Markov Localization

EE468/CE468: Mobile Robotics

Dr. Basit Memon

Electrical and Computer Engineering
Habib University

November 6, 2023

Table of Contents

Table of Contents

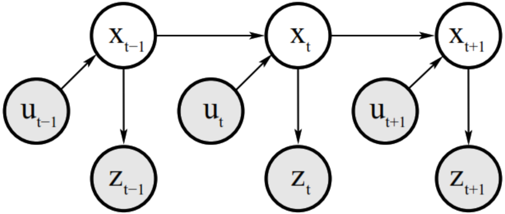


Figure: Dead Reckoning

Find $p(x_t|z_t, x_{t-1})$.

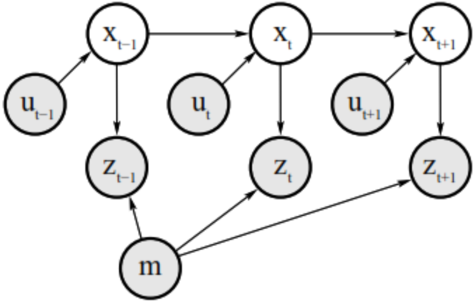


Figure: Pose Fixing

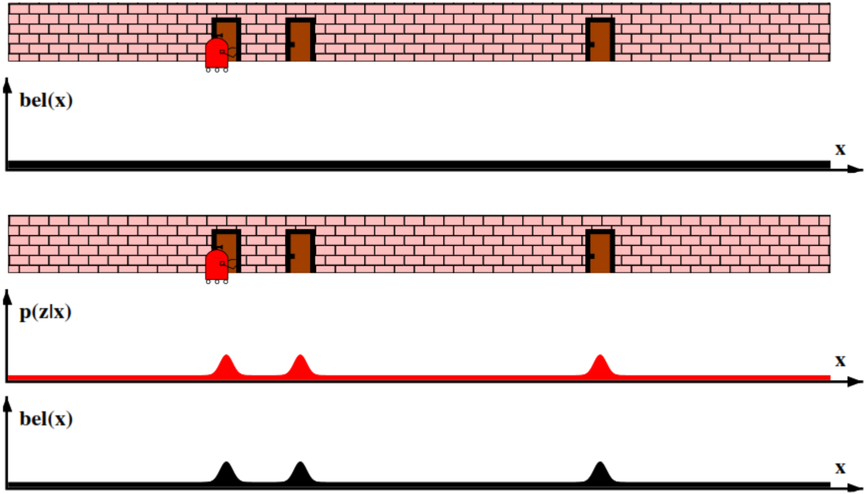
Find $p(x_t|z_t, x_{t-1}, m)$.

Markov Localization: Bayes algorithm applied as it is

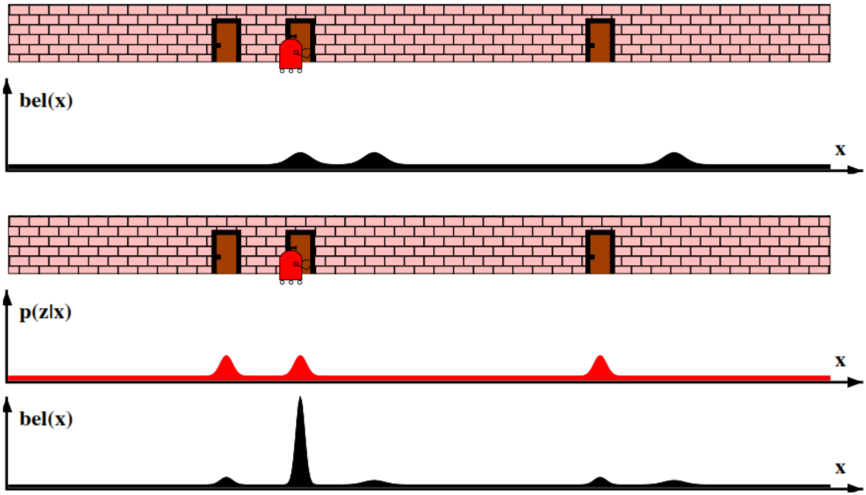
- Bayes Algorithm applied without any modification is Markov localization.

