

# The Roles of Monoarticular and Biarticular Muscles of the Lower Limbs in Terrestrial Locomotion

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Received June 10, 2003

**Abstract**—Specific features of the functioning of mono- and biarticular muscles were studied using a multi-joint movement (a high jump) as an example. The powers of the knee and ankle joint extensors are insufficient for a strong and quick movement such as a high jump. Biarticular muscles (*m. rectus femoris*) transfer forces/powers from one joint to another, thereby compensating for the physiological shortcoming of monoarticular muscles, that is, a decrease in the tractive force with increasing contraction rate. In a high jump, a power of 300 W may be transferred from the hip to the knee joint via the *m. rectus femoris*; 230 W, from the knee to the hip joint via the hamstring muscle; 210 W, from the knee joint to the ankle via the *m. gastrocnemius*; and 15 W, from the metatarsophalangeal joint to the ankle via the *mm. flexors*.

A large number of studies have recently been conducted to determine the tractive force of muscles by solving the inverse problem of dynamics (IPD), i.e., when the forces producing a movement are inferred from known kinematics. The muscle forces developed in human locomotion have been determined using the following methods: (1) mathematical modeling (linear, nonlinear, and dynamic programming) [1–32], (2) construction of regression models linking the electromyographic muscular activity with its force [33–40], (3) measurement of the intramuscular pressure [41, 42], and (4) recording of the muscular tractive force with tendon dynamometric transducers (tendometry) [43–47].

The interest in the tractive forces of muscles is not accidental. In order to understand how the brain controls motion, we can either use neurophysiological methods or try to solve a similar task indirectly, i.e., to guess at the mechanisms of motion control by the velocity and tractive force of muscles. The latter method implies the need for a mathematical model estimating the velocity and tractive force of muscles.

The aim of this work was to assess the contraction power of mono- and biarticular muscles of the human lower extremity. The power of muscular contraction was studied under the following kinematic conditions:

- (i) The movement under study must be multijoint; i.e., it must be performed in several joints.
- (ii) The movement must involve both mono- and biarticular agonists/antagonists.
- (iii) To simplify mathematical calculations in solving the IPD, the movement must be performed predominantly in one plane.

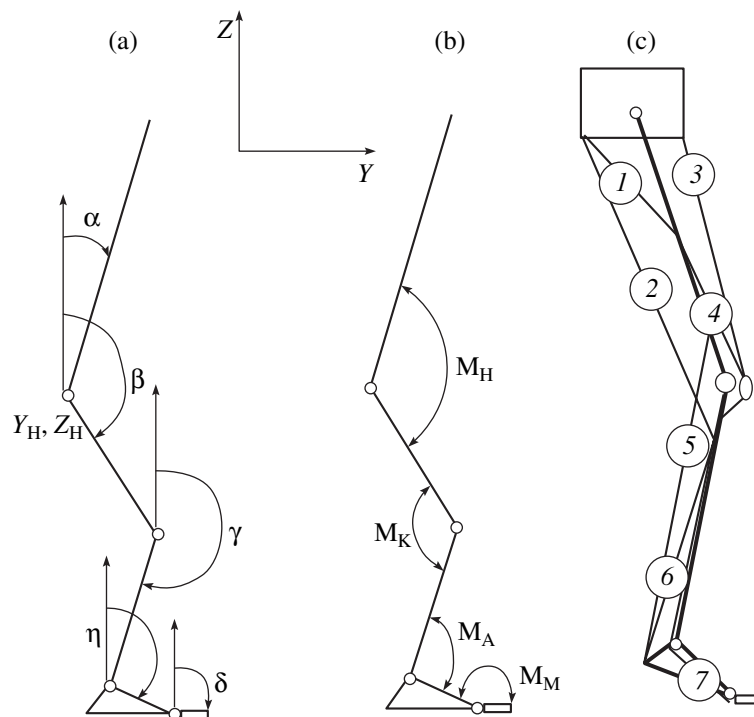
Of the great number of locomotions, such as walking, running, swimming, and jumping, a standing high jump meets the above requirements to the greatest degree because (1) the movement is a multijoint one,

(2) a high jump activates both synergists and antagonists, and (3) a high jump is performed predominantly in one plane.

## METHODS

An athlete (age 19, height 172 cm, body weight 65 kg) was enrolled in the study, which included two tests. In the first test (day 1), kinematic and dynamic characteristics of high jumps were recorded. In the second test (day 2), the force–velocity characteristics of the lower limb muscles were determined using isokinetic dynamometry.

