Localization

EE468/CE468: Mobile Robotics

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Robot motion is imprecise. Odometry is not enough.

Figure: Motion Error. Time lapse of robot executing the same square motion on a carpeted floor.

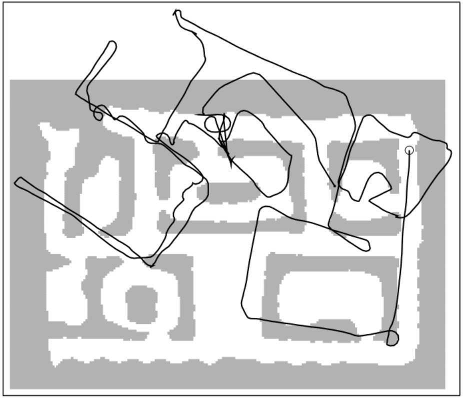
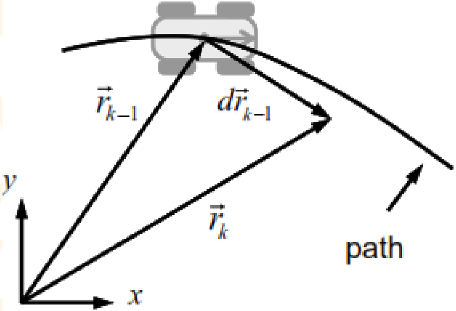
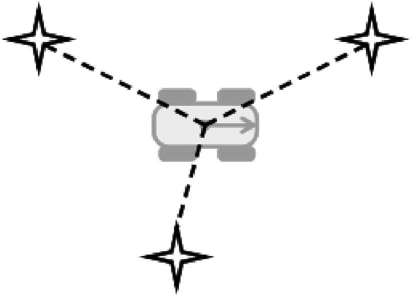


Figure: Dead Reckoning Error. Pose is obtained by integrating odometry data.



Dead Reckoning



Pose Fixing

Most practical systems employ both.

Attribute	Dead Reckoning	Pose Fixing
Process	Integration	Nonlinear System Solvers
Initial Conditions	Required	Not required
Errors	(Often) Time Dependent	Position Dependent
Update Frequency	Determined by Required Accuracy	Determined only by the application
Error Propagation	Dependent on Previous Estimates	Independent of Previous Estimates
Requires a map.	No	Yes

Figure: Comparison of two methods

Localization problems, by type of available knowledge: [2]

Local Localization

- Initial robot pose is known.
- Pose error is typically assumed to be small.
- Pose uncertainty is usually modeled by unimodal distribution.

Global Localization

- Initial robot pose is unknown.
- Boundedness of pose error is unreasonable here.
- Pose uncertainty cannot be modeled by unimodal distributions.

Localization problems, by type of available knowledge:

Kidnapped Robot Problem

- During operation, robot could get kidnapped and transported to new location.
- Robot does not know it has been kidnapped.
- Different from global localization, as robot pose is inaccurate but it believes it to be accurate.
- Tests a robot's ability to recover from global localization failures.

