

Statistics & Inferencing Homework #04

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P9.3) (a) $H_0: \mu = 25$ $H_a: \mu \neq 25$

$\bar{x} = 28.1$, $n = 57$ $\sigma = 8.46$ $\alpha = 0.1$

$$z = (28.1 - 25) / (8.46 / \sqrt{57}) = 2.77$$

$$\alpha = 0.1 \Rightarrow \alpha/2 = 0.05 \quad z_{\alpha/2} = 2.5758$$

$z > z_{\alpha/2}$ hence we reject H_0 .

(b) p-value = $P[z > 2.77] \times 2$

$$= 2 \times [1 - P[z < 2.77]] \times 2$$

$$= 2 \times [1 - 0.9972] = 0.0056$$

Since p-value $< \alpha/2$, we reject H_0 .

(c) $z_c = (\bar{x}_c - \mu) / (\sigma / \sqrt{n})$

$$\pm 2.5758 = (\bar{x}_c - 25) / (8.46 / \sqrt{57})$$

$$\bar{x}_c = 25 \pm 2.5758 (8.46 / \sqrt{57})$$

Upper $\bar{x}_c = 27.886$, Lower $\bar{x}_c = 22.114$

P9.5) (a) $H_0: \mu = 1200$ $H_a: \mu > 1200$

$\bar{x} = 1215$, $n = 113$ $\sigma = 100$ $\alpha = 0.10$

$$z = (1215 - 1200) / (100 / \sqrt{113}) = 1.59 \quad z_{\alpha} = 1.2816$$

$z > z_{\alpha}$, reject H_0 .

(b) p-value = $P[z > 1.59]$

$$= 1 - P[z < 1.59] = 1 - 0.9441 = 0.0559$$

$p < \alpha$, reject H_0 .

(c) $z_c = (\bar{x}_c - \mu) / (\sigma / \sqrt{n}) \Rightarrow 1.2816 = (\bar{x}_c - 1200) / (100 / \sqrt{113})$

$$\bar{x}_c = 1212.06$$

P9.15) $n = 11$ $\alpha = 0.05$ $H_0: \mu = 1160$ $H_a: \mu > 1160$

$t_{0.05, 10} = 1.8125$ $t = (\bar{x} - \mu) / (s / \sqrt{n})$

$\bar{x} = 1236.364$

$$t = (1236.364 - 1160) / (103.812 / \sqrt{11}) = 2.44$$

$t > t_{\alpha}$, hence reject H_0 .

P9.17) $n = 12$ $\alpha = 0.10$ $H_0: \mu = 1.84$ $H_a: \mu \neq 1.84$

$t_{0.05, 11} = 1.8125$

$\bar{x} = 1.851$

$s = 0.0235$

$$t = (1.851 - 1.84) / (0.0235 / \sqrt{12}) = 1.6$$

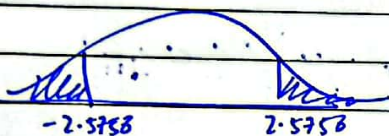
p-value = $2 \times P(t > 1.6) = 2 \times P(1.6, 11)$

$$= 2 \times 0.065 = 0.13$$

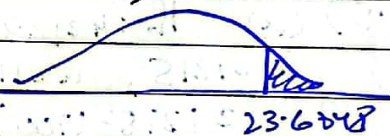
$p > \alpha$, fail to reject H_0 .

P9.25) $H_0: p \geq 0.63$ $H_a: p < 0.63$ $n = 100$
 $x = 55 \rightarrow$ characteristic of interest. $\alpha = 0.01$
 $z_{\alpha} = 2.3263$ $\hat{p} = 0.55$ $q = 0.45$ $z = (\hat{p} - p) / (\sqrt{pq/n})$
 $z = (0.55 - 0.63) / \sqrt{0.55(0.45)/100} = -1.61$
 $z < z_{\alpha} \rightarrow$ we fail to reject.

