Weekly Challenge 02: Logical Thinking
CS/MATH 113 Discrete Mathematics

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1. Logical Hikers

A group of n hikers is standing at the base of a mountain. Their guide joins them and hangs a compass from each hiker's neck. At their current position, all the compasses should point North. But some are faulty. A hiker cannot see their own compass but can see everyone else's. For the hike to go smoothly, it is important for every hiker to know the condition of their compass, i.e., if it is faulty or functions correctly. The guide is going around asking each hiker in turn about the condition of their compass. Each hiker reports either "Working", "Faulty", or "Can't tell". The hikers are highly logical and always respond truthfully.

(a) Given that the guide has announced that *exactly* k compasses are faulty where $0 \leq k \leq n$, prove that every hiker can correctly tell the condition of their compass just by looking at the compasses of the other hikers.

(b) Suppose instead that the guide announces *at least* k faulty compasses where $1 \leq k \leq (n-1)$. Prove the following statement about the responses: As soon as a hiker responds, "Faulty", no subsequent hiker will respond, "Can't tell".

(c) Suppose instead that the guide announces *at most* k faulty compasses where $1 \leq k \leq (n-1)$. Provide a statement about the responses similar to the one given in the previews part. Show that it is correct.

Solution: