DLD Homework 2
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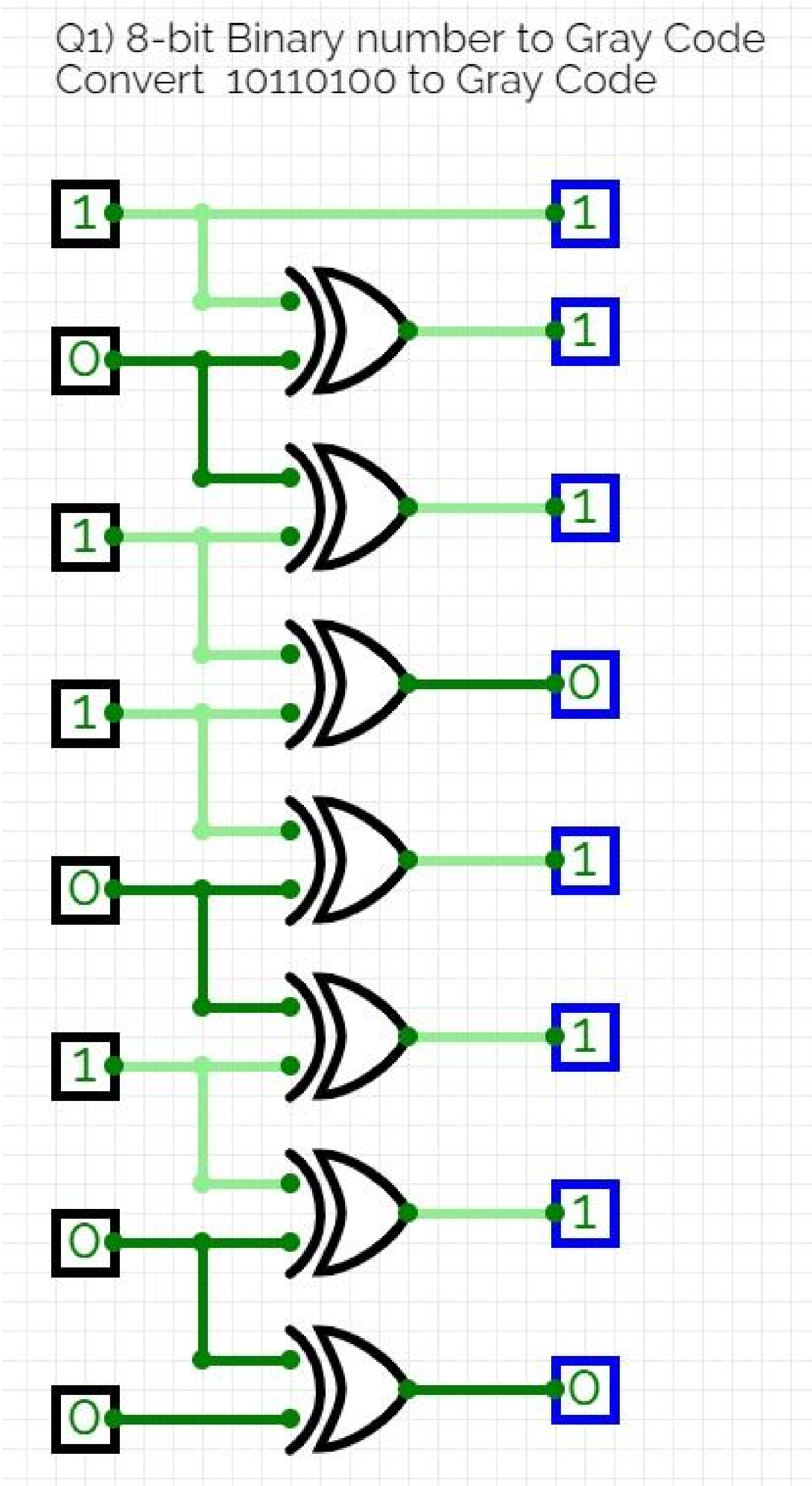
aa07190



(Made on Circuitverse)

Dated: DLD Homework 2

Ali Muhammad Asad

07190

Q2) (a) $F = wx + yz$

$F' = (wx+yz)' = (wx)' \cdot (yz)'$

$= (w'+x') \cdot (y'+z') = w'y' + w'z' + x'y' + x'z'$

$F \cdot F' = 0 \Rightarrow (wx+yz)(w'+x')(y'+z')$

~~[illegible]~~

$\Rightarrow (w'wx + wxx' + yz(w'+x'))(y'+z')$

$[x \cdot x' = 0]$

$\Rightarrow (0 + 0 + yz(w'+x'))(y'+z')$ $[x+0=x]$

$\Rightarrow yz(w'+x')(y'+z')$

$\Rightarrow yz(y'+z')(w'+x')$ [Commutative]

$\Rightarrow (y'yz + yzz')(w'+x')$ $[x \cdot x' = 0]$

$\Rightarrow (0 + 0)(w'+x')$

$\Rightarrow 0(w'+x')$ $[x \cdot 0 = 0]$

$\Rightarrow \underline{0}$

[redacted]

Q2) (a) $F+F'=1 = (wx+yz) + (w'+x')(y'+z')$

$= wx+yz + (w'y'+w'z'+x'y'+x'z')$ [Distributive Law]

$= wx+yz + w'(y'+z') + x'(y'+z')$ (Distributive]

$= wx+yz+w'(yz)'+x'(yz)'$ [De Morgan's Law]

$= wx+yz+w'+x'$ [$a+a'b = a+b$ Absorption]

$= x'+wx+w'+yz$ [Associative Law]

~~= [illegible]~~ ~~[Distributive]~~

~~= [illegible]~~ ~~[Postulate 2(a) x+1=1]~~

~~= [illegible]~~ ~~[Postulate~~

$= x'+wx+w'(x+x')+yz$ [Postulate 5(a) $x+x'=1$]

$= x'+wx+w'x+w'x'+yz$ [Distributive]

