Set theory Problem

February 19, 2023

1. Union and intersection involving infinite collections of sets are defined as follows: Let $\{S_{\alpha} | \alpha \in A\}$ be a collection of sets, where A is an index set that can be of any size. Then

$$\bigcup_{\alpha \in A} S_{\alpha} = \{x | \exists \alpha \in A \text{ s.t. } (x \in S_{\alpha})\}$$

and

$$\bigcap_{\alpha \in A} S_{\alpha} = \{x | \forall \alpha \in A, \ x \in S_{\alpha}\}$$

Show that the distributed laws holds in the following forms:

$$\bigcup_{\alpha \in A} (T \cap S_{\alpha}) = T \cap \left(\bigcup_{\alpha \in A} S_{\alpha}\right)$$

$$\bigcap_{\alpha \in A} (T \cup S_{\alpha}) = T \cup \left(\bigcap_{\alpha \in A} S_{\alpha}\right)$$

Solution: