Computational Intelligence

Unit # 13 -1

Being Imprecise

"As complexity rises, precise statements lose meaning and meaningful statements lose precision."

Albert Einstein

What is Fuzzy?

What is Fuzzy?

- Definition of Fuzzy
  - Fuzzy: “not clear, distinct, or precise; blurred”
- Examples:
  - It is partly cloudy.
  - He is very tall.
  - It is really hot today.

What is Fuzzy?

- Definition of Fuzzy
  - Fuzzy: “not clear, distinct, or precise; blurred”
- Examples:
  - It is partly cloudy.
  - He is very tall.
  - It is really hot today.

What is Fuzzy Logic?

- Definition of Fuzzy Logic
  - A form of knowledge representation suitable for notations that cannot be defined precisely but which depend upon their contexts.
- The term was coined by Lotfi Zadeh in 1965 with his mathematics of fuzzy set theory.

Successful Applications

- Automatic Control of dam gates for hydroelectric-power plants
- Camera aiming
- Compensation against vibration in camcorders
- Cruise-control for automobiles
- Controlling air-conditioning systems
- …..
- Many others

Application

- One of the most famous applications of fuzzy logic is that of the Sendai Subway system in Sendai, Japan. This control of the Nanboku line, developed by Hitachi, used a fuzzy controller to run the train all day long. This made the line one of the smoothest running subway systems in the world and increased efficiency as well as stopping time. This is also an example of the earlier acceptance of fuzzy logic in the east since the subway went into operation in 1988.

Application

- The most tangible applications of fuzzy logic control have appeared in commercial appliances. Specifically, but not limited to
  - heating ventilation and air conditioning (HVAC) systems.
- These systems use fuzzy logic thermostats to control the heating and cooling, this saves energy by making the system more efficient. It also keeps the temperature more steady than a traditional thermostat.

Fuzzy Logic

- With fuzzy logic, domains are characterized by linguistic terms, rather than by numbers.
  - For example, in the phrases *"it is partly cloudy",* or *"Stephan is very tall",* both *partly* and *very* are linguistic terms describing the *magnitude* of the fuzzy (or linguistic) variables *cloudy* and *tall.*
- The human brain has the ability to understand these terms, and infer from them.
- Fuzzy logic and fuzzy sets give the tools to also write software which enables computing systems to understand such vague terms, and to reason with these terms.

Examples of Linguistic Impression

- How was the weather like yesterday?
  - Oh! It was rainy with 98% humidity and hot with temperature of 35.5 deg C
  - Oh! It was <u>very</u> humid and <u>really</u> hot.

* Source: University Malaysian Pahang

Examples of Linguistic Impression (Cont'd)

- When you are at 10 meters from the junction start braking at 50% pedal level.
- When you are <u>near</u> the junction, start braking <u>slowly</u>.

* Source: University Malaysian Pahang

Uncertainty vs Fuzziness

- Certainty – degree of belief
  - There is a 50% probability of rain today
  - I am 30% sure the patient is suffering from pneumonia
- Fuzziness – the degree to which an item belongs to a category
  - The man is tall
  - Move the wheel slightly to the left
  - The patient’s lungs are highly congested

