Scheme0 Core Typing Relation

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The Scheme0 Core static semantics is given as a three-place relation between a variable typing context Γ, expression e, and type T, written $\Gamma \vdash e : T$, pronounced "under Γ, e has type T". Formally, the static semantics is taken to be the smallest relation closed under the following rules:

Variables and values

$$\text{T-NUM}$$
$$\frac{}{\Gamma \vdash n : \text{num}}$$

$$\text{T-FALSE}$$
$$\frac{}{\Gamma \vdash false : \text{bool}}$$

$$\text{T-TRUE}$$
$$\frac{}{\Gamma \vdash true : \text{bool}}$$

$$\text{T-VAR}$$
$$\frac{(x : \text{T}) \in \Gamma}{\Gamma \vdash x : \text{T}}$$

Unary operators

$$\text{T-NOT}$$
$$\frac{\Gamma \vdash e : \text{bool}}{\Gamma \vdash (\text{not } e) : \text{bool}}$$

Binary operators

$$\text{T-BINOP-ARITH}$$
$$\frac{\Gamma \vdash e_1 : \text{num} \qquad \Gamma \vdash e_2 : \text{num} \qquad b \in \{+, *, -, /\}}{\Gamma \vdash (b\ e_1\ e_2) : \text{num}}$$

$$\text{T-BINOP-COMP}$$
$$\frac{\Gamma \vdash e_1 : \text{num} \qquad \Gamma \vdash e_2 : \text{num} \qquad b \in \{=, <\}}{\Gamma \vdash (b\ e_1\ e_2) : \text{bool}}$$

Let expressions

$$\text{T-LET}$$
$$\frac{\Gamma \vdash e_1 : T_1 \qquad (\Gamma, x : T_1) \vdash e_2 : T_2}{\Gamma \vdash (\text{let } x\ e_1\ e_2) : T_2}$$

Conditionals

$$\frac{\text{T-IF} \quad \Gamma \vdash e_{cond} : \text{bool} \qquad \Gamma \vdash e_1 : T \qquad \Gamma \vdash e_2 : T}{\Gamma \vdash (\text{if } e_{cond}\ e_1\ e_2) : T}$$