

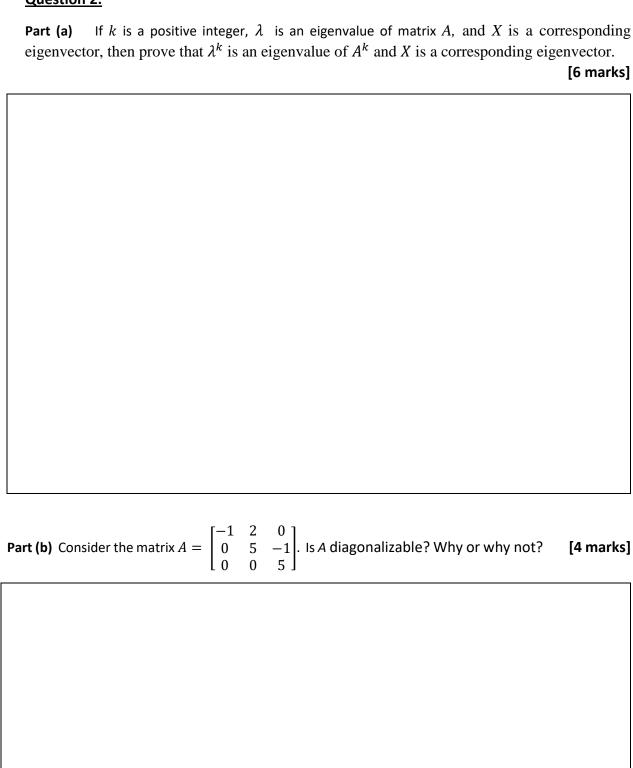
LINEAR ALGEBRA

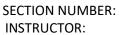
FINAL EXAM [Total Marks: 100]

(SPRING 2022)

Question 1: [10 Mar	rksj
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Question 2:







Question 3: [10 Marks]

Gram-Schmidt process to transform the standard basis $S = \{1, x, x^2\}$ into an orthonormal basis.

Let the vector space P_2 have the inner product $\langle \boldsymbol{p}, \boldsymbol{q} \rangle = \int_{-1}^1 p(x) q(x) dx$, apply the



Question 4: [10 Marks]

Indicate whether each statement is always True, or sometimes False. Justify your answer by giving a logical argument or a counterexample:

(a) The intersection of two subspaces of a vector space V is also a subspace of V .
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(b) A set that contains the zero vector is linearly dependent.
(c)	Every nonzero finite dimensional inner product space has an orthonormal basis.
(0)	2 Every Honzero Hinte dimensional liner produce space has an orthonormal basis.
(d) If $Span(S_1) = Span(S_2)$, then $S_1 = S_2$.
10) If W is a set of one or more vectors from a vector space \emph{V} , and if $\emph{k} \emph{u} + \emph{v}$ is a vecto
(0)	in W for all vectors u and v in W and for all scalars k , then W is a subspace of V .
	in W for all vectors & and V in W and for all sealars to, then W is a subspace of V.



vectors, then S is a	 	- C. J. 10 III Cu	,	
Part (b) Prove: If Wi				
Part (b) Prove: If W i $\dim(W) \leq \dim(V)$; m				
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Question 6						
Given that A is diagonalizable and that $A^5=0$, show that A must also be a zero matrix.						
[Warning: Do not assume that $(A^n = 0) \Rightarrow (A = 0)$ holds true for matrices just because it holds true for real numbers. That is a major error.] [10 marks]						

Question 7

x =	$\langle x, x \rangle$ for all x	V which has a $\in V$, then $ \langle x \rangle $				
	same vector s $\ \leq \ a\ + \ b\ $		t (a) above,	show that fo	r any $oldsymbol{a},oldsymbol{b}\in V$, we have
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			t (a) above, s	show that fo	r any $oldsymbol{a},oldsymbol{b}\in V$, we have
			t (a) above, s	show that fo	r any $oldsymbol{a}, oldsymbol{b} \in V$, we have
			t (a) above, s	show that fo	r any a, b ∈ <i>V</i>	, we have
			t (a) above, s	show that fo	r any a, b ∈ V	, we have
			t (a) above, s	show that fo	r any a, b ∈ V	, we have
			t (a) above, s	show that fo	r any a, b ∈ V	, we have

Question 8

(a) Prove that if $Ax=b$ is consistent for every $n\times 1$ matrix b , then the $n\times n$ matrix A is invertible. [7 marks]	
(b) Prove that if an $n \times n$ matrix A is invertible, then $Ax = b$ is consistent for every $n \times 1$ matrix b . [3 marks]	

Question 9

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	(a) If $T\left(\underline{e_1}\right) = (1,1)$, $T\left(\underline{e_2}\right) = (3,0)$, and $T\left(\underline{e_3}\right) = (4,7)$ then find $T(2,4,6)$, given that T is a linear transformation. [3 marks]
	(b) For a general $(x, y, z) \in \mathbb{R}^3$, find $T(x, y, z)$. [3 marks]
	(c) Find the matrix of the linear transformation in the parts above. [4 marks]

Question 10

Let $A\underline{x} = 0$ be a homogenous system where A is some $m \times n$ matrix. Prove that $\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ is a

solution of the system if and only if \underline{x} is orthogonal to every row vector of A (using the Euclidean inner product). [10 marks]

