Dynamics of Nonlinear Robotic Systems Assignment 2

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Abstract

This report is part of Dynamics of Nonlinear Robotic Systems course for 1st year master students at Innopolis University.

In this report I will derive the Dynamics for this robot in order to calculate the toques needed to achieve a specific trajectory. Full implementation will be on Matlab and the code will be uploaded to repository.

1. Dynamic.

The dynamic for a robot helps us to calculate the required torques to achieve a specific trajectory $(q, \dot{q}, \ddot{q})$ and those are the main input to robot control system.

The dynamic equation for any robot system has the following form:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau_{tot} \tag{1}$$

Where:

$$\begin{aligned}
&q: generalized\ joint\ coordinates. \\
&M(q): Mass\ or\ inertia\ matrix. \\
&C(q, \dot{q}): Centrifugal\ and\ Coriolis\ forces. \\
&g(q): Gravittioal\ forces\ (due\ to\ the\ weight\ of\ each\ link). \\
&\tau_{tot}: Total\ generalized\ forces\ (non\ conservative).
\end{aligned}$$

In order to derive each and every component of this equation we need to calculate the generalized Jacobian matrix for the linear and angular velocity and for all joints (n). The Jacobian matrix for the linear velocity (3, n):

$$J_v^i = \begin{bmatrix} \frac{\partial x_i}{\partial q_1} & \frac{\partial x_i}{\partial q_2} & \cdots & \frac{\partial x_i}{\partial q_n} \\ \frac{\partial y_i}{\partial q_1} & \frac{\partial y_i}{\partial q_2} & \cdots & \frac{\partial y_i}{\partial q_n} \\ \frac{\partial z_i}{\partial q_1} & \frac{\partial z_i}{\partial q_2} & \cdots & \frac{\partial z_i}{\partial q_n} \end{bmatrix} \tag{2}$$

Jacobian matrix for the angular velocity (3, n):

$$J_w^1 = [u_0 \quad 0 \quad 0 \quad \cdots \quad 0] \tag{3}$$
$$J_w^2 = [u_0 \quad u_1 \quad 0 \quad \cdots \quad 0] \tag{4}$$
$$J_w^3 = [u_0 \quad u_1 \quad u_2 \quad 0 \quad \cdots \quad 0] \tag{5}$$
$$\vdots \quad \vdots \quad \vdots$$
$$J_w^n = [u_0 \quad u_1 \quad u_2 \quad \cdots \quad u_n] \tag{6}$$

It worth to illustrate that, if the joint is prismatic the corresponding column in Jacobian for the angular velocity would be zero.

I will start the solution taking the first two links Figure 1.

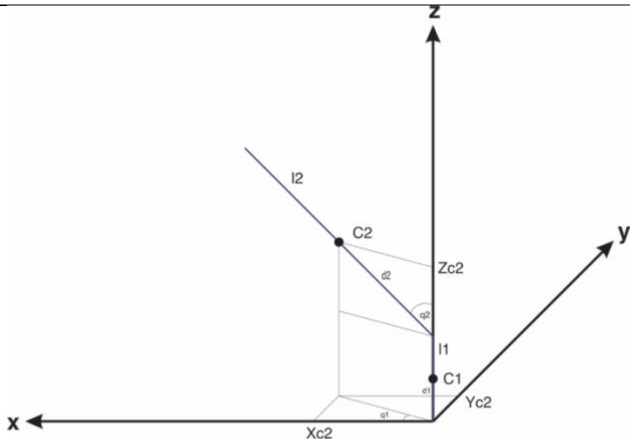

Figure 1. the first two robot links.

I assume that the center of mass for each link "i" is shifted by d_i then the coordinates for the first two centers of mass will be in Equations (7,8):

$$Pc_1 = \begin{bmatrix} 0 \\ 0 \\ d1 \end{bmatrix} \tag{7}$$

$$Pc_2 = \begin{bmatrix} d_2 \sin(q_2)\cos(q1) \\ d_2 \sin(q2)\sin(q1) \\ l1 + d_2 \cos(q_2) \end{bmatrix} \tag{8}$$

Now we can calculate Jacobians as in Equation (2…6) taking into consideration that both joints are revolt:

$$J_v^1 = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \end{bmatrix} \tag{9}$$

$$J_v^2 = \begin{bmatrix} -d_2 \sin(q_2)\sin(q_1) & d_2 \cos(q_2)\cos(q_1) & 0 & \cdots & 0 \\ d_2 \sin(q_2)\cos(q_1) & d_2 \cos(q_2)\sin(q_1) & 0 & \cdots & 0 \\ 0 & -d_2 \sin(q_2) & 0 & \cdots & 0 \end{bmatrix} \tag{10}$$

The first joint revolves around z axis:

$$J_w^1 = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \end{bmatrix} \tag{11}$$

The second joint revolves around y axis:

$$J_w^2 = \begin{bmatrix} 0 & -\sin(q1) & 0 & \cdots & 0 \\ 0 & \cos(q1) & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \end{bmatrix} \tag{12}$$

I will use the direct kinematic to find the position of center of mass for each joint and from the R_i matrix I can derive the u_i.

