Computational Intelligence

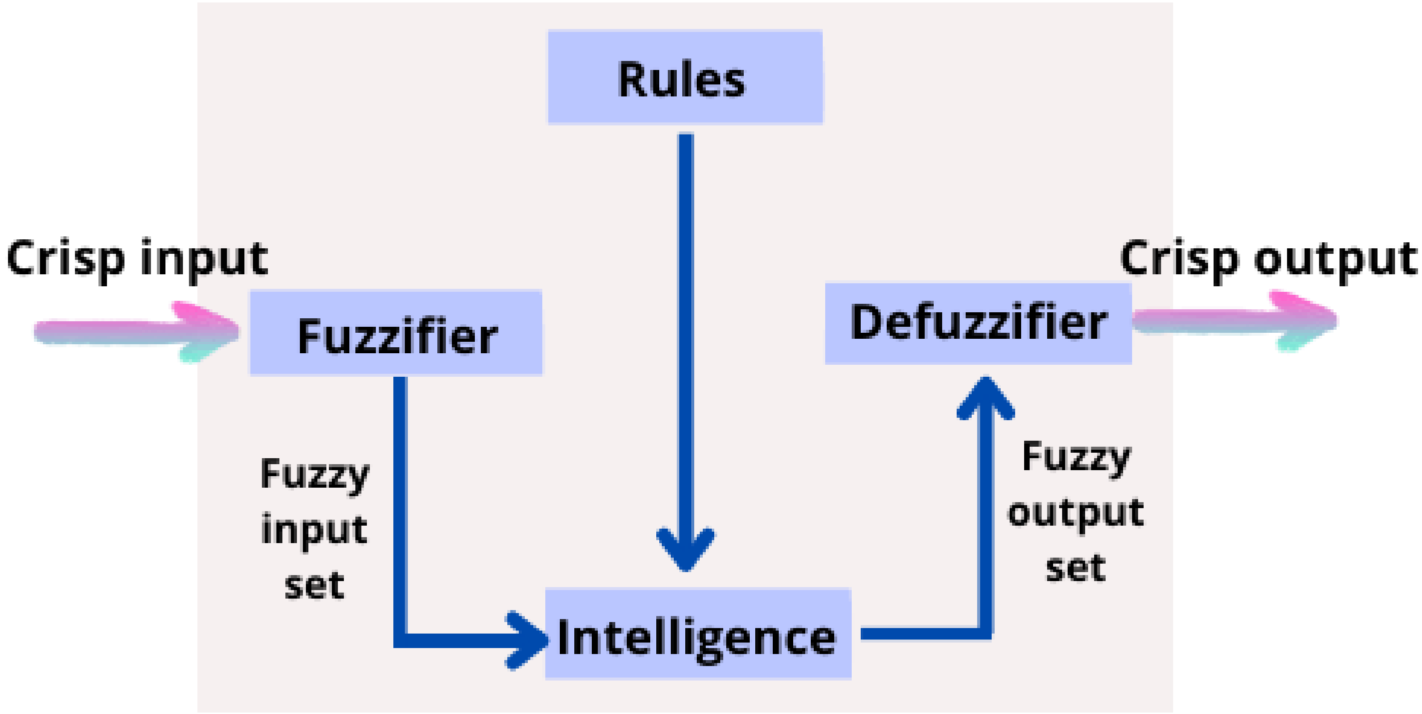
Fuzzy Reasoning

Unit # 13-2

Fuzzy Reasoning

- A fuzzy reasoning system consists of three other components, each performing a specific task in the reasoning process:
  - Fuzzification
  - Inferencing
  - Defuzzification

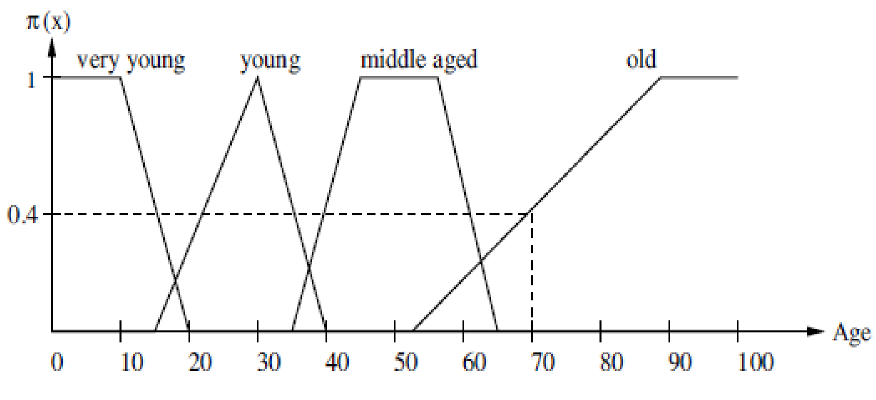
Fuzzy Inference System



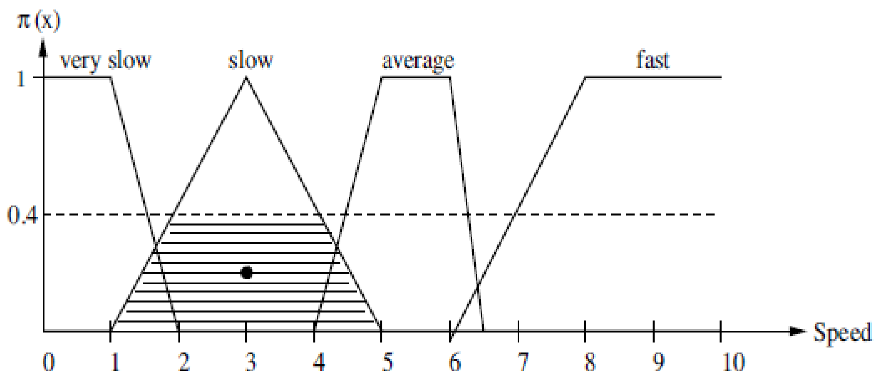
[Fuzzy Logic Explained | Baeldung on Computer Science]

Fuzzy Reasoning: Example 1

- Rule
  - If *Age* is *Old* the *Speed* is *Slow*
- What can be said about *Speed* if *Age* has the value of 70?



(a) *Age* Membership Functions



(b) *Speed* Membership Functions

Fuzzy Reasoning: Example 2

Let us suppose that we are designing a simple braking system for a car, which is designed to cope when the roads are icy and the wheels lock.

The rules for our system might be as follows:

Rule 1 IF pressure on brake pedal is med
THEN apply the brake

Rule 2 IF pressure on brake pedal is high
AND car speed is fast
AND wheel speed is fast
THEN apply the brake

Rule 3 IF pressure on brake pedal is high
AND car speed is fast
AND wheel speed is slow
THEN release the brake

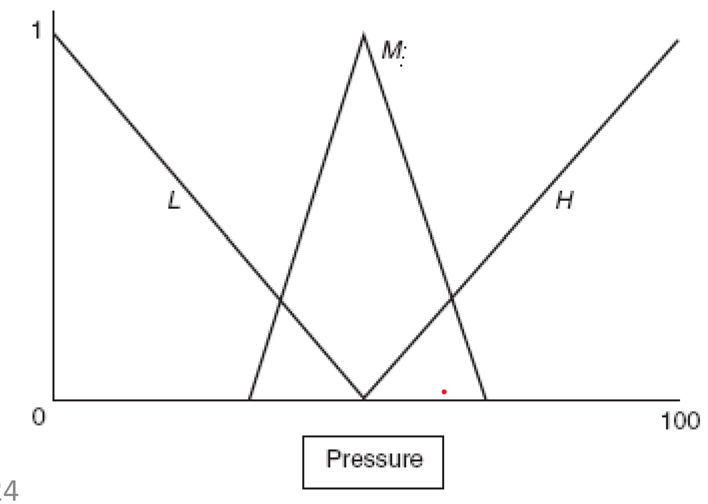
Rule 4 IF pressure on brake pedal is low
THEN release the brake

For this simple example, we will assume that brake pressure is measured from 0 (no pressure) to 100 (brake fully applied). We will define brake pressure as having three linguistic values: high (H), medium (M), and low (L), which we will define as follows:

$$H = \{(50, 0), (100, 1)\}$$

$$M = \{(30, 0), (50, 1), (70, 0)\}$$

$$L = \{(0, 1), (50, 0)\}$$



Example 2 (Cont'd)

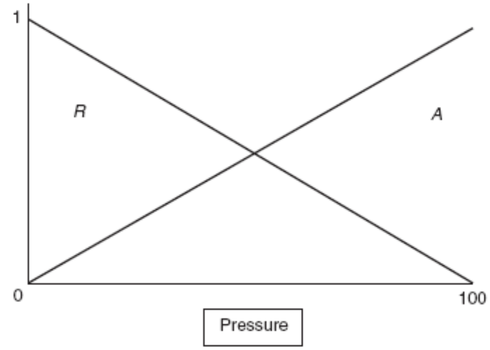
Similarly, we must consider the wheel speed. We will define the wheel speed as also having three linguistic values: slow, medium, and fast. We will define the membership functions for these values for a universe of discourse of values from 0 to 100:

$$S = \{(0, 1), (60, 0)\}$$

$$M = \{(20, 0), (50, 1), (80, 0)\}$$

$$F = \{(40, 0), (100, 1)\}$$

For the sake of simplicity, we will define the linguistic variable *car speed* using the same linguistic values (S, M, and F for slow, medium, and fast), using the same membership functions. Clearly, in a real system, the two would be entirely independent of each other.



