Weekly Challenge 02: Deterministic Finite Automata (DFA)

CS 212 Nature of Computation
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1. The Complement Language

Consider the following finite automata and their languages.

- $M_1 = (Q, \{0, 1\}, \delta, q_o, F)$ and its language, $L_1 = L(M_1)$, and
- $M_2 = (Q, \{0, 1\}, \delta, q_o, Q - F)$ and its language, $L_2 = L(M_2)$.

Prove or disprove the following claim,

$$\boxed{L_1 = L_2'}$$

where L' is the set-complement of L.

Solution:

We have the following finite automata and their languages:
- $M_1 = (Q, \{0, 1\}, \delta, q_o, F)$ and its language $L1 = L(M_1)$, and
- $M_2 = (Q, \{0, 1\}, \delta, q_o, Q - F)$ and its language $L2 = L(M_2)$

For both the given finite automata M_1 and M_2, they have the same set of states Q, alphabet $\{0, 1\}$, transition function δ, and starting state q_o. However, they have different set of final states, that is F and $Q - F$ respectively.

Considering the languages L_1 and L_2, L_1 is the language that leads M_1 to an accepting state, but not M_2 since $F \subseteq Q$, so F and $Q - F$ would be disjoint sets, and by the definition of set complement, $F = (Q - F)'$ or $F' = Q - F$. Similarly, L_2 is the language that leads M_2 to an accepting state but not M_1.

Then for any arbitrary string $w \in \{0, 1\}^*$:
- If w is accepted by M_1, then $w \in L_1$. However, since L_1 contains the strings not accepted by M_2, we can conclude that $w \notin L_2$ which implies $w \in L_2'$.
  Thus $w \in L_1 \implies w \in L_2'$. So $L_1 \subseteq L_2'$.
- If w is not accepted by M_2, then $w \notin L_2 \implies w \in L_2'$. However, $w \in L_2'$ implies that the string w leads M_2 to a rejecting state $Q - (Q - F)$ which is equal to F. From this we can conclude that the string w would then lead to a state in F which is an accept state for M_1. Therefore, $L_2' \subseteq L_1$.

Thus, for any arbitrary string w, we've shown that it either belongs to L_1 and L_2', or it does not. In either case, $w \in L_1 \iff w \in L_2'$. Therefore, $L_1 = L_2'$.

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