



Q no. 1

(a) Under what conditions $AB = BA$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$\begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix} = \begin{bmatrix} b_{11}a_{11} + b_{12}a_{21} & b_{11}a_{12} + b_{12}a_{22} \\ b_{21}a_{11} + b_{22}a_{21} & b_{21}a_{12} + b_{22}a_{22} \end{bmatrix}$$

Comparing corresponding entries

$$\begin{aligned} a_{11}b_{11} + a_{12}b_{21} &= b_{11}a_{11} + b_{12}a_{21} \\ a_{11}b_{12} + a_{12}b_{22} &= b_{11}a_{12} + b_{12}a_{22} \\ a_{21}b_{11} + a_{22}b_{21} &= b_{21}a_{11} + b_{22}a_{21} \\ a_{21}b_{12} + a_{22}b_{22} &= b_{21}a_{12} + b_{22}a_{22} \end{aligned}$$

$$\begin{aligned} a_{12}b_{21} &= b_{12}a_{21} \\ a_{11}b_{12} + a_{12}b_{22} &= b_{11}a_{12} + b_{12}a_{22} \\ a_{21}b_{11} + a_{22}b_{21} &= b_{21}a_{11} + b_{22}a_{21} \\ a_{21}b_{12} &= b_{21}a_{12} \end{aligned}$$

(b) $A^{r+s} = A^r A^s$ is valid for negative integers r, s

Solution: Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ and let $r = 1$ and $s = -1$. It shows, it would be

$$\begin{aligned} A^{1-1} &= A^1 A^{-1} \\ A^0 &= AA^{-1} \\ I &= \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}^{-1} \end{aligned}$$

Inverse not exist, hence it is not valid for negative integers.

(c) Solution: Since A^{-1} does not exist, that's why $AB = AC$ but $B \neq C$. (Fill the details for the verification)

Q.no 2

Solution: Using $[A|I] \rightarrow [I|A^{-1}]$

$$\begin{aligned}
 & \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 2 & -1 & 3 & 0 & 1 & 0 \\ 4 & 1 & 8 & 0 & 0 & 1 \end{array} \right] \\
 R_2 - 2R_1 & \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & -1 & -1 & -2 & 1 & 0 \\ 4 & 1 & 8 & 0 & 0 & 1 \end{array} \right] & R_3 - 4R_1 & \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & -1 & -1 & -2 & 1 & 0 \\ 0 & 1 & 0 & -4 & 0 & 1 \end{array} \right] \\
 R_3 \rightleftharpoons R_2 & \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & -4 & 0 & 1 \\ 0 & -1 & -1 & -2 & 1 & 0 \end{array} \right] & R_3 + R_2 & \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & -4 & 0 & 1 \\ 0 & 0 & -1 & -6 & 1 & 1 \end{array} \right] \\
 R_1 + 2R_3 & \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -15 & 2 & 2 \\ 0 & 1 & 0 & -4 & 0 & 1 \\ 0 & 0 & -1 & -6 & 1 & 1 \end{array} \right] & -1R_3 & \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -15 & 2 & 2 \\ 0 & 1 & 0 & -4 & 0 & 1 \\ 0 & 0 & 1 & 6 & -1 & -1 \end{array} \right]
 \end{aligned}$$

Q. no. 3

$$\begin{aligned}
 x + y + 2z &= 9 \\
 2x + 4y - 3z &= 1 \\
 3x + 6y - 5z &= 0
 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 2 & 4 & -3 & 1 \\ 3 & 6 & -5 & 0 \end{array} \right]$$

$$\begin{aligned}
 R_2 - 2R_1 & \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & -7 & 17 \\ 3 & 6 & -5 & 0 \end{array} \right] & R_3 - 3R_1 & \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & -7 & 17 \\ 0 & 3 & -11 & -27 \end{array} \right] \\
 \frac{1}{2}R_2 & \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 1 & -7/2 & 17/2 \\ 0 & 3 & -11 & -27 \end{array} \right] & R_1 - R_2 & \left[\begin{array}{ccc|c} 1 & 0 & 11/2 & 18 \\ 0 & 1 & -7/2 & 17/2 \\ 0 & 3 & -11 & -27 \end{array} \right] \\
 \dots & \dots & \dots & \dots
 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

Exercise Set 1.2 Solution**Question 18**

Reduce

$$\left[\begin{array}{ccc} 2 & 1 & 3 \\ 0 & -2 & -29 \\ 3 & 4 & 5 \end{array} \right]$$

to reduced row-echelon form.