From the robot scheme illustrated in Figure 2. And from Equations (2…6), the Matlab Code give the following results:

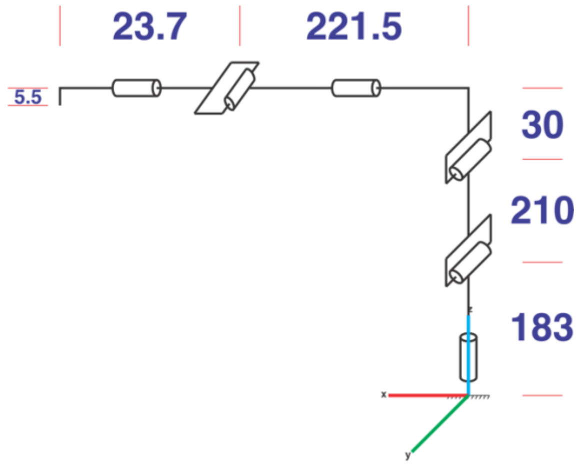

Figure 2 Kinematic scheme for Niryo one manipulator

$$T0c_1 = R_z(q_1) * T_z(d_1) = \begin{bmatrix} \cos(q_1) & -\sin(q_1) & 0 & 0 \\ \sin(q_1) & \cos(q_1) & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad 13$$

$$J_v^1 = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \end{bmatrix} \quad 14$$

$$J_w^1 = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \end{bmatrix} \quad 15$$

$$T0c_1 = R_z(q_1) * T_z(l_1) * R_y(q_2) * T_z(d_2) = \begin{bmatrix} \mathrm{c}_1 * \mathrm{c}_2 & -\mathrm{s}_1 & \mathrm{c}_1 * \mathrm{s}_2 & \mathrm{d}_2 * \mathrm{c}_1 * \mathrm{s}_2 \\ \mathrm{c}_2 * \mathrm{s}_1 & \mathrm{c}_1 & \mathrm{s}_1 * \mathrm{s}_2 & \mathrm{d}_2 * \mathrm{s}_1 * \mathrm{s}_2 \\ -\mathrm{s}_2 & 0 & \mathrm{c}_2 & \mathrm{d}_2 * \mathrm{c}_2 + (\mathrm{l1} + \mathrm{l2}) \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad 16$$

$$J_v^2 = \begin{bmatrix} -d_2 \sin(q_2)\sin(q_1) & d_2 \cos(q_2)\cos(q_1) & 0 & \cdots & 0 \\ d_2 \sin(q_2)\cos(q_1) & d_2 \cos(q_2)\sin(q_1) & 0 & \cdots & 0 \\ 0 & -d_2 \sin(q_2) & 0 & \cdots & 0 \end{bmatrix} \quad 17$$

$$J_w^2 = \begin{bmatrix} 0 & -\sin(q1) & 0 & 0 & 0 & 0 \\ 0 & \cos(q1) & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad 18$$

for joints from 3 to 6 Jacobean matrices will be the following:

$$J_v^3 = \begin{bmatrix} -s_1(d_3c_{23} + ds_{23} + l_3s_2) & c_1(dc_{23} - d_3s_{23} + l_3c_2) & c_1(dc_{23} - d_3s_{23}) & 0 & 0 & 0 \\ c_1(d_3c_{23} + ds_{23} + l_3s_2) & s_1(dc_{23} - d_3s_{23} + l_3c_2) & s_1(dc_{23} - d_3s_{23}) & 0 & 0 & 0 \\ 0 & -d_3c_{23} - ds_{23} & -d_3c_{23} - d\_s_{23} & 0 & 0 & 0 \end{bmatrix} \quad 19$$

$$J_w^3 = \begin{bmatrix} 0 & -\sin(q1) & -\sin(q_1) & 0 & 0 & 0 \\ 0 & \cos(q1) & \cos(q_1) & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad 20$$

$$J_v^4 = \begin{bmatrix} -s_1(d_4c_{23} + l_4c_{23} + ds_{23} + l_3s_2) & c_1(dc_{23} - d_4s_{23} - l_4s_{23} + l_3c_2) & -c_1(d_4s_{23} - dc_{23} + l_4s_{23}) & 0 & 0 & 0 \\ c_1(d_4c_{23} + l_4c_{23} + ds_{23} + l_3s_2) & s_1(dc_{23} - d_4s_{23} - l_4s_{23} + l_3c_2) & -s_1(d_4s_{23} - dc_{23} + l_4s_{23}) & 0 & 0 & 0 \\ 0 & -d_4c_{23} - l_4c_{23} - ds_{23} - l_3s_2 & -d_4c_{23} - l_4c_{23} - ds_{23} & 0 & 0 & 0 \end{bmatrix} \quad 21$$