**The first test.** The subject performed five high jumps from the original position, a vertical stance with both hands on the waist. The jump height (approximately 33–38 cm) was determined by the impulse of the pushing off force. The rest between the jumps lasted from 1 to 2 min. The kinematic and dynamic characteristics were recorded using a Selspot-2 system (Switzerland). The error in measuring the coordinates along the horizontal and vertical axes was less than 3 mm. The Selspot-2 system included two infrared chambers, seven infrared transducers (of the LED 8 type), and an AMTI dynamometric platform (United States). The kinematics of the jumps was recorded from the right side of the body. Actively reflecting markers were fixed on the head (the temporal region); the shoulder, hip, knee, and ankle joints; the toe; and the heel. The kinematic information and the ground reaction were recorded with a sampling rate of 400 Hz. The kinematic data were smoothed by the moving average technique using five points: the frequency was decreased to 100 Hz, i.e., every fourth point was retained. Smoothing was then performed using a second-order Butterworth filter with a cutoff frequency of 6 Hz [48]. The



**Fig. 1.** Model of the human body. (a) The direction of angle counting from the vertical axis of the inertial basis:  $\alpha$ , trunk slope angle;  $\beta$ , thigh slope angle;  $\gamma$ , shin slope angle;  $\eta$ , foot slope angle;  $\delta$ , toe slope angle. (b) Positive direction of moment and angle counting at the joints:  $M_H$ , hip joint moment;  $M_K$ , knee joint moment;  $M_A$ , ankle joint moment;  $M_M$ , metatarsophalangeal joint moment. (c) The string model of (1) *m. gluteus maximus*, (2) the hamstring, (3) *m. rectus femoris*, (4) *m. vastus*, (5) *m. gastrocnemius*, (6) *m. soleus*, and (7) *mm. flexors*.

angular and linear velocities and accelerations were obtained with the finite difference method.

**The second test.** The force–velocity characteristics of the lower limb muscles (velocity–moment–power) were investigated using an isokinetic dynamometer of the Cybex-2 type. Joint moments were assessed for the hip (extension), knee, and ankle (extension and flexion) joints. The angular velocities were 5.23, 4.19, 3.14, 2.09, 1.05, and 0.52 rad/s. Measurements were performed in two or three replications at each velocity. The period of rest between the measurements was about 1 min. The force–velocity characteristics were calculated with the maximal results obtained in a series.

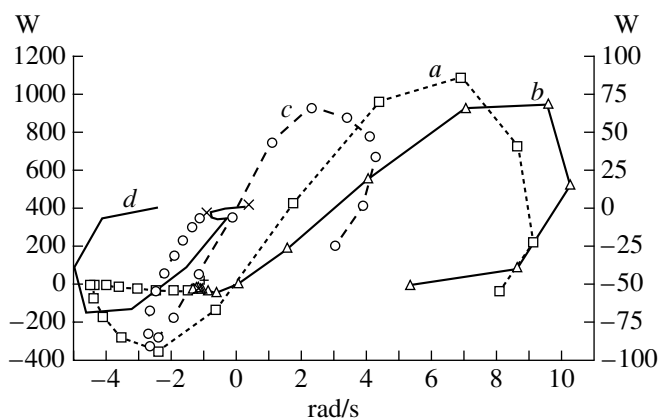
**Mathematical model.** In our model, the human body was considered to consist of five rigid segments: trunk + arms + head, thighs, shins, feet, and toes. The mass–inertia characteristics and the biomechanical segment lengths were computed using published regression equations [49, 50]. The right hip joint was taken to be the pole of the model; segment slope angles were counted clockwise (positive direction) from the vertical axis  $Z$  (Fig. 1a). The movement was performed in the  $YZ$  plane symmetrically for the left and right sides of the body.

To find the control moments in the lower limb joints, the equations of motion were set up in the form of secondary Lagrangian equations. Since the ground reac-

tion force and the reaction moment were determined with force platforms, only control joint moments were obtained by solving the system of dynamic equations [51]. The positive direction of the moments and the angle counting direction are indicated with arrows in Fig. 1b. The control moments and the joint power were calculated for the highest jump (38 cm).

The muscular model included seven muscles: *m. gluteus maximus*, the hamstring, *m. rectus femoris*, *m. vastus*, *m. gastrocnemius*, *m. soleus*, and *m. flexor digitorum longus* + *m. flexor hallucis longus* (the last two muscles were combined to form one group and designated as *mm. flexors*). When describing the anatomical position of the lower limb muscles, we used a “string” model [52]. The string analogue of a muscle implies that the muscle tractive force is directed along the straight line connecting the points of muscle attachment to bones (Fig. 1b). The points of muscle attachment to the lower limb skeleton were determined from [52]. Knowing the angles in the lower limb joints and the points of attachment, we calculated the muscle lengths.

The tractive forces of the lower limb muscles in a high jump were calculated by nonlinear quadratic programming [30].



**Fig. 2.** Power-angular velocity at the lower limb joints in a standing high jump (0.4 s before separation from the support). Ordinate, power (W) at the knee and ankle joints (on the left) or at the hip and metatarsophalangeal joints (on the right). Abscissa, angular velocity (rad/s) at the (a) knee, (b) ankle, (c) hip, and (d) metatarsophalangeal joints.