$= x'+wx'+wx+w'x+yz$ [Associative]

$= x'(1+w)+x(w+w')+yz$ [Distributive]

$= x'(1)+x(1)+yz$ [Theorem 2(a) $x+1=1$]

$= x'+x+yz$ [Postulate 2(b) $x.1=x$]

$= 1+yz = \underline{\underline{1}}$

Dated:

(b) i) $(x+y)(x+y')(xz')'$

$\Rightarrow (x+y)(x+y')(x'+z)$

$\Rightarrow (x + xy' + xy)(x'+z)$ [Distributive Law]

$\Rightarrow 0 + 0 + 0 + xz + xy'z + xyz$ [Post 5 (b) $x \cdot x' = 0$]

$\Rightarrow xz + xy'z + xyz$ [Post 2 (a) $x + 0 = x$]

$\Rightarrow xz + xz(y'+y)$ [Distributive Law]

$\Rightarrow xz + xz \Rightarrow xz$ [has 2 literals]

[Post 5 (a) $x + x' = 1$] [Theorem 1 (a) $x + x = x$]

ii) $w'(wxyz)'$

$\Rightarrow (w + wxyz)'$ [DeMorgan's Law]

$\Rightarrow (w(1+xyz))'$ [Distributive Law]

$\Rightarrow (w(1))'$ [$x \cdot 1 = x$]

$\Rightarrow (w)' \Rightarrow w'$ [1 Literal]

[Involution]

iii) $(ab + c')' + ac'b + b$

$\Rightarrow (ab)'(c')' + b(ac'+1)$ [DeMorgan's Law, Distributive]

$\Rightarrow (a'+b')(c) + b$ [De Morgan's, Theorem 2 (a) $x + 1 = x$]

$\Rightarrow a'c + b'c + b$ [redacted]

[Distributive Law]

$\Rightarrow a'c + b'c + b$

$= a'c + c + b$ [Absorption $a + a'b = a + b$]

$= c(a'+1) + b$ Distributive]

$= c(1) + b$ [Theor 2 (a) $x+1 = 1$]

$= c + b$ [Postulate 2 (b) $x \cdot 1 = x$]

[2 literals]

(c) A = 1 0 1 1 0 0 0 1

B = 1 0 1 0 1 1 0 0

i) A OR B = 1 0 1 1 1 1 0 1

ii) A XOR B = 0 0 0 1 1 1 0 1

iii) Not B = 0 1 0 1 0 0 1 1

Dated:

Q3) (a) $(x + x')'$

$\Rightarrow (x')\cdot(x')'$ [De Morgan's Law]

$\Rightarrow x' \cdot x$ [Double Complement → Theorem 3]

$\Rightarrow x \cdot x'$ [Commutative with respect to '·']

$\Rightarrow 0$ [Postulate 5 (b)]

(b) $w' \cdot (wxyz)'$

$\Rightarrow (w + wxyz)'$ [De Morgan's Law]

$\Rightarrow (w(1 + xyz))'$ ['·' is Distributive over '+']

$\Rightarrow (w(1))'$ [Theorem 2 (a) : $x+1=1$]

$\Rightarrow (w)'$ [Postulate 2 (b) : $x\cdot 1 = x$]

$\Rightarrow w'$ [Complement]

(c) $(x+y)\cdot(x+y')\cdot(xz')'$

$\Rightarrow (x + xy' + yx + yy')\cdot(xz')'$ [Distributive Law]

$\Rightarrow (x + xy' + xy + 0)\cdot(xz')'$ [$y\cdot y' = 0$: Postulate 5 (b)]

$\Rightarrow (x(1 + y' + y))\cdot(x\cdot z')'$ [$x+0=x$: Postulate 2(a), Distributive Law]

$\Rightarrow (x(1+1))\cdot(xz')'$ [Postulate 5 (a)]

$\Rightarrow (x(1))\cdot(xz')'$ [$1+1=1$]

$\Rightarrow x\cdot(xz')'$ [$x\cdot 1 = x$: Postulate 2 (b)]

$\Rightarrow x\cdot(x' + z'')$ [De Morgan's Law]

$\Rightarrow x\cdot(x' + z)$ [Theorem 3: $(x')' = x$]

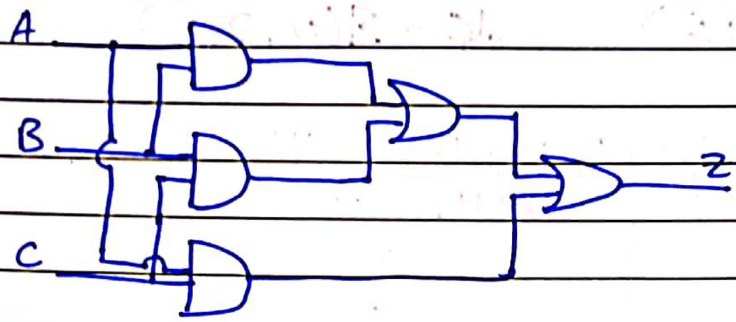
$\Rightarrow x\cdot x' + xz$ [Distributive Law]

$\Rightarrow 0 + xz$ [Postulate 5 (b) : $x\cdot x' = 0$]

$\Rightarrow xz$ [$x + 0 = x$: Postulate 2 (a)]

Dated:

Q4) (a)



A	B	C	Z
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

(b) Minterms:

$f_1 = x'yz + xy'z + xyz' + xyz = m_3 + m_5 + m_6 + m_7$

$F(x, y, z) = \Sigma(3, 5, 6, 7)$

Maxterms:

