2 \ (shift) \\
&= \frac{1}{4\pi^2} \int\int X(\omega_1,\omega_2) X^*(\omega_1,\omega_2) d\omega_1 d\omega_2
\end{aligned} \tag{12}
$$

4 Problem 4

In this problem, we are asked to de-noise a a noisy version of *Lena* image which is shown in Figure 7. De-noising the image was performed using Fourier transform.

Figure 7: *Lena* noisy image.

We zeroed out the frequencies above a certain threshold in the noise area of the transformed version of the image. Figure 8. shows the transformed image, the selected noise regions, and the zeroed out components for different thresholds.

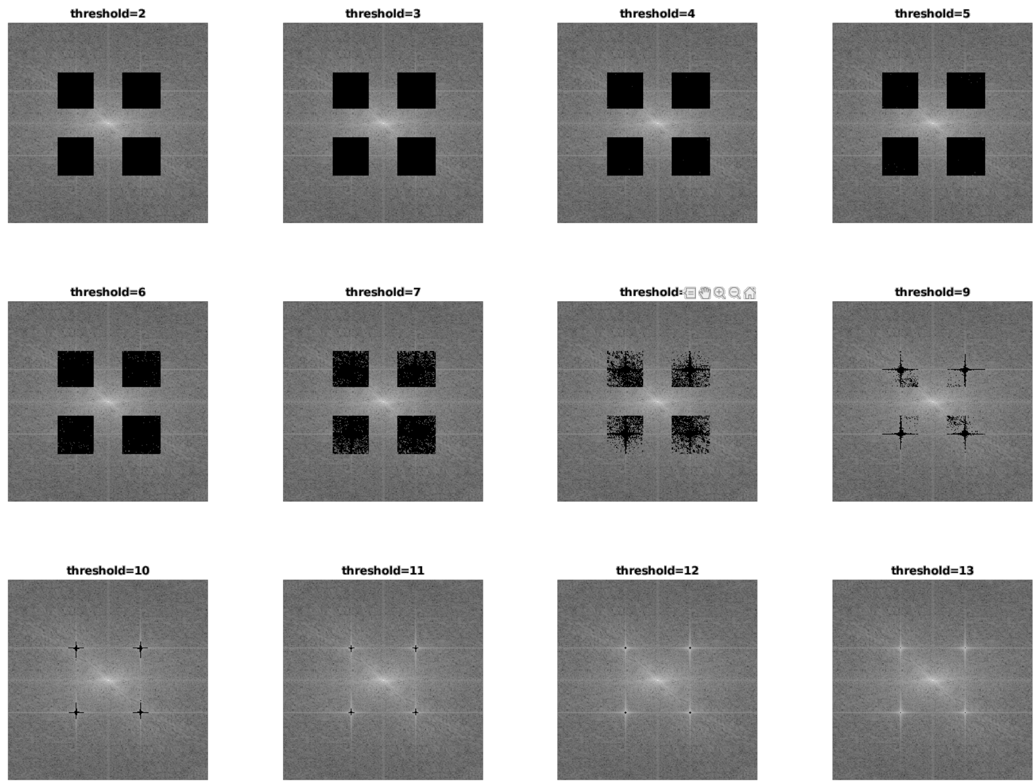


Figure 8: Zeroing out the noise region of transformed noisy *Lena* with different thresholds.

The resulting image from the de-noising technique that we used is illustrated in Figure 9. As we can see in Figure 9, for lower thresholds, due to removing more frequency components of the image, the image loses more details. The threshold equal to 10 results in the best de-noised version of the image.

Figure 9: De-noised version of noisy *Lena* with different zeroing out thresholds.