INNOPOLIS UNIVERSITY

Differential equation

Order reduction for second order deferential equation

Ali Jnadi

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Overview

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Introduction

Introduction

Definition

$$y'' = f(x, y, y') \tag{1}$$

is a second order differential equation in explicit form.

Introduction

Special cases

It is not easy to solve these kind of equations, but we have a special cases where we can reduce the order of the equation to first order then we can solve it using the methods we already now.

- f doesn't depend on $x \iff y" = f(y, y')$.
- f doesn't depend on $y \iff y" = f(x, y')$.

In both cases we try variable substitution $p = y'$.

Example1

Example1

Solve the following differential equation

$$y'' = \frac{y'^2}{y} \tag{2}$$

The equation can be written as $y'' = f(y', y)$ to solve it we assume that:
$y' = p(y(x)) \longrightarrow y'' = \frac{dp}{dy}\frac{dy}{dx} = p'p$ and we substitute in the equation.

Example1

Cont...

$$pp' = \frac{p^2}{y}$$

$$\frac{dp}{dy} = \frac{p}{y}$$

$$\frac{dp}{p} = \frac{dy}{y}$$

$$p = C_1 y = y'$$

Example1

Cont...

$$\frac{dy}{dx} = C_1 y$$

$$\frac{dy}{y} = C_1 dx$$

$$y = C_2 e^{C_1 x}$$

Example 2

Example 2

Solve the following differential equation:

$$y'' = \frac{1}{y^3} \tag{3}$$

Example 3

Example 3

Solve the following (IVP) differential equation:

$$y'' = \frac{1}{y^3} \tag{4}$$

If you know that $y(1) = 0$ and $y'(1) = 1$.

Example 4

Example 4

Solve the following (IVP) differential equation:

$$xy'' = 2x - y' \tag{5}$$

If you know that $y(1) = \frac{1}{2}$ and $y'(1) = 1$.

The equation can be written as $y'' = f(x, y')$ to solve it we assume that:
$y' = p(x) \longrightarrow y'' = p'$ and we substitute in the equation.

Example 4

Cont...

$$xp' + p = 2x$$

$$p' + \frac{p}{x} = 2$$

$$u = \frac{p}{x} \longrightarrow p = ux \longrightarrow p' = u + xu'$$

$$u + u'x + u = 2 \longrightarrow 2u - 2 = -\frac{xdu}{dx}$$

$$\frac{du}{u-1} = \frac{-2dx}{x} \longrightarrow ln(u-1) = -2ln(x) + C_1$$

Example 4

Cont...

$$u = \frac{C_1}{x^2} + 1 = \frac{p}{x}$$

$$p = \frac{C_1}{x} + x$$

$$y' = \frac{C_1}{x} + x = \frac{dy}{dx}$$

$$y = C_1 ln(x) + \frac{x^2}{2} + C_2$$

Example 4

Cont...

$$y'(1) = 1 \longrightarrow 1 = C_1 + 1 \longrightarrow C_1 = 0$$

$$y(1) = \frac{1}{2} = \frac{1}{2} + C_2 = 0 \longrightarrow C_2 = 0$$

$$y = \frac{x^2}{2}$$

QUESTIONS