

Dynamics of Nonlinear Robotic Systems Assignment1

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Abstract

This report is part of Dynamics of Nonlinear Robotic Systems course for 1st year master students at Innopolis University.

In this report I am working in Niryo one manipulator designed by Niryo. I will develop the kinematic model of the robot, solve both forward and inverse kinematics problems, implement the proposed solution using Matlab and check the validity of my solution by making a test file.

1. Kinematic Scheme

Figure 1 is taken from robot datasheet to illustrate dimension of the robot, Figure 2 describe the kinematic scheme of Niryo robot manipulator, and Figure 3 is coordinates assignment to robot joints.

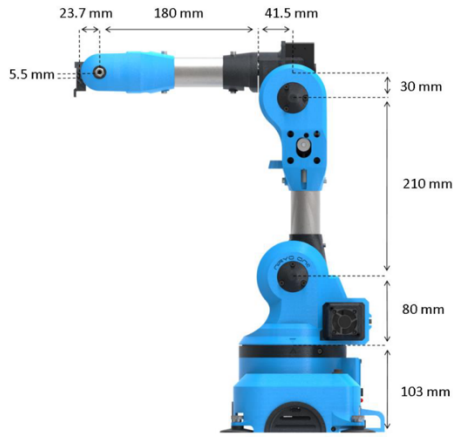


Figure 1

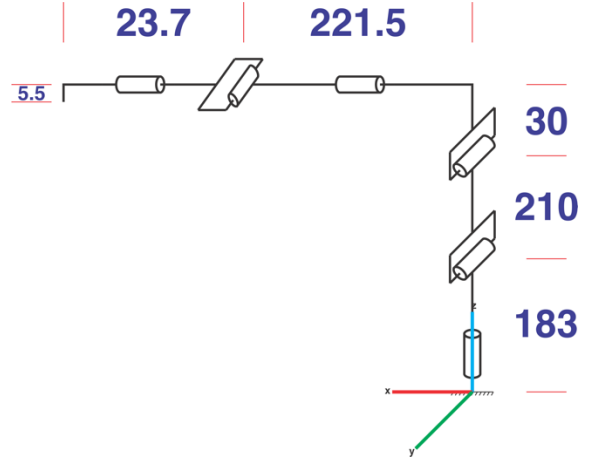


Figure 2

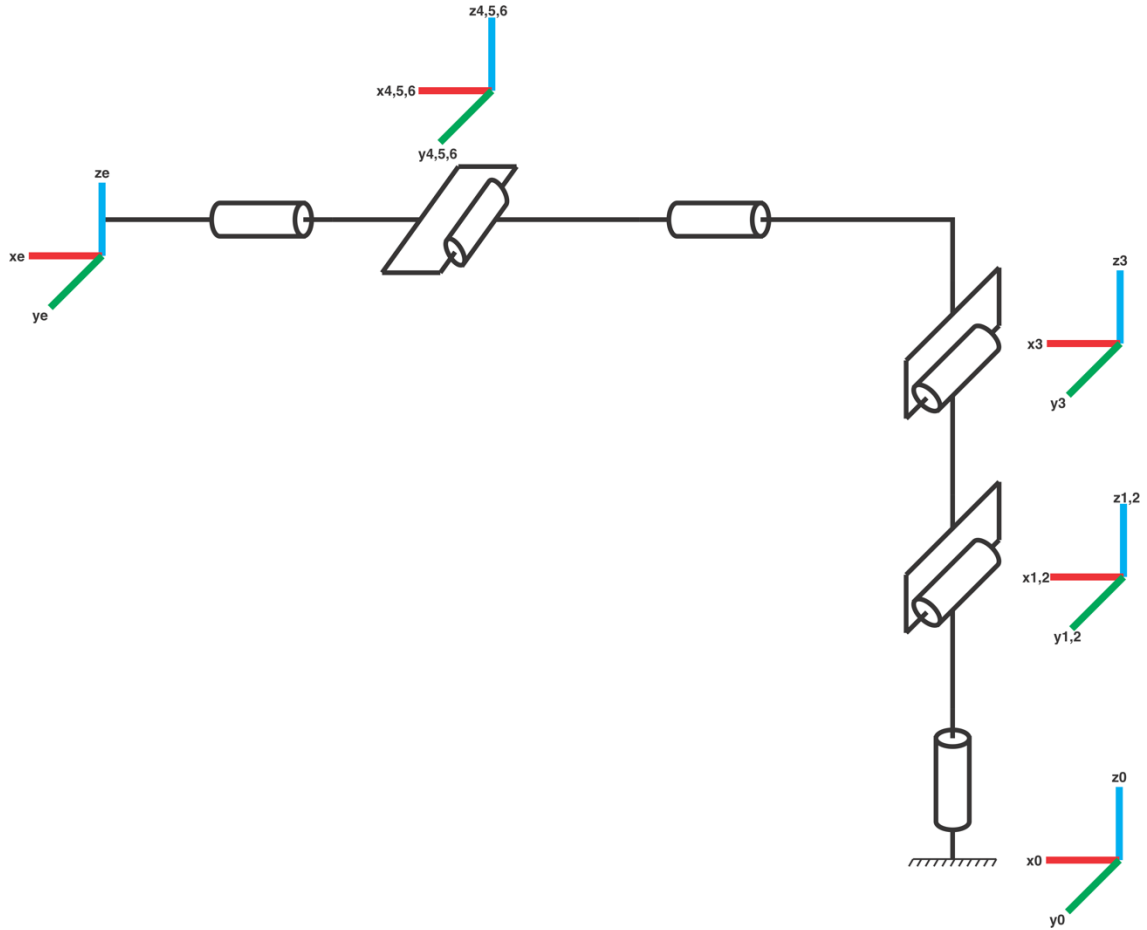


Figure 3

2. Forward Kinematics

Using transformation between two frames, forward kinematics for the robot is represented by Equation 1, 2, 3, 4:

$$T = T_w^0 * T_6^w * T_e^6 \quad (1)$$

$$T_w^0 = T_z(0.183) * R_z(\theta_1) * R_y(\theta_2) * T_z(0.210) * R_y(\theta_3) * T_z(0.03) * T_x(0.2215) \quad (2)$$

$$T_6^w = R_x(\theta_4) * R_y(\theta_5) * R_x(\theta_6) \quad (3)$$

$$T_e^6 = T_x(0.0237) * T_z(-0.0055) \quad (4)$$

The final result will have the shape illustrated in Equation 5.

$$T = \begin{bmatrix} n_x & s_x & a_x & P_x \\ n_y & s_y & a_y & P_y \\ n_z & s_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

After writing the forward kinematic code in Matlab I test it on the initial robot alignment Figure4,5. This function has the following structure Equation 6:

$$T = DKM(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, draw\ robot, show\ symbolic) \quad (6)$$

```
>> T = DKM(0, 0, 0, 0, 0, 0, true, false);

T =

    1.0000         0         0    0.2452
         0    1.0000         0         0
         0         0    1.0000    0.4175
         0         0         0    1.0000
```

