Linear Algebra, Quiz 10, Section L2

Name:____________________________ Student ID:______________________

Prove that if S is a finite set of vectors in a finite-dimensional vector space V such that $Span(S) = V$, then if S is not a basis for V, then it can be reduced to a basis for V by removing appropriate vectors from S.

Proof:
If S is a set of vectors that spans V but is not a basis for V, then S' is a linearly dependent set. Thus some vector v in S' is expressible as a linear combination of the other vectors in S. By the Plus/Minus Theorem (Theorem 5.4.4b), we can remove v from S, and the resulting set will still span V. If S' is linearly independent, then is a basis for V, and we are done. If s' is linearly dependent, then we can remove some appropriate vector from S' to produce a set that still spans V. We can continue removing vectors in this way until we finally arrive at a set of vectors in S that is linearly independent and spans V. This subset of S is a basis for V.