## RESULTS

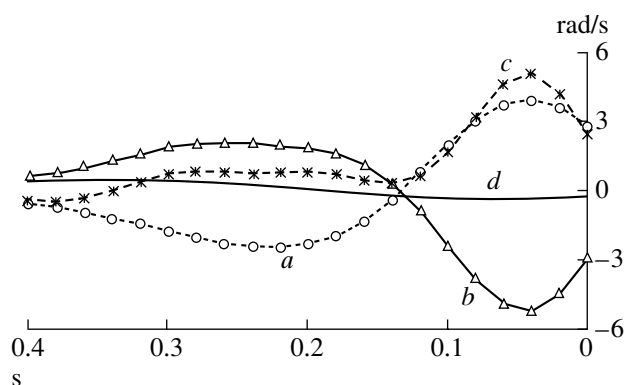
Analysis of the kinematic and dynamic characteristics of a standing high jump was carried out for the 0.4-s period preceding foot separation from the support (point 0 in Figs. 2–8).

### 1. Kinematic and Dynamic Analyses of a Standing High Jump (Multijoint Movement)

**High jump kinematics.** At the beginning of a jump, the athlete bends the legs at the hip, knee, and ankle joints. The angular velocities in the joints are negative (Fig. 2). Half-squatting before pushing off is of certain biomechanical importance: first, elastic muscular elements stretch; second, wavelike folding of successive elastic muscular elements disappears [53, 54]; third, muscular rigidity increases when an active muscle extends owing to reflex mechanisms (the stretch reflex) and the specific functioning of actin-myosin complexes [55, 56]; fourth, the hip, knee, and ankle joint extensors begin to develop forces at the angles where the greatest static forces occur; and, fifth, a greater bending at the knee and ankle joints increases the time of the subsequent extension and, consequently, the force impulse applied to the support.

**The power of the control moments of the hip joint.** The hip joint extensors *m. gluteus maximus* and the hamstring develop a power of 70 W when the angular velocity of extension in the hip joint is 2 rad/s (Fig. 2). With the extension velocity in the hip joint increasing to 4 rad/s, the joint power decreases to 40–50 W, becoming negative at the very end of pushing off.

**The power of the control moments of the knee joint.** The maximum joint power in the knee is developed at an angular velocity of 4–7 rad/s and attains 1100 W (Fig. 2).



**Fig. 3.** Angular velocities (rad/s) of the body segments in a standing high jump. Segment angles were counted from the vertical line for the (a) thigh, (b) shin, (c) foot, and (d) trunk. Abscissa: time, s.

**The power of the control moments of the ankle and metatarsophalangeal joints.** The maximum joint power of the *m. triceps surae* is developed 0.05 s before the end of pushing off and attains approximately 900 W (Fig. 2). The power in the metatarsophalangeal joints is negative because the directions of the moment and the angular velocity do not coincide: the *mm. flexors* moment is aimed at extension,\* while the joint angle decreases. Hence, the *mm. flexors* store energy, which attains 60 W at maximum.

### 2. Kinematic and Dynamic Analyses of the Results of Isokinetic Dynamometry (Single-Joint Movement)

**Analysis of joint kinematics.** The studies with the isokinetic dynamometer showed that, at angular velocities exceeding 6 rad/s, the moments at the lower limb joints are close to zero. In a high jump (multijoint movement), the maximum angular velocity attained in the knee and ankle joints is 9–10 rad/s (Fig. 2). What is the reason for the almost twofold change in the angular velocity in the same joints depending on the type of movement? The angular velocities ( $\dot{U}_{v,i}$ ) in the lower limb joints were determined using the following equations:

$$(a) \dot{U}_{H,i} = \dot{\beta}_i - \dot{\alpha}_i \text{ for the hip joint;}$$

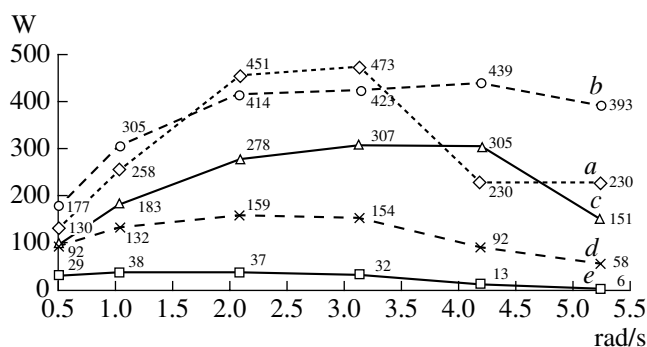
$$(b) \dot{U}_{K,i} = \dot{\beta}_i - \dot{\gamma}_i \text{ for the knee joint; and} \quad (1)$$

$$(c) \dot{U}_{A,i} = \dot{\eta}_i - \dot{\gamma}_i \text{ for the ankle joint, (1)}$$

where the subscripts H, K, and A denote the hip, knee, and ankle joints, respectively, and  $i$  denotes time.

