Probabilistic Graph Models CS-452

Assignment 01

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Question:	1	2	3	4	Total
Points:	8	12	10	20	50
Score:					

Q1 - [08 Points] Playing with Joint Probabilities

Given the bayesian network below:

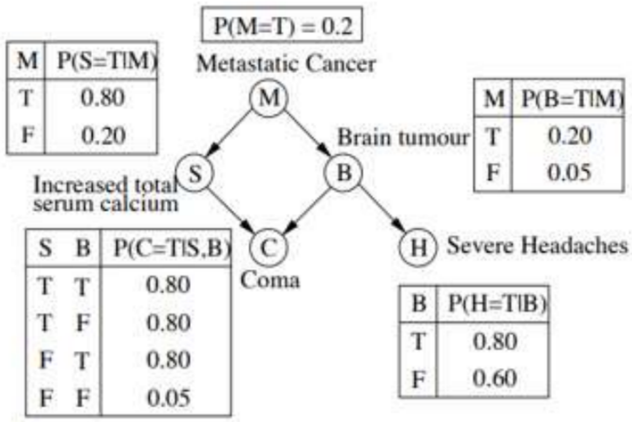


Figure 1- Metastatic Cancer BN

a- Generate complete joint problem distribution table for the variables modelled in this Bayesian networks. Compute their joint probabilities using the Conditional Probability Table (CPT) given above.

b- Compute Marginal Probabilities of all variables

c- Given a person has 'severe headache' and his serum calcium is not increased as per the tests, what are the chances now that the person will have

  a. Metastic Cancer

  b. Brain Tumor

d- Prove, from JPT that $C \perp H \mid B$

**Note: You have to show your working along with all the formulas that you have used. You can do this working on paper by hand OR in excel.*

Solution:

(a) We first compute all the remaining probabilities using the given CPTs. Then our remaining probabilities can be found as so:

P(M = F) = 1 - P(M = T) = 1 - 0.2 = 0.8

P(S = F | M = T) = 1 - P(S = T | M = T) = 1 - 0.8 = 0.2
P(S = F | M = F) = 1 - P(S = T | M = F) = 1 - 0.2 = 0.8

P(B = F | M = T) = 1 - P(B = T | M = T) = 1 - 0.2 = 0.8
P(B = F | M = F) = 1 - P(B = T | M = F) = 1 - 0.05 = 0.95

P(C = F | S = T, B = T) = 1 - P(C = T | S = T, B = T) = 1 - 0.8 = 0.2
P(C = F | S = T, B = F) = 1 - P(C = T | S = T, B = F) = 1 - 0.8 = 0.2
P(C = F | S = F, B = T) = 1 - P(C = T | S = F, B = T) = 1 - 0.8 = 0.2
P(C = F | S = F, B = F) = 1 - P(C = T | S = F, B = F) = 1 - 0.05 = 0.95

P(H = F | B = T) = 1 - P(H = T | B = T) = 1 - 0.8 = 0.2
P(H = F | B = F) = 1 - P(H = T | B = F) = 1 - 0.6 = 0.4

Then to find the complete joint probability table, we need to find P(M, S, B, C, H) = P(M) * P(S | M) * P(B | M) * P(C | S, B) * P(H | B)

For example, the first 6 rows of the table would be found as so:

