

Q1 [70]: Consider i.i.d. random variables X_1, \dots, X_n sampled from the distribution with probability density

$$f(x | \theta) = K\theta^3\sqrt{x}e^{-\theta^2x}, \quad x > 0$$

and 0 elsewhere.

- (a) Find K .
- (b) Compute the method of moment estimator for θ ; denote it as $\hat{\theta}_{\text{MoM}}$.
- (c) Using Taylor's series, compute the bias of the estimator, $\hat{\theta}_{\text{MoM}}$.
- (d) Using Taylor's series, compute the variance of the estimator, $\hat{\theta}_{\text{MoM}}$.
- (e) Compute the maximum likelihood estimator for θ ; denote it as $\hat{\theta}_{\text{ML}}$.
- (f) Using Taylor's series, compute the bias of the estimator, $\hat{\theta}_{\text{ML}}$.
- (g) Compute the asymptotic variance of the estimator $\hat{\theta}_{\text{ML}}$.