$f_2 = (x+y+z)(x+y+z')(x+y'+z)(x'+y+z)$

$F(x, y, z) = \Pi(0, 1, 2, 4)$

Q5) i) $[(a' + cd')(abc' + bd)(ad' + b + c')]'$

$= (a' + cd')' + (abc' + bd)' + (ad' + b + c')'$

$= (a')' \cdot (cd')' + (abc') \cdot (bd)' + (ad')' \cdot b' \cdot (c')'$

$= a \cdot (c' + d) + (a' + b' + c) \cdot (b' + d') + (a' + d) \cdot b' \cdot c$

$= a \cdot (c' + d) + (b' + d') \cdot (a' + b' + c) + b'c(a' + d)$

ii) $[(xyz' + x'z + yz')(xy)]'$

$= (xyz' + x'z + yz')' + (xy)'$

$= (xyz')'(x'z)'(yz')' + (x' + y')$

$= (x' + y' + z)(x + z')(y' + z) + (x' + y')$

Q6) Canonical POS

$$Y = \overline{(AB + A'B')(CD' + C'D)}$$

$$= (ABCD' + ABC'D + A'B'CD' + A'B'C'D)'$$

$$= (A'+B'+C'+D'') \cdot (A'+B'+C''+D') \cdot (A''+B''+C'+D'') \cdot (A''+B''+C''+D')$$

$$= (A'+B'+C'+D) \cdot (A'+B'+C+D') \cdot (A+B+C'+D) \cdot (A+B+C+D')$$

Q7

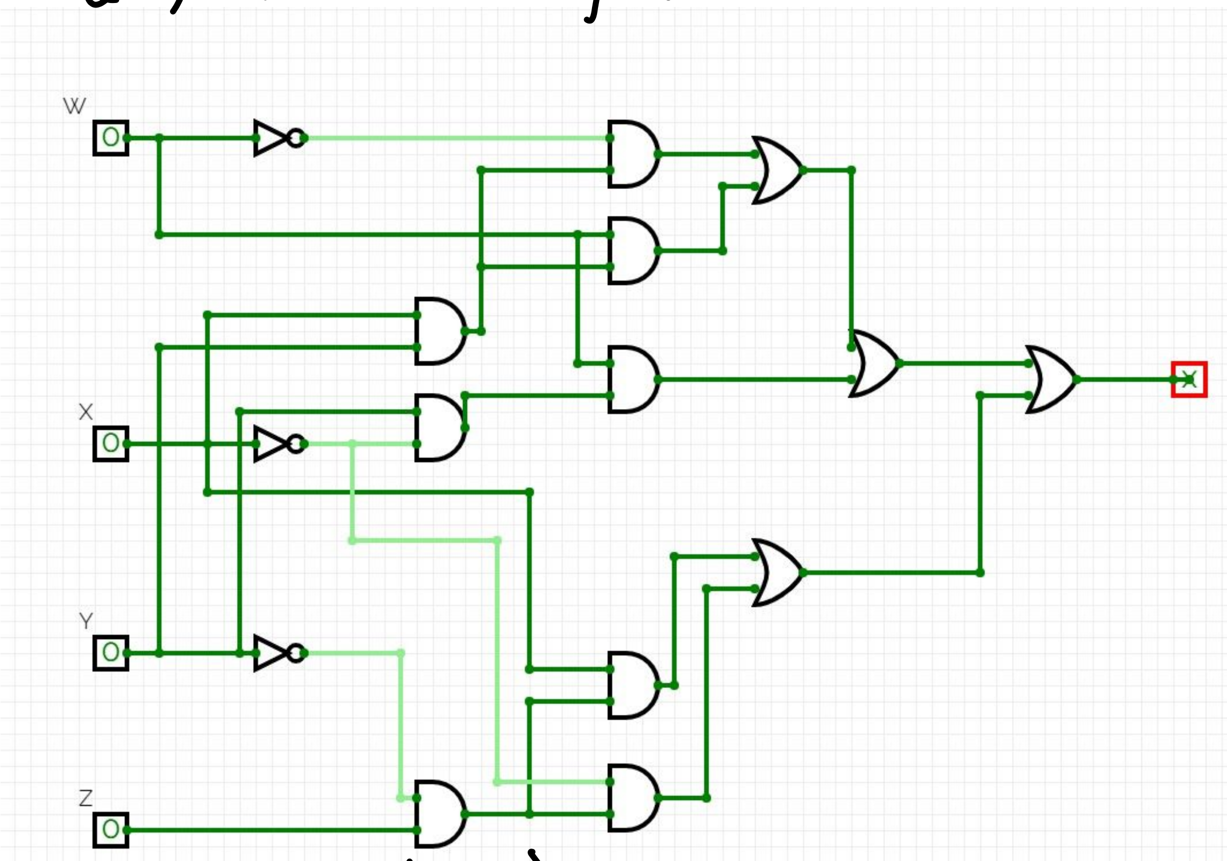
F = xy'z + x'y'z + w'xy + wx'y + wxy

(a) Obtain the truth table for F

w	x	y	z	w'	x'	y'	z'	$xy'z$	$x'y'z$	$w'xy$	$wx'y$	wxy	F
F	F	F	F	T	T	T	T	F	F	F	F	F	F
F	F	F	T	T	T	T	F	F	T	F	F	F	T
F	F	T	F	T	T	F	T	F	F	F	F	F	F
F	F	T	T	T	T	F	F	F	F	F	F	F	F
F	T	F	F	T	F	T	T	F	F	F	F	F	F
F	T	F	T	T	F	T	F	T	F	F	F	F	T
F	T	T	F	T	F	F	T	F	F	T	F	F	T
F	T	T	T	T	F	F	F	F	F	T	F	F	T
T	F	F	F	F	T	T	T	F	F	F	F	F	F
T	F	F	T	F	T	T	F	F	T	F	F	F	T
T	F	T	F	F	T	F	T	F	F	F	T	F	T
T	F	T	T	F	T	F	F	F	F	F	T	F	T
T	T	F	F	F	F	T	T	F	F	F	F	F	F
T	T	F	T	F	F	T	F	T	F	F	F	F	T
T	T	T	F	F	F	F	T	F	F	F	F	T	T
T	T	T	T	F	F	F	F	F	F	F	F	T	T

Truth table was typed on LaTeX

Q7) (b) Circuit Diagram



(Made on Circuitverse)

Dated:

Q7) $F = xy'z + x'y'z + w'xy + wx'y + wxy$

(c) $F = (x+x')(y'z) + w'xy + wxy + wx'y$

$[x + x' = 1]$

$= 1(y'z) + (w'+w)xy + wx'y$

$= y'z + xy + wx'y$

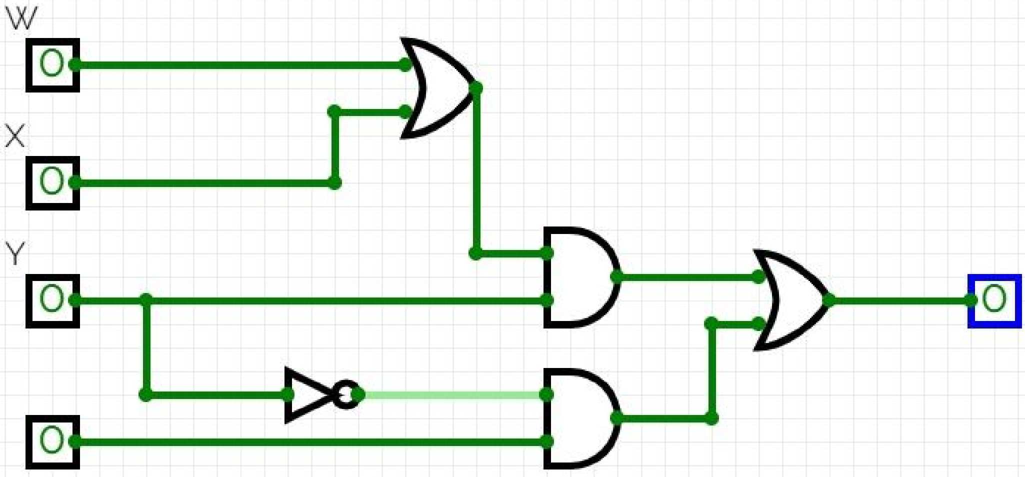
$= y'z + y(x + wx')$

* ORing x OR wx' would give the same truth table as x OR w, as when x will be 0, x' will be 1 so wx' will only be false when x & w are false, else it will be true. So $x + wx'$

$= x + w$. [$x + x'w = x + w$ Absorption Law]

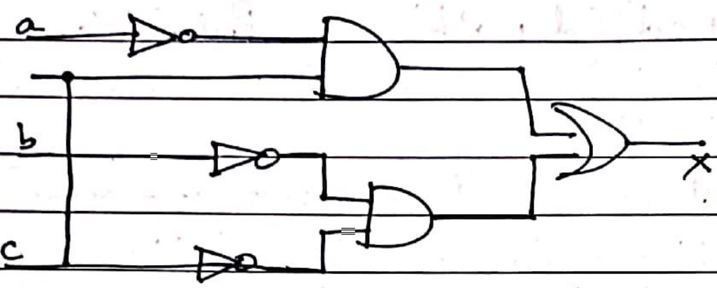
$= y'z + y(x+w)$

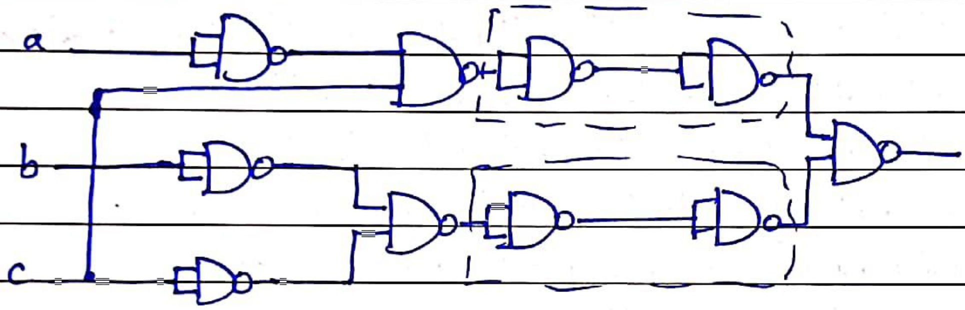
Q7) (d) Simplified Circuit Diagram

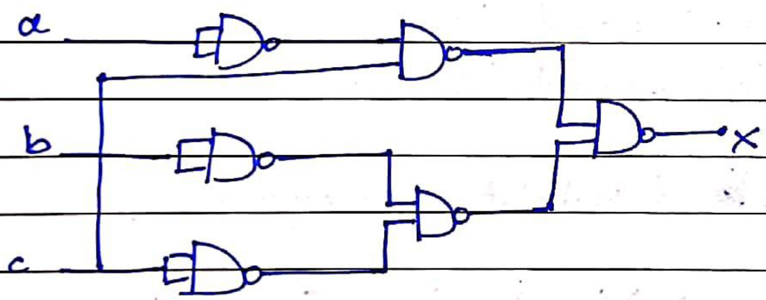


Dated:

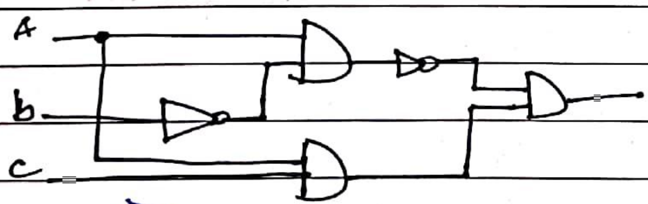
Q3) (a)