$$\begin{aligned}
P(M=T,S=T,B=T,C=T,H=T) &= P(M=T)*P(S=T\mid M=T)*P(B=T\mid M=T)*\\
&\quad P(C=T\mid S=T,B=T)*P(H=T\mid B=T)\\
&= 0.2*0.8*0.2*0.8*0.8\\
&= 0.02048\\
P(M=T,S=T,B=T,C=T,H=F) &= P(M=T)*P(S=T\mid M=T)*P(B=T\mid M=T)*\\
&\quad P(C=T\mid S=T,B=T)*P(H=F\mid B=T)\\
&= 0.2*0.8*0.2*0.8*0.2\\
&= 0.00512\\
P(M=T,S=T,B=T,C=F,H=T) &= P(M=T)*P(S=T\mid M=T)*P(B=T\mid M=T)*\\
&\quad P(C=F\mid S=T,B=T)*P(H=T\mid B=T)\\
&= 0.2*0.8*0.2*0.2*0.8\\
&= 0.00512\\
P(M=T,S=T,B=T,C=F,H=F) &= P(M=T)*P(S=T\mid M=T)*P(B=T\mid M=T)*\\
&\quad P(C=F\mid S=T,B=T)*P(H=F\mid B=T)\\
&= 0.2*0.8*0.2*0.2*0.2\\
&= 0.00128\\
P(M=T,S=T,B=F,C=T,H=T) &= P(M=T)*P(S=T\mid M=T)*P(B=F\mid M=T)*\\
&\quad P(C=T\mid S=T,B=F)*P(H=T\mid B=F)\\
&= 0.2*0.8*0.8*0.8*0.4\\
&= 0.04096
\end{aligned}$$

Then our complete joint probability table would be as follows:

M	**S**	**B**	**C**	**H**	**P(M)**	**P(S|M)**	**P(B|M)**	**P(C|S,B)**	**P(H|B)**	**P(M, S, B, C, H)**
T	T	T	T	T	0.2	0.8	0.2	0.8	0.8	0.02048
T	T	T	T	F	0.2	0.8	0.2	0.8	0.2	0.00512
T	T	T	F	T	0.2	0.8	0.2	0.2	0.8	0.00512
T	T	T	F	F	0.2	0.8	0.2	0.2	0.2	0.00128
T	T	F	T	T	0.2	0.8	0.8	0.8	0.6	0.06144
T	T	F	T	F	0.2	0.8	0.8	0.8	0.4	0.04096
T	T	F	F	T	0.2	0.8	0.8	0.2	0.6	0.01536
T	T	F	F	F	0.2	0.8	0.8	0.2	0.4	0.01024
T	F	T	T	T	0.2	0.8	0.2	0.8	0.8	0.00512
T	F	T	T	F	0.2	0.8	0.2	0.8	0.2	0.00128
T	F	T	F	T	0.2	0.2	0.2	0.2	0.8	0.00128
T	F	T	F	F	0.2	0.2	0.2	0.2	0.2	0.00032
T	F	F	T	T	0.2	0.2	0.8	0.05	0.6	0.00096
T	F	F	T	F	0.2	0.2	0.8	0.05	0.4	0.00064
T	F	F	F	T	0.2	0.2	0.8	0.95	0.6	0.01824
T	F	F	F	F	0.2	0.2	0.8	0.95	0.4	0.01216
F	T	T	T	T	0.8	0.2	0.05	0.8	0.8	0.00512
F	T	T	T	F	0.8	0.2	0.05	0.8	0.2	0.00128
F	T	T	F	T	0.8	0.2	0.05	0.2	0.8	0.00128
F	T	T	F	F	0.8	0.2	0.05	0.2	0.2	0.00032
F	T	F	T	T	0.8	0.2	0.95	0.8	0.6	0.07296
F	T	F	T	F	0.8	0.2	0.95	0.8	0.4	0.04864
F	T	F	F	T	0.8	0.2	0.95	0.2	0.6	0.01824
F	T	F	F	F	0.8	0.8	0.95	0.2	0.4	0.01216
F	F	T	T	T	0.8	0.8	0.05	0.8	0.8	0.02048
F	F	T	T	F	0.8	0.8	0.05	0.8	0.2	0.00512
F	F	T	F	T	0.8	0.8	0.05	0.2	0.8	0.00512
F	F	T	F	F	0.8	0.8	0.05	0.2	0.2	0.00128
F	F	F	T	T	0.8	0.8	0.95	0.05	0.6	0.01824
F	F	F	T	F	0.8	0.8	0.95	0.05	0.4	0.01216
F	F	F	F	T	0.8	0.8	0.95	0.95	0.6	0.34656
F	F	F	F	F	0.8	0.8	0.95	0.95	0.4	0.23104

(b) To find the marginal probabilities of each of the variables, we can sum over all the other variables in the joint probability table. For example, to find P(M = T), we would sum over all the other variables in the table. This would be done for all the variables.