Example 2: Fuzzification

- In a given situation, pressure value is 60, wheel speed is 55, and the car speed is 80.
  - $M_{P\_L}(60) = 0.0$
  - $M_{P\_M}(60) = 0.5$
  - $M_{P\_H}(60) = 0.2$
  - $M_{W\_S}(55) = 0.083$
  - $M_{W\_M}(55) = 0.833$
  - $M_{W\_F}(55) = 0.250$
  - $M_{C\_S}(80) = 0.0$
  - $M_{C\_M}(80) = 0.0$
  - $M_{C\_F}(80) = 0.667$

Inferencing

- Calculate the firing strength of each rule in the rule-base(min operator)
- Accumulate all activated outcomes to determine one single fuzzy value for that outcome (max or union operator)

Example 2: Inference

- Let's compute the firing strength of each rule:

Let us suppose that we are designing a simple braking system for a car, which is designed to cope when the roads are icy and the wheels lock.

The rules for our system might be as follows:

Rule 1 IF pressure on brake pedal is medium
THEN apply the brake

Rule 2 IF pressure on brake pedal is high
AND car speed is fast
AND wheel speed is fast
THEN apply the brake

Rule 3 IF pressure on brake pedal is high
AND car speed is fast
AND wheel speed is slow
THEN release the brake

Rule 4 IF pressure on brake pedal is low
THEN release the brake

- $M_{P\_L}(60) = 0.0$
- $M_{P\_M}(60) = 0.5$
- $M_{P\_H}(60) = 0.2$

- $M_{W\_S}(55) = 0.083$
- $M_{W\_M}(55) = 0.833$
- $M_{W\_F}(55) = 0.250$

- $M_{C\_S}(80) = 0.0$
- $M_{C\_M}(80) = 0.0$
- $M_{C\_F}(80) = 0.667$

Example 2: Inference

- Fuzzy values obtained from the four rules are:
  - Rule 1: 0.5
  - Rule 2: Min(0.2, 0.667, 0.25) = 0.2
  - Rule 3: Min(0.2, 0.667, 0.083) = 0.083
  - Rule 4: 0

- Apply break should be:

  (Rule 1 U Rule 2) - > 0.5 + 0.2 – 0.5*0.2 = 0.6
- Release break should be 0.083.

Let us suppose that we are designing a simple braking s
which is designed to cope when the roads are icy and the w

The rules for our system might be as follows:

Rule 1 IF pressure on brake pedal is medium
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Rule 2 IF pressure on brake pedal is high
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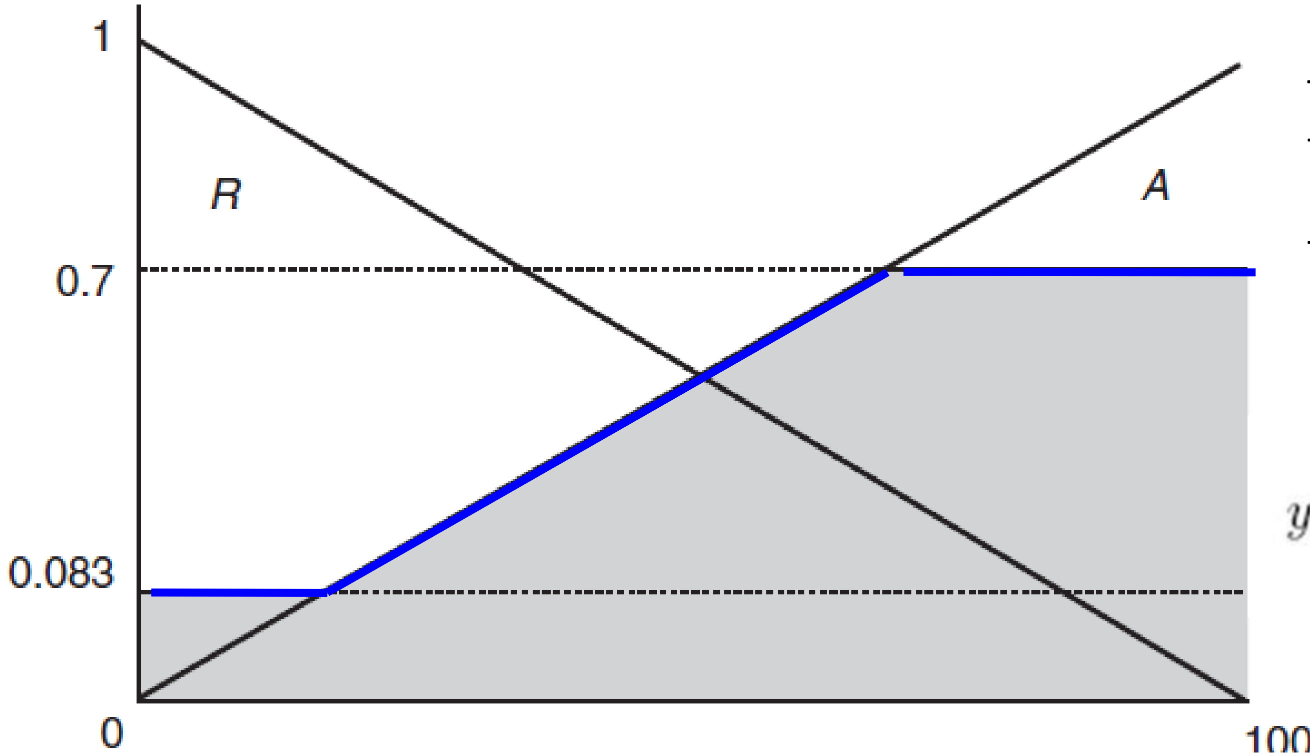
THEN apply the brake

Rule 3 IF pressure on brake pedal is high
AND car speed is fast
AND wheel speed is slow
THEN release the brake

Rule 4 IF pressure on brake pedal is low
THEN release the brake

Example 2: Defuzzification



There is a mistake in the figure and this line should be at 0.6 as explained on the previous slide.

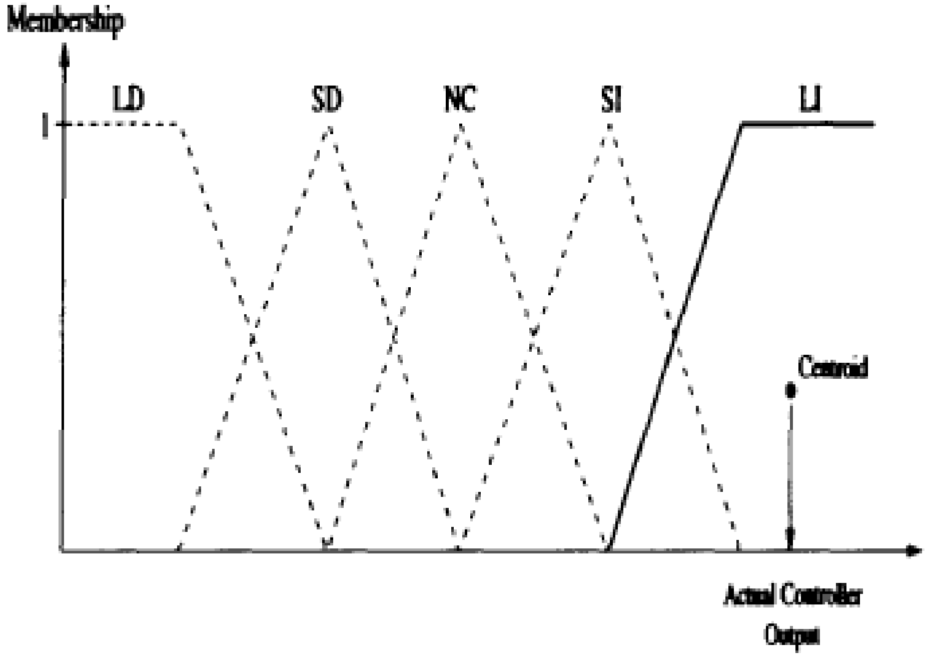
$$y = \frac{\sum_{i=1}^{n} \mu_o(x_i) x_i}{\sum_{i=1}^{n} \mu_o(x_i)}$$

- Center of Gravity = (5 x 0.083 + 10x0.1 + 15x0.15 +……+ 70x0.6 + 75x0.6 + 80x0.6 + …. + 100x0.6)/ (0.083 + 0.1 + 0.15 + …… + 0.6)
- Center of Gravity = 63.97

Defuzzification

- The **max-min method**: The rule with the largest firing strength is selected, and it is determined which consequent membership function is activated. The centroid of the area under that function is calculated and the horizontal coordinate of that centroid is taken as the output of the controller. For our example, the largest firing strength is 0.8, which corresponds to the *large_increase* membership function.

