

# Robot Control

EE468/CE468: Mobile Robotics

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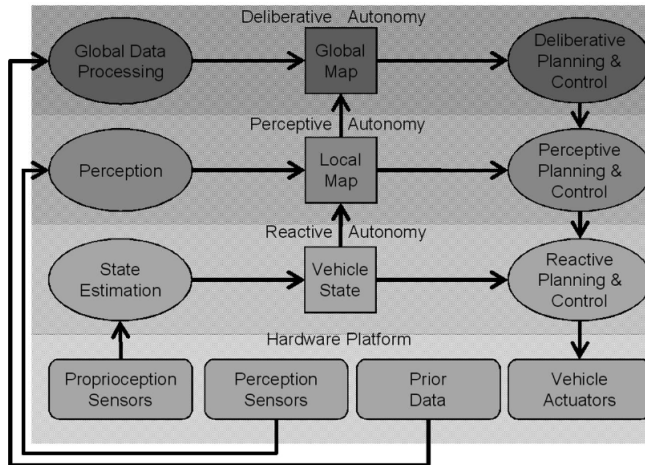
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**Figure 1.9 Layers of Autonomy.** The entire mobile system can be described in terms of three nested perceive-think-act loops.



## Definitions of autonomy layers. [2, 1.3.2]

- **Reactive Autonomy.** Responsible for controlling the motion of robot with respect to the environment. Requires motion state (position, velocity, heading, etc.) Examples: Follow a trajectory, Go to a desired pose, etc.
- **Perceptive Autonomy.** Responsible for responding to immediately perceivable environment. Requires feedback of state of the environment. Examples: Obstacle avoidance, Wall following, Line following, etc.
- **Deliberative Autonomy.** Responsible for achieving long-term goals, i.e. mission. Requires global position estimate and prediction extend far into future. Examples: Localization, Mapping, etc.

# Control is the process of converting intentions into actions.

- Control exists at all layers.
- This module is about low-level control.
- Reactive autonomy and some perceptive autonomy.

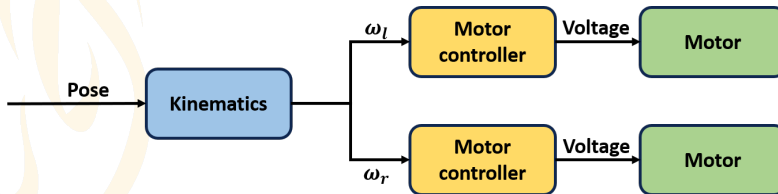


Figure: Motor controller ensures motor speeds and generates voltage commands.

# Divide & Conquer: Organize control in terms of simple behaviors.

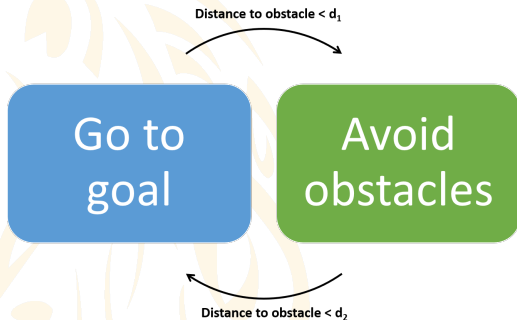


Figure: Switching between behaviors based on guard conditions.

- Go to goal
- Avoid obstacles
- Follow wall
- Track target
- ...
- Switch between behaviors as needed



# Pure reactive control connects sense directly to act.

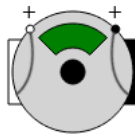
- Inspired from biology.
- Walter Grey's Tortoises (1949)



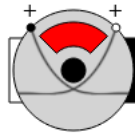


Figure: [Simulator](#)

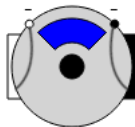
MIT Braitenberg  
Creatures



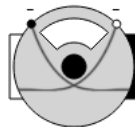
*Fear*



*Aggression*



*Love*



*Exploration*

Figure: Four original modes of Braitenberg vehicles



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- We need control for precise motion despite modeling errors and perturbations.

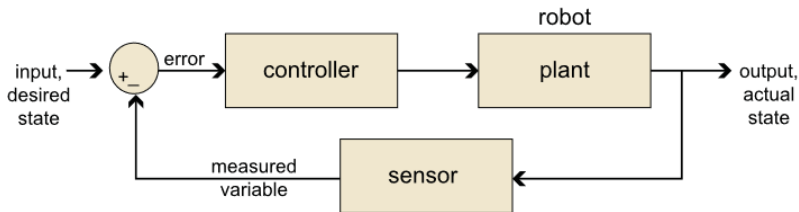


Figure: Basic model of feedback control [1]



- **Reference Signal or Desired state ( $y_r$ ):** It specifies goal or target for the control system.
- **Output Signal ( $y$ ):** What is the system actually doing.
- **Error Signal ( $e$ ):** Difference between desired and actual state,  $e = y_r - y$ .
- **Feedback Controller or Closed Loop:** Control signal is based on error signal.
- **Plant:** Robot body (system) plus actuators.
- **Control Input ( $u$ ):** Component(s) of system we can control.