Figure 4

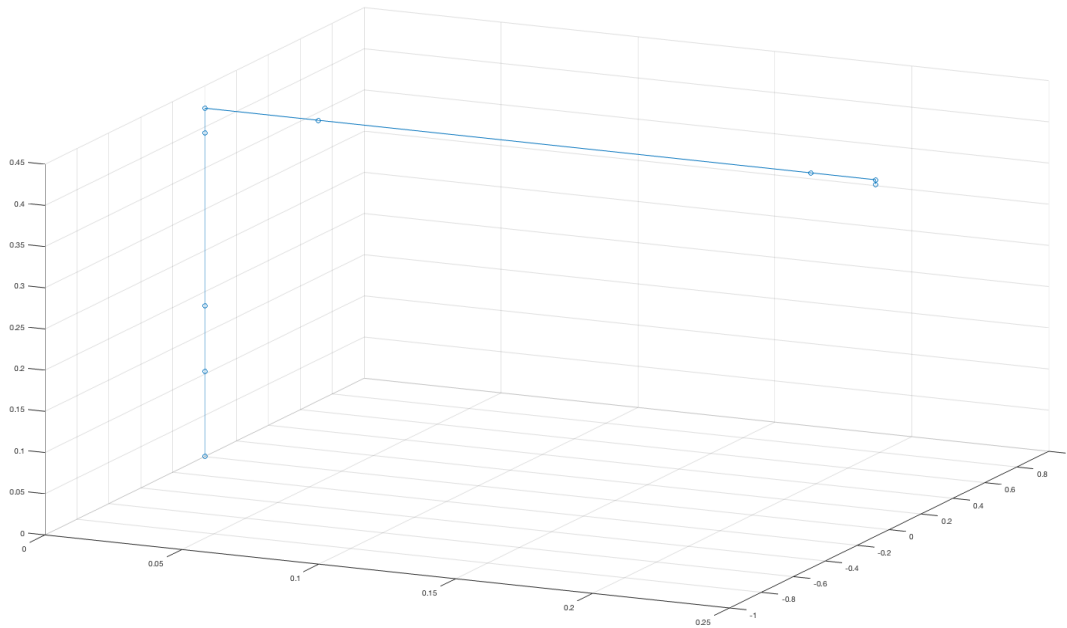


Figure 5

3. Inverse Kinematics

This problem can be separated into two parts: position and orientation.

3.1.Position.

First we find the center of spherical wrist by retracting from end effector using Equation 6

