CS/MATH 113 – Problem Set 1

Habib University – Spring 2023

Week 01

1 Definitions

Definition 1. (Integer) An integer is a number with no decimal or fractional part and it includes negative and positive numbers, including zero.

Definition 2. (Even Integer) An integer is even if it can be written as $2k$ where k is an integer.

Definition 3. (Odd Integer) An integer is even if it can be written as $2k + 1$ where k is an integer.

Definition 4. (Parity) The parity of an integer is its property of being even or odd.

Definition 5. (Natural Numbers) Natural numbers are a set of positive numbers from 1 to ∞

Definition 6. (Rational Numbers) Rational numbers are any numbers that can be expressed in the form $\frac{a}{b}$ where a and b are integers, and $b \neq 0$

Definition 7. (Divisiblity) A nonzero integer m divides an integer n provided that there is an integer q such that $n = mq$. We say that m is a divisor of n and that m is a factor of n and use the notation $m|n$

2 Problems

Using the definitions above solve the following problems.

Problem 1. Prove that the sum of two odd integers is even.

Problem 2. Prove that the product of two even integers is even.

Problem 3. Prove that the product of any two rational numbers is also a rational number.

Problem 4. Prove that the square of any natural number is also a natural number.

Problem 5. Prove that the square of any rational number is also a rational number.

Problem 6. In each case either prove the statement or find a counterexample.

(a) The sum of any three consecutive integers (positive or negative) is divisible by 3.

(b) The product any two even integers is divisible by 4.

(c) The product of any four consecutive integers (positive or negative) is divisible by 8.

(d) If $a - b$ has remainder 0 when divided by m, then a and b have remainders 0 when divided by m.

(e) If n is an odd integer, then $3n + 3$ is divisible by 6

Problem 7. Prove that the product of five consecutive integers is divisible by 120.

Problem 8. Prove that the sum of two postive integers of the same parity (odd/even) is even.

Problem 9. Prove or disprove that if $a + b$ is an odd integer, then both $a + x$ and $b + x$ are odd integers, where a, b, and x are integers.