Recitation 3: Introductory Propositional Logic

Dead TA's Society

Discrete Mathematics
Habib University
Karachi, Pakistan

Febuary 4, 2022

Godel must be proud

Question: Is the assertion "This statement is false" a proposition?

Godel must be proud

Question: Is the assertion "This statement is false" a proposition?

Answer: **No**. A proposition is a statement that can have either be true or false but not both. This can not therefore this is not a proposition.

Question

What is the negation of each of these propositions?

Statement: Chika is the best girl.

Question

What is the negation of each of these propositions?

Statement: Chika is the best girl.
Negation: Chika is not the best girl.

Question

What is the negation of each of these propositions?

Statement: Everyone loves me.

Question

What is the negation of each of these propositions?

Statement: Everyone loves me.
Negation: There is someone who doesn't love me.

Question

What is the negation of each of these propositions?

Statement: There are 42 items in a Mujtaba's dozen.

Question

What is the negation of each of these propositions?

Statement: There are 42 items in a Mujtaba's dozen.
Negation: There aren't 42 items in a Mujtaba's dozen.

Question

What is the negation of each of these propositions?

Statement: Everyday, Blingblong sends more than 100 text messages to the guy who delivered him pizza 2 months ago.

Question

What is the negation of each of these propositions?

Statement: Everyday, Blingblong sends more than 100 text messages to the guy who delivered him pizza 2 months ago.
Negation: On someday, Blingblong doesn't send more than 100 text messages to the guy who delivered him pizza 2 months ago.

Question

What is the negation of each of these propositions?

Statement: 121 is a perfect square.

Question

What is the negation of each of these propositions?

Statement: 121 is a perfect square.
Negation: 121 is not a perfect square.

Question

What is the negation of each of these propositions?

Statement: Karen took the kids and the dog

Question

What is the negation of each of these propositions?

Statement: Karen took the kids and the dog
Negation: Either Karen didn't take the kids or Karen didn't take the dog, or Karen took neither.

Question

Let p and q be the propositions

$$p = \text{We live in a society}$$
$$q = \text{I am an emo kid}$$

Write these propositions using p and q and logical connectives (including negations).
Proposition: We live in a society and I am an emo kid.

Question

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Proposition: We live in a society and I am an emo kid.
Answer: $p \wedge q$

Question

Let p and q be the propositions

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Proposition: We live in a society but I am not an emo kid.

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Proposition: We live in a society but I am not an emo kid.
Answer: $p \wedge \neg q$

Question

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$$p = \text{We live in a society}$$
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Write these propositions using p and q and logical connectives (including negations).
Proposition: Either we live in a society or I am an emo kid or both.

Question

Let p and q be the propositions

$$p = \text{We live in a society}$$
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Write these propositions using p and q and logical connectives (including negations).
Proposition: Either we live in a society or I am an emo kid or both.
Answer: $p \vee q$

Question

Let p and q be the propositions

$$p = \text{We live in a society}$$
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Proposition: If we live in a society, I am an emo kid.

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Proposition: If we live in a society, I am an emo kid.
Answer: $p \implies q$

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Write these propositions using p and q and logical connectives (including negations).
Proposition: Either we live in a society or I am an emo kid, but not both.
Answer: $p \oplus q$

Question

Let p and q be the propositions

$$p = \text{We live in a society}$$
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Write these propositions using p and q and logical connectives (including negations).
Proposition: We living in a society is necessary for me to be an emo kid.

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Let p and q be the propositions

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Answer: $q \implies p$

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$$p = \text{We live in a society}$$
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Write these propositions using p and q and logical connectives (including negations).
Proposition: We living in a society is sufficient for me to be an emo kid.

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Let p and q be the propositions

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Proposition: We living in a society is sufficient for me to be an emo kid.
Answer: $p \implies q$

Question

Let p and q be the propositions

$$p = \text{We live in a society}$$
$$q = \text{I am an emo kid}$$

Write these propositions using p and q and logical connectives (including negations).
Proposition: We living in a society is necessary and sufficient for me to be an emo kid.

