

Let $A_{DFA} = \{ (D, x) \mid \text{DFA } D, \text{ accepts string } x \}$. Show that $A_{DFA} \in L^1$.

DFA = Deterministic Finite Automata

For since it's a DFA, for every bit in string x there is a state it corresponds to. ~~and there is only one accept state~~
worktape must have $O(\log n)$ bits used at all times

~~There is a logspace transducer T~~
~~For every bit x_i that has a valid transition in the set for DFA D ,~~
~~the work tape~~

~~The logspace transducer T computes whether the i th bit of string x~~
~~has a valid transition in the DFA D . in this case it outputs a 1 onto~~
~~the work tape~~

We create a config graph for the transitions on DFA D with input x . Since it's a deterministic finite automata, we know that it will have at most 2 branches at most 2 branches. Since each config graph node has a size of $O(\log n)$ and the length of the branch can be at most $\frac{\text{height of tree} \times \log n}{\text{the total transitions}}$, we can say that the total space the work space occupies is $2^{n \times \log n} \times \text{height of tree} \times \log n$.

(This takes $n \log n$ space now that I look at it)

height of the tree will be linear, not logarithmic

"length of a branch can be the total transitions?"

¹ Refers to a particular space complexity class.
Logarithmic space

Quiz - 3

Consider the language TQBF = True Quantified Boolean Formula.

A fully quantified Boolean formula is a formula where every variable is quantified, using either existential or universal quantifiers, at the beginning of the sentence. Such a formula is equivalent to either true or false (since there are no free variables). If such a formula evaluates to true, then that formula is in the language TQBF.

E.g, $\forall x \exists y \forall z ((x \vee z) \wedge y) \in \text{TQBF}$, but $\forall x \exists y \forall z ((x \vee y) \wedge z) \notin \text{TQBF}$.

Prove that TQBF is coNP-Hard.

similar to Boolean expressions in 3SAT.

CoNP-Hard

Find all quantifiers that evaluate to TQBF = true

~~NP-Hard~~ NP-Hard equivalent will be all expressions that evaluate to false.

~~3SAT~~ 3SAT is a problem which involves satisfiability and is a known NP-Complete problem (NP-Hard + NP)

Solution:

How can a language be a member of 3SAT?

TQBF \in 3SAT

~~Since TQBF can be of size n with n variables, we need (n) clauses to result in a false output.~~

To ~~make~~ TQBF must have at least 1 pair of "and" clauses to result in a false output. If there are ~~n~~ variables, we need $\binom{n}{2}$ clauses to be of result false for TQBF to be false. and hence Since this is also a satisfiability problem which is decided in $\binom{n}{2}$ time like 3SAT, we can say that TQBF is at least as difficult as 3SAT because TQBF has at least as many clauses in 3SAT.

But 3SAT isn't known to be decidable in $\binom{n}{2}$ time!

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Quiz 1

Prove that the following language on graphs is in P. (You may pick any appropriate representation for graphs)

CONNECTED—The set of all connected graphs. That is, $G \in \text{CONNECTED}$ if every pair of vertices u, v in G is connected by a path.

G :

To check if ~~the~~ every pair of vertices is connected in G , full connectivity, we can use the Ford Fulkerson Algorithm to check for s.c.c's (if I'm not wrong). ~~wrong also~~

This problem is in P b/c ~~FF~~ FF algorithm uses DFS on a ~~G~~ G and G_{reversed} to determine if the

We can use ~~the~~ DAG's to check whether the graph ~~is~~ fully connected or strongly connected.

~~We run a DFS on G to obtain a set of vertices~~
We run a DFS on G and G_{rev} (given G is a directed graph), if both result in the same answer and the total number of vertices ~~in the~~ in the graph is equal to the total number of vertices in the result then the graph is fully connected.

This has a upper bound of $O(n^2)$ hence this problem is in P.

