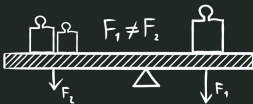
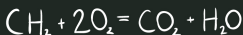
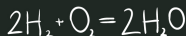




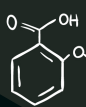
$$\begin{pmatrix} 1001 \\ 1110 \\ 1010 \\ 0001 \end{pmatrix}$$



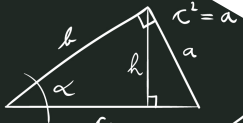
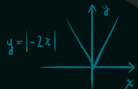
$$(\pi k, 0); k \in \mathbb{Z}$$

$$ax^2 + bx + c = 0$$

$$\sin^2 \alpha + \cos^2 \alpha = 1$$



$$\phi(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{x^2}{2\sigma^2}}$$



$$y = \cos x$$

$$\frac{\cos \alpha}{\sin \alpha} = \cot \alpha$$

$$f(\omega) = \int_{-\infty}^{\infty} f(x) \cdot e^{-2\pi i x \omega} dx$$



Differential equation

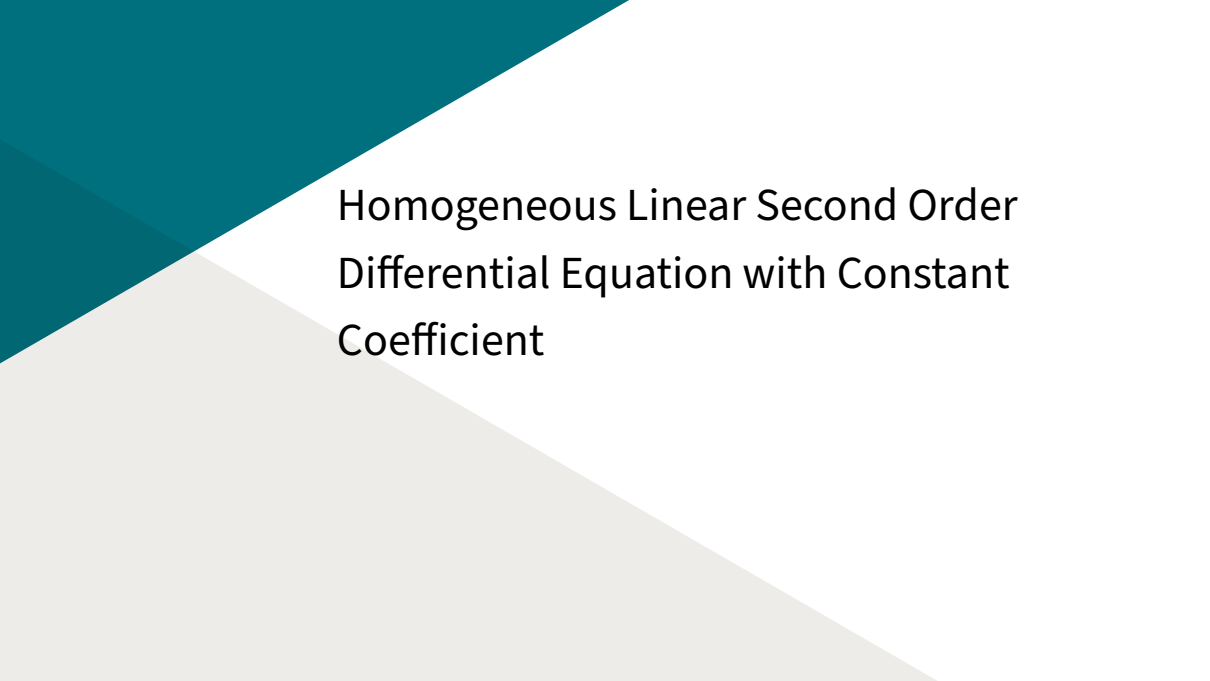
Second order differential equation

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Overview

1. Homogeneous Linear Second Order Differential Equation with Constant Coefficient
2. Non Homogeneous Linear Second Order Differential Equation with Constant Coefficient
3. QUESTIONS

The background of the slide is composed of two large, overlapping geometric shapes. A teal-colored shape, consisting of two triangles meeting at a diagonal line, occupies the upper-left portion of the frame. The remaining area is a light gray shape, also composed of triangles, which fills the lower-left and extends towards the bottom right. The text is positioned in the white space between these two colored areas.

Homogeneous Linear Second Order Differential Equation with Constant Coefficient

Homogeneous Linear Second Order Differential Equation with Constant Coefficient

Definition

$$a_0y^{(n)} + a_1y^{(n-1)} + \dots + a_ny = 0 \quad (1)$$

is a homogeneous n^{th} order differential equation.

Homogeneous Linear Second Order Differential Equation with Constant Coefficient

Solution

To solve this equation we just suppose that $y = e^{\lambda x}$ is a solution and we substitute in the equation, this will convert the equation to algebraic equation, we solve it and we will have the following:

- ▶ Real Solution: then the solution for the DE is $y = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x}$.
- ▶ Complex Solution: then the solution for the DE is $y = e^{ax} * (C_1 \cos(bx) + C_2 \sin(bx))$.
- ▶ Repeated Solution: then the solution for the DE is $y = (C_1 + C_2 x) e^{\lambda x}$.