Question

Let p and q be the propositions

$$p = \text{We live in a society}$$
$$q = \text{I am an emo kid}$$

Write these propositions using p and q and logical connectives (including negations).
Proposition: We living in a society is necessary and sufficient for me to be an emo kid.
Answer: $p \iff q$

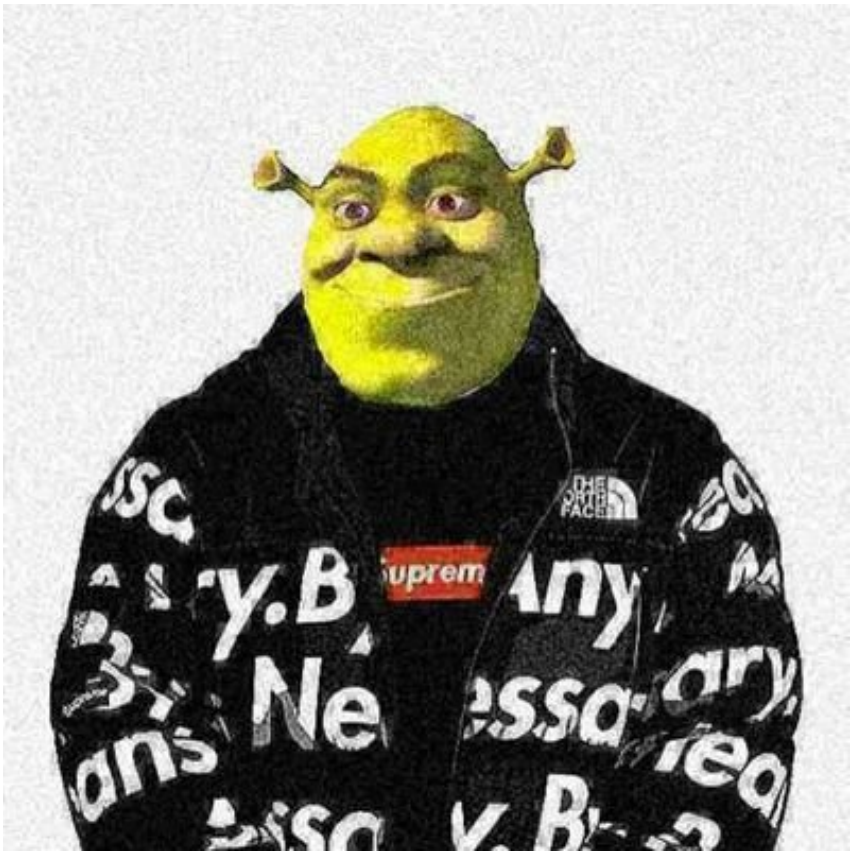
Question

Given

p : Shrek is adorable.

q : Kermit is in gamer rage.

r : The TAs get enough sleep



Question

Let
p : Shrek is adorable.
q : Kermit is in gamer rage.
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Express each of these compound propositions as an English sentence.

Compound propositions: $\neg r$

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English sentence: The TAs don't get enough sleep

Question

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Express each of these compound propositions as an English sentence.

Compound propositions: $q \wedge \neg r$

Question

Let
p : Shrek is adorable.
q : Kermit is in gamer rage.
r : The TAs get enough sleep
Express each of these compound propositions as an English sentence.

Compound propositions: $q \wedge \neg r$

English sentence: Kermit is in gamer rage and The TAs don't get enough sleep

Question

Let p, q, r be the propositions "Shrek is adorable", "Kermit is in gamer Let
p : Shrek is adorable.
q : Kermit is in gamer rage.
r : The TAs get enough sleep
Express each of these compound propositions as an English sentence.

Compound propositions: $p \Rightarrow \neg r$

Question

Let p, q, r be the propositions "Shrek is adorable", "Kermit is in gamer Let
p : Shrek is adorable.
q : Kermit is in gamer rage.
r : The TAs get enough sleep
Express each of these compound propositions as an English sentence.

Compound propositions: $p \Rightarrow \neg r$

English sentence: If Shrek is adorable then the TAs don't get enough sleep

Question

Let
p : Shrek is adorable.
q : Kermit is in gamer rage.
r : The TAs get enough sleep
Express each of these compound propositions as an English sentence.