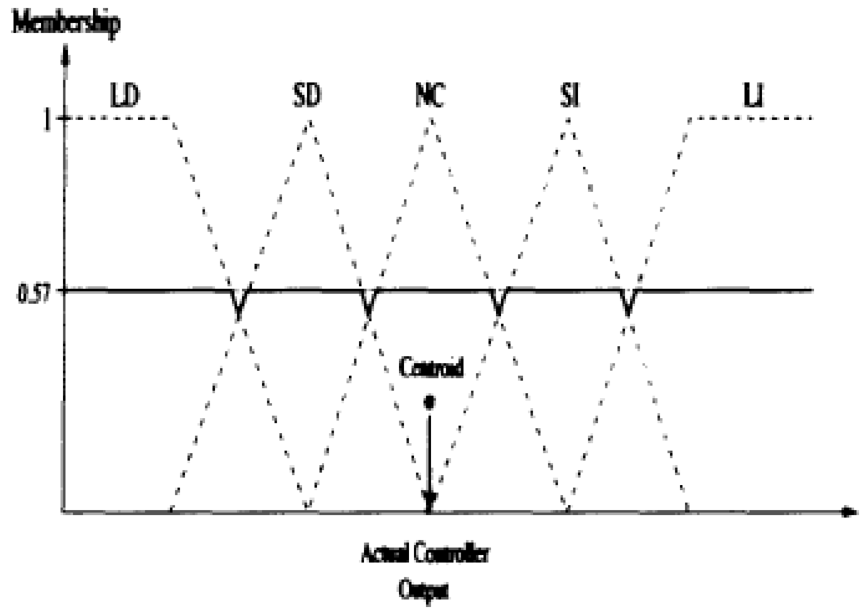
$\mu_{LI} = 0.8, \mu_{SI} = 0.6$ and $\mu_{NC} = 0.3$.



Defuzzification

- The **averaging method**: For this approach, the average rule firing strength is calculated, and each membership function is clipped at the average. The centroid of the composite area is calculated and its horizontal coordinate is used as output of the controller. All rules therefore play a role in determining the action of the controller.

$$\mu_{LI} = 0.8, \mu_{SI} = 0.6 \text{ and } \mu_{NC} = 0.3.$$

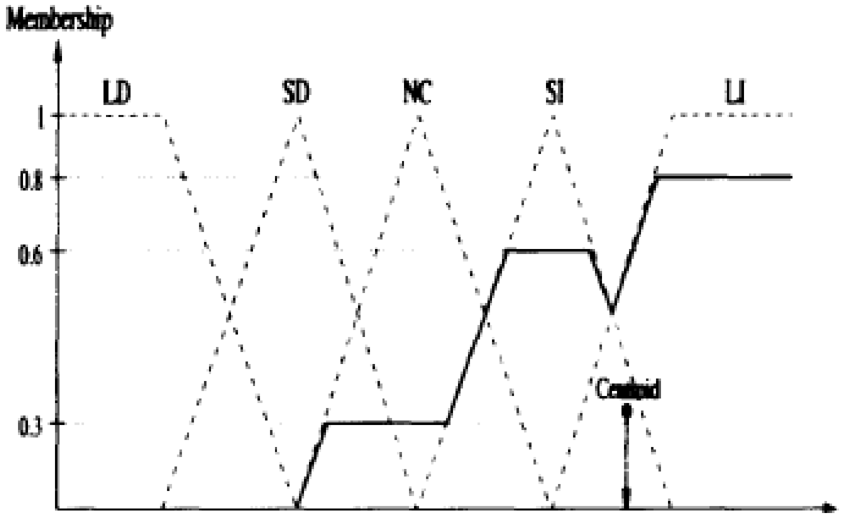


Defuzzification

- The **clipped center of gravity method**: For this approach, each membership function is clipped at the corresponding rule firing strengths. The centroid of the composite area is calculated and the horizontal coordinate is used as the output of the controller.

$$output = \frac{\sum_{i=1}^{n} x_i \mu_C(x_i)}{\sum_{i=1}^{n} \mu_C(x_i)}$$

$\mu_{LI} = 0.8, \mu_{SI} = 0.6$ and $\mu_{NC} = 0.3.$



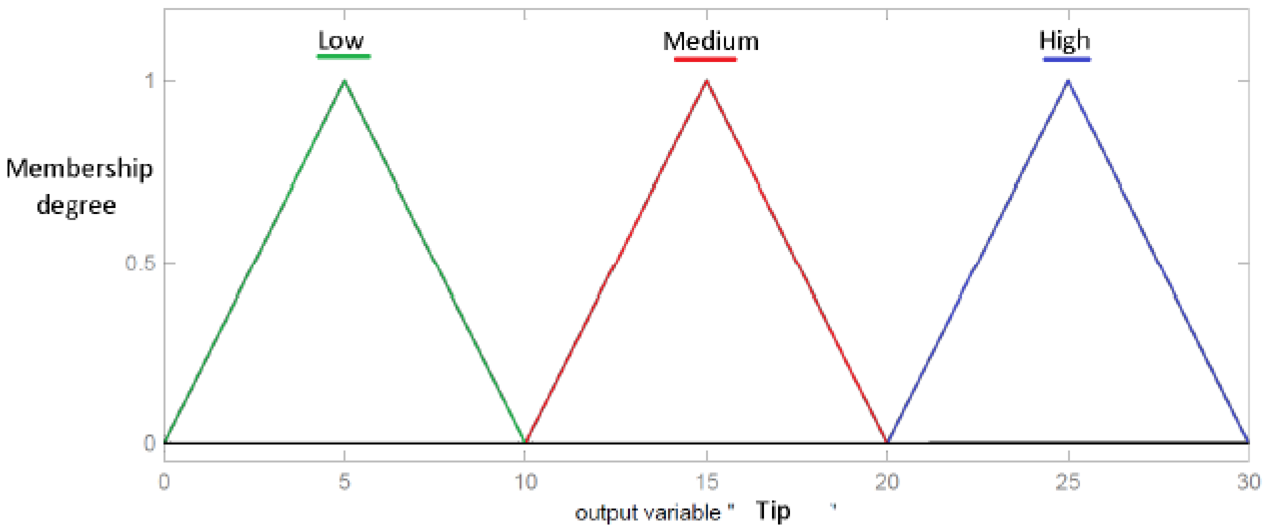
Defuzzification Techniques

- Maxima Based
- Centroid Based

- http://cse.iitkgp.ac.in/~dsamanta/courses/sca/resources/slides/FL-03%20Defuzzification.pdf

