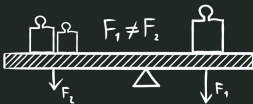
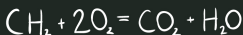
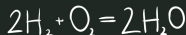




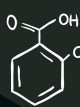
$$\begin{pmatrix} 1001 \\ 1110 \\ 1010 \\ 0001 \end{pmatrix}$$



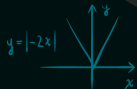
$$(\pi k, 0); k \in \mathbb{Z}$$

$$ax^2 + bx + c = 0$$

$$\sin^2 \alpha + \cos^2 \alpha = 1$$



$$\phi(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{x^2}{2\sigma^2}}$$



$$y = \cos x$$

$$\frac{\cos \alpha}{\sin \alpha} = \cot \alpha$$

$$f(\omega) = \int_{-\infty}^{\infty} f(x) \cdot e^{-2\pi i x \omega}$$



# Differential equation

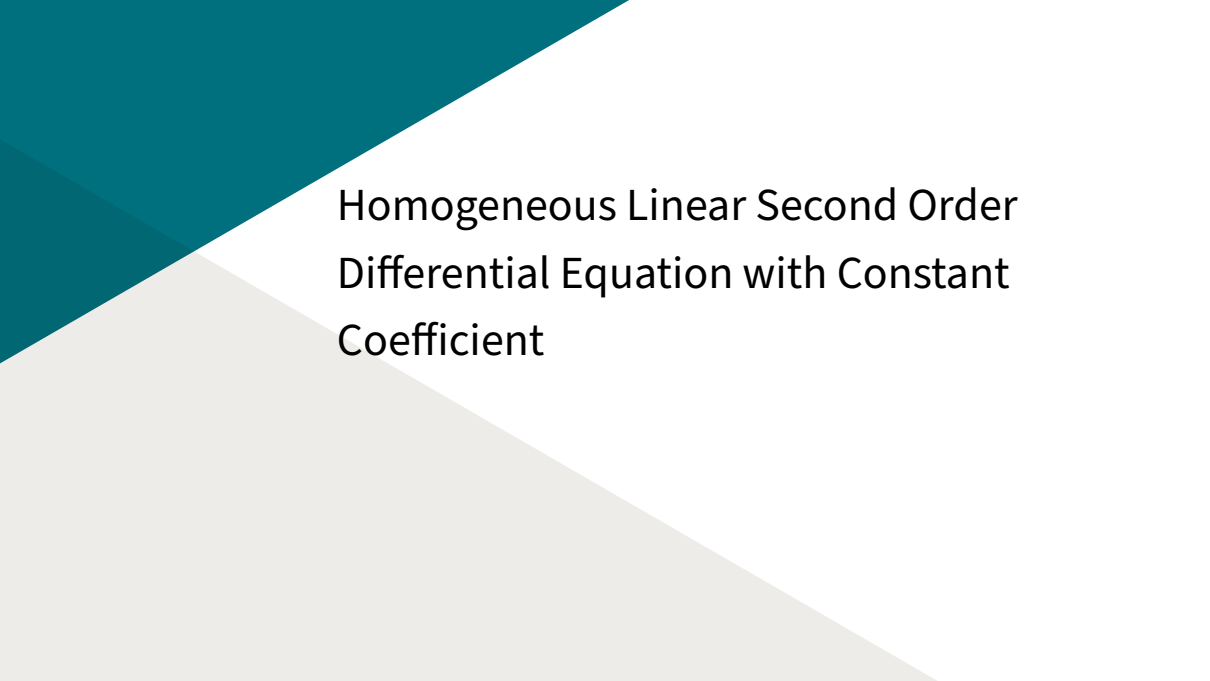
## Second order differential equation

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# Overview

1. Homogeneous Linear Second Order Differential Equation with Constant Coefficient
2. Non Homogeneous Linear Second Order Differential Equation with Constant Coefficient
3. QUESTIONS

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# Homogeneous Linear Second Order Differential Equation with Constant Coefficient

# Homogeneous Linear Second Order Differential Equation with Constant Coefficient

Definition

$$a_0y^n + a_1y^{n-1} + \dots + a_ny = 0 \tag{1}$$

is a homogeneous  $n^{th}$  order differential equation.

# Homogeneous Linear Second Order Differential Equation with Constant Coefficient

## Solution

To solve this equation we just suppose that  $y = e^{\lambda x}$  is a solution and we substitute in the equation, this will convert the equation to algebraic equation, we solve it and we will have the following:

- ▶ Real Solution: then the solution for the DE is  $y = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x}$ .
- ▶ Complex Solution: then the solution for the DE is  $y = e^{ax} * (C_1 \cos(bx) + C_2 \sin(bx))$ .
- ▶ Repeated Solution: then the solution for the DE is  $y = (C_1 + C_2 x) e^{\lambda x}$ .