Algorithm Markov_localization($bel(x_{t-1}), u_t, z_t, m$)**:**
    for all x_t *do*
        $\overline{bel}(x_t) = \int p(x_t \mid u_t, x_{t-1}, m)\ bel(x_{t-1})\ dx$
        $bel(x_t) = \eta\ p(z_t \mid x_t, m)\ \overline{bel}(x_t)$
    endfor
    return $bel(x_t)$

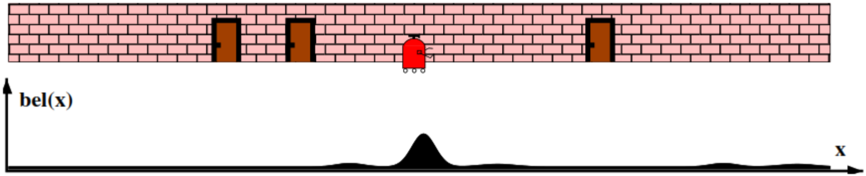
- It can address global localization, local, and kidnapped robot problem.


bel(x)
x
p(z|x)
x
bel(x)
x

Markov localization: Belief updated after action



Markov: Belief provides good pose estimate



- **Position Tracking:** Let $\bar{x}_0$ be the known initial pose. Then,

$$bel(x_0) = \begin{cases} 1 & \text{if } x_0 = \bar{x}_0 \\ 0 & \text{otherwise.} \end{cases}$$

  Practically, $\bar{x}_0$ is known in approximation. So, belief is initialized as a Gaussian with mean at $\bar{x}_0$ and small covariance, Σ, i.e.

$$bel(x_0) = \det(2\pi\Sigma)^{-1/2} \exp\left\{-\frac{1}{2}(x_0 - \bar{x}_0)^T \Sigma^{-1}(x_0 - \bar{x}_0)\right\}$$

Initial Belief - $bel(x_0)$

- **Global Localization:** Uniform distribution over the space of all legal poses in the map:

$$bel(x_0) = \frac{1}{|X|},$$

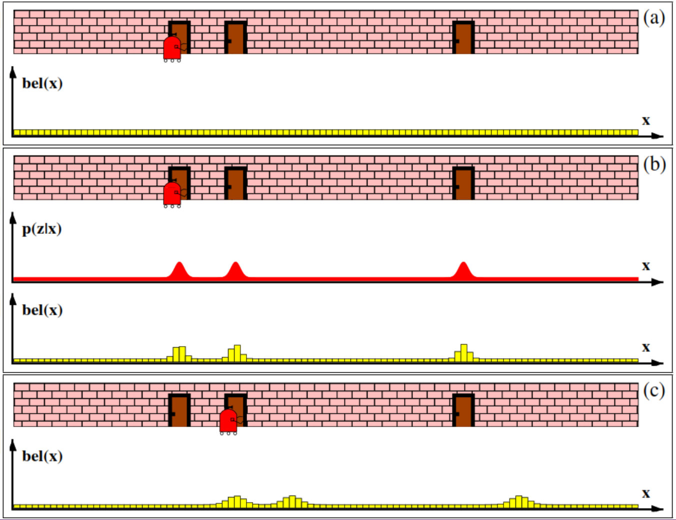
where $|X|$ is volume of space of all poses in the map.

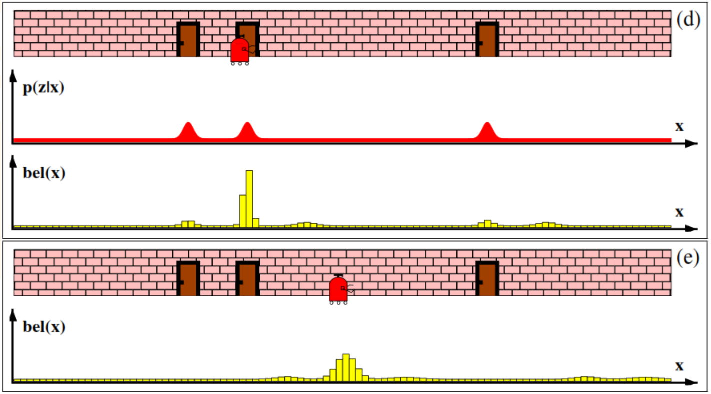
HUGE Problem: Markov localization is computationally intractable.

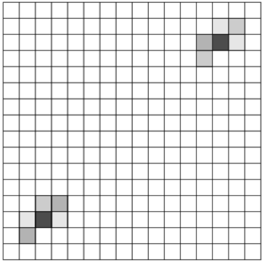
- Approximate posterior by finite number of values, each corresponding to a region in state space.
- Decompose state space:
  - **Grid Localization:** Approximate posterior using *histogram filter* over grid decomposition of pose space.