Solution:

$$\begin{aligned}
 & \begin{bmatrix} 2 & 1 & 3 \\ 0 & -2 & -29 \\ 3 & 4 & 5 \end{bmatrix} \xrightarrow{\frac{1}{2}R_1} \begin{bmatrix} 1 & 1/2 & 3/2 \\ 0 & -2 & -29 \\ 3 & 4 & 5 \end{bmatrix} \xrightarrow{R_3-3R_1} \begin{bmatrix} 1 & 1/2 & 3/2 \\ 0 & -2 & -29 \\ 0 & 5/2 & 1/2 \end{bmatrix} \\
 & \xrightarrow{-\frac{1}{2}R_2} \begin{bmatrix} 1 & 1/2 & 3/2 \\ 0 & 1 & 29/2 \\ 0 & 5/2 & 1/2 \end{bmatrix} \xrightarrow{R_3-\frac{5}{2}R_2} \begin{bmatrix} 1 & 1/2 & 3/2 \\ 0 & 1 & 29/2 \\ 0 & 0 & -143/4 \end{bmatrix} \xrightarrow{R_1-\frac{1}{2}R_2} \begin{bmatrix} 1 & 0 & -13 \\ 0 & 1 & 29/2 \\ 0 & 0 & -143/4 \end{bmatrix} \\
 & \xrightarrow{-\frac{4}{143}R_3} \begin{bmatrix} 1 & 0 & -13 \\ 0 & 1 & 29/2 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_2-\frac{2}{29}R_2} \begin{bmatrix} 1 & 0 & -13 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_1-(-13)R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
 \end{aligned}$$

Question 19

Find two different row-echelon forms of

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$$

Solution: One possibility is

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} \xrightarrow{R_2-2R_1} \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

Another possibility is

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} \xrightarrow{R_1 \rightleftharpoons R_2} \begin{bmatrix} 2 & 7 \\ 1 & 3 \end{bmatrix} \xrightarrow{R_1-\frac{1}{2}R_2} \begin{bmatrix} 1 & 7/2 \\ 1 & 3 \end{bmatrix} \xrightarrow{R_1-R_2} \begin{bmatrix} 1 & 7/2 \\ 0 & 1/2 \end{bmatrix}$$

Question 25

Find the coefficients a , b , c , and d so that the curve shown in the accompanying figure is the graph of the equation $y = ax^3 + bx^2 + cx + d$.

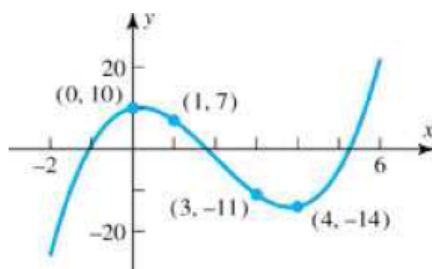


Figure Ex-25

Solution: Using the given points $(0, 10)$, $(1, 7)$, $(3, -11)$, and $(4, -14)$ we obtain the equations by substituting in $y = ax^3 + bx^2 + cx + d$.

$$d = 10 \quad (1)$$

$$a + b + c + d = 7 \quad (2)$$

$$27a + 9b + 3c + d = -11 \quad (3)$$

$$64a + 16b + 4c + d = -14 \quad (4)$$

Now put $Eq(1)$ in $Eq(2, 3, 4)$, we have a system of linear equations(3 equations 3 unknown).

$$\begin{aligned}a + b + c + 10 &= 7 \\27a + 9b + 3c + 10 &= -11 \\64a + 16b + 4c + 10 &= -14\end{aligned}$$

Now simplifying

$$\begin{aligned}a + b + c &= -3 \\27a + 9b + 3c &= -21 \\64a + 16b + 4c &= -24\end{aligned}$$

$$\begin{aligned}a + b + c &= -3 \\9a + 3b + c &= -7 \\16a + 4b + c &= -6\end{aligned}$$

In form of Matrix

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 9 & 3 & 1 & -7 \\ 16 & 4 & 1 & -6 \end{array} \right]$$

If we solve this system by ERO, we find that $a = 1, b = -6, c = 2$, and $d = 10$.

Q2

(b)

So, augment the matrix with the identity matrix:

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -4 & 1 & 0 & 0 \\ -1 & -1 & 5 & 0 & 1 & 0 \\ 2 & 7 & -3 & 0 & 0 & 1 \end{array} \right]$$

Add row 1 to row 2: $R_2 = R_2 + R_1$.

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -4 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 2 & 7 & -3 & 0 & 0 & 1 \end{array} \right]$$

Subtract row 1 multiplied by 2 from row 3: $R_3 = R_3 - 2R_1$.

