INNOPOLIS UNIVERSITY

Differential equation

Stability, Laplace transforms

Ali Jnadi

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Overview

1. Stability
2. Laplace Transforms
3. QUESTIONS

Stability

Stability

Definition

Stability is a fundamental property of every control system.

It also could be generalised to other systems, like chemical, medical or financial.

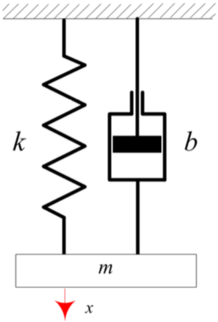
You could study if the solution of the equation you have will diverge or converge.

Stability

Mass Spring Damper

Let’s find the equation of the following mechanical system!

1. What are the equilibrium points for the system?
2. Is this system stable?



Stability

Mass Spring Damper: Lyapunov Solution

To apply the Lyapunov solution, we simply find a function which have two main properties:

1. It is strictly positive unless both state variables are zeros ($x = 0, \dot{x} = 0$).
2. The function is monotonically decreasing when the state variables vary according to the initial dynamic equation.

Stability

Mass Spring Damper: Lyapunov Solution

To do that easily, find a good Lyapunov function, and check the following:

1. $V(x) > 0 : x \neq 0$
2. $\dot{V} \leq 0$

Then the system is **Lyapunov** stable, otherwise it is not stable, that mean the solution that start near x_0 stay near x_0, if all the solutions that start near x_0 end at x_0 then the system is **asymptotically** stable.

Stability

Mass Spring Damper: Lambda Solution

After we solve the equation and find $\lambda = Re + i * Im$ we have three posibilities:

- $Re = 0$ **Lyapunov** stable.
- $Re < 0$ **Asymptotically** stable.
- $Re > 0$ **Unstable**.

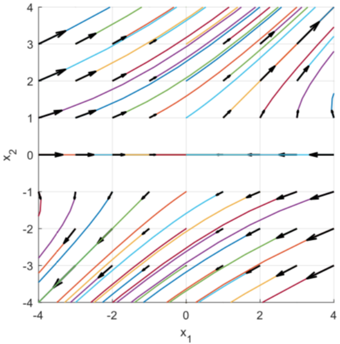
Stability

Example1:

Study the stability of the following system:

$$\begin{aligned} \dot{x}_1 &= -x_1 + 3x_2 \\ \dot{x}_2 &= +2x_2 \end{aligned} \tag{1}$$

Stability



Saddle

Stability

Example2:

Study the stability of the following system:

$$\ddot{x} + 10\dot{x} + 5x = 0 \tag{2}$$

Laplace Transforms

Laplace Transforms

Definition

If we have the following function f(x) defined in $[0, \infty[$ then the Laplace transform can be defined as:

$$L[f(t)] = \int_0^\infty e^{-st} f(t)\, dt = F(s) \tag{3}$$

Laplace Transforms

Laplace transform properties:

- $L[Cf(x)] = CL[f(x)]$
- $L[f(x) + g(x)] = L[f(x)] + L[g(x)] = F(s) + G(s)$

Laplace Transforms

Famous Laplace transformation:

$L[1] = \frac{1}{s}$
$L[c] = \frac{c}{s}$

Laplace Transforms

Famous Laplace transformation:

$$L[e^(ax)] = \frac{1}{s-a}$$
$$L[x^n] = \frac{n!}{s^{n+1}}$$
$$L[sin(ax)] = \frac{a}{s^2+a^2}$$
$$L[cos(ax)] = \frac{s}{s^2+a^2}$$

Laplace Transforms

Famous Laplace transformation:

$L[sinh(ax)] = \frac{a}{s^2 - a^2}$
$L[cosh(ax)] = \frac{s}{s^2 - a^2}$

Laplace Transforms

Famous Laplace transformation:

$L[sinh(ax)] = \frac{a}{s^2-a^2}$
$L[cosh(ax)] = \frac{s}{s^2-a^2}$

Laplace Transforms

Example:

Find Laplace transformation for the following function:

$$f(x) = 4e^{5x} + 6x^3 - 3sin(4x) + cos(2x) \tag{4}$$

Laplace Transforms

Example:

Find Laplace transformation for the following function:

$$f(x) = sin^2(x) \tag{5}$$

Laplace Transforms

Example:

Find Laplace transformation for the following function:

$$f(x) = e^{-2x} sinh(5x) \tag{6}$$

Laplace Transforms

Example:

Find Laplace transformation for the following function:

$$f(x) = e^{5x}x^2 \tag{7}$$

Laplace Transforms

Example:

Find Laplace transformation for the following function:

$$f(x) = cos(2x)cos(3x) \tag{8}$$

QUESTIONS