Quiz 3C: NFAs

CS 212 Nature of Computation

Habib University — Fall 2023

Total Marks: 10 | Date: September 6, 2023
Duration: 15 minutes | Time: 830–845h

Student ID: _____

Student Name: _____

1. Given DFAs $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$, that recognize the languages L_1 and L_2 respectively, we construct the NFA $M_3 = (Q_3, \Sigma_\epsilon, \delta_3, q_3, F_3)$ as follows.
   1. $Q_3 = Q_1 \cup Q_2$
   2. $F_3 = F_2$
   3. $q_3 = q_1$
   4. $\delta_3(q_3, \epsilon) = \{q_1, q_2\}$
   5. $\forall q \in Q_1 \forall a \in \Sigma\ (\delta_3(q, a) = \{\delta_1(q, a)\})$
   6. $\forall q \in F_1\ (\delta_3(q, \epsilon) = \{q_2\})$
   7. $\forall q \in Q_2 \forall a \in \Sigma\ (\delta_3(q, a) = \{\delta_2(q, a)\})$

   (a) (7 points) Prove or disprove the following claim.
   Claim 1. $L(M_3) = L_1 \circ L_2$

   (b) (3 points) If the above claim is true, what does it establish about the closure of the class of regular languages?

Solution:

(a) *Proof.* We disprove the claim by presenting a counter example.
Consider the string $w \in L_2$ and $L_1 \neq \{\epsilon\}$.
Then $w \notin (L_1 \circ L_2)$.
Then, from the construction, there exists a sequence of states, $r_0 r_1 r_2 \dots r_m$, that M_3 traverses for w such that:

1. $r_0 = q_3$,
2. $r_1 = q_2$, and
3. $r_m \in F_2$.

$\therefore w \in L(M_3)$. □

(b) The truth of the claim establishes that the class of regular languages is closed under concatenation.