Scheme1 Core Small-step Operational Semantics

November 7, 2021

The Scheme1 core small-step operational semantics is given as a two-place relation between expressions e and e', written $e \longrightarrow e'$, pronounced "e steps to e'". Formally, the small-step semantics is taken to be the smallest relation closed under the following rules:

Values

The Scheme1 Core values are the two Boolean values true and false, numbers, and functions of the form (fun x $e1$). The notation $[x \mapsto v]e$ denotes capture-avoiding substitution of value v for variable x in expression e. We let metavariables b and n (variously n_1, n_2, etc.) range over Boolean values and numbers, respectively.

Unary operators

$$\frac{\text{E-NOT1}}{e \longrightarrow e'} \qquad \frac{\text{E-NOT}}{(\text{not } e) \longrightarrow (\text{not } e')} \qquad \frac{\text{E-NOT}}{(\text{not } b) \longrightarrow \neg b}$$

Binary operators

$$\frac{\text{E-BINOP1}}{e1 \longrightarrow e1'} \qquad \frac{\text{E-BINOP2}}{e2 \longrightarrow e2'}$$
$$\frac{}{(op\ e_1\ e_2) \longrightarrow (op\ e_1'\ e_2)} \qquad \frac{}{(op\ n_1\ e_2) \longrightarrow (op\ n_1\ e_2')}$$

$$\frac{\text{E-BINOP}}{n_1\ op\ n_2 = v \qquad op \in \{+, *, -, /, =, <\}}{(op\ n_1\ n_2) \longrightarrow v}$$

Conditionals

$$\frac{\text{E-IF1}}{e_1 \longrightarrow e_1'} \qquad \frac{\text{E-IF-TRUE}}{} \qquad \frac{\text{E-IF-FALSE}}{}$$
$$\frac{}{(\text{if } e_1\ e_2\ e_3) \longrightarrow (\text{if } e_1'\ e_2\ e_3)} \qquad \frac{}{(\text{if true } e_2\ e_3) \longrightarrow e_2} \qquad \frac{}{(\text{if false } e_2\ e_3) \longrightarrow e_3}$$

Functions

E-APP1
$$\frac{e1 \longrightarrow e1'}{(e_1\ e_2) \longrightarrow (e'_1\ e_2)}$$

E-APP2
$$\frac{e2 \longrightarrow e2'}{((\text{fun } x\ e_1)\ e_2) \longrightarrow ((\text{fun } x\ e_1)\ e'_2)}$$

E-APP
$$\frac{}{((\text{fun } x\ e_1)\ v_2) \longrightarrow [x \mapsto v_2]e_1}$$