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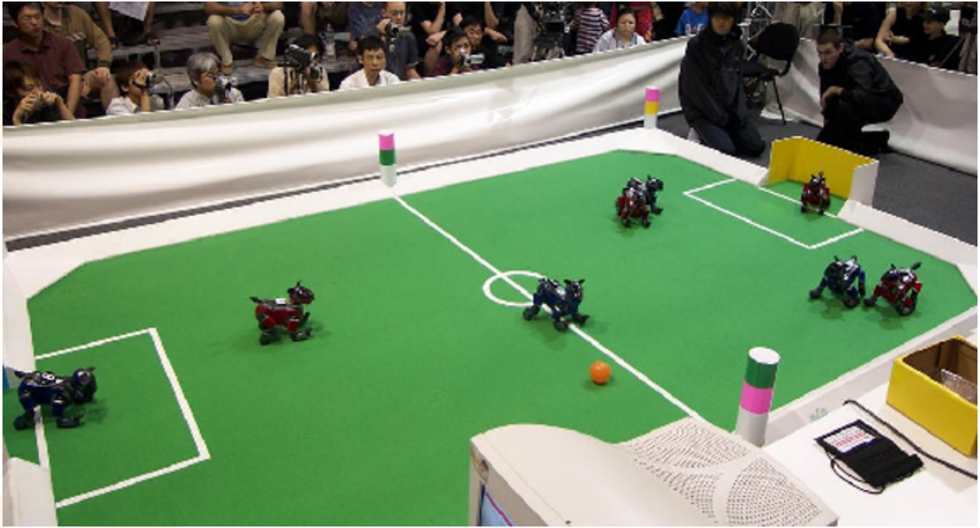
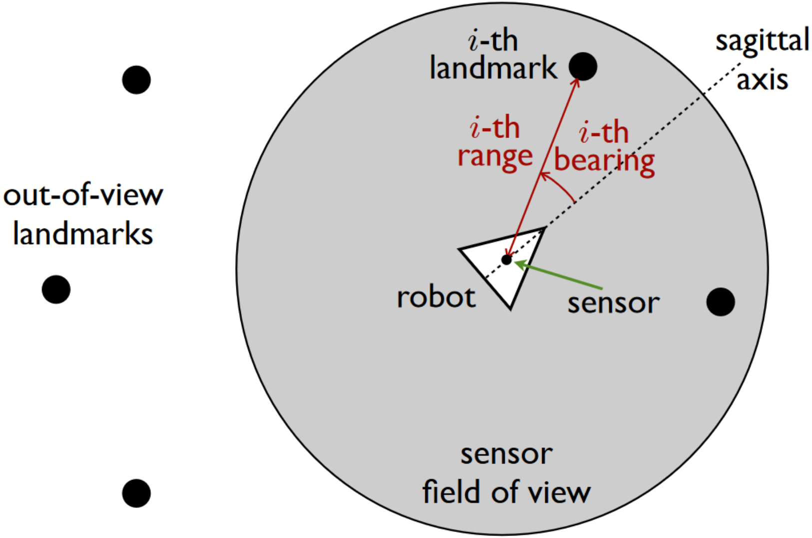


Our problem setup

- Unicycle-type robot
- Equipped with a sensor that measures range (relative distance) and bearing (relative orientation) to some landmarks.
- Landmarks may be artificial or natural.
- Positions of the landmarks are fixed and known, (x_{l_i}, y_{l_i}) for $i = 1, \cdots, n$. Map is known.
- At any time, the robot can see a subset of landmarks that are in view of its sensors.



i-th
landmark
sagittal
axis
i-th
range
i-th
bearing
out-of-view
landmarks
robot
sensor
sensor
field of view

The motion model is:

- If (x, y, θ) is the pose of the robot, then

$$\begin{bmatrix} x_k \\ y_k \\ \theta_k \end{bmatrix} = \begin{bmatrix} x_{k-1} + v_k T \cos\theta_{k-1} \\ y_{k-1} + v_k T \sin\theta_{k-1} \\ \theta_{k-1} + \omega_k T \end{bmatrix} + w_k.$$

- The input, $(v_k, \omega_k)^T$, denotes the linear velocity and angular velocity respectively.

- $w_k \in \mathbb{R}^{3\times1}$ is a random vector from a zero-mean Gaussian distribution with covariance Q_k.

The measurement model is:

- At time k, we receive measurement z_k, which contains range and bearing readings to m_k landmarks.

$$z_k = \begin{bmatrix} h_1(\mathbf{x}_k, l(1)) \\ h_2(\mathbf{x}_k, l(2)) \\ \vdots \\ h_{m_k}(\mathbf{x}_k, l(m_k)) \end{bmatrix} + n_k,$$

$$h_i(x_k, j) = \begin{bmatrix} \sqrt{\left(x_k - x_{l_j}\right)^2 + \left(y_k - y_{l_j}\right)^2} \\ \operatorname{arctan2}\left(y_{l_j} - y_k, x_{l_j} - x_k\right) - \theta_k \end{bmatrix},$$

where l associates the reading to a landmark index, and n_k is a random vector from a zero-mean Gaussian with covariance R_k.