Compound propositions: $\neg p \vee \neg r$

Question

Let
p : Shrek is adorable.
q : Kermit is in gamer rage.
r : The TAs get enough sleep
Express each of these compound propositions as an English sentence.

Compound propositions: $\neg p \vee \neg r$

English sentence: Either the TAs don't get enough sleep or Shrek is not adorable or both.

Question

Let
p : Shrek is adorable.
q : Kermit is in gamer rage.
r : The TAs get enough sleep
Express each of these compound propositions as an English sentence.

Compound propositions: $\neg p \Leftrightarrow q$

Question

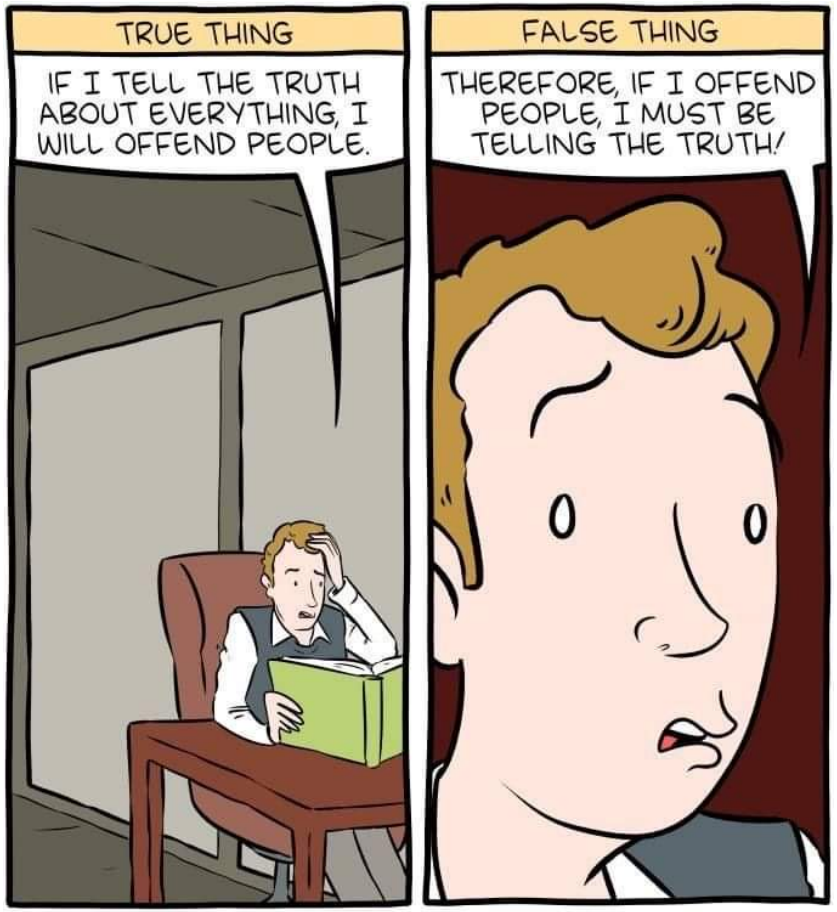
Let
p : Shrek is adorable.
q : Kermit is in gamer rage.
r : The TAs get enough sleep
Express each of these compound propositions as an English sentence.

Compound propositions: $\neg p \Leftrightarrow q$

English sentence: Kermit is in gamer rage if and only if Shrek is not adorable.

Context

"If I tell the truth about everything then I will offend someone."



Among scholars, this is known as the Youtube Commentator's Fallacy.

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Write this statement in propositional logic and its converse, contrapositive, and inverse in English and propositional logic

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$$p \implies q$$

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$$p \implies q$$

Converse: q $\implies$ p

Question

"If I tell the truth about everything then I will offend someone."

Write this statement in propositional logic and its converse, contrapositive, and inverse in English and propositional logic

p: I tell the truth about everything
q: I will offend someone

$$p \implies q$$

Converse: q $\implies$ p
If I will offend someone then I'm telling the truth about everything

Question

"If I tell the truth about everything then I will offend someone."