# Create a wall-following robot [1]

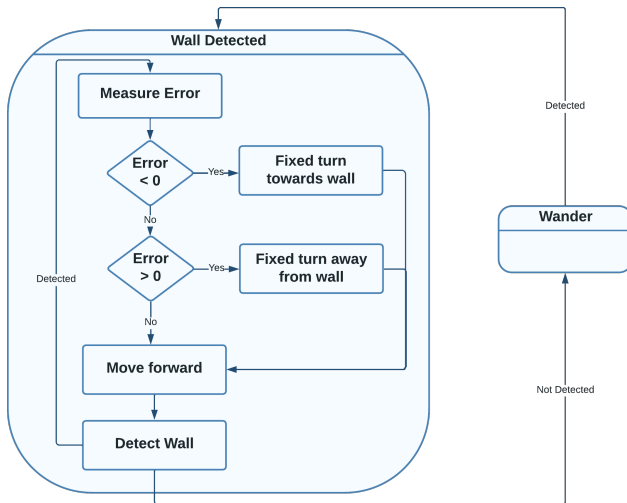
- Maintain a distance of 5 in from a wall.
- If robot loses the wall, it randomly wanders till it detects the wall.



# What will be open-loop control strategy?

- Given model of world (robot + environment), decide a path, determine wheel control commands using inverse kinematics, execute.
- Will it work?
  - Only if we had a perfect world model.
  - Modeling errors, Parameter changes, Unwanted inputs

# Will this closed-loop strategy succeed?



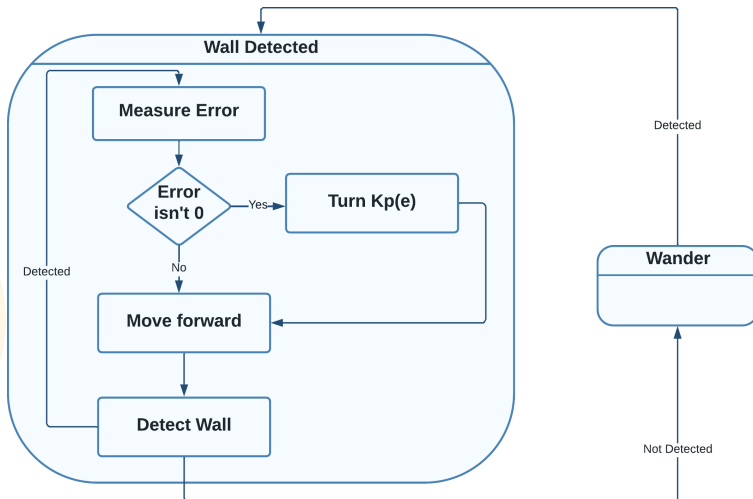


# This is bang-bang or on-off control.

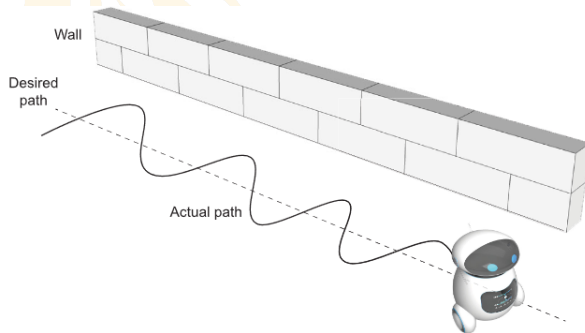
- Difficult to maintain exact distance.
- Odometry errors, Sensor errors, Wheel slippage cause robot to overturn.
- Oscillations may make system unstable.



# How can we do better?

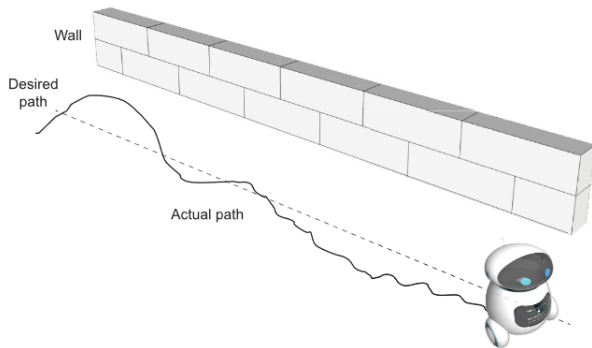


# This is proportional control.



- Amount of turn is proportional to error.
- We still have oscillations.
- One way to reduce them is to introduce deadband - no action is taken for error values in this band.

