Recitation 5: Onto Predicates and Inference

Discrete Mathematics

Habib University
Karachi, Pakistan

February 19, 2022

Question 1: Animal Farm

Let

$$C(x) = \text{“}x \text{ has a cat”}$$
$$D(x) = \text{“}x \text{ has a dog”}$$
$$F(x) = \text{“}x \text{ has a ferret”}$$

Express each of these statements in terms of $C(x), D(x), F(x)$, quantifiers, and logical connectives. Let the domain consist of all students in your class.

1. A student in your class has a cat, a dog, and a ferret.
   Solution:
2. All students in your class have a cat, a dog, or a ferret.
   Solution:
3. Some student in your class has a cat and a ferret, but not a dog.
   Solution:

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1. A student in your class has a cat, a dog, and a ferret.
   Solution: $\exists x(C(x) \wedge D(x) \wedge F(x))$
2. All students in your class have a cat, a dog, or a ferret.
   Solution: $\forall x(C(x) \vee D(x) \vee F(x))$
3. Some student in your class has a cat and a ferret, but not a dog.
   Solution: $\exists x(C(x) \wedge F(x) \wedge \neg D(x))$

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Express each of these statements in terms of $C(x), D(x), F(x)$, quantifiers, and logical connectives. Let the domain consist of all students in your class.

4. No student in your class has a cat, a dog, and a ferret.
   Solution:
5. For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has this animal as a pet.
   Solution:

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$$C(x) = \text{“}x \text{ has a cat”}$$
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Express each of these statements in terms of $C(x), D(x), F(x)$, quantifiers, and logical connectives. Let the domain consist of all students in your class.

4. No student in your class has a cat, a dog, and a ferret.
   Solution: $\neg\exists x(C(x) \land D(x) \land F(x))$
5. For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has this animal as a pet.
   Solution: $(\exists x C(x)) \land (\exists x D(x)) \land (\exists x F(x))$

Question 2: What is true?

Let $P(x)$ be the statement "$x = x^2$" If the domain consists of the integers, what are these truth values?

1. $P(0) =$
2. $P(1) =$
3. $P(2) =$
4. $P(-1) =$
5. $\exists x P(x) =$
6. $\forall x P(x) =$

Question 2: What is true?

Let $P(x)$ be the statement "$x = x^2$" If the domain consists of the integers, what are these truth values?

1. $P(0) \quad = \textbf{T}$
2. $P(1) \quad = \textbf{T}$
3. $P(2) \quad = \textbf{F}$
4. $P(-1) = \textbf{F}$
5. $\exists x P(x) = \textbf{T}$
6. $\forall x P(x) = \textbf{F}$

Question 3: For the love of Nick

Let $P(x, y)$ be the statement "x has sent a box of Nick Wilde photos to y," where the domain for both x and y consists of all students in your class. Let the domain U be the students in your class. Express each of these quantifications in English.

1. $\exists x \exists y P(x, y)$
2. $\exists x \forall y P(x, y)$

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1. $\exists x \exists y P(x, y)$

   Possible Translation: There is a student in your class x who has sent a box of Nick Wilde photos to a student y in your class

2. $\exists x \forall y P(x, y)$

   Possible Translation: There is a student x in your class who has sent a box of Nick Wilde photos to every student in your class.

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3. $\forall x \exists y P(x, y)$

4. $\forall x \forall y P(x, y)$

Question 3: For the love of Nick

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3. $\forall x \exists y P(x, y)$

   Possible Translation: Every student in your class has sent a student in your class a box of Nick Wilde photos.

4. $\forall x \forall y P(x, y)$

   Possible Translation: Every student in class has sent a box of Nick Wilde photos to every student in class

Question 3.5: Loved by Few

Let $L(x)$ denote that x is in love with Nick Wilde. Let $M(x)$ denote that student x is a male. Let the domain U be the students in your class.

5 $\forall x(M(x) \implies L(x))$

6 $\exists x(M(x) \wedge L(x))$

Question 3.5: Loved by Few

Let $L(x)$ denote that x is in love with Nick Wilde. Let $M(x)$ denote that student x is a male. Let the domain U be the students in your class.

5. $\forall x(M(x) \implies L(x))$

   Possible Translation: All males in your class love Nick Wilde

6. $\exists x(M(x) \wedge L(x))$

   Possible Translation: There is a male in your class who loves Nick Wilde.

Question 4: Inferencing

What rule of inference is used in each of these arguments?

- Affan plays Genshin. Therefore, Affan plays Genshin or helps starving kids in Africa.
- Josh is a niche internet micro-celebrity and everyone loves Josh. Therefore, everyone loves Josh.
- If I partake in embezzlement, then I own an air fryer. I partake in embezzlement. Therefore, I own an air fryer.
- If red is sus, red is the imposter. red is not the imposter. Therefore, red is not sus.
- If Spongebob committed ✨ tax fraud ✨ , then my childhood is a lie. If my childhood is a lie, then I have a tragic backstory.
  Therefore, if spongebob committed ✨ tax fraud ✨ , then I have a tragic backstory.

