Recitation 6: Predicate Logic

Discrete Mathematics

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Recap of the Great Rules

TABLE 1 Rules of Inference.

Rule of Inference	*Tautology*	*Name*
$\begin{array}{rl} & p \\ & p \to q \\ \hline \therefore & q \end{array}$	$(p \land (p \to q)) \to q$	Modus ponens
$\begin{array}{rl} & \neg q \\ & p \to q \\ \hline \therefore & \neg p \end{array}$	$(\neg q \land (p \to q)) \to \neg p$	Modus tollens
$\begin{array}{rl} & p \to q \\ & q \to r \\ \hline \therefore & p \to r \end{array}$	$((p \to q) \land (q \to r)) \to (p \to r)$	Hypothetical syllogism
$\begin{array}{rl} & p \lor q \\ & \neg p \\ \hline \therefore & q \end{array}$	$((p \lor q) \land \neg p) \to q$	Disjunctive syllogism
$\begin{array}{rl} & p \\ \hline \therefore & p \lor q \end{array}$	$p \to (p \lor q)$	Addition
$\begin{array}{rl} & p \land q \\ \hline \therefore & p \end{array}$	$(p \land q) \to p$	Simplification
$\begin{array}{rl} & p \\ & q \\ \hline \therefore & p \land q \end{array}$	$((p) \land (q)) \to (p \land q)$	Conjunction
$\begin{array}{rl} & p \lor q \\ & \neg p \lor r \\ \hline \therefore & q \lor r \end{array}$	$((p \lor q) \land (\neg p \lor r)) \to (q \lor r)$	Resolution

Sample Question

Question: State which rule of inference is the basis of the following argument: "It is below freezing now. Therefore, it is either below freezing or raining now."

Sample Question

Solution: Solution: Let p be the proposition "It is below freezing now" and q the proposition "It is raining now." Then this argument is of the form

$$\frac{p}{p \vee q}$$

This is an argument that uses the addition rule.

Sample Question

State which rule of inference is the basis of the following argument: “It is below freezing and raining now. Therefore, it is below freezing now.”

Sample Question

Solution: Solution: Let p be the proposition "It is below freezing now," and let q be the proposition "It is raining now." This argument is of the form

$$\frac{p \wedge q}{p}$$

This argument uses the simplification rule.

Sample Question

Q2, Show that the premises "The HW is not due this afternoon and it is was due yesterday," "We will go to the TA only if HW is due this afternoon," "If we do not go to the TA, then we will cry," and "If we cry, then there will be tears" leads to the conclusion "There will be tears."

Sample Question

Solution: Let

p : "The HW is due this afternoon"

q : "The HW was due yesterday"

r : "We will cry"

s : "We will go to TA"

t : "There will be tears"

Sample Question

Solution:

Let

p : "The HW is due this afternoon"

q : "The HW was due yesterday"

r : "We will cry"

s : "We will go to TA"

t : "There will be tears"

1. $\neg p \wedge q$ Premise
2. $\neg p$ Simplification using (1)
3. $s \implies p$ Premise
4. $\neg s$ Modus tollens using (2) and (3)
5. $\neg s \implies r$ Premise
6. r Modus ponens using (4) and (5)
7. $r \implies t$ Premise
8. t Modus ponens using (6) and (7)

DIY

The following questions are for you to do yourself rather then me explaining. I'm here to help and support you, I am here for if you wanna talk about it.

Question 1: What is true?

The following questions are for you to do yourself. I'm here to help and support you **For each of these arguments determine whether the argument is valid or invalid and explain why.**

1. a) All students in this class understand logic. Affan is a student in this class. Therefore, Affan understands logic.
2. b) Every computer science major takes discrete mathematics. Khubaib is taking discrete mathematics. Therefore, Khubaib is a computer science major.
3. c) All TAs are lazy. Some students are not TAs. Therefore, those students are not lazy.
4. d) Everyone who eats Andey wala burger every day wears a kameez. Mujtaba does not wear a kameez. Therefore, Mujtaba does not eat Andey wala burger every day.

Question 1: What is true?

a) All students in this class understand logic. Affan is a student in this class. Therefore, Affan understands logic.

Solution: Valid. We can universally instantiate the implication on Affan and by Modus ponens, Affan understands logic.

Question 1: What is true?

b) Every computer science major takes discrete mathematics. Khubaib is taking discrete mathematics. Therefore, Khubaib is a computer science major.
Solution: Invalid. If we universally instantiate Khubaib, then the implication is if Khubaib is a CS major then he is taking Discrete Mathematics. This does not mean that CS major implies Discrete Mathematics. For example, Khubaib can have a CS minor.

Question 1: What is true?

c) All TAs are lazy. Some students are not TAs. Therefore, those students are not lazy.
Solution: Invalid. The statement is if TAs then Lazy. It doesn't say if not TA then not lazy.

