Localization

ASSIGNMENT 2

Motion and Measurement Models

Vehicle state consists of position, velocity and parametrization of orientation using a unit quaternion:

$$\mathbf{x}_k = \begin{bmatrix} \mathbf{p}_k \\ \mathbf{v}_k \\ \mathbf{q}_k \end{bmatrix} \in R^{10}$$

Motion model input will consist of specific force and rotational rates from our IMU:

$$\mathbf{u}_k = \begin{bmatrix} \mathbf{f}_k \\ \boldsymbol{\omega}_k \end{bmatrix} \in R^6$$

Position $$\mathbf{p}_k = \mathbf{p}_{k-1} + \Delta t \mathbf{v}_{k-1} + \frac{\Delta t^2}{2}\left(\mathbf{C}_{ns}\mathbf{f}_{k-1} - \mathbf{g}\right)$$

Velocity $$\mathbf{v}_k = \mathbf{v}_{k-1} + \Delta t\left(\mathbf{C}_{ns}\mathbf{f}_{k-1} - \mathbf{g}\right)$$

Orientation $$\mathbf{q}_k = \boldsymbol{\Omega}\left(\mathbf{q}(\boldsymbol{\omega}_{k-1}\Delta t)\right)\mathbf{q}_{k-1}$$

where…

$$\mathbf{C}_{ns} = \mathbf{C}_{ns}(\mathbf{q}_{k-1}) \qquad \boldsymbol{\Omega}\left(\begin{bmatrix} q_w \\ \mathbf{q}_v \end{bmatrix}\right) = q_w \mathbf{1} + \begin{bmatrix} 0 & -\mathbf{q}_v^T \\ \mathbf{q}_v & -\{\mathbf{q}_v\}_\times \end{bmatrix} \qquad \mathbf{q}(\boldsymbol{\theta}) = \begin{bmatrix} \sin\frac{|\boldsymbol{\theta}|}{2} \\ \frac{\boldsymbol{\theta}}{|\boldsymbol{\theta}|}\cos\frac{|\boldsymbol{\theta}|}{2} \end{bmatrix}$$

Linearized Motion Model

Error State

$$\delta \mathbf{x}_k = \begin{bmatrix} \delta \mathbf{p}_k \\ \delta \mathbf{v}_k \\ \delta \boldsymbol{\phi}_k \end{bmatrix} \in R^9$$

3x1 rotation error

Error Dynamics

$$\delta \mathbf{x}_k = \mathbf{F}_{k-1} \delta \mathbf{x}_{k-1} + \mathbf{L}_{k-1} \mathbf{n}_{k-1}$$

measurement noise

where...

$$\mathbf{F}_{k-1} = \begin{bmatrix} \mathbf{1} & \mathbf{1}\Delta t & 0 \\ 0 & \mathbf{1} & -[\mathbf{C}_{ns} \mathbf{f}_{k-1}]_\times \Delta t \\ 0 & 0 & \mathbf{1} \end{bmatrix} \qquad \mathbf{L}_{k-1} = \begin{bmatrix} 0 & 0 \\ \mathbf{1} & 0 \\ 0 & \mathbf{1} \end{bmatrix}$$

$$\mathbf{n}_k \sim \mathcal{N}\left(\mathbf{0}, \mathbf{Q}_k\right) \sim \mathcal{N}\left(\mathbf{0}, \Delta t^2 \begin{bmatrix} \sigma^2_{\text{acc}} & \\ & \sigma^2_{\text{gyro}} \end{bmatrix}\right)$$

$\mathbf{1}$ is the 3×3 identity matrix

Measurement Model

GNSS Measurement Model

$$
\begin{aligned}
\mathbf{y}_k &= \mathbf{h}(\mathbf{x}_k) + \boldsymbol{\nu}_k \\
&= \mathbf{H}_k \mathbf{x}_k + \boldsymbol{\nu}_k = [1 \;\; 0 \;\; 0]\, \mathbf{x}_k + \boldsymbol{\nu}_k \\
&= \mathbf{p}_k + \boldsymbol{\nu}_k
\end{aligned}
$$

$$\boldsymbol{\nu}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_{\mathrm{GNSS}})$$

Lidar Measurement Model

$$
\begin{aligned}
\mathbf{y}_k &= \mathbf{h}(\mathbf{x}_k) + \boldsymbol{\nu}_k \\
&= \mathbf{H}_k \mathbf{x}_k + \boldsymbol{\nu}_k = [1 \;\; 0 \;\; 0]\, \mathbf{x}_k + \boldsymbol{\nu}_k \\
&= \mathbf{p}_k + \boldsymbol{\nu}_k
\end{aligned}
$$

$$\boldsymbol{\nu}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_{\mathrm{LIDAR}})$$