Ordinary Differential Equations ODE

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Outline

1st Week

- ODEs Classification.
- General and Particular Solutions.
- Linear ODE Systems.
- Attendance, Rules and Grading (Negotiable).

Exercise 1

ODEs Classification

For each and every of the following differential equations, determine: Linear or nonlinear, homogeneous or nonhomogeneous and which kind of coefficients are involved:

1. $\frac{d^2y}{dx^2} + y = 0$
2. $y\frac{d^2y}{dx^2} + y = 0$
3. $x\frac{d^2y}{dx^2} + y = 0$
4. $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = \sin(x)$
5. $\frac{dy}{dx} + \ln(y) = \ln(x)$
6. $\frac{d^2y}{dx^2} + \sin(y) = 0$
7. $(x+1)^2\frac{d^2y}{dx^2} - 3(x+1)\frac{dy}{dx} + 2y = 3$
8. $\frac{dy}{dx} = \frac{3x^2}{2y}$

Exercise 2

General and Particular Solutions

Verify that $y(x) = ce^{x^2}$ is <u>the general solution</u> of the following ODE, what information we need to exclude the particular solution.

$$\frac{dy}{dx} = 2xy$$

Exercise 3

General and Particular Solutions

Verify that $y(x) = ce^{-\frac{1}{x}}$ is the general solution of the following ODE.

$$\frac{dy}{dx} - \frac{1}{x^2}y = 0$$

Exercise 4

General and Particular Solutions

Verify that $y(t) = \frac{3}{2} + ce^{-2t}$ is the general solution of the following ODE.

$$\frac{dy}{dt} + 2y = 3$$

Exercise 5

General and Particular Solutions

Verify that $\boldsymbol{y(t) = \frac{t^2}{2}}$ is solution of the following ODE.

$$\begin{cases} \dfrac{dy}{dt} = \dfrac{y}{t} + \dfrac{t}{2} \\ y(1) = 1 \end{cases}$$

Exercise 6

Linear ODE Systems

Check that the functions

$$
\begin{aligned}
C_0 &= e^{-k_0 t} \\
C_1 &= k_0 \frac{e^{-k_0 t} - e^{-(k_1+k_2)t}}{k_1 + k_2 - k_0} \\
C_2 &= \frac{k_0 k_1}{k_1 + k_2 - k_0} \left(\frac{1 - e^{-k_0 t}}{k_0} - \frac{1 - e^{-(k_1+k_2)t}}{k_1 + k_2} \right) \\
C_3 &= \frac{k_0 k_2}{k_1 + k_2 - k_0} \left(\frac{e^{-k_0 t} - e^{-k_3 t}}{k_3 - k_0} - \frac{e^{-(k_1+k_2)t} - e^{-k_3 t}}{k_3 - (k_1 + k_2)} \right) \\
C_4 &= 1 - (C_0 + C_1 + C_2 + C_3)
\end{aligned}
$$

satisfy the following linear ODE system:

$$
\begin{aligned}
\frac{dC_0}{dt} &= -k_0 C_0 \\
\frac{dC_1}{dt} &= k_0 C_0 - (k_1 + k_2) C_1 \\
\frac{dC_2}{dt} &= k_1 C_1 \\
\frac{dC_3}{dt} &= k_2 C_1 - k_3 C_3 \\
\frac{dC_4}{dt} &= k_3 C_3
\end{aligned}
$$

Attendance, Rules and Grading.