P(M = T) = 0.02048 + 0.00512 + 0.00512 + 0.00128 + 0.06144 + 0.04096 + 0.01536 + 0.01024 + 0.00512 + 0.00128 + 0.00128 + 0.00032 + 0.00096 + 0.00064 + 0.01824 + 0.01216 = 0.2
P(M = F) = 1 - P(M = T) = 0.8

P(S = T) = 0.02048 + 0.00512 + 0.00512 + 0.00128 + 0.06144 + 0.04096 + 0.01536 + 0.01024 + 0.00512 + 0.00128 + 0.00128 + 0.00032 + 0.07296 + 0.04864 + 0.01824 + 0.01216 = 0.32

P(S = F) = 1 - P(S = T) = 0.68

P(B = T) = 0.02048 + 0.00512 + 0.00512 + 0.00128 + 0.00512 + 0.00128 + 0.00128 + 0.00032 + 0.00512 + 0.00128 + 0.00128 + 0.00032 + 0.02048 + 0.00512 + 0.00512 + 0.00128 = 0.08
P(B = F) = 1 - P(B = T) = 0.92

P(C = T) = 0.02048 + 0.00512 + 0.06144 + 0.04096 + 0.00512 + 0.00128 + 0.00096 + 0.00064 + 0.00512 + 0.00128 + 0.07296 + 0.04864 + 0.02048 + 0.00512 + 0.01824 + 0.01216 = 0.32
P(C = F) = 1 - P(C = T) = 0.68

P(H = T) = 0.02048 + 0.00512 + 0.06144 + 0.01536 + 0.00512 + 0.00128 + 0.00096 + 0.01824 + 0.00512 + 0.00128 + 0.07296 + 0.01824 + 0.02048 + 0.00512 + 0.01824 + 0.34656 = 0.616
P(H = F) = 1 - P(H = T) = 0.384

So our marginal probabilities are as shown in the table below:

Variable	**Marginal Probability**
M	0.2
S	0.32
B	0.08
C	0.32
H	0.616

(c) To find the probability that a person has metastic cancer given that they have a severe headache and their serum calcium is not increased, we can use the formula for conditional probability. Then their probabilities would be noted from the joint probability distribution table, and the final probability would be computed. This would be done for both metastic cancer and brain tumor.

a. $\text{P(M = T} \mid \text{H = T, S = F)} = \dfrac{\text{P(M = T, H = T, S = F)}}{\text{P(H = T, S = F)}}$

   $\text{P(M = T, H = T, S = F)} = \frac{0.00512+0.00128+0.00096+0.01824}{0.00512+0.00128+0.00096+0.01824+0.02048+0.00512+0.01824+0.34656}$
   P(M = T, H = T, S = F) = 0.0615

b. $\text{P(B = T} \mid \text{H = T, S = F)} = \dfrac{\text{P(B = T, H = T, S = F)}}{\text{P(H = T, S = F)}}$

   $\text{P(B = T, H = T, S = F)} = \frac{0.00512+0.00128+0.02048+0.00512}{0.00512+0.00128+.02048+0.00512+0.00096+0.01824+0.01824+0.3456}$
   P(B = T, H = T, S = F) = 0.0769

(d) To prove that $C \perp H \mid B$, we need to show that P(C, H | B) = P(C | B)P(H | B). We can use the joint probability distribution table to compute the probabilities.