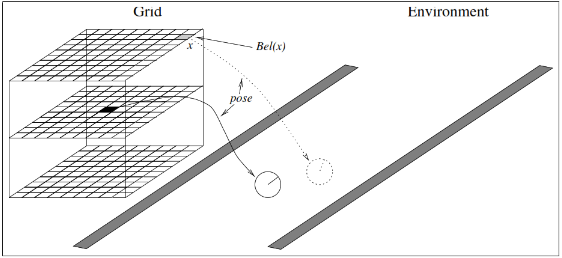
Table of Contents

Grid Localization Example



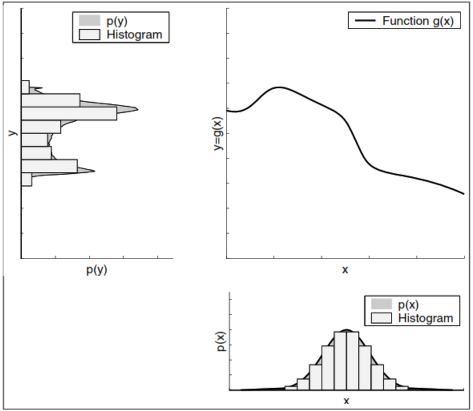

(d)
p(z|x)
x
bel(x)
x
(e)
bel(x)
x





- For planar pose, (x, y, θ), grid is 3D array.
- Cells are of same size and decomposition is time-invariant.
- Grid cells, $\mathbf{x}_k$, form a partition of pose space:

$$\text{domain} X_t = \mathbf{x}_{1,t} \cup \mathbf{x}_{2,t} \cup \cdots \mathbf{x}_{k,t}$$



- Each region $\mathbf{x}_{k,t}$ is assigned a probability $p_{k,t}$.
- Within each region, PDF is assumed to be uniform.

Grid Localization Algorithm

Algorithm Grid_localization($\{p_{k,t-1}\}, u_t, z_t, m$):

for all k *do*

$\bar{p}_{k,t} = \sum_i p_{i,t-1}$ **motion_model**$(\text{mean}(\mathbf{x}_k), u_t, \text{mean}(\mathbf{x}_i))$

$p_{k,t} = \eta\, \bar{p}_{k,t}$ **measurement_model**$(z_t, \text{mean}(\mathbf{x}_k), m)$

endfor

return $\{p_{k,t}\}$

- Note that mean is being used as representative for each region.

$$\hat{x}_{k,t} = |\mathbf{x}_{k,t}|^{-1} \int_{\mathbf{x}_{k,t}} x_t \, dx_t .$$

- Proof at [1, 4.1.3].

What should be size of a grid cell? [1, 8.2.2]

- Obvious that the smaller the grid cell, the better the localization accuracy but greater the computational load.
- Typical indoors granularity: 15cm for $x-$, $y-$, and 5° for θ
- Math lab has dimensions $12.2m \times 9.5m$. If we use the above granularity for the grid, how many updates are required for each time step? 370,880!
- How do you reduce computation?

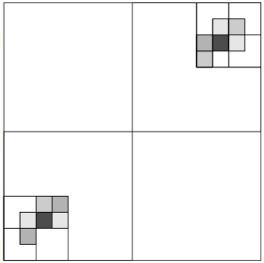
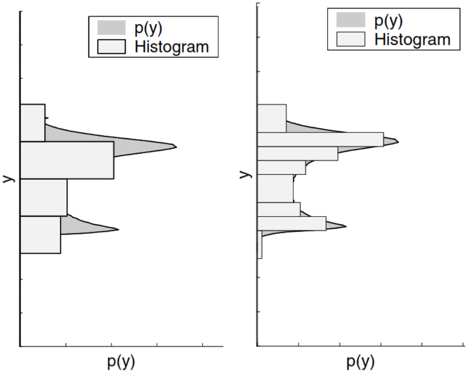


Figure: Grid cell size is depended on the posterior probability

- Dynamic decomposition adapt decomposition to shape of the posterior distribution. The less likely a region, coarser the decomposition.



Static vs Dynamic decomposition of state space [1, 4.1.4]

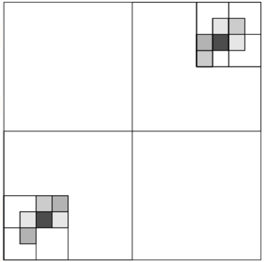


Figure: Grid cell size is depended on the posterior probability

- Dynamic decomposition adapt decomposition to shape of the posterior distribution. The less likely a region, coarser the decomposition.
  - Density Trees
  - Selective updating
- Burgard, Wolfram, et al. "Integrating global position estimation and position tracking for mobile robots: the Dynamic Markov Localization approach." IEEE, 1998.

