# Differential equation

## First order differential equations

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# Overview

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# Introduction

# Introduction

## Differential Equations of the first order

**First order DE** is a Differential equation which has the following general form:

$$F(x, y, y') = 0 \tag{1}$$

The solution of the Differential equation of the first order could be:

- Unique solution.
- Many solutions.
- No solution.

# Separable 1st order DE

# Separable 1st order DE

Suppose we have the following differential equation:

$$y' = f(x, y) \tag{2}$$

if we can rewrite it in the following form:

$$g(y)\frac{dy}{dx} + h(x) = 0 \tag{3}$$

or the equivalent form:

$$g(y)dy + h(x)dx = 0 \tag{4}$$

then we have a separable differential equation because we could separate variable x and y totally.

# Separable 1st order DE

To solve the equation lets integrate:

$$\int g(y)dy + \int h(x)dx = C \tag{5}$$

Where C is a constant, we have only one constant because the equation is from the first order. After calculating the integration we get:

$$G(y) + H(x) = C \tag{6}$$

Which is the general solution of the differential equation.

# Separable 1st order DE

The 1st order differential equation with separable variables could have other forms:

- $$g_1(y)h_1(x)dy + g_2(y)h_2(x)dx = 0 \tag{7}$$
- $$\frac{dy}{dx} + g(y)h(x) = 0 \tag{8}$$

Can you find the trick the converts the previous equations to the equation 4.

# Separable 1st order DE

## Example

Solve the following DE:

$$x^2y' + y = 0 \tag{9}$$

# Separable 1st order DE

## Example

Solve the following DE:

$$xy' + ln(x)y = 0 \tag{10}$$

# Separable 1st order DE

## Example

Solve the following DE:

$$x(y^2 - 1) + y(x^2 + 1)\frac{dy}{dx} = 0 \tag{11}$$

# Separable 1st order DE

## Initial value

Let's consider the following DE:

$$xy' = 2y \tag{12}$$

Check if $y = Ax^2$ is a solution?

- How many solutions if y(1) = 1?
- How many solutions if y(0) = 0?
- How many solutions if y(0) = 1?

# Separable 1st order DE

## Example

Solve the following DE:

$$y' = 4 - y \tag{13}$$

considering the following:

1. $y(0) = 1$.
2. $y(0) = 5$.

# Separable 1st order DE

## Example

Solve the following DE:

$$(x^2 + 1)y' + (y^2 + 1) = 0 : y(0) = 1 \tag{14}$$

# Separable 1st order DE

## Example

Solve the following DE:

$$xy' + (1 + \frac{1}{ln(x)})y = 0 : y(e) = 1 \tag{15}$$

# Separable 1st order DE

## Self practice

Solve the following DE:

$$e^{(2x-y)}dx + e^{(x+y)}dy = 0 \tag{16}$$