

Algorithms: Design and Analysis - CS 412

Problem Set 02: Asymptotic Analysis

1. Let $f(n)$ and $g(n)$ be asymptotically non-negative functions. Using the basic definition of Θ -notation, prove that $\max\{f(n), g(n)\} = \Theta(f(n) + g(n))$.

Solution:

2. Prove or disprove the statements below.

Solution:

- (a) $(n + 1)^2 = n^2 + O(n)$
- (b) $n + O(\sqrt{n})(n + O(\log(n)))^2 = n^3 + O(\sqrt{n^5})$
- (c) $\exp(O(1)) = O(e^n)$
- (d) $n^{\log(n)} = O((\log n)^n)$
- (e) $2^{2n} = O(2^n)$

3. Prove that for any two functions $f(n)$ and $g(n)$, we have $f(n) = \Theta(g(n))$ if and only if $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$.

Solution:

4. Prove that for $S \subseteq \mathbb{Z}$,

$$\sum_{k \in S} \Theta(f(k)) = \Theta\left\{\sum_{k \in S} f(k)\right\}$$

assuming both sums converge.

Solution:

5. Is $2^{n+1} = O(2^n)$? Is $2^{2n} = O(2^n)$?