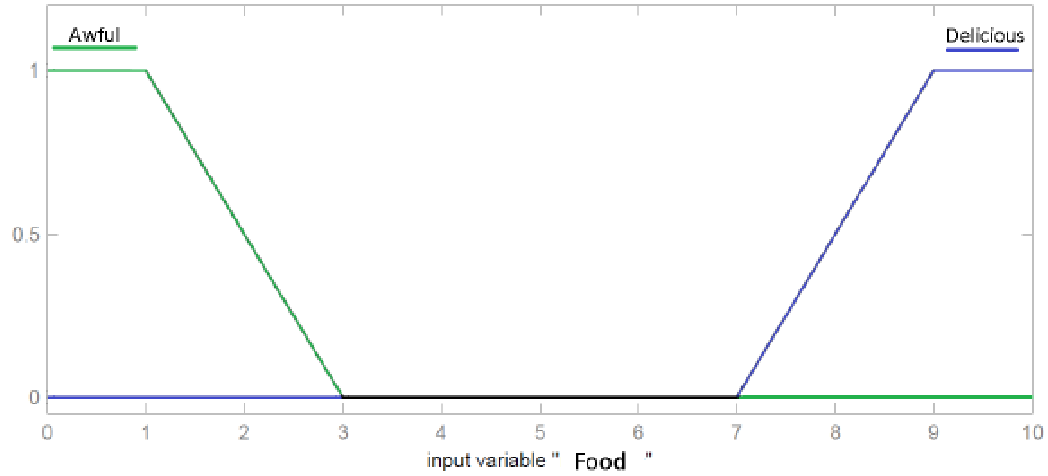
Examples 3 – Eating out

Tip



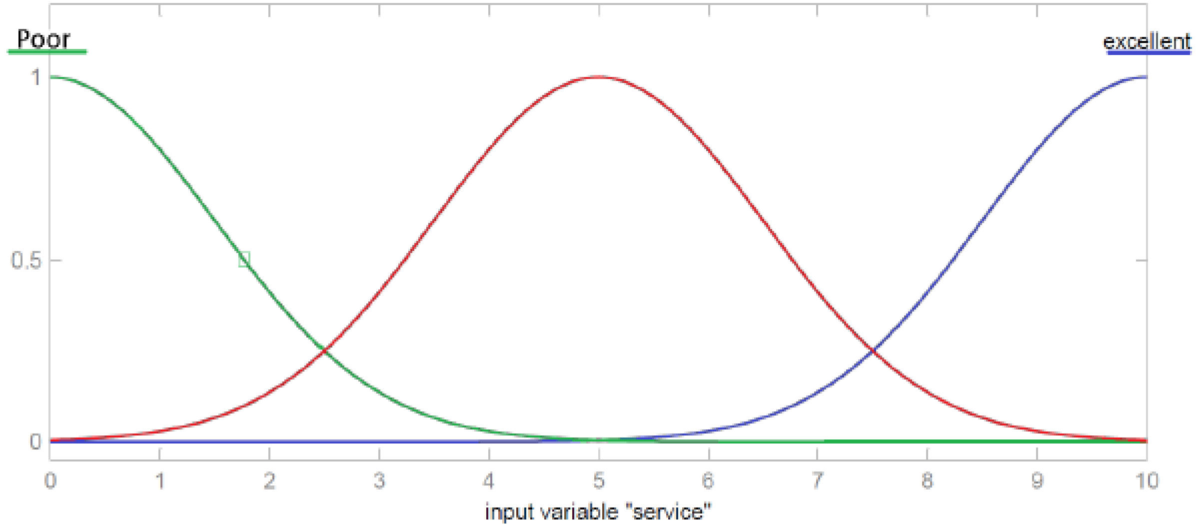
Examples 3 – Eating out

Quality of Food



Examples 3 – Eating out

Quality of Service



Examples 3 – Eating out

- 'If (the food quality is delicious) then (tip is high)'
- Food quality is rated 8.31 out of 10

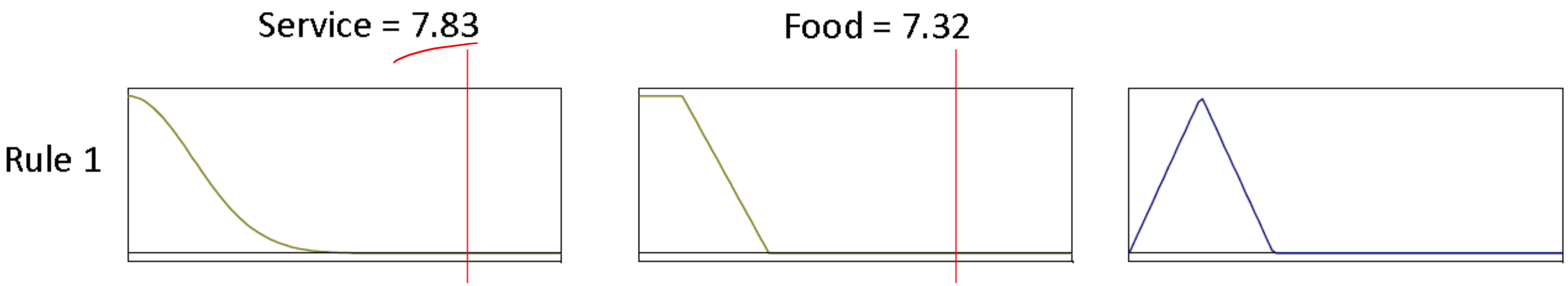
Figure 2.9: Example of fuzzy implication

Fuzzy Rules

If the service is bad or the food is awful	then the tip is low
If the service is good	then the tip is average
If the service is excellent or the food is delicious	then the tip is high

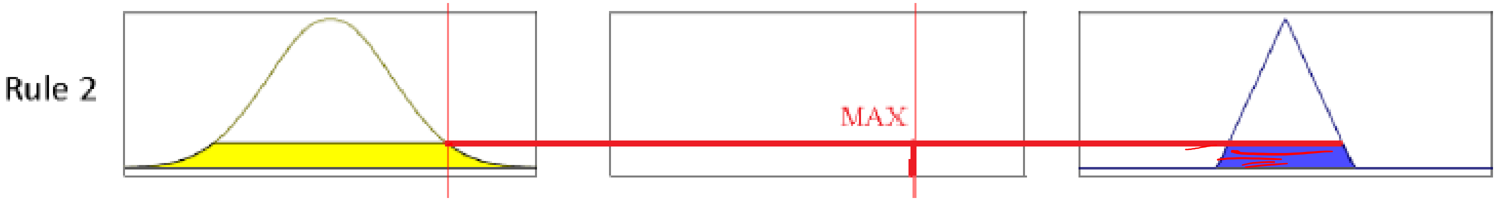
Rule 1

If the service is bad or the food is awful then the tip is low



Rule 2

If the service is good then the tip is average.



Rule 3

If the service is excellent or the food is delicious then the tip is high.

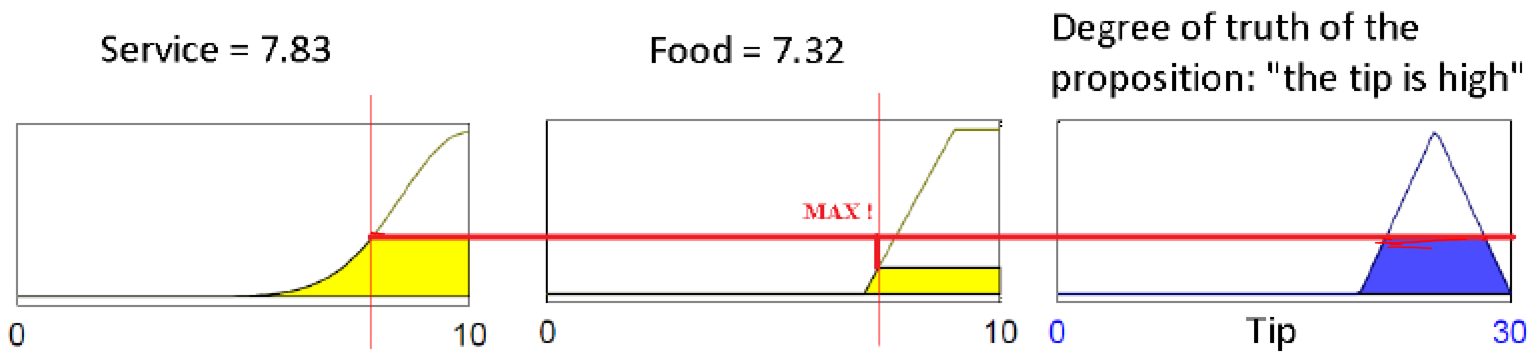


Figure 2.10: Example of fuzzy implication with conjunction OR translated into a MAX