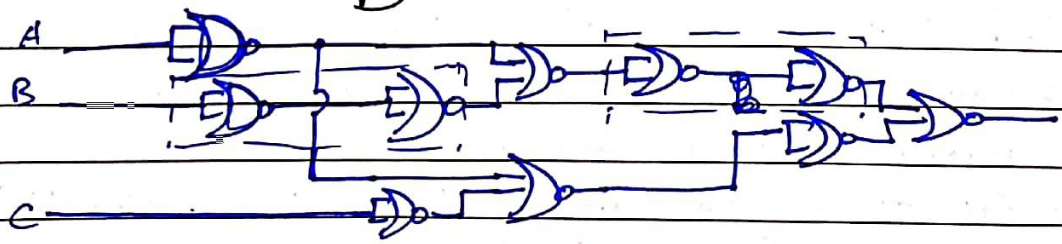


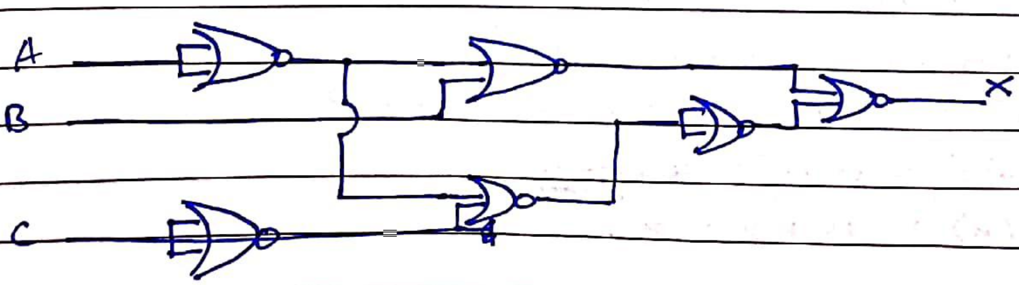




(b)







Dated:

Q9) (a) $F(a,b,c,d) = (a+cd)(d+a'b)$ Prod. of Max.

$= ((a+c)(a+d))((d+a')(d+b))$ [Theorem 4 Distributive (b)]

$= (a+c)(a+d)(d+a')(d+b)$

$= (a+c)(a+d)(a'+d)(b+d)$ [Post. 3: $x+y = y+x$]

$= (a+c+bb'+dd')(a+d+bb'+cc')(a'+d+bb'+cc')(b+d+aa'+cc')$

$= (a+c+b+dd')(a+c+b'+dd')(a+d+b+cc')(a+d+b'+cc')$
$(a'+d+b+cc')(a'+d+b'+cc')(b+d+a+cc')(b+d+a'+cc')$

$= (a+c+b+d)(a+c+b+d')(a+c+b'+d)(a+c+b'+d')$
$(a+d+b+c)(a+d+b+c')(a+d+b'+c)(a+d+b'+c')$
$(a'+d+b+c)(a'+d+b+c')(a'+d+b'+c)(a'+d+b'+c')$
$(b+d+a+c)(b+d+a+c')(b+d+a'+c)(b+d+a'+c')$

[Theorem 4 Distributive (b)]

$= (a+b+c+d)(a+b+c+d')(a+b'+c+d)(a+b'+c+d')$
$(a+b+c'+d)(a+b'+c'+d)(a'+b+c+d)(a'+b+c'+d)$
$(a'+b'+c+d)(a'+b'+c'+d)$

[Theorem 1 (b): $x \cdot x = x$ & Postulate 3: $x+y = y+x$]

b) $F(w,x,y,z) = (w'z+x)(y'+wy'x)(xz'+w)$ Sum of Min.

$= (w'y'z + w'wy'x + xy' + xwy'x)(xz'+w)$

[Theorem 4 Distributive (a): $x(y+z) = xy+xz$]

$= (w'y'z + 0 + xy' + wxy')(xz'+w)$

[Postulate 5(b): $x \cdot x' = 0$, Theorem 1 (b): $x \cdot x = x$]

$= xz'w'y'z + xz'xy' + xz'wxy' + ww'y'z + wxy' + ww'xy'$

$= 0 + xy'z' + wxy'z + 0 + wxy' + wxy'$

[Post. 5 (b): $x \cdot x' = 0$, Theorem 1 (b): $x \cdot x = x$, Theorem 4 Associative (b)]

$= xy'z' + wxy'z + wxy'$ [Theorem 1 (a): $x+x = x$]

$= xy'z'(w+w') + wxy'z + wxy'(z+z')$

$= wxy'z' + w'xy'z' + wxy'z + wxy'z + wxy'z'$

$= wxy'z' + w'xy'z' + wxy'z$

[Theorem 1 (a): $x+x = x$]

Dated:

(c) $F(i,j,k,l) = ij'l + j'kl' + i'j'k'$ Sum of Mint

$= ij'l(k+k') + j'kl'(i+i') + i'j'k'(l+l')$

$= ij'lk + ij'lk' + j'kl'i + j'kl'i' + i'j'k'l + i'j'k'l'$

$= ij'kl + ij'k'l + ij'kl' + i'j'kl' + i'j'k'l + i'j'k'l'$

[Theorem 4 Associative (ab) $x(yz) = (xy)z$]

(d) $F'(p,q,r,s) = p'q'r + p'rs' + pq'r's$ Prod of Max $\rightarrow F$

$F = (p'q'r + p'rs' + pq'r's)'$

$= (p'q'r)' \cdot (p'rs')' \cdot (pq'r's)'$ [Theorem 5 (a) De Morgan's]

$= (p'' + q'' + r') \cdot (p'' + r' + s'')(p' + q'' + r'' + s')$ [Thm 5 (a) De Morgan's]

$= (p + q + r') \cdot (p + r' + s)(p' + q + r + s')$ [Thm 3 $(x')' = x$]

$= (p + q + r' + ss') \cdot (p + r' + s + qq')(p' + q + r + s')$

$= (p+q+r'+s)(p+q+r'+s')(p+r'+s+q)(p+r'+s+q')(p'+q+r+s')$

[Theorem 4 Distributive (b) $x + yz = (x+y)(x+z)$]

$= (p+q+r'+s)(p+q+r'+s')(p+q'+r'+s)(p'+q+r+s')$

[Theorem 1 (b) $x \cdot x = x$]

Dated:

