Habib University
Course Code: EE 468/CE 468: Mobile Robotics
Course Title: Mobile Robotics
Instructor name: Dr. Basit Memon
Examination: Quiz 3 Exam Date: November 29, 2023
Total Marks: 100 Duration: 30 minutes

Instructions

1. This quiz is open book/notes. Use of internet is not permitted.
2. You are welcome to write computer programs to assist your computation.
3. The exam will be administered under HU student code of conduct (see Chapter 3 of `https://habibuniversity.sharepoint.com/sites/Student/code-of-conduct`).
4. Make sure that you show your work (intermediate steps). Most of the points for any question will be given based on the followed process.
5. Kindly use a black/blue pen to write your hand-written solutions, so that they are legible.

Questions

Problem 1
CLO2-C4
100 points

We're going to model a robot vacuum cleaner in the Bayesian framework. To simplify this problem, we'll define the state of the robot, X, as *the room of the house in which it is located*. There are three possible rooms in this house: Kitchen, Hallway, Dining room. It is known for certain that the initial state of the robot is the *Dining Room*. The three states are shown in Figure 1.

You'll notice U, D, R, L on top of the arrows. Our primitive robot only can take one of these four possible control actions at any time - *U: Up; D: Down; R: Right; L: Left*. The behavior of the robot in response to actions is not certain, e.g. if the robot is in the hallway and the control action is 'R', it only may end up in the Dining room with some probability. This is captured in Table 1.

The robot is equipped with a light sensor, which provides a measurement Z_t at every time instant, where $Z_t \in \{dark, medium, light\}$. The sensor is also not precise and the probability of getting a reading in any given room is captured in Table 2.

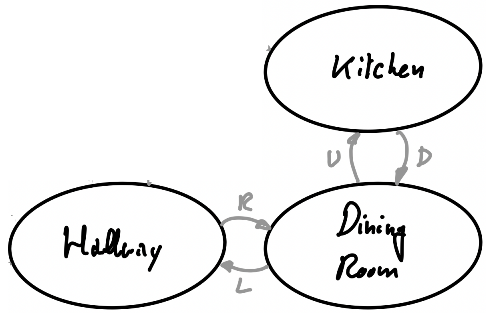

Figure 1: Dynamic Bayes Network with Actions

X_1	A	Kitchen	Hallway	Dining Room
Kitchen	L	1	0	0
Kitchen	R	1	0	0
Kitchen	U	1	0	0
Kitchen	D	0.2	0	0.8
Hallway	L	0	1	0
Hallway	R	0	0.2	0.8
Hallway	U	0	1	0
Hallway	D	0	1	0
Dining Room	L	0	0.8	0.2
Dining Room	R	0	0	1
Dining Room	U	0.8	0	0.2
Dining Room	D	0	0	1

Table 1: State Transition Table

X_1	dark	medium	light
Kitchen	0.1	0.3	0.6
Hallway	0.6	0.2	0.2
Dining Room	0.1	0.7	0.2

Table 2: Measurement Model

Finally, our robot is moody and at any given time step, it is equally likely to take an action from the list of four actions, (U, D, L, R). Given that the sensor measurement at time 2 is *light*, what is the most likely action, u_1, that the robot decided to take at time 1? If you're using an expression that is not included in the book or notes, then provide a derivation for it.

Solution 1

We need to find $p(u|z_2, x_1)$. Let's derive an expression for it. We know from the Bayes Network that z_2 is related to x_2, so let's first try to add that to our expression.

$$p(u|z_2, x_1) = \sum_{x_2} p(x_2, u|z_2, x_1)$$

Since we have probabilities $(z|x)$ available to us, let's convert this expression using Bayes rule:

$$p(u|z_2, x_1) = \sum_{x_2} \frac{p(z_2|x_1, x_2, u)\, p(x_2, u|x_1)}{p(z_2|x_1)}$$

The denominator is just a constant as z_2 and x_1 are given and no longer random. So, let's treat it as a scaling factor. The second term in the numerator is joint probability of x_2 and u, and it would be convenient if we could express it in the form $(x_2|u)$. We can do this by using definition of conditional probability.

$$p(u|z_2, x_1) = \eta \sum_{x_2} p(z_2|x_1, x_2, u)\, p(x_2|u, x_1)\, p(u|x_1)$$

The first term in the product can be simplified using the Markov assumption: $p(z_2|x_1, x_2, u) = p(z_2|x_2)$. Our u affects the state x_2, but is independent of x_1.

$$p(u|z_2, x_1) = \eta \sum_{x_2} p(z_2|x_2)\, p(x_2|u, x_1)\, p(u)$$

Since u is a uniform RV, $p(u) = 1/4$ for all possible actions. So, we can combine it with our scaling factor to simplify our numerical computations.

$$p(u|z_2, x_1) = \eta \sum_{x_2} p(z_2|x_2)\, p(x_2|u, x_1)$$

Let's compute numerical probabilities now:

$\mathbf{u}$	$\mathbf{p}(\mathbf{u}\|\text{light, dining})$
L	$\eta(0.6 \times 0 + 0.2 \times 0.8 + 0.2 \times 0.2) = 0.2\eta$
R	$\eta(0.6 \times 0 + 0.2 \times 0 + 0.2 \times 1) = 0.2\eta$
U	$\eta(0.6 \times 0.8 + 0.2 \times 0 + 0.2 \times 0.2) = 0.52\eta$
D	$\eta(0.6 \times 0 + 0.2 \times 0 + 0.2 \times 1) = 0.2\eta$

Let's define the most likely action as the one with the highest probability. In this case then, the most likely action is U.