

# Quiz 10C: Compute with Turing Machine

CS 212 Nature of Computation

Habib University — Fall 2023

Total Marks: 10  
Duration: 10 minutes

Date: October 26, 2023  
Time: 830–840h

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1. (10 points) So far, we have encountered Turing machines which solve decision problems, i.e., we are interested in whether they accept or reject an input. For other types of problems, we are interested in the content of the tape when the Turing machine halts. Let us call these, “compute” problems.

We know that if a machine can perform the Boolean operations, then it can perform all the computations that we are used to with contemporary computers.

*Show that a Turing machine can compute the NOT of an  $n$ -bit string.*

That is, if the tape contains an  $n$ -bit string,  $a = a_1a_2 \dots a_n$ , a Turing machine can read the tape and halt leaving only an  $n$ -bit string,  $b = b_1b_2 \dots b_n$ , on the tape. Each bit of  $b$  is the bitwise NOT of the corresponding bit of  $a$ , i.e.,  $b_i = \neg a_i$  for  $1 \leq i \leq n$ .

**Solution:** We provide a proof by construction, i.e., we define a Turing machine for the computation.

*Proof.* The machine reads the input and writes the corresponding output one cell after the input. The input bits are blanked out (replaced with  $\sqcup$ ) as they are processed. When the input is exhausted, the tape contains only the result surrounded by  $\sqcup$ .

We provide an intermediate- or implementation- level description of the Turing machine. The machine starts with the head at  $a_1$ . We use “skip” to denote one or more moves of the head that do not replace any tape symbols.

1. Remember the current symbol, call it  $a_i$ , and replace it with  $\sqcup$ .
2. Move right (next bit of  $a$  to process).
3. If the current symbol is  $\sqcup$  (input is exhausted)
  - (a) Skip right to find  $\sqcup$  (next free space for output).
  - (b) Write  $\neg a_i$ .
  - (c) Accept.
4. If the current symbol is 0 or 1 (input remains to be processed)
  - (a) Skip right to find  $\sqcup$  (end of  $a$ ). Skip right to find  $\sqcup$  (next free space for output).
  - (b) Write  $\neg a_i$ .

- (c) Skip left to find  $\perp$  (end of input). Skip left to find  $\perp$  (just-processed bit of  $a$ ).
- (d) Move right.

5. Repeat.



Sample Solution