

Practice Problems - Sets

DM Course Staff

January 2022

Questions

Question 1: There are 2000 students on campus who own Team Fortress 2, Plants vs Zombies, or Kerbal Space Program. If 500 students own all three games, 200 own only Team Fortress 2, 350 own only Plants vs. Zombies, and 150 own only Kerbal Space Program, how many of these games in total are owned by Florida Tech students?

Question 2:

- Explain what it means for two sets to be equal.
- Describe as many of the ways as you can to show that two sets are equal.
- Show in at least two different ways that the sets $A - (B \cap C)$ and $(A - B) \cup (A - C)$ are equal.

Question 3: Which of the following set descriptions gives the set $\{2, 8, 14, 20, 26, 32\}$?

- a) $\{n \in \mathbb{N} \mid n = 2x + 6 \text{ for some integer } x \text{ such that } 1 < x < 6\}$
- b) $\{n \in \mathbb{N} \mid n = 6x + 2 \text{ for some integer } x \text{ such that } 1 < x < 6\}$
- c) $\{n \in \mathbb{N} \mid n = 6x + 2 \text{ for some integer } x \text{ such that } 0 < x < 6\}$
- d) None of the above

Question 4: Let $B = \{2, 3, 6, 9, 11\}$ and $C = \{1, 4, 6, 11, 15\}$. Which of the following sets are not any of $B \cup C$, $B \cap C$, and $B - C$?

- a) $\{1, 6, 9, 15\}$
- b) $\{6, 11\}$
- c) $\{2, 3, 9\}$
- d) None of the above

Question 5: Describe each of the following sets in set builder notation.

- $B = \{1, 2, 5, 10, 17, 26, 37, 50, \dots\}$
- $C = \{1, 5, 9, 13, 17, 21, \dots\}$
- $E = \{\text{lemon, lime, 1, 3, 5, 7, } \dots\}$

Question 6: The defining property of an ordered pair is that two ordered pairs are equal if and only if their first elements are equal and their second elements are equal. Surprisingly, instead of taking the ordered pair as a primitive concept, we can construct ordered pairs using basic notions from set theory. Show that if we define the ordered pair (a, b) to be $\{\{a\}, \{a, b\}\}$, then $(a, b) = (c, d)$ if and only if $a = c$ and $b = d$. [Hint: First show that $\{\{a\}, \{a, b\}\} = \{\{c\}, \{c, d\}\}$ if and only if $a = c$ and $b = d$.]

Question 7: Can you conclude that $A = B$ if A and B are two sets with the same power set?

Question 8: Explain why $A \times B \times C$ and $(A \times B) \times C$ are not the same.

Question 9: Show that if $A \subseteq C$ and $B \subseteq D$, then $A \times B \subseteq C \times D$.

Question 10: Show that if A , B , and C are sets, then $\overline{A \cap B \cap C} = \overline{A} \cup \overline{B} \cup \overline{C}$ by

- showing each side is a subset of the other side.
- using a membership table.

Question 11: Prove the first distributive law by showing that if A , B , and C are sets, then $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Answer Key

Answer 1: 3800

Answer 3: c

Answer 4: a

Answer 5:

$$B = \{x^2 + 1 \mid x \in \mathbb{N}\}$$

$$C = \{4x + 1 \mid x \in \mathbb{N}\}$$

$$C = \{x \mid x \in \{\text{lemon, lime}\} \vee (x \in \mathbb{N} \wedge x \bmod 2 = 1)\}$$

Answer 7: “Yes. Yes we can.” - Probably Phineas