Q no. 1

(a) Under what conditions $AB = BA$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$\begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix} = \begin{bmatrix} b_{11}a_{11} + b_{12}a_{21} & b_{11}a_{12} + b_{12}a_{22} \\ b_{21}a_{11} + b_{22}a_{21} & b_{21}a_{12} + b_{22}a_{22} \end{bmatrix}$$

Comparing corresponding entries

$$\begin{aligned} a_{11}b_{11} + a_{12}b_{21} &= b_{11}a_{11} + b_{12}a_{21} \\ a_{11}b_{12} + a_{12}b_{22} &= b_{11}a_{12} + b_{12}a_{22} \\ a_{21}b_{11} + a_{22}b_{21} &= b_{21}a_{11} + b_{22}a_{21} \\ a_{21}b_{12} + a_{22}b_{22} &= b_{21}a_{12} + b_{22}a_{22} \end{aligned}$$

$$\begin{aligned} a_{12}b_{21} &= b_{12}a_{21} \\ a_{11}b_{12} + a_{12}b_{22} &= b_{11}a_{12} + b_{12}a_{22} \\ a_{21}b_{11} + a_{22}b_{21} &= b_{21}a_{11} + b_{22}a_{21} \\ a_{21}b_{12} &= b_{21}a_{12} \end{aligned}$$

(b) $A^{r+s} = A^r A^s$ is valid for negative integers r, s

Solution: Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ and let $r = 1$ and $s = -1$. It shows, it would be

$$\begin{aligned} A^{1-1} &= A^1 A^{-1} \\ A^0 &= AA^{-1} \\ I &= \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}^{-1} \end{aligned}$$

Inverse not exist, hence it is not valid for negative integers.

(c) Solution: Since A^{-1} does not exist, that's why $AB = AC$ but $B \neq C$. (Fill the details for the verification)

Q.no 2

Solution: Using $[A|I] \to [I|A^{-1}]$

$$\left[\begin{array}{rrr|rrr} 1 & 0 & 2 & 1 & 0 & 0 \\ 2 & -1 & 3 & 0 & 1 & 0 \\ 4 & 1 & 8 & 0 & 0 & 1 \end{array}\right]$$

$$R_2 - 2R_1 \left[\begin{array}{rrr|rrr} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & -1 & -1 & -2 & 1 & 0 \\ 4 & 1 & 8 & 0 & 0 & 1 \end{array}\right] \quad R_3 - 4R_1 \left[\begin{array}{rrr|rrr} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & -1 & -1 & -2 & 1 & 0 \\ 0 & 1 & 0 & -4 & 0 & 1 \end{array}\right]$$

$$R_3 \leftrightarrows R_2 \left[\begin{array}{rrr|rrr} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & -4 & 0 & 1 \\ 0 & -1 & -1 & -2 & 1 & 0 \end{array}\right] \quad R_3 + R_2 \left[\begin{array}{rrr|rrr} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & -4 & 0 & 1 \\ 0 & 0 & -1 & -6 & 1 & 1 \end{array}\right]$$

$$R_1 + 2R_3 \left[\begin{array}{rrr|rrr} 1 & 0 & 0 & -15 & 2 & 2 \\ 0 & 1 & 0 & -4 & 0 & 1 \\ 0 & 0 & -1 & -6 & 1 & 1 \end{array}\right] \quad -1R_3 \left[\begin{array}{rrr|rrr} 1 & 0 & 0 & -15 & 2 & 2 \\ 0 & 1 & 0 & -4 & 0 & 1 \\ 0 & 0 & 1 & 6 & -1 & -1 \end{array}\right]$$

Q. no. 3

$$\begin{aligned} x + y + 2z &= 9 \\ 2x + 4y - 3z &= 1 \\ 3x + 6y - 5z &= 0 \end{aligned}$$

$$\left[\begin{array}{rrr|r} 1 & 1 & 2 & 9 \\ 2 & 4 & -3 & 1 \\ 3 & 6 & -5 & 0 \end{array}\right]$$

$$R_2 - 2R_1 \left[\begin{array}{rrr|r} 1 & 1 & 2 & 9 \\ 0 & 2 & -7 & 17 \\ 3 & 6 & -5 & 0 \end{array}\right] \quad R_3 - 3R_1 \left[\begin{array}{rrr|r} 1 & 1 & 2 & 9 \\ 0 & 2 & -7 & 17 \\ 0 & 3 & -11 & -27 \end{array}\right]$$

$$\frac{1}{2}R_2 \left[\begin{array}{rrr|r} 1 & 1 & 2 & 9 \\ 0 & 1 & -7/2 & 17/2 \\ 0 & 3 & -11 & -27 \end{array}\right] \quad R_1 - R_2 \left[\begin{array}{rrr|r} 1 & 0 & 11/2 & 18 \\ 0 & 1 & -7/2 & 17/2 \\ 0 & 3 & -11 & -27 \end{array}\right]$$

$\dots \qquad \dots \qquad \dots$

$$\left[\begin{array}{rrr|r} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

Exercise Set 1.2 Solution

Question 18

Reduce

$$\begin{bmatrix} 2 & 1 & 3 \\ 0 & -2 & -29 \\ 3 & 4 & 5 \end{bmatrix}$$

to reduced row-echelon form.