$$T_w^0 = T_e^0 * (Tx(0.0237)^{-1}Tz(-0.0055)^{-1}) \quad (6)$$

From last matrix we can extract the positions of the wrist center

$$X_w = T_w^0(1, 4)$$

$$Y_w = T_w^0(2, 4)$$

$$Z_w = T_w^0(3, 4)$$

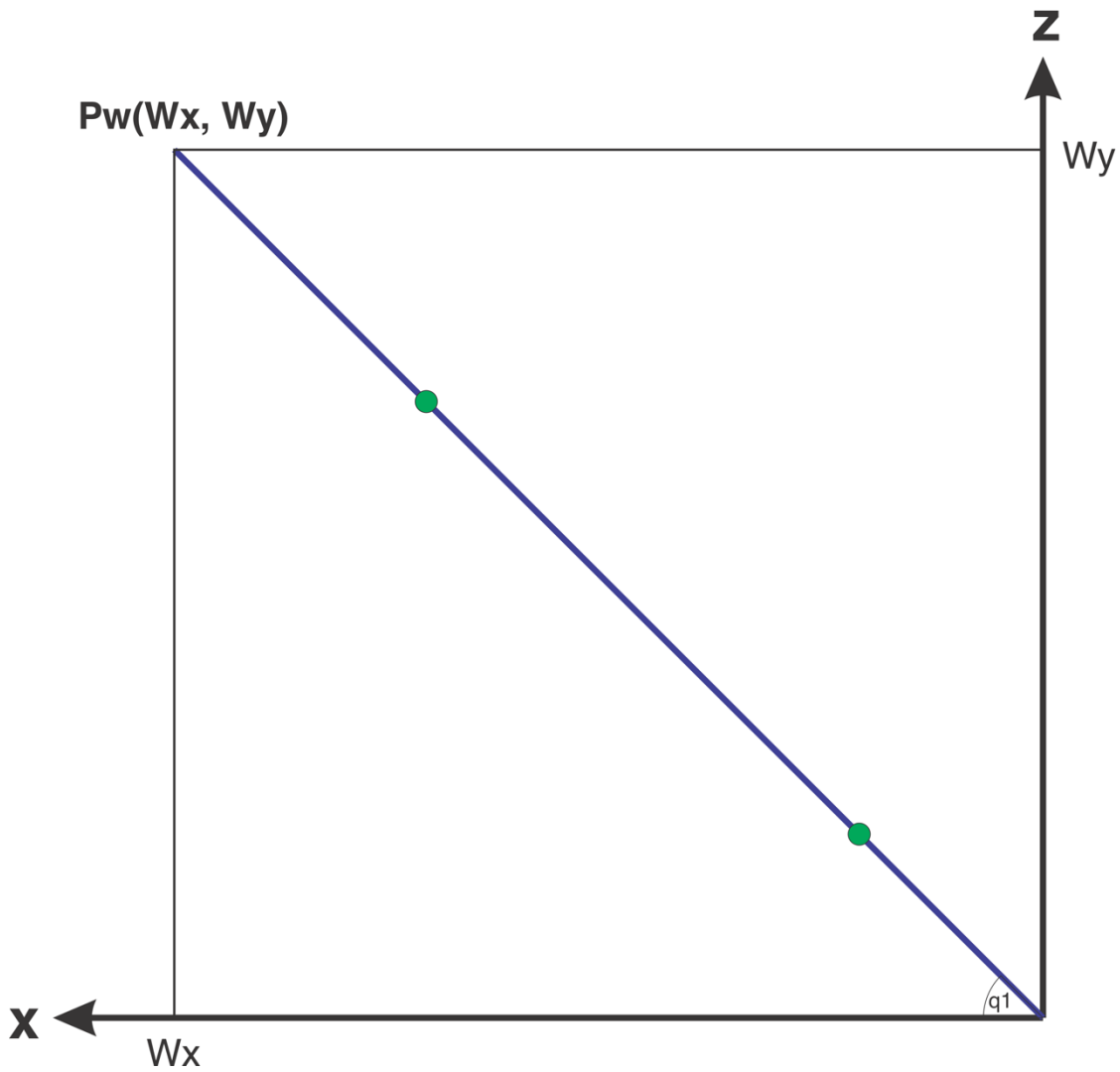


Figure 6

From Figure 6 (top view) we can calculate q1 which has two solutions:

$$q1(1) = atan2(W_y, W_x)$$

$$q1(2) = atan2(-W_y, -W_x)$$

There is a singularity if $(x = y = 0)$ because the end effector will intersect with z axis, which means any value given to q1 the end effector will keep its position.

