Habib University
Course Code: EE 468/CE 468: Mobile Robotics
Course Title: Mobile Robotics
Instructor name: Dr. Basit Memon
Examination: Quiz 1 Exam Date: October 21, 2022
Total Marks: 100 Duration: 30 minutes

Instructions

1. You can refer to course slides, your notes, homework solutions, or any of the books included in the course syllabus. You're not permitted to specifically search for responses to any of the exam questions online. Where appropriate, you can cite the slides and don't have to redo what has already been done.
2. You are welcome to utilize MATLAB in whichever way you find useful. You can submit the response to a question in the form of a MATLAB live script as well, but you cannot use numerical methods where question explicitly asks you to employ analytical methods.
3. The exam will be administered under HU student code of conduct (see