Homework 1: Regular Languages: Automata and Expressions

CS 212 Nature of Computation
Habib University

Fall 2023

General instructions

- For drawing finite automata, see this TikZ guide or the JFLAP tool. Hand drawn diagrams will not be accepted.
- Please ensure that your solutions are neatly formatted and organized, and use clear and concise language.
- Please consult Canvas for a rubric containing the breakdown of points for each problem.
- For all the problems below, $\Sigma = \{a, b\}$.
- Some of the problems below make use of the following count function.

$$n_a(w) = \text{the number of occurences of } a \text{ in } w, \text{ where } a \in \Sigma, w \in \Sigma^*.$$

Problems

1. $\boxed{\text{15 points}}$ List 2 members and 2 non-members of the language, $(a \cup ba \cup bb)\Sigma^*$.

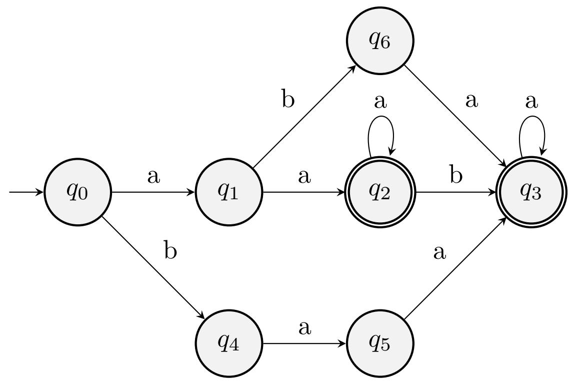
   Solution: Members: ba, bb
   Non-members: ϵ, b

2. $\boxed{\text{20 points}}$ Provide the state diagram of a simplified DFA that recognizes the language,

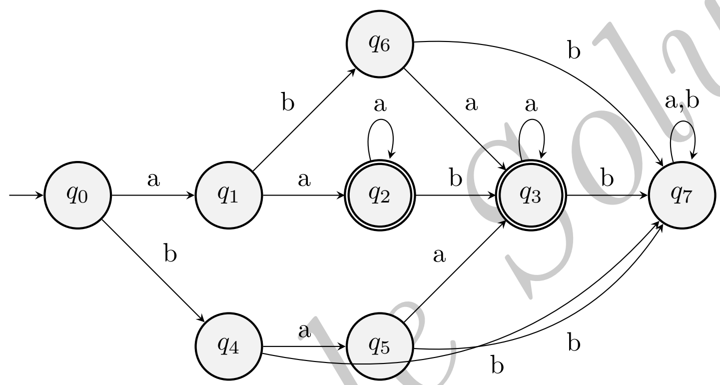
$$A = \{w \in \Sigma^* \mid n_a(w) \geq 2, n_b(w) \leq 1\}.$$

Solution: The language can be expressed as: $aaa^* + baaa^* + abaa^* + aaa^*ba^*$.

This helps to design the following NFA.



Converting this NFA to a DFA results in a the addition of a *dead* state, q_7.



3. $\boxed{\text{30 points}}$ Given the languages, A and B, we derive the language, $C = \{w \in A \mid w \in B\}$.

Prove or disprove the following claim.

Claim 1. *If A and B are regular languages, then so is C.*

Solution: We prove the claim by referring to a construction described in our textbook.

Proof. We notice that $C = A \cap B$.

The construction of the corresponding machine is described in footnote 3 in the proof of Theorem 1.25 in the textbook. □

4. $\boxed{\text{35 points}}$ Given the languages, A and B, we define the following operation.

$$A \smile_a B = \{u \in A \mid \exists v \in B \ni n_a(u) = n_a(v)\}$$

Prove or disprove the following claim.

Claim 2. *The class of regular languages is closed under $\smile_a$.*

Solution: We prove the claim by using the result in the previous question, i.e. closure of regular languages under intersection.

Proof. We notice that $A \smile_a B$ can be expressed as $A \cap B'$ where

$$B' = \{u \in \Sigma^* \mid \exists v \in B \ni n_a(u) = n_a(v)\}.$$

Given regular languages, A and B, if B' is regular then so is $A \cap B'$ or $A \smile_a B$.

It remains to show that B' is regular. We do so by deriving a regular expression for B'.

The strings in B' contain the same number of as as the strings in B, and may contain an arbitrary number of bs. Given a string, v in B, we can consider the following cases for a corresponding string, u, in B'. That is, u contains the same number of as as v.

1. v contains as: each a in v can be surrounded by 0 or more bs to obtain u.
2. v contains no as and some bs: each b in v can be replaced with 0 or more bs to obtain u.
3. v contains no as and no bs: u contains 0 or more bs.

The following table shows how a regular expression, R', for B' can be obtained from the regular expression. R, for B. It contains substitution rules to be applied to expressions in R in order to obtain the corresponding expression in R'.

R	R'
$\emptyset$	b^*
ϵ	b^*
a	b^*ab^*
b	b^*
$R_1 \cup R_2$	$R_1 \cup R_2$
$R_1 \circ R_2$	$R_1 \circ R_2$
R_1^*	R_1^*

□

We note that the obtained grammar may leave a lot of room for simplification. That is not our concern here.