L

Q3

a) $(A+A^T)$ is symmetric

$\Rightarrow$ $(A)^T = A$ symmetric

$\Rightarrow$ $(A + A^T)^T = (A^T + (A^T)^T)$ $\quad \therefore (A+B)^T = A^T + B^T$

$\therefore (A^T)^T = A$

$\Rightarrow$ $(A^T + A) = (A + A^T)$ $\quad \therefore (A+B) = (B+A)$

Hence

$$(A+A^T)^T = (A+A^T)$$

$(A-A^T)^T = (A^T - (A^T)^T)$ $\quad \therefore (A+B)^T = A^T + B^T$

$(A-A^T)^T = (A^T - A)$ $\quad \therefore (A^T)^T = A$

$(A-A^T)^T = -(A - A^T)$

Hence $A^T = -A$ it is skew symmetric

b) $AA^T$ ~~is~~ & $A^TA$ are symmetric

$(AA^T)^T = (A^T)^T (A)^T$ $\quad \therefore (AB)^T = B^T A^T$

$= A A^T$ $\quad \therefore (A^T)^T = A$

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$(A^TA)^T = (A^T (A^T)^T)$
$= (A^T \ A)$

Hence they are symbolic

c) $A^2 = A$ , $A^{-1}$ Exist then $A = I$

$A^{-1}A^2 = A^{-1}A$
$\underbrace{(A^{-1}A)}A = (A^{-1}A)$
$IA = I$
$A = I$

d) $(A^{-1})^T = (A^T)^{-1}$
we know
$I = I$

$(A A^{-1}) = I$
$(A A^{-1})^T = I^T$
$(A^{-1})^T A^T = I$ $\quad \therefore I^T = I$ & $(AB)^T = B^T$
$(A^{-1})^T A^T (A^T)^{-1} = I \cdot (A^T)^{-1}$
$(A^{-1})^T \cdot I = (A^T)^{-1}$

$$\boxed{(A^{-1})^T = (A^T)^{-1}}$$

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$$AX = XB$$

$$AX \Rightarrow \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x & y \\ z & t \end{bmatrix}$$

$$\begin{bmatrix} x & y \\ x+2z & y+2t \end{bmatrix}$$

$$XB = \begin{bmatrix} x & y \\ z & t \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2x-y & -x+2y \\ 2z-t & -z+2t \end{bmatrix}$$

$$AX = XB$$

$$\begin{bmatrix} x & y \\ x+2z & y+2t \end{bmatrix} = \begin{bmatrix} 2x-y & -x+2y \\ 2z-t & -z+2t \end{bmatrix}$$

$x = 2x-y$ — (1) ; $y = -x+2y$ — (2)

$\cancel{x+2z}$ $x+2z = 2z-t$ (3) ; $y+2t = -z+2t$ (4)

(1) $\Rightarrow$ $x = y$ (2) $\Rightarrow$ $x = y$

(3) $\Rightarrow$ $x = -t$ (4) $\Rightarrow$ $y = -z$

So, every variable in x $\begin{bmatrix} x & y \\ z & t \end{bmatrix} = \begin{bmatrix} x & x \\ -x & -x \end{bmatrix}$

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$\Rightarrow \quad \times \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$

Hence proved!

Q6

$A^t = -A$ . then $a_{ii} = 0$

$\Downarrow$

$A^t + A = O \leftarrow$ Zero matrix

$$\begin{bmatrix} 2a_{11} & a_{12}+a_{21} & a_{13}+a_{31} \\ a_{21}+a_{12} & 2a_{22} & a_{23}+a_{32} \\ a_{31}+a_{13} & a_{32}+a_{32} & 2a_{33} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Comparing co-efficients

$2a_{11} = 0, \quad 2a_{22} = 0, \quad 2a_{33} = 0$

$a_{11} = 0, \quad a_{22} = 0, \quad a_{33} = 0$

Hence diagonals of $A = 0$

$PX = XP$ (Commutative Property)

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} d & e & f \\ g & h & i \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & a & b \\ 0 & d & e \\ 0 & g & h \end{bmatrix}$$

$\Downarrow$ deduce by equating

$d=0$, $a=e$, $b=f$

$g=0$, $h=d$, $e=i$

$0=0$ $g=0$, $h=0$

$$\begin{bmatrix} a & b & c \\ 0 & a & b \\ 0 & 0 & a \end{bmatrix}$$