Recitazione 4: Propositional Logic (Now in HD!)

Discrete Mathematics

Habib University
Karachi, Pakistan

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Question 1: Tips on being successful in DM

Consider the following statements:

1. If I attend recitations and participate then TA will award me full marks.
2. If I attend recitations or participate then TA will award me full marks.

Given the above statements:

- Translate the statements into propositional logic.
- Check if (1) and (2) are equal.

Translation

Translation in Propositional logic:

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$$p: \text{I attend the recitations}$$
$$q: \text{I participate in the recitations}$$
$$r: \text{TA awards me full marks}$$

"Therefore If I attend recitations and participate then TA will award me full marks." is

Translation

Translation in Propositional logic:

$$p : \text{I attend the recitations}$$
$$q : \text{I participate in the recitations}$$
$$r : \text{TA awards me full marks}$$

"Therefore If I attend recitations and participate then TA will award me full marks." is

$$(p \land q) \implies r$$

and "If I attend recitations or participate then TA will award me full marks." is

Translation

Translation in Propositional logic:

$$p : \text{I attend the recitations}$$
$$q : \text{I participate in the recitations}$$
$$r : \text{TA awards me full marks}$$

"Therefore If I attend recitations and participate then TA will award me full marks." is

$$(p \land q) \implies r$$

and "If I attend recitations or participate then TA will award me full marks." is

$$(p \lor q) \implies r$$

Are they equal?

Check if (1) and (2) are equal:

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- Or find an instance where they differ

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$$p = \mathsf{T}, q = \mathsf{F}, r = \mathsf{F}$$

(1) will yield T.

Are they equal?

Check if (1) and (2) are equal:

To check:

- We can either use truth table
- Or find an instance where they differ

$$p = \mathsf{T}, q = \mathsf{F}, r = \mathsf{F}$$

(1) will yield T.
(2) will yield F.

Are they equal?

Check if (1) and (2) are equal:

To check:

- We can either use truth table
- Or find an instance where they differ

$$p = \mathsf{T}, q = \mathsf{F}, r = \mathsf{F}$$

(1) will yield T.
(2) will yield F.
Therefore not equal

Question 2 : Communication is key

Consider the following statement:
If I share my concern with the TAs and sharing the concerns with them is sufficient for them to understand me, then they will understand me. Show that this statement is a tautology.

Solution: Translation

A statement that is always true is a tautology.

Statement: If I share my concern with the TAs and sharing the concerns with them is sufficient for them to understand me, then they will understand me.
The propositions are:

$p =$ I share my concern with the TAs.

$q =$ TAs will understand me.

Solution: Translation

A statement that is always true is a tautology.

Statement: If I share my concern with the TAs and sharing the concerns with them is sufficient for them to understand me, then they will understand me.
The propositions are:

$$p = \text{I share my concern with the TAs.}$$
$$q = \text{TAs will understand me.}$$

The statement in propositional logic will be:

Solution: Translation

A statement that is always true is a tautology.

Statement: If I share my concern with the TAs and sharing the concerns with them is sufficient for them to understand me, then they will understand me.
The propositions are:

$$p = \text{I share my concern with the TAs.}$$
$$q = \text{TAs will understand me.}$$

The statement in propositional logic will be:

$$[p \wedge (p \implies q)] \implies q$$

Solution: Equivalence

Using equivalence laws, we can show $[p \land (p \implies q)] \implies q$ is always true

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$$(p \land (p \implies q)) \implies q \equiv$$

Solution: Equivalence

Using equivalence laws, we can show $[p \land (p \implies q)] \implies q$ is always true

$(p \land (p \implies q)) \implies q \equiv$ by the Implication Equivalence

Solution: Equivalence

Using equivalence laws, we can show $[p \wedge (p \implies q)] \implies q$ is always true

$(p \wedge (p \implies q)) \implies q \equiv p \wedge (\neg p \vee q) \implies q$ by the Implication Equivalence

Solution: Equivalence

Using equivalence laws, we can show $[p \wedge (p \implies q)] \implies q$ is always true

$(p \wedge (p \implies q)) \implies q \equiv p \wedge (\neg p \vee q) \implies q$ by the Implication Equivalence

by second distributive law

Solution: Equivalence

Using equivalence laws, we can show $[p \wedge (p \implies q)] \implies q$ is always true

$$\begin{aligned} (p \wedge (p \implies q)) \implies q &\equiv p \wedge (\neg p \vee q) \implies q && \text{by the Implication Equivalence} \\ &\equiv ((p \wedge \neg p) \vee (p \wedge q)) \implies q && \text{by second distributive law} \end{aligned}$$

Solution: Equivalence

Using equivalence laws, we can show $[p \wedge (p \implies q)] \implies q$ is always true

$$(p \wedge (p \implies q)) \implies q \equiv p \wedge (\neg p \vee q) \implies q \qquad \text{by the Implication Equivalence}$$

$$\equiv ((p \wedge \neg p) \vee (p \wedge q)) \implies q \qquad \text{by second distributive law}$$

by law of Contradiction

Solution: Equivalence

Using equivalence laws, we can show $[p \wedge (p \implies q)] \implies q$ is always true

