

# Weekly Challenge 05: Predicate Logic and Natural Language

## CS/MATH 113 Discrete Mathematics

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### 1. Limit of a Function

**Definition 1** (Limit of a function at a point). Let  $f(x)$  be a real valued function defined on an interval that contains  $x = a$ , except possibly at  $x = a$ . For  $\epsilon$  and  $\delta$  real numbers, we say that limit of the function  $f(x)$  at  $x = a$  is a real number  $L$  and write  $\lim_{x \rightarrow a} f(x) = L$  if

$$\forall \epsilon > 0, \exists \delta > 0 \ni (0 < |x - a| < \delta \implies |f(x) - L| < \epsilon).$$

The  $\ni$  symbol denotes, “such that”.

- (a) Express in English language, the definition of the limit of a function at a point.

**Solution:**

- (b) Express in English language, what it means for the limit of a function  $f(x)$  to not exist at point  $x = a$ .

**Solution:**

- (c) Express in predicate logic, what it means for the limit of a function  $f(x)$  to not exist at point  $x = a$ .

**Solution:**

- (d) If in the definition of limit we swap the two quantifiers like this:

$$\exists \delta > 0 \ni \forall \epsilon > 0, (|x - a| < \delta \implies |f(x) - L| < \epsilon).$$

How does the new statement read? Does it still define the limit of a function? Discuss.

**Solution:**