Scheme1 Core Typing Relation

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The Scheme1 Core static semantics is given as a three-place relation between a variable typing context Γ, expression e, and type T, written $\Gamma \vdash e : T$, pronounced "under Γ, e has type T". Formally, the static semantics is taken to be the smallest relation closed under the following rules:

Variables and values

$$\frac{}{\Gamma \vdash n : \text{num}} \text{T-\textsc{num}}$$

$$\frac{}{\Gamma \vdash \mathit{false} : \text{bool}} \text{T-\textsc{false}}$$

$$\frac{}{\Gamma \vdash \mathit{true} : \text{bool}} \text{T-\textsc{true}}$$

$$\frac{(x : \text{T}) \in \Gamma}{\Gamma \vdash x : \text{T}} \text{T-\textsc{var}}$$

Unary operators

$$\frac{\Gamma \vdash e : \text{bool}}{\Gamma \vdash (\text{not } e) : \text{bool}} \text{T-\textsc{not}}$$

Binary operators

$$\frac{\Gamma \vdash e_1 : \text{num} \qquad \Gamma \vdash e_2 : \text{num} \qquad b \in \{+, *, -, /\}}{\Gamma \vdash (b \; e_1 \; e_2) : \text{num}} \text{T-\textsc{binop-arith}}$$

$$\frac{\Gamma \vdash e_1 : \text{num} \qquad \Gamma \vdash e_2 : \text{num} \qquad b \in \{=, <\}}{\Gamma \vdash (b \; e_1 \; e_2) : \text{bool}} \text{T-\textsc{binop-comp}}$$

Conditionals

$$\frac{\Gamma \vdash e_{cond} : \text{bool} \qquad \Gamma \vdash e_1 : T \qquad \Gamma \vdash e_2 : T}{\Gamma \vdash (\text{if } e_{cond} \; e_1 \; e_2) : T} \text{T-\textsc{if}}$$

Functions

T-FUN
$$\frac{(\Gamma, x : T_1) \vdash e : T_2}{\Gamma \vdash (\text{fun } x \ T_1 \ e) : T_2}$$

T-APP
$$\frac{\Gamma \vdash e_1 : T_1 \rightarrow T_2 \qquad \Gamma \vdash e_2 : T_1}{\Gamma \vdash (e_1 \ e_2) : T_2}$$

Recursion

T-REC
$$\frac{(\Gamma, x : T) \vdash e : T}{\Gamma \vdash (\text{rec } x \ T \ e) : T}$$