Homogeneous Linear Second Order Differential Equation with Constant Coefficient

Example1

Solve the following DE:

$$y'' - 5y' + 6y = 0 \quad (2)$$

Homogeneous Linear Second Order Differential Equation with Constant Coefficient

Example2

Solve the following DE:

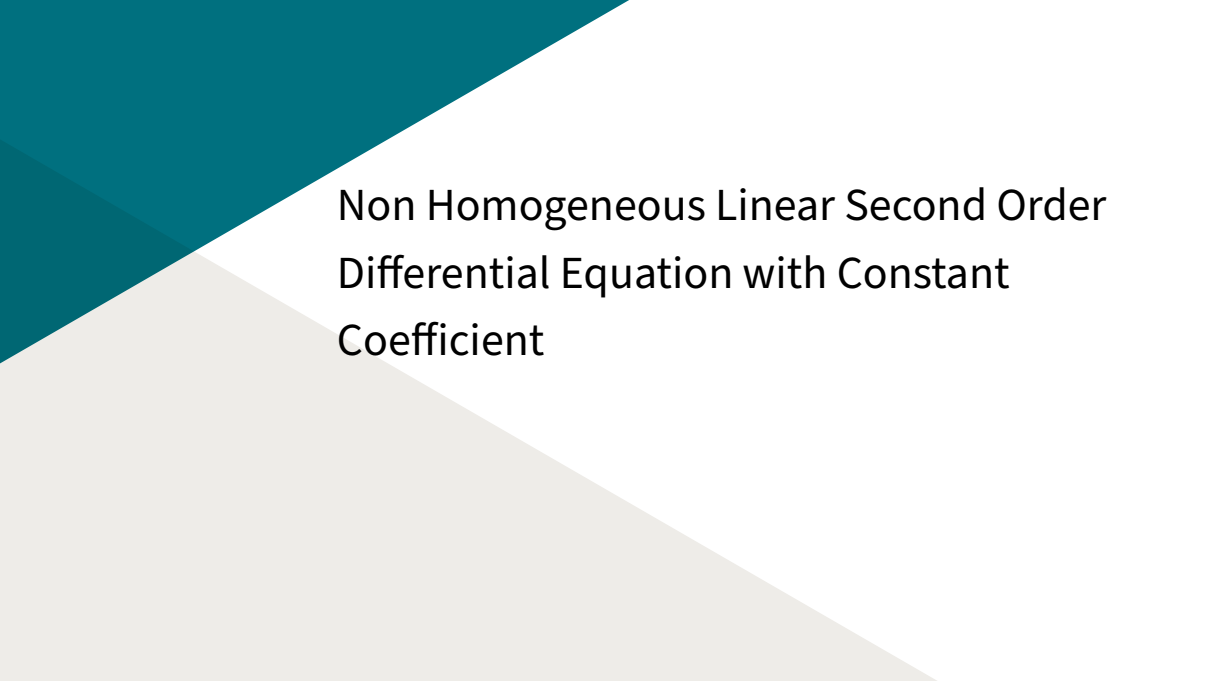
$$\frac{d^2y}{dx^2} + 4y = 0 \quad (3)$$

Homogeneous Linear Second Order Differential Equation with Constant Coefficient

Example3

Solve the following DE:

$$y'' + 6y' + 9y = 0 \quad (4)$$

The background of the slide is composed of two large, overlapping geometric shapes. A teal-colored shape occupies the upper-left portion, while a light gray shape occupies the lower-left portion. The rest of the slide is white. The text is positioned in the white area on the right side.

Non Homogeneous Linear Second Order Differential Equation with Constant Coefficient

Non Homogeneous Linear Second Order Differential Equation with Constant Coefficient

Definition

$$a_0y^{(n)} + a_1y^{(n-1)} + \dots + a_ny = f(x) \quad (5)$$

is a non homogeneous n^{th} order differential equation.

Non Homogeneous Linear Second Order Differential Equation with Constant Coefficient

Solution

The partial solution of this equation must be supposed with respect to $f(x)$, where:

- ▶ $f(x) = ax^2 + bx + c$ then the particular solution could be $z = Ax^2 + Bx + C$.
- ▶ $f(x) = ae^{mx}$ then the particular solution could be $z = Ae^{mx}$.
- ▶ $f(x) = m\cos(wx) + m\sin(wx)$ then the particular solution could be $z = A\cos(wx) + B\sin(wx)$.

We substitute in the DE and calculate the constant and we will have the partial solution.

Non Homogeneous Linear Second Order Differential Equation with Constant Coefficient

Example1

Solve the following DE:

$$y'' - 4y' + 13y = 2x + 1 \quad (6)$$

Non Homogeneous Linear Second Order Differential Equation with Constant Coefficient

Example2

Solve the following DE:

$$y'' - 5y' + 6y = e^x \quad (7)$$

Non Homogeneous Linear Second Order Differential Equation with Constant Coefficient

Example3

Find the partial solution for the following DE:

$$\frac{y''}{x} - \frac{y'}{x} + \frac{4y}{x} = e^{-2x} \quad (8)$$

Non Homogeneous Linear Second Order Differential Equation with Constant Coefficient

Example4

Find the partial solution for the following DE:

$$y'' - y = e^x + \sin(x) \quad (9)$$

The background consists of two large, overlapping geometric shapes. A teal-colored shape is in the upper-left corner, and a light gray shape is in the lower-left corner. They meet at a diagonal line that runs from the top-left towards the bottom-right. The rest of the background is white.

QUESTIONS