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\begin{aligned}
(p \wedge (p \implies q)) \implies q &\equiv p \wedge (\neg p \vee q) \implies q && \text{by the Implication Equivalence} \\
&\equiv ((p \wedge \neg p) \vee (p \wedge q)) \implies q && \text{by second distributive law} \\
&\equiv (F \vee (p \wedge q)) \implies q && \text{by law of Contradiction}
\end{aligned}
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& && \text{by Identity law}
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&\equiv (p \wedge q) \implies q && \text{by Identity law} \\
&\equiv \neg(p \wedge q) \vee q && \text{by the Implication Equivalence} \\
& && \text{by Demorgan's law}
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Solution: Equivalence

Using equivalence laws, we can show $[p \land (p \implies q)] \implies q$ is always true

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\begin{aligned}
(p \land (p \implies q)) \implies q &\equiv p \land (\neg p \lor q) \implies q && \text{by the Implication Equivalence} \\
&\equiv ((p \land \neg p) \lor (p \land q)) \implies q && \text{by second distributive law} \\
&\equiv (F \lor (p \land q)) \implies q && \text{by law of Contradiction} \\
&\equiv (p \land q) \implies q && \text{by Identity law} \\
&\equiv \neg(p \land q) \lor q && \text{by the Implication Equivalence} \\
&\equiv \neg p \lor \neg q \lor q && \text{by Demorgan's law}
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Using equivalence laws, we can show $[p \wedge (p \implies q)] \implies q$ is always true

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&\equiv (F \vee (p \wedge q)) \implies q && \text{by law of Contradiction} \\
&\equiv (p \wedge q) \implies q && \text{by Identity law} \\
&\equiv \neg(p \wedge q) \vee q && \text{by the Implication Equivalence} \\
&\equiv \neg p \vee \neg q \vee q && \text{by Demorgan's law} \\
& && \text{by the law of Excluded Middle}
\end{aligned}
$$

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(p \wedge (p \implies q)) \implies q &\equiv p \wedge (\neg p \vee q) \implies q && \text{by the Implication Equivalence} \\
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&\equiv (F \vee (p \wedge q)) \implies q && \text{by law of Contradiction} \\
&\equiv (p \wedge q) \implies q && \text{by Identity law} \\
&\equiv \neg(p \wedge q) \vee q && \text{by the Implication Equivalence} \\
&\equiv \neg p \vee \neg q \vee q && \text{by Demorgan's law} \\
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&\equiv (p \wedge q) \implies q && \text{by Identity law}\\
&\equiv \neg(p \wedge q) \vee q && \text{by the Implication Equivalence}\\
&\equiv \neg p \vee \neg q \vee q && \text{by Demorgan's law}\\
&\equiv \neg p \vee T && \text{by the law of Excluded Middle}\\
& && \text{by the law of Domination}
\end{aligned}
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Solution: Equivalence

Using equivalence laws, we can show $[p \land (p \implies q)] \implies q$ is always true

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\begin{aligned}
(p \land (p \implies q)) \implies q &\equiv p \land (\neg p \lor q) \implies q && \text{by the Implication Equivalence}\\
&\equiv ((p \land \neg p) \lor (p \land q)) \implies q && \text{by second distributive law}\\
&\equiv (F \lor (p \land q)) \implies q && \text{by law of Contradiction}\\
&\equiv (p \land q) \implies q && \text{by Identity law}\\
&\equiv \neg(p \land q) \lor q && \text{by the Implication Equivalence}\\
&\equiv \neg p \lor \neg q \lor q && \text{by Demorgan's law}\\
&\equiv \neg p \lor T && \text{by the law of Excluded Middle}\\
&\equiv T && \text{by the law of Domination}
\end{aligned}
$$

Solution: Equivalence

Using equivalence laws, we can show $[p \wedge (p \implies q)] \implies q$ is always true

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&\equiv (p \wedge q) \implies q && \text{by Identity law} \\
&\equiv \neg(p \wedge q) \vee q && \text{by the Implication Equivalence} \\
&\equiv \neg p \vee \neg q \vee q && \text{by Demorgan's law} \\
&\equiv \neg p \vee T && \text{by the law of Excluded Middle} \\
&\equiv T && \text{by the law of Domination}
\end{aligned}
$$

Since we have simplified the statement to True, hence it is a tautology.

