Using Packages

Week 3 SEL Activity 3

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n	0	1	2	3	4	5	6	7	8	9	10	11	12	13+
F	14	30	36	68	43	43	30	14	10	6	4	1	1	0

Poisson distibution has been used to model the traffic situation. The estimation for λ for observations $X_1, X_2, X_3, \cdots, X_N$ is given by,

$$\hat{\lambda} = \frac{1}{N}\sum_{i=1}^{N} X_i$$

Where X_i is Poisson random varriable, whose pdf is given by,

$$f_X(n;\lambda) = \begin{cases} \dfrac{\lambda^n e^{-\lambda}}{n!} & n \geq 0 \\ 0 & n < 0 \end{cases}$$

We can use the method of moments to estimate the value of λ.

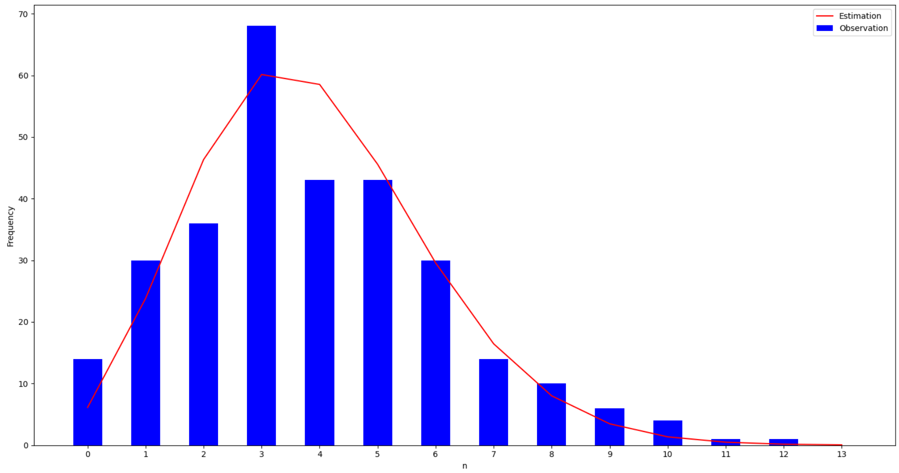
$$\begin{aligned}
\mathrm{E}[X] &= \sum_{n\geq 0} n f_X(n;\lambda) \\
\mathrm{E}[X] &= \sum_{n\geq 0} n \frac{\lambda^n e^{-\lambda}}{n!} \\
\mathrm{E}[X] &= e^{-\lambda} \sum_{n\geq 1} \frac{\lambda^n}{(n-1)!} \\
\mathrm{E}[X] &= e^{-\lambda} \sum_{n\geq 0} \frac{\lambda^{n+1}}{n!} \\
\mathrm{E}[X] &= \lambda e^{-\lambda} \sum_{n\geq 0} \frac{\lambda^n}{n!} \\
\mathrm{E}[X] &= \lambda e^{-\lambda} e^{\lambda} \\
\mathrm{E}[X] &= \lambda
\end{aligned}$$

Our first moment is the function of the variable we want to estimate. This mean the estimation of λ will be,

$$\begin{aligned}
\hat{\lambda} &= \hat{\mu}_1 \\
\hat{\lambda} &= \frac{(0)(14) + (1)(30) + (2)(36) + \cdots + (11)(1) + (12)(1) + (13)(0)}{300} \\
\hat{\lambda} &= 3.893
\end{aligned}$$

The values for $\hat{\lambda}$ are,

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13+
$\hat{F}$	6.12	23.81	46.34	60.13	58.52	45.57	29.57	16.44	8.0	3.46	1.35	0.48	0.15	0.05



As most of the values are in the vicinity of 3 and 4 and around those values our estimator is not estimating good. Hence we made a bad estimator.