UFAZ
Mathematics
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EXERCISES

1 Polynomials

Exercise 1 — Find the polynomial of degree 3 which has a root at -1, a double root at 3 and whose value at $x = 2$ is 12.

Exercise 2 — Find the solutions of the following equations.

1. $4x^4 - x^2 - 18 = 0$,
2. $x^3 - 8 = 0$,
3. $8x^3 - 27 = 0$,
4. $x^4 - 1 = 0$,
5. $81x^4 - 64 = 0$.

Exercise 3 — Verify that $(x^4 - 2x^3 + x^2 + x - 1)$ has of factor $(x - 1)$.

Exercise 4 — Given that $x - 2$ is a factor of the polynomial $x^3 - kx^2 - 24x + 28$, find k and the roots of this polynomial.

Exercise 5 — Find the remainder of the following division
$(x^5 - 2x^2 - 3) \div (x - 1)$

Exercise 6 — Carry out the following divisions and write your answer in the form $p(x) = f(x)q(x) + r(x)$.

1. $(3x^3 - x^2 + 4x + 7) \div (x + 2)$
2. $(3x^3 - x^2 + 4x + 7) \div (x^2 + 2)$
3. $(x^4 - 3x^2 - 2x + 4) \div (x - 1)$
4. $(5x^4 + 30x^3 - 6x^2 + 8x) \div (x^2 - 3x + 1)$

Exercise 7 — Show that the set $\mathbb{R}[X]$ carries a ring structure.

2 Lagrange Interpolation and Hermite interpolation

Exercise 8 — Find the unique Lagrange polynomial $P(x)$ of degree 2 that satisfies

$$P(1) = 1,\ P(2) = 4,\ P(3) = 10.$$

Compute $P(1.6)$.

Exercise 9 — Find the Lagrange polynomial passing through the points :

1. $(1, 3),\ (4, 5)$.
2. $(1, -1),\ (3, 2),\ (4, 7)$.
3. $(0, -1),\ (2, 2),\ (3, 9),\ (5, 6)$.

Exercise 10 — Given $ln(529) = 6.270988$ and $ln(530) = 6.272977$, compute $ln(529.62)$.

Exercise 11 — Let f be a function defined by $f(x) = \frac{1}{1+x^2}$.

1. Find the Lagrange interpolating polynomial P on points $x = 0, 1, 3, 5$.
2. Compute $P(4)$ and compare to $f(4)$.

recall that the error is given by the formula

$$E_n = f(x) - P_n(x) = (x - x_0)\cdots(x - x_n)\frac{f^{n+1}(\xi)}{(n+1)!}$$

where $\xi \in [x_0, x_n]$.

Exercise 12 — Find the Hermite interpolating polynomial P that satisfies

$$\begin{aligned} P(0) = 1, &\quad P'(0) = -0.5, \\ P(1) = 1, &\quad P'(0) = -2, \\ P(2) = 3, &\quad P'(2) = 2, \end{aligned}$$

3 Bezier curves

Exercise 13 — Consider the control polygon determined by the sequence of points

$$P_0(2, 2), P_1(0, 1), P_2(3, -1), P_3(4, 1).$$

1. Establish the De Casteljau scheme and find the corresponding Bezier curve.
2. Find the Bezier curve using Bernstein polynomials.
3. Find the tangents at extremal points and graph the curve together with the control polygon.
4. Find and graph the hodograph (curve corresponding to the derivative)

Exercise 14 — Same exercice with thhe following sequence of points

$$P_0(0, -2), P_1(4, 0), P2(0, 2).$$
$$P_0(0, 0), P_1(2, 1), P_2(3, -2), P_3(4, 0).$$
$$P_0(0, 0), P_2(3, -2), P_1(2, 1), P_3(4, 0).$$