Homework–1
Fall 2023: CS 313: Computational Complexity Theory

Due: Thursday, September 28, 2023. Total Marks: 40

This homework can be discussed in groups of two, but must be **attempted individually**.

Question 1 [20 points]

Prove, for any **one** of the following languages, that it is **NP**-Complete, by giving a suitable mapping reduction. Consider only undirected graphs and improper subsets, and research the exact definitions of unknown terms.

- **Dominating Set** $= \{ (G, k) \mid$ Graph G contains a dominating set of size at most k $\}$.
- **Subset Sum** $= \{ (S, t) \mid S$ is a multiset of positive integers, and some subset of S sums to t $\}$.
- **Graph 3-Coloring** $= \{ G \mid$ Graph G is 3-colorable$\}$.
- **0/1 Integer Programming** $= \{ L \mid L$ is a list of inequalities with rational coefficients, satisfiable using an assignment of 0s and 1s to the variables only$\}$.
- **Comparative Divisibility** $= \{ (A, B) \mid A$ and B are strictly increasing sequences of positive integers, and some number c divides more elements of A than $B\}$.
- **Traveling Salesperson** $= \{ (G, l) \mid$ Positive-weighted graph G has a tour that visits every vertex once in at most L weight combined$\}$.
- **Bipartite Subgraph** $= \{ (G, k) \mid$ There is a bipartite spanning subgraph of G with at least k edges$\}$.
- **Monochromatic Triangle** $= \{ G \mid$ The edges of graph G can be partitioned into two disjoint sets, such that neither of the spanning subgraphs formed using the sets contains a triangle $\}$.
- **Set Splitting** $= \{ (S, C) \mid C$ is a collection of subsets of S, such that for some disjoint partition-into-two of S, no element of C is a subset of either partition$\}$.

Question 2 [10 points]

Define a *coding* κ to be a mapping, $\kappa : \Sigma^* \to \Sigma^*$ (not necessarily one-to-one).

For some string x, $x = \sigma_1 \cdots \sigma_n \in \Sigma^*$, we define $\kappa(x) = \kappa(\sigma_1) \cdots \kappa(\sigma_n)$ and for a language $L \subseteq \Sigma^*$, we define $\kappa(L) = \{\kappa(x) : x \in L\}$.

Show that the class **NP** is closed under *codings*.

Question 3 [10 points]

Let A be an **NP**-complete problem and B be a **coNP**-complete problem.

Show that, "**NP** $=$ **coNP** if and only if $A \leq_{\mathbf{P}} B$ and $B \leq_{\mathbf{P}} A$."