# Homogeneous Linear Second Order Differential Equation with Constant Coefficient

## Example1

Solve the following DE:

$$y'' - 5y' + 6 = 0 \quad (2)$$

# Homogeneous Linear Second Order Differential Equation with Constant Coefficient

Example2

Solve the following DE:

$$\frac{d^2y}{dx^2} + 4y = 0 \quad (3)$$

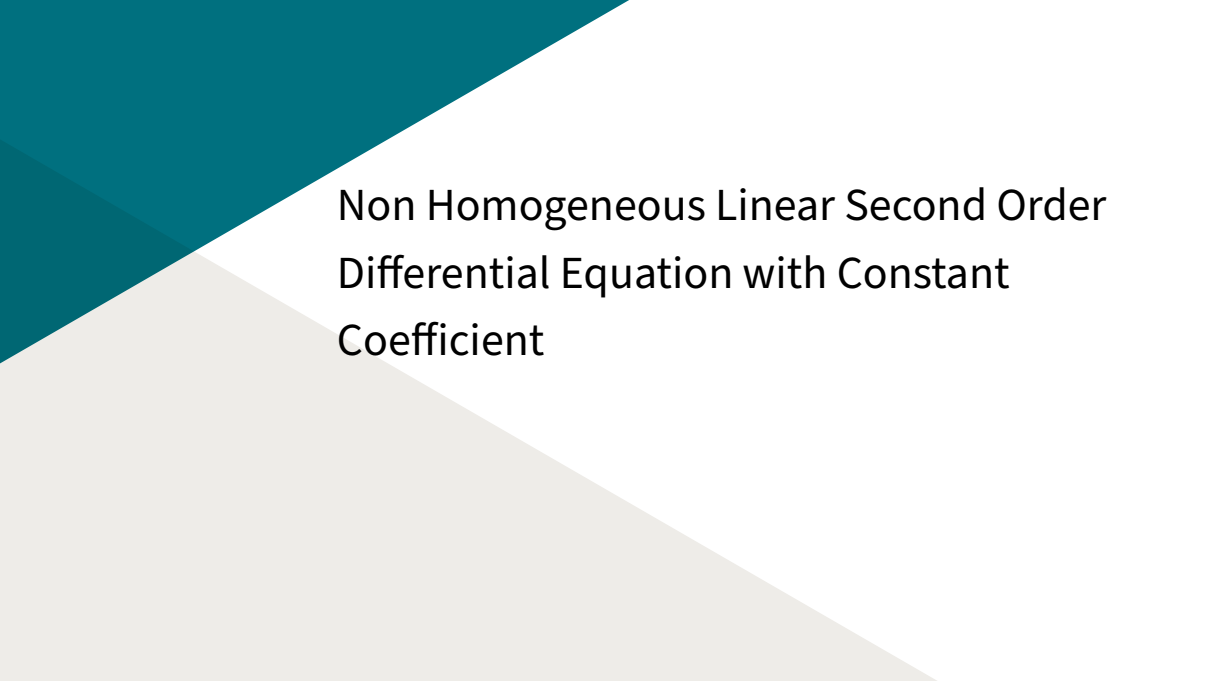
# Homogeneous Linear Second Order Differential Equation with Constant Coefficient

Example3

Solve the following DE:

$$y'' + 6y' + 9y = 0 \quad (4)$$



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# Non Homogeneous Linear Second Order Differential Equation with Constant Coefficient

# Non Homogeneous Linear Second Order Differential Equation with Constant Coefficient

Definition

$$a_0y^n + a_1y^{n-1} + \dots + a_ny = f(x) \tag{5}$$

is a non homogeneous  $n^{th}$  order differential equation.

# Non Homogeneous Linear Second Order Differential Equation with Constant Coefficient

## Solution

The partial solution of this equation must be supposed with respect to  $f(x)$ , where:

- ▶  $f(x) = ax^2 + bx + c$  then the partial solution could be  $z = Ax^2 + Bx + C$ .
- ▶  $f(x) = ae^{mx}$  then the partial solution could be  $z = Ae^{mx}$ .
- ▶  $f(x) = m\cos(wx) + m\sin(wx)$  then the partial solution could be  $z = A\cos(wx) + B\sin(wx)$ .

We substitute in the DE and calculate the constant and we will have the partial solution.

# Non Homogeneous Linear Second Order Differential Equation with Constant Coefficient

## Example1

Solve the following DE:

$$y'' - 4y' + 13y = 2x + 1 \quad (6)$$

# Non Homogeneous Linear Second Order Differential Equation with Constant Coefficient

Example2

Solve the following DE:

$$y'' - 5y' + 6y = e^x \quad (7)$$

# Non Homogeneous Linear Second Order Differential Equation with Constant Coefficient

## Example3

Find the partial solution for the following DE:

$$\frac{y''}{x} - \frac{y'}{x} + \frac{4y}{x} = e^{-2x} \quad (8)$$

# Non Homogeneous Linear Second Order Differential Equation with Constant Coefficient

## Example4

Find the partial solution for the following DE:

$$y'' - y = e^x + \sin(x) \quad (9)$$

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QUESTIONS