Recitation: Functions

Discrete Mathematics TAs
(plaigarised primarily from the book)

April 15, 2022

Questions

1. $\boxed{\text{10 points}}$ Determine whether $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ is onto if
   (a) $f(m, n) = 2m - n$
   (b) $f(m, n) = m^2 - n^2$
   (c) $f(m, n) = m + n + 1$
   (d) $f(m, n) = |m| - |n|$
   (e) $f(m, n) = m^2 - 4$

2. $\boxed{\text{5 points}}$ if $f : A \to B$ is a one to one function show that there exists a function $g : B \to A$ such that g is onto

3. $\boxed{\text{5 points}}$ if $f : A \to B$ is a onto function show that there exists a function $g : B \to A$ such that g is one to one

4. $\boxed{\text{5 points}}$ Show that for every one-to-one and onto function $f : A \to B$ it has an inverse function g, such that $\forall a \in A, g(f(a)) = a$ and $\forall b \in A, f(g(b)) = b$

5. $\boxed{\text{7 points}}$ 1. Let I be the set of decimals of the form $0.d_1d_2d_3d_4...$ Construct a one to one function from I to $I \times I$
   2. Find either an onto function from I to $I \times I$ or a one to one function from for $I \times I$ to I
   3. Do I **and** $I \times I$ have the same cardinality?

6. $\boxed{\text{5 points}}$ Let $f : A \to B$ and $g : B \to C$
   If $C_0 \subseteq C$, show that $(g \circ f)^{-1}(C_0) = f^{-1}(g^{-1}(C_0))$

7. $\boxed{\text{5 points}}$ Let $f : A \to B$ and $g : B \to C$
   If f and g are injective, show that $g \circ f$ is injective.

8. $\boxed{\text{5 points}}$ Let $f : A \to B$ and $g : B \to C$
   If f and g are surjective, show that $g \circ f$ is surjective.

9. $\boxed{\text{5 points}}$ In mathematics and computer science, the floor function is the function that takes as input a real number x, and gives as output the greatest integer less than or equal to x.
   Let g be a function from the set A to the set B. Let S be a subset of B. We define the inverse image of S to be the subset of A whose elements are precisely all pre-images of all elements of S. We denote the inverse image of S by $g^{-1}(S)$, so $g^{-1}(S) = \{a \in A \mid g(a) \in S\}$. (Beware: The notation g^{-1} is used in two different ways. Do not confuse the notation introduced here with the notation $g^{-1}(y)$ for the value at y of the inverse of the invertible function g. Notice also that $g^{-1}(S)$, the inverse image of the set S, makes sense for all functions g , not just invertible functions.)
   Let $g(x) = \lfloor x \rfloor$. Find

(a) $g^{-1}(\{0\})$.

(b) $g^{-1}(\{-1, 0, 1\})$

(c) $g^{-1}(\{x \mid 0 < x < 1\})$.

10. $\boxed{\text{5 points}}$ Let f be a function from the set A to the set B. Let S be a subset of B. We define the inverse image of S to be the subset of A whose elements are precisely all pre-images of all elements of S. We denote the inverse image of S by $f^{-1}(S)$, so $f^{-1}(S) = \{a \in A \mid f(a) \in S\}$. (Beware: The notation f^{-1} is used in two different ways. Do not confuse the notation introduced here with the notation $f^{-1}(y)$ for the value at y of the inverse of the invertible function f. Notice also that $f^{-1}(S)$, the inverse image of the set S, makes sense for all functions f , not just invertible functions.)
Let f be a function from A to B. Let S and T be subsets of B. Show that
$f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$.