Fuzzy Implication

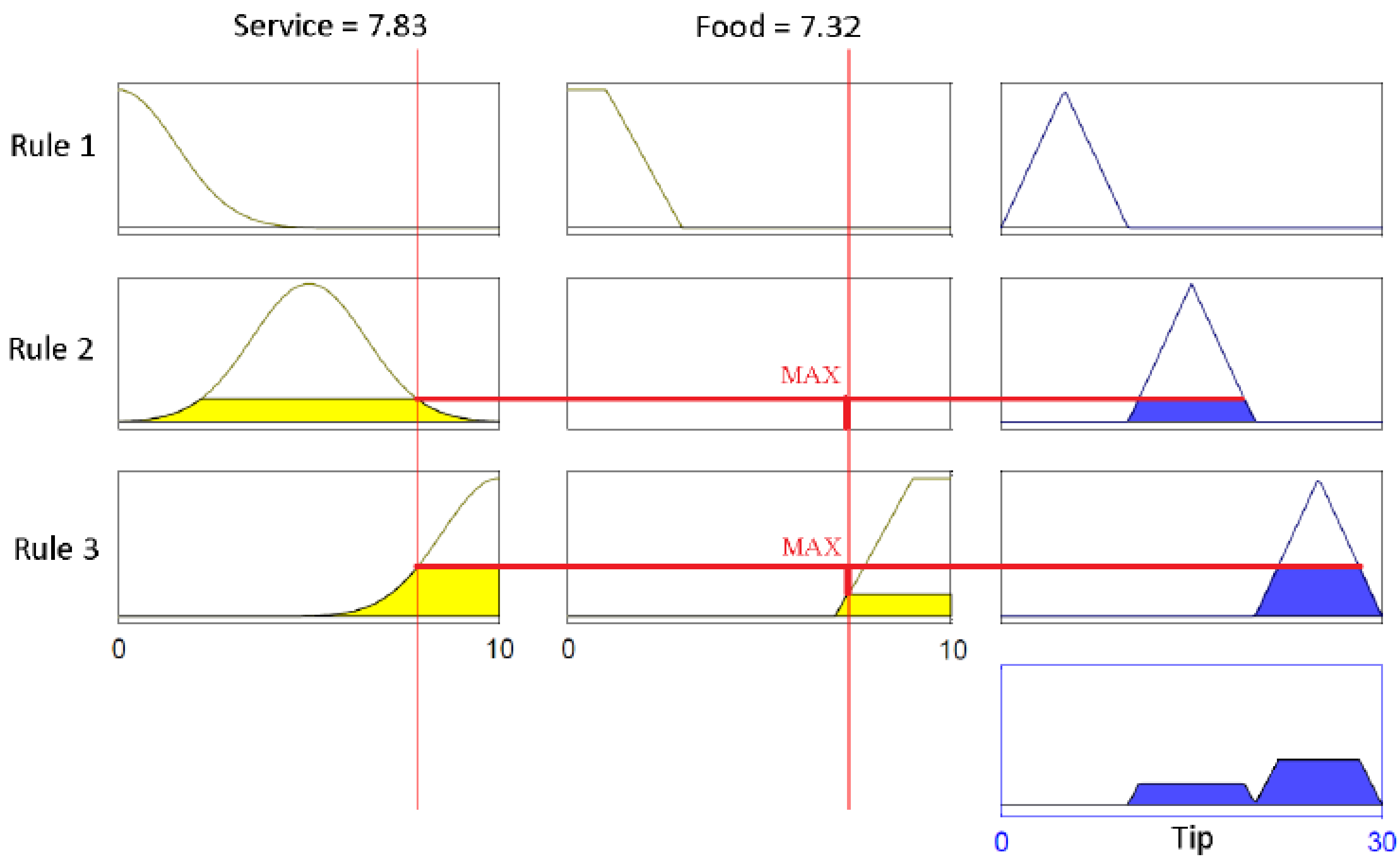
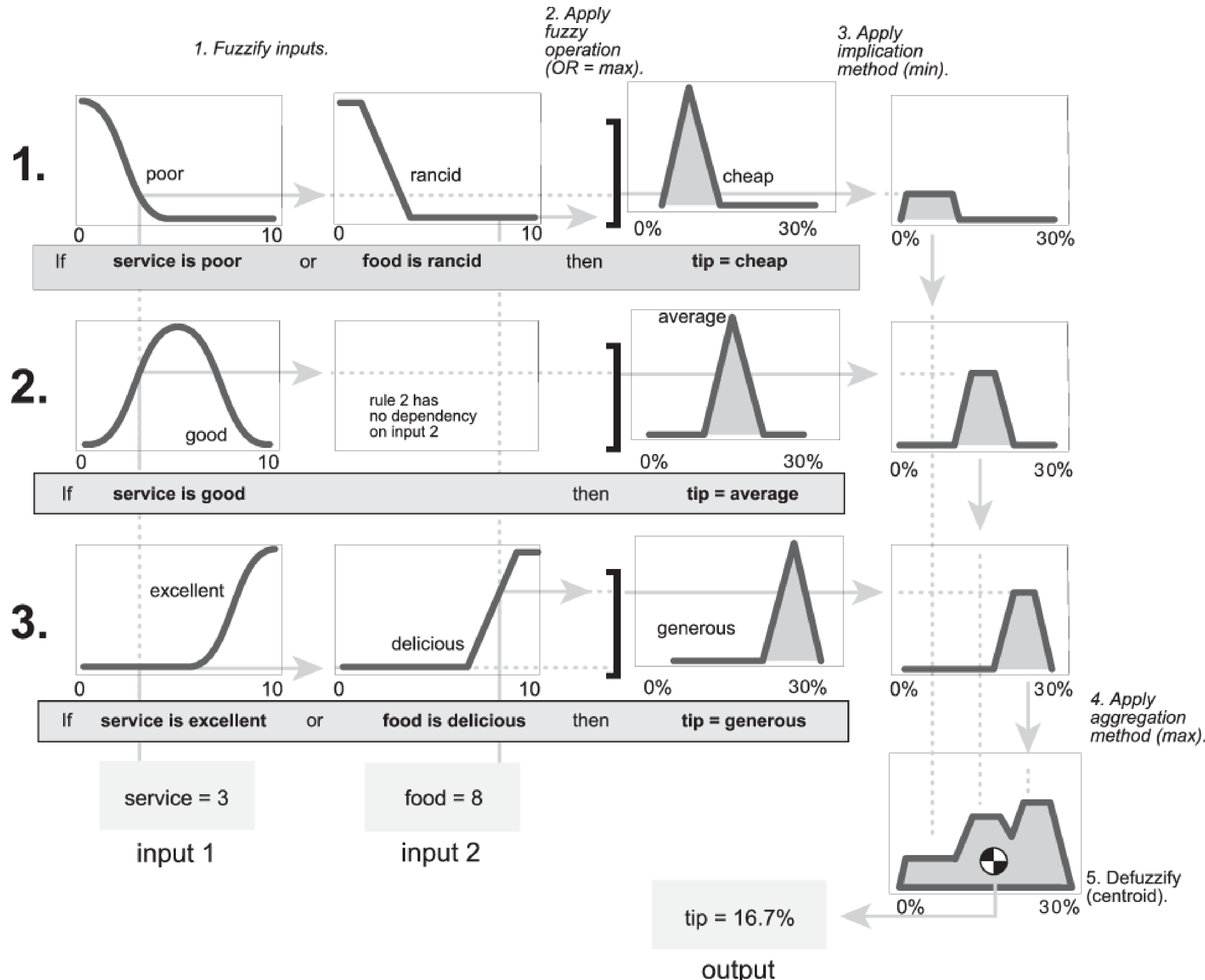


Figure 2.11: Example of fuzzy implication using the decision matrix


1. Fuzzify inputs.
2. Apply fuzzy operation (OR = max).
3. Apply implication method (min).
1.
poor
0
10
rancid
0
10
cheap
0%
30%
0%
30%
If service is poor or food is rancid then tip = cheap
2.
good
0
10
rule 2 has no dependency on input 2
average
0%
30%
0%
30%
If service is good then tip = average
3.
excellent
0
10
delicious
0
10
generous
0%
30%
0%
30%
If service is excellent or food is delicious then tip = generous
4. Apply aggregation method (max).
service = 3
input 1
food = 8
input 2
0%
30%
5. Defuzzify (centroid).
tip = 16.7%
output

Decision in classical vs fuzzy logic

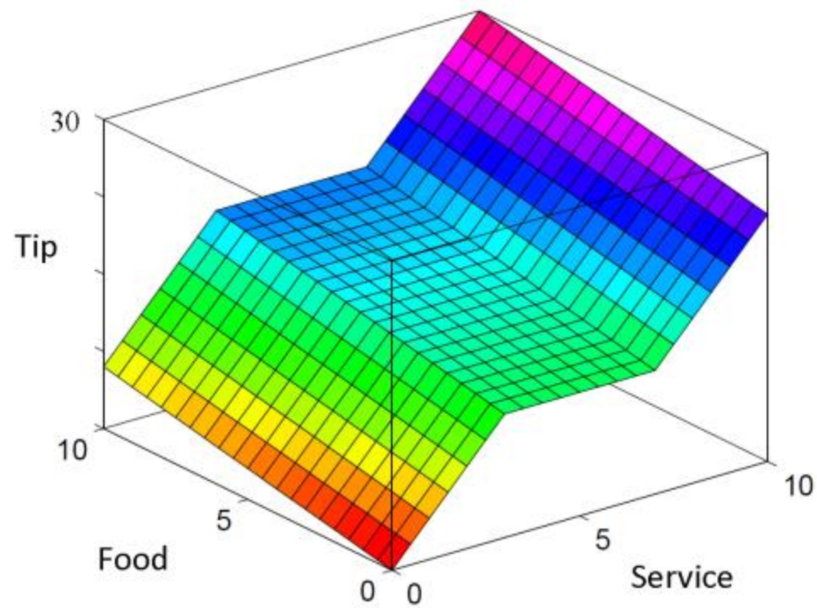


Figure 2.16: Decisions of a system based on classical logic

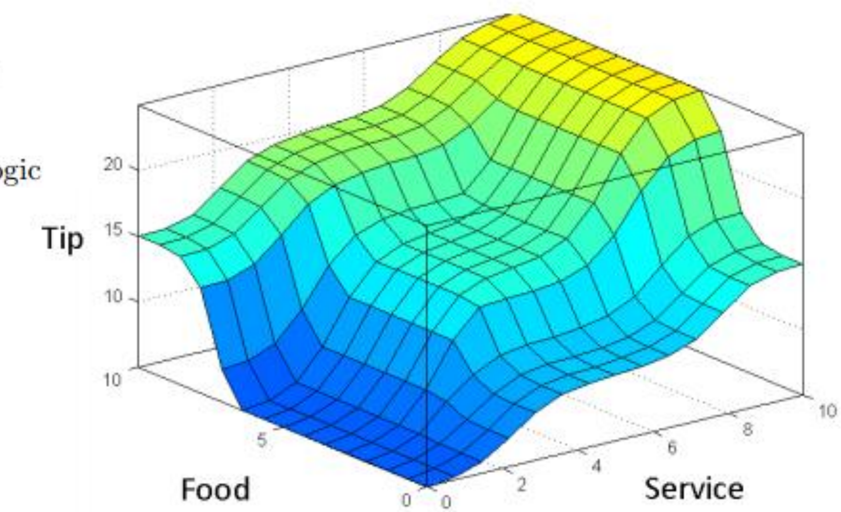


Figure 2.15: Decisions of a system based on fuzzy system

Example 4 - Susceptible to Malaria

- Consider an imaginary medical system designed to recommend a dose of quinine to a patient or doctor based on the likelihood that that patient might catch malaria while on vacation.