$$\text{P(C,H|B)} = \frac{\text{P(C,H,B)}}{\text{P(B)}} = \frac{0.02048 + 0.00512 + 0.00512 + 0.02048}{0.08} = \frac{0.0512}{0.08} = 0.64$$

$$\text{P(C|B)} = \frac{\text{P(C,B)}}{\text{P(B)}} = \frac{0.02048 + 0.00512 + 0.00512 + 0.00128 + 0.00512 + 0.00128 + 0.2048 + 0.00512}{0.08}$$
$$= \frac{0.064}{0.08} = 0.8$$

$$\text{P(H|B)} = \frac{\text{P(H,B)}}{\text{P(B)}} = \frac{0.02048 + 0.00512 + 0.00512 + 0.00128 + 0.00512 + 0.00128 + 0.02048 + 0.00512}{0.08}$$
$$= \frac{0.064}{0.08} = 0.8$$

P(C|B) * P(H|B) = 0.8 * 0.8 = 0.64 = P(C,H|B)

Hence proved that C $\perp$ H | B (C is conditionally independent of H given B)!

Q2 - [12 points] Applying Markov Property and Conditional Independence

This question refers to the graphical models shown in Figure 2, which encode a set of independencies among the following variables: Season (S), Flu (F), Dehydration (D), Chills (C), Headache (H), Nausea (N), Dizziness (Z).

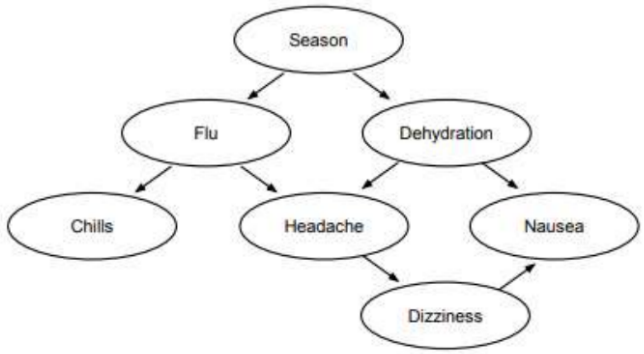


Figure 2- Seasonal Flu BN

Indicate whether the following independence statements are true or false according to this model. Provide a very brief justification of your answer (no more than 1 sentence).

Solution:

a) Season $\perp$ Chills

   False, since having evidence about Chills influences our belief about Flu which in turn influences our belief about Season.

b) Season $\perp$ Chills | Flu

   True, as having evidence about Flu blocks any flow of influence from Chills. So knowing Flu, our belief about Season is only influenced by Season and not Chills anymore as having evidence about Chills wouldn't affect our belief about Season.

c) Season $\perp$ Headache | Flu

   False, since having evidence about Headache influences our belief about Dehydration as well, which can influence our belief about Season.

d) Season $\perp$ Headache | Flu, Dehydration

   True, since knowing both Flu and Dehydration, any flow of influence is blocked from Headache to Season, that is, having evidence about Headache does not affect our belief on Season.

e) Season $\perp$ Nausea | Dehydration

False, since Nausea is also influenced by Dizziness, having evidence (or changed belief) about Nausea updates our belief on Dizziness, which also influences our belief on Headaches, which influences our belief on Flu, which in turn influences our belief on Season.

f) Season $\perp$ Nausea | Dehydration, Headache

True, given both Dehydration and Headache, no influence can flow from Nausea to Season, that is, our belief on Season is no longer influenced by Nausea.

g) Flu $\perp$ Dehydration

False, since both Flu and Dehydration have a common parent, thus having evidence about one can influence our belief about the other.

h) Flu $\perp$ Dehydration | Season, Headache

False, the diverging path thorough season is blocked. However, evidence on Headache makes flu and dehydration dependent.

i) Flu $\perp$ Dehydration | Season

True, given Season, no influence can flow from Dehydration to Flu since path through headache and nausea are already blocked.

j) Flu $\perp$ Dehydration | Season, Nausea

False, since Nausea is also influenced by Dizziness, having evidence about Nausea updates our belief on Dizziness, which also influences our belief on Headaches, which influences our belief on Flu, thus having evidence about Nausea can influence our belief about Flu so they are not conditionally independent.

k) Chills $\perp$ Nausea

False, since influence can flow from Chills to Nausea through Flu to Season to Dehydration to Nausea or Flu to headache to dizziness to nausea.

l) Chills $\perp$ Nausea | Headache

False, since having evidence on Headache opens up influence path from Chills to Nausea through headache and dehydration. They are also commonly influenced by Season.