Testing on the isokinetic dynamometer shows that, when the hip joint is extended, the trunk is fixed

\*The moment of the plantar muscles is neglected by virtue of its being infinitesimal a fortiori.



**Fig. 4.** Power-velocity at the lower limb joints as recorded with the isokinetic dynamometer for (a) hip joint extensors, (b) knee joint extensors, (c) knee joint flexors, (d) ankle joint extensors, and (e) ankle joint flexors. Abscissa: angular velocity, rad/s.

( $\dot{\alpha}_i = 0$ ) and the thigh is in motion ( $\dot{\beta}_i \neq 0$ ). Hence, the angular velocity of the joint extension is determined only by the movement of the thigh:  $\dot{U}_{H,i} = \dot{\beta}_i$ . Similar results were obtained for the other joints in testing on the Cybex-2. For the knee joint,  $\dot{\beta}_i = 0$  and, therefore,  $\dot{U}_{K,i} = \dot{\gamma}_i$ . For the ankle joint,  $\dot{\gamma}_i = 0$  and, therefore,  $\dot{U}_{A,i} = \dot{\eta}_i$ .

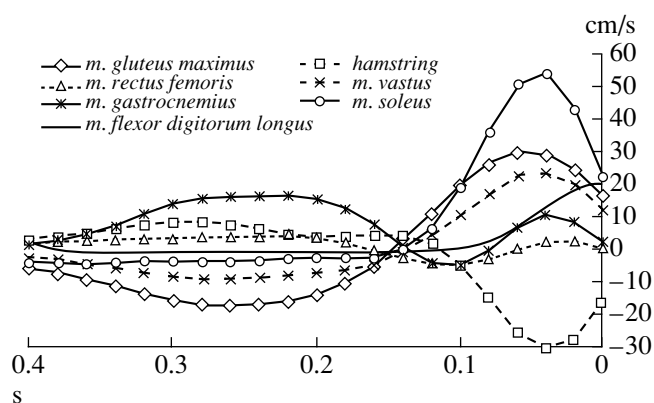
Figure 3 shows the angular velocities of the segments relative to inertial basis axis Z (Fig. 1) during a high jump. In the process, the maximum velocity of each segment does not exceed 6 rad/s. High velocities of 9–10 rad/s (Fig. 2), which were calculated using equation (1), are attained in a jump as a result of the simultaneous movement of neighboring segments at the lower limb joints.

**Joint powers.** We will consider the powers at maximum voluntary single-joint extension/flexion of the lower limb joints at different angular velocities established on the Cybex-2.

**The hip joint.** Testing the force-velocity characteristics of the *m. gluteus maximus* and hamstring on the isokinetic dynamometer showed that, in this joint, the maximum power of 450–470 W was developed at an angular velocity of  $2 \leq \dot{U}_H \leq 3$  rad/s (Fig. 4).

**The knee joint.** The maximum power of the extensor muscles (*m. quadriceps*) was 400–440 W and was attained at an angular velocity of  $2 \leq \dot{U}_K \leq 5$  rad/s; the power of the flexors was approximately 300 W at an angular velocity of  $2 \leq \dot{U}_K \leq 4$  rad/s (Fig. 4).

**The ankle joint.** An extensor power of 155–160 W was attained at an angular velocity of 2–3 rad/s. The flexor power did not exceed 40 W.