Example 4 - Susceptible to Malaria

- Average temperature of destination (*T*)
- Average humidity of destination (*H*)
- Proximity to large bodies of water (*P*)
- Industrialization of destination (*I*)
- Dose of Quinine *(Q)*

$$M_{TH}(x) = \begin{cases} \dfrac{x-25}{75} & for\ x \geq 25 \\ 0 & for\ x < 25 \end{cases}$$

$$M_{HH}(x) = \frac{x}{100}$$

$$M_{TL}(x) = \begin{cases} 1 - \dfrac{x}{75} & for\ x \leq 75 \\ 0 & for\ x > 75 \end{cases}$$

$$M_{HL}(x) = 1 - \frac{x}{100}$$

Example 4 - Susceptible to Malaria

$$M_{PN}(x) = \begin{cases} 1 & for\ x < 10 \\ \dfrac{40 - x}{30} & for\ 10 \le x < 40 \\ 0 & for\ x \ge 40 \end{cases}$$

$$M_{IH}(x) = \begin{cases} 0 & for\ x < 10 \\ \dfrac{x - 10}{10} & for\ 10 \le x < 20 \\ 1 & for\ x \ge 20 \end{cases}$$

$$M_{PF}(x) = \begin{cases} 0 & for\ x < 10 \\ \dfrac{x - 10}{30} & for\ 10 \le x < 40 \\ 1 & for\ x \ge 40 \end{cases}$$

$$M_{IL}(x) = \begin{cases} 1 & for\ x < 10 \\ \dfrac{20 - x}{10} & for\ 10 \le x < 20 \\ 0 & for\ x \ge 20 \end{cases}$$

$$M_{TH}(x) = \begin{cases} \dfrac{x-25}{75} & for\ x \geq 25 \\ 0 & for\ x < 25 \end{cases}$$

$$M_{HH}(x) = \frac{x}{100}$$

$$M_{TL}(x) = \begin{cases} 1 - \dfrac{x}{75} & for\ x \leq 75 \\ 0 & for\ x > 75 \end{cases}$$

$$M_{HL}(x) = 1 - \frac{x}{100}$$

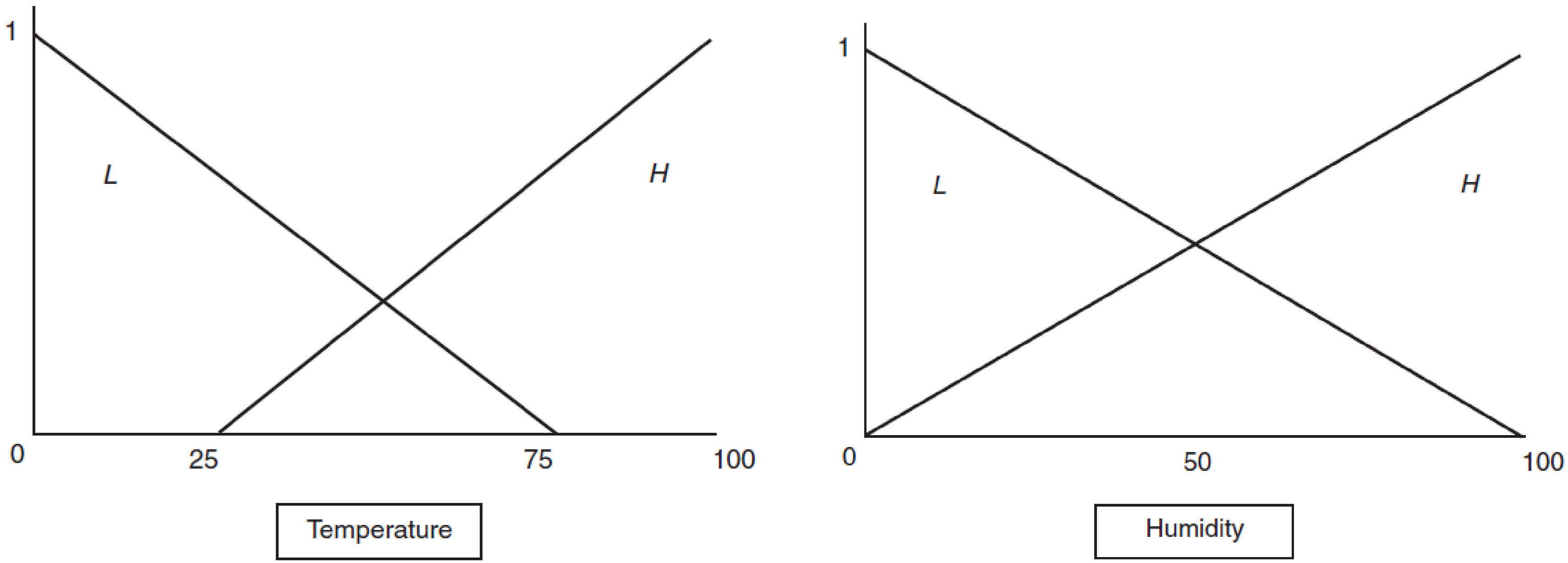
$$M_{PN}(x) = \begin{cases} 1 & for\ x < 10 \\ \dfrac{40-x}{30} & for\ 10 \leq x < 40 \\ 0 & for\ x \geq 40 \end{cases}$$

$$M_{IH}(x) = \begin{cases} 0 & for\ x < 10 \\ \dfrac{x-10}{10} & for\ 10 \leq x < 20 \\ 1 & for\ x \geq 20 \end{cases}$$

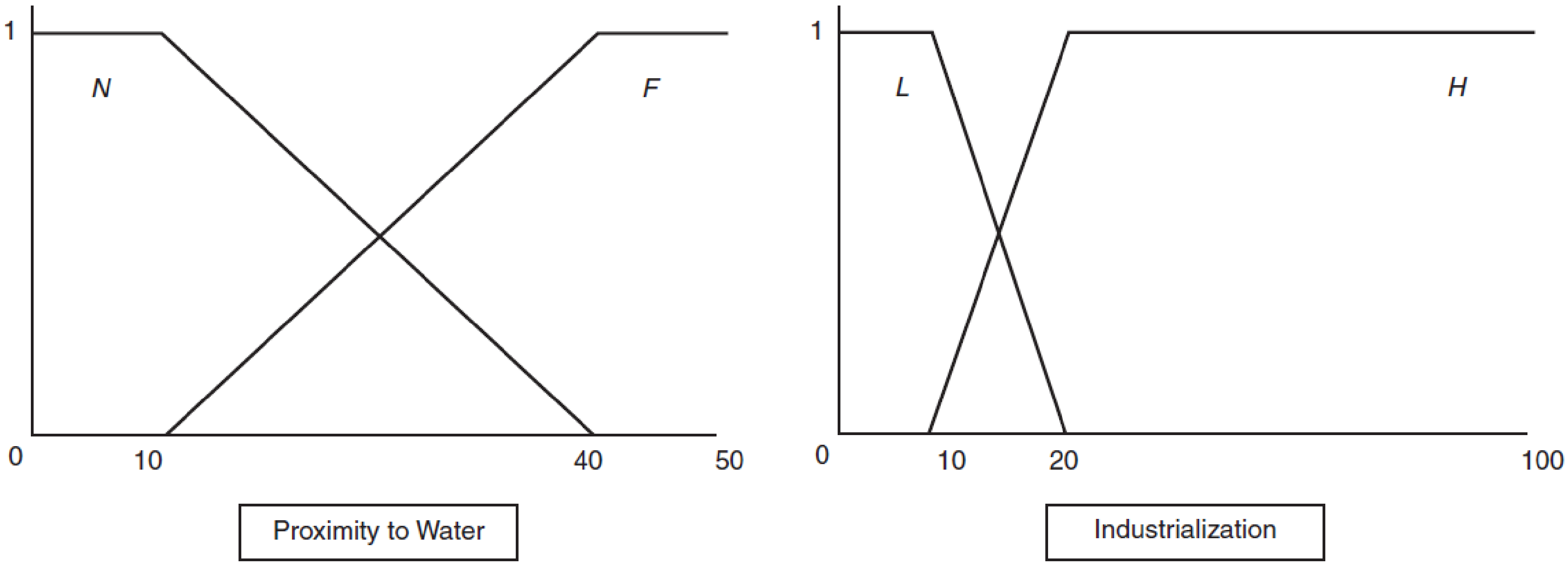
$$M_{PF}(x) = \begin{cases} 0 & for\ x < 10 \\ \dfrac{x-10}{30} & for\ 10 \leq x < 40 \\ 1 & for\ x \geq 40 \end{cases}$$

$$M_{IL}(x) = \begin{cases} 1 & for\ x < 10 \\ \dfrac{20-x}{10} & for\ 10 \leq x < 20 \\ 0 & for\ x \geq 20 \end{cases}$$

