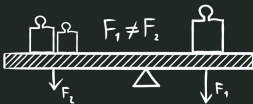
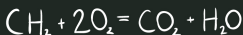
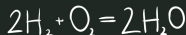




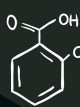
$$\begin{pmatrix} 1001 \\ 1110 \\ 1010 \\ 0001 \end{pmatrix}$$



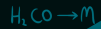
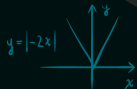
$$(\pi k, 0); k \in \mathbb{Z}$$

$$ax^2 + bx + c = 0$$

$$\sin^2 \alpha + \cos^2 \alpha = 1$$



$$\phi(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{x^2}{2\sigma^2}} \quad R = \mathbb{R}$$



$$y = \cos x$$

$$\frac{\cos \alpha}{\sin \alpha} = \cot \alpha$$

$$f(\omega) = \int_{-\infty}^{\infty} f(x) \cdot e^{-2\pi i x \omega} dx$$



Differential equation

Numerical Methods

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Overview

1. Euler Method

2. Runge Kutta

The background consists of two large, overlapping geometric shapes. A teal-colored shape occupies the top-left corner, while a light gray shape occupies the bottom-left corner. The rest of the background is white. The text 'Euler Method' is centered in the white area.

Euler Method

Euler Method

Input, Output.

- ▶ Input: Initial value problem:
 - ▶ $y' = f(x, y)$.
 - ▶ $x \in [a, b]$.
 - ▶ $y(a) = c$.
- ▶ Output:
 - ▶ Tabulated function $(x_m, u_m) \big|_{m=0}^{m=n}$.
 - ▶ A real value u_n approximating $y(b)$.

Euler Method

Steps.

1. Choose an integer $n \geq 1$ such that $h = \frac{b-a}{n}$ "What is h , can we choose h first?".
2. Let $x_0 = a, u_0 = y(a) = c$.
3. if $x_m (0 \leq m < n)$ is defined then:
 - ▶ $x_{m+1} = x_m + h,$
 - ▶ $d_m = h \cdot f(x_m, u_m),$
 - ▶ $u_{m+1} = u_m + d_m.$

Euler Method

Example

Find using Euler an approximation for the function $y = x^2$ for $x \in [1, 1.5]$ and $y(1) = 1$, The question could also be find the approximate value of $y(1.5)$ using the Euler method if you know that $y(1) = 1$.

Euler Method

Solution

First we have to find the derivative:

$$y(x) = x^2 \Rightarrow y' = 2x$$

Then we have to calculate n or h depend on the information in the question:

$$\text{if } h = 0.1 \text{ then } n = \frac{b-a}{h} = \frac{1.5-1}{0.1} = 5.$$

Euler Method

Solution cont...

Now we can start to calculate:

n	x_n	$y_n = y_{n-1} + d_n$	$f(x_n, y_n) = 2x$	$d_n = h.f(x_n, y_n)$
0	1	1	2	0.2
1	1.1	1.2	2.2	0.22
2	1.2	1.42	2.4	0.24
3	1.3	1.66	2.6	0.26
4	1.4	1.92	2.8	0.28
5	1.5	2.2	-	-

Euler Method

Example

Find using Euler an approximation for the solution of the equation $y' = x + y$ for $x \in [0, 1]$ and $y(0) = 0$ and ($h = 0.2$).

Euler Method

Solution

Now we can start to calculate:

n	x_n	$y_n = y_{n-1} + d_n$	$f(x_n, y_n) = 2x$	$d_n = h.f(x_n, y_n)$
0	0	0	0	0
1	0.2	0	0.2	0.04
2	0.4	0.04	0.44	0.088
3	0.6	0.128	0.728	0.1456
4	0.8	0.2736	1.0736	0.21472
5	1	0.48832	-	-

Euler Method

Example

Find using Euler an approximation for square root for real values $[1, \infty[$, use $h = 0.1$ and calculate $\sqrt{1.5}$.

The background of the slide is composed of two large, overlapping geometric shapes. A teal-colored shape occupies the top-left corner, while a light gray shape occupies the bottom-left corner. The rest of the slide is white. The text 'Runge Kutta' is centered in the white area.

Runge Kutta

Runge Kutta

Input, Output.

- ▶ Input: Initial value problem:
 - ▶ $y' = f(x, y)$.
 - ▶ $x \in [a, b]$.
 - ▶ $y(a) = c$.
- ▶ Output:
 - ▶ Tabulated function $(x_m, u_m) \big|_{m=0}^{m=n}$.
 - ▶ A real value u_n approximating $y(b)$.

Runge Kutta

Steps.

1. Choose an integer $n \geq 1$ such that $h = \frac{b-a}{n}$ "What is h , can we choose h first?".
2. Let $x_0 = a, u_0 = y(a) = c$.
3. if $x_m (0 \leq m < n)$ is defined then:
 - ▶ $y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4),$
 - ▶ $k_1 = h.y'(x_n, y_n),$
 - ▶ $k_2 = h.y'(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}).$
 - ▶ $k_3 = h.y'(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}).$
 - ▶ $k_4 = h.y'(x_n + h, y_n + k_3).$

Runge Kutta

Example

Using Runge Kutta method calculate an approximation for the solution of differential equation $y' = y - x$, where $y(0) = 2$ and $h = 0.1$, one step is enough.