* Source: Susan Bridges @ Mississippi State University

Bivalent Logic

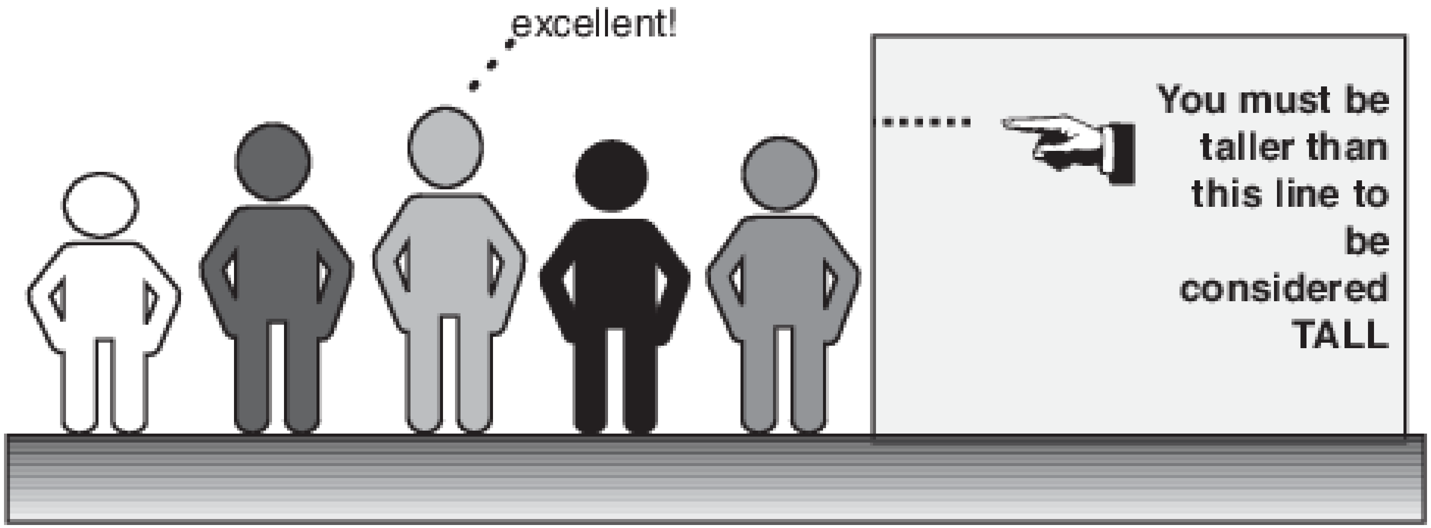
- In classical logic, propositions are either true or false. Such systems are known as **bivalent logics because they involve two logical values.**
- The logic employed in Bayesian reasoning and other probabilistic models is also bivalent: each fact is either true or false, but it is often unclear whether a given fact is true or false.
- Probability is used to express the likelihood that a particular proposition will turn out to be true.

Fuzzy Sets

- In traditional two-valued set theory, an element either belongs to a set or not. That is, set membership is precise.
- In fuzzy sets, an element belongs to a set to a degree, indicating the certainty (or uncertainty) of membership.

(a) (b)

