Complexity Theory Quiz 6

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Q) $LADDER_{DFA} = \{\langle M, s, t\rangle \mid M$ is a DFA & $L(M)$ contains a Ladder of string starting with s & ending with t$\}$.

LAD-- $\in$ PSPACE

/* LD = $LADDER_{DFA}$ for my ease of writing /

* We can use the fact that PSPACE = NPSPACE to show that LD $\in$ PSPACE by showing LD $\in$ NPSPACE.

* Given that $\langle M, s, t\rangle$, reject if $|s| \neq |t|$. Otherwise we can consider a graph G of exponential size whose vertices are indexed by the strings in $\Sigma^{|s|}$ & there is a directed edge from w_1 to w_2 if & only if w_1 & w_2 differ in exactly one character & $w_1, w_2 \in L(M)$.

Then $\langle M, s, t\rangle \in LD$ if & only if there is a path from s to t in G.

↳ We can check this in NPSPACE by guessing the path at each step, & only store the index of the current vertex we are at. So we nondeterministically select a new vertex w_2 that differs from w_1 by exactly one character, & verify that M accepts w_2.

↳ We can verify ~~the Machine~~ that M always halts by ensuring a counter to keep track of traversed nodes, which would again take PSPACE as we can overwrite the current counter, & stopping when the counter reaches $|\Sigma^{|s|}|$. As if a ladder exists, then it is of at most $|\Sigma^{|s|}|$ length.

Thus LD $\in$ NPSPACE, & since NPSPACE = PSPACE, therefore LD $\in$ PSPACE.