Homework 9b: Exercise Set 5.3 Solution

Question 11

Show that if $S : \{v_1, v_2, \ldots, v_r\}$ is a linearly independent set of vectors, then so is every nonempty subset of S
Solution: Please take a look at the solution manual.

Question 13

Show that if $\{v_1, v_2, \ldots, v_r\}$ is a linearly dependent set of vectors in a vector space V, and if $v_{r+1}, \ldots, v_n$ are any vectors in V, then $\{v_1, v_2, \ldots, v_r, v_{r+1}, \ldots v_n\}$ is also linearly dependent.
Solution: Please take a look at the solution manual.

Question 15

Show that if is $\{v_1, v_2\}$ linearly independent and v_3 does not lie in span $\{v_1, v_2\}$, then $\{v_1, v_2, v_3\}$ is linearly independent.
Solution: Please take a look at the solution manual.

Question 18

Under what conditions is a set with one vector linearly independent?
Solution: The vector $\vec{v_1}$ should be nonzero. Let's take zero vector...

$$\vec{0} = k_1\vec{v_1} \Rightarrow k_1 = 0 \qquad\qquad \because \vec{v_1} \neq 0$$

Question 24

Indicate whether each statement is always true or sometimes false. Justify your answer by giving a logical argument or a counterexample.
(a) *The set 2×2 of matrices that contain exactly two 1's and two 0's is a linearly independent set in M_{22}.*
Solution: False. Let the set of 2×2 matrices be

$$\left\{E_1 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, E_2 = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, E_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, E_4 = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, E_5 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\right\}$$

$$\vec{0} = k_1E_1 + k_2E_2 + k_3E_3 + k_4E_4 + k_5E_5$$

Take $k_1 = 0, k_2 = k_4 = 1, k_3 = k_5 = -1$ Hence it is not linearly independent.
(b) *If $\{v_1, v_2\}$ is a linearly dependent set, then each vector is a scalar multiple of the other.*
Solution: True. By using linear dependent definition

$$\vec{0} = k_1v_1 + k_2v_2 \Rightarrow k_1, k_2 \neq 0$$

It implies one vector can written as the scalar multiplication of other vector.
(c) *If $\{v_1, v_2, v_3\}$ is a linearly independent set, then so is the set $\{kv_1, kv_2, k_3\}$ for every nonzero scalar k.*
Solution: True. By definition of linear independent, any vector v_i can be written as the linear combination of the $\{kv_1, kv_2, k_3\}$.
(d) *The converse of Theorem 5.3.2a is also true.*
Solution: True. A set of vectors is linearly dependent if there is a nontrivial linear combination of the vectors that equals 0, i.e, part (a).

Question 25

Show that if $\{v_1, v_2, v_3\}$ is a linearly dependent set with nonzero vectors, then each vector in the set is expressible as a linear combination of the other two.
Solution: False. The definition only requires that at least one vector be expressible as a linear combination of the other two. Here's an easy example in R^2 that shows we don't have to have each vector expressible in terms of the other two (even when all the vectors are nonzero): $v_1 = (1, 0), v_2 = (2, 0), v_3 = (0, 1)$. Obviously the vector v_3 cannot be expressed in terms of v_1 and v_2.