



**Habib University**  
shaping futures

# Longest Common Subsequence (LCS): an example

**CS 412**

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Source: *CLRS*

The LCS problem has an optimal-substructure property, however, as the following theorem shows. As we shall see, the natural classes of subproblems correspond to pairs of “prefixes” of the two input sequences. To be precise, given a sequence  $X = \langle x_1, x_2, \dots, x_m \rangle$ , we define the  $i$ th *prefix* of  $X$ , for  $i = 0, 1, \dots, m$ , as  $X_i = \langle x_1, x_2, \dots, x_i \rangle$ . For example, if  $X = \langle A, B, C, B, D, A, B \rangle$ , then  $X_4 = \langle A, B, C, B \rangle$  and  $X_0$  is the empty sequence.

***Theorem 15.1 (Optimal substructure of an LCS)***

Let  $X = \langle x_1, x_2, \dots, x_m \rangle$  and  $Y = \langle y_1, y_2, \dots, y_n \rangle$  be sequences, and let  $Z = \langle z_1, z_2, \dots, z_k \rangle$  be any LCS of  $X$  and  $Y$ .

1. If  $x_m = y_n$ , then  $z_k = x_m = y_n$  and  $Z_{k-1}$  is an LCS of  $X_{m-1}$  and  $Y_{n-1}$ .
2. If  $x_m \neq y_n$ , then  $z_k \neq x_m$  implies that  $Z$  is an LCS of  $X_{m-1}$  and  $Y$ .
3. If  $x_m \neq y_n$ , then  $z_k \neq y_n$  implies that  $Z$  is an LCS of  $X$  and  $Y_{n-1}$ .

Let  $c[i, j]$  be the **length** of the longest common subsequence (LCS) of two sequences  $X_i$  and  $Y_j$ . If  $i = 0$  or  $j = 0$ , then  $c[i, j] = 0$ .

$$c[i, j] = \begin{cases} 0 & , \text{if } i = 0 \text{ or } j = 0 \\ c[i - 1, j - 1] + 1 & , \text{if } i, j > 0 \text{ and } x_i = y_j \\ \max(c[i, j - 1], c[i - 1, j]) & , \text{if } i, j > 0 \text{ and } x_i \neq y_j \end{cases}$$

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**Example:**  $X = \text{ABCB}$  and  $Y = \text{BDCAB}$ . We will fill-in the table of sub-problems, starting with the smallest optimal sub-problem.

	$j$	0	1	2	3	4	5
$i$		$y_j$	<b>B</b>	<b>D</b>	<b>C</b>	<b>A</b>	<b>B</b>
0	$x_i$						
1	<b>A</b>						
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4	<b>B</b>						

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Smallest sub-problems with an optimal solution.

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$i$	$y_j$		<b>B</b>	<b>D</b>	<b>C</b>	<b>A</b>	<b>B</b>
0	$x_i$	0	0	0	0	0	0
1	<b>A</b>	0	0	0	0	1	1
2	<b>B</b>	0	1	1	1	1	2
3	<b>C</b>	0	1	1	2	2	2
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1	<b>A</b>	0	0	0	0	1	1
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3	<b>C</b>	0	1	1	2	2	2
4	<b>B</b>	0	1	1	2	2	

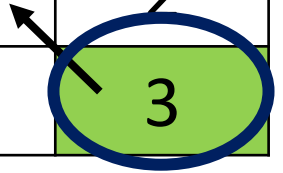


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0	$x_i$	0	0	0	0	0	0
1	<b>A</b>	0	0	0	0	1	1
2	<b>B</b>	0	1	1	1	1	2
3	<b>C</b>	0	1	1	2	2	2
4	<b>B</b>	0	1	1	2	2	3



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0	$x_i$	0	0	0	0	0	0
1	<b>A</b>	0	0	0	0	1	-1
2	<b>B</b>	0	1	1	-1	1	2
3	<b>C</b>	0	1	1	2	2	2
4	<b>B</b>	0	1	1	2	2	3