Dynamics of Nonlinear Robotic Systems Assignment1

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Abstract

This report is part of Dynamics of Nonlinear Robotic Systems course for 1st year master students at Innopolis University.
In this report I am working in Niryo one manipulator designed by Niryo. I will develop the kinematic model of the robot, solve both forward and inverse kinematics problems, implement the proposed solution using Matlab and check the validity of my solution by making a test file.

1. Kinematic Scheme

Figure 1 is taken from robot datasheet to illustrate dimension of the robot, Figure 2 describe the kinematic scheme of Niryo robot manipulator, and Figure 3 is coordinates assignment to robot joints.

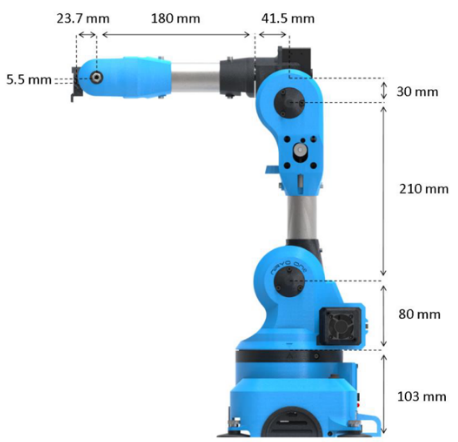

Figure 1

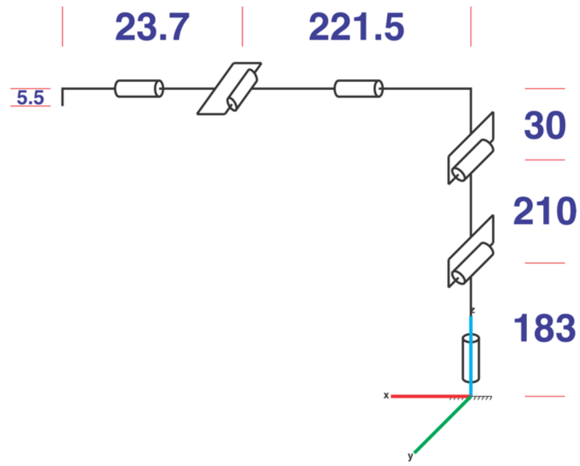

Figure 2

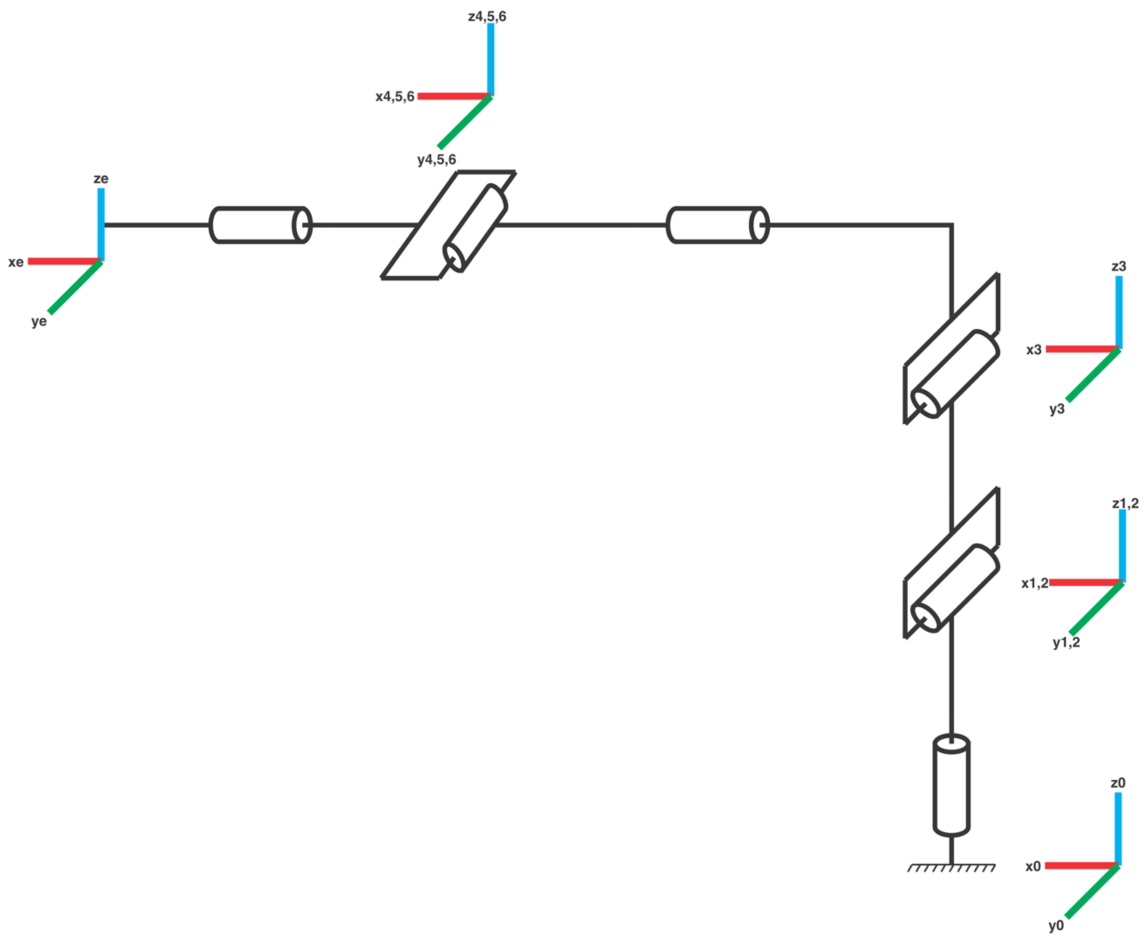

Figure 3

2. Forward Kinematics

Using transformation between two frames, forward kinematics for the robot is represented by Equation 1, 2, 3, 4:

$$T = T_w^0 * T_6^w * T_e^6 \tag{1}$$

$$T_w^0 = T_z(0.183) * R_z(\theta_1) * R_y(\theta_2) * \; T_z(0.210) * R_y(\theta_3) * T_z(0.03) * T_x(0.2215) \tag{2}$$

$$T_6^w = R_x(\theta_4) * R_y(\theta_5) * R_x(\theta_6) \tag{3}$$

$$T_e^6 = T_x(0.0237) * T_z(-0.0055) \tag{4}$$

The final result will have the shape illustrated in Equation 5.

$$T = \begin{bmatrix} n_x & s_x & a_x & P_x \\ n_y & s_y & a_y & P_y \\ n_z & s_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

After writing the forward kinematic code in Matlab I test it on the initial robot alignment Figure4,5. This function has the following structure Equation 6:

$$T = DKM(\theta_1\, ,\theta_2\, ,\theta_3\, ,\theta_4\, ,\theta_5\, ,\theta_6\, , draw\ robot, show\ symbolic) \tag{6}$$

```
>> T = DKM(0, 0, 0, 0, 0, 0, true, false);

T =

    1.0000         0         0    0.2452
         0    1.0000         0         0
         0         0    1.0000    0.4175
         0         0         0    1.0000
```

