Quiz 15B: The class NP

CS 212 Nature of Computation

Habib University — Fall 2023

Total Marks: 10 | Date: November 29, 2023
Duration: 15 minutes | Time: 830–845h

Student ID: _____

Student Name: _____

1. (10 points) Show that the class NP is closed under concatenation.

Solution: We prove the closure by constructing a non-deterministic polynomial-time decider for the concatenation of two languages in NP.

Proof. Consider the languages $L_1, L_2 \in \text{NP}$ and let N_1 and N_2 be their respective non-deterministic polynomial-time deciders.
Construct N to decide $L_1 \circ L_2$ as follows.
On input $w = w_1 w_2 w_3 \dots w_n$:

1. for i in 0 to n:
   (a) Simulate N_1 on $w_1 w_2 \dots w_i$.
   (b) If N_1 accepts,
      i. simulate N_2 on $w_{i+1} w_{i+2} \dots w_n$.
      ii. If N_2 accepts, *accept*.
2. *Reject*.

N utilizes N_1 and N_2 so is non-deterministic.
The steps in the loop altogether take polynomial time, say $O(n^k)$, in the worst case.
The loop runs $n + 1$ times in the worst case.
The worst case running time of the algorithm is therefore $O(n^{k+1})$.
N halts in all cases. □