- m_k can vary with k.

Crank the EKF machinery

- Linearize the process and measurement models:

$$F_k = \left.\frac{\partial f}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{x}_{k-1}}$$

$$= \begin{bmatrix} 1 & 0 & -v_k T \sin\theta \\ 0 & 1 & v_k T \cos\theta \\ 0 & 0 & 1 \end{bmatrix}_{\mathbf{x}=\hat{x}_{k-1}}$$

$$= \begin{bmatrix} 1 & 0 & -v_k T \sin\hat{\theta}_{k-1} \\ 0 & 1 & v_k T \cos\hat{\theta}_{k-1} \\ 0 & 0 & 1 \end{bmatrix}$$

Crank the EKF machinery (2)

- Linearize the process and measurement models:

$$H_k = \begin{bmatrix} H_k^1 \\ \vdots \\ H_k^{m_k} \end{bmatrix}$$

$$H_k^i = \left.\frac{\partial h_i(x_k, j)}{\partial \mathbf{x}}\right|_{\mathbf{x}=\bar{x}_k}$$

$$= \begin{bmatrix} \frac{\bar{x}_k - x_{l_j}}{\sqrt{\left(\bar{x}_k - x_{l_j}\right)^2 + \left(\bar{y}_k - y_{l_j}\right)^2}} & \frac{\bar{y}_k - y_{l_j}}{\sqrt{\left(\bar{x}_k - x_{l_j}\right)^2 + \left(\bar{y}_k - y_{l_j}\right)^2}} & 0 \\ -\frac{\bar{y}_k - y_{l_j}}{\left(\bar{x}_k - x_{l_j}\right)^2 + \left(\bar{y}_k - y_{l_j}\right)^2} & \frac{\bar{x}_k - x_{l_j}}{\left(\bar{x}_k - x_{l_j}\right)^2 + \left(\bar{y}_k - y_{l_j}\right)^2} & -1 \end{bmatrix}$$

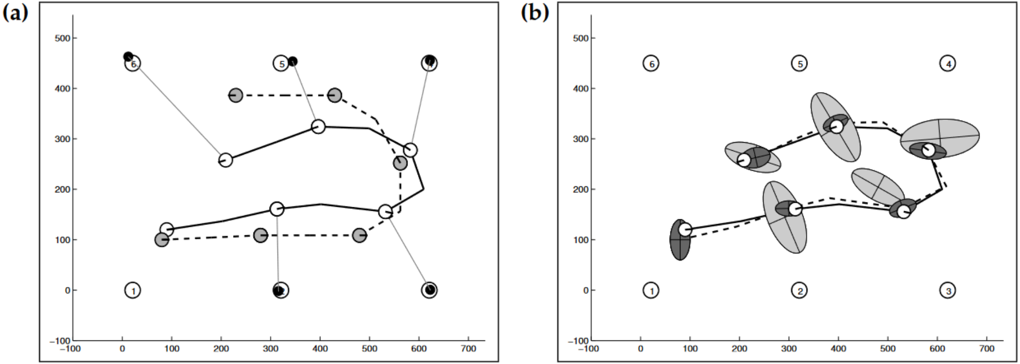
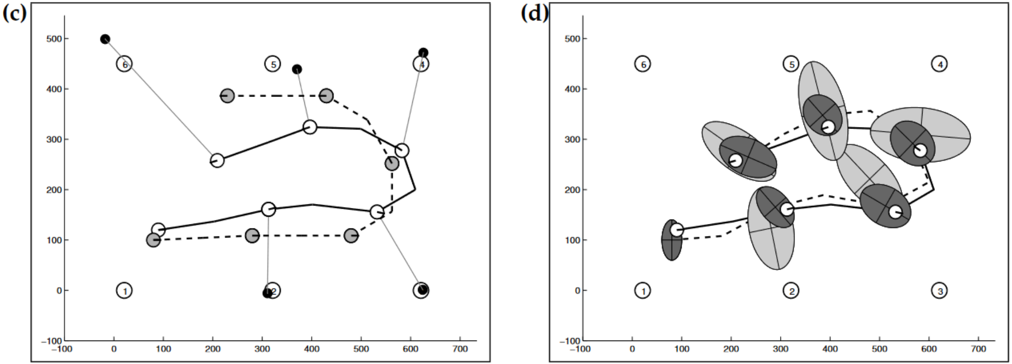


