Lecture 5

Monday, January 24, 2022 11:05 PM

1) LECTURE NO. 5 — LINEAR ALGEBRA

GAUSSIAN ELIMINATION: (ECHELON FORM)

A MATRIX HAVING THE FOLLOWING PROPERTIES IS SAID TO BE IN ROW-ECHELON FORM.

(1) IF A ROW DOES NOT CONSIST ENTIRELY OF ZEROS, THEN THE FIRST NONZERO NUMBER IN THE ROW IS A $\boxed{1}$. (WE CALL THIS A LEADING 1).

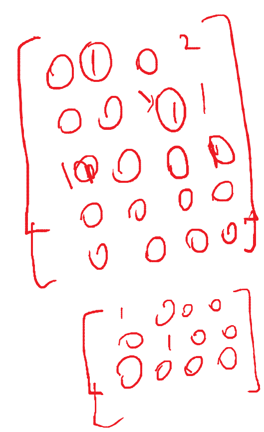
(2) IF THERE ARE ANY ROWS THAT CONSIST ENTIRELY OF ZEROS, THEN THEY ARE GROUPED TOGETHER AT THE BOTTOM OF THE MATRIX.

(3) IN ANY TWO SUCCESSIVE ROWS THAT DO NOT CONSIST ENTIRELY OF ZEROS, THE LEADING $\boxed{1}$ IN THE LOWER ROW OCCURS FARTHER TO THE RIGHT THAN THE LEADING $\boxed{1}$ IN THE HIGHER ROW.

Not in REF

$$\begin{bmatrix} 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



IF ○ then ___

EXAMPLES: THE FOLLOWING MATRICES ARE IN ROW-ECHELON FORM.

LEADING 1

$$\begin{bmatrix} 1 & 4 & 3 & 7 \\ 0 & 1 & 6 & 2 \\ 0 & 0 & 1 & 5 \end{bmatrix}, \quad \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\begin{bmatrix} 0 & 1 & 2 & 8 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

NOTE: IN THIS SECTION WE SHALL DISCUSS A PROCEDURE FOR SOLVING SYSTEMS OF LINEAR EQUATIONS BY REDUCING THE AUGMENTED MATRIX TO ROW-ECHELON FORM. CONSIDER THE FOLLOWING EXAMPLE:

$\boxed{3}$

EXAMPLE: SOLVE THE FOLLOWING SYSTEM BY GAUSSIAN ELIMINATION (ECHELON FORM) METHOD.

$$3x_1 + 4x_2 + 5x_3 = 12$$
$$x_1 - x_2 + 2x_3 = 2$$
$$2x_1 + x_2 + 3x_3 = 6$$

Gauss-Jordan

$$\begin{bmatrix} 3 & 4 & 5 & 12 \\ 1 & -1 & 2 & 2 \\ 2 & 1 & 3 & 6 \end{bmatrix}$$

$$R_1 \leftrightarrow R_3 \quad \begin{bmatrix} 2 & 1 & 3 & 6 \\ 1 & -1 & 2 & 2 \\ 3 & 4 & 5 & 12 \end{bmatrix}$$

$$R_1 \to R_1 - R_2 \quad \begin{bmatrix} 1 & 2 & 1 & 4 \\ 1 & -1 & 2 & 2 \\ 3 & 4 & 5 & 12 \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - 3R_1$

SOLUTION: WE SHALL REDUCE THE AUGMENTED MATRIX TO ECHELON FORM, CONSIDER

$$\begin{bmatrix} 3 & 4 & 5 & 12 \\ 1 & -1 & 2 & 2 \\ 2 & 1 & 3 & 6 \end{bmatrix}$$

$$\sim R_1 \longleftrightarrow R_2 \quad \begin{bmatrix} 1 & -1 & 2 & 2 \\ 3 & 4 & 5 & 12 \\ 2 & 1 & 3 & 6 \end{bmatrix}$$

$$\sim \begin{array}{l} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 2R_1 \end{array} \quad \begin{bmatrix} 1 & -1 & 2 & 2 \\ 0 & 7 & -1 & 6 \\ 0 & 3 & -1 & 2 \end{bmatrix}$$

