By definition:

$$Q^\pi(s,a) = \mathop{E}_{\gamma^\pi}\Big[\sum_{t\leq T} \gamma^t r(s_t,a_t) \mid s_0=s, A_0=a\Big]$$

Show that $Q^\pi(s,a) = \mathop{E}_{(s',a')\sim \gamma(.|s,a)}\big[r(s,a)+\gamma\, Q^\pi(s',a')\big]$

$$Q^\pi(s,a) = \mathop{E}_{\gamma^\pi}\Big[\sum_{0\leq t\leq T} \gamma^t r(s_t,a_t) \mid s_0=s, A_0=a\Big]$$

$$= \mathop{E}_{\gamma^\pi}\Big[r(s,a) + \gamma \sum_{0\leq t\leq T} \gamma^t r(s_{t+1},a_{t+1}) \mid s_0=s, A_0=a\Big]$$

$$= \mathop{E}_{\gamma^\pi}\Big[r(s,a) + \gamma E\Big[\sum_{0\leq t\leq T} \gamma^t r(s_{t+1},a_{t+1}) \mid s_{t+1}, A_{t+1}, s_0=s, A_0=a\Big] \Big| \substack{s_0=s \\ A_0=a}\Big]$$

As the process $(S_t, A_t)_{t\geq 0}$ is a markovien, thus the following holds:

$$Q^\pi(s,a) = \mathop{E}_{\gamma^\pi}\big[r(s,a)+\gamma\, Q^\pi(S_{t+1},A_{t+1}) \mid s_0=s, A_0=a\big]$$

Show that $Q^*(s,a) = \mathop{E}_{s'\sim \pi^*(.|s,a)}\big[r(s,a)+\gamma \max_{a'} Q^*(s',a')\big]$

$$Q^*(s,a) = \max_\pi Q^\pi(s,a)$$

$$= \max_\pi \mathop{E}_{\gamma^\pi}\big[r(s,a)+\gamma\, Q^\pi(S_{t+1},A_{t+1}) \mid s_0=s, A_0=a\big]$$

By definition of π^* the optimal policy:

$$Q^*(s,a) = \mathop{E}_{\gamma^{\pi^*}}\big[r(s,a)+\gamma \max_\pi Q^\pi(S_{t+1},A_{t+1}) \mid s_0=s, A_0=a\big]$$

$$= \mathop{E}_{\gamma^{\pi^*}}\big[r(s,a)+\gamma \max_{a'} Q^*(S_{t+1},a') \mid s_0=s, A_0=a\big]$$