Quiz 13A: Mapping Reduction

CS 212 Nature of Computation

Habib University — Fall 2023

Total Marks: 10
Duration: 15 minutes

Date: November 16, 2023
Time: 830–845h

Student ID: _____

Student Name: _____

1. (10 points) Let us extend mapping reduction to arbitrary sets.

> A set A is *mapping reducible* to a set B, written $A \leq_m B$, if there is a computable function f, where for every u,
> $$u \in A \iff f(u) \in B.$$
> The function f is called the *reduction* from A to B.

Prove or disprove the following claim.

Claim 1. *The following function, $f : P(\Sigma^*) \to P(\Sigma^*)$, is a reduction from the class of regular languages to the class of context-free languages.*

$$f(L) = \{w^R \mid w \in L\} \text{ where } w^R \text{ is the reverse of the string } w.$$

Solution: We disprove the claim by showing that f does not meet the required condition for a reduction. For ease of description, we denote the class of regular languages as RL and the class of context-free languages as CFL.

Proof. For f to be a reduction from RL to CFL, the following must hold.

$$\forall L\ (L \in RL \iff f(L) \in CFL).$$

This further reduces to 2 cases, one of which we show through a counterexample to not hold.

<u>Case</u>: $\forall L\ (f(L) \in CFL \implies L \in RL)$
Consider a language, $A \in CFL - RL$.
Then, $f(A)$ is a counterexample to the claim. $\qquad\square$