Midterm-I Exam (max pt. 100) Dr. Shafayat Abrar

CE 362: Statistics and Inferencing

Timing: 05:00 - 07:00 PM Dated: Dec. 08, 2023 Duration: 120 min

Q1 [70]: Consider i.i.d. random variables  $X_1, \ldots, X_n$  sampled from the distribution with probability density

$$f(x \mid \theta) = K\theta^3 \sqrt{x} e^{-\theta^2 x}, \quad x > 0$$

and 0 elsewhere.

- (a) Find K.
- (b) Compute the method of moment estimator for  $\theta$ ; denote it as  $\widehat{\theta}_{\text{MoM}}$ .
- (c) Using Taylor's series, compute the bias of the estimator,  $\widehat{\theta}_{\text{MoM}}$ .
- (d) Using Taylor's series, compute the variance of the estimator,  $\widehat{\theta}_{\text{MoM}}$ .
- (e) Compute the maximum likelihood estimator for  $\theta$ ; denote it as  $\widehat{\theta}_{ML}$ .
- (f) Using Taylor's series, compute the bias of the estimator,  $\widehat{\theta}_{\mathrm{ML}}$ .
- (g) Compute the asymptotic variance of the estimator  $\widehat{\theta}_{ML}$ .