Figure 4

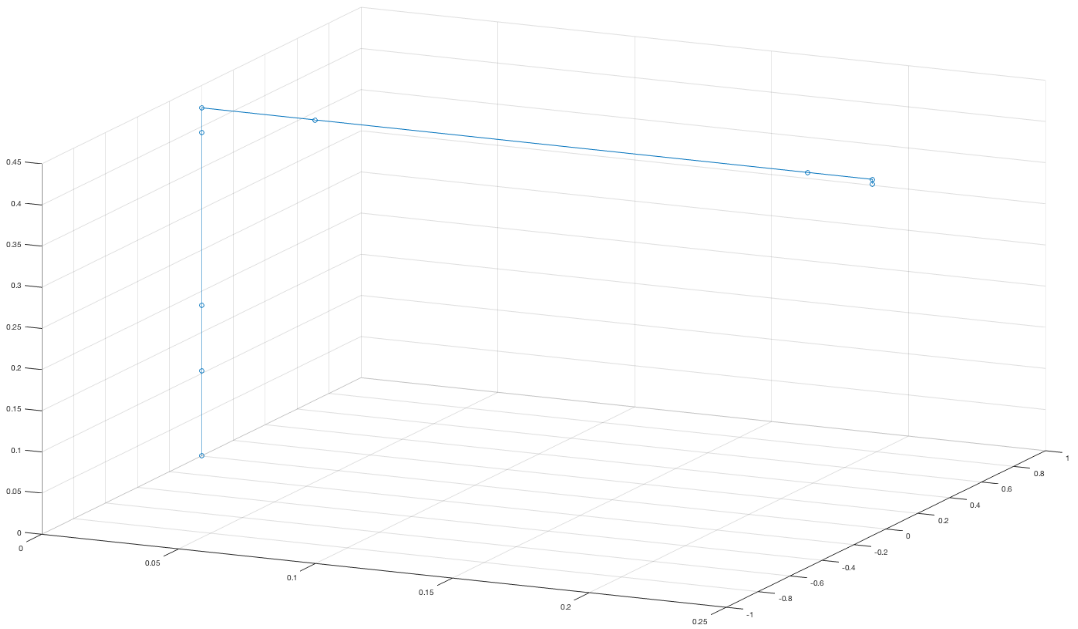

Figure 5

3. Inverse Kinematics

This problem can be separated into two parts: position and orientation.

3.1. Position.

First we find the center of spherical wrist by retracting from end effector using Equation 6

