Statistics & Inferencing - Activity 01

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Q1) Show mgf of Bernoulli (p) is: $M_x(t) = E[e^{tx}] = 1-p+e^tp$

$M_x(t) = E[e^{tx}] = p(e^{t(1)}) + (1-p)(e^{t(0)})$
$= pe^t + 1-p = 1-p+pe^t$
$\Rightarrow M_x(t) = E[e^{tx}] = 1-p+pe^t$ shown!

Q2) Using mgf of Bernoulli (p), find its mean & variance

$\mu_1 = M_x'(0)$ $\mu_2 = M_x''(0)$ $\sigma^2 = \mu_2 - (\mu_1)^2$

$M_x'(t) = 0 - 0 + pe^t \Rightarrow M_x'(0) = p \Rightarrow \mu_1 = p.$

$M_x'(t) = pe^t$

$M_x''(t) = pe^t \Rightarrow M_x''(0) = p = \mu_2$

$\sigma^2 = p - (p)^2 \Rightarrow \sigma^2 = p(1-p).$

Mean $= \mu_1 = p$ Var $= \sigma^2 = p(1-p)$

Q3) Use the fact ~~in question~~ & Theorem to derive mean & variance for Binomial Distribution.

$\rightarrow$ Binomial Distribution $\rightarrow$ Sum of Independent Bernoulli

$M_Y(t) = M_{x_1}(t) \cdot M_{x_2}(t)\, M_{x_3}(t) \cdots M_{x_n}(t)$
where $Y = X_1 + X_2 + \cdots + X_n$

$\Rightarrow M_Y(t) = (1-p+pe^t)(1-p+pe^t)(1-p+pe^t)\cdots(1-p+pe^t)$

$M_Y(t) = (1-p+pe^t)^n$

For Y: $\mu_1 = M_Y'(0)$ $\mu_1 = \mu$ $\mu_2 = M_Y''(0)$ $\sigma^2 = \mu_2 - (\mu_1)^2$

$M_Y'(t) = n(1-p+pe^t)^{n-1}(pe^t)$

$M_Y'(0) = n(1-p+p)^{n-1}(p) = n(p) \Rightarrow M_Y'(0) = np = \mu = \mu_1$

$M_Y''(t) = n[(n-1)(1-p+pe^t)^{n-2}(pe^t)(pe^t) + (1-p+pe^t)^{n-1}(pe^t)]$

$M_Y''(0) = n[(n-1)(1-p+p)(p)(p) + (1-p+p)^{n-1}(p)]$

$= n[(n-1)(p^2) + p]$

$M_Y''(0) = np[(n-1)p+1] = \mu_2$

$\sigma^2 = np[(n-1)p+1] - n^2p^2$

$= np[(n-1)p+1-np] = np[\cancel{np}-p+1-\cancel{np}]$

$\Rightarrow \sigma^2 = np(1-p)$

For Y where $Y \sim Bin(n,p)$ Mean $= np$ & Var $= np(1-p)$.

Hence shown!