Quiz 3B: NFAs

CS 212 Nature of Computation

Habib University — Fall 2023

Total Marks: 10 | Date: September 6, 2023
Duration: 15 minutes | Time: 830–845h

Student ID: _____

Student Name: _____

1. Given DFAs $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$, that recognize the languages L_1 and L_2 respectively, we construct the NFA $M_3 = (Q_3, \Sigma_\epsilon, \delta_3, q_3, F_3)$ as follows.
   1. $Q_3 = Q_1 \cup Q_2 \cup \{q_3\}$
   2. $F_3 = F_1 \cup F_2$
   3. $\delta_3(q_3, \epsilon) = \{q_1, q_2\}$
   4. $\forall q \in Q_1 \forall a \in \Sigma \ (\delta_3(q, a) = \{\delta_1(q, a)\})$
   5. $\forall q \in Q_2 \forall a \in \Sigma \ (\delta_3(q, a) = \{\delta_2(q, a)\})$

   (a) (7 points) Prove or disprove the following claim.
   Claim 1. $L(M_3) = L_1 \cup L_2$

   (b) (3 points) If the above claim is true, what does it establish about the closure of the class of regular languages?

Solution:

(a) *Proof.* $L(M_3) = (L_1 \cup L_2)$

We prove that each side is a subset of the other.

Case 1: $L(M_3) \subseteq (L_1 \cup L_2)$
Assume $w \in L(M_3)$.
Then, there exists a state $q \in F_3$ such that reading w leaves M_3 in the state q.
Then, from the construction, $q \in F_1 \vee q \in F_2$.
Then, from the construction, $w \in L_1 \vee w \in L_2$.
$\therefore w \in (L_1 \cup L_2)$.

Case 2: $(L_1 \cup L_2) \subseteq L(M_3)$
Assume $w \in (L_1 \cup L_2)$.
Then, reading w leaves M_1 in a state $r_1 \in F_1$ or M_2 in a state $r_2 \in F_2$.
Then, from the construction, there exists a sequence of state transitions for w that leaves M_3 in r_1 or r_2.
$\therefore w \in L(M_3)$. □

(b) The truth of the claim establishes that the class of regular languages is closed under union.