Quiz 1C: Parity

CS/MATH 113 Discrete Mathematics L1

Habib University — Spring 2023

Total Marks: 5 | Date: January 16, 2023
Duration: 15 minutes | Time: 1715–1730h

Student ID: _____

Student Name: _____

1 Problems

1. (5 points) Given the following definitions, prove the claim below.

   Definition 1.1 (Even integer)**.** An integer is *even* if it can be written as $2k$ where k is an integer.

   Definition 1.2 (Odd integer)**.** An integer is *odd* if it can be written as $2k + 1$ where k is an integer.

   Definition 1.3 (Parity)**.** The *parity* of an integer is its property of being even or odd.

   Claim 1. *Given integers, m, n, of the same parity, $m \cdot n$ has the same parity.*

Solution: There are 2 cases to consider: both m and n are even, and both are odd. We prove below that their product shares their parity in each case.

Proof. Case 1: m and n are even.
Let $m = 2p$ and $n = 2q$ where p and q are integers.
Then $mn = 4pq = 2(2pq)$.
$2pq$ is an integer.
$\therefore mn$ is even.
Case 2: m and n are odd.
Let $m = 2p + 1$ and $n = 2q + 1$ where p and q are integers.
Then $mn = 4pq + 2p + 2q + 1 = 2(2pq + p + q) + 1$.
$2pq + p + q$ is an integer.
$\therefore mn$ is odd. □