Crisp boundaries



Fuzzy

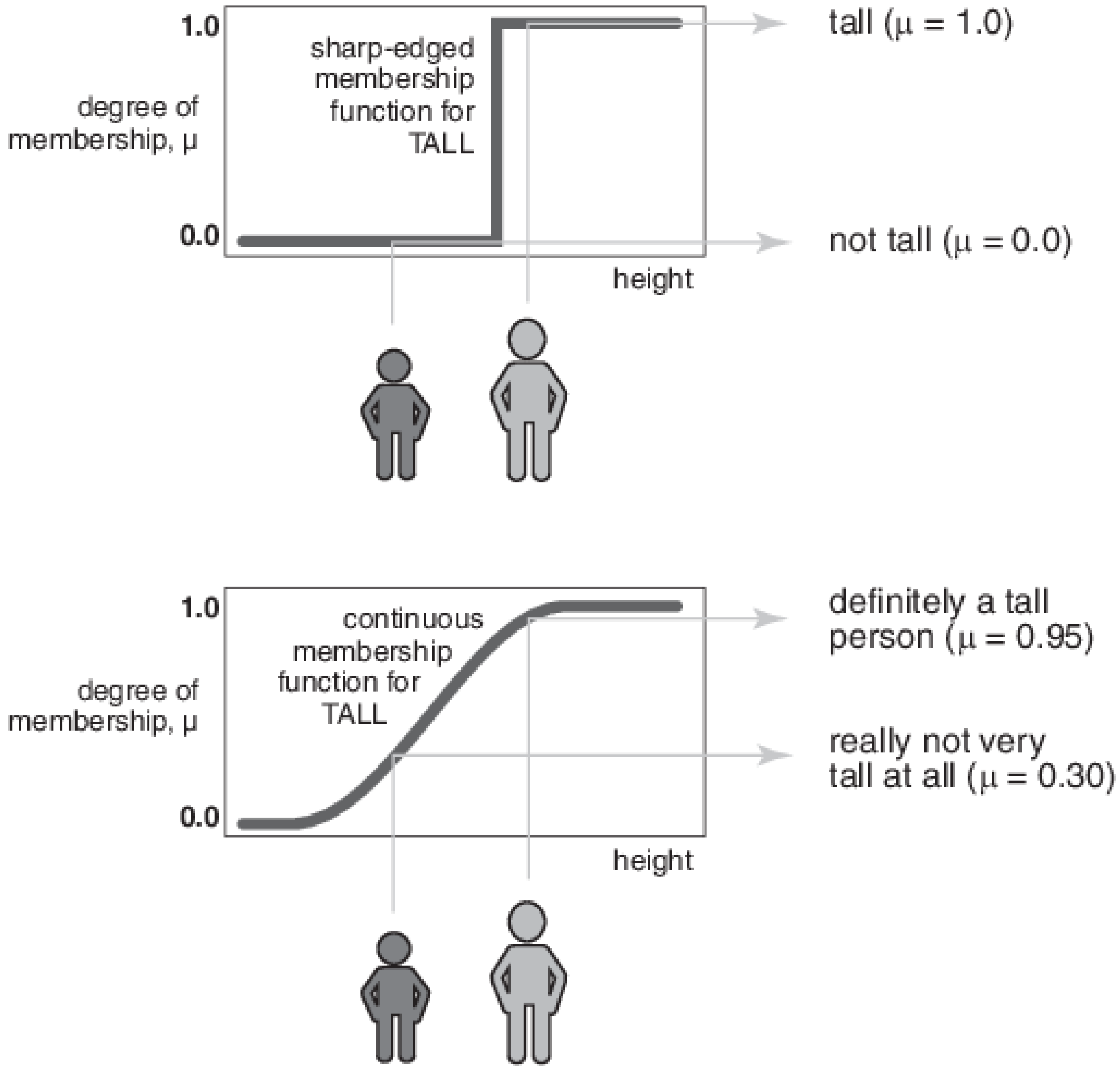
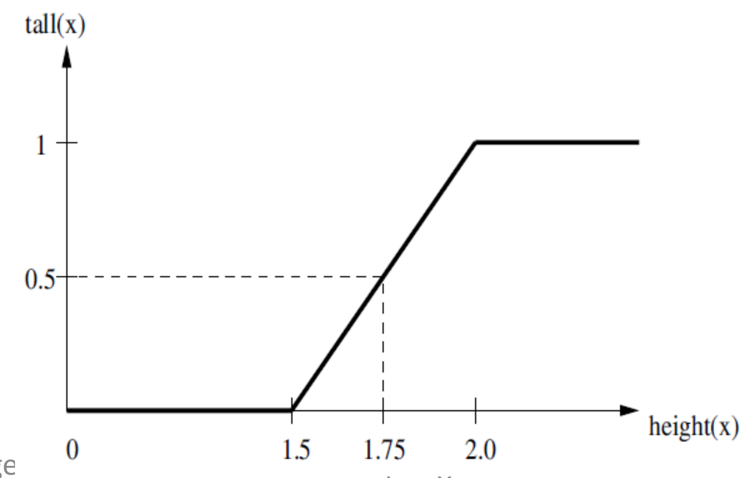

1.0
degree of membership, μ
0.0
sharp-edged membership function for TALL
height
tall (μ = 1.0)
not tall (μ = 0.0)
1.0
degree of membership, μ
0.0
continuous membership function for TALL
height
definitely a tall person (μ = 0.95)
really not very tall at all (μ = 0.30)