Question 4: Genshin addiction and micro-celebrity - Solution

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- **Statement**: Affan plays Genshin. Therefore, Affan plays Genshin or helps starving kids in Africa.

  p : Affan plays Genshin, q : Affan helps starving kids in Africa.
  The statement can be written as $p \implies (p \vee q)$

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- **Statement**: Josh is a niche internet micro-celebrity and everyone loves Josh. Therefore, everyone loves Josh.

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  Rule used: Simplification

Question 4: Air fryers and imposters - Solution

What rule of inference is used in each of these arguments?

- **Statement**: If I partake in embezzlement, then I own an air fryer. I partake in embezzlement. Therefore, I own an air fryer.

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  The statement can be written as $((p \implies q) \land p) \implies q$

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  Rule used: ✨ Modus tollens ✨

Question 4: Spongebob Fraudpants - Solution

What rule of inference is used in each of these arguments?

- **Statement**: If Spongebob committed ✨ tax fraud ✨ , then my childhood is a lie. If my childhood is a lie, then I have a tragic backstory.
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$$p : \text{Spongebob committed ✨ tax fraud ✨}$$
$$q : \text{my childhood is a lie}$$
$$r : \text{I have a tragic backstory}$$

The statement can be written as $((p \implies q) \wedge (q \implies r)) \implies (p \implies r)$

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p : Spongebob committed ✨ tax fraud ✨
q : my childhood is a lie
r : I have a tragic backstory

The statement can be written as $((p \implies q) \wedge (q \implies r)) \implies (p \implies r)$

Rule used: Hypothetical Syllogism

Question 5: Validity

Determine whether each of these arguments is valid. If an argument is correct, what rule of inference is being used? If it is not, what logical error occurs?

1. If n is a real number such that $n > 1$, then $n^2 > 1$. Suppose that $n^2 > 1$. Then $n > 1$.
2. If n is a real number with $n > 3$, then $n^2 > 9$. Suppose that $n^2 \leq 9$. Then $n \leq 3$.
3. If n is a real number with $n > 2$, then $n^2 > 4$. Suppose that $n \leq 2$. Then $n^2 \leq 4$.

Question 5: Validity - Solution

1. If n is a real number such that $n > 1$, then $n^2 > 1$. Suppose that $n^2 > 1$. Then $n > 1$.

$$p = n \text{ is a real number such that } n > 1$$
$$q = n^2 > 1$$

The argument is

$$\begin{array}{l} p \implies q \\ q \\ \hline p \end{array}$$

There is no information about any inference from q and the conclusion p cannot be inferred from the premises. Therefore, this argument is invalid. (The argument is saying that if $p \implies q$ then $q \implies p$, which is not true)

Question 5: Validity - Solution

2 If n is a real number with $n > 3$, then $n^2 > 9$. Suppose that $n^2 \leq 9$. Then $n \leq 3$.

$$p = n \text{ is a real number such that } n > 3$$
$$q = n^2 > 9$$

The argument is

$$\begin{array}{l} p \implies q \\ \neg q \\ \hline \neg p \end{array}$$

This argument is valid, and is just the use of Modus Tollens.

Question 5: Validity - Solution

3. If n is a real number with $n > 2$, then $n^2 > 4$. Suppose that $n \leq 2$. Then $n^2 \leq 4$.

$$p = n \text{ is a real number such that } n > 2$$
$$q = n^2 > 4$$

The argument is

$$\begin{array}{l} p \implies q \\ \neg p \\ \hline \neg q \end{array}$$

There is no information about any inference from $\neg p$ and the conclusion $\neg q$ cannot be inferred from the premises. Therefore, this argument is invalid. (This argument is saying that $p \implies q$ implies $q \implies p$ which is not true)

Question 6: Who lied about their hours?

"There seems to be a fraud! a TA has lied in their timesheets" claimed SW. *"How?"* asked AT surprised, thinking he trusted those innocent faces. *"Elementary my dear Amin. Either a TA worked the said hours for Discrete math or spent all night watching degenerate isekais"*, explained SW, knowing too well the antics of youth. *"If a TA has not worked the said hours for DM, then they have lied on their timesheet"*, SW elaborated, setting up the stage for his flawless deduction. *"Our culprit here Mujtaba is a TA. It is logically proven that Mujtaba either adds a question on degenerate isekais in discrete math reciation or talks about 42. If Mujtaba doesn't watch dengerate isekais, her humour is at peak, which is quite rare of a a sight. When Mujtaba's humour is at peak, she talks about 42, like the woman of culture she is. But Mujtaba added a question on degenerate isekais in discrete math reciation. Therefore Mujtaba lied on her timesheet!"*

Write down this argument in predicate logic.

