Weekly Challenge 10: Turing Machine

CS 212 Nature of Computation
Habib University

Fall 2023

1. Stay Put

A *stay-put* Turing machine is similar to an ordinary Turing machine, but the transition function has the form

$$\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{\text{L, R, S}\}.$$

At each point, the machine can move its head left or right, or let it stay in the same position.

Prove or disprove the following claim,

Claim 1. *The stay-put Turing machine is equivalent to the usual version.*

Solution:

Consider two Turing Machines; M_1 and M_2 where M_1 is an ordinary Turing Machine and M_2 is a stay-put Turing Machine.

Let $M_1 = \{Q, \Sigma, \Gamma, \delta_1, q_o, q_{\text{accept}}, q_{\text{reject}}\}$ where $\delta_1 : Q \times \Gamma \rightarrow Q \times \Gamma \times \{\text{L, R}\}$

Let $M_2 = \{Q, \Sigma, \Gamma, \delta_2, q_o, q_{\text{accept}}, q_{\text{reject}}\}$ where $\delta_2 : Q \times \Gamma \rightarrow Q \times \Gamma \times \{\text{L, R, S}\}$

We can show that M_1 and M_2 are equivalent by showing that M_1 can simulate M_2 and vice versa, that is, $L(M_1) = L(M_2)$.

1. M_2 can simulate M_1; $L(M_1) \subseteq L(M_2)$
   This is trivial as all languages that are accepted by M_2 which don't involve the head staying in its position at any given state will also be accepted by M_1. More formally, for any arbitrary language accepted by M_1, we can construct M_2 as follows: for every transition in M_1 of the form $(q, a) \rightarrow (p, b, L)$ or $(q, a) \rightarrow (p, b, R)$, we add the exact same transition in M_2. Therefore, M_2 can simulate M_1 and $L(M_1) \subseteq L(M_2)$.

2. M_1 can simulate M_2; $L(M_2) \subseteq L(M_1)$
   We can construct M_1 to simulate M_2 such that transitions of the form $(q, a) \rightarrow (p, b, L)$ or $(q, a) \rightarrow (p, b, R)$ are added the same to M_1. But for all transitions of the form $(q, a) \rightarrow (p, a, S)$, we add two transitions in M_1: $(q, a) \rightarrow (p, b, R)$ and $(p, b) \rightarrow (q, a, L)$ which essentially means we move one transition right, and then immediately one transition left which simulates being in place. Therefore M_1 can simulate M_2 and $L(M_2) \subseteq L(M_1)$.

Since $L(M_1) \subseteq L(M_2)$ and $L(M_2) \subseteq L(M_1)$, we can conclude that $L(M_1) = L(M_2)$ and therefore M_1 and M_2 are equivalent.

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