Scheme1 Core Evaluation Semantics

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The Scheme1 core evaluation semantics is given as a three-place relation between a variable environment ρ, expression e, and value v, written $\rho \vdash e \Downarrow v$, pronounced "under ρ, e evaluates to v". Formally, the evaluation semantics is taken to be the smallest relation closed under the following rules:

Variables and values

$$\text{E-VAL}\ \frac{}{\rho \vdash v \Downarrow v} \qquad \text{E-VAR}\ \frac{\rho(x) = e \qquad \rho \vdash e \Downarrow v}{\rho \vdash x \Downarrow v}$$

Unary operators

$$\text{E-NOT}\ \frac{\rho \vdash e \Downarrow b}{\rho \vdash (\text{not } e) \Downarrow \neg b}$$

Binary operators

$$\text{E-BINOP}\ \frac{\rho \vdash e_1 \Downarrow n_1 \qquad \rho \vdash e_2 \Downarrow n_2 \qquad n_1\ op\ n_2 = v \qquad op \in \{+, *, -, /, =, <\}}{\rho \vdash (op\ e_1\ e_2) \Downarrow v}$$

Conditionals

$$\text{E-IF-TRUE}\ \frac{\rho \vdash e_{cond} \Downarrow \text{true} \qquad \rho \vdash e_1 \Downarrow v_1}{\rho \vdash (\text{if } e_{cond}\ e_1\ e_2) \Downarrow v_1} \qquad \text{E-IF-FALSE}\ \frac{\rho \vdash e_{cond} \Downarrow \text{false} \qquad \rho \vdash e_2 \Downarrow v_2}{\rho \vdash (\text{if } e_{cond}\ e_1\ e_2) \Downarrow v_2}$$

Functions

$$\text{E-FUN}\quad \frac{}{\rho \vdash (\text{fun } x\ e) \Downarrow clos(\rho, x, e)}$$

$$\text{E-APP}\quad \frac{\rho \vdash e_1 \Downarrow \text{clos}(\rho', x, e_{body}) \qquad \rho \vdash e_2 \Downarrow v_2 \qquad \rho'[x \mapsto v_2] \vdash e_{body} \Downarrow v}{\rho \vdash (e_1\ e_2) \Downarrow v}$$

Recursion

$$\text{E-REC}\quad \frac{\rho[x \mapsto (\text{rec } x\ e)] \vdash e \Downarrow v}{\rho \vdash (\text{rec } x\ e) \Downarrow v}$$