$$J_w^4 = \begin{bmatrix} 0 & -s_1 & -s_1 & c_{23}c_1 & 0 & 0 \\ 0 & c_1 & c_1 & c_{23}s_1 & 0 & 0 \\ 1 & 0 & 0 & -s_{23} & 0 & 0 \end{bmatrix} \quad 22$$

$$J_v^5(:,1) = \begin{bmatrix} -l_4c_{23}s_1 - l_5c_{23}s_1 - ds_{23}s_1 - d_5c_1c_4 - l_3s_1s_2 - d_5c_2s_1s_3s_4 - d_5c_3s_1s_2s_4 \\ l_4(c_1c_2c_3 - c_1s_2s_3) + l_5(c_1c_2c_3 - c_1s_2s_3) - d_5\big(c_4s_1 - s_4(c_1c_2s_3 + c_1c_3s_2)\big) + d(c_1c_2s_3 + c_1c_3s_2) + l_3c_1s_2 \\ 0 \end{bmatrix}$$

$$J_v^5(:,2{:}3) = \begin{bmatrix} -c_1\left(\frac{d_5s_{23-4}}{2} - dc_{23} + l_4s_{23} + l_5s_{23} - l_3c_2 - l_3c_2 - \frac{d_5s_{234}}{2}\right) & -c_1\left(\frac{d_5s_{23-4}}{2} - dc_{23} + l_4s_{23} + l_5s_{23} - \frac{d_5s_{234}}{2}\right) \\ -s_1\left(\frac{d_5s_{23-4}}{2} - dc_{23} + l_4s_{23} + l_5s_{23} - l_3c_2 - \frac{d_5s_{234}}{2}\right) & -s_1\left(\frac{d_5s_{23-4}}{2} - dc_{23} + l_4s_{23} + l_5s_{23} - \frac{d_5s_{234}}{2}\right) \\ -l_4c_{23} - l_5c_{23} - ds_{23} - l_3s_2 - d_5s_{23}s_4 & -l_4c_{23} - l_5c_{23} - d\,s_{23} - d_5s_{23}s_4 \end{bmatrix} \quad 23$$

$$J_v^5(:,4{:}6) = \begin{bmatrix} d_5\big(s_1s_4 + c_4(c_1c_2s_3 + c_1c_3s_2)\big) & 0 & 0 \\ -d_5\big(c_1s_4 - c_4(c_2s_1s_3 + c_3s_1s_2)\big) & 0 & 0 \\ d_5c_{23}c_4 & 0 & 0 \end{bmatrix}$$

$$J_w^5 = \begin{bmatrix} 0 & -s_1 & -s_1 & c_{23}c_1 & s_4(c_1c_2s_3 + c_1c_3s_2) - c_4s_1 & 0 \\ 0 & c_1 & c_1 & c_{23}s_1 & c_1c_4 + s_4(c_2s_1s_3 + c_3s_1s_2) & 0 \\ 1 & 0 & 0 & -s_{23} & c_{23}s_4 & 0 \end{bmatrix} \quad 24$$

$$J_v^6(:,1) = \begin{bmatrix} d_6c_2c_4s_1s_3s_5 - l_5c_{23}s_1 - ds_{23}s_1 - l_3s_1s_2 - d_6c_{23}c_5s_1 - d_6c_1s_4s_5 - l_4c_{23}s_1 + d_6c_3c_4s_1s_2s_5 \\ l_4c_{23}c_1 + l_5c_{23}c_1 + ds_{23}c_1 + l_3c_1s_2 + d_6c_{23}c_1c_5 - d_6s_1s_4s_5 - d_6c_1c_2c_4s_3s_5 - d_6c_1c_3c_4s_2s_5 \\ 0 \end{bmatrix}$$

$$J_v^6(:,2) = \begin{bmatrix} -c_1(l_4s_{23} - dc_{23} + l_5s_{23} - l_3c_2 + d_6s_{23}c_5 + d_6c_2c_3c_4s_5 - d_6c_4s_2s_3s_5) \\ l_3c_2s_1 - d_6\big(c_5(c_2s_1s_3 + c_3s_1s_2) - c_4s_5(s_1s_2s_3 - c_2c_3s_1)\big) - l_4(c_2s_1s_3 + c_3s_1s_2) - l_5(c_2s_1s_3 + c_3s_1s_2) - d(s_1s_2s_3 - c_2c_3s_1) \\ \frac{d_6s_{23}s_{45}}{2} - l_5c_{23} - d\,s_{23} - l_3s_2 - l_4c_{23} - d_6c_{23}c_5 - \frac{d_6s_{45}s_{23}}{2} \end{bmatrix} \quad 25$$

