

Differential equation

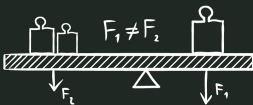
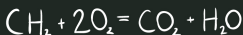
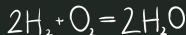
First order differential equations

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September 6, 2022



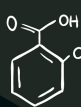
$$\begin{pmatrix} 1001 \\ 1110 \\ 1010 \\ 0001 \end{pmatrix}$$



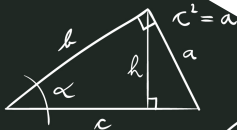
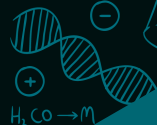
$$(\pi k, 0); k \in \mathbb{Z}$$

$$ax^2 + bx + c = 0$$

$$\sin^2 \alpha + \cos^2 \alpha = 1$$



$$\phi(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{x^2}{2\sigma^2}}$$



$$y = \cos x$$

$$\frac{\cos \alpha}{\sin \alpha} = \cot \alpha$$

$$f(\omega) = \int_{-\infty}^{\infty} f(x) \cdot e^{-2\pi i x \omega}$$



Overview

1. Introduction

2. Separable 1st order DE

The background of the slide is composed of two large, overlapping geometric shapes. A teal-colored shape occupies the top-left corner, while a light beige shape occupies the bottom-left corner. The rest of the slide is white. The word "Introduction" is centered in the white area.

Introduction

Introduction

Differential Equations of the first order

First order DE is a Differential equation which has the following general form:

$$F(x, y, y') = 0 \quad (1)$$

The solution of the Differential equation of the first order could be:

- ▶ Unique solution.
- ▶ Many solutions.
- ▶ No solution.

Separable 1st order DE

Separable 1st order DE

Suppose we have the following differential equation:

$$y' = f(x, y) \quad (2)$$

if we can rewrite it in the following form:

$$g(y) \frac{dy}{dx} + h(x) = 0 \quad (3)$$

or the equivalent form:

$$g(y)dy + h(x)dx = 0 \quad (4)$$

then we have a separable differential equation because we could separate variable x and y totally.

Separable 1st order DE

To solve the equation lets integrate:

$$\int g(y)dy + \int h(x)dx = C \quad (5)$$

Where C is a constant, we have only one constant because the equation is from the first order. After calculating the integration we get:

$$G(y) + H(x) = C \quad (6)$$

Which is the general solution of the differential equation.

Separable 1st order DE

The 1st order differential equation with separable variables could have other forms:



$$g_1(y)h_1(x)dy + g_2(y)h_2(x)dx = 0 \quad (7)$$



$$\frac{dy}{dx} + g(y)h(x) = 0 \quad (8)$$

Can you find the trick that converts the previous equations to the equation 4.

Separable 1st order DE

Example

Solve the following DE:

$$x^2 y' + y = 0 \quad (9)$$

Separable 1st order DE

Example

Solve the following DE:

$$xy' + \ln(x)y = 0 \quad (10)$$

Separable 1st order DE

Example

Solve the following DE:

$$x(y^2 - 1) + y(x^2 + 1)\frac{dy}{dx} = 0 \quad (11)$$

Separable 1st order DE

Initial value

Let's consider the following DE:

$$xy' = 2y \quad (12)$$

Check if $y = Ax^2$ is a solution?

- ▶ How many solutions if $y(1) = 1$?
- ▶ How many solutions if $y(0) = 0$?
- ▶ How many solutions if $y(0) = 1$?

Separable 1st order DE

Example

Solve the following DE:

$$y' = 4 - y \quad (13)$$

considering the following:

1. $y(0) = 1.$
2. $y(0) = 5.$

Separable 1st order DE

Example

Solve the following DE:

$$(x^2 + 1)y' + (y^2 + 1) = 0 : y(0) = 1 \quad (14)$$

Separable 1st order DE

Example

Solve the following DE:

$$xy' + (1 + \frac{1}{\ln(x)})y = 0 : y(e) = 1 \quad (15)$$

Separable 1st order DE

Self practice

Solve the following DE:

$$e^{(2x-y)}dx + e^{(x+y)}dy = 0 \quad (16)$$