**Fig. 5.** Velocities (cm/s) of the lower limb muscles in a high jump. Abscissa: time, s.

### 3. Joint Power in Single-Joint and Multijoint Movements

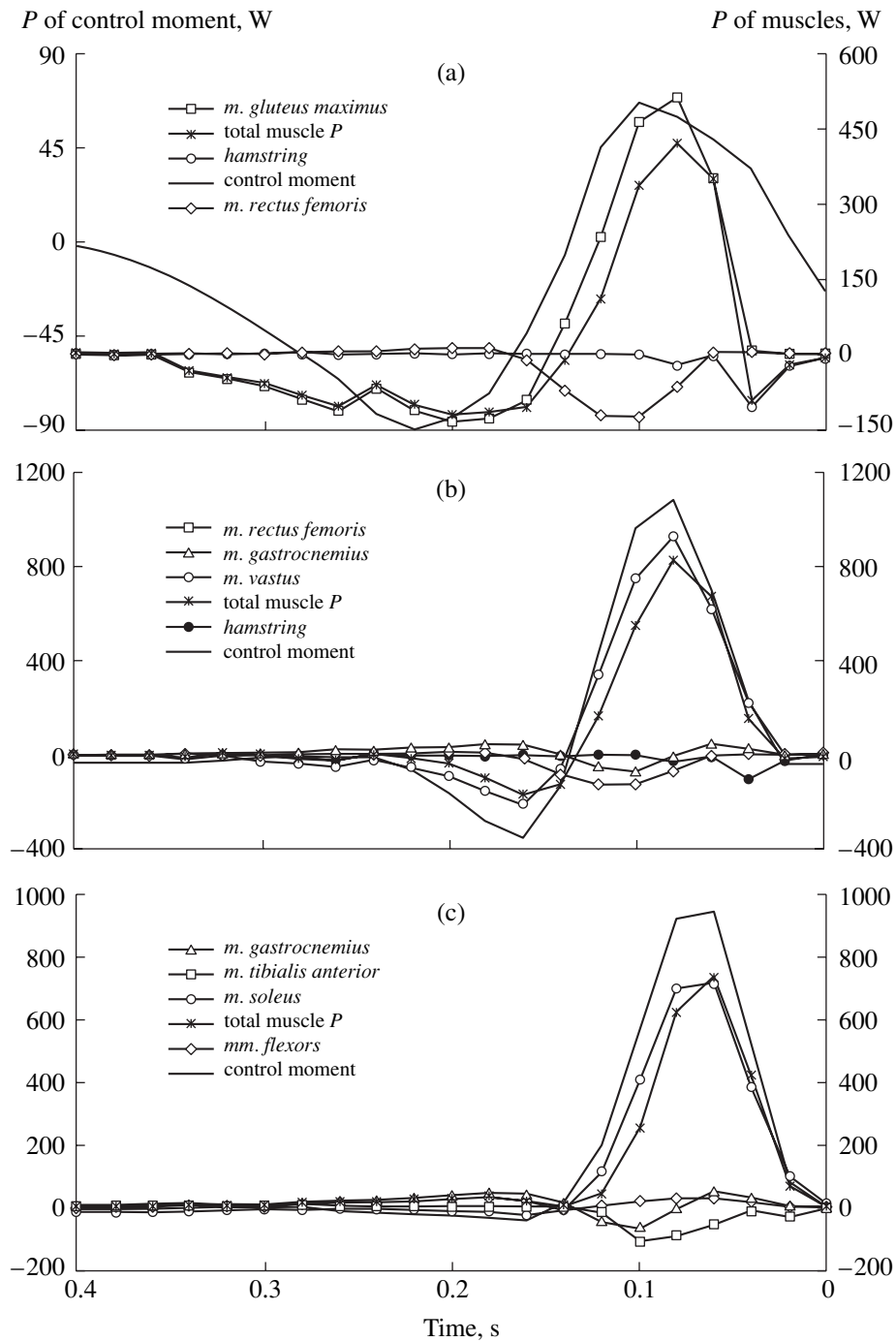
Now, we compare the powers developed in the multijoint movement (standing high jump) with the analogous parameters developed in single-joint movements under the conditions of isokinetic testing.

**The hip joint.** The power did not exceed 70 W at an angular velocity of 2.5 rad/s in a high jump (Fig. 2), whereas the extensor power at the same angular velocity was 400–450 W as measured with the isokinetic dynamometer (Fig. 4). Hence, the force-velocity characteristics of the extensors are sufficient for developing the hip joint moment required to enable the subject to jump as high as about 38 cm.

**The knee joint (extensors).** In testing on the Cybex-2, the power of the *m. quadriceps* at the knee joint was 440 W at an angular velocity of 4.0–5.0 rad/s (Fig. 4). In a high jump, the maximum power of extension of the knee joint was 1100 W at angular velocities ranging from 4 to 9 rad/s (Fig. 2). Let us consider the kinematic and dynamic factors that contribute an “additional” knee joint power of about 700 W in a high jump as compared to the isokinetic testing.

Extension of the knee joint is accompanied by a clockwise movement of the thigh and a simultaneous counterclockwise movement of the shin. The thigh moves clockwise owing to the forces developed by the *m. gluteus maximus* and hamstring. At an angular velocity of  $<3.5$  rad/s, the power produced to move the thigh attains 400 W (Fig. 4).

Let us transform the moment acting at the hip joint into the knee joint moment. For this purpose, let  $\vec{l}_T$  be the distance between the pivots in the knee and hip joints,  $\vec{r}_1$  be the distance between the pivot in the knee joint and the origin of the *m. gluteus maximus* force in the thigh, and  $\vec{r}_2$  be the distance between the origin of the *m. gluteus maximus* tractive force in the thigh and



**Fig. 6.** Powers ( $P$ ) of the muscles and the control moments at the (a) hip, (b) knee, and (c) ankle joints in a high jump. Abscissa: time, s.

the pivot in the hip joint. Hence,  $\dot{r}_1 = \dot{l}_T - \dot{r}_2$ . The *m. gluteus maximus* tractive force ( $F^{GL}$ ) produces a moment relative to the knee joint:  $F^{GL} \times \dot{r}_1 = F^{GL} \times \dot{l}_T - F^{GL} \times \dot{r}_2$ . The upper limit of  $F^{GL} \times \dot{r}_2$  will be considered to be equal to the hip joint extensor moment recorded on the Cybex-2 isokinetic dynamometer. It follows that

