CS412 Algorithms: Design & Analysis

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Practice Problems

Week 11

1. Why do we analyze the expected running time of a randomized algorithm and not its worst-case running time?

> **Solution:** We analyze the expected run time because it represents the more typical time cost. Also, we are doing the expected run time over the possible randomness used during computation because it can't be produced adversarially, unlike when doing expected run time over all possible inputs to the algorithm.

2. When RANDOMIZED-QUICKSORT runs, how many calls are made to the random number generator RANDOM in the worst case? How about in the best case? Give your answer in terms of Θ-notation.

> **Solution:** In either case, $\Theta(n)$ calls are made to RANDOM. PARTITION will run faster in the best case because the inputs will generally be smaller, but RANDOM is called every single time RANDOMIZED-PARTITION is called, which happens $\Theta(n)$ times.

3. Show that RANDOMIZED-SELECT never makes a recursive call to a 0-length array.

> **Solution:** Calling a zero length array would mean that the second and third arguments are equal. So, if the call is made on line 8, we would need that $p = q - 1$, which means that $q - p + 1 = 0$. However, i is assumed to be a nonnegative number, and to be executing line 8, we would need that $i < k = q - p + 1 = 0$, a contradiction. The other possibility is that the bad recursive call occurs on line 9. This would mean that $q + 1 = r$. To be executing line 9, we need that $i > k = q - p + 1 = r - p$. This would be a nonsensical original call to the array though because we are asking for the ith element from an array of strictly less size.

4. Argue that the indicator random variable X_k and the value $max(k-1, n-k)$ are independent.

> **Solution:** The probability that X_k is equal to 1 is unchanged when we know the max of $k-1$ and $n-k$. In other words, $P(X_k = a | max(k-1, n-k) = m) = P(X_k = a)$ for $a = 0$, 1 and $m = k-1$, $n-k$ so X_k and $max(k-1, n-k)$ are independent. By C.3-5, so are X_k and $T(max(k-1, n-k))$.

5. Write an iterative version of RANDOMIZED-SELECT.

Solution:

Algorithm 1 ITERATIVE-RANDOMIZED-SELECT

```
while p < r do
    q = RANDOMIZED − PARTITION(A, p, r)
    k = q − p + 1
    if i=k then
        return A[q]
    end if
    if i < k then
        r = q − 1
    else
        p = q
        i = i − k
    end if
end while
return A[p]
```

6. Suppose we use RANDOMIZED-SELECT to select the minimum element of the array $A = \{3, 2, 9, 0, 7, 5, 4, 8, 6, 1\}$. Describe a sequence of partitions that results in a worst-case performance of RANDOMIZED-SELECT.

Solution: When the partition selected is always the maximum element of the array we get worst-case performance. In the example, the sequence would be 9, 8, 7, 6, 5, 4, 3, 2, 1, 0.