Question 3: Satisfiability

Determine if the following proposition is satisfiable:

$$(p \iff q) \wedge (\neg p \iff q)$$

See if a scenario where propostion is True

p	q	$p \Leftrightarrow q$	$\neg p \Leftrightarrow q$	$p \Leftrightarrow q \wedge \neg p \Leftrightarrow q$

See if a scenario where propostion is True

p	q	$p \Leftrightarrow q$	$\neg p \Leftrightarrow q$	$p \Leftrightarrow q \land \neg p \Leftrightarrow q$
T	T			
T	F			
F	T			
F	F			

See if a scenario where propostion is True

p	q	$p \Leftrightarrow q$	$\neg p \Leftrightarrow q$	$p \Leftrightarrow q \wedge \neg p \Leftrightarrow q$
T	T	T		
T	F	F		
F	T	F		
F	F	T		

See if a scenario where propostion is True

p	q	$p \Leftrightarrow q$	$\neg p \Leftrightarrow q$	$p \Leftrightarrow q \wedge \neg p \Leftrightarrow q$
T	T	T	F	
T	F	F	T	
F	T	F	T	
F	F	T	F	

See if a scenario where propostion is True

p	q	$p \Leftrightarrow q$	$\neg p \Leftrightarrow q$	$p \Leftrightarrow q \wedge \neg p \Leftrightarrow q$
T	T	T	F	F
T	F	F	T	F
F	T	F	T	F
F	F	T	F	F

Question 4: Graduating and Growing Apart

Consider the following propositions:

$$p = \text{I graduate}$$
$$q = \text{I grow apart from people}$$
$$r = \text{I become sad}$$

Question 4: Graduating and Growing Apart

Consider the following propositions:

$p =$ I graduate

$q =$ I grow apart from people

$r =$ I become sad

Using the propositions given above, translate the following statements into propositional logic and check if these are equal.

1. If I graduate then I'll become sad or if I lose my friends then I'll become sad.
2. If I graduate and lose my friends then I'll become sad.

Translation

Statements:

1. If I graduate then I'll become sad or if I lose my friends then I'll become sad.
2. If I graduate and lose my friends then I'll become sad.

Translating the statements in propositional logic,

Translation

Statements:

1. If I graduate then I'll become sad or if I lose my friends then I'll become sad.
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Translating the statements in propositional logic,

$$\text{Statement 1: } (p \implies r) \vee (q \implies r)$$

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Statements:

1. If I graduate then I'll become sad or if I lose my friends then I'll become sad.
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Translating the statements in propositional logic,

$$\text{Statement 1: } (p \implies r) \vee (q \implies r)$$
$$\text{Statement 2: } (p \wedge q) \implies r$$

Translation

Statements:

1. If I graduate then I'll become sad or if I lose my friends then I'll become sad.
2. If I graduate and lose my friends then I'll become sad.

Translating the statements in propositional logic,

$$\text{Statement 1: } (p \implies r) \vee (q \implies r)$$
$$\text{Statement 2: } (p \wedge q) \implies r$$

We want to prove $(p \implies r) \vee (q \implies r) \equiv (p \wedge q) \implies r$

Proving: *Give me a reason to believe*

We want to prove $(p \implies r) \vee (q \implies r) \equiv (p \wedge q) \implies r$

Proving: *Give me a reason to believe*

We want to prove $(p \implies r) \vee (q \implies r) \equiv (p \wedge q) \implies r$

$$((p \implies r) \vee (q \implies r)) \equiv$$

Proving: *Give me a reason to believe*

We want to prove $(p \implies r) \vee (q \implies r) \equiv (p \wedge q) \implies r$

$((p \implies r) \vee (q \implies r)) \equiv$ by the Implication Equivalence

Proving: *Give me a reason to believe*

We want to prove $(p \implies r) \vee (q \implies r) \equiv (p \wedge q) \implies r$

$$((p \implies r) \vee (q \implies r)) \equiv (\neg p \vee r) \vee (\neg q \vee r) \qquad \text{by the Implication Equivalence}$$

Proving: *Give me a reason to believe*

We want to prove $(p \implies r) \vee (q \implies r) \equiv (p \wedge q) \implies r$

$$((p \implies r) \vee (q \implies r)) \equiv (\neg p \vee r) \vee (\neg q \vee r)$$ by the Implication Equivalence

by the commutativity

Proving: *Give me a reason to believe*

We want to prove $(p \implies r) \lor (q \implies r) \equiv (p \land q) \implies r$

$$\begin{aligned} ((p \implies r) \lor (q \implies r)) &\equiv (\neg p \lor r) \lor (\neg q \lor r) && \text{by the Implication Equivalence} \\ &\equiv (\neg p \lor r) \lor (r \lor \neg q) && \text{by the commutativity} \end{aligned}$$

Proving: *Give me a reason to believe*

We want to prove $(p \implies r) \vee (q \implies r) \equiv (p \wedge q) \implies r$

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Proving: *Give me a reason to believe*

We want to prove $(p \implies r) \vee (q \implies r) \equiv (p \wedge q) \implies r$

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((p \implies r) \vee (q \implies r)) &\equiv (\neg p \vee r) \vee (\neg q \vee r) && \text{by the Implication Equivalence} \\
&\equiv (\neg p \vee r) \vee (r \vee \neg q) && \text{by the commutativity} \\
&\equiv \neg p \vee (r \vee r) \vee \neg q && \text{by Associativity}
\end{aligned}$$