(e) $F(A,B,C) = (A'+C'+D')(A'+C')(C'+D')$

$= (A'\cdot A' + A'C' + C'A' + C'C' + D'A' + D'C')(C'+D')$

$= (A' + A'C' + C' + A'D' + C'D')(C'+D')$

[Post 5 (b): $x \cdot x = x$, Theorem 4 Assosiation & Distributive]

$= (A'(1+C'+D') + C'(1+D'))(C'+D')$

$= (A'(1) + C'(1))(C'+D')$ [Theorem 2 (a) $x+1=1$]

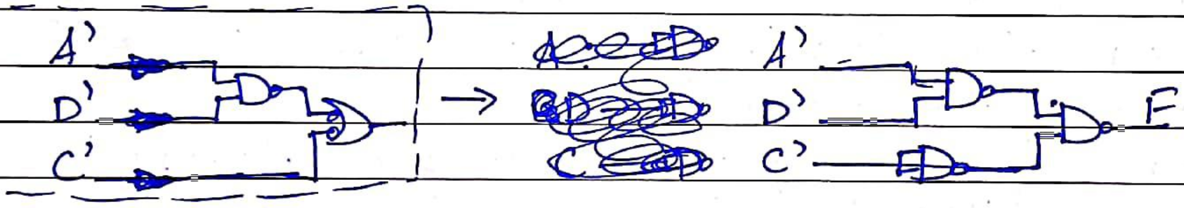
$= (A' + C')(C'+D')$ [Postulate 2(b) $x \cdot 1 = x$]

$= A'C' + A'D' + C' + C'D'$

$= A'D' + C'(1 + D' + A')$

$= A'D' + C'(1)$ [Theorem 2(a) $x+1=1$]

$= A'D' + C'$ [Post. 2(b) $x \cdot 1 = x$]



Q10) (a) $(A+B')(A+C')(A'+B'+C')$ 2, 3, 1, 6

$\Pi(1, 2, 3, 6)$

$\Sigma(0, 4, 5, 7)$

A \ BC	00	01	11	10
0	1			
1	1	1	1	

$= B'C' + AC$

Dated:

b) $(A'+B)(A'+B'+C').(B+C'+D)(A+B'+C+D')$

$A'+B+C \rightarrow A'+B+C+D,\ A'+B+C+D'$ 8, 9

$A'+B+C' \rightarrow A'+B+C'+D,\ A'+B+C'+D'$ 10, 11

$A'+B'+C' \rightarrow A'+B'+C'+D,\ A'+B'+C'+D'$ 14, 15

$B+C'+D \rightarrow A+B+C'+D,\ A'+B+C'+D$ 2, 10

$A+B'+C+D'$ 5

$\Pi(2, 5, 8, 9, 10, 11, 14, 15)$

$\Sigma(0, 1, 3, 4, 6, 7, 12, 13)$

AB \ CD	00	01	11	10
00	m_0 1	m_1 1	m_3 1	m_2
01	m_4 1	m_5	m_7 1	m_6 1
11	m_{12} 1	m_{13} 1	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

A, B, C, D

$SOP = A'C'D' + A'B'D + A'BC + ABC'$

$= ABC' + A'BC + A'B'C + A'C'D'$

Q11) (a) $X(A,B,C,D) = \Sigma(1, 3, 7, 9, 11, 13, 15)$

AB \ CD	00	01	11	10
00		1	1	
01			1	
11		1	1	
10		1	1	

A, B, C, D

$X(A,B,C,D) = B'D + AD + CD$

$= AD + B'D + CD.$

$Y(A,B,C,D) = \Pi(0, 1, 3, 15, 11)$

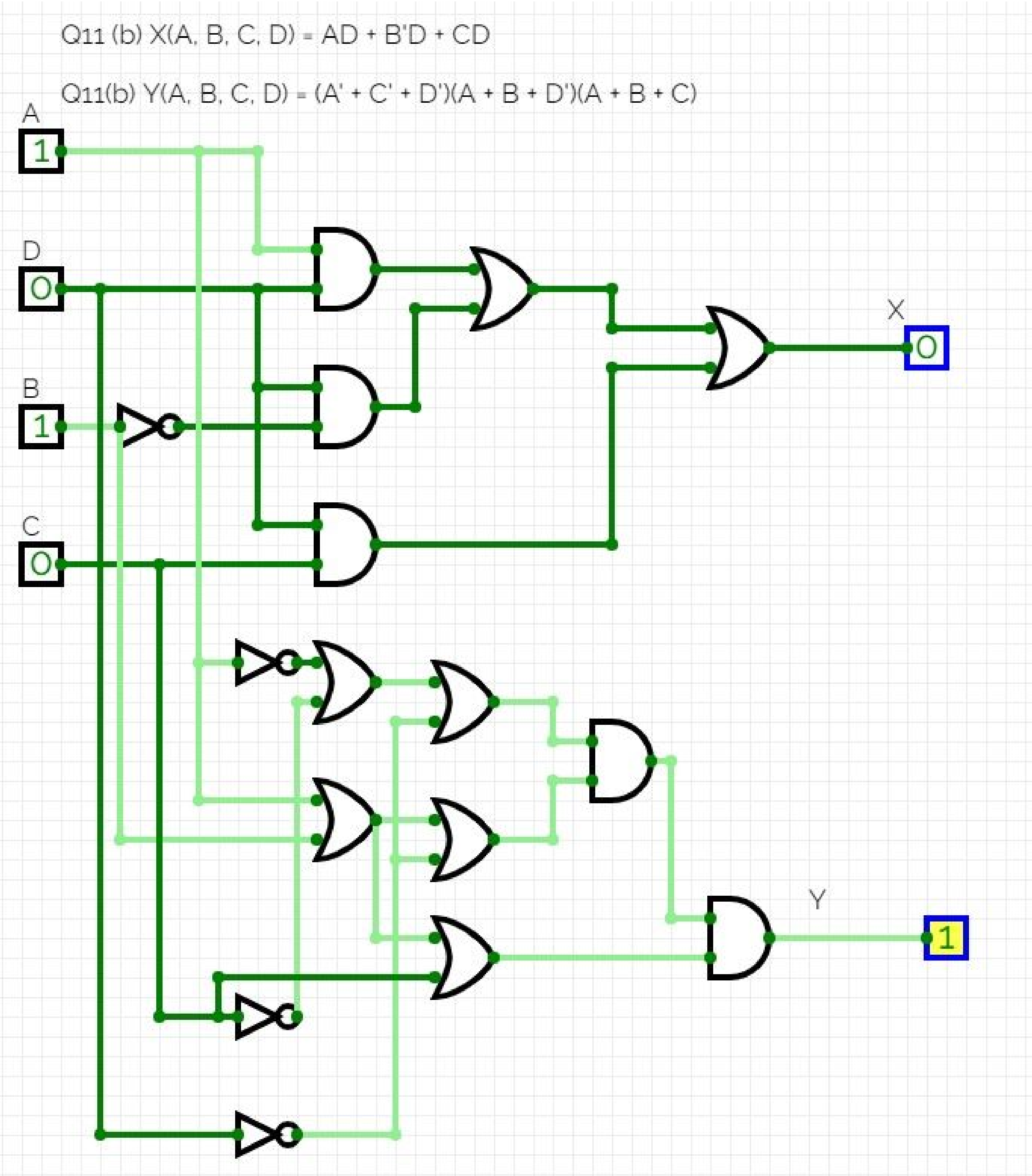
AB \ CD	00	01	11	10
00	0	0	0	
01				
11			0	
10			0	