P9.27) $n = 380 \rightarrow$ purchased policy, always record: 164
 $\alpha = 0.01$ 48% record: $p = 0.48$
 $H_0: \mu = 0.48$ $H_1: \mu \neq 0.48$
 $\hat{p} = 164/380 = 0.431$ $\alpha/2 = 0.005$ $z_{\alpha/2} = 2.5758$
 $z = (0.431 - 0.48) / \sqrt{0.48(0.52)/380}$
 $z = -1.911$
 ~~$z > z_{\alpha/2} \rightarrow$ fail to reject H_0~~
 $z > z_{\alpha/2}$
 \hookrightarrow Fail to reject H_0 .

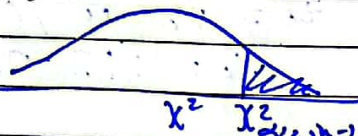


P9.33) (a) $H_0: \sigma^2 \leq 20$ $H_a: \sigma^2 > 20$ $\alpha = 0.05$, $n = 15$ $s^2 = 32$
 $\chi^2_{\alpha, n-1} = 23.6848$ $\chi^2 = (n-1)s^2/\sigma^2$
 $\chi^2 = (14)(32)/20 = 22.4$
 $\chi^2 < \chi^2_{\alpha, n-1} \rightarrow$ fail to reject H_0 .

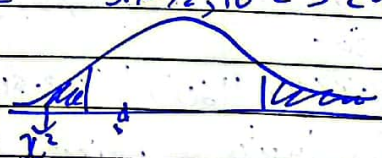


(b) $H_0: \sigma^2 \leq 8.5$ $H_a: \sigma^2 > 8.5$ $\alpha = 0.1$ $n = 22$ $s^2 = 17$
 $\alpha = 0.05$ $\chi^2_{\alpha, n-1} = 32.6706$
 $\chi^2 = (21)(17)/8.5 = 42$ $\chi^2 > \chi^2_{\alpha, n-1} \rightarrow$ reject H_0 .

(c) $H_0: \sigma^2 = 45$ $H_a: \sigma^2 < 45$ $\alpha = 0.01$, $n = 8$ $s^2 = 4.12$
 $s^2 = 16.9744$ $\chi^2_{\alpha, n-1} = 2.184753$
 $\chi^2 = (7)(16.9744)/45 = 2.64$



$\chi^2 < \chi^2_{\alpha, n-1} \rightarrow$ fail to reject H_0 .
 (d) $H_0: \sigma^2 \leq 5$ $H_a: \sigma^2 \neq 5$ $\alpha = 0.05$, $n = 11$ $s^2 = 1.2$
 $\alpha/2 = 0.025$ $\chi^2_{\alpha/2, n-1} = 20.4832$ $\chi^2_{1-\alpha/2, n-1} = 3.2470$
 $\chi^2 = (10)(1.2)/5 = 2.4$



$\chi^2 < \chi^2_{1-\alpha/2, n-1}$
 \hookrightarrow reject H_0 .

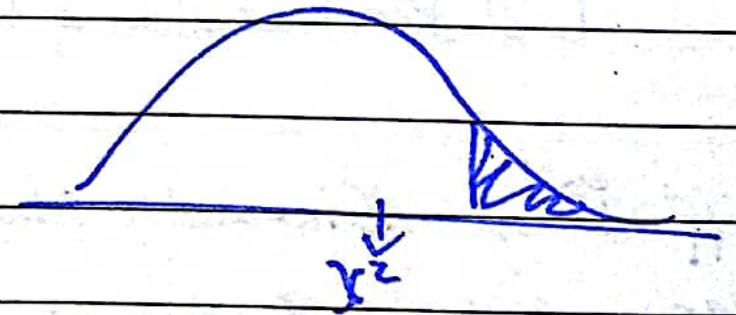
P9.35) $n=16$ $\alpha=0.001$ $H_0: \sigma^2 \leq 0.001$ $H_a: \sigma^2 > 0.001$

$\chi^2_{\alpha, 15} = 30.5779$ ~~See~~ $\bar{s} = 0.038035$

$$\bar{s}^2 = 0.001447$$

$$\chi^2 = (15)(0.001447)/0.001 =$$

~~$\chi^2 = 21.6999$~~ $\chi^2 = 21.6999$



$\chi^2 < \chi^2_{\alpha, n-1} \rightarrow \text{Fail to reject } H_0$