Using Packages

SEL Activity 1

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1 Activity 1a

We know that $X \backsim \text{Ber}(p)$, and the expected/mean value of X will be $\mu = p$. Variance is defined as,

$$\begin{aligned}
\text{Var}[X] &:= \text{E}[(X-\mu)^2] \\
\text{Var}[X] &:= \sum_{x\in X}(x-\mu)^2 \Pr[X=x] \\
\text{Var}[X] &:= \sum_{x\in X}(x-p)^2 \Pr[X=x] \\
\text{Var}[X] &:= (0-p)^2 \Pr[X=0] + (1-p)^2 \Pr[X=1] \\
\text{Var}[X] &:= p^2(1-p) + (1-p)^2 p \\
\text{Var}[X] &:= p(1-p)\blacksquare
\end{aligned} \tag{1}$$

2 Activity 1b

A random experiment has been caried out as independent trail of Bernoulli Random Variable with same probability p. The estimation for $\hat{p}$ is given by,

$$\hat{p} = \frac{X_1 + X_2 + \cdots + X_n}{n}$$

The variance of $\hat{p}$ will be,

$$\begin{aligned}
\text{Var}[\hat{p}] &= \text{Var}\left[\frac{X_1 + X_2 + \cdots + X_n}{n}\right] \\
\text{Var}[\hat{p}] &= \frac{1}{n^2}\text{Var}\left[X_1 + X_2 + \cdots + X_n\right]
\end{aligned} \tag{2}$$

Variance of sum of independent random variable is equals to the sum of variance of those random variable.

$$\text{Var}[\hat{p}] = \frac{1}{n^2}\text{Var}[X_1] + \frac{1}{n^2}\text{Var}[X_2] + \cdots + \frac{1}{n^2}\text{Var}[X_n]$$

Variance of all X_i will be same.

$$
\begin{aligned}
\operatorname{Var}[\hat{p}] &= \frac{1}{n^2}\operatorname{Var}[X_1] + \frac{1}{n^2}\operatorname{Var}[X_1] + \cdots + \frac{1}{n^2}\operatorname{Var}[X_1] \\
\operatorname{Var}[\hat{p}] &= \frac{n}{n^2}\operatorname{Var}[X_1] \\
\operatorname{Var}[\hat{p}] &= \frac{1}{n}\operatorname{Var}[X_1] \\
\operatorname{Var}[\hat{p}] &= \frac{p(1-p)}{n} \blacksquare
\end{aligned} \tag{3}
$$