Solution:

$$\begin{bmatrix} 2 & 1 & 3 \\ 0 & -2 & -29 \\ 3 & 4 & 5 \end{bmatrix} \xrightarrow{\frac{1}{2}R_1} \begin{bmatrix} 1 & 1/2 & 3/2 \\ 0 & -2 & -29 \\ 3 & 4 & 5 \end{bmatrix} \xrightarrow{R_3-3R_1} \begin{bmatrix} 1 & 1/2 & 3/2 \\ 0 & -2 & -29 \\ 0 & 5/2 & 1/2 \end{bmatrix}$$

$$\xrightarrow{-\frac{1}{2}R_2} \begin{bmatrix} 1 & 1/2 & 3/2 \\ 0 & 1 & 29/2 \\ 0 & 5/2 & 1/2 \end{bmatrix} \xrightarrow{R_3-\frac{5}{2}R_2} \begin{bmatrix} 1 & 1/2 & 3/2 \\ 0 & 1 & 29/2 \\ 0 & 0 & -143/4 \end{bmatrix} \xrightarrow{R_1-\frac{1}{2}R_2} \begin{bmatrix} 1 & 0 & -13 \\ 0 & 1 & 29/2 \\ 0 & 0 & -143/4 \end{bmatrix}$$

$$\xrightarrow{-\frac{4}{143}R_3} \begin{bmatrix} 1 & 0 & -13 \\ 0 & 1 & 29/2 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_2-\frac{2}{29}R_2} \begin{bmatrix} 1 & 0 & -13 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_1-(-13)R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Question 19

Find two different row-echelon forms of

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$$

Solution: One possibility is

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} \xrightarrow{R_2-2R_1} \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

Another possibility is

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} \xrightarrow{R_1 \rightleftarrows R_2} \begin{bmatrix} 2 & 7 \\ 1 & 3 \end{bmatrix} \xrightarrow{R_1-\frac{1}{2}R_2} \begin{bmatrix} 1 & 7/2 \\ 1 & 3 \end{bmatrix} \xrightarrow{R_1-R_1} \begin{bmatrix} 1 & 7/2 \\ 0 & 1/2 \end{bmatrix}$$

Question 25

Find the coefficients a, b, c, and d so that the curve shown in the accompanying figure is the graph of the equation $y = ax^3 + bx^2 + cx + d$.

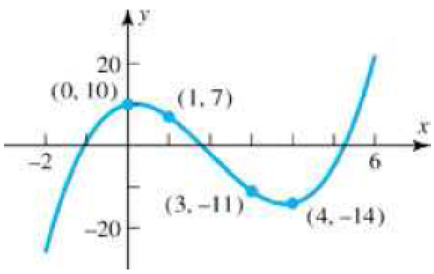

Figure Ex-25

Solution: Using the given points $(0, 10), (1, 7), (3, -11)$, and $(4, -14)$ we obtain the equations by substituting in $y = ax^3 + bx^2 + cx + d$.

$$d = 10 \quad (1)$$
$$a + b + c + d = 7 \quad (2)$$
$$27a + 9b + 3c + d = -11 \quad (3)$$
$$64a + 16b + 4c + d = -14 \quad (4)$$

Now put $Eq(1)$ in $Eq(2,3,4)$, we have a system of linear equations(3 equations 3 unknown).

$$\begin{aligned} a+b+c+10 &= 7 \\ 27a+9b+3c+10 &= -11 \\ 64a+16b+4c+10 &= -14 \end{aligned}$$

Now simplifying

$$\begin{aligned} a+b+c &= -3 \\ 27a+9b+3c &= -21 \\ 64a+16b+4c &= -24 \end{aligned}$$

$$\begin{aligned} a+b+c &= -3 \\ 9a+3b+c &= -7 \\ 16a+4b+c &= -6 \end{aligned}$$

In form of Matrix

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 9 & 3 & 1 & -7 \\ 16 & 4 & 1 & -6 \end{array}\right]$$

If we solve this system by ERO, we find that $a=1, b=-6, c=2$, and $d=10$.