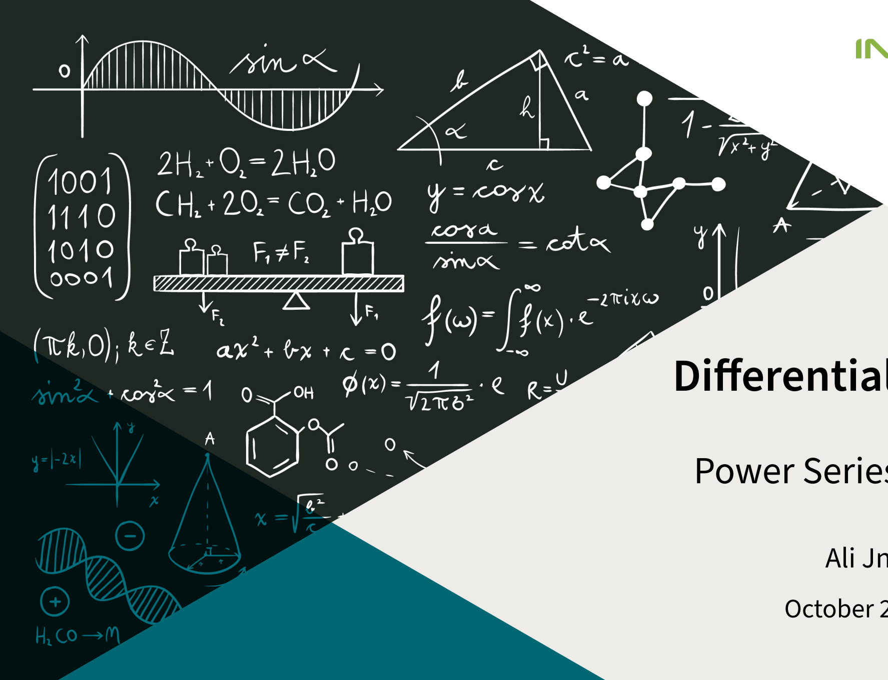

Differential equation

Power Series Approach

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Overview

1. Basic types of series

2. The Power Series Methods

Basic types of series

Basic types of series

Famous Series

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots \tag{1}$$

Basic types of series

Famous Series

$$cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots \tag{2}$$

$$sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^6}{6!} - \dots \tag{3}$$

Basic types of series

Famous Series

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \dots \tag{4}$$

Something looks wrong about the last expression. The expression is correct **IFF** $|x| < 1$

Basic types of series

Taylor

The Taylor series with center x = a of the function f is series:

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots \tag{5}$$

assuming that f is infinitely differentiable at x = a.

Basic types of series

Maclaurin

The Taylor series with a = 0 is Maclaurin series.

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}(x)^n = f(0) + f'(0)(x) + \frac{f''(0)}{2!}(x)^2 + \dots \tag{6}$$

assuming that f is infinitely differentiable at x = a.

The Power Series Methods

The Power Series Methods

Power Series Operation

Let f(x) and g(x) have power series representations:

$$f(x) = \sum_{n=0}^{\infty} a_n x^n \quad and \quad g(x) = \sum_{n=0}^{\infty} b_n x^n \tag{7}$$

then

$$f(x) + g(x) = \sum_{n=0}^{\infty} c_n x^n \quad where \quad c_n = a_n + b_n \tag{8}$$

and

$$f(x)g(x) = \sum_{n=0}^{\infty} c_n x^n \quad where \quad c_n = \sum_{k=0}^{\infty} a_k a_{n-k} \tag{9}$$

The Power Series Methods

Radius of Convergence

If we have the following power series:

$$\sum_{n=0}^{\infty} c_n x^n \tag{10}$$

suppose that the limit:

$$\rho = \lim_{n\to\infty} \left| \frac{c_n}{c_{n+1}} \right| \tag{11}$$

we have:

- if $\rho = 0$ then the series **Diverges** for all $x \neq 0$.
- if $0 < \rho < \infty$ then the series: **Converges** if $|x| < \rho$ and **Diverges** if $|x| > \rho$.

The Power Series Methods

Solving

To solve the differential equation using power series approach we follow the following procedure:

1. We suppose that $y = \sum_{n=0}^{\infty} C_n X^n$ is a solution for the DE.
2. We make all the sequences have the same power X^m and start $\sum_{m=?}$.
3. We find the constants C_i.
4. We substitute in (1) to have general solution for the DE.

The Power Series Methods

Example1

$$\frac{dy}{dx} - 2xy = 0 \tag{12}$$

The Power Series Methods

Example2

$$(x-3)y' + 2y = 0 \tag{13}$$