# Differential equation

## System of Linear Homogeneous Equations

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# Overview

1. System of linear homogeneous equations

2. QUESTIONS

# System of linear homogeneous equations

# System of linear homogeneous equations

## Definition

If the equations have the following form:

$$\begin{cases} y_1'(x) = \left(\sum_{i\in[1..n]} a_{1i} y_i(x)\right) + f_1(x) \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ y_n'(x) = \left(\sum_{i\in[1..n]} a_{ni} y_i(x)\right) + f_n(x) \end{cases}$$

Then we say that they are system of linear equations.

If we set $f_1(x), \dots f_n(x)$ to zeros then we have a system of linear homogeneous equations.

# System of linear homogeneous equations

## Solution using elimination

The easiest way to solve this system id to suppose that $D = \frac{d}{dx}$ then we substitute in the equations, and try to isolate on of the variables, re-substitute D solve the resulting differential equation, and finally find the other variables.

# System of linear homogeneous equations

## Solution using elimination: Example1

Solve the following system:

$$\begin{aligned} y_1' &= 2y_1 + 3y_2 \\ y_2' &= 4y_1 - 2y_2 \end{aligned} \tag{1}$$

# System of linear homogeneous equations

## Solution using Lambda

we can rewrite the system illustrated in 1:

$$y' = \begin{bmatrix} 2 & 3 \\ 4 & -2 \end{bmatrix} y \tag{2}$$

Then we can find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 2 & 3 \\ 4 & -2 \end{bmatrix}$

# System of linear homogeneous equations

## Solution using Lambda: Eigenvalues and Eigenvectors

Calculate the eigenvalues and eigenvectors for the following matrix:

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \tag{3}$$

# System of linear homogeneous equations

## Solution using Lambda: cont.

After finding the eigenvalues $\lambda_1, \lambda_2$ and the eigenvectors $u_1, u_2$ the solution of the system of equation will be:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = C_1 e^{\lambda_1 t} u_1 + C_2 e^{\lambda_2} u_2 \tag{4}$$

**Note:** $u_1$ is the eigenvector related to $\lambda_1$ and $u_2$ is the eigenvector related to $\lambda_2$

# System of linear homogeneous equations

## Solution using Lambda: Example1

Solve the system illustrated in 2 using eigenvalues and eigenvectors.

# System of linear homogeneous equations

## Solution using Lambda: Example2

Solve the folowing system:

$$x'(t) = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix} x(t) \tag{5}$$

# QUESTIONS