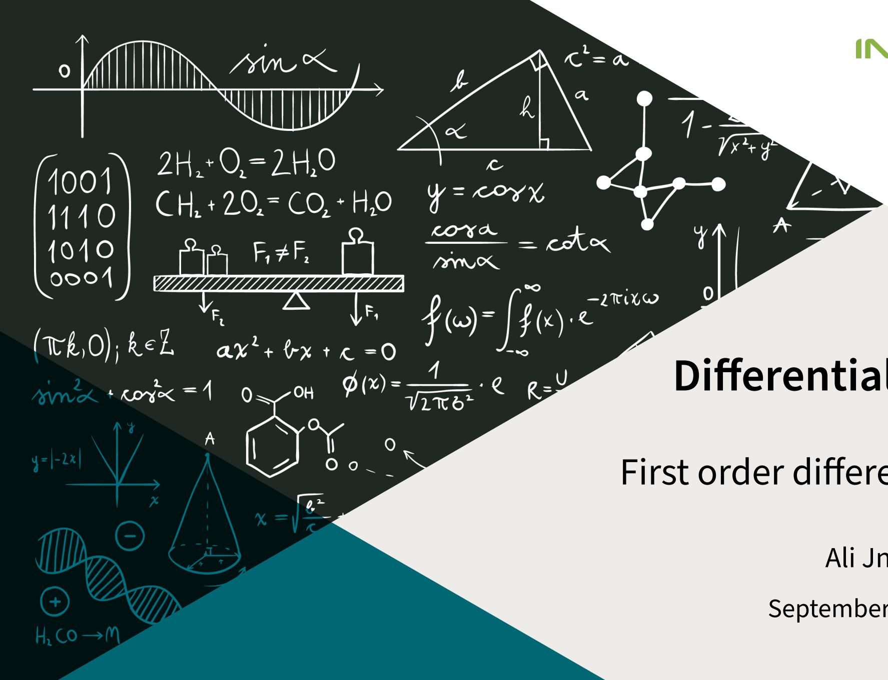

INNOPOLIS UNIVERSITY

Differential equation

First order differential equations

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Overview

1. Variable Variation Technique

Variable Variation Technique

Variable Variation Technique

Homogeneous Function

Consider that we have F which is function for two arguments:

F is homogeneous with degree n.

$$F(\lambda x, \lambda y) = \lambda^n F(x, y) \tag{1}$$

For example:

$$f(x, y) = x^3 - x^2 y + 2xy^2 + 7y^3 \tag{2}$$

is homogeneous function with degree (n = ?).

Variable Variation Technique

Examples

Determine if the following are homogeneous and with which degree?

$$f(x, y) = e^{\frac{y}{x}} + sin(\frac{y}{x}) \tag{3}$$

$$f(x, y) = \frac{x - y + 1}{x + y + 2} \tag{4}$$

Variable Variation Technique

FO ODE with homogeneous coefficient

If we have the following differential equation:

$$M(x, y)dx + N(x, y)dy = 0 \tag{5}$$

If N(x,y) and M(x,y) are homogeneous with the same degree n

then the equation is homogeneous differential equation.

we can reduce it to separable one using $u = \frac{x}{y}$ or $v = \frac{y}{x}$.

Variable Variation Technique

Example

Solve the following DE:

$$(x + y)dx - xdy = 0 \tag{6}$$

Variable Variation Technique

Example

Solve the following DE:

$$x^2y' = x^2 + xy + y^2 \tag{7}$$

Variable Variation Technique

Bernoulli Equation

Bernoulli equation is an equation that have the following form(pattern):

$$y' + g(x)y = f(x)y^k \tag{8}$$

Variable Variation Technique

Bernoulli Equation cont...

$K = 0$

The equation become linear non-homogeneous equation.

$$y' + g(x)y = f(x) \tag{9}$$

$k = 1$

The equation become linear homogeneous separable equation.

$$y' + (g(x) - f(x))y = 0 \tag{10}$$

Variable Variation Technique

Bernoulli Equation cont...

$f(x) = 0$

then we have what called complementary equation.

$$y' + g(x)y = 0 \tag{11}$$

Can be solved by variable separation(The equation have a trivial solution ($y = 0$) and when ($y \neq 0$) it can solved by variable separation.

Variable Variation Technique

Example

Solve the following DE:

$$y' - y = xy \tag{12}$$

Variable Variation Technique

Example

Solve the following DE:

$$y' - y = xy^2 \tag{13}$$

Hint: use the substitution $y = e^x u$.