# Weekly Challenge 13: Decidability

CS 212 Nature of Computation
Habib University

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## 1. Bit flip

Given $\Sigma = \{0, 1\}$, consider the function, $f$, defined over a character, $a$, and extended to a string, $u = u_1 u_2 u_3 \dots u_n$, as follows.

$$f(a) = \begin{cases} 0 & a = 1 \\ 1 & a = 0 \end{cases}$$
$$f(u) = f(u_1)f(u_2)f(u_3)\dots f(u_n)$$

Assuming the results in Chapter 4, prove or disprove that the following language is decidable.

$$S = \{\langle M \rangle \mid M \text{ is a DFA}, u \in L(M) \implies f(u) \in L(M)\}.$$

**Solution:** The language $S$ is decidable as we can construct a Turing Machine, $M_S$, that decides $S$ as follows:

On input $\langle M \rangle$

1. Construct a DFA $M'$ using $M$. Let $M = (Q, \Sigma, \delta, q_o, F)$.
   Then $M' = (Q, \Sigma, \delta', q_o, F)$ where we make a transition function by flipping the alphabets in the transition: $\delta(q, a) = p \implies \delta'(q, f(a)) = p$.
2. Run $\langle M, M' \rangle$ on $EQ_{\text{DFA}}$
3. If $EQ_{\text{DFA}}$ accepts, then accept. If it rejects, then reject.

If $M$ is a DFA which satisfies the condition described in $S$, then flipping the transition function will not change the language it accepts. Therefore, $M'$ will accept the string that $M$ accepts. So, if their languages are the same, then we know $M \in S$ else it rejects.

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