Matrix Chain Multiplication

CS-6th

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Matrix Multiplication

- Let A be an n by m matrix and B be an m by p matrix, then C=AB is an n by p matrix.
- Total number of multiplications= nmp

Example (parenthesizations)

- $Z=A_1XA_2XA_3$
- =(A1XA2)XA3
- =A1X(A2XA3)

$$\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\7 & 8 & 9\end{bmatrix} X \begin{bmatrix}10 & 11 & 12\\13 & 14 & 15\\16 & 17 & 18\end{bmatrix} X \begin{bmatrix}19 & 20\\21 & 22\\23 & 24\end{bmatrix}$$

Dimensions=[(3X3)*(3X3)]*(3X2)=(3X3)*(3X2)=3X2

#Mult-operations=27+18=45

Example (parenthesizations)

- $Z=A_1XA_2XA_3$
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- =A1X(A2XA3)

$$\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\7 & 8 & 9\end{bmatrix} X \begin{bmatrix}10 & 11 & 12\\13 & 14 & 15\\16 & 17 & 18\end{bmatrix} X \begin{bmatrix}19 & 20\\21 & 22\\23 & 24\end{bmatrix}$$

Dimensions=(3X3)*[(3X3)*(3X2)]=(3X3)*(3X2)=3X2

#Mult-operations=18+18=36

Example (4 matrices)

$$
\begin{aligned}
&(A_1(A_2(A_3A_4))) \ , \\
&(A_1((A_2A_3)A_4)) \ , \\
&((A_1A_2)(A_3A_4)) \ , \\
&((A_1(A_2A_3))A_4) \ , \\
&(((A_1A_2)A_3)A_4) \ .
\end{aligned}
$$

$$m[i,j] = \begin{cases} 0 & \text{if } i = j\ , \\ \min\{m[i,k] + m[k+1,j] + p_{i-1}p_k p_j : i \le k < j\} & \text{if } i < j\ . \end{cases}$$

$$m[i,j] = \begin{cases} 0 & \text{if } i = j\ , \\ \min\{m[i,k] + m[k+1,j] + p_{i-1}p_k p_j : i \le k < j\} & \text{if } i < j\ . \end{cases}$$

A1 X A2 X A3 x A4

p0*p1*p1*p2*p2*p3*p3*p4

2*3 * 3*4 * 4*2 * 2*2

j →

i ↓

	1	2	3	4
1	0			
2		0		
3			0	
4				0

S matrix

	1	2	3	4
1				
2				
3				
4				

$$m[i,j] = \begin{cases} 0 & \text{if } i = j\ , \\ \min\{m[i,k] + m[k+1,j] + p_{i-1}p_k p_j : i \le k < j\} & \text{if } i < j\ . \end{cases}$$

A1 X A2 X A3 x A4

p0*p1Xp1*p2Xp2*p3Xp3*p4

2*3 * 3*4 * 4*2 * 2*2

j →

i ↓

	1	2	3	4
1	0	24		
2		0	24	
3			0	16
4				0

S matrix

	1	2	3	4
1		1		
2			2	
3				3
4				

$$m[i,j] = \begin{cases} 0 & \text{if } i = j\ , \\ \min\{m[i,k] + m[k+1,j] + p_{i-1}p_k p_j : i \le k < j\} & \text{if } i < j\ . \end{cases}$$

A1 X A2 X A3 x A4

p0*p1Xp1*p2Xp2*p3Xp3*p4

2*3 * 3*4 * 4*2 * 2*2

j →

i ↓

	1	2	3	4
1	0	24	36	
2		0	24	
3			0	16
4				0

min{ m[1,3]=for k=1{0+24+2*3*2=36}
m[1,3]=for k=2{24+0+2*4*2=40}

$$m[i,j] = \begin{cases} 0 & \text{if } i = j\ , \\ \min\{m[i,k] + m[k+1,j] + p_{i-1}p_k p_j : i \le k < j\} & \text{if } i < j\ . \end{cases}$$

A1 X A2 X A3 x A4

p0*p1Xp1*p2Xp2*p3Xp3*p4

2*3 * 3*4 * 4*2 * 2*2

j →

i ↓

	1	2	3	4
1	0	24	36	
2		0	24	36
3			0	16
4				0

min{ m[2,4]=for k=2{0+16+3*4*2=40}
m[2,4]=for k=3{24,0,3*2*2=36}

$$m[i,j] = \begin{cases} 0 & \text{if } i = j\ , \\ \min\{m[i,k] + m[k+1,j] + p_{i-1}p_k p_j : i \le k < j\} & \text{if } i < j\ . \end{cases}$$

A1 X A2 X A3 x A4

p0*p1Xp1*p2Xp2*p3Xp3*p4

2*3 * 3*4 * 4*2 * 2*2

j →

i ↓

	1	2	3	4
1	0	24	36	
2		0	24	36
3			0	16
4				0

S matrix

	1	2	3	4
1		1	1	
2			2	3
3				3
4				

$$m[i,j] = \begin{cases} 0 & \text{if } i = j\ , \\ \min\{m[i,k] + m[k+1,j] + p_{i-1}p_k p_j : i \le k < j\} & \text{if } i < j\ . \end{cases}$$