Figure 7.11 EKF-based localization with an accurate (upper row) and a less accurate (lower row) landmark detection sensor. The dashed lines in the left panel indicate the robot trajectories as estimated from the motion controls. The solid lines represent the true robot motion resulting from these controls. Landmark detections at five locations are indicated by the thin lines. The dashed lines in the right panels show the corrected robot trajectories, along with uncertainty before (light gray, $\bar{\Sigma}_t$) and after (dark gray, Σ_t) incorporating a landmark detection.



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Which landmark corresponds to a particular measurement?

- **Data association** is the process of associating uncertain measurements to known features (landmarks).
- Landmarks have similar properties, making them good features but difficult to distinguish.
- Estimate an association map, which associates a landmark with each measurement, i.e. $a(i) = j$, where jth landmark is associated with ith measurement.
- **Idea:** Associate a measurement to the landmark that minimizes the magnitude of difference between measurement and expected measurement, $z_k - h(x_k)$.

Euclidean distance doesn't consider uncertainty in dimensions.

- **Use Mahalanobis Distance:** For the ith measurement and jth landmark,

$$d_{ij}^2 = \nu_{ij} S_{ij}^{-1} \nu_{ij}^T,$$
$$\nu_{ij} = z_k^i - h_i(\bar{x}_k, j)$$
$$S_{ij} = H_k(i,j) \bar{\Sigma}_k H_k(i,j)^T + Q_k$$

- For each i, loop over all j or subset of j, and associate

$$a(i) = \arg\min_j d_{ij}^2.$$

- https://youtu.be/K-Hk1haIrXE?si=aFrd23JpmfUW6Nh8

- False data association
- Equally likely candidates
- KF are not robust to data association errors. One mistake can cause underestimation of uncertainty in a state, chain reaction of misassociations, and complete failure (filter divergence).
- Solutions:
  - Gating
  - Choosing unique and sufficiently apart landmarks
  - Better techniques (MHT)

Validation Gate Technique [1, 5.3.4.7]

- Measurements are only incorporated in KF if Mahalanobis distance (d^2) to at least one landmark is within confidence threshold:

$$\{z : d^2 \leq \gamma\}$$

- γ is obtained from the inverse CDF of chi-square distribution for confidence level α
- d^2 is chi-square distributed.
- Validation gate is a region of acceptance such that $100(1-\alpha)\%$ of true measurements are rejected.

Last thoughts

- Note that z and $h(x)$ should both be in the same frame and coordinates for you to be able to compute difference.

- **Reading:** Siegwart 5.6.8.4

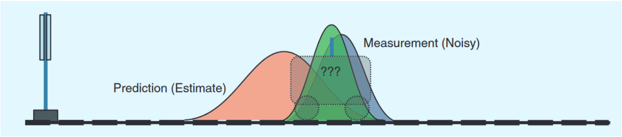

Figure: Green Gaussian is obtained by fusing the measurement and prediction Gaussians.

- What if this assumption is not true? Distribution is not Gaussian or unimodal?
- Bayesian Estimation provides a richer framework that can address global localization or kidnapped robot problem.

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[1] **Alonzo Kelly.**
Mobile robotics: mathematics, models, and methods.
Cambridge University Press, 2013.

[2] **Sebastian Thrun, Wolfram Burgard, and Dieter Fox.**
Probabilistic robotics.
2006.