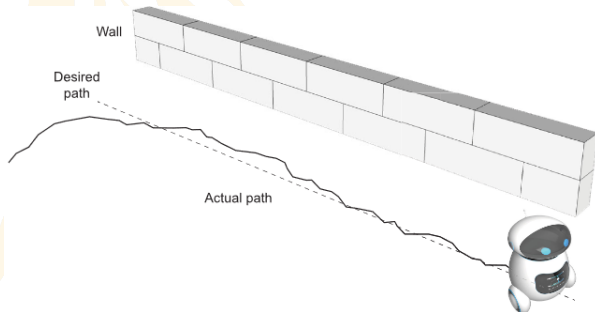
# How can we decrease the oscillations?



- Add derivative control. PD control is:

$$u(t) = K_p e(t) + K_d \frac{de(t)}{dt}.$$

- Slows down rate of change in position as robot gets closer to desired state.
- Decreases sharpness of turns and reduces overshoot.
- Still has steady-state error. The rate of change of error and error are too small to remove this.



- Adding integral terms removes steady-state error:

$$u(t) = K_p e(t) + K_d \frac{de(t)}{dt} + K_i \int e(s) ds.$$



## ■ Issues with differential term

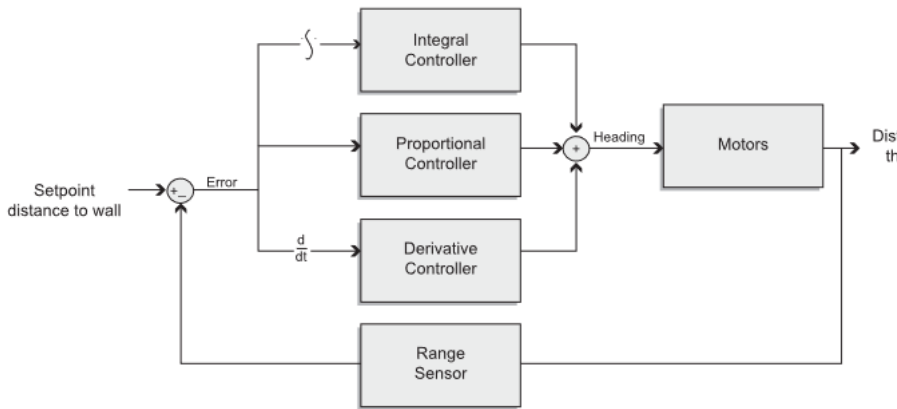
- Sudden change in the reference or current state (e.g. brakes) causes abrupt unwanted changes in control.
- Current state measurements are noisy, resulting in high derivatives.

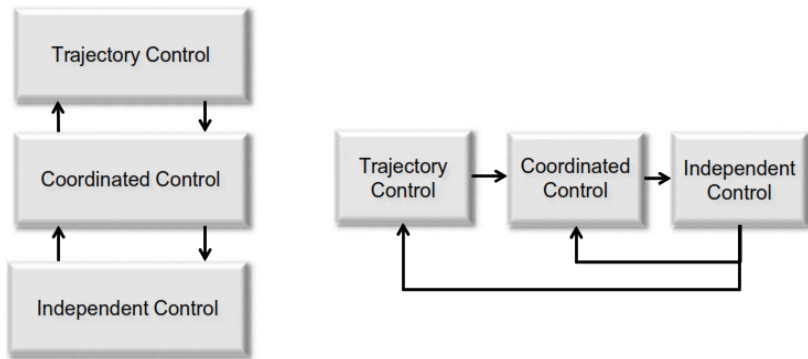
## ■ Issues with integral term

- *Windup*: The integral term is non-zero even when the desired state is achieved.



# PID controller is sending heading commands down to motors!





**Figure 7.3 A Controller Hierarchy.** Left: Higher levels produce the reference signals for lower levels. Right: The equivalent block diagram is a cascade configuration.

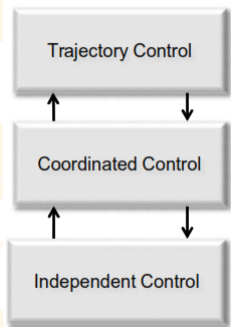


Figure: Hierarchical Control

- Causes robot to follow a trajectory over a period of time.
- Requires observations or predictions of the robot with respect to the environment.
- Examples: Path following, Move robot to specified pose, Follow leader robot, Line following



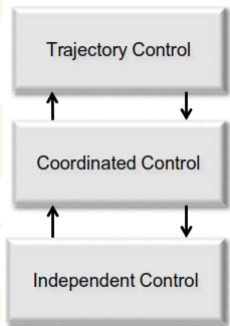


Figure: Hierarchical Control

- Instantaneous control of entire robot as an entity.
- MIMO: Consistent control of independent dof.

