$$p[\mu] := \frac{1}{\sqrt{2\,\pi\,\sigma0^2}}\,e^{-\frac{(\mu-\mu0)^2}{2\,\sigma0^2}}$$

(*p(x|μ)*)

$$p\mu[x] := \frac{1}{\sqrt{2\,\pi\,\sigma^2}}\,e^{-\frac{(x-\mu)^2}{2\,\sigma^2}}$$

(*p(μ|x)= $\frac{p(x|\mu)P(\mu)}{P(x)}$ *)

In[•]:= $p[\mu]\ p\mu[x]$

Out[•]= $$\frac{e^{-\frac{(x-\mu)^2}{2\,\sigma^2}-\frac{(\mu-\mu0)^2}{2\,\sigma0^2}}}{2\,\pi\,\sqrt{\sigma^2}\,\sqrt{\sigma0^2}}$$

In[•]:= $$\text{Together}\left[-\frac{(x-\mu)^2}{2\,\sigma^2}-\frac{(\mu-\mu0)^2}{2\,\sigma0^2}\right]$$

Out[•]= $$\frac{-\mu^2\,\sigma^2+2\,\mu\,\mu0\,\sigma^2-\mu0^2\,\sigma^2-x^2\,\sigma0^2+2\,x\,\mu\,\sigma0^2-\mu^2\,\sigma0^2}{2\,\sigma^2\,\sigma0^2}$$

In[•]:= $$\text{Collect}\left[\frac{-\mu^2\,\sigma^2+2\,\mu\,\mu0\,\sigma^2-\mu0^2\,\sigma^2-x^2\,\sigma0^2+2\,x\,\mu\,\sigma0^2-\mu^2\,\sigma0^2}{2\,\sigma^2\,\sigma0^2},\mu\right]$$

Out[•]= $$\frac{\mu^2\,\left(-\sigma^2-\sigma0^2\right)}{2\,\sigma^2\,\sigma0^2}+\frac{\mu\,\left(2\,\mu0\,\sigma^2+2\,x\,\sigma0^2\right)}{2\,\sigma^2\,\sigma0^2}+\frac{-\mu0^2\,\sigma^2-x^2\,\sigma0^2}{2\,\sigma^2\,\sigma0^2}$$

(*p(μ|x)∝ $\frac{1}{\sqrt{2\pi\ \sigma1^2}}e^{\left(-\frac{(\mu-\mu1)^2}{2\sigma1^2}\right)}$ *)

$$\mu1\ =\ \left(\frac{\sigma^2\,\sigma0^2}{\sigma^2+\sigma0^2}\right)\left(\frac{\mu0}{\sigma0^2}+\frac{x}{\sigma2}\right)$$

$$\sigma1^2\ =\ \frac{\sigma^2\,\sigma0^2}{\sigma^2+\sigma0^2}$$

In[•]:= $$\text{FullSimplify}\left[\left(\frac{\sigma^2\,\sigma0^2}{\sigma^2+\sigma0^2}\right)\left(\frac{\mu0}{\sigma0^2}+\frac{x}{\sigma^2}\right)\right]$$

Out[•]= $$\frac{\mu0\,\sigma^2+x\,\sigma0^2}{\sigma^2+\sigma0^2}$$

In[•]:= $$\text{FullSimplify}\left[\mu0+\frac{\sigma0^2}{\sigma0^2+\sigma^2}\,(x-\mu0)\right]$$

Out[•]= $$\frac{\mu0\,\sigma^2+x\,\sigma0^2}{\sigma^2+\sigma0^2}$$

(*Multivariate Gaussian Function*)

In[•]:= $$f[\mu\_,\ \sigma\_] := \frac{1}{\sqrt{2\,\pi\,\sigma^2}}\,e^{\left(-\frac{(x-\mu)^2}{2\,\sigma^2}\right)}$$

In[•]:=
```
f[μ1, σ1] f[μ2, σ2]
```

Out[•]=
$$\frac{e^{-\frac{(x-\mu 1)^2}{2\,\sigma 1^2}-\frac{(x-\mu 2)^2}{2\,\sigma 2^2}}}{2\,\pi\,\sqrt{\sigma 1^2}\,\sqrt{\sigma 2^2}}$$

In[•]:=
```
M = {{μ1, μ2}};
MatrixForm[M]
```

Out[•]//MatrixForm=
$$\begin{pmatrix} \mu 1 & \mu 2 \end{pmatrix}$$

In[•]:=
```
Σ = {{σ1², 0}, {0, σ2²}};
MatrixForm[Σ]
```

Out[•]//MatrixForm=
$$\begin{pmatrix} \sigma 1^2 & 0 \\ 0 & \sigma 2^2 \end{pmatrix}$$

In[•]:=
$$\frac{1}{(2\,\pi)^{2\times\frac{1}{2}}\,(\mathtt{Det}[\Sigma])^{\frac{1}{2}}}\,e^{-\frac{1}{2}\,((\mathtt{x-M}).\mathtt{Inverse}[\Sigma].(\mathtt{x-M})^{\mathsf{T}})}$$

Out[•]=
$$\left\{\left\{\frac{e^{\frac{1}{2}\left(-\frac{(x-\mu 1)^2}{\sigma 1^2}-\frac{(x-\mu 2)^2}{\sigma 2^2}\right)}}{2\,\pi\,\sqrt{\sigma 1^2\,\sigma 2^2}}\right\}\right\}$$

```
[μ2, σ2] f[μ3, σ3] f[μ4, σ4] f[μ5, σ5]
```

Out[•]=
$$\frac{e^{-\frac{(x-\mu 1)^2}{2\,\sigma 1^2}-\frac{(x-\mu 2)^2}{2\,\sigma 2^2}-\frac{(x-\mu 3)^2}{2\,\sigma 3^2}-\frac{(x-\mu 4)^2}{2\,\sigma 4^2}-\frac{(x-\mu 5)^2}{2\,\sigma 5^2}}}{4\,\sqrt{2}\,\pi^{5/2}\,\sqrt{\sigma 1^2}\,\sqrt{\sigma 2^2}\,\sqrt{\sigma 3^2}\,\sqrt{\sigma 4^2}\,\sqrt{\sigma 5^2}}$$

In[•]:=
```
Plot3D[PDF[MultinormalDistribution[{0, 0}, {{1, 0}, {0, 1}}], {x, y}],
 {x, -3, 3}, {y, -3, 3}, PlotRange → All]
```

Out[•]=