Question 1: What is true?

d) Everyone who eats Andey wala burger every day wears a kameez. Mujtaba does not wear a kameez. Therefore, Mujtaba does not eat Andey wala burger every day.
Solution: Valid. Universally instantiate the implication for Mujtaba and apply modus tollens.

Question 2 : Is this Real life?

The following questions are for you to do yourself rather then me explaining. I'm here to help and support you.

Use resolution to show the hypotheses "Affan is bad at beamer or Mujtaba likes anime" and "Affan is not bad at beamer or Ifrah is best TA" imply the conclusion "Mujtaba likes anime or Ifrah is best TA."

Question 2 : Is this Real life?

BadAtBeamer(x) : Person x is bad at Beamer
Anime(x) : Person x likes anime
BestTA(x) : Person x is the best TA

$$\text{BadAtBeamer(Affan)} \lor \text{Anime(Mujtaba)}$$
$$\neg\text{BadAtBeamer(Affan)} \lor \text{BestTA(Ifrah)}$$

We can apply resolution on this taking p as BadAtBeamer(Affan), q as Anime(Mujtaba) and r as BestTA(Ifrah) hence we get Anime(Mujtaba) $\lor$ BestTA(Ifrah) which is our desired conclusion.

Question 3: Or is this Fanta-sea?

The following questions are for you to do yourself rather then me explaining. I'm here to help and support you.

1. What is wrong with this argument? Let $H(x)$ be "x can't wait for the new Demon Slayer season." Given the premise $\exists x H(x)$, we conclude that $H(\text{Ifrah})$. Therefore, "Ifrah can't wait for the new Demon Slayer season."
2. What is wrong with this argument? Let $S(x, y)$ be "x is smarter than y." Given the premise $\exists s S(s, \text{Kanye West})$, it follows that $S(\text{Kanye West}, \text{Kanye West})$. Then by existential generalization, it follows that $\exists x S(x, x)$, so that Kanye West is smarter than himself.

Question 3: Or is this Fanta-sea?

What is wrong with this argument? Let $H(x)$ be "x can't wait for the new Demon Slayer season." Given the premise $\exists x H(x)$, we conclude that $H(\text{Ifrah})$. Therefore, "Ifrah can't wait for the new Demon Slayer season."

Solution: We know there exists some person but this doesn't mean that person is necessarily Ifrah. (It might be someone else)

Question 3: Or is this Fanta-sea?

What is wrong with this argument? Let $S(x, y)$ be "x is smarter than y." Given the premise $\exists s S(s, \text{Kanye West})$, it follows that $S(\text{Kanye West}, \text{Kanye West})$. Then by existential generalization, it follows that $\exists x S(x, x)$, so that Kanye West is smarter than himself.

Solution: We know there is someone smarter than Kanye West but this doesn't mean that person is necessarily Kanye West. (It might be someone else)

Question 4: Compound are scary

The following questions are for you to do yourself. I'm here to help and support you, Send me the solution for each in our MS Teams channel.

Use resolution to show that the compound proposition $(p \vee q) \wedge (\neg p \vee q) \wedge (p \vee \neg q) \wedge (\neg p \vee \neg q)$ is not satisfiable.

Question 4: Compound are scary

$$(p \vee q) \wedge (\neg p \vee q) \wedge (p \vee \neg q) \wedge (\neg p \vee \neg q)$$

$(p \vee q) \wedge (\neg p \vee q)$ by resolution gives us $q \vee q$ which is q by idempotent laws, similarly $(p \vee \neg q) \wedge (\neg p \vee \neg q)$ gives us $\neg q$. Therefore our expression implies $q \wedge \neg q$ which by negation law is F and hence unsatisfiable.

Question 5: Proof.exe is not responding

Identify the error or errors in this argument that supposedly shows that if $\exists x P(x) \wedge \exists x Q(x)$ is true then $\exists x (P(x) \wedge Q(x))$ is true

1. 1. $\exists x P(x) \wedge \exists x Q(x)$ Premise
2. 2. $\exists x P(x)$ Simplification from (1)
3. 3.$P(c)$ Existential instantiation from (2)
4. 4. $\exists x Q(x)$ Simplification from (1)
5. 5. $Q(c)$ Existential instantiation from (4)
6. 6. $P(c) \wedge Q(c)$ Conjunction from (3) and (5)
7. 7. $\exists x (P(x) \wedge Q(x))$ Existential generalization

The error occurs in step (5), because we cannot assume, as is being done here, that the c that makes P true is the same as the c that makes Q true.

Conclusion

That's all folks! Attendance will be checked through your submission.

1. Read the book!
2. Practice more!
3. Please start the assignment if you haven't
4. Don't forget to hit the like button and subscribe to our youtube channel.
5. Remember that the TA's hours can be seen on canvas and TAs can be found in their hours on EHSAS Group (MS Teams)