$$J_v^6(:,3{:}4) = \begin{bmatrix} -c_1(l_4s_{23} - dc_{23} + l_5s_{23} + d_6s_{23}c_5 + d_6c_2c_3c_4s_5 - d_6c_4s_2s_3s_5) & d_6s_5(c_1c_2s_3s_4 - c_4s_1 + c_1c_3s_2s_4) \\ -s_1(l_4s_{23} - dc_{23} + l_5s_{23} + d_6s_{23}c_5 + d_6c_{23}c_4s_5 & d_6s_5\big(c_1c_4 + s_4(c_2s_1s_3 + c_3s_1s_2)\big) \\ d_6s_{23}c_4s_5 - l_5c_{23} - ds_{23} - d_6c_{23}c_5 - l_4c_{23} & d_6c_{23}s_4s_5 \end{bmatrix}$$

$$J_v^6(:,5{:}6) = \begin{bmatrix} -d_6c_{23}c_1s_5 - d_6c_5s_1s_4 - d_6c_1c_2c_4c_5s_3 - d_6c_1c_3c_4c_5s_2 & 0 \\ d_6\left(c_5\big(c_1s_4 - c_4(c_2s_1s_3 + c_3s_1s_2)\big)s_5(s_1s_2s_3 - c_2c_3s_1)\right) & 0 \\ d_6s_{23}s_5 - d_6c_{23}c_4c_5 & 0 \end{bmatrix}$$

$$J_w^6 = \begin{bmatrix} 0 & -s_1 & -s_1 & c_{23}c_1 & s_4(c_1c_2s_3 + c_1c_3s_2) - c_4s_1 & c_5(c_1c_2c_3 - c_1s_2s_3) - s_5\big(s_1s_4 + c_4(c_1c_2s_3 + c_1c_3s_2)\big) \\ 0 & c_1 & c_1 & c_{23}s_1 & c_1c_4 + s_4(c_2s_1s_3 + c_3s_1s_2) & s_5\big(c_1s_4 - c_4(c_2s_1s_3 + c_3s_1s_2)\big) - c_5(s_1s_2s_3 - c_2c_3s_1) \\ 1 & 0 & 0 & -s_{23} & c_{23}s_4 & -s_{23}c_5 - c_{23}c_4c_5 \end{bmatrix} \tag{26}$$

After deriving Jacobian, we can now calculate M (n x n) using Equation 27.

$$M = \sum_{i=1}^{n} m_i {J_v^i}^T J_v^i + {J_w^i}^T R_i I_i R_i^T J_w^i \tag{27}$$

After deriving M, we can now calculate C (n x n) using Equation 28.

$$C = \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k+1}^{n} \frac{1}{2} * \left(\frac{\partial M(i,j)}{\partial M(k)} + \frac{\partial M(i,k)}{\partial M(j)} - \frac{\partial M(j,k)}{\partial M(i)}\right) \tag{28}$$

The previous matrices output will be implemented in Matlab only due to its complicated output.

The last matrix is G (1,6) matrix and has the following equation:

$$G = -\sum_{i=1}^{n}\sum_{k=1}^{n} \left(J_{vi}^k\right)^T m_k g_0 \tag{29}$$

$$G = \begin{bmatrix} 0 \\ -gm_3(d_3c_{23} + ds_{23} + l_3s_2) - gm_6\left(l_4c_{23} + l_5c_{23} + ds_{23} + l_3s_2 - \frac{d_6s_{23}s_{45}}{2} + d_6c_{23}c_5 + \frac{d_6s_{45}s_{23}}{2} - gm_4(d_4c_{23} + l_4c_{23} + ds_{23} + l_3s_2) - gm_5(l_4c_{23} + l_5c_{23} + ds_{23} + l_3s_{22} + d_5s_{23}s_4) - d_2gm_2s_2\right) \\ -gm_5(l_4c_{23} + l_5c_{23} + ds_{23} + d_5s_{23}s_4) - gm_6(l_4c_{23} + l_5c_{23} + ds_{23} + d_6c_{23}c_5 - d_6s_{23}c_4s_5) - gm_3(d_3c_{23} + ds_{23}) - gm_4(d_4c_{23} + l_4c_{23} + ds_{23}) \\ gc_{23}(d_5m_5c_4d_6m_6s_4s_5) \\ d_6gm_6(s_{23}s_5 - c_{23}c_4c_5) \\ 0 \end{bmatrix} \tag{30}$$

2. GitHub

The project can be found [here]()