Illustration of *tall* Membership Function

- A possible membership function for *tall* fuzzy set can be defined as

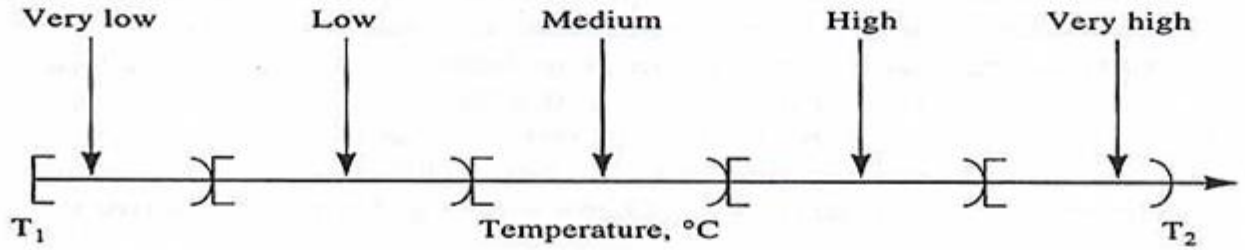
$$\text{tall}(x) = \begin{cases} 0 & \text{if length}(x) < 1.5 \\ (\text{length}(x) - 1.5) * 2 & \text{if } 1.5 < \text{length}(x) < 2 \\ 1 & \text{if length}(x) > 2 \end{cases}$$



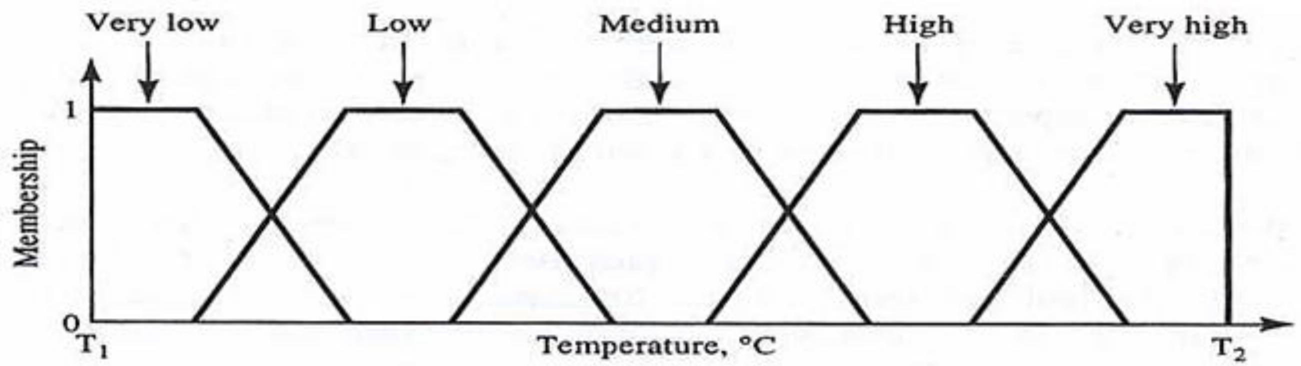
Membership Function

- The function is used to associate a degree of membership of each of the elements of the domain to the corresponding fuzzy set.
- Conditions
  - A membership function must be bounded below from 0 and from above by 1.
  - The range of a membership function must therefore be [0, 1].
  - For each $x \varepsilon X$, $\mu_A(x)$ must be unique. That is, the same element cannot map to different degrees of membership for the same fuzzy set.

Crisp vs Fuzzy variable

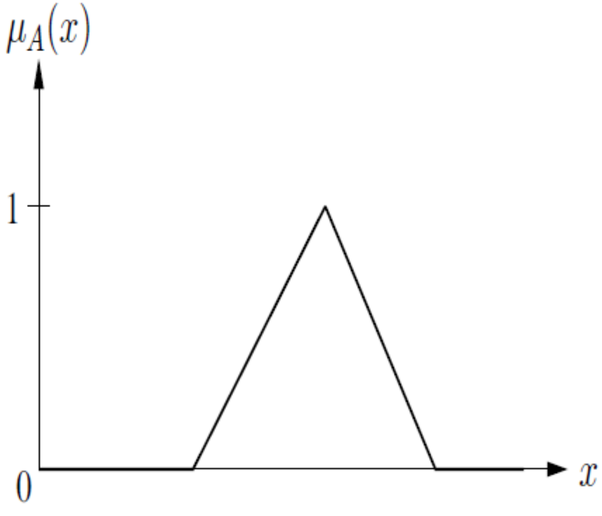


(b)