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((p \implies r) \vee (q \implies r)) &\equiv (\neg p \vee r) \vee (\neg q \vee r) && \text{by the Implication Equivalence} \\
&\equiv (\neg p \vee r) \vee (r \vee \neg q) && \text{by the commutativity} \\
&\equiv \neg p \vee (r \vee r) \vee \neg q && \text{by Associativity} \\
& && \text{by Idempotence}
\end{aligned}$$

Proving: *Give me a reason to believe*

We want to prove $(p \implies r) \lor (q \implies r) \equiv (p \land q) \implies r$

$$
\begin{aligned}
((p \implies r) \lor (q \implies r)) &\equiv (\neg p \lor r) \lor (\neg q \lor r) && \text{by the Implication Equivalence} \\
&\equiv (\neg p \lor r) \lor (r \lor \neg q) && \text{by the commutativity} \\
&\equiv \neg p \lor (r \lor r) \lor \neg q && \text{by Associativity} \\
&\equiv \neg p \lor r \lor \neg q && \text{by Idempotence}
\end{aligned}
$$

Proving: *Give me a reason to believe*

We want to prove $(p \implies r) \vee (q \implies r) \equiv (p \wedge q) \implies r$

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& && \text{by Associativity}
\end{aligned}
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&\equiv \neg p \vee (r \vee r) \vee \neg q && \text{by Associativity} \\
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&\equiv \neg p \vee \neg q \vee r && \text{by Commutativity} \\
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Proving: *Give me a reason to believe*

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&\equiv \neg p \vee r \vee \neg q && \text{by Idempotence}\\
&\equiv \neg p \vee \neg q \vee r && \text{by Commutativity}\\
&\equiv (\neg p \vee \neg q) \vee r && \text{by Associativity}\\
& && \text{by Demorgan's law}
\end{aligned}
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&\equiv \neg p \vee \neg q \vee r && \text{by Commutativity} \\
&\equiv (\neg p \vee \neg q) \vee r && \text{by Associativity} \\
&\equiv \neg(p \wedge q) \vee r && \text{by Demorgan's law}
\end{aligned}
$$

Proving: *Give me a reason to believe*

We want to prove $(p \implies r) \vee (q \implies r) \equiv (p \wedge q) \implies r$

$$
\begin{aligned}
((p \implies r) \vee (q \implies r)) &\equiv (\neg p \vee r) \vee (\neg q \vee r) && \text{by the Implication Equivalence} \\
&\equiv (\neg p \vee r) \vee (r \vee \neg q) && \text{by the commutativity} \\
&\equiv \neg p \vee (r \vee r) \vee \neg q && \text{by Associativity} \\
&\equiv \neg p \vee r \vee \neg q && \text{by Idempotence} \\
&\equiv \neg p \vee \neg q \vee r && \text{by Commutativity} \\
&\equiv (\neg p \vee \neg q) \vee r && \text{by Associativity} \\
&\equiv \neg(p \wedge q) \vee r && \text{by Demorgan's law} \\
& && \text{by the Implication Equivalence}
\end{aligned}
$$

Proving: *Give me a reason to believe*

We want to prove $(p \implies r) \lor (q \implies r) \equiv (p \land q) \implies r$

$$
\begin{aligned}
((p \implies r) \lor (q \implies r)) &\equiv (\neg p \lor r) \lor (\neg q \lor r) && \text{by the Implication Equivalence} \\
&\equiv (\neg p \lor r) \lor (r \lor \neg q) && \text{by the commutativity} \\
&\equiv \neg p \lor (r \lor r) \lor \neg q && \text{by Associativity} \\
&\equiv \neg p \lor r \lor \neg q && \text{by Idempotence} \\
&\equiv \neg p \lor \neg q \lor r && \text{by Commutativity} \\
&\equiv (\neg p \lor \neg q) \lor r && \text{by Associativity} \\
&\equiv \neg (p \land q) \lor r && \text{by Demorgan's law} \\
&\equiv (p \land q) \implies r && \text{by the Implication Equivalence}
\end{aligned}
$$

Question 5: HW submission

Consider the following statements:

1. Your correct teams names are necessary and sufficient for your homework to get checked.
2. Your team names are correct and your homework will get checked, or your team names are incorrect and your homework will not get checked.

Translate them into propositional logic and show that they are logically equivalent.

Translation

1. Your correct teams names are necessary and sufficient for your homework to get checked.
2. Your team names are correct and your homework will get checked, or your team names are incorrect and your homework will not get checked.

In Propositional Logic:

Translation

1. Your correct teams names are necessary and sufficient for your homework to get checked.
2. Your team names are correct and your homework will get checked, or your team names are incorrect and your homework will not get checked.