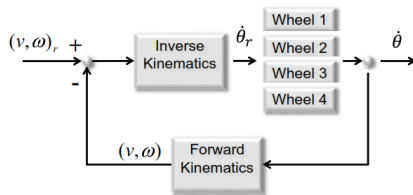


Figure: Wheels are coordinated to produce desire robot linear and angular velocity.

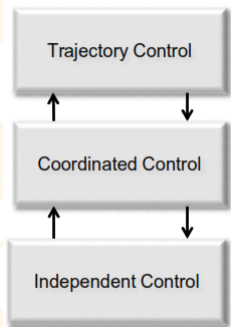


Figure: Hierarchical Control

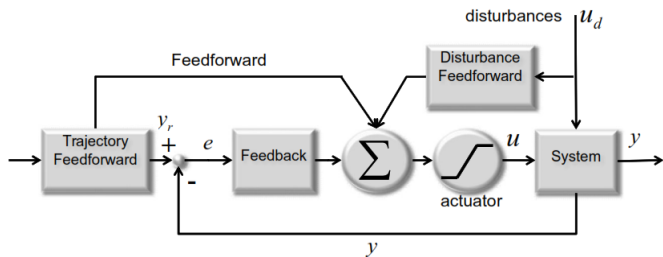
- SISO: Controls a single degree of freedom.
- Relies only on sensing connected to that DOF.
- Connected directly to actuators.
- Example: Controllers for motor of differential drive that accepts wheel velocity and converts to torque/voltage signals.



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# Feedforward control makes system reach goal faster.



**Figure 7.2 Generic Controller Block Diagram.** This diagram summarizes most cases of interest.



- **Disturbance** ( $u_d$ ): System inputs we cannot control.
- Actuators have saturation limits.
- **Feedforward Control or Open loop:** Control signal is based on system model.



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# Mathematical model of system for control

- Recall that continuous-time control requires a differential model of the system, which is called *dynamical system model* in control literature.
- The word *dynamical* here simply refers to time-evolving nature of system captured by differential equations.
- We can use kinematic model of robot (also differential equations) for control, and don't require model to include forces (dynamics model).
- Kinematic model is sufficient, if we have good velocity control and dynamic effects are not vital.

$$\dot{q} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \frac{r}{2} \cos \phi & \frac{r}{2} \cos \phi \\ \frac{r}{2} \sin \phi & \frac{r}{2} \sin \phi \\ -r/L & r/L \end{bmatrix} \begin{bmatrix} u_L \\ u_R \end{bmatrix}$$

■ We can rewrite it as:

$$\dot{x} = v \cos \phi$$

$$\dot{y} = v \sin \phi$$

$$\dot{\phi} = \omega,$$

if

$$u_L = \frac{2v - \omega l}{2r} \text{ and } u_R = \frac{2v + \omega l}{2r}.$$



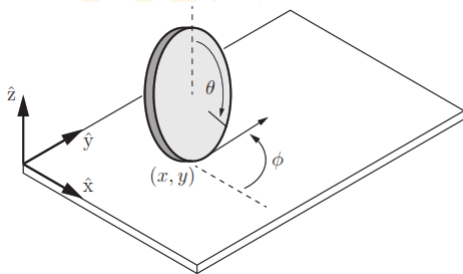


Figure: Wheel rolling without slipping

$$\dot{q} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} r \cos \phi & 0 \\ r \sin \phi & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

- $u_1$  is the wheel's driving speed and  $u_2$  is heading direction turning speed.

$$\dot{x} = v \cos \phi$$

$$\dot{y} = v \sin \phi$$

$$\dot{\phi} = \omega$$

- The model for car can also be written as:

$$\dot{x} = v \cos \phi$$

$$\dot{y} = v \sin \phi$$

$$\dot{\phi} = \omega,$$

with the expressions

$$v = v$$

$$\psi = \arctan \left( \frac{\omega d}{v} \right)$$

converting the controls  $(v, \omega)$  to actual controls  $(v, \psi)$ .



# Assumptions while using classical control for robots

- **Assumption:** We're able to measure the controlled variables, typically position and orientation of the robot, with respect to either a fixed frame or a path that the robot should follow.
- **Assumption:** Observations are continuous in time.
- **Assumption:** Observations are not corrupted by noise.



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