A1 X A2 X A3 x A4

p0*p1Xp1*p2Xp2*p3Xp3*p4

2*3 * 3*4 * 4*2 * 2*2

j →

i ↓

	1	2	3	4
1	0	24	36	44
2		0	24	36
3			0	16
4				0

min{ m[1,4]=for k=1{0+36+2*3*2=48}
m[1,4]=for k=2{24+16+2*4*2=56}
m[1,4]=for k=3{36+0+2*2*2=44}

$$m[i,j] = \begin{cases} 0 & \text{if } i = j\ , \\ \min\{m[i,k] + m[k+1,j] + p_{i-1}p_k p_j : i \le k < j\} & \text{if } i < j\ . \end{cases}$$

A1 X A2 X A3 x A4

p0*p1Xp1*p2Xp2*p3Xp3*p4

2*3 * 3*4 * 4*2 * 2*2

j →

i ↓

	1	2	3	4
1	0	24	36	44
2		0	24	36
3			0	16
4				0

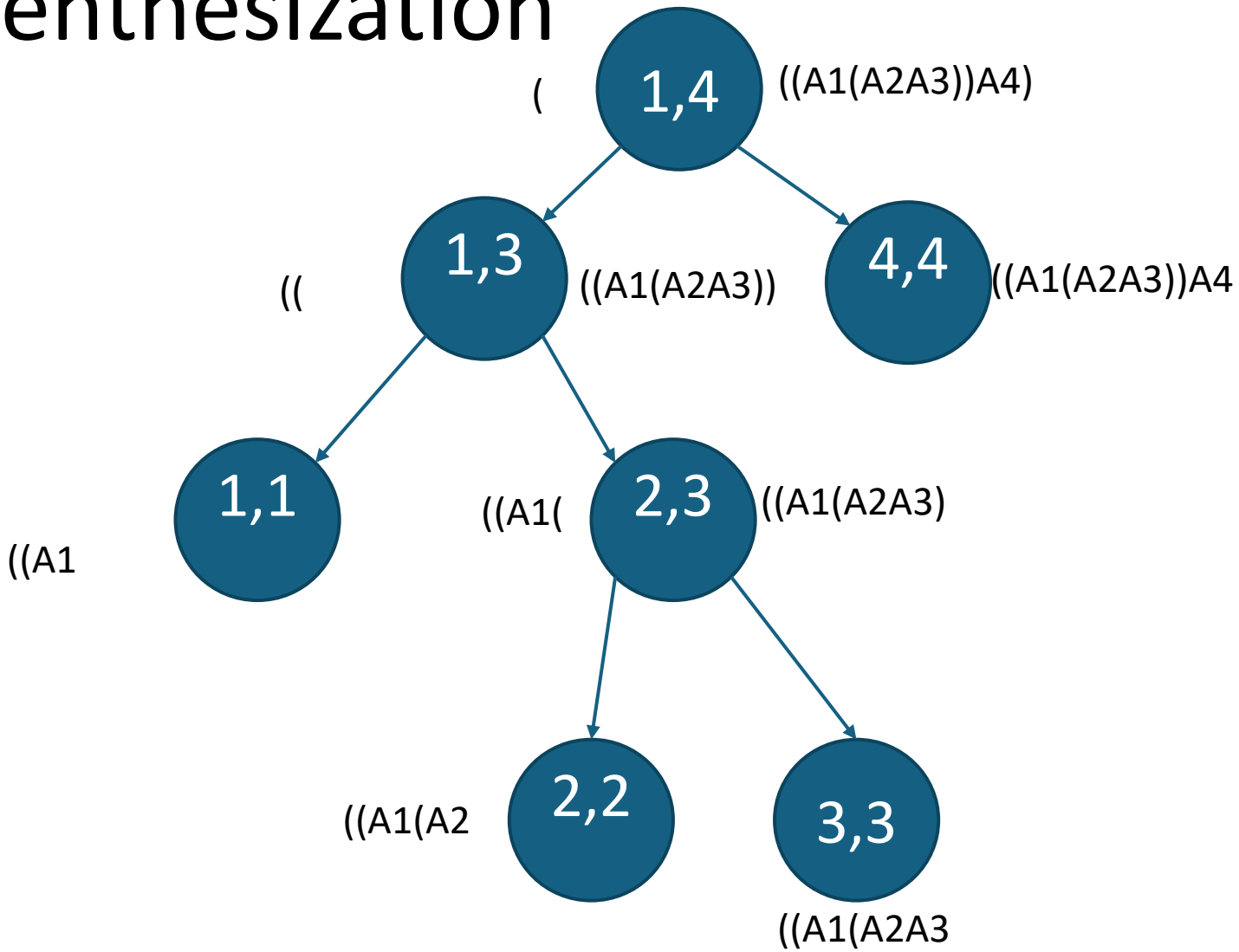
S matrix

	1	2	3	4
1		1	1	3
2			2	3
3				3
4				

The initial call PRINT-OPTIMAL-PARENS (s, 1, n) prints an optimal parenthesization of the full matrix chain product $A_1A_2\ldots A_n$

```
PRINT-OPTIMAL-PARENS(s, i, j)
1  if i == j
2       print "A"_i
3  else print "("
4       PRINT-OPTIMAL-PARENS(s, i, s[i, j])
5       PRINT-OPTIMAL-PARENS(s, s[i, j] + 1, j)
6       print ")"
```

Parenthesization



Parenthesization

A1 X A2 X A3 x A4

Output: ((A1(A2A3))A4)

j →

i ↓

	1	2	3	4
1	0	24	36	44
2		0	24	36
3			0	16
4				0

S matrix

	1	2	3	4
1		1	1	3
2			2	3
3				3
4				