## 1. An example for zombie number of Cartesian product of two graphs

**Example 1.** $z(P_3 \square P_4) = 2$

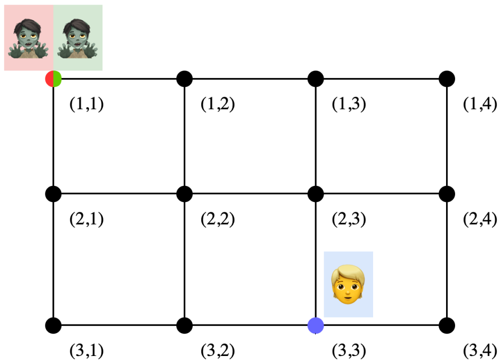


Figure 1: $P_3 \square P_4$ and initial vertices

It is easy to show that $z(P_3) = z(P_4) = 1$. On each of these path graphs, zombie's initial position could be any vertex of the graph. For this example, we put the *G-zombie* and *H-zombie* ($G = P_3$ and $H = P_4$) both on vertex $(1, 1)$. We show the survivor with blue color, *H-zombie* with red, and *G-zombie* with green. *G-zombie* will try to get to the same $G_i$ as the survivor's which is $G_3$ using an *H-edge*. *H-zombie* will try to get to $H_3$ (See figure 2).

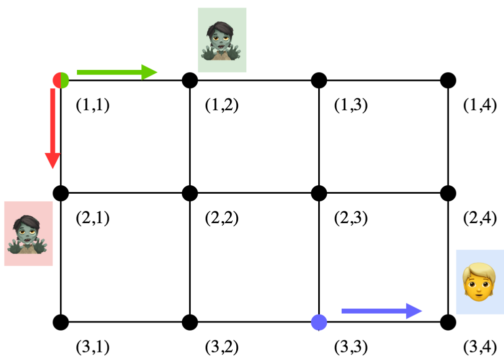


Figure 2: First move of players

After zombies' move the survivor must move. No matter what move he makes, either *G-zombie* has made itself closer to $H_x$ or *H-zombie* has made itself closer to $G_y$. In this case, *H-zombie* got closer to $H_x$. Since neither *H or G-zombie*s share $H_x$ or $G_y$ with the survivor, they will still try to achieve that (See figure 3).

Now *H-zombie* shares the same copy of $H$ as the survivor and it is the survivor's turn. If the survivor moves to another $H_i$, *H-zombie* will mimic the move. If the survivor makes an *H-move*, *H-zombie* will do whatever it did on a single $H$ for capturing the survivor.

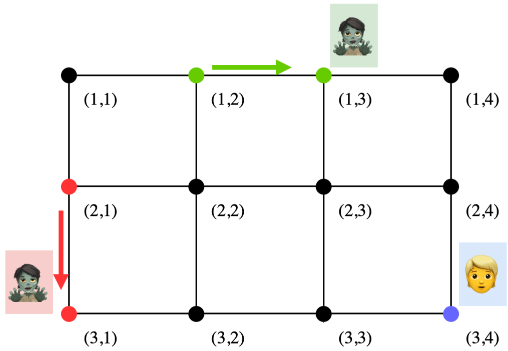


Figure 3: Second move made by zombies, third in total

This means the survivor cannot do infinite *H-move*s. Thus for him being able to survive he has to do infinite *G-move*s, which again leads to *G-zombie* capturing him. For other moves, you can see figure 4.

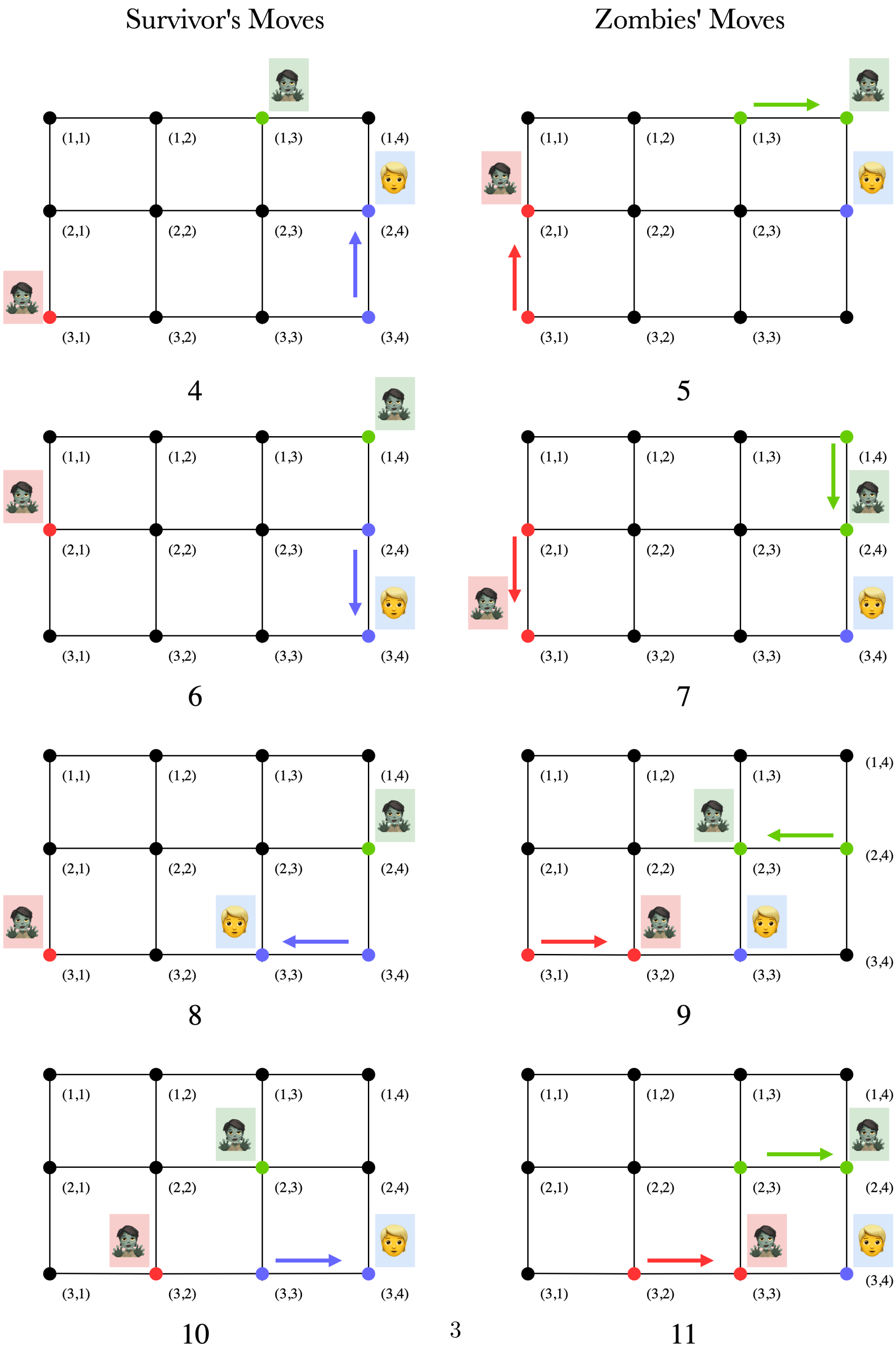




Figure 4: Other moves made by players