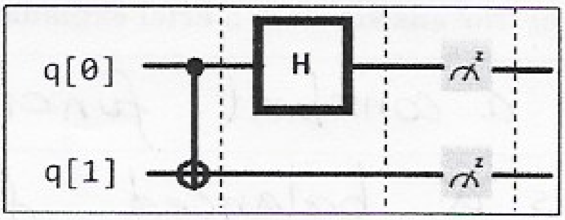
SOLUTION

Name: ______________________ **ID:** ________ L1

Section:

Q. 1: [2 marks] Consider the following quantum circuit:

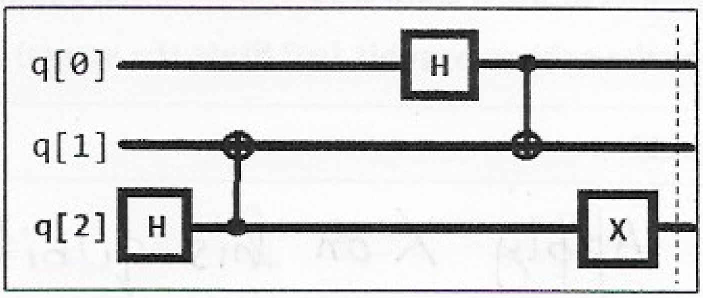


$|\psi1\rangle$ $|\psi2\rangle$ $|\psi3\rangle$

What is the state at $|\psi1\rangle$, $|\psi2\rangle$ and $|\psi3\rangle$ if the input at q[0] and q[1] is:

Input at q[0] & q[1]	Output at $\lvert\psi1\rangle$	Output at $\lvert\psi2\rangle$	Output at $\lvert\psi3\rangle$ (There is measurement here)
$\frac{\lvert 00\rangle - \lvert 11\rangle}{\sqrt{2}}$	$\frac{\lvert 00\rangle - \lvert 10\rangle}{\sqrt{2}}$	$\lvert 10\rangle$	q[0]= 1
			q[1]= 0
$\frac{\lvert 01\rangle + \lvert 10\rangle}{\sqrt{2}}$	$\frac{\lvert 01\rangle + \lvert 11\rangle}{\sqrt{2}}$	$\lvert 01\rangle$	q[0]= 0
			q[1]= 1

Q. 2: [2 marks] Consider the following quantum circuit:



$|\psi\rangle$

Suppose that the input to the circuit is $|000\rangle$. What is the output of this circuit?

$\lvert\psi\rangle = \frac{1}{2}\left(\lvert 001\rangle + \lvert 010\rangle + \lvert 100\rangle + \lvert 111\rangle\right)$

Q. 3: [2 marks] Consider the functions on bit strings $f1, f2: \Sigma^n \rightarrow \Sigma$, where $\Sigma = \{0, 1\}$

$f1(x) = \frac{1 + (-1)^{2x}}{2}$ for all x, $f2(x) = \frac{1 + (-1)^{x}}{2}$ for all x, where x is an arbitrary bit string.

Can the Deutsch-Jozsa Algorithm be used to distinguish between these two functions. Give a 'Yes' or 'No' answer with a brief explanation.

f1(x) is a constant function.
f2(x) is a balanced function.
Hence DJ- Algorithm can be used.

Q. 4: [4 marks] In the following scenario, Alice and Bob share the following state:

$$\frac{1}{2} \quad (|\phi^+\rangle \otimes X|\psi\rangle + |\phi^-\rangle \otimes XZ|\psi\rangle + |\psi^+\rangle \otimes |\psi\rangle + |\psi^-\rangle \otimes Z|\psi\rangle)$$

Alice has the first and second qubits, Bob has the third [Note that $|\psi\rangle$ is an arbitrary qubit, i.e., $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, while $|\phi+\rangle$, $|\phi^-\rangle$, $|\psi+\rangle$ and $|\psi^-\rangle$ are Bell States.].

Consider the following steps in a protocol:

1. **Suppose that Alice measures her two qubits using a circuit to distinguish between one of the four Bell states $|\phi+\rangle$, $|\phi^-\rangle$, $|\psi+\rangle$ and $|\psi^-\rangle$. She sends this result to Bob.**
2. **Based on the results of Alice state what operation(s) should Bob apply on his qubit to recover the arbitrary qubit $|\psi\rangle$? State the step(s) in the table below:**

Alice's result	Bob should
$\lvert\phi+\rangle$	(1) Apply X on his qubit.
$\lvert\phi^-\rangle$	(2) Apply X followed by Z (i.e. ZX)
$\lvert\psi+\rangle$	(3) Do nothing (or apply identity)
$\lvert\psi^-\rangle$	(4) Apply Z on his qubit.