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Q5 let $u = (x_1, 0)$ & $v = (x_2, 0)$

① $u + v$ is in V.

$(x_1, 0) + (x_2, 0)$

$= (x_1 + x_2, 0)$

let $x_3 = x_1 + x_2$

$= (x_3, 0) \in V$ (closed under addition)

② ku is in V

$k(x_1, 0) = (kx_1, 0)$

let $kx_1 = x_2$

$(x_2, 0) \in V$ (closed under multiplication)

③ $u + v = v + u$

$(x_1 + x_2, 0) = (x_2 + x_1, 0)$

$(x_1 + x_2, 0) = (x_1 + x_2, 0)$

④ let $w = (x_3, 0)$

$u + (v + w) = (u + v) + w$

$u + v = (x_1 + x_2, 0)$

$v + w = (x_2 + x_3, 0)$

Taking LHS $(u + v) + w$

$(x_1 + x_2, 0) + (x_3, 0)$

$(x_1 + x_2 + x_3, 0)$

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$(x_1, 0) + (x_2 + x_3, 0)$
$u + (v + w)$

(5) $u + 0 = u$
$(x_1, 0) + (0, 0) = (x_1 + 0, 0)$
$= (x_1, 0)$

(6) $u + (-u) = 0$
$u = (x_1, 0)$
$-u = (-x_1, 0)$

$(x_1, 0) + (-x_1, 0)$
$(x - x_1, 0)$
$(0, 0)$

(7) $c(u + v) = cu + cv$
$c(x_1 + x_2, 0)$
$(cx_1, 0) + (cx_2, 0)$

(8) $(c + d)u = cu + du$

$(c + d)(x_1, 0)$
$(cx_1, 0) + (dx_1, 0)$
$cu + du$

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(9) $c(du) = cd(u)$

Taking LHS.

$c(dx_1, 0)$

$(cdx_1, 0)$

$cd(x_1, 0)$

(10) $1u = u$

$1(x_1, 0)$

$(x_1, 0)$

It is a vector space.

Q6) For the set of all real numbers of the form (x, y) where $x \geq 0$, with standard operations on $\mathbb{R}^2$.

→ Not a vector space as one axiom fails here.
↳ there exists an object (x_K, y_K) such that
$(x, y) + (x_K, y_K) = 0$
this can't be true as $x \in \mathbb{R}^+$ hence $x_K \in \mathbb{R}^-$
which does not exist in the vector space.

11. The set of all real-valued functions f defined everywhere on the real line and such that $f(1) = 0$, with the operations defined in Example 4.

$$(\mathbf{f} + \mathbf{g})(x) = f(x) + g(x) \quad \text{and} \quad (k\mathbf{f})(x) = kf(x)$$

→ Checking all the axioms:-

→ $f(x) \in F(-\infty, \infty)$ {such a vector field is denoted like this, dw}.

and $g(x) \in F(-\infty, \infty)$
then $(f+g)(x) \in F(-\infty, \infty)$ → This must be true as
$(f+g)(x) = f(x) + g(x)$ and the sum of 2 real valued functions is a real valued function.

→ $(f+g)(x) = (g+f)(x)$

$f(x) + g(x) = g(x) + f(x)$

LHS = RHS. (commutative addition).

$\rightarrow (k+f)(x) = (f+k)(x) = f(x)$.

if $k(x) = 0 \quad \forall x$

Then the axiom is True and we know that $k(x) \in F(-\infty, \infty)$ as it is also a real valued function.

$\rightarrow f(x) + (g+h)(x) = (f+g)(x) + h(x)$.

$f(x) + g(x) + h(x) = f(x) + g(x) + h(x) \rightarrow$ True.

$\rightarrow \exists\, g(x) \in F(-\infty, \infty)$ such that $g(x) + f(x) = f(x) + g(x) = 0$

if $g(x) = -f(x)$ The axiom is True and we know that $-f(x) \in F(-\infty, \infty)$ as it is a real valued function.

$\rightarrow kf(x) \in F(-\infty, \infty)$ true as $kf(x)$ is just another real valued function given that k is a real scalar.

$\rightarrow k((f+g)(x)) = kf(x) + kg(x)$.

$k(f(x) + g(x)) = kf(x) + kg(x)$.

$kf(x) + kg(x) = kf(x) + kg(x)$. ✓

$\rightarrow (k+m) f(x) = kf(x) + mf(x)$.

$kf(x) + mf(x) = kf(x) + mf(x)$. ✓

$k f(x) + m f(x) = k f(x) + m f(x)$.

$\rightarrow k(m f(x)) = (km) f(x)$.

↳ True.

$\rightarrow 1 \cdot f(x) = f(x)$

1 Times any real valued function is the function itself.

$\therefore f(x) \in F(-\infty, \infty)$ as it qualifies through all axioms.

17. Show that the following sets with the given operations fail to be vector spaces by identifying all axioms that fail to hold.

(a) The set of all triples of real numbers with the standard vector addition but with scalar multiplication defined by $k(x, y, z) = (k^2x, k^2y, k^2z)$.