Q3 - [10 Points] Bayesian Networks in Action

At Habib University, there is concern over a possible unauthorized access incident in the university's restricted server room. The investigation centers around several key factors: the occurrence of unauthorized access (U), signs of a forced entry into the server room (F), activation of the security alarm system (A), detection of movement by campus security cameras (C), and the completion of routine security checks by campus personnel (S). The server room door was found damaged, suggesting a potential forced entry. Around 2:00 AM, the security alarm was activated, but there was a noticeable delay in response, which could indicate a malfunction. At the same time, security cameras recorded unusual movement in the server room area. Additionally, the security personnel log showed an unexplained absence during the critical period when the cameras detected activity. This scenario presents a complex situation involving potential breaches and lapses in security measures, making it an ideal example for applying Bayesian networks to evaluate the likelihood of unauthorized access based on observed evidence and security system status.

a) Construct in GeNIE a Bayesian Network to model this scenario

b) Specify prior and conditional probabilities as per your understanding of the domain

c) Compute given probabilities from your model:

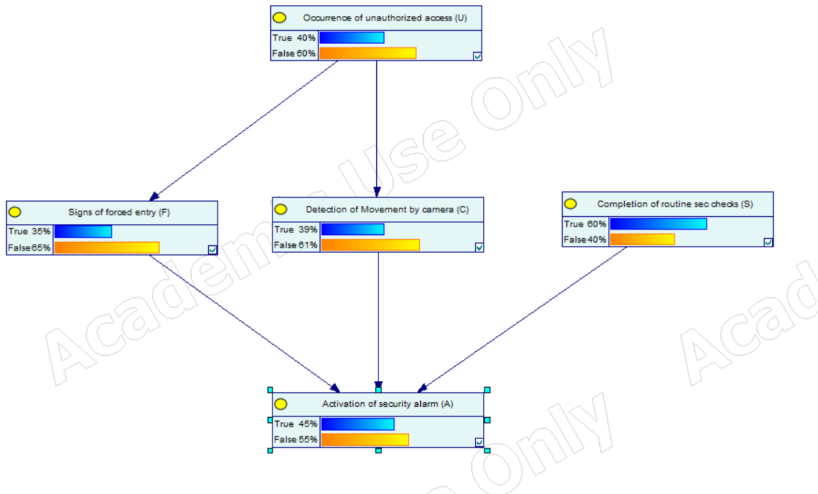
   i. If the motion sensors detected movement, what is the probability that the lab door was forced open?

   ii. What is the probability that the security guard was not alert if an unauthorized access occurred and the door was found forced open?

   iii. What is the probability that the alarm was triggered if no forced entry was detected but motion was observed?

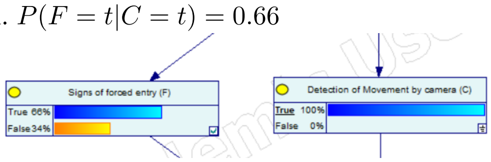
d) Give some examples of predictive, diagnostic and explaining-away reasoning that you can perform in this scenario.

Solution: (a) The image below shows the Genie model for the problem. The node (U) is taken as the parent node for (F) and (C) since the occurrence of unauthorized access can be detected thorugh camera or evident through sign of forced entry. We assume that the camera and the sensors/door-locks in the server room are connected to an alarm triggering system such that they send a signal to trigger the alarm in case they detect suspicious activity. Moreover, we assume that the routine security checks (S) are periodic and occur over short intervals. A completion of routine security check could influence triggering of an alarm (A) since the security personnel can detect a suspicious activity and trigger the alarm, however, the absence of personnel can cause security check to not be completed and decrease likelihood of alarn being triggered.

