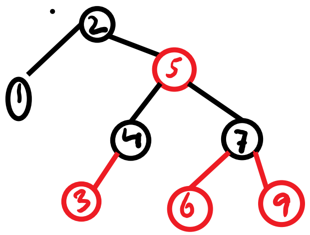

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CS 201 Data Structure II (L2 / L5)

Height of AVL Tree

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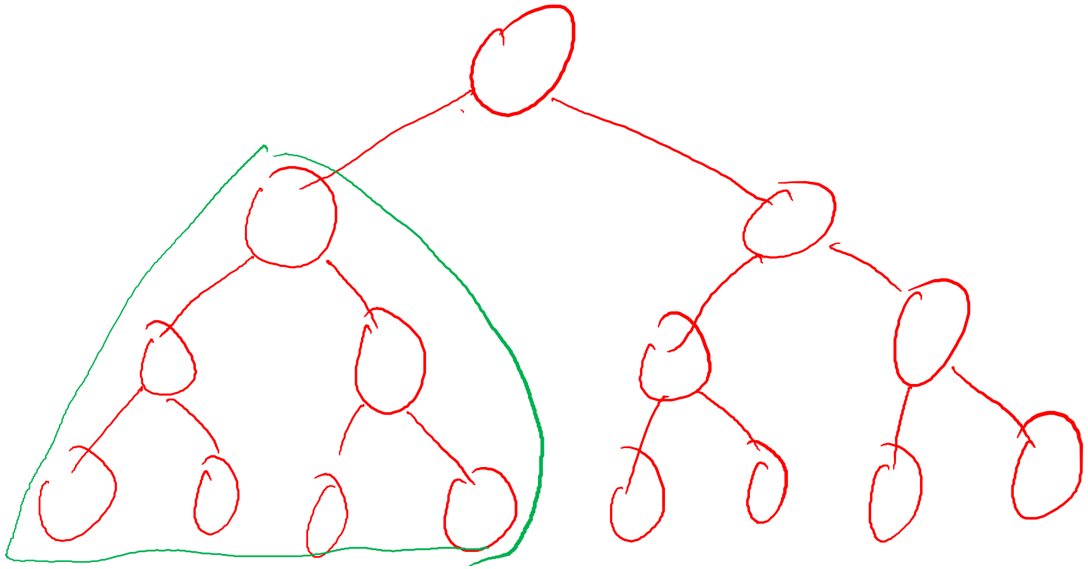
Refer the worksheet on LMS for this Lecture

Height of an AVL Tree

- number of nodes in a complete binary tree with height h?
- Minimum number of nodes for an AVL tree with height h?
  - h=0?
  - h=1?
- N_h= minimum number of nodes
  in an AVL tree of height h
- $N_h = N_{h-2} + N_{h-1} + 1$
- Fibonacci analysis states:
  - $N_h \geq \phi^h$ ($\phi \approx 1.62$)



h	0	1	2	3	4	5
n(h)	1	2	4	7	12	20

Proof for height

- $N_h = N_{h-2} + N_{h-1} + 1$ (Eq. 1)

Base Case: $N_0 = 1, N_1 = 2$

$$N_h = N_{h-1} + N_{h-2} + 1$$
$$N_4 = N_3 + N_2 + 1 = (N_2 + N_1 + 1) + (N_1 - N_0 + 1) + 1$$
$$N_4 = N_3 + N_2 + 1 = ((N_1 + N_0 + 1) + N_1 + 1) + (N_1 - N_0 + 1) + 1$$

We know $N_{h-1} > N_{h-2}$

$$N_h > N_{h-2} + N_{h-2} + 1$$
$$N_h > 2 \cdot N_{h-2}$$
$$N_h > 2 \cdot N_{h-2} > 4 \cdot N_{h-4} > \cdots > 2^i \cdot N_0$$
$$N_h > 2^i N_{h-2 \cdot i}$$
$$N_h > 2^{\frac{h}{2}}$$

$$h - 2, h - 4, \ldots, h - 2 \cdot i \Rightarrow h - 2 \cdot i = 0 \Rightarrow i = \frac{h}{2}$$