In Propositional Logic:

p : Your team names are correct
q : Your homework gets checked

Translation

1. Your correct teams names are necessary and sufficient for your homework to get checked.
2. Your team names are correct and your homework will get checked, or your team names are incorrect and your homework will not get checked.

In Propositional Logic:

$$\begin{aligned} p &: \text{Your team names are correct} \\ q &: \text{Your homework gets checked} \end{aligned}$$

1. $p \leftrightarrow q$

Translation

1. Your correct teams names are necessary and sufficient for your homework to get checked.
2. Your team names are correct and your homework will get checked, or your team names are incorrect and your homework will not get checked.

In Propositional Logic:

$$\begin{aligned} p &: \text{ Your team names are correct} \\ q &: \text{ Your homework gets checked} \end{aligned}$$

1. $p \leftrightarrow q$
2. $(p \wedge q) \vee (\neg p \wedge \neg q)$

Proving Equality (stay with us please)

$p \leftrightarrow q \equiv$

Proving Equality (stay with us please)

$p \leftrightarrow q \equiv$ Definition of biconditional

Proving Equality (stay with us please)

$p \leftrightarrow q \equiv (p \implies q) \land (q \implies p)$ Definition of biconditional

Proving Equality (stay with us please)

$$p \leftrightarrow q \equiv (p \implies q) \land (q \implies p)$$

Definition of biconditional

$$p \implies q \equiv \neg p \lor q$$

Proving Equality (stay with us please)

$$\begin{aligned} p \leftrightarrow q &\equiv (p \implies q) \land (q \implies p) \\ &\equiv (\neg p \lor q) \land (\neg q \lor p) \end{aligned}$$

Definition of biconditional

$p \implies q \equiv \neg p \lor q$

Proving Equality (stay with us please)

$$p \leftrightarrow q \equiv (p \implies q) \land (q \implies p)$$

Definition of biconditional

$$\equiv (\neg p \lor q) \land (\neg q \lor p)$$

$p \implies q \equiv \neg p \lor q$

Let $A = \neg p \lor q, B = \neg q, C = p$

Proving Equality (stay with us please)

$$p \leftrightarrow q \equiv (p \implies q) \wedge (q \implies p) \quad \text{Definition of biconditional}$$

$$\equiv (\neg p \vee q) \wedge (\neg q \vee p) \quad p \implies q \equiv \neg p \vee q$$

$$\equiv A \wedge (B \vee C) \quad \text{Let } A = \neg p \vee q, B = \neg q, C = p$$

Proving Equality (stay with us please)

$$p \leftrightarrow q \equiv (p \implies q) \land (q \implies p)$$ Definition of biconditional

$$\equiv (\neg p \lor q) \land (\neg q \lor p)$$ $p \implies q \equiv \neg p \lor q$

$$\equiv A \land (B \lor C)$$ Let $A = \neg p \lor q, B = \neg q, C = p$

$A \land (B \lor C) \equiv (A \land B) \lor (A \land C)$

Proving Equality (stay with us please)

$p \leftrightarrow q \equiv (p \implies q) \wedge (q \implies p)$	Definition of biconditional
$\equiv (\neg p \vee q) \wedge (\neg q \vee p)$	$p \implies q \equiv \neg p \vee q$
$\equiv A \wedge (B \vee C)$	Let $A = \neg p \vee q, B = \neg q, C = p$
$\equiv (A \wedge B) \vee (A \wedge C)$	$A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$

Proving Equality (stay with us please)

$$
\begin{aligned}
p \leftrightarrow q &\equiv (p \implies q) \land (q \implies p) && \text{Definition of biconditional} \\
&\equiv (\neg p \lor q) \land (\neg q \lor p) && p \implies q \equiv \neg p \lor q \\
&\equiv A \land (B \lor C) && \text{Let } A = \neg p \lor q, B = \neg q, C = p \\
&\equiv (A \land B) \lor (A \land C) && A \land (B \lor C) \equiv (A \land B) \lor (A \land C)
\end{aligned}
$$

Substitution

Proving Equality (stay with us please)

$p \leftrightarrow q \equiv (p \implies q) \land (q \implies p)$	Definition of biconditional
$\equiv (\neg p \lor q) \land (\neg q \lor p)$	$p \implies q \equiv \neg p \lor q$
$\equiv A \land (B \lor C)$	Let $A = \neg p \lor q, B = \neg q, C = p$
$\equiv (A \land B) \lor (A \land C)$	$A \land (B \lor C) \equiv (A \land B) \lor (A \land C)$
$\equiv ((\neg p \lor q) \land \neg q) \lor ((\neg p \lor q) \land p)$	Substitution

Proving Equality (stay with us please)

