Probability & Statistics

Date:.....

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Assignment #01

Q1) 5 Red, 10 White, 10 Black, 5 Orange

Total Balls = 30

$P(R) = 5/30$ $P(W) = 10/30$ $P(B) = 10/30$ $P(O) = 5/30$

(a) Orange or Red $= P(O) + P(R) = \frac{5}{30} + \frac{5}{30} = \frac{10}{30}$

$= 1/3$

(b) Neither Red nor Black $= 1 - [P(R) + P(B)]$

$= 1 - [5/30 + 10/30]$

$= 1 - [15/30] = 15/30$

$= 1/2$

(c) Not Black $= 1 - P(B) = 1 - \frac{10}{30} = \frac{20}{30}$

$= 2/3$

(d) Red, White or Black $= P(R) + P(W) + P(B)$

$= 5/30 + 10/30 + 10/30 = 25/30$

$= 5/6$

Q2) 5 Red, 5 Black, 5 White

a Red Ball is twice as likely than white or black.

(a) $P(R) + P(B) + P(W) = 1$

$\Rightarrow 2x + x + x = 1$

$\Rightarrow 4x = 1 \Rightarrow x = 1/4$

Let $P(W) = P(B) = x$

$P(R) = 2P(W) = 2x$

$P(R) = 2(1/4) = 1/2$

(b) Neither Red nor Black $= 1 - [P(R) + P(B)] = P(W)$

$= 1/4$

(c) Not Black $= 1 - [P(B)] = 1 - \frac{1}{4}$

$= 3/4$

(d) Red, White or Black $= 1$ [all elements of the sample space]

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Q3) Two 4-sided die thrown simultaneously
If Sum is 5, what is the probability at least one of the dice had 3.

D_2 \ D_1	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

Sum is 5: {(1,4), (4,1), (2,3), (3,2)}
Die had 3: {(2,3), (3,2)}.

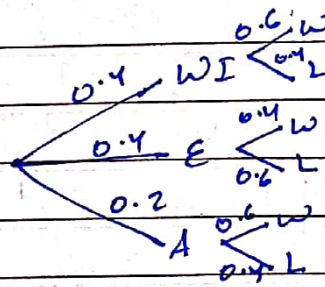
$P(E) = P(3)$ given that sum is 5.
$P(E) = 2/4 \Rightarrow P(E) = 1/2$

Q4) 5 Red, 10 White, 10 Black, 5 Orange Total = 30
Two Balls chosen without replacement.

(a) Red & Red
$P(R \& R) = \frac{5}{30} \times \frac{4}{29} = 2/87$

(b) Red & Orange
$P(R \& O) = 5/30 \times 5/29 = 5/174$

Q5) $P(WI) = 0.4$ $P(E) = 0.4$ $P(A) = 0.2$
$P(P_w W) = 0.6$ $P(P_w E) = 0.4$ $P(P_w A) = 0.6$



$P(\text{Pak Wins}) = 0.4(0.6) + 0.4(0.4) + 0.2(0.6)$
$= 0.52$

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Q6) Taxis, 1 Blue & 99 Green

Witness, saw blue: 99% as blue saw green: 2% as blue

Prob, taxi was blue given that the witness saw blue

P(Witness said blue | blue) = 0.99

P(" " " | green) = 0.02

Using Bayes's Rule

$$P(\text{Blue} \mid \text{Witness Said Blue}) = \frac{P(W.S.B \mid \text{Blue})\, P(\text{Blue})}{P(W.S.B)}$$

$$= \frac{0.99 \times 0.01}{[(0.99 \times 0.01) + (0.99 \times 0.02)]}$$

$$= 1/3$$

The Probability that taxi was actually blue is 33%.

Q7) Coin "A" → P(H) = 0.49 P(T) = 0.51

Coin "B" → P(H) = 0.52 P(T) = 0.48

Coin "C" → P(H) = 0.5 P(T) = 0.5

Equally likely to select any coin. Flip 7 times.

(a) $P(A \mid 7H) = \frac{P(A \cap 7H)}{P(7H)}$

Using Bayes' Rule: $P(A \mid 7H) = \frac{P(A)P(7H \mid A)}{P(7H)}$

$$= \frac{1/3\,[0.49]^7}{\frac{1}{3}[0.49^7 + 0.52^7 + 0.50^7]} = 0.272$$

(b) $P(B \mid 7H) = \frac{1}{3}[0.52^7] / \frac{1}{3}[0.49^7 + 0.52^7 + 0.50^7]$

$= 0.413$

(c) $P(C \mid 7H) = \frac{1}{3}[0.50^7] / \frac{1}{3}[0.49^7 + 0.52^7 + 0.50^7]$

$= 0.314$

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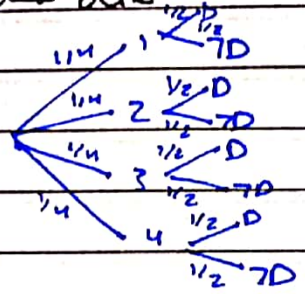
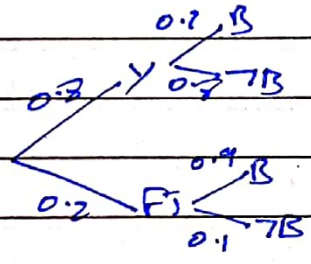
Q8) 4 Trails, each with 2 sub trails.
4 sided die flipping a coin



$P(D) = 4\left[\frac{1}{4} \times \frac{1}{2}\right]$

$\Rightarrow P(D) = 1/2$

Q9) Yorker = 80% ↓ Full toss = 1 − 0.8 = 20%
Boundary = 20% ↓ Boundary = 90% ↑



$P(\text{Boundary}) = 0.8(0.2) + 0.2(0.9)$

$= 0.34$

Q10) A = 1st Roll ⇒ 1, 2 or 4
B = 1st Roll ⇒ 1, 2 or 5

$P(A) = 3/6 = 1/2$

$P(B) = 3/6 = 1/2$

Independant ⇒ $P(A \cap B) = P(A)P(B)$

$P(A \cap B) = 2/6 = 1/3$

$P(A)P(B) = 1/2 \cdot 1/2 = 1/4$

$P(A \cap B) \neq P(A)P(B)$

Therefore they are not independant.

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