Example 4 - Susceptible to Malaria



Example 4 - Susceptible to Malaria

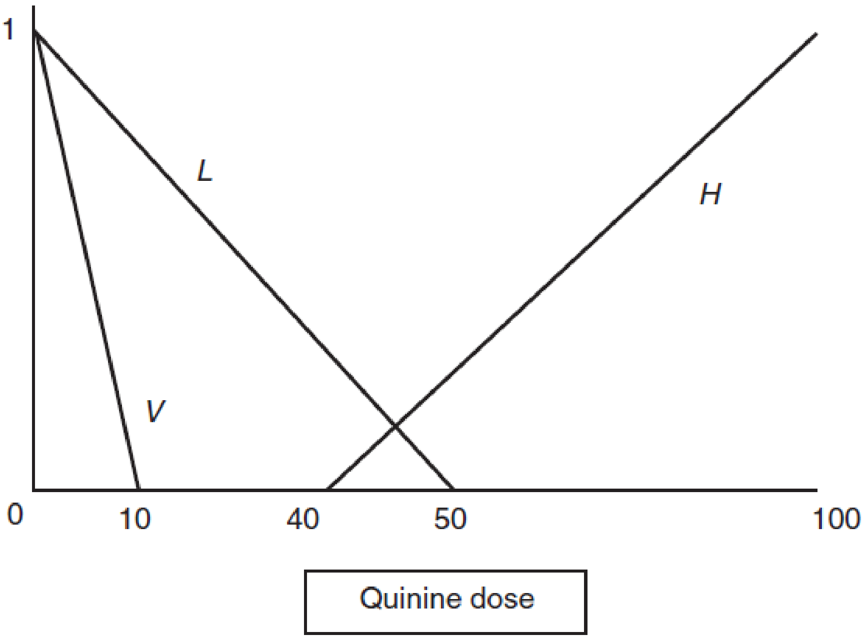


Example 4 - Susceptible to Malaria

$$M_{QV}(x) = \begin{cases} \dfrac{10-x}{10} & for\ x \le 10 \\ 0 & for\ x > 10 \end{cases}$$

$$M_{QL}(x) = \begin{cases} \dfrac{50-x}{50} & for\ x \le 50 \\ 0 & for\ x > 50 \end{cases}$$

$$M_{QH}(x) = \begin{cases} 0 & for\ x \le 40 \\ \dfrac{x-40}{60} & for\ x > 40 \end{cases}$$



V: Very Low Dose
L: Low Dose
H: High Dose

Example 4 - Susceptible to Malaria

- Rule 1
  - IF temperature is high
  - AND humidity is high
  - AND proximity to water is near
  - AND industrialization is low
  - THEN quinine dose is high
- Rule 2
  - IF industrialization is high
  - THEN quinine dose is low
- Rule 3
  - IF humidity is high
  - AND temperature is high
  - AND (industrialization is low
  - OR proximity to water is near)
  - THEN quinine dose is high
- Rule 4
  - IF temperature is low
  - AND humidity is low
  - THEN quinine dose is very low

Example 4 - Susceptible to Malaria

- Rule 3
  - IF humidity is high
  - AND temperature is high
  - AND (industrialization is low
  - OR proximity to water is near)
  - THEN quinine dose is high
- Rule 4
  - IF temperature is low
  - AND humidity is low
  - THEN quinine dose is very low

Example 4 - Susceptible to Malaria

- We will examine five sets of data, for five individuals, each of whom is traveling to a country that is at risk from malaria.
- The crisp data are as follows:
  - temperature = {80, 40, 30, 90, 85}
  - humidity = {10, 90, 40, 80, 75}
  - proximity to water = {15, 45, 20, 5, 45}
  - industrialization = {90, 10, 15, 20, 10}
- Hence, for example, person three is traveling to an area where the average temperature is 30, the humidity is 40, the distance to water is 20, and the level of industrialization is 15.

Example 4 - Fuzzification

For traveler # 1

- M_{TH} (80) = 0.733
- M_{TL} (80) = 0

- M_{HH} (10) = 0.1
- M_{HL} (10) = 0.9

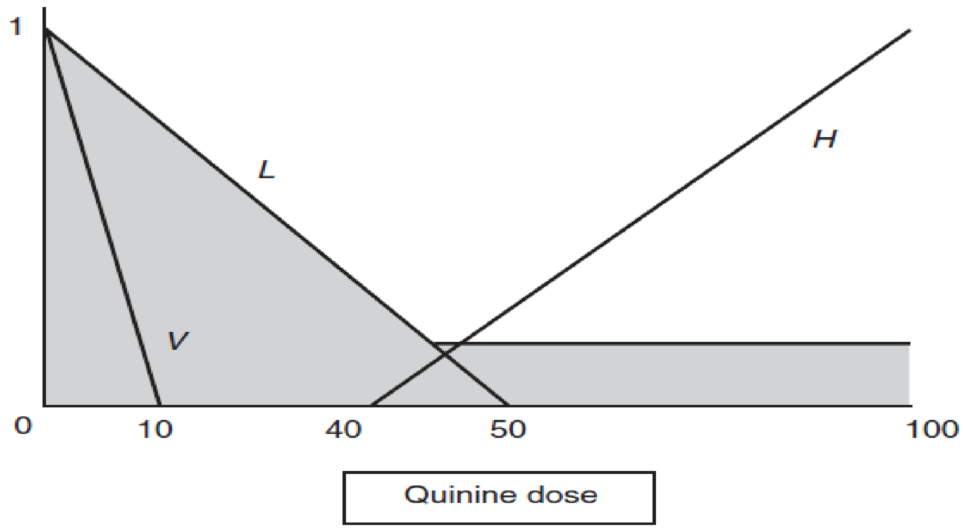
- M_{PN} (15) = 0.833
- M_{PF} (15) = 0.167

- M_{IH} (90) = 1
- M_{IL} (90) = 0

Example 4 - Implication

- Rule 1: 0
- Rule 2: 1
- Rule 3: 0.1
- Rule 4: 0
- In other words
  - Very low dose (V) = 0
  - Low does (L) = 1
  - High dose (H) = 0.1

Example 4: Defuzzification



$$C = \frac{(0.9 \times 5) + (0.8 \times 10) + (0.7 \times 15) + (0.6 \times 20) + (0.5 \times 25) + (0.4 \times 30) + (0.3 \times 35) + (0.2 \times 40) + (0.1 \times 45) + (0.1 \times 50) + (0.1 \times 55) + (0.1 \times 60) + (0.1 \times 65) + (0.1 \times 70) + (0.1 \times 75) + (0.1 \times 80) + (0.1 \times 85) + (0.1 \times 90) + (0.1 \times 95) + (0.1 \times 100)}{0.9 + 0.8 + 0.7 + 0.6 + 0.5 + 0.4 + 0.3 + 0.2 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1}$$

$= 165 / 5.6$

$= 29.46$

Fuzzification for all 5 Cases

$$M_{TH} = \{0.733, 0.2, 0.067, 0.867, 0.8\}$$

$$M_{TL} = \{0, 0.467, 0.6, 0, 0\}$$

$$M_{HH} = \{0.1, 0.9, 0.4, 0.8, 0.75\}$$

$$M_{HL} = \{0.9, 0.1, 0.6, 0.2, 0.25\}$$

$$M_{PN} = \{0.833, 0, 0.667, 1, 0\}$$

$$M_{PF} = \{0.167, 1, 0.333, 0, 1\}$$

$$M_{IH} = \{1, 0, 0.5, 1, 0\}$$

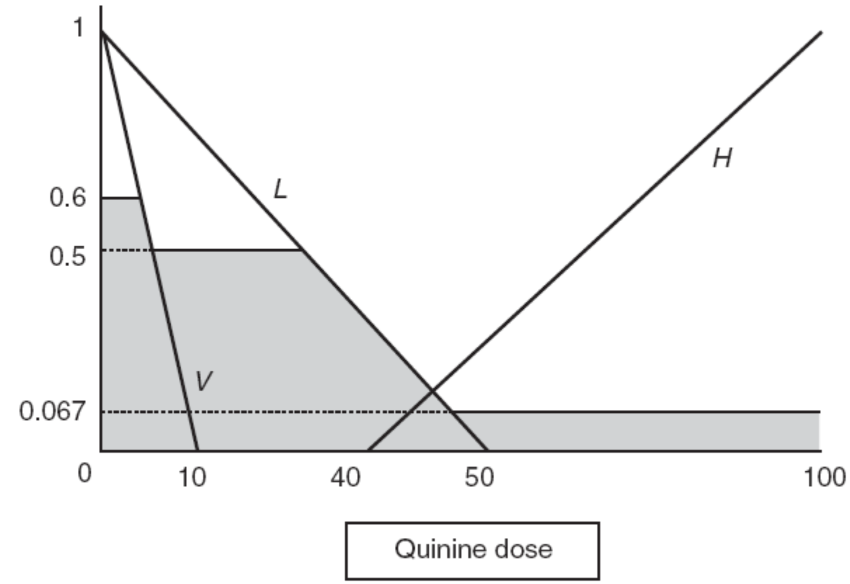
$$M_{IL} = \{0, 1, 0.5, 0, 1\}$$

Inferencing for all 5 Cases

- Rule 1
  - (high dose): {0, 0, 0.067, 0, 0}
- Rule 2
  - (low dose): {1, 0, 0.5, 1, 0}
- Rule 3
  - (high dose): {0.1, 0.2, 0.067, 0.8, 0.75}
- Rule 4
  - (very low dose): {0, 0.1, 0.6, 0, 0}
- Since both Rule 1 and 3 suggest high dose, we can take max of them. Thus
  - high dose: {0.1, 0.2, 0.067, 0.8, 0.75}