$p \leftrightarrow q \equiv (p \implies q) \wedge (q \implies p)$	Definition of biconditional
$\equiv (\neg p \vee q) \wedge (\neg q \vee p)$	$p \implies q \equiv \neg p \vee q$
$\equiv A \wedge (B \vee C)$	Let $A = \neg p \vee q, B = \neg q, C = p$
$\equiv (A \wedge B) \vee (A \wedge C)$	$A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$
$\equiv ((\neg p \vee q) \wedge \neg q) \vee ((\neg p \vee q) \wedge p)$	Substitution
	$p \wedge q \equiv q \wedge p$

Proving Equality (stay with us please)

$p \leftrightarrow q \equiv (p \implies q) \wedge (q \implies p)$	Definition of biconditional
$\equiv (\neg p \vee q) \wedge (\neg q \vee p)$	$p \implies q \equiv \neg p \vee q$
$\equiv A \wedge (B \vee C)$	Let $A = \neg p \vee q, B = \neg q, C = p$
$\equiv (A \wedge B) \vee (A \wedge C)$	$A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$
$\equiv ((\neg p \vee q) \wedge \neg q) \vee ((\neg p \vee q) \wedge p)$	Substitution
$\equiv (\neg q \wedge (\neg p \vee q)) \vee (p \wedge (\neg p \vee q))$	$p \wedge q \equiv q \wedge p$

Proving Equality (stay with us please)

$p \leftrightarrow q \equiv (p \implies q) \wedge (q \implies p)$	Definition of biconditional
$\equiv (\neg p \vee q) \wedge (\neg q \vee p)$	$p \implies q \equiv \neg p \vee q$
$\equiv A \wedge (B \vee C)$	Let $A = \neg p \vee q, B = \neg q, C = p$
$\equiv (A \wedge B) \vee (A \wedge C)$	$A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$
$\equiv ((\neg p \vee q) \wedge \neg q) \vee ((\neg p \vee q) \wedge p)$	Substitution
$\equiv (\neg q \wedge (\neg p \vee q)) \vee (p \wedge (\neg p \vee q))$	$p \wedge q \equiv q \wedge p$
	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

Proving Equality (stay with us please)

$p \leftrightarrow q \equiv (p \implies q) \wedge (q \implies p)$	Definition of biconditional
$\equiv (\neg p \vee q) \wedge (\neg q \vee p)$	$p \implies q \equiv \neg p \vee q$
$\equiv A \wedge (B \vee C)$	Let $A = \neg p \vee q, B = \neg q, C = p$
$\equiv (A \wedge B) \vee (A \wedge C)$	$A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$
$\equiv ((\neg p \vee q) \wedge \neg q) \vee ((\neg p \vee q) \wedge p)$	Substitution
$\equiv (\neg q \wedge (\neg p \vee q)) \vee (p \wedge (\neg p \vee q))$	$p \wedge q \equiv q \wedge p$
$\equiv ((\neg q \wedge \neg p) \vee (\neg q \wedge q)) \vee ((p \wedge \neg p) \vee (p \wedge q))$	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

Proving Equality (stay with us please)

$p \leftrightarrow q \equiv (p \implies q) \wedge (q \implies p)$	Definition of biconditional
$\equiv (\neg p \vee q) \wedge (\neg q \vee p)$	$p \implies q \equiv \neg p \vee q$
$\equiv A \wedge (B \vee C)$	Let $A = \neg p \vee q, B = \neg q, C = p$
$\equiv (A \wedge B) \vee (A \wedge C)$	$A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$
$\equiv ((\neg p \vee q) \wedge \neg q) \vee ((\neg p \vee q) \wedge p)$	Substitution
$\equiv (\neg q \wedge (\neg p \vee q)) \vee (p \wedge (\neg p \vee q))$	$p \wedge q \equiv q \wedge p$
$\equiv ((\neg q \wedge \neg p) \vee (\neg q \wedge q)) \vee ((p \wedge \neg p) \vee (p \wedge q))$	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
	$p \wedge \neg p \equiv F$

Proving Equality (stay with us please)

$p \leftrightarrow q \equiv (p \implies q) \wedge (q \implies p)$	Definition of biconditional
$\equiv (\neg p \vee q) \wedge (\neg q \vee p)$	$p \implies q \equiv \neg p \vee q$
$\equiv A \wedge (B \vee C)$	Let $A = \neg p \vee q, B = \neg q, C = p$
$\equiv (A \wedge B) \vee (A \wedge C)$	$A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$
$\equiv ((\neg p \vee q) \wedge \neg q) \vee ((\neg p \vee q) \wedge p)$	Substitution
$\equiv (\neg q \wedge (\neg p \vee q)) \vee (p \wedge (\neg p \vee q))$	$p \wedge q \equiv q \wedge p$
$\equiv ((\neg q \wedge \neg p) \vee (\neg q \wedge q)) \vee ((p \wedge \neg p) \vee (p \wedge q))$	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
$\equiv ((\neg q \wedge \neg p) \vee F) \vee (F \vee (p \wedge q))$	$p \wedge \neg p \equiv F$

