Scheme2 Core Evaluation Semantics

November 16, 2021

The Scheme2 core evaluation semantics is given as a three-place relation between a variable environment ρ, expression e, and value v, written $\rho \vdash e \Downarrow v$, pronounced "under ρ, e evaluates to v". Formally, the evaluation semantics is taken to be the smallest relation closed under the following rules:

Variables and values

$$\text{E-VAL}\ \frac{}{\rho \vdash v \Downarrow v} \qquad \text{E-VAR}\ \frac{\rho(x) = e \qquad \rho \vdash e \Downarrow v}{\rho \vdash x \Downarrow v}$$

Unary operators

$$\text{E-NOT}\ \frac{\rho \vdash e \Downarrow b}{\rho \vdash (\text{not } e) \Downarrow \neg b}$$

Binary operators

$$\text{E-BINOP}\ \frac{\rho \vdash e_1 \Downarrow n_1 \qquad \rho \vdash e_2 \Downarrow n_2 \qquad n_1\ op\ n_2 = v \qquad op \in \{+, *, -, /, =, <\}}{\rho \vdash (op\ e_1\ e_2) \Downarrow v}$$

Conditionals

$$\text{E-IF-TRUE}\ \frac{\rho \vdash e_{cond} \Downarrow \text{true} \qquad \rho \vdash e_1 \Downarrow v_1}{\rho \vdash (\text{if } e_{cond}\ e_1\ e_2) \Downarrow v_1} \qquad \text{E-IF-FALSE}\ \frac{\rho \vdash e_{cond} \Downarrow \text{false} \qquad \rho \vdash e_2 \Downarrow v_2}{\rho \vdash (\text{if } e_{cond}\ e_1\ e_2) \Downarrow v_2}$$

Functions

$$\textsc{E-fun}\ \frac{}{\rho \vdash (\text{fun } x\ e) \Downarrow clos(\rho, x, e)}$$

$$\textsc{E-app}\ \frac{\rho \vdash e_1 \Downarrow \text{clos}(\rho', x, e_{body}) \qquad \rho \vdash e_2 \Downarrow v_2 \qquad \rho'[x \mapsto v_2] \vdash e_{body} \Downarrow v}{\rho \vdash (e_1\ e_2) \Downarrow v}$$

Recursion

$$\textsc{E-rec}\ \frac{\rho[x \mapsto (\text{rec } x\ e)] \vdash e \Downarrow v}{\rho \vdash (\text{rec } x\ e) \Downarrow v}$$

Products

$$\textsc{E-pair}\ \frac{\rho \vdash e_1 \Downarrow v_1 \qquad \rho \vdash e_2 \Downarrow v_2}{\rho \vdash (\text{pair } e_1\ e_2) \Downarrow \text{vpair}(v_1, v_2)} \qquad \textsc{E-fst}\ \frac{\rho \vdash e \Downarrow \text{vpair}(v_0, v_1)}{\rho \vdash (\text{fst } e) \Downarrow v_0} \qquad \textsc{E-snd}\ \frac{\rho \vdash e \Downarrow \text{vpair}(v_0, v_1)}{\rho \vdash (\text{snd } e) \Downarrow v_1}$$

Sums

$$\textsc{E-inl}\ \frac{\rho \vdash e \Downarrow v}{\rho \vdash (\text{inl } e) \Downarrow \text{vinl}(v)} \qquad \textsc{E-inr}\ \frac{\rho \vdash e \Downarrow v}{\rho \vdash (\text{inr } e) \Downarrow \text{vinr}(v)}$$

$$\textsc{E-case-l}\ \frac{\rho \vdash e_1 \Downarrow \text{vinl}(v_1) \qquad \rho \vdash (e_2\ v_1) \Downarrow v_2}{\rho \vdash (\text{case } e_1\ e_2\ e_3) \Downarrow v_2} \qquad \textsc{E-case-r}\ \frac{\rho \vdash e_1 \Downarrow \text{vinr}(v_1) \qquad \rho \vdash (e_3\ v_1) \Downarrow v_3}{\rho \vdash (\text{case } e_1\ e_2\ e_3) \Downarrow v_3}$$

Lists

$$\textsc{E-nil}\ \frac{}{\rho \vdash (\text{nil } T) \Downarrow \text{vnil}} \qquad \textsc{E-cons}\ \frac{\rho \vdash e_1 \Downarrow v_1 \qquad \rho \vdash e_2 \Downarrow v_2}{\rho \vdash (\text{cons } e_1\ e_2) \Downarrow \text{vcons}(v_1, v_2)}$$

$$\textsc{E-fold-base}\ \frac{\rho \vdash e_1 \Downarrow \text{nil} \qquad \rho \vdash e_3 \Downarrow v}{\rho \vdash (\text{fold } e_1\ e_2\ e_3) \Downarrow v}$$

$$\textsc{E-fold-rec}\ \frac{\begin{array}{c}\rho \vdash e_l \Downarrow \text{cons}(v_h, v_t) \\ \rho \vdash e_c \Downarrow v_c \qquad \rho \vdash e_n \Downarrow v_n \qquad \rho \vdash (\text{fold } v_t\ v_c\ v_n) \Downarrow v \qquad \rho \vdash ((v_c\ v_h)\ v) \Downarrow v'\end{array}}{\rho \vdash (\text{fold } e_l\ e_c\ e_n) \Downarrow v'}$$