$F^{GL} \times \dot{r}_1 = F^{GL} \times \dot{l}_T - M_H = F^{GL} \times 7 \times \dot{r}_2 - F^{GL} \times \dot{r}_2 = 6 \times F^{GL} \times \dot{r}_2$ , where  $\dot{l}_T$  (the biomechanical length of the thigh, or the distance between the pivots in the knee and hip joints) is expressed in terms of the *m. gluteus maximus* traction arm— $\dot{l}_{UL} \approx 7 \times \dot{r}_2$ —and  $M_H$  is the hip joint extensor moment. The hip joint extensor power at

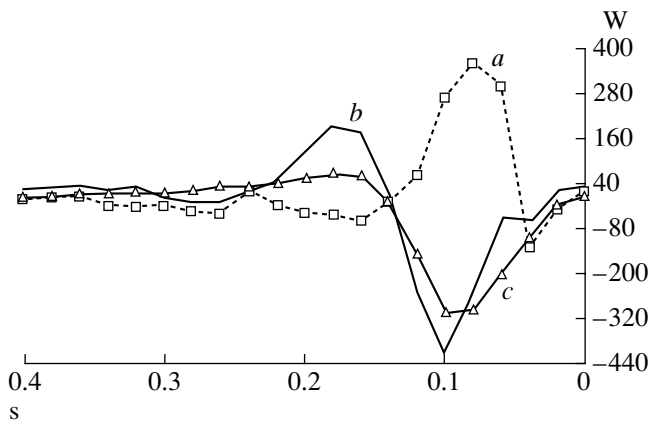


Fig. 7. Difference between the powers of the muscles and the control moments in a high jump as estimated for the (a) hip, (b) knee, and (c) ankle joints. Abscissa: time, s.

an angular velocity of 3–3.5 rad/s (corresponds to the angular velocity observed in a high jump) is approximately  $6 \times F^{GL} \dot{r}_2 \times \dot{U}_C = 6 \times 400 \times 0.08 \approx 200$  W, where  $F^{GL} \times \dot{U}_C \approx 400$  W was obtained from the results of the isokinetic testing (Fig. 4) and  $\dot{r}_2 \approx 0.08$  cm [52].

The counterclockwise movement of the shin is performed at the expense of the moment generated by the *m. quadriceps*, whose power varies in the range 400–440 W at a shin angular velocity of 4–5.5 rad/s (Fig. 4).

If we sum the powers developed by the *m. gluteus maximus*, hamstring, and *m. quadriceps* at the knee joint as evidenced by testing on the Cybex-2, we will see that, according to the isokinetic testing, the power of the knee joint moment cannot exceed 650 W in a high jump. The IPD study of the high jump dynamics showed that the highest knee joint power was 1100 W (Fig. 2), 300–400 W higher than the total power generated by the *m. gluteus maximus*, hamstring, and *m. quadriceps*.

**The knee joint (flexors).** When the subject half-squatted before pushing off, the angle at the knee joint decreased by  $45^\circ$  and the knee joint flexor power attained 390 W (the angular velocity was about 2.5 rad/s, Fig. 2). By isokinetic dynamometry, the knee joint flexor power in a single-joint movement was 270–300 W at the same angular velocity (Fig. 4). Half-squatting, as well as the knee joint flexion, entails a 100-W deficiency in the joint flexor power.

**The ankle joint.** During testing on the isokinetic dynamometer, we recorded a maximal power of 160 W at an angular velocity of 2–3 rad/s (Fig. 4). In a jump, when the angular velocity varied between 4 and 10 rad/s, the power of the control moment attained 900 W (Fig. 2). The causes of generation of the additional power in the ankle joint are as follows: (a) extension of the ankle joint is effected by simultaneous movements of the shin (counterclockwise) and the foot (clockwise); (b) the shin moves counterclockwise

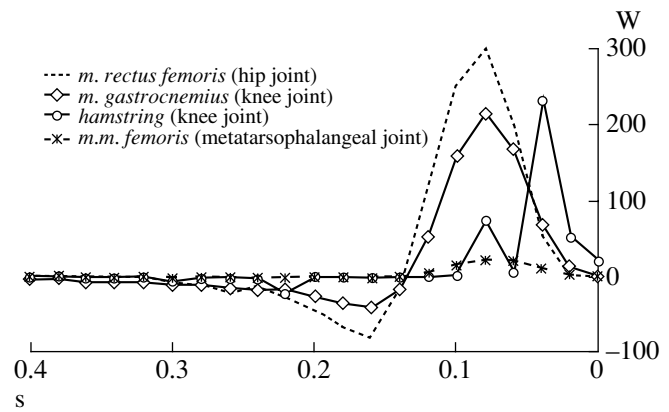


Fig. 8. Power values transferred by the biarticular muscles in a high jump (the joints where additional power is generated are shown in parentheses). Abscissa: time, s.

owing to the *m. quadriceps* moment, whose power is 400–440 W at an angular velocity of 5 rad/s (Fig. 4); (c) the foot moves clockwise owing to the moment (about 20 N m) developed by the *m. triceps surae*, whose power is less than 100 W at an angular velocity of 4–5 rad/s in the single-joint movement (Fig. 4).