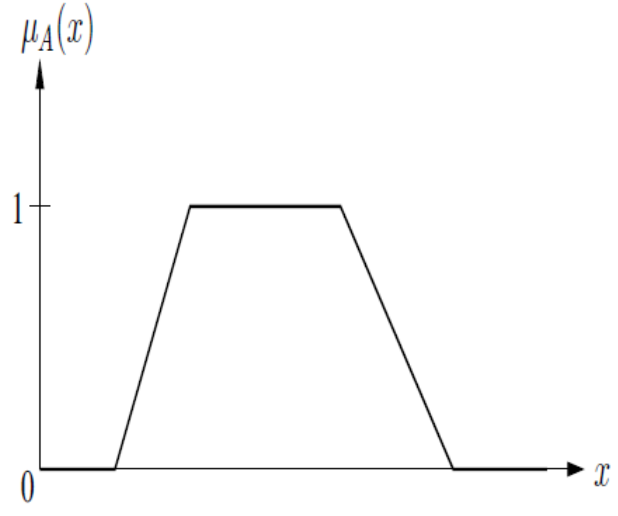


(a)

Other Function Types



(a) Triangular Function



(b) Trapezoidal Function

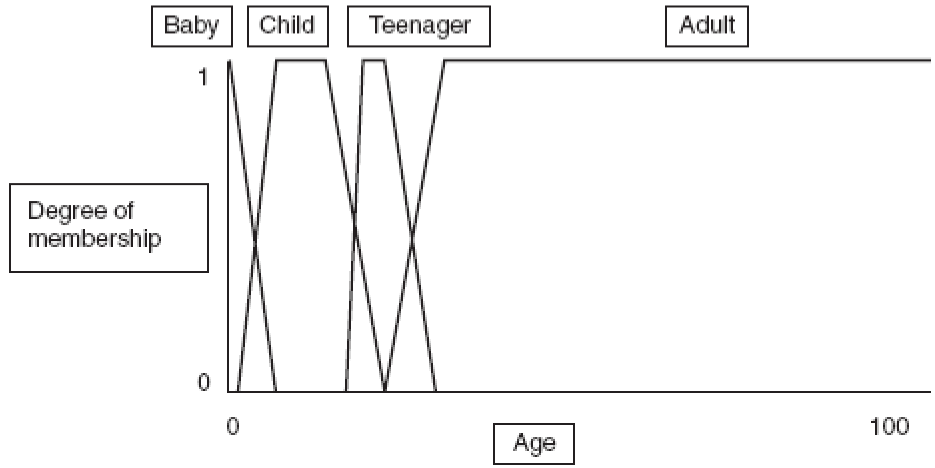
$$\mu_A(x) = \begin{cases} 0 & \text{if } x \leq \alpha_{min} \\ \frac{x-\alpha_{min}}{\beta-\alpha_{min}} & \text{if } x \in (\alpha_{min}, \beta] \\ \frac{\alpha_{max}-x}{\alpha_{max}-\beta} & \text{if } x \in (\beta, \alpha_{max}) \\ 0 & \text{if } x \geq \alpha_{max} \end{cases}$$

$$\mu_A(x) = \begin{cases} 0 & \text{if } x \leq \alpha_{min} \\ \frac{x-\alpha_{min}}{\beta_1-\alpha_{min}} & \text{if } x \in [\alpha_{min}, \beta_1) \\ \frac{\alpha_{max}-x}{\alpha_{max}-\beta_2} & \text{if } x \in (\beta_2, \alpha_{max}) \\ 0 & \text{if } x \geq \alpha_{max} \end{cases}$$