Proving Equality (stay with us please)

$$
\begin{aligned}
p \leftrightarrow q &\equiv (p \implies q) \land (q \implies p) && \text{Definition of biconditional} \\
&\equiv (\neg p \lor q) \land (\neg q \lor p) && p \implies q \equiv \neg p \lor q \\
&\equiv A \land (B \lor C) && \text{Let } A = \neg p \lor q, B = \neg q, C = p \\
&\equiv (A \land B) \lor (A \land C) && A \land (B \lor C) \equiv (A \land B) \lor (A \land C) \\
&\equiv ((\neg p \lor q) \land \neg q) \lor ((\neg p \lor q) \land p) && \text{Substitution} \\
&\equiv (\neg q \land (\neg p \lor q)) \lor (p \land (\neg p \lor q)) && p \land q \equiv q \land p \\
&\equiv ((\neg q \land \neg p) \lor (\neg q \land q)) \lor ((p \land \neg p) \lor (p \land q)) && p \land (q \lor r) \equiv (p \land q) \lor (p \land r) \\
&\equiv ((\neg q \land \neg p) \lor F) \lor (F \lor (p \land q)) && p \land \neg p \equiv F \\
& && p \lor F \equiv p
\end{aligned}
$$

Proving Equality (stay with us please)

$$
\begin{aligned}
p \leftrightarrow q &\equiv (p \implies q) \wedge (q \implies p) && \text{Definition of biconditional} \\
&\equiv (\neg p \vee q) \wedge (\neg q \vee p) && p \implies q \equiv \neg p \vee q \\
&\equiv A \wedge (B \vee C) && \text{Let } A = \neg p \vee q, B = \neg q, C = p \\
&\equiv (A \wedge B) \vee (A \wedge C) && A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C) \\
&\equiv ((\neg p \vee q) \wedge \neg q) \vee ((\neg p \vee q) \wedge p) && \text{Substitution} \\
&\equiv (\neg q \wedge (\neg p \vee q)) \vee (p \wedge (\neg p \vee q)) && p \wedge q \equiv q \wedge p \\
&\equiv ((\neg q \wedge \neg p) \vee (\neg q \wedge q)) \vee ((p \wedge \neg p) \vee (p \wedge q)) && p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r) \\
&\equiv ((\neg q \wedge \neg p) \vee F) \vee (F \vee (p \wedge q)) && p \wedge \neg p \equiv F \\
&\equiv (\neg q \wedge \neg p) \vee (p \wedge q) && p \vee F \equiv p
\end{aligned}
$$

Proving Equality (stay with us please)

$p \leftrightarrow q \equiv (p \implies q) \wedge (q \implies p)$	Definition of biconditional
$\equiv (\neg p \vee q) \wedge (\neg q \vee p)$	$p \implies q \equiv \neg p \vee q$
$\equiv A \wedge (B \vee C)$	Let $A = \neg p \vee q, B = \neg q, C = p$
$\equiv (A \wedge B) \vee (A \wedge C)$	$A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$
$\equiv ((\neg p \vee q) \wedge \neg q) \vee ((\neg p \vee q) \wedge p)$	Substitution
$\equiv (\neg q \wedge (\neg p \vee q)) \vee (p \wedge (\neg p \vee q))$	$p \wedge q \equiv q \wedge p$
$\equiv ((\neg q \wedge \neg p) \vee (\neg q \wedge q)) \vee ((p \wedge \neg p) \vee (p \wedge q))$	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
$\equiv ((\neg q \wedge \neg p) \vee F) \vee (F \vee (p \wedge q))$	$p \wedge \neg p \equiv F$
$\equiv (\neg q \wedge \neg p) \vee (p \wedge q)$	$p \vee F \equiv p$
	$p \wedge q \equiv q \wedge p, p \vee q \equiv q \vee p$

Proving Equality (stay with us please)

$p \leftrightarrow q \equiv (p \implies q) \wedge (q \implies p)$	Definition of biconditional
$\equiv (\neg p \vee q) \wedge (\neg q \vee p)$	$p \implies q \equiv \neg p \vee q$
$\equiv A \wedge (B \vee C)$	Let $A = \neg p \vee q, B = \neg q, C = p$
$\equiv (A \wedge B) \vee (A \wedge C)$	$A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$
$\equiv ((\neg p \vee q) \wedge \neg q) \vee ((\neg p \vee q) \wedge p)$	Substitution
$\equiv (\neg q \wedge (\neg p \vee q)) \vee (p \wedge (\neg p \vee q))$	$p \wedge q \equiv q \wedge p$
$\equiv ((\neg q \wedge \neg p) \vee (\neg q \wedge q)) \vee ((p \wedge \neg p) \vee (p \wedge q))$	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
$\equiv ((\neg q \wedge \neg p) \vee F) \vee (F \vee (p \wedge q))$	$p \wedge \neg p \equiv F$
$\equiv (\neg q \wedge \neg p) \vee (p \wedge q)$	$p \vee F \equiv p$
$\equiv (p \wedge q) \vee (\neg p \wedge \neg q)$	$p \wedge q \equiv q \wedge p, p \vee q \equiv q \vee p$

The other way

We can also see it through a truth table:

p	q	$p \leftrightarrow q$	$(p \wedge q) \vee (\neg p \wedge \neg q)$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	T	T

Question 6: Truth table

Show that $((p \implies q) \implies r)$ and $(p \implies (q \implies r))$ are not logically equivalent using truth table.

