Kinematic Constraints

EE468/CE468: Mobile Robotics

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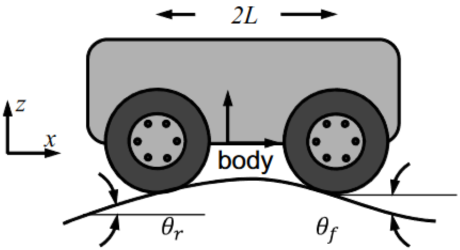
Form of the **Differential Kinematic** model

- Kinematic equations for WMRs are of the form

$$\begin{aligned} \dot{x} &= f(x, u) \\ w(x)\dot{x} &= 0. \end{aligned}$$

- **Assumption:** Wheel motions are already consistent with rigid body motion.
- **Assumption:** Idealized rolling wheel model.
  - Motion by rolling only.
  - No slip in the driving or lateral direction.



- Terrain contact is a **holonomic constraint**.
- It is of the form $c(x) = 0$, i.e. constraint is expressed in terms of state.
- For example, terrain contact constraint for situation in figure can be expressed as:

$$\xi(x_f) - z_f = 0.$$

Rolling without slipping

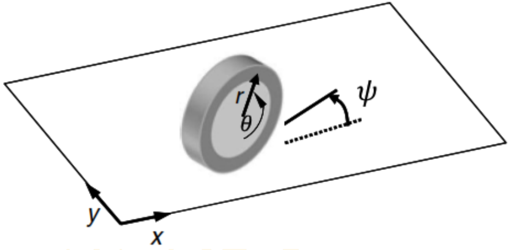


Figure: No slipping

- $\vec{v}_c \cdot \hat{y}_c = 0$
- This is constraint of type $c(x, \dot{x}) = 0$, and is called **nonholonomic constraint**.
- Could we not integrate the nonholonomic constraint to obtain a constraint in x?
  - These will be non-integrable expressions, for which closed-form integration solution will not exist.

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All velocity pairs from unconstrained ODE are not valid [2].

Recall general equation for wheel:

$$\mathbf{v}_c^w = \mathbf{v}_v^w + \omega_v^w \times \mathbf{r}_c^v,$$

with no wheel offset, no suspension, fixed contact point.

Say $\vec{w}$ is unallowed direction for a wheel, then

$$\vec{w}_c \cdot \vec{v}_c^w = 0$$

Assigning coordinates to terms, it can be rewritten as:

$$w_c^T \, v_c^w = 0$$
$$w_c^T \, (\mathbf{v}_v^w + \omega_v^w \times \mathbf{r}_c^v) = 0$$

Motion due to translational velocity in disallowed direction is canceled by that due to rotational velocity.

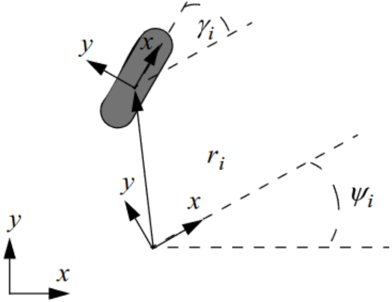

Figure: Wheel Model

- $r_c^v = \begin{bmatrix} x_c & y_c & 0 \end{bmatrix}^T$
- $w_c = {}^v\hat{y}_c = \begin{bmatrix} -\sin\gamma & \cos\gamma \end{bmatrix}^T$
- $\omega_v^w = \begin{bmatrix} 0 & 0 & \omega \end{bmatrix}^T$
- $\omega_v^w \times r_c^v = \omega \begin{bmatrix} -y_c & x_c & 0 \end{bmatrix}^T$
- $w_c^T(\omega_v^w \times r_c^v) = \omega(y_c \sin\gamma + x_c \cos\gamma) = \omega(r_c^v \cdot {}^v\hat{x}_c)$
- $\begin{bmatrix} -\sin\gamma & \cos\gamma & (r_c \cdot \hat{x}_c) \end{bmatrix} \begin{bmatrix} v_x & v_y & \omega \end{bmatrix}^T = 0$