4)

$$\sim \begin{bmatrix} 1 & -1 & 2 & 2 \\ 0 & 1 & 1 & 2 \\ 0 & 3 & -1 & 2 \end{bmatrix} \begin{matrix} R_2 \rightarrow \\ R_2 - 2R_3 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -1 & 2 & 2 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & -4 & -4 \end{bmatrix} \begin{matrix} R_3 \rightarrow \\ R_3 - 3R_2 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -1 & 2 & 2 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix} R_3 \rightarrow -\frac{1}{4} R_3$$

WHICH IS THE REQUIRED ECHELON FORM. SO THE GIVEN LINEAR SYSTEM IS $\boxed{\text{REDUCED}}$ TO

$$x_1 - x_2 + 2x_3 = 2 \rightarrow (1)$$

$$x_2 + x_3 = 2 \rightarrow (2)$$

$$\boxed{x_3 = 1} \rightarrow (3)$$

$(2) \Rightarrow \boxed{x_2} = 2 - x_3 = 2 - 1 = 1$

$(1) \Rightarrow \boxed{x_1} = 2 + x_2 - 2x_3 = 2 + 1 - 2 = 1$

$= 1$

NOTES:

(1) IN ROW REDUCTION PROCESS DON'T $\boxed{\text{PERFORM}}$ ANY STEPS BY WHICH YOU LOSE $\boxed{\text{ZEROS}}$ OR $\boxed{\text{1's}}$ (OBTAINED ALREADY).

(2) IF POSSIBLE THEN $\boxed{\text{AVOID}}$ THE FORMATION OF FRACTIONS.

$\boxed{\text{ASSIGNMENT NO. 2}}$

$\boxed{\text{Q.no.1}}$

(a) UNDER WHAT CONDITIONS $\boxed{AB = BA}$, WHERE

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

(b) IF $\boxed{A}$ IS A MATRIX THEN $A^r A^s = A^{r+s}$ FOR $\boxed{r, s}$ POSI–

6) TIVE INTEGERS.

IS THIS RESULT TRUE FOR NEGATIVE INTEGERS ALSO? JUSTIFY YOUR ANSWER.

(c) IF $A = \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}$

AND $C = \begin{bmatrix} 2 & 5 \\ 3 & 4 \end{bmatrix}$ THEN

$AB = AC = \begin{bmatrix} 3 & 4 \\ 6 & 8 \end{bmatrix}$ BUT $\boxed{B \neq C}$,

$\boxed{\text{WHY?}}$

$\boxed{\text{Q. no. 2}}$

USING THE TECHNIQUE OF FORMING A BLOCK MATRIX $[A \mid I]$ AND PERFORMING E.R.O.S SUCH THAT

$$[A \mid I] \xrightarrow{E.R.O.S} [I \mid A^{-1}]$$

FIND THE INVERSE OF THE FOLLOWING WHERE $\boxed{A}$ IS GIVEN BY

7)

(a) $\begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 & -4 \\ -1 & -1 & 5 \\ 2 & 7 & -3 \end{bmatrix}$

Q.no.3

SOLVE THE FOLLOWING SYSTEM OF EQUATIONS BY REDUCING THEM TO ECHELON FORM (GAUSSIAN ELIMINATION METHOD)

$$x + y + 2z = 9$$
$$2x + 4y - 3z = 1$$
$$3x + 6y - 5z = 0$$

Q.no.4

SOLVE THE FOLLOWING SYSTEM BY GAUSS-JORDAN ELIMINATION (REDUCED ROW-ECHELON FORM)

$$2x_1 + 2x_2 + 2x_3 = 0$$
$$-2x_1 + 5x_2 + 2x_3 = 1$$
$$8x_1 + x_2 + 4x_3 = -1$$

8)

Q.no.5

REDUCE $\begin{bmatrix} 2 & 1 & 3 \\ 0 & -2 & 7 \\ 3 & 4 & 5 \end{bmatrix}$ TO REDUCED ROW ECHELON FORM WITHOUT INTRODUCING ANY FRACTIONS.

Q.no.6

FIND TWO DIFFERENT ROW-ECHELON FORMS OF

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$$

Q.no.7

Exercise Set 1.3, Question no. 25

Q.no.8

Exercise set 1.3, Questions 18 and 19