

# Differential equation

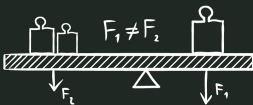
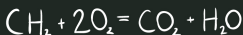
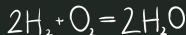
## Power Series Approach

Ali Jnadi

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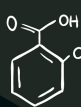
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$$(\pi k, 0); k \in \mathbb{Z}$$

$$ax^2 + bx + c = 0$$

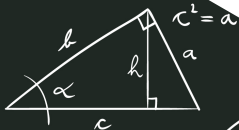
$$\sin^2 \alpha + \cos^2 \alpha = 1$$



$$\phi(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{x^2}{2\sigma^2}}$$



$$x = \sqrt{\frac{b^2}{c}}$$



$$y = \cos x$$

$$\frac{\cos \alpha}{\sin \alpha} = \cot \alpha$$

$$f(\omega) = \int_{-\infty}^{\infty} f(x) \cdot e^{-2\pi i x \omega}$$



# Overview

1. Basic types of series
2. The Power Series Methods

The background of the slide is composed of two large, overlapping geometric shapes. A teal-colored shape occupies the top-left corner, while a light gray shape occupies the bottom-left corner. The rest of the slide is white. The text is centered in the white area.

## Basic types of series

# Basic types of series

## Famous Series

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \quad (1)$$

# Basic types of series

## Famous Series

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots \quad (2)$$

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \quad (3)$$

# Basic types of series

## Famous Series

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} = 1 + x + x^2 + x^3 + \dots \quad (4)$$

Something looks wrong about the last expression. The expression is correct **IFF**  $|x| < 1$

# Basic types of series

## Taylor

The Taylor series with center  $x = a$  of the function  $f$  is series:

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x - a)^n = f(a) + f'(a)(x - a) + \frac{f''(a)}{2!} (x - a)^2 + \dots \quad (5)$$

assuming that  $f$  is infinitely differentiable at  $x = a$ .

# Basic types of series

## Maclaurin

The Taylor series with  $a = 0$  is Maclaurin series.

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} (x)^n = f(0) + f'(0)(x) + \frac{f''(0)}{2!} (x)^2 + \dots \quad (6)$$

assuming that  $f$  is infinitely differentiable at  $x = a$ .



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# The Power Series Methods

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## Power Series Operation

Let  $f(x)$  and  $g(x)$  have power series representations:

$$f(x) = \sum_{n=0}^{\infty} a_n x^n \quad \text{and} \quad g(x) = \sum_{n=0}^{\infty} b_n x^n \quad (7)$$

then

$$f(x) + g(x) = \sum_{n=0}^{\infty} c_n x^n \quad \text{where} \quad c_n = a_n + b_n \quad (8)$$

and

$$f(x)g(x) = \sum_{n=0}^{\infty} c_n x^n \quad \text{where} \quad c_n = \sum_{k=0}^{\infty} a_k a_{n-k} \quad (9)$$

# The Power Series Methods

## Radius of Convergence

If we have the following power series:

$$\sum_{n=0}^{\infty} c_n x^n \quad (10)$$

suppose that the limit:

$$\rho = \lim_{n \rightarrow \infty} \left| \frac{c_n}{c_{n+1}} \right| \quad (11)$$

we have:

- ▶ if  $\rho = 0$  then the series **Diverges** for all  $x \neq 0$ .
- ▶ if  $0 < \rho < \infty$  then the series: **Converges** if  $|x| < \rho$  and **Diverges** if  $|x| > \rho$ .

# The Power Series Methods

## Solving

To solve the differential equation using power series approach we follow the following procedure:

1. We suppose that  $y = \sum_{n=0}^{\infty} C_n X^n$  is a solution for the DE.
2. We make all the sequences have the same power  $X^m$  and start  $\sum_{m=?}$ .
3. We find the constants  $C_i$ .
4. We substitute in (1) to have general solution for the DE.

# The Power Series Methods

## Example1

$$\frac{dy}{dx} - 2xy = 0 \quad (12)$$

# The Power Series Methods

## Example2

$$(x - 3)y' + 2y = 0 \quad (13)$$