Scheme1 Core Evaluation Semantics

November 1, 2021

The Scheme1 core evaluation semantics is given as a three-place relation between a variable environment ρ, expression e, and value v, written $\rho \vdash e \Downarrow v$, pronounced "under ρ, e evaluates to v". Formally, the evaluation semantics is taken to be the smallest relation closed under the following rules:

Variables and values

$$\text{E-VAL}\ \frac{}{\rho \vdash v \Downarrow v} \qquad \text{E-VAR}\ \frac{\rho(x) = v}{\rho \vdash x \Downarrow v}$$

Unary operators

$$\text{E-NOT}\ \frac{\rho \vdash e \Downarrow b}{\rho \vdash (\text{not } e) \Downarrow \neg b}$$

Binary operators

$$\text{E-BINOP}\ \frac{\rho \vdash e_1 \Downarrow n_1 \qquad \rho \vdash e_2 \Downarrow n_2 \qquad n_1\, b\, n_2 = v \qquad b \in \{+, *, -, /, =, <\}}{\rho \vdash (b\, e_1\, e_2) \Downarrow v}$$

Functions

$$\text{E-FUN}\ \frac{}{\rho \vdash (\text{fun } x\ e) \Downarrow clos(\rho, x, e)}$$

$$\text{E-APP}\ \frac{\rho \vdash e_1 \Downarrow \text{clos}(\rho', x, e_{body}) \qquad \rho \vdash e_2 \Downarrow v_2 \qquad \rho'[x \mapsto v_2] \vdash e_{body} \Downarrow v}{\rho \vdash (e_1\, e_2) \Downarrow v}$$

Conditionals

$$\textsc{E-if-true}\quad \frac{\rho \vdash e_{cond} \Downarrow \text{true} \qquad \rho \vdash e_1 \Downarrow v_1}{\rho \vdash (\text{if } e_{cond}\, e_1\, e_2) \Downarrow v_1}$$

$$\textsc{E-if-false}\quad \frac{\rho \vdash e_{cond} \Downarrow \text{false} \qquad \rho \vdash e_2 \Downarrow v_2}{\rho \vdash (\text{if } e_{cond}\, e_1\, e_2) \Downarrow v_2}$$