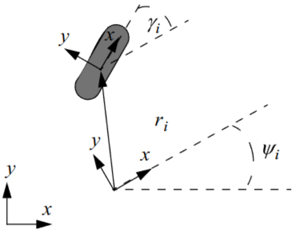

Figure: Wheel Model

- $r_c^v = \begin{bmatrix} x_c & y_c & 0 \end{bmatrix}^T$
- $w_c = {}^wR_v \, {}^v\hat{y}_c = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix} \begin{bmatrix} -\sin\gamma & \cos\gamma \end{bmatrix}^T$
- $\omega_v^w \times r_c^v = \omega \begin{bmatrix} -y_c & x_c & 0 \end{bmatrix}^T$
- $w_c^T(\omega_v^w \times r_c^v) = \omega(y_c \sin\gamma + x_c \cos\gamma) = \omega(r_c^v \cdot {}^v\hat{x}_c)$
- $\begin{bmatrix} -\sin(\gamma+\psi) & \cos(\gamma+\psi) & (r_c \cdot \hat{x}_c) \end{bmatrix} \begin{bmatrix} {}^wv_x & {}^wv_y & {}^w\omega \end{bmatrix}^T = 0$

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Degrees of freedom

- Recall that pose of wheeled mobile robot is expressed as (x, y, θ).

Degrees of Freedom

Degrees of freedom of a robot is the dimension of the space of all poses achievable by a robot.

Controllable Degrees of Freedom

Number of Degrees of freedom of a robot that are controllable, i.e. there is an actuator for such a DOF.

Holonomic Robot [1, Chapter 3]

A robot is **holonomic** iff CDOF = DOF.

- A holonomic robot does not have any nonholonomic kinematic constraints.
- A nonholonomic constraint is a constraint on the derivatives of pose variables that cannot be integrated to a corresponding constraint on the pose variables.
- The no-sliding constraint is nonholonomic.

Robots with high CDOF to DOF ratio are easier to control.

- **Holonomic:** CDOF = DOF
- **Nonholonomic:** CDOF < DOF
- **Redundant:** CDOF > DOF

Nonholonomic robot is not limited in achieving global poses.

- A nonholonomic robot is restrained in its local movements, i.e. it cannot instantaneously move in any direction.
- But, it may be possible to achieve all poses, e.g. parallel parking.
- A nonholonomic robot will have a complicated trajectory with continuous position and discontinuous velocity to achieve every pose in its space.

Joints positions don't uniquely determine robot pose.

- For nonholonomic robots, it's not enough to know distance traveled per wheel, but we need speed of each wheel as a function of time.
- Integrated over time to obtain robot pose, which turns out to be a major source of uncertainty.
- This is not the case for robot arms.
- *A system is nonholonomic when closed trajectories in its configuration space may not return it to its original state.*
- We use differential kinematic model.

Need to know how movement was executed for wheeled robots.

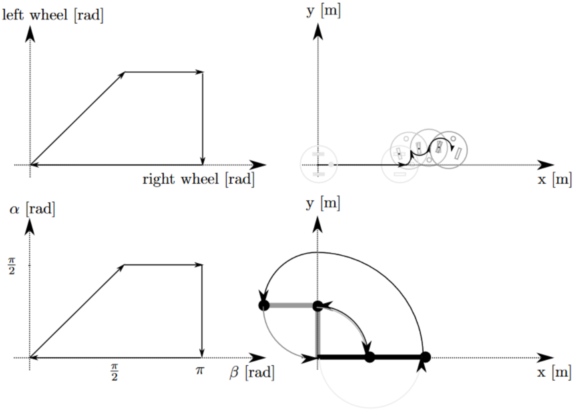

Figure 3.4. Configuration or joint space (left) and workspace or operational space (right) for a non-holonomic mobile robot (top) and a holonomic manipulator (bottom). Closed trajectories in configuration space result in closed trajectories in the workspace if the robot's kinematics is holonomic.

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[1] Nikolaus Correll, Bradley Hayes, Christoffer Heckman, and Alessandro Roncone.
Introduction to autonomous robots: mechanisms, sensors, actuators, and algorithms.
Mit Press, 2022.

[2] Alonzo Kelly.
Mobile robotics: mathematics, models, and methods.
Cambridge University Press, 2013.