Example of a Fuzzy Variable

$$M_B(x) = \begin{cases} 1 - \dfrac{x}{2} & \text{for } x \le 2 \\ 0 & \text{for } x > 2 \end{cases}$$

$$M_C(x) = \begin{cases} \dfrac{x-1}{6} & \text{for } x \le 7 \\ 1 & \text{for } x > 7 \text{ and } x \le 8 \\ \dfrac{14-x}{6} & \text{for } x > 8 \end{cases}$$



- We represent a fuzzy set using a list of pairs, where each pair represents a value and the fuzzy membership value for that value.
- For example, we might define *B*, the fuzzy set of babies as follows:
  - *B* = {(0, 1), (2, 0)}
- Similarly,we could define the fuzzy set of children,*C*, as follows:
  - *C* = {(1, 0), (7, 1), (8, 1), (14, 0)}

A fuzzy set

Example

Consider the two fuzzy sets:

$$\text{long pencils} = \{pencil1/0.1, pencil2/0.2, pencil3/0.4, pencil4/0.6, pencil5/0.8, pencil6/1.0\}$$

$$\text{medium pencils} = \{pencil1/1.0, pencil2/0.6, pencil3/0.4, pencil4/0.3, pencil5/0.1\}$$

Fuzzy Operators

- **Equality:** Two fuzzy sets A and B are equal if and only if the sets have the same domain, and $\mu_A(x) = \mu_B(x)$ for all $x \in X$.

- **Complement of fuzzy set:** Let A^c denote the complement of set A. Then for all $x \in X$, $\mu_{A^c}(x) = 1 - \mu_A(x)$.

Fuzzy Operators (Cont'd)

- Intersection of fuzzy sets: If A and B are two fuzzy sets, then
  - Min-operator: $\mu_{A\cap B}(x)=\min\{\mu_A(x),\mu_B(x)\}, \forall x \in X$
  - Product Operator: $\mu_{A\cap B}(x)=\mu_a(x)\mu_B(x), \forall x \in X$
- Union of fuzzy sets: If A and B are two fuzzy sets, then
  - Max-operator: $\mu_{A\cup B}(x)=\max\{\mu_B(x),\mu_B(x)\}, \forall x \in X$
  - Summation Operator: $\mu_{A\cup B}(x)=\mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x), \forall x \in X$

Exercise 1

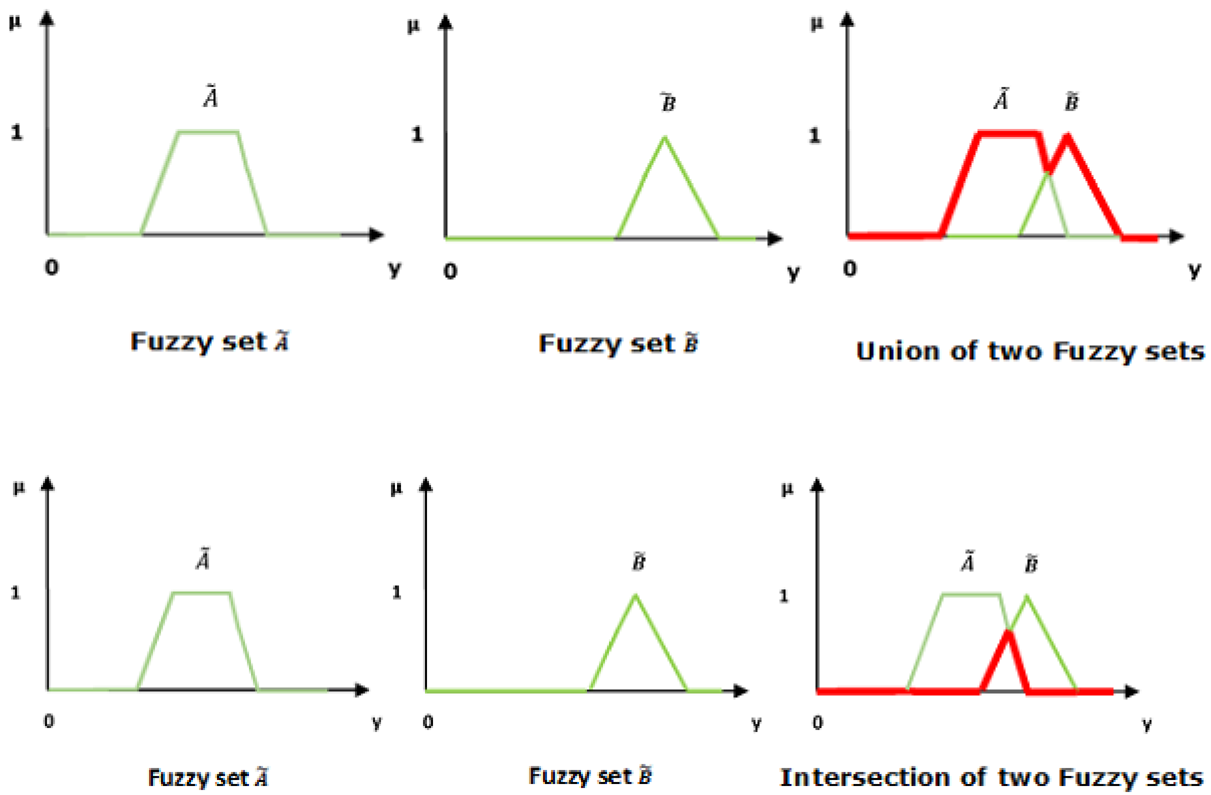
Consider the two fuzzy sets:

$$\begin{aligned} \text{long pencils} &= \{pencil1/0.1, pencil2/0.2, pencil3/0.4, pencil4/0.6, \\ &\quad pencil5/0.8, pencil6/1.0\} \\ \text{medium pencils} &= \{pencil1/1.0, pencil2/0.6, pencil3/0.4, pencil4/0.3, \\ &\quad pencil5/0.1\} \end{aligned}$$

(a) Determine the union of the two sets.

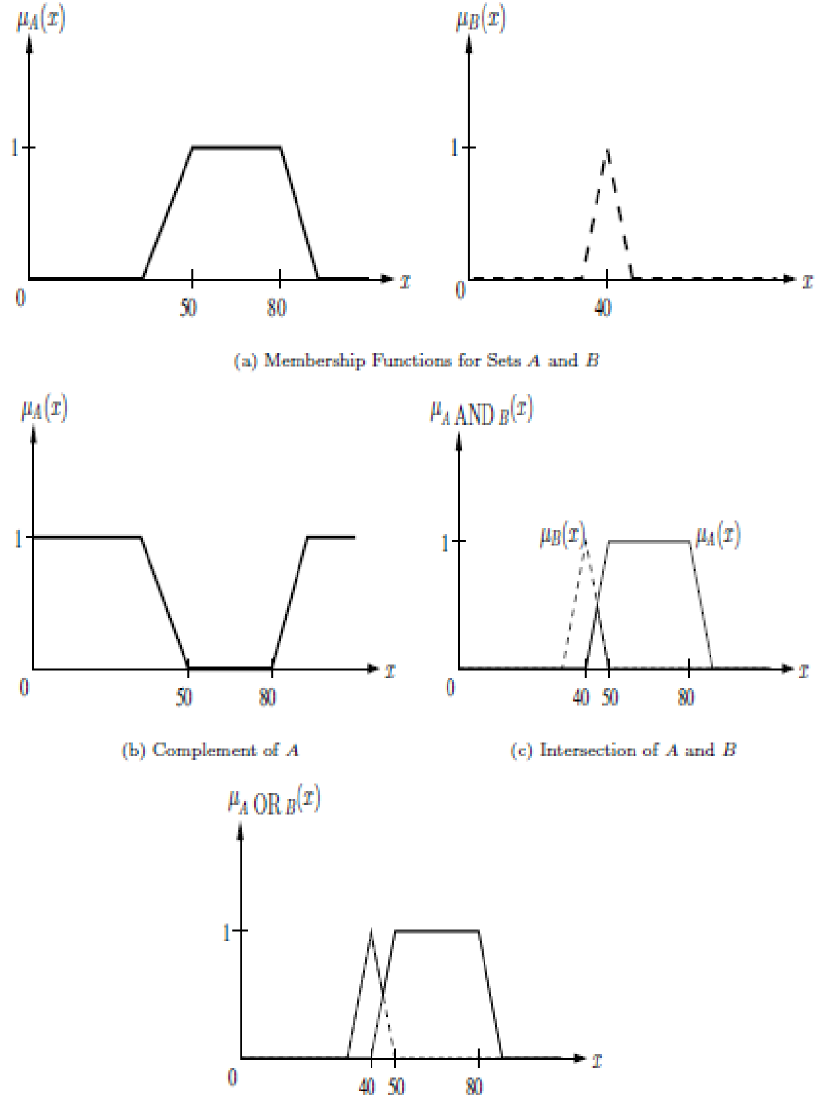
(b) Determine the intersection of the two sets.