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -4 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 3 & 5 & -2 & 0 & 1 \end{array} \right]$$

Subtract row 2 multiplied by 2 from row 1: $R_1 = R_1 - 2R_2$.

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -6 & -1 & -2 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 3 & 5 & -2 & 0 & 1 \end{array} \right]$$

Subtract row 2 multiplied by 3 from row 3: $R_3 = R_3 - 3R_2$.

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -6 & -1 & -2 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 2 & -5 & -3 & 1 \end{array} \right]$$

Divide row 3 by 2: $R_3 = \frac{R_3}{2}$.

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -6 & -1 & -2 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & -\frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{array} \right]$$

Add row 3 multiplied by 6 to row 1: $R_1 = R_1 + 6R_3$.

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -16 & -11 & 3 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & -\frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{array} \right]$$

Subtract row 3 from row 2: $R_2 = R_2 - R_3$.

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -16 & -11 & 3 \\ 0 & 1 & 0 & \frac{7}{2} & \frac{5}{2} & -\frac{1}{2} \\ 0 & 0 & 1 & -\frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{array} \right]$$

Q4)

Divide row 1 by 2: $R_1 = \frac{R_1}{2}$.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & -1 \end{array} \right]$$

Add row 1 multiplied by 2 to row 2: $R_2 = R_2 + 2R_1$.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 7 & 4 & 1 \\ 8 & 1 & 4 & -1 \end{array} \right]$$

Subtract row 1 multiplied by 8 from row 3: $R_3 = R_3 - 8R_1$.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 7 & 4 & 1 \\ 0 & -7 & -4 & -1 \end{array} \right]$$

Divide row 2 by 7: $R_2 = \frac{R_2}{7}$.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & \frac{4}{7} & \frac{1}{7} \\ 0 & -7 & -4 & -1 \end{array} \right]$$

Subtract row 2 from row 1: $R_1 = R_1 - R_2$.

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{3}{7} & -\frac{1}{7} \\ 0 & 1 & \frac{4}{7} & \frac{1}{7} \\ 0 & -7 & -4 & -1 \end{array} \right]$$

Add row 2 multiplied by 7 to row 3: $R_3 = R_3 + 7R_2$.

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{3}{7} & -\frac{1}{7} \\ 0 & 1 & \frac{4}{7} & \frac{1}{7} \\ 0 & 0 & 0 & 0 \end{array} \right]$$

Since the element at row 3 and column 3 (pivot element) equals 0, we need to swap the rows.

Find the first nonzero element in column 3 under the pivot entry.

As can be seen, there are no such entries.

Q1 continued

$$x_1 + \frac{3}{7}x_3 = -\frac{1}{7}$$

$$x_2 + \frac{4}{7}x_3 = \frac{1}{7}$$

$$-(x_1 + \frac{3}{7}x_3) = \frac{1}{7} \quad \text{--- (1)}$$

$$x_2 + \frac{4}{7}x_3 = \frac{1}{7} \quad \text{--- (2)}$$

substituting 1 & 2

$$-x_1 - \frac{3}{7}x_3 = x_2 + \frac{4}{7}x_3$$

$$-x_1 - x_2 = \frac{4}{7}x_3 + \frac{3}{7}x_3$$

$$\boxed{-x_1 - x_2 = \frac{10}{7}x_3}$$

$$x_3 = \frac{7}{10}(-x_1 - x_2)$$

$$\text{let } x_1 = s \quad x_2 = t$$

$$x_3 = \frac{7}{10}(-s - t)$$

Q5)

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 0 & -2 & 7 \\ 3 & 4 & 5 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - R_1 \quad \begin{bmatrix} 2 & 1 & 3 \\ 0 & -2 & 7 \\ 1 & 3 & 2 \end{bmatrix}$$

$$R_1 \rightarrow R_1 - 2R_3 \quad \begin{bmatrix} 0 & -5 & -1 \\ 0 & -2 & 7 \\ 1 & 3 & 2 \end{bmatrix}$$

$$R_2 \rightarrow R_2 + 7R_3 \quad \begin{bmatrix} 0 & -5 & -1 \\ 0 & -37 & 0 \\ 1 & 3 & 2 \end{bmatrix}$$

$$R_2 \rightarrow R_2 / -37 \quad \begin{bmatrix} 0 & -5 & -1 \\ 0 & 1 & 0 \\ 1 & 3 & 2 \end{bmatrix}$$

$$R_1 \rightarrow R_1 - 5R_2 \quad \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 3 & 2 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - 3R_2 + 2R_1 \quad \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$R_1 \rightarrow -1R_1 \quad \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\text{row exchange} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$