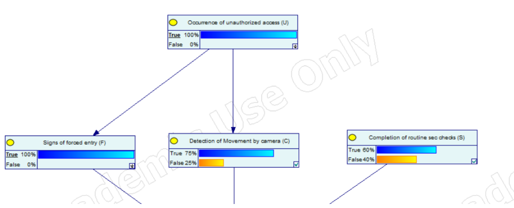


(b) Each node in the Genie model is assigned prior and conditional probabilities. Please refer to the model file.

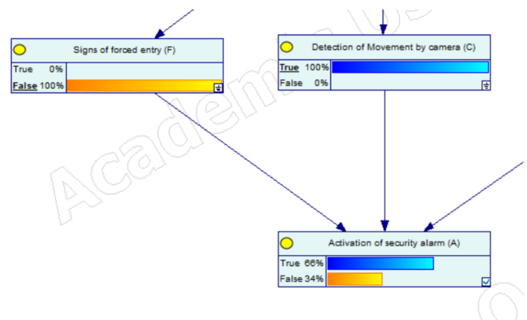
(c) For this part, we set evidences on the nodes and update beliefs instead of manual calculation.
i. $P(F = t|C = t) = 0.66$



ii. We think of security guard being alert or not alert in the sense that he was actively doing surveillance or not. Since the signs of forced entry or an unauthorized access does not affect if he was alert or not, therefore, it is independent and probability is not affected.
$P(S = f|U = t, F = t) = P(S = f) = 0.4$



iii. $P(A = t|F = f, C = t) = 0.66$



d) **Predictive Reasoning**
1. If we have evidence that an unauthorized access (U) has occurred, we can predict that it is likely that movement has been detected by camera (C) and that the signs of forced entry (F) are likely and eventually that the probability of triggering of the alarm (A) is high.
2. Given that there are no signs of forced entry (F), we can reason on the probability of activation of security alarm (A) being less than the prior belief.
3. Similarly, if we have evidence on detection of movement by security camera (C) and evidence that security check was completed (S), and evidence that there were no signs of forced entry (F), then we can predict the triggering of security alarm (A) as being somewhat more probable.

Diagnostic Reasoning
1. Given activation of security alarm, we can diagnostically predict that the chances of (C) or (S), or (F) or a subset of them triggering it are high.
2. If we have evidence on the detection of movement by camera and signs of forced entry, then we can diagnostically reason that the chances that an unauthorized access has occurred are fairly high.

Explaining Away Reasoning
1. Given that movement was detected by the camera (C) and as a result, alarm was triggered (A), then it slightly reduces the chances of security checks being completed.

**Note: Please mention any assumptions that you may have taken regarding this scenario. Your assumptions must not contradict with the actual scenario.*

Q4 - [20 Points] Bayesian Networks in the real-world

Come up with a real and contextualized problem involving reasoning with evidence and uncertainty.

a) Write down a text description of the problem. Make the problem sufficiently complex that your network has at least 8 nodes and is multiply-connected (i.e., not a tree or a polytree).

b) Model the problem using a Bayesian network. Use GeNIE to construct the Bayesian network.

c) Specify meaningful probabilities for all nodes.

d) Show the beliefs for each node in the network before any evidence is added.

e) Which nodes are d-separated with no evidence added?

f) Show how the beliefs change in a form of "diagnostic reasoning" when evidence about at least one of the domain variables is added. Which nodes are d-separated with this evidence?

g) Show how the beliefs change in a form of predictive reasoning when evidence about at least one of the domain variables is added. Which nodes are d-separated with this evidence added?

h) Show how the beliefs change through explaining away when particular combinations of evidence are added.

i) Show how the beliefs change when you change the priors for a root node (rather than adding evidence).