Example 4 (Cont'd)

- For Traveler 3
  - V: 0.6
  - L: 0.5
  - H: 0.067



$$C = \frac{(0.6 \times 5) + (0.5 \times 10) + (0.5 \times 15) + (0.5 \times 20) + (0.5 \times 25) + (0.4 \times 30) + (0.3 \times 35) + (0.2 \times 40) + (0.1 \times 45) + (0.067 \times 50) + (0.067 \times 55) + (0.067 \times 60) + (0.067 \times 65) + (0.067 \times 70) + (0.067 \times 75) + (0.067 \times 80) + (0.067 \times 85) + (0.067 \times 90) + (0.067 \times 95) + (0.067 \times 100)}{0.6 + 0.5 + 0.5 + 0.5 + 0.5 + 0.4 + 0.3 + 0.2 + 0.1 + 0.067 + 0.067 + 0.067 + 0.067 + 0.067 + 0.067 + 0.067 + 0.067 + 0.067 + 0.067 + 0.067}$$

$$= 128 \, / \, 4.3$$

$$= 29.58$$

Applications

Product	Company	Fuzzy Logic
Anti-lock brakes	Nissan	Use fuzzy logic to controls brakes in hazardous cases depend on car speed, acceleration, wheel speed, and acceleration
Auto transmission	NOK/Nissan	Fuzzy logic is used to control the fuel injection and ignition based on throttle setting, cooling water temperature, RPM, etc.
Auto engine	Honda, Nissan	Use to select geat based on engine load, driving style, and road conditions.
Copy machine	Canon	Using for adjusting drum voltage based on picture density, humidity, and temperature.
Cruise control	Nissan, Isuzu, Mitsubishi	Use it to adjusts throttle setting to set car speed and acceleration
Dishwasher	Matsushita	Use for adjusting the cleaning cycle, rinse and wash strategies based depend upon the number of dishes and the amount of food served on the dishes.
Elevator control	Mitsubishi Toshiba	Use it to reduce waiting for time-based on passenger traffic
Golf diagnostic system	Maruman Golf	Selects golf club based on golfer's swing and physique.
Fitness management	Omron	Fuzzy rules implied by them to check the fitness of their employees.
Kiln control	Nippon Steel	Mixes cement
Microwave oven	Mitsubishi Chemical	Sets lunes power and cooking strategy
Palmtop computer	Hitachi, Sharp, Sanyo, Toshiba	Recognizes handwritten Kanji characters
Plasma etching	Mitsubishi Electric	Sets etch time and strategy

Strength

- It is very tolerant on Imprecise Data
- It can take vague information and give a reasonable suggestion
- It is also conceptually easy to understand because it is based on linguistic terms.
- Fuzzy logic also is good with expert and common sense knowledge.

Weaknesses

- Conventional approaches may be formally verified to work, where as fuzzy logic can not be empirically verified.

Applications

- Fuzzy Logic - Applications (tutorialspoint.com)

Neuro-Fuzzy System

- Neuro-fuzzy hybridization results in a hybrid intelligent system that synergizes the two techniques by combining the human-like reasoning style of fuzzy systems with the learning and connectionist structure of neural networks.

Neuro-Fuzzy Systems

- This term is also used to describe some other configurations including:
  - Deriving fuzzy rules from trained networks.
  - Fuzzy logic based tuning of neural network training parameters.
  - Fuzzy logic criteria for increasing a network size.
  - Realizing fuzzy membership function through clustering algorithms in unsupervised learning in SOMs and neural networks.
  - Representing fuzzification, fuzzy inference and defuzzification through multi-layers feed-forward networks.

Mamdani Neuro-fuzzy System

- A Mamdani neuro-fuzzy system uses a supervised learning technique (backpropagation learning) to learn the parameters of the membership functions.

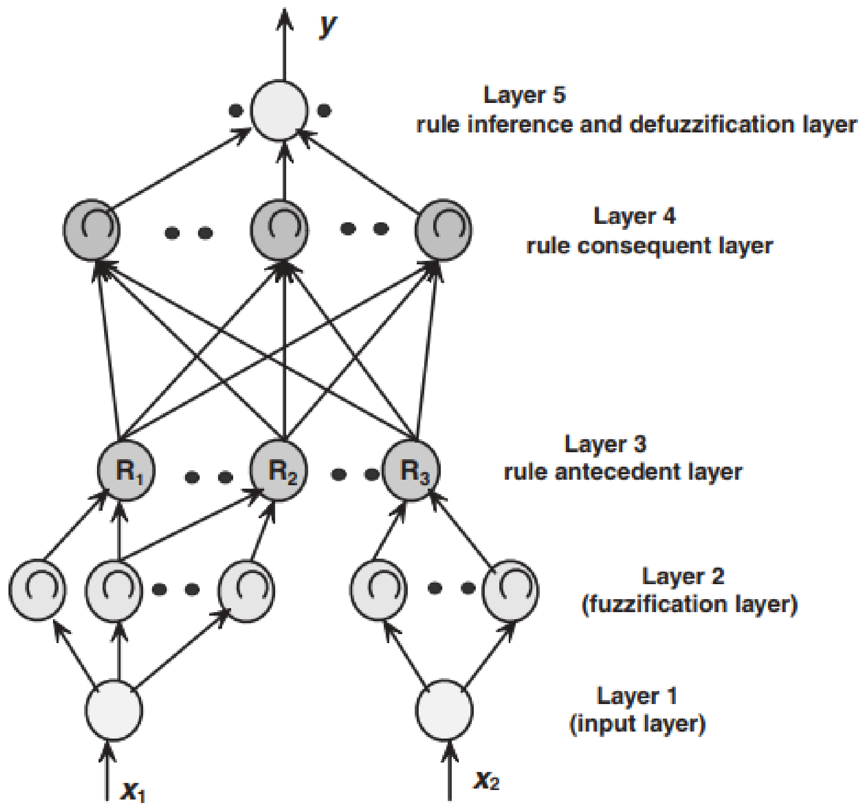


Fig. 3.3. Mamdani neuro-fuzzy system

Mamdani Neuro-fuzzy System…

- **Layer-1(input layer):** No computation is done in this layer. Each node in this layer, which corresponds to one input variable, only transmits input values to the next layer directly. The link weight in layer 1 is unity.
- **Layer-2 (fuzzification layer):** Each node in this layer corresponds to one linguistic label (excellent, good, etc.) to one of the input variables in layer 1. In other words, the output link represent the membership value, which specifies the degree to which an input value belongs to a fuzzy set, is calculated in layer 2. A clustering algorithm will decide the initial number and type of membership functions to be allocated to each of the input variable. The final shapes of the MFs will be fine tuned during network learning.
- **Layer-3 (rule antecedent layer):** A node in this layer represents the antecedent part of a rule. The output of a layer 3 node represents the firing strength of the corresponding fuzzy rule.

Mamdani Neuro-fuzzy System…

- **Layer-4 (rule consequent layer):** This node basically has two tasks. To combine the incoming rule antecedents and determine the degree to which they belong to the output linguistic label (high, medium, low, etc.). The number of nodes in this layer will be equal to the number of rules.
- **Layer-5 (combination and defuzzification layer)**: This node does the combination of all the rules consequents and finally computes the crisp output after defuzzification.

Resources

- https://www.researchgate.net/publication/267041266_Introduction_to_fuzzy_logic
- http://cse.iitkgp.ac.in/~dsamanta/courses/sca/resources/slides/FL-03%20Defuzzification.pdf
- https://www.researchgate.net/publication/267041266_Introduction_to_fuzzy_logic
- Fuzzy Logic Explained | Baeldung on Computer Science