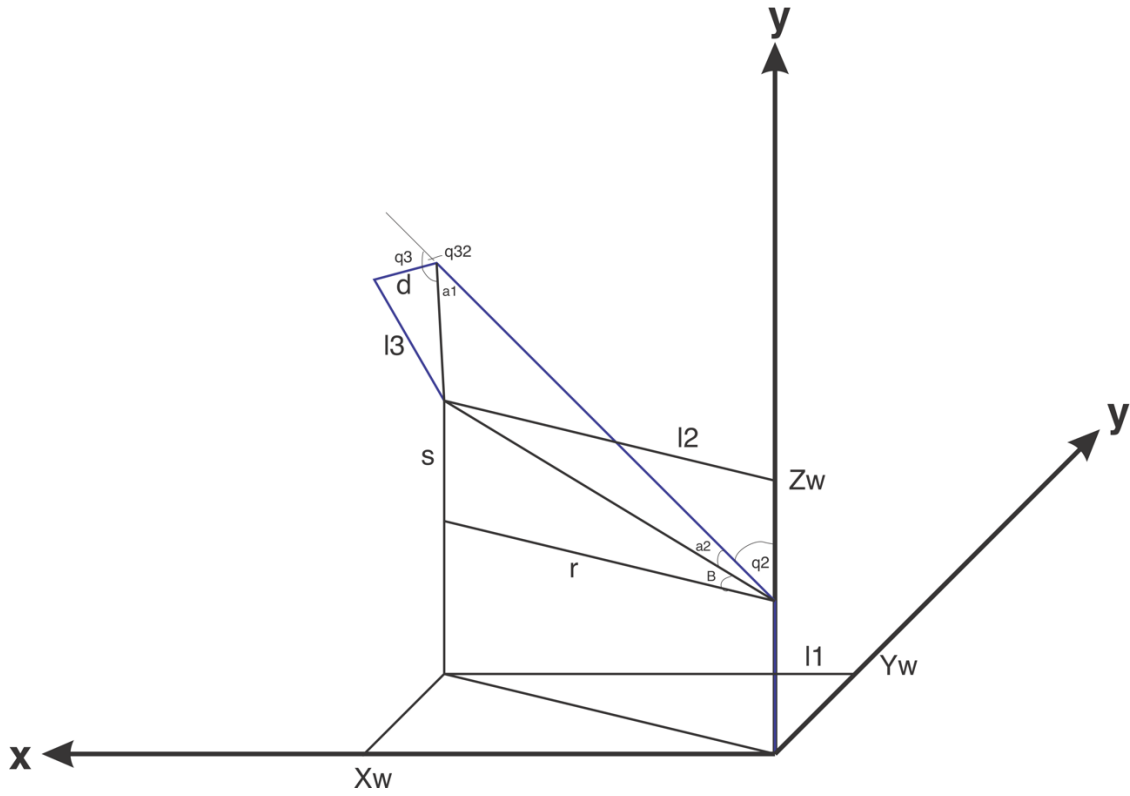


Figure 7

By studying the side view of the manipulator, it is possible to obtain q_2 and q_3 . The following equations describe the calculations:

$$r = \sqrt{X_w^2 + Y_w^2} \quad (7)$$

$$s = Z_w - l_1 \quad (8)$$

If we apply Cosine theorem:

$$r^2 + s^2 = l_2^2 + l_3^2 + d^2 - 2l_2\sqrt{d^2 + l_3^2} \cos(a_1) \quad (9)$$

To write the equation in form of q_{32} :

$$r^2 + s^2 = l_2^2 + l_3^2 + d^2 + 2l_2\sqrt{d^2 + l_3^2} \cos(q_{23}) \quad (11)$$

From the previous equation we can calculate q_3 as follow:

$$q_3 = q_{32} - \text{atan2}(l_3, d) \quad (12)$$

In Equation (11) can yield a complex solution in some cases:

$$\text{abs}\left(\frac{r^2 + s^2 - l_2^2 - l_3^2 - d^2}{2l_2\sqrt{d^2 + l_3^2}}\right) > 1 \quad (13)$$

The previous mean the point is outside of the workspace of the robot. Also, if $q_3 = 0$ then robot losses 1 DOF.

By applying Sine theorem:

$$\frac{\sin(a_1)}{\sqrt{r^2 + s^2}} = \frac{\sin(a_2)}{\sqrt{l_3^2 + d^2}} \quad (14)$$

And by finding B we can calculate q_2

$$B = \text{atan2}(s, r) \quad (15)$$

$$q_2 = \frac{\pi}{2} - (B + a_2) \quad (16)$$

We must take in consideration the probability of atan2 parameters so we will have all the possible solutions.

3.2.Orientation.

After calculating q_1 , q_2 and q_3 we can write:

$$T_e^0 = T_w^0 * T_e^w \quad (17)$$

$$T_e^w = (T_w^0)^{-1} * T_e^0 \quad (18)$$

$$T_e^w = \begin{bmatrix} c_5 & s_5 s_6 & c_6 s_5 & P_X \\ s_4 s_5 & c_4 c_6 - c_5 s_4 s_6 & -c_4 s_6 - c_5 c_6 s_4 & P_Y \\ -c_4 s_5 & c_6 s_4 + c_4 c_5 s_6 & c_4 c_5 c_6 - s_4 s_6 & P_Z \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (19)$$

For each value of q1,q2,q3 we have two possible solution, but we have two situations:

1. Abs(C5) != 1:

```
theta41 = atan2(T456(2,1), -T456(3,1));
theta61 = atan2(T456(1,2), T456(1,3));
theta51 = atan2(sqrt(T456(1,3)^2 + T456(1,2)^2), T456(1,1));

theta42 = atan2(-T456(2,1), T456(3,1));
theta62 = atan2(-T456(1,2), -T456(1,3));
theta52 = atan2(-sqrt(T456(1,3)^2 + T456(1,2)^2), T456(1,1));
```

2. Abs(C5) = 1:

```
theta41 = 0;
theta61 + theta51 = atan2(T456(2,3), T456(3,3));
```

When trying the code I got a lot of solution (32) possibilities (8) valid and the others not valid, so the code need optimization.

4. GitHub

The project can be found [here](#)