Let us transform the moment of the knee joint extensors into the ankle joint moment:  $F^{QDR} \times \dot{r}_2 = F^{QDR} \times \dot{l}_s - F^{QDR} \times \dot{r}_1$ , where  $F^{QDR}$  is the *m. quadriceps* tractive force,  $\dot{l}_s$  is the shin mechanical length (the distance between the pivots in the knee and ankle joints),  $\dot{r}_1$  is the distance from the ankle joint pivot to the point of attachment of the patella tendon in the shin, and  $\dot{r}_2$  is the distance from the point of attachment of the patella tendon in the shin to the knee joint pivot. Then  $F^{QDR} \times \dot{r}_1 \times \dot{U}_K = (F^{QDR} \times \dot{l}_s - M_K) \dot{U}_K = (F^{QDR} \times 7 \times \dot{r}_2 - F^{QDR} \times \dot{r}_2) \dot{U}_K = 6 \times F^{QDR} \times \dot{r}_2 \times \dot{U}_K \approx 150$  W, where  $\dot{r}_2 \approx 6$  cm (the 3-cm distance from the point of attachment of the patella tendon to the joint space + 3 cm to the knee joint pivot) [52] and  $M_K$  is the moment in the knee joint. According to isokinetic testing, when the angular velocity in the ankle joint is 9–10 rad/s in a high jump, the *m. soleus* and the *m. gastrocnemius* contribute only 10% (100 W) and the *m. quadriceps*, 15% (150 W) to the control moment power at the ankle joint.

Calculations of the joint power for a high jump showed that the ankle joint power in a multijoint movement is three times higher than the power recorded on the Cybex-2. This discrepancy can be explained as follows. The unusual position of the foot extensors in testing negatively affects the muscle force; consequently, the extensor muscles are unable to develop a power exceeding 100 W at an angular velocity higher than 4 rad/s (Fig. 4). In half-squatting before pushing off, the angle at the ankle joint decreases from  $110^\circ$  to  $95^\circ$ ,

which aids in stretching the successive and parallel elastic components in the *m. triceps surae* with a subsequent elastic recoil on pushing off. The foot extensors develop a power of 750 W, which exceeds the Cybex-2 testing value by a factor of 3.

#### 4. Biomechanical Causes of the Generation of Additional Power in the Joints

Consider the power and the force-velocity characteristics of the monoarticular (*m. gluteus maximus*, *m. vastus*, and *m. soleus*) and biarticular (*m. rectus femoris*, hamstring, and *m. gastrocnemius*) muscles. The muscular velocities (Fig. 5) of monoarticular synergistic muscles (*m. vastus* and *m. soleus*) are higher than the velocities of biarticular synergistic muscles (*m. rectus femoris* and *m. gastrocnemius*). Consequently, the monoarticular synergists work in the overcoming mode, while the biarticular synergists work in the semi-static (as *m. rectus femoris* and *m. gastrocnemius*) or even in the yielding (as the hamstring) mode.

The power of the monoarticular (MA) or biarticular (BA)  $j$ th muscle is determined as

$$P_i^{j, \text{MA, BA}} = F_i^{j, \text{MA, BA}} \times \dot{L}_i^{j, \text{F, MA, BA}}, \quad (2)$$

where  $F_i^{j, \text{MA, BA}}$  is the tractive force of the  $j$ th muscle and  $\dot{L}_i^{j, \text{F, MA, BA}}$  is the fiber velocity of the  $j$ th muscle at the  $i$ th time point.

The power ( $P_i^{j, \text{BA}, \nu}$ ) that can be transferred by the biarticular  $j$ th muscle via the movement in the  $\nu$ th joint is [26, 57]

$$P_i^{j, \text{BA}, \nu} = d_i^{\nu, j} \times F_i^{j, \text{BA}} \times \dot{U}_{\nu, i}, \quad (3)$$

where  $d_i^{\nu, j}$  is the traction arm of the biarticular muscle in the  $\nu$ th joint,  $F_i^{j, \text{BA}}$  is the tractive force of the biarticular  $j$ th muscle, and  $\dot{U}_{\nu, i}$  is the angular velocity in the  $\nu$ th joint. In the case under consideration,  $\nu = 1, 2, 3, 4$  (the hip, knee, ankle, and metatarsophalangeal joints).

