Quiz 1B: Parity

CS/MATH 113 Discrete Mathematics L1

Habib University — Spring 2023

Total Marks: 5   Date: January 16, 2023
Duration: 15 minutes   Time: 1715–1730h

Student ID: _____

Student Name: _____

1   Problems

1. (5 points) Given the following definitions, prove the claim below.

   Definition 1.1 (Even integer). An integer is *even* if it can be written as $2k$ where k is an integer.

   Definition 1.2 (Odd integer). An integer is *odd* if it can be written as $2k + 1$ where k is an integer.

   Definition 1.3 (Parity). The *parity* of an integer is its property of being even or odd.

   Claim 1. *Given integers, m and n of different parity, $m \cdot n$ is odd.*

   Solution: There are 2 cases to consider: m is even and n is odd, and vice versa. The proof proceeds similarly in both cases, so only the case of even m and odd n is explored below.

   Proof. Let $m = 2p$ and $n = 2q + 1$ where p and q are integers.
   Then $mn = 4pq + 2p = 2(2pq + p)$.
   $2pq + p$ is an integer.
   $\therefore mn$ is even.
   $\therefore$ the claim not true and cannot be proved. $\square$