Theorem 7.1.5 (Master Theorem for this course upto this point)

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THEOREM 7.1.5

Equivalent Statements

If A is an $n \times n$ matrix, and if $T_A: R^n \longrightarrow R^n$ is multiplication by A, then the following are equivalent.

(a) A is invertible.

(b) $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.

(c) The reduced row-echelon form of A is I_n.

(d) A is expressible as a product of elementary matrices.

(e) $A\mathbf{x} = \mathbf{b}$ is consistent for every $n \times 1$ matrix $\mathbf{b}$.

(f) $A\mathbf{x} = \mathbf{b}$ has exactly one solution for every $n \times 1$ matrix $\mathbf{b}$.

(g) $\det(A) \neq 0$.

(h) The range of T_A is R^n.

(i) T_A is one-to-one.

(j) The column vectors of A are linearly independent.

(k) The row vectors of A are linearly independent.

(l) The column vectors of A span R^n.

(m) The row vectors of A span R^n.

(n) The column vectors of A form a basis for R^n.

(o) The row vectors of A form a basis for R^n.

(p) A has rank n.

(p) *A has rank n.*

(q) *A has nullity 0.*

(r) *The orthogonal complement of the nullspace of A is R^n.*

(s) *The orthogonal complement of the row space of A is {**0**}.*

(t) *$A^T A$ is invertible.*

(u) *$\lambda = 0$ is not an eigenvalue of A.*