Question

p	q	r	$p \implies q$	$(p \implies q) \implies r$	$q \implies r$	$p \implies (q \implies r)$

Question

p	q	r	$p \implies q$	$(p \implies q) \implies r$	$q \implies r$	$p \implies (q \implies r)$
T						
T						
T						
T						
F						
F						
F						
F						

Question

p	q	r	$p \implies q$	$(p \implies q) \implies r$	$q \implies r$	$p \implies (q \implies r)$
T	T					
T	T					
T	F					
T	F					
F	T					
F	T					
F	F					
F	F					

Question

p	q	r	$p \implies q$	$(p \implies q) \implies r$	$q \implies r$	$p \implies (q \implies r)$
T	T	T				
T	T	F				
T	F	T				
T	F	F				
F	T	T				
F	T	F				
F	F	T				
F	F	F				

Question

p	q	r	$p \implies q$	$(p \implies q) \implies r$	$q \implies r$	$p \implies (q \implies r)$
T	T	T	T			
T	T	F	T			
T	F	T	F			
T	F	F	F			
F	T	T	T			
F	T	F	T			
F	F	T	T			
F	F	F	T			

Question

p	q	r	$p \implies q$	$(p \implies q) \implies r$	$q \implies r$	$p \implies (q \implies r)$
T	T	T	T	T		
T	T	F	T	F		
T	F	T	F	T		
T	F	F	F	T		
F	T	T	T	T		
F	T	F	T	F		
F	F	T	T	T		
F	F	F	T	F		

Question

p	q	r	$p \implies q$	$(p \implies q) \implies r$	$q \implies r$	$p \implies (q \implies r)$
T	T	T	T	T	T	
T	T	F	T	F	F	
T	F	T	F	T	T	
T	F	F	F	T	T	
F	T	T	T	T	T	
F	T	F	T	F	F	
F	F	T	T	T	T	
F	F	F	T	F	T	

Question

p	q	r	$p \implies q$	$(p \implies q) \implies r$	$q \implies r$	$p \implies (q \implies r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	T	T	T	T
F	T	F	T	F	F	T
F	F	T	T	T	T	T
F	F	F	T	F	T	T

Question 7: What I negate can't hurt me?

Timmy got a B grade in Discrete Mathematics. Timmy is sick of his dad grumbling about him getting bad grades.
So out of fun he negates the statements to make himself feel better.

How will he negate the following statement?

Your Cousin Tommy is a Doctor and he is only nine years old.

Solution

Your Cousin Tommy is a Doctor and he is only nine years old.

Solution

Your Cousin Tommy is a Doctor and he is only nine years old.

Propositions:

- p : Timmy is a doctor
- q: Timmy is only 9 years old.

Solution

Your Cousin Tommy is a Doctor and he is only nine years old.

Propositions:

- p : Timmy is a doctor
- q: Timmy is only 9 years old.

Original proposition

$$p \wedge q$$

Solution

Your Cousin Tommy is a Doctor and he is only nine years old.

Propositions:

- p : Timmy is a doctor
- q: Timmy is only 9 years old.

Original proposition

$$p \wedge q$$

Hence negation is

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

.

Solution

Your Cousin Tommy is a Doctor and he is only nine years old.

Propositions:

- p : Timmy is a doctor
- q: Timmy is only 9 years old.

Original proposition

$$p \wedge q$$

Hence negation is

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

. **Either your cousin Tommy isn't a doctor or he is not only 9 years old or neither.**

Question: *Just a normal office*

Consider yourself working in an office.
Few of your colleagues are named Michael.
Let $P(x)$ be the statement "x wants people to be afraid of how much they love x"
where the domain of x consists of all the *Michaels* in the office.
Express each of the following quantifications in English.

Solution

$$\forall x P(x)$$

Solution

$$\forall x P(x)$$

Each Michael wants people to be afraid of how much they love him.

Solution

$$\forall x P(x)$$

Each Michael wants people to be afraid of how much they love him.

$$\exists x \neg P(x)$$

Solution

$$\forall x P(x)$$

Each Michael wants people to be afraid of how much they love him.

$$\exists x \neg P(x)$$

There exists a Michael who does not want people to be afraid of how much they love him.

Conclusion

That's all folks! Attendance time.

1. Read the book!
2. Practice more! (Practice problems on Sets are available on Canvas)
3. Don't forget to hit the like button and subscribe to our youtube channel.
4. Remember that the TA's hours can be seen on canvas and TAs can be found in their hours on EHSAS Group (MS Teams)