

Weekly Challenge 08: Context-Free Languages

CS 212 Nature of Computation
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1. Closure

Given a language, L , the operation, f is defined as $f(L) = \{w \mid vw \in L \text{ and } v, w \in \Sigma^*\}$.

Prove or disprove the given claim.

Claim 1. *The class of context-free languages is closed under f .*

Solution:

The operation f generates a language consisting of all strings w such that for some string v , the concatenation of v and w (vw) is in the original language.

In essence, it extracts a suffix w from a string vw that is in the original language L .

Given a context-free language L , let the PDA for L be P . Let the resulting language $f(L)$ be L' . Then we can construct a PDA P' for L' as follows:

- Create an identical copy of P and name it as P_c , having the same transitions as P . P and P_c will be combined to form P' ; P_c recognizes the prefix v .
- Modify the input part of transitions in P_c to ε without changing the stack symbol. For example, if the input transition has $0, 1 \rightarrow \varepsilon$ modify it to $\varepsilon, 1 \rightarrow \varepsilon$. The input in the transition $0, 1 \rightarrow \varepsilon$ is 0 and it is changed $\varepsilon, 1 \rightarrow \varepsilon$ where the stack symbol ε is unchanged. This step ignores the input symbols when transitioning in P_c as we want P_c to recognize the prefix v , so the input symbols don't matter to it and it doesn't consume the input symbols.
- For each state in P_c , add a new transition $\varepsilon, \varepsilon \rightarrow \varepsilon$ to the corresponding state in P . This step connects each state in P_c to its corresponding state in P , essentially connecting P_c to P . It says that when P_c is done recognizing the prefix v and is ready to look at the suffix w , it transitions to the corresponding state in P .
- The start state of P_c should be the start state of the whole PDA P' . So when we start processing an input string, first we go through P_c which recognizes the prefix v and then we go through P which recognizes the suffix w . Thus, P' is the combination of the two PDAs P and P_c .

Then, by this construction, P_c 'ignores' some prefix of the input string and then processes the rest of the string normally through P . Hence recognizing all strings w after removing some prefix v from the original string vw in the language.

Since a PDA recognizes $f(L)$, the class of context-free languages is closed under f .

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