- Union: {pencil1/1.0, pencil2/0.6,pencil3/0.4……}
- Intersection: {pencil1/0.1, pencil2/0.2,pencil3/0.4……}



https://www.tutorialspoint.com/fuzzy_logic/fuzzy_logic_set_theory.htm

Illustration of Fuzzy Operators



(a) Membership Functions for Sets A and B

(b) Complement of A

(c) Intersection of A and B

(d) Union of A

A fuzzy set

Exercise 1

Consider the two fuzzy sets:

$$\begin{aligned} \text{long pencils} &= \{pencil1/0.1, pencil2/0.2, pencil3/0.4, pencil4/0.6, \\ &\quad pencil5/0.8, pencil6/1.0\} \\ \text{medium pencils} &= \{pencil1/1.0, pencil2/0.6, pencil3/0.4, pencil4/0.3, \\ &\quad pencil5/0.1\} \end{aligned}$$

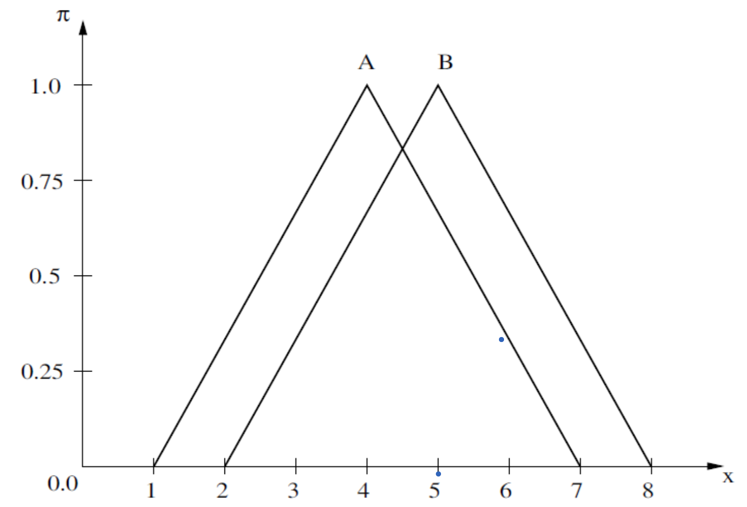
(a) Determine the union of the two sets.

(b) Determine the intersection of the two sets.

- Union: {pencil1/1.0, pencil2/0.6m,pencil3/0.4……}
- Intersection: {pencil1/0.1, pencil2/0.2,pencil3/0.4……}
-

Exercise 2

- Consider the membership function of two fuzzy sets, A and B, as given in the figure.
  - Draw the membership function for the fuzzy set C = $A \cap B^c$, using the min-operator.
  - Compute $\mu_C(5)$.



Resources

- https://www.researchgate.net/publication/267041266_Introduction_to_fuzzy_logic
- https://www.tutorialspoint.com/fuzzy_logic/fuzzy_logic_quick_guide.htm