Solution:

a) Mr. X, a high-profile underground hacker, is planning to buy cutting-edge Nvidia AI chips from Mr. Y, a chip dealer operating in Karachi's black market. Mr. X is highly cautious and does not trust Mr. Y, fearing he might be an undercover operative from an intelligence agency setting up a trap. To minimize risk, Mr. X wants to assess all possible outcomes and calculate the risks associated with this deal before arriving in Karachi.

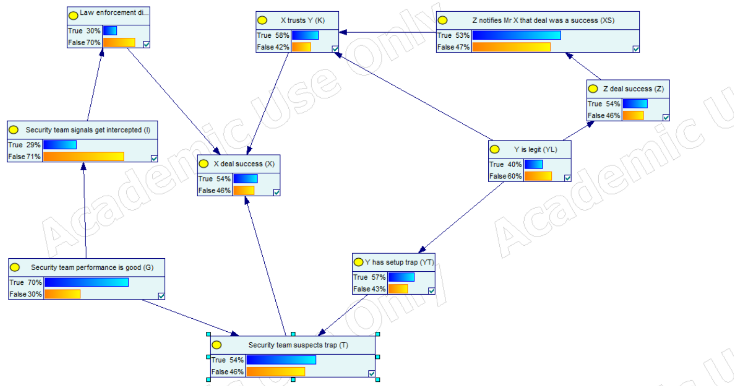
   Mr. X learns from a third party ally Mr Z that he is also planning a deal with Mr. Y. Mr X asks Mr.Z to inform about the deal's success. But it is also probable that Mr Z might give wrong information about his deal's success. If Mr. Zs deal is successful, then it would indicate that Mr Y might not be a fraud. However, failure or intervention in Mr. Zs deal could raise suspicion that Mr. Y is a fraud/undercover intelligence.

   Mr. X is also suspicious that Mr.Y might become greedy and setup a trap. For this, he has setup a security team in Karachi to notify him if they suspect a trap. Mr X is fairly confident that his security team performs good. Given that there are network firewalls in Karachi, it depends on the team's performance to prevent the message from getting intercepted by law enforcement agencies. Their task is to notify Mr X of any trap while making sure that thier message is not intercepted by law enforcement agencies which might otherwise result in discovery of the location of the deal.

   Mr.X now wants to model the situation to see if the deal with Mr. Y is worth or not based on how several factors might affect its chances of success.

(b),(c) Please refer to the genie files

d) The model with initial beliefs is shown below



e) If no evidence is added, then (L), (K) are d-seperated since there is no unblocked collision-free path between them. The path (L - X - K) is blocked due to collision at X and path (L - I - G - T - YT - ...) is blocked due to collision at T.

f) Consider the example where "Y has setup a trap (YT)" is true, that is, Mr. Y has setup a trap. Then using diagnostic reasoning, our belief in the network that Mr. Y is legit decreases i.e he is a fraud. Consequently, a large number of nodes get affected. Specifically, node (T), (YL), (Z), (XS), (K), and (X) get affected by this evidence.

1 The probability that the security team detects trap (T) increases

2 The probability of legitimacy of Mr.y (YL) decreases

Mr Z's deal's success, the trust of Mr.X on Mr.Y and the probability of Mr.X's deal's success with Mr.Y decreases.

3 Consequently, there are high chances of Mr.Y being a fraud, the probability that Mr Z's deal being a success also decreases.

4 Due to this, the probability that Mr Z tells Mr X about his deal's success (XS) also decreases

6 As a result, the trust of Mr X on Mr Y (K) decreases.

7 Due to increase in probability of (T) and decrease in probability of (K), the chances of deal being a success also decreases (X).