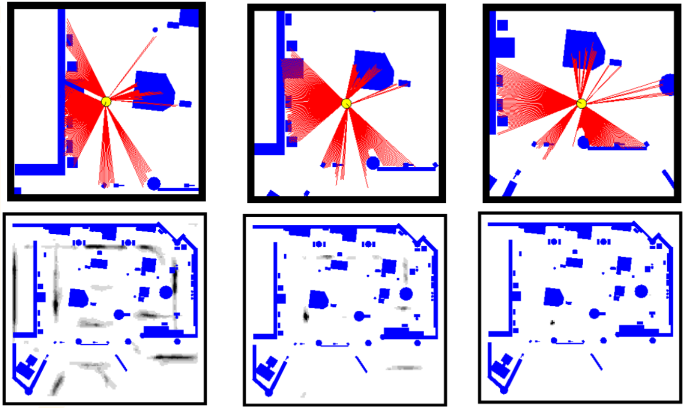
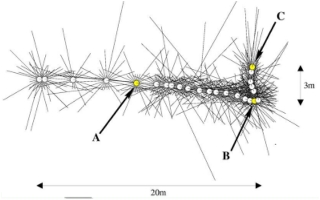
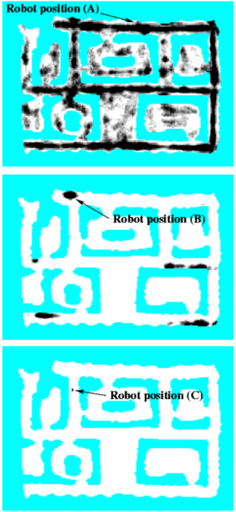


Figure: Grid-based localization using laser scans. Black regions in bottom row indicate robot's belief about its position. Photo Credit: Burgard slides

Grid-based localization using noisy sonar scans. Black regions indicate robot's belief about its position. Photo Credit: Burgard slides





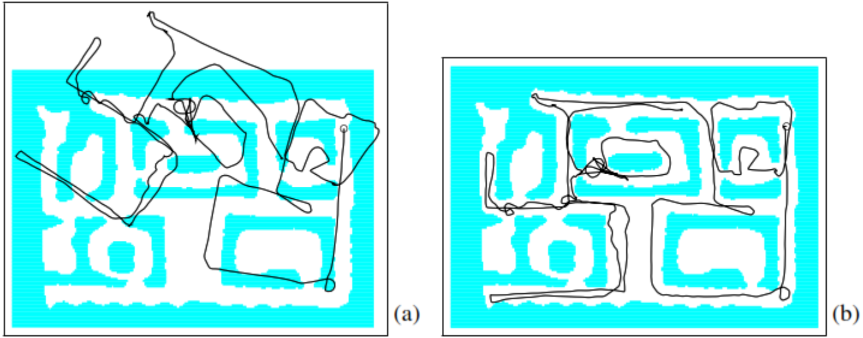
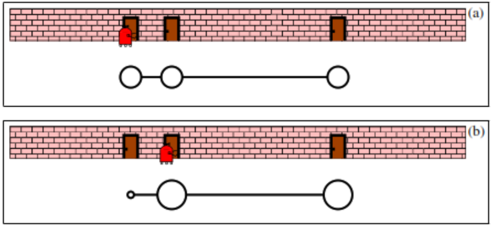


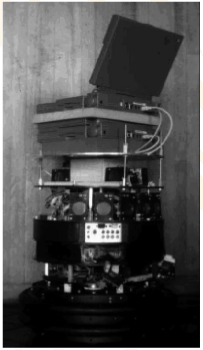
Figure: (a) Odometry Information (b) Corrected Path

A different static decomposition requiring less computations.



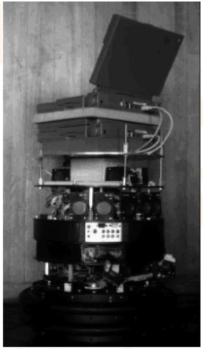
- Dynamic Decomposition
- Static Decomposition: Fixed Grids
  - Fine grid cells of the same size (**Metric representation**)
  - **Topological Grid:** A coarse grid obtained by decomposing pose space into regions that correspond to significant places in environment, e.g. doors, windows, T-junctions, intersections, dead ends, etc.

Dervish, a case study of Markov localization using topological grid



Read Section 5.6.7.5 in Autonomous Mobile Robots and answer the following questions:

1. What is distance between two doors in this example?
2. What sensors were used by Dervish?
3. How many node types are there in this example?
4. Does the algorithm make use of distance readings from sonar? If not, what does it use?
5. What is difference between probability and likelihood?



Read Section 5.6.7.5 in Autonomous Mobile Robots and answer the following questions:

6. How frequently does Dervish update its belief about its position?
7. Explain source of all numbers utilized in computing the likelihood of example situation in this case study.
8. How does Dervish decide its position, given computed likelihoods of all nodes?
9. Are there any disadvantages to this approach?

Table of Contents

[1] **Sebastian Thrun, Wolfram Burgard, and Dieter Fox.**
Probabilistic robotics.
2006.