$$T_w^0 = T_e^0 * (Tx(0.0237)^{-1} Tz(-0.0055)^{-1}) \tag{6}$$

From last matrix we can extract the positions of the wrist center

$$X_w = T_W^0(1,4)$$
$$Y_w = T_W^0(2,4)$$
$$Z_w = T_W^0(3,4)$$

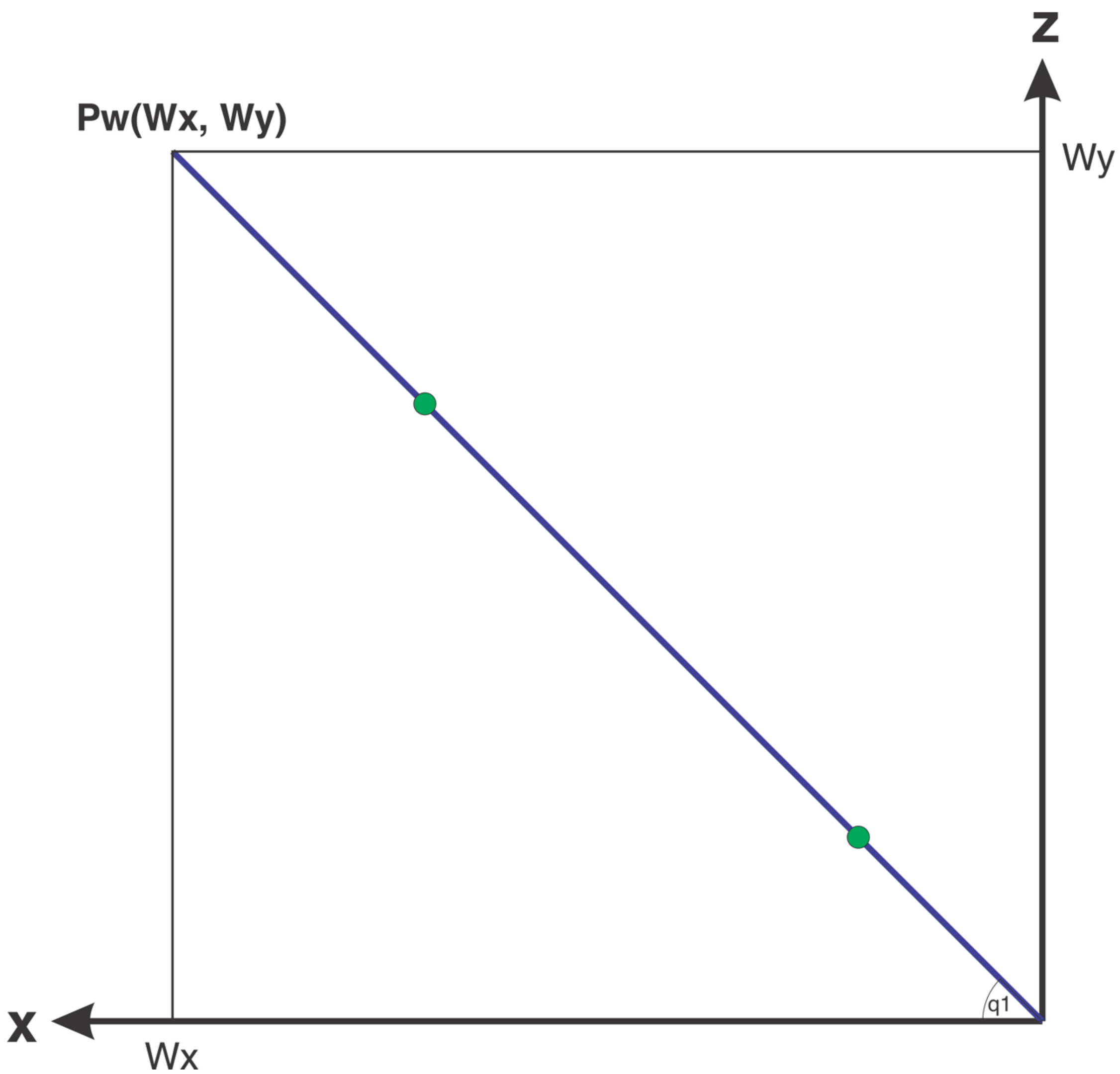

Figure 6

From Figure 6 (top view) we can calculate q1 which has two solutions:

$$q1(1) = atan2(W_y, W_x)$$
$$q1(2) = atan2(-W_y, -W_x)$$

There is a singularity if(x = y = 0) because the end effector will intersect with z axis, which means any value given to q1 the end effector will keep its position.

