

Scheme2 Core Typing Relation

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The Scheme2 Core static semantics is given as a three-place relation between a variable typing context Γ , expression e , and type T , written $\Gamma \vdash e : T$, pronounced “under Γ , e has type T ”. Formally, the static semantics is taken to be the smallest relation closed under the following rules:

Variables and values

$$\begin{array}{c} \text{T-NUM} \\ \hline \Gamma \vdash n : \text{num} \end{array} \quad \begin{array}{c} \text{T-FALSE} \\ \hline \Gamma \vdash \text{false} : \text{bool} \end{array} \quad \begin{array}{c} \text{T-TRUE} \\ \hline \Gamma \vdash \text{true} : \text{bool} \end{array} \quad \begin{array}{c} \text{T-VAR} \\ (x : T) \in \Gamma \\ \hline \Gamma \vdash x : T \end{array}$$

Unary operators

$$\begin{array}{c} \text{T-NOT} \\ \Gamma \vdash e : \text{bool} \\ \hline \Gamma \vdash (\text{not } e) : \text{bool} \end{array}$$

Binary operators

$$\begin{array}{c} \text{T-BINOP-ARITH} \\ \Gamma \vdash e_1 : \text{num} \quad \Gamma \vdash e_2 : \text{num} \quad b \in \{+, *, -, /\} \\ \hline \Gamma \vdash (b \ e_1 \ e_2) : \text{num} \end{array}$$

$$\begin{array}{c} \text{T-BINOP-COMP} \\ \Gamma \vdash e_1 : \text{num} \quad \Gamma \vdash e_2 : \text{num} \quad b \in \{=, <\} \\ \hline \Gamma \vdash (b \ e_1 \ e_2) : \text{bool} \end{array}$$

Conditionals

$$\begin{array}{c} \text{T-IF} \\ \Gamma \vdash e_{\text{cond}} : \text{bool} \quad \Gamma \vdash e_1 : T \quad \Gamma \vdash e_2 : T \\ \hline \Gamma \vdash (\text{if } e_{\text{cond}} \ e_1 \ e_2) : T \end{array}$$

Functions

$$\frac{\text{T-FUN}}{\Gamma, x : T_1 \vdash e : T_2} \quad \frac{\text{T-APP}}{\Gamma \vdash e_1 : T_1 \rightarrow T_2 \quad \Gamma \vdash e_2 : T_1} \quad \Gamma \vdash (e_1 e_2) : T_2$$

Recursion

$$\frac{\text{T-REC}}{\Gamma, x : T \vdash e : T} \quad \Gamma \vdash (\text{rec } x T e) : T$$

Products

$$\frac{\text{T-PAIR}}{\Gamma \vdash e_1 : T_1 \quad \Gamma \vdash e_2 : T_2} \quad \Gamma \vdash (\text{pair } e_1 e_2) : T_1 * T_2 \quad \frac{\text{T-FST}}{\Gamma \vdash e : T_1 * T_2} \quad \Gamma \vdash (\text{fst } e) : T_1 \quad \frac{\text{T-SND}}{\Gamma \vdash e : T_1 * T_2} \quad \Gamma \vdash (\text{snd } e) : T_2$$

Sums

$$\frac{\text{T-INL}}{\Gamma \vdash e : T_1} \quad \Gamma \vdash (\text{inl } T_2 e) : T_1 + T_2 \quad \frac{\text{T-INR}}{\Gamma \vdash e : T_2} \quad \Gamma \vdash (\text{inl } T_1 : e) : T_1 + T_2 \quad \frac{\text{T-CASE}}{\Gamma \vdash e_1 : T_1 + T_2 \quad \Gamma \vdash e_2 : T_1 \rightarrow T \quad \Gamma \vdash e_3 : T_2 \rightarrow T} \quad \Gamma \vdash (\text{case } e_1 e_2 e_3) : T$$

Lists

$$\frac{\text{T-NIL}}{\Gamma \vdash (\text{nil } T) : \text{list } T} \quad \frac{\text{T-CONS}}{\Gamma \vdash e_1 : T \quad \Gamma \vdash e_2 : \text{list } T} \quad \Gamma \vdash (\text{cons } e_1 e_2) : \text{list } T \quad \frac{\text{T-FOLD}}{\Gamma \vdash e_1 : \text{list } T_1 \quad \Gamma \vdash e_2 : T_1 \rightarrow T_2 \rightarrow T_2 \quad \Gamma \vdash e_3 : T_2} \quad \Gamma \vdash (\text{fold } e_1 e_2 e_3) : T_2$$