

## Quiz 12: Reducibility

CS 212 Nature of Computation

Habib University — Fall 2023

Total Marks: 10  
Duration: 15 minutes

Date: November 14, 2023  
Time: 830–845h

Student ID: \_\_\_\_\_

Student Name: \_\_\_\_\_

1. (10 points) We are given that the language  $A$  reduces to the language  $B$ . That is, a decider,  $R$ , for  $B$  can be used to make a decider,  $S$ , for  $A$ .

Show that the language  $A'$  reduces to the language  $B'$ . That is, a decider,  $R1$ , for  $B'$  can be used to make a decider,  $S1$ , for  $A'$ .

**Solution:** We provide a proof by construction. We observe that  $R1$  can be constructed from  $R$  by simply inverting the output of  $R$ . Similarly  $S1$  can be constructed from  $S$ ;  $R$  from  $R1$ ; and  $S$  from  $S1$ .

*Proof.* Construct  $S1$  from  $R1$  as follows:

- Invert the output of  $R1$  to obtain  $R$ .
- Use  $R$  to construct  $S$  as given.
- Output the inverse of  $S$ .

□