D-seperated nodes
Given (YT), Node (T) gets d-seperated from node (YL), (Z), (XS), (K)

g) Suppose we add evidence that "Y is Legit" is false, then our belief changes as follows:

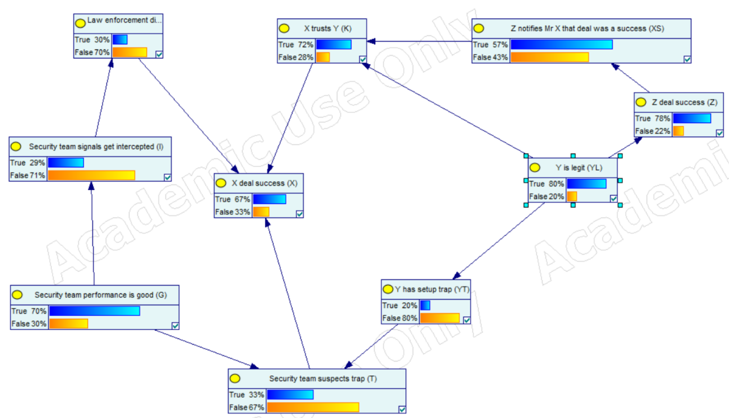
1 Our belief in Z Deal Outcome (Z) being good decreases since Mr.Y is not legit, this also decreases the chance of Mr.Z notifying Mr.X (XS) which can decrease Mr.X's trust in Mr.Y (K) which can lower Mr.X deal success (X).

2 Our belief that Y has setup a trap (YT) increases, which increases the chances of security team detecting a trap planted by Mr.Y (T) which decreases the chances of deal succeeding.

D-seperated nodes
Given (YL), Node (Z), (XS) and (K) are d-seperated pairwise with (YT) and (T).

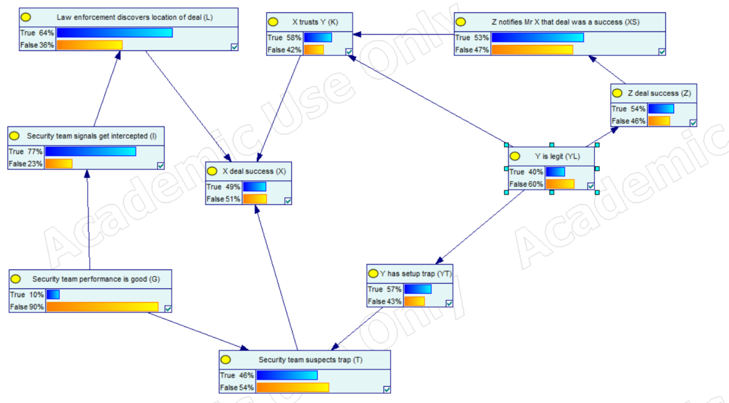
h) If we assume that Mr. X's deal with Mr. Y was a success (X) then the likelihood of Mr.X trusting Mr. Y increases. If additionally, the security team is sure there is no trap (T), then the chances of X trusting Y (K) slightly decreases. This might see counter intuitive at first, but since there was not trap detected, this means that it is highly likely that there was no trap setup by Mr.Y. This increases legitimacy of Mr Y which in-turn increases chances of Mr. Z's deal being a success too. But since he might be prone to giving false information if his deal has succeed, he might tell Mr.X otherwise about his deal. This will not create a significant impact given that Mr X deal was a success, but reduces the chances that Mr X had trusted Mr Y (K).

i) If we increase our prior belief in Y being legit (YL) to a high value, we get the following beliefs:



This shows that the change in prior belief in root node (YT) changes the belief (Z), (K), (X), (YT), (T), (XS). The overall propagated effect is that the chances of Mr. X deal being a success increases

Moreover, If we decrease our prior belief in the security team's performance (G) to a low value, we get the following probabilities:



This shows that if the Security team has poor performance, then the probability that the signal gets intercepted (I) becomes high, which in turn increases our belief in law enforcement finding the location (L), thus lowering X deal success (X). Also, the chance that security team suspects trap (T) also lowers, which again lowers our belief in X Deal being a success.