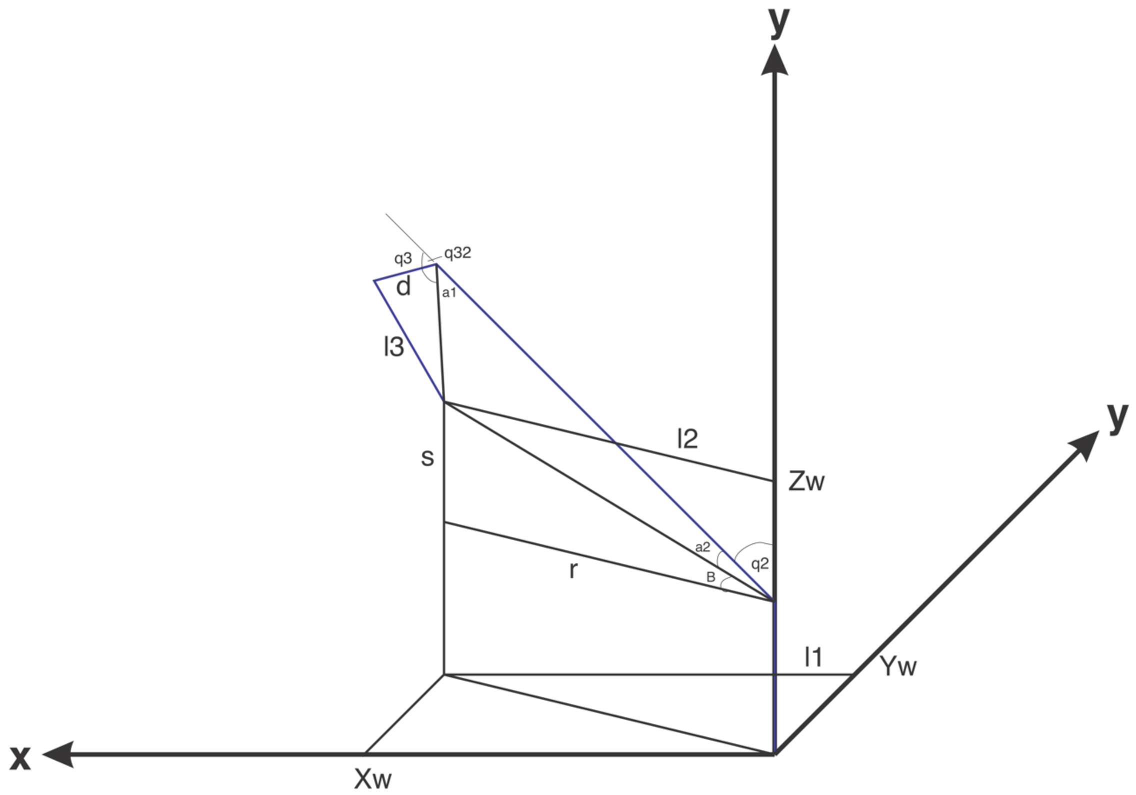

Figure 7

By studying the side view of the manipulator, it is possible to obtain q2 and q3.
The following equations describe the calculations:

$$r = \sqrt{Xw^2 + Yw^2} \tag{7}$$

$$s = Z_w - l_1 \tag{8}$$

If we apply Cosine theorem:

$$r^2 + s^2 = l_2^2 + l_3^2 + d^2 - 2l_2\sqrt{d^2 + l_3^2}\cos(a1) \tag{9}$$

To write the equation in form of q32:

$$r^2 + s^2 = l_2^2 + l_3^2 + d^2 + 2l_2\sqrt{d^2 + l_3^2}\cos(q_{23}) \tag{11}$$

From the previous equation we can calculate q3 as follow:

$$q3 = q_{32} - atan2(l_3, d) \tag{12}$$

In Equation (11) can yield a complex solution in some cases:

$$abs\left(\frac{r^2 + s^2 - l_2^2 - l_3^2 - d^2}{2l_2\sqrt{d^2 + l_3^2}}\right) > 1 \tag{13}$$

The previous mean the point is outside of the workspace of the robot. Also, if q3 = 0 then robot losses 1 DOF.
By applying Sine theorem:

$$\frac{\sin(a1)}{\sqrt{r^2 + s^2}} = \frac{\sin(a2)}{\sqrt{l_3^2 + d^2}} \tag{14}$$

And by finding B we can calculate q2

$$B = atan2(s, r) \tag{15}$$

$$q2 = \frac{\pi}{2} - (B + a2) \tag{16}$$

We must take in consideration the probability of atan2 parameters so we will have all the possible solutions.

3.2.Orientation.

After calculating q1, q2 and q3 we can write:

$$T_e^0 = T_w^0 * T_e^w \tag{17}$$

$$T_e^w = (T_w^0)^{-1} * T_e^0 \tag{18}$$

$$T_e^w = \begin{bmatrix} c_5 & s_5 s_6 & c_6 s_5 & P_X \\ s_4 s_5 & c_4 c_6 - c_5 s_4 s_6 & -c_4 s_6 - c_5 c_6 s_4 & P_Y \\ -c_4 s_5 & c_6 s_4 + c_4 c_5 s_6 & c_4 c_5 c_6 - s_4 s_6 & P_Z \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{19}$$

For each value of q1,q2,q3 we have two possible solution, but we have two situations:

1. Abs(C5) != 1:

```
theta41 = atan2(T456(2,1), -T456(3,1));
theta61 = atan2(T456(1,2), T456(1,3));
theta51 = atan2(sqrt(T456(1,3)^2 + T456(1,2)^2),T456(1,1));

theta42 = atan2(-T456(2,1), T456(3,1));
theta62 = atan2(-T456(1,2), -T456(1,3));
theta52 = atan2(-sqrt(T456(1,3)^2 + T456(1,2)^2),T456(1,1));
```

2. Abs(C5) = 1:

```
theta41 = 0;
theta61 + theta51 = atan2(T456(2,3), T456(3,3));
```

When trying the code I got a lot of solution (32) possibilities (8) valid and the others not valid, so the code need optimization.