Question 6: The future

1. If you ask the TA a question in their hours, they would reply
2. If the TA is asked a question outside their hours, then DM HW 2 is due in the next 5 hours.
3. You ask the TA a question and they didn't reply

Based on this information, what can you say about HW2? Also express this in terms of propositions

Question 6: The future

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3. You ask the TA a question and they didn't reply

Based on this information, what can you say about HW2? Also express this in terms of propositions

$$p = \text{You asked the TA a question in their hours}$$
$$q = \text{You asked the TA a question outside their hours}$$
$$r = \text{The TA replies}$$
$$s = \text{DM HW2 is due in the next 5 hours}$$

Question 6

What can you say about HW2?

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$$p \implies r \tag{1}$$
$$\tag{2}$$
$$\tag{3}$$

Question 6

What can you say about HW2?

$$p \implies r \tag{1}$$

$$q \implies s \tag{2}$$

(3)

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What can you say about HW2?

$$p \implies r \tag{1}$$

$$q \implies s \tag{2}$$

$$(p \lor q) \land \neg r \tag{3}$$

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$$(p \vee q) \wedge \neg r \tag{3}$$

Using this

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$$p \implies r \tag{1}$$
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Using this

$$(p \vee q) \wedge \neg r \implies \neg r \text{ Simplification}$$

Question 6

What can you say about HW2?

$$p \implies r \tag{1}$$
$$q \implies s \tag{2}$$
$$(p \lor q) \land \neg r \tag{3}$$

Using this

$$(p \lor q) \land \neg r \implies \neg r \text{ Simplification}$$
$$\neg r \land (p \implies r) \implies \neg p \text{ Modus tollens}$$

Question 6

What can you say about HW2?

$$p \implies r \tag{1}$$
$$q \implies s \tag{2}$$
$$(p \lor q) \land \neg r \tag{3}$$

Using this

$$(p \lor q) \land \neg r \implies \neg r \text{ Simplification}$$
$$\neg r \land (p \implies r) \implies \neg p \text{ Modus tollens}$$
$$\neg p \land (p \lor q) \implies q \text{ Disjunctive syllogism}$$

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Using this

$$(p \vee q) \wedge \neg r \implies \neg r \text{ Simplification}$$
$$\neg r \wedge (p \implies r) \implies \neg p \text{ Modus tollens}$$
$$\neg p \wedge (p \vee q) \implies q \text{ Disjunctive syllogism}$$
$$q \wedge (q \implies s) \implies s \text{ Modus Ponens}$$

Question 6

What can you say about HW2?

$$p \implies r \tag{1}$$
$$q \implies s \tag{2}$$
$$(p \vee q) \wedge \neg r \tag{3}$$

Using this

$$(p \vee q) \wedge \neg r \implies \neg r \text{ Simplification}$$
$$\neg r \wedge (p \implies r) \implies \neg p \text{ Modus tollens}$$
$$\neg p \wedge (p \vee q) \implies q \text{ Disjunctive syllogism}$$
$$q \wedge (q \implies s) \implies s \text{ Modus Ponens}$$

The assignment is due in the next 5 hours

Question 7: Uniqueness Quantifier

Show that each of these statements can be used to express the fact that there is a unique element x such that $P(x)$ is true. [Note that we can also write this statement as $\exists! x P(x)$.]

1. $\exists x \forall y (P(y) \leftrightarrow x = y)$
2. $\exists x P(x) \wedge \forall x \forall y (P(x) \wedge P(y) \implies x = y)$

Question 7: Solution

1. $\exists x \forall y (P(y) \leftrightarrow x = y)$
   This statement asserts the existence of x with a certain property. If we let $y = x$, we can see that $P(x)$ is true. If y is anything other than x then P(x) is not true. Thus x is the unique element that makes $P(x)$ true.

Question 7: Solution

1. $\exists x \forall y (P(y) \leftrightarrow x = y)$
   This statement asserts the existence of x with a certain property. If we let $y = x$, we can see that $P(x)$ is true. If y is anything other than x then P(x) is not true. Thus x is the unique element that makes $P(x)$ true.
2. $\exists x P(x) \wedge \forall x \forall y (P(x) \wedge P(y) \implies x = y)$
   The first clause here says that there is an element that makes P true. The second clause says that whenever two elements both make P true, they are in fact the same element. Together these say that P is satisfied by exactly one element.

Question 7: Solution

1. $\exists x \forall y (P(y) \leftrightarrow x = y)$
   This statement asserts the existence of x with a certain property. If we let $y = x$, we can see that $P(x)$ is true. If y is anything other than x then P(x) is not true. Thus x is the unique element that makes $P(x)$ true.
2. $\exists x P(x) \land \forall x \forall y (P(x) \land P(y) \implies x = y)$
   The first clause here says that there is an element that makes P true. The second clause says that whenever two elements both make P true, they are in fact the same element. Together these say that P is satisfied by exactly one element.

Conclusion

That's all folks! Attendance time.

1. Read the book!
2. Practice more!
3. Please start the assignment if you haven't 🙂
4. Don't forget to hit the like button and subscribe to our youtube channel.
5. Remember that the TA's hours can be seen on canvas and TAs can be found in their hours on EHSAS Group (MS Teams)