

# Differential equation

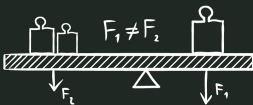
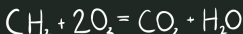
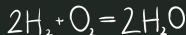
## First order differential equations

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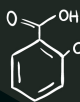
$$\begin{pmatrix} 1001 \\ 1110 \\ 1010 \\ 0001 \end{pmatrix}$$



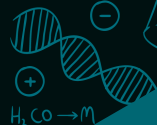
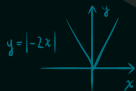
$$(\pi k, 0); k \in \mathbb{Z}$$

$$ax^2 + bx + c = 0$$

$$\sin^2 \alpha + \cos^2 \alpha = 1$$



$$\phi(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{x^2}{2\sigma^2}}$$



$$y = \cos x$$

$$\frac{\cos \alpha}{\sin \alpha} = \cot \alpha$$

$$f(\omega) = \int_{-\infty}^{\infty} f(x) \cdot e^{-2\pi i x \omega}$$



$$1 - \frac{1}{\sqrt{x^2 + y^2}}$$

A

$$x = \sqrt{\frac{a^2}{c}}$$

# Overview

## 1. Variable Variation Technique

The background of the slide is composed of two large, overlapping geometric shapes. A teal-colored shape occupies the top-left corner, while a light gray shape occupies the bottom-left corner. The rest of the slide is white. The text is centered in the white area.

# Variable Variation Technique

# Variable Variation Technique

## Homogeneous Function

Consider that we have  $F$  which is function for two arguments:

**$F$  is homogeneous with degree  $n$ .**

$$F(\lambda x, \lambda y) = \lambda^n F(x, y) \quad (1)$$

For example:

$$f(x, y) = x^3 - x^2y + 2xy^2 + 7y^3 \quad (2)$$

is homogeneous function with degree ( $n = ?$ ).

# Variable Variation Technique

## Examples

Determine if the following are homogeneous and with which degree?

$$f(x, y) = e^{\frac{y}{x}} + \sin\left(\frac{y}{x}\right) \quad (3)$$

$$f(x, y) = \frac{x - y + 1}{x + y + 2} \quad (4)$$

# Variable Variation Technique

FO ODE with homogeneous coefficient

If we have the following differential equation:

$$M(x,y)dx + N(x,y)dy = 0 \quad (5)$$

If  $N(x,y)$  and  $M(x,y)$  are homogeneous with the same degree  $n$   
then the equation is homogeneous differential equation.

we can reduce it to separable one using  $u = \frac{x}{y}$  or  $v = \frac{y}{x}$ .

# Variable Variation Technique

## Example

Solve the following DE:

$$(x + y)dx - xdy = 0 \quad (6)$$

# Variable Variation Technique

## Example

Solve the following DE:

$$x^2y' = x^2 + xy + y^2 \quad (7)$$



# Variable Variation Technique

## Bernoulli Equation

Bernoulli equation is an equation that have the following form(pattern):

$$y' + g(x)y = f(x)y^k \quad (8)$$

# Variable Variation Technique

Bernoulli Equation cont...

$K = 0$

The equation become linear non-homogeneous equation.

$$y' + g(x)y = f(x) \quad (9)$$

$k = 1$

The equation become linear homogeneous separable equation.

$$y' + (g(x) - f(x))y = 0 \quad (10)$$

# Variable Variation Technique

Bernoulli Equation cont...

$$f(x) = 0$$

then we have what called complementary equation.

$$y' + g(x)y = 0 \tag{11}$$

Can be solved by variable separation(The equation have a trivial solution ( $y = 0$ ) and when ( $y \neq 0$ ) it can solved by variable separation.

# Variable Variation Technique

## Example

Solve the following DE:

$$y' - y = xy \quad (12)$$

# Variable Variation Technique

## Example

Solve the following DE:

$$y' - y = xy^2 \quad (13)$$

Hint: use the substitution  $y = e^x u$ .