(b) The set of all triples of real numbers with addition defined by $(x, y, z) + (u, v, w) = (z + w, y + v, x + u)$ and standard scalar multiplication.

(c) The set of all 2×2 invertible matrices with the standard matrix addition and scalar multiplication.

Q17) (a) $(k+m)\vec{u} = k\vec{u} + m\vec{u}$ fails here

$\Rightarrow (k+m)(x, y, z) = ((k^2+2km+m^2)x, (k^2+2km+m^2)y, (k^2+2km+m^2)z)$

while

$k\vec{u} + m\vec{u} = ((k^2+m^2)x, (k^2+m^2)y, (k^2+m^2)z)$

Q17) (b) $u + (v+w) = (u+v)+w$.

$(a_1, b_1, c_1) + ((a_2, b_2, c_2) + (a_3, b_3, c_3))$

$(a_1, b_1, c_1) + (c_2+c_3, b_2+b_3, a_2+a_3)$

$\rightarrow (a_2+a_3+c_1, b_1+b_2+b_3, c_2+c_3+a_1)$

$\hookrightarrow (u+v)+w$.

$(c_1+c_2, b_1+b_2, a_1+a_2) + (a_3, b_3, c_3)$

$\hookrightarrow (a_1+a_2+c_3, b_1+b_2+b_3, c_1+c_2+a_3)$ not a vector space

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Q19

(a) let $u = (x_1, y_1)$, $V = (x_2, y_2)$

$y = mx \rightarrow$ line passing through origin.

$u + v \in V$, $(x_1 + x_2, y_1 + y_2)$
$\downarrow x_3 \qquad \downarrow y_3$

$y_1 + y_2 = m(x_1 + x_2)$

$y_3 = mx_3 \rightarrow$ proved

(a) $y = mx$

$ky = mkx$ $\qquad$ let $ky = y_1$

$y_1 = mx_1$ $\qquad$ $kx = x_2$

$ku \in V$ proved

(b) $ax + by + cz = d$

$a(x_1 + x_2) + b(y_1 + y_2) + c(z_1 + z_2) = d$

$u + v \in V$

$kax + kby + kcz = d$

$ku \in V$ $\qquad$ closed under both

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25. We showed in Example 6 that every plane in $\overline{R^3}$ that passes through the origin is a vector space under the standard operations on R^3. Is the same true for planes that do not pass through the origin? Explain your reasoning.

26. It was shown in Exercise 14 above that the set of polynomials of degree 1 or less is a vector space under the operations stated in that exercise. Is the set of polynomials whose degree is exactly 1 a vector space under those operations? Explain your reasoning.

Q25) The planes that donot pass through The origin donot qualify for 1 axiom :-

$$0 + u = u + 0 = u.$$

as They don't have such a zero element (coordinate

Q26) Yes it should constitute a vector field as The decrease in the degree 0 Terms occurs for all sides and all other axioms apply on what is left.

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Q27

moon + moom = moon

Kmoon = moon.

It is only a vector space if moon is zero for an $u + 0 = u$ to be satisfied. Without zero vector, it does not qualify to be vector space.

Q28

No, because the space must contain a vector, its inverse and zero vector it makes 3 vectors.

Q29

- Ax 7
- Hypothesis
- Ax 5
- Ax 4

$\hookrightarrow K0 + K4 = K4$

$0 + \underbrace{K4}_{Z} = \underbrace{K4}_{Z}$

$0 + Z = Z$

- Ax 4
- Ax 4
- if K = 1, then Ax 10 else hypothesis.

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Q3d

- Ax 1
- Ax5
- hypothesis
- hypothesis

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(32) No, it is not possible.

Consider a vector space with two zero vectors $\vec{O}_1$ and $\vec{O}_2$ such that for a vector $\vec{u}$

$$\vec{u}+\vec{O}_1=\vec{u} \quad -(1)$$

$$\vec{u}+\vec{O}_2=\vec{u} \quad -(2)$$

$(1) \Rightarrow \quad O_1=\vec{u}-\vec{u}$

$(2) \Rightarrow \quad O_2=\vec{u}-\vec{u}$

Hence, $\quad \vec{u}-\vec{u}=\vec{u}-\vec{u}$

$$\vec{O}_1=\vec{O}_2$$

(33) No, it is not possible.

Consider a vector space ~~with~~ containing a vector $\vec{u}$ and a zero vector $\vec{O}$.

Consider that $\vec{u}$ has two negatives $\overrightarrow{(-u)}_1$ and $\overrightarrow{(-u)}_2$ such that

$$\vec{u}+\overrightarrow{(-u)}_1=\vec{O} \quad -(1)$$

$$\vec{u}+\overrightarrow{(-u)}_2=\vec{O} \quad -(2)$$

$(1) \Rightarrow (-u)_1=\vec{O}-\vec{u}$

$(2) \Rightarrow \overrightarrow{(-u)}_2=\vec{O}-\vec{u}$

$\therefore \quad \vec{O}-\vec{u}=\vec{O}-u \Rightarrow \overrightarrow{(-u)}_1=\overrightarrow{(-u)}_2$

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