INNOPOLIS UNIVERSITY

Differential equation

Second order differential equation

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Overview

1. Homogeneous Linear Second Order Differential Equation with Constant Coefficient

2. Non Homogeneous Linear Second Order Differential Equation with Constant Coefficient

3. QUESTIONS

Homogeneous Linear Second Order Differential Equation with Constant Coefficient

Homogeneous Linear Second Order Differential Equation with Constant Coefficient

Definition

$$a_0 y^{(n)} + a_1 y^{(n-1)} + ... + a_n y = 0 \tag{1}$$

is a homogeneous n^{th} order differential equation.

Homogeneous Linear Second Order Differential Equation with Constant Coefficient

Solution

To solve this equation we just suppose that $y = e^{\lambda x}$ is a solution and we substitute in the equation, this will convert the equation to algebraic equation, we solve it and we will have the following:

- Real Solution: then the solution for the DE is $y = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x}$.
- Complex Solution: then the solution for the DE is $y = e^{ax} * (C_1 cos(bx) + C_2 sin(bx))$.
- Repeated Solution: then the solotion for the DE is $y = (C_1 + C_2 x)e^{\lambda x}$.

Homogeneous Linear Second Order Differential Equation with Constant Coefficient

Example1

Solve the following DE:

$$y'' - 5y' + 6y = 0 \tag{2}$$

Homogeneous Linear Second Order Differential Equation with Constant Coefficient

Example2

Solve the following DE:

$$\frac{d^2y}{dx^2} + 4y = 0 \tag{3}$$

Homogeneous Linear Second Order Differential Equation with Constant Coefficient

Example3

Solve the following DE:

$$y'' + 6y' + 9y = 0 \tag{4}$$

Non Homogeneous Linear Second Order Differential Equation with Constant Coefficient

Non Homogeneous Linear Second Order Differential Equation with Constant Coefficient

Definition

$$a_0y^{(n)} + a_1y^{(n-1)} + ... + a_ny = f(x) \tag{5}$$

is a non homogeneous n^{th} order differential equation.

Non Homogeneous Linear Second Order Differential Equation with Constant Coefficient

Solution

The partial solution of this equation must be supposed with respect to f(x), where:

- $f(x) = ax^2 + bx + c$ then the particular solution could be $z = Ax^2 + Bx + C$.
- $f(x) = ae^{mx}$ then the particular solution could be $z = Ae^{mx}$.
- $f(x) = mcos(wx) + msin(wx)$ then the particular solution could be $z = Acos(wx) + Bsin(wx)$.

We substitute in the DE and calculate the constant and we will have the partial solution.

Non Homogeneous Linear Second Order Differential Equation with Constant Coefficient

Example1

Solve the following DE:

$$y'' - 4y' + 13y = 2x + 1 \tag{6}$$

Non Homogeneous Linear Second Order Differential Equation with Constant Coefficient

Example2

Solve the following DE:

$$y'' - 5y' + 6y = e^x \tag{7}$$

Non Homogeneous Linear Second Order Differential Equation with Constant Coefficient

Example3

Find the partial solution for the following DE:

$$\frac{y''}{x} - \frac{y'}{x} + \frac{4y}{x} = e^{-2x} \tag{8}$$

Non Homogeneous Linear Second Order Differential Equation with Constant Coefficient

Example4

Find the partial solution for the following DE:

$$y'' - y = e^x + sin(x) \tag{9}$$

QUESTIONS