Write this statement in propositional logic and its converse, contrapositive, and inverse in English and propositional logic

p: I tell the truth about everything
q: I will offend someone

$$p \implies q$$

Converse: q $\implies$ p
If I will offend someone then I'm telling the truth about everything
Contrapositive: $\neg$ q $\implies$ $\neg$ p

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p: I tell the truth about everything
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$$p \implies q$$

Converse: q $\implies$ p
If I will offend someone then I'm telling the truth about everything
Contrapositive: $\neg$ q $\implies$ $\neg$ p
If I will offend no one then I'm not telling the truth about something

Question

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q: I will offend someone

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Converse: q $\implies$ p
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Converse: q $\implies$ p
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If I will offend no one then I'm not telling the truth about something
Inverse: $\neg$ p $\implies$ $\neg$ q
If I'm not telling the truth about something then I will offend no one

Question

Construct a truth table for the following compound proposition

$$(p \Leftrightarrow q) \oplus (p \Leftrightarrow \neg q)$$

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T	T			
T	F			
F	T			
F	F			

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T	T	T		
T	F	F		
F	T	F		
F	F	T		

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T	F	F	T	
F	T	F	T	
F	F	T	F	

Question

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T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	T	F	T

Question

"Only a Sith deals in Absolutes" - Obi-Wan Kenobi

Translate the above statement in propositional logic. Is the statement above an absolute statement? If the statement above is true, what does this say about Obi-Wan Kenobi.

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$p =$ The person is a Sith

$q =$ The person deals in absolutes

Question

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Translate the above statement in propositional logic. Is the statement above an absolute statement? If the statement above is true, what does this say about Obi-Wan Kenobi.

$p =$ The person is a Sith

$q =$ The person deals in absolutes

$$q \implies p$$

Question

Note that the statement isn't bidirectional. Bidirectional refers to if and only if.

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It is also not $p \implies q$ since it is not necessary for a Sith to deal in absolutes. A dead sith can not deal in anything for example. If a person deals in absolutes then we are sure he is a sith therefore $q \implies p$.

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Note that the statement isn't bidirectional. Bidirectional refers to if and only if.
It is also not $p \implies q$ since it is not necessary for a Sith to deal in absolutes. A dead sith can not deal in anything for example. If a person deals in absolutes then we are sure he is a sith therefore $q \implies p$.
This statement is an absolute statement since it uses the word **Only**.
If Obi-wan is telling the truth, since he is saying an absolute statement he is a sith and should be removed from the council to be replaced by Anakin.

Question

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Binary search runs in $O(\log n)$ if and only if π is irrational

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Let P : π is irrational.

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Binary search runs in $O(\log n)$ if and only if π is irrational

Let P : π is irrational.
Let B : Binary search runs in $O(\log n)$.

Question

Is the following statement valid?
Binary search runs in $O(\log n)$ if and only if π is irrational

Let P : π is irrational.
Let B : Binary search runs in $O(\log n)$.
Statement

$$B \Leftrightarrow P$$

Question

Is the following statement valid?
Binary search runs in $O(\log n)$ if and only if π is irrational

Let P : π is irrational.
Let B : Binary search runs in $O(\log n)$.
Statement

$$B \Leftrightarrow P$$

As P is always true:

$$B \Rightarrow P \text{ is true}$$

Question

Is the following statement valid?
Binary search runs in $O(\log n)$ if and only if π is irrational

Let P : π is irrational.
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Statement

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Question

Is the following statement valid?
Binary search runs in $O(\log n)$ if and only if π is irrational

Let P : π is irrational.
Let B : Binary search runs in $O(\log n)$.
Statement

$$B \Leftrightarrow P$$

As P is always true:

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As B is always true:

$$P \Rightarrow B \text{ is true}$$

Thus

$$B \Leftrightarrow P \text{ is true}$$

Its True, as both statements are true.

Conclusion

That's all folks! Attendance time.

1. Read the book!
2. Practice more! (Practice problems on Sets are available on Canvas)
3. Don't forget to hit the like button and subscribe to our youtube channel.
4. Remember that the TA's hours can be seen on canvas and TAs can be found in their hours on EHSAS Group (MS Teams)