The power of the control moment in the  $\nu$ th muscle is

$$P_i^{\text{MMA}, \nu} = \theta_{\nu, i} \times \dot{U}_{\nu, i}, \quad (4)$$

where  $\theta_{\nu, i}$  is the control muscle moment.

The power developed by the mono- and biarticular muscles in the lower limb joints was compared with the power of the control moments (Fig. 6).

(a) In the hip joint, the power of the control moment did not exceed 60 W; the power of the hip joint extensors *m. gluteus maximus* and the hamstring attained 400 W (Fig. 6a).

(b) In the knee joint, the power of the control moment was about 1100 W, 200 W higher than the total

power developed by the shin extensor muscles (mainly *m. vastus*, Fig. 6b).

(c) In the ankle joint, the power of the control moment constituted almost 960 W, while the total muscle power did not exceed 720 W (Fig. 6c).

The differences between the powers of the control moments and the muscle powers at the lower limb joints in a high jump are shown in Fig. 7. The synergistic muscles acting at the knee and ankle joints are unable to furnish the control moment power necessary for a high jump (Figs. 6b, 6c, 7). Hence, an additional power must be transmitted from some other joint. The powers that could be gained by the biarticular muscles owing to simultaneous leg extension (a multijoint movement) in a standing high jump were calculated according to (3). The peak velocities constituted 300, 210, and 210 W for the *m. rectus femoris*, hamstring, and *m. gastrocnemius*, respectively (Fig. 8).

**Inflow of power to the knee joint.** Figures 6a and 7 show that the hip joint muscles develop a power that is many times greater than the power of the joint moment. Consequently, this power can be transferred to the knee joint. The maximum power inflow from the hip to the knee joint via the *m. rectus femoris* (as assessed by (3)) was 300 W (Fig. 8) and almost completely compensated for the *m. vastus* power deficiency of 400 W (Fig. 7). Hence, movement at the hip joint with a velocity of 2.5–4 rad/s creates the dynamic conditions necessary for the extension of the hip joint with a high angular velocity in a high jump (Fig. 6b).

**Inflow of power to the hip joint.** An increase in the angle of the knee joint stretches the distal end of the hamstring, which increases the power of the hip joint extensors by approximately 200 W at the final stage of pushing off (Fig. 8). This means that the transfer of forces from the knee joint extensors to the hip joint occurs via the biarticular muscles of the posterior aspect of the thigh.

**Inflow of power to the ankle joint.** In a standing high jump, the maximum power of the ankle joint extensors at an angular velocity of 9–10 rad/s was 720 W (Fig. 6c), almost 240 W lower than the joint moment power (Fig. 7,  $t = 0.05$  s). The inflow of power (a) from the knee joint to the ankle joint through the *m. gastrocnemius* (maximum 210 W, Fig. 8) and (b) from the metatarsophalangeal joint to the ankle joint through *mm. flexors* (maximum 15 W) creates the dynamic conditions required for a high jump.

Dynamic analysis of the lower limb joints during a high jump showed that the power of the monoarticular muscles (actuators) is insufficient for a quick vigorous stretching of the lower limb joints. During a high jump, the hip joint donates mechanical power to the knee and ankle joints. The maximum muscular power of 400 W (mainly created by the *m. gluteus maximus*, Fig. 6a) is transferred to the knee and ankle joints via the biarticular *m. rectus femoris* and *m. gastrocnemius*. Then the power of the monoarticular synergists and the power

transferred via the biarticular muscles of the adjacent joints are summed and the resulting muscular power of the knee and ankle joints becomes equivalent to the control moment power obtained as a solution of the IPD (Figs. 6a, 6b).

### CONCLUSIONS

Biomechanical analysis of the functioning of mono- and biarticular muscles in isokinetic testing (single-joint movement) and a high jump (multijoint movement) revealed the following.

(1) The maximum angular velocities of the body segments constitute no more than 6 rad/s. Summation of the angular velocities of the body segments increases the joint velocities twofold in multijoint movements.

(2) Biarticular muscles transfer forces/powers from one joint to another, thereby compensating for the physiological drawback of the monoarticular muscles, that is, a decrease in the tractive force of a muscle with an increasing contraction rate. In a high jump, 300 W may be transferred from the hip to the knee joint; 240 W, from the knee to the hip joint; 200 W, from the knee to the ankle joint; and 15 W, from the metatarsophalangeal to the ankle joint.

(3) The monoarticular muscles change their length and, hence, the length of their intrafusal fibers in proportion to the joint angle; i.e., not only do they perform the function of actuators (propellers), but they also correct moving actions, signaling to the CNS about the amplitude and velocity of joint angles.

(4) The lengths of the biarticular muscles compared to the monoarticular ones do not change in a high jump; hence, the *m. rectus femoris*, hamstring, *m. gastrocnemius*, and *mm. flexors* mainly perform the force function, redistributing forces/powers among the joints.

### ACKNOWLEDGMENTS

I am grateful to D.Yu. Bravaya for isokinetic investigations.

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