$Y' = ACD + A'B'D + A'B'C'$

$Y = (Y')'$ [Involution]

$Y = (ACD + A'B'D + A'B'C')'$

$Y = (A'+C'+D')(A+B+D')(A+B+C)$



(Made on Circuitverse)

Dated:

Q12) (a) $(W'.X + W.X').Y' + (X+Y') + Y.Z'$

$= W'XY' + WX'Y' + X + Y' + Y.Z'$ (Theorem 4 Distributive (a))

$= X + W'XY' + Y' + WX'Y' + YZ'$ (Theorem 4 Associative (a))

$= X(1 + W'Y') + Y'(1 + WX') + YZ'$ (Theorem 4 Distributive (a))

$= X(1) + Y'(1) + YZ'$ (Theorem 2 (a) $x + 1 = 1$)

$= X + Y' + YZ'$ (Postulate 2 (b) $x.1 = x$)

$= X + Y' + Z'$ (Absorption Law $a + a'b = a + b$)

Q12 (b)

Make the truth table of the above circuit given in question

w	x	y	z	$(w'.x + w.x').y' + x + y' + y.z'$
F	F	F	F	T
F	F	F	T	T
F	F	T	F	T
F	F	T	T	F
F	T	F	F	T
F	T	F	T	T
F	T	T	F	T
F	T	T	T	T
T	F	F	F	T
T	F	F	T	T
T	F	T	F	T
T	F	T	T	F
T	T	F	F	T
T	T	F	T	T
T	T	T	F	T
T	T	T	T	T

Truth table typed on LaTeX
[Too messy on paper]

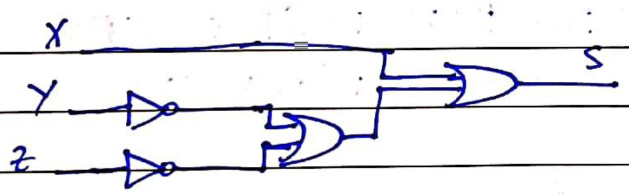
(c) [redacted] $\Sigma(0, 1, 2, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15)$

WX \ YZ	00	01	11	10
00	1	1		1
01	1	1	1	1
11	1	1	1	1
10	1	1		1

$\Rightarrow Y' + X + Z'$

$\Rightarrow X + Y' + Z'$ by Associativity $\Rightarrow$ Theorem 4

(d)



The number of gates used after simplification has reduced significantly.

Dated:

Q13) (a) $F(w, x, y, z) = \Sigma(0, 2, 8, 12, 15)$

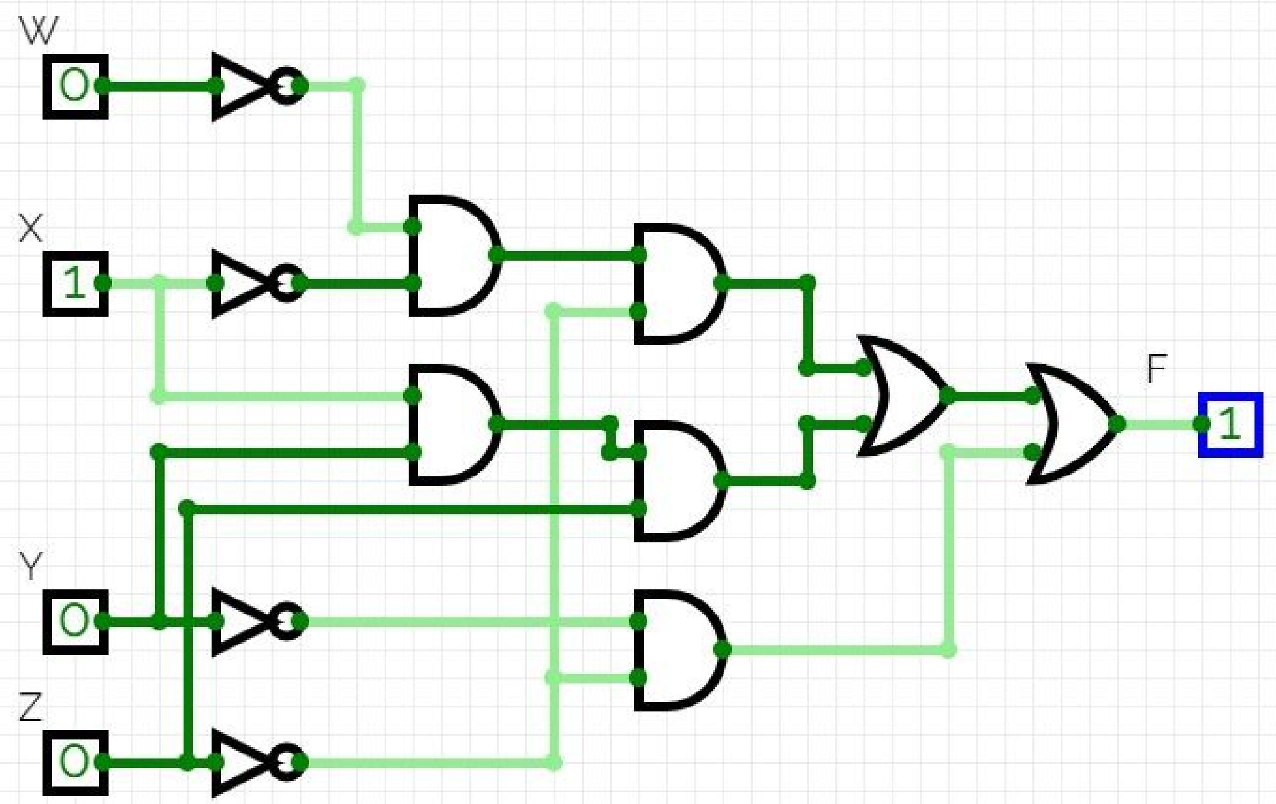
$d(w, x, y, z) = \Sigma(3, 4, 7, 14)$ -> don't care conditions

wx \ yz	00	01	11	10
00	1		X	1
01	X		X	
11	1		1	X
10	1			

$F = y'z' + w'x'z' + xyz$

$F = w'x'z' + xyz + y'z'$

Q13 (b) $F = W'X'Z' + XYZ + Y'Z'$



(Made on Circuitverse)

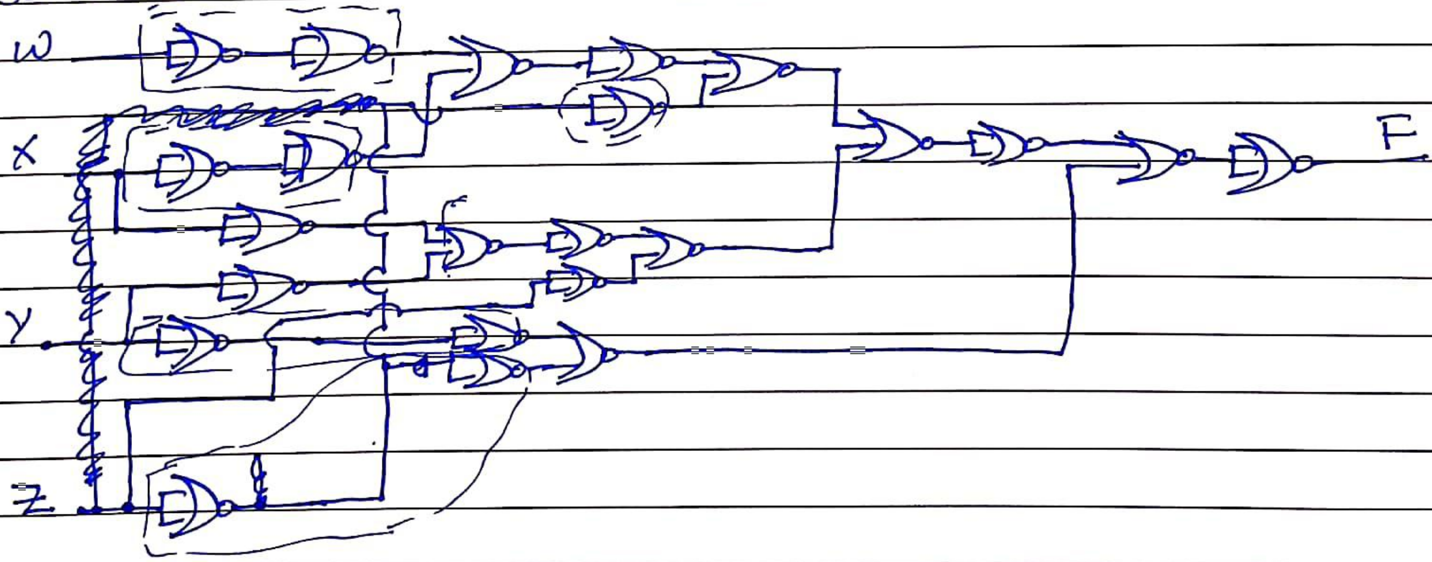
(c) Original: $F = w'x'y'z' + w'x'yz' + wx'y'z' + wxy'z' + wxyz$

Terms = 5 Literals = 20

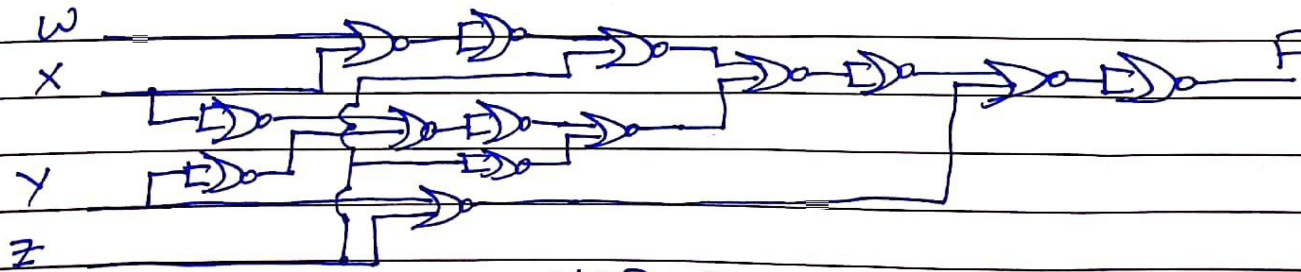
Reduced: $F = w'x'z' + xyz + y'z'$

Terms = 3 Literals = 8

(d) From (b):



⇒



NOR Representation