Scheme2 Core Typing Relation

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The Scheme2 Core static semantics is given as a three-place relation between a variable typing context Γ, expression e, and type T, written $\Gamma \vdash e : T$, pronounced "under Γ, e has type T". Formally, the static semantics is taken to be the smallest relation closed under the following rules:

Variables and values

$$\textsc{T-num}\quad \frac{}{\Gamma \vdash n : \text{num}} \qquad \textsc{T-false}\quad \frac{}{\Gamma \vdash \mathit{false} : \text{bool}} \qquad \textsc{T-true}\quad \frac{}{\Gamma \vdash \mathit{true} : \text{bool}} \qquad \textsc{T-var}\quad \frac{(x : \mathrm{T}) \in \Gamma}{\Gamma \vdash x : \mathrm{T}}$$

Unary operators

$$\textsc{T-not}\quad \frac{\Gamma \vdash e : \text{bool}}{\Gamma \vdash (\text{not } e) : \text{bool}}$$

Binary operators

$$\textsc{T-binop-arith}\quad \frac{\Gamma \vdash e_1 : \text{num} \qquad \Gamma \vdash e_2 : \text{num} \qquad b \in \{+, *, -, /\}}{\Gamma \vdash (b\, e_1\, e_2) : \text{num}}$$

$$\textsc{T-binop-comp}\quad \frac{\Gamma \vdash e_1 : \text{num} \qquad \Gamma \vdash e_2 : \text{num} \qquad b \in \{=, <\}}{\Gamma \vdash (b\, e_1\, e_2) : \text{bool}}$$

Conditionals

$$\textsc{T-if}\quad \frac{\Gamma \vdash e_{cond} : \text{bool} \qquad \Gamma \vdash e_1 : T \qquad \Gamma \vdash e_2 : T}{\Gamma \vdash (\text{if } e_{cond}\, e_1\, e_2) : T}$$

Functions

$$\textsc{T-fun}\quad \frac{\Gamma, x : T_1 \vdash e : T_2}{\Gamma \vdash (\text{fun } x\, T_1\, e) : T_1 \to T_2}$$

$$\textsc{T-app}\quad \frac{\Gamma \vdash e_1 : T_1 \to T_2 \qquad \Gamma \vdash e_2 : T_1}{\Gamma \vdash (e_1\, e_2) : T_2}$$

Recursion

$$\textsc{T-rec}\quad \frac{\Gamma, x : T \vdash e : T}{\Gamma \vdash (\text{rec } x\, T\, e) : T}$$

Products

$$\textsc{T-pair}\quad \frac{\Gamma \vdash e_1 : T_1 \qquad \Gamma \vdash e_2 : T_2}{\Gamma \vdash (\text{pair } e_1\, e_2) : T_1 * T_2}$$

$$\textsc{T-fst}\quad \frac{\Gamma \vdash e : T_1 * T_2}{\Gamma \vdash (\text{fst } e) : T_1}$$

$$\textsc{T-snd}\quad \frac{\Gamma \vdash e : T_1 * T_2}{\Gamma \vdash (\text{snd } e) : T_2}$$

Sums

$$\textsc{T-inl}\quad \frac{\Gamma \vdash e : T_1}{\Gamma \vdash (\text{inl } T_2\, e) : T_1 + T_2}$$

$$\textsc{T-inr}\quad \frac{\Gamma \vdash e : T_2}{\Gamma \vdash (\text{inl } T_1 : e) : T_1 + T_2}$$

$$\textsc{T-case}\quad \frac{\Gamma \vdash e_1 : T_1 + T_2 \qquad \Gamma \vdash e_2 : T_1 \to T \qquad \Gamma \vdash e_3 : T_2 \to T}{\Gamma \vdash (\text{case } e_1\, e_2\, e_3) : T}$$

Lists

$$\textsc{T-nil}\quad \frac{}{\Gamma \vdash (\text{nil } T) : \text{list } T}$$

$$\textsc{T-cons}\quad \frac{\Gamma \vdash e_1 : T \qquad \Gamma \vdash e_2 : \text{list } T}{\Gamma \vdash (\text{cons } e_1\, e_2) : \text{list } T}$$

$$\textsc{T-fold}\quad \frac{\Gamma \vdash e_1 : \text{list } T_1 \qquad \Gamma \vdash e_2 : T_1 \to T_2 \to T_2 \qquad \Gamma \vdash e_3 : T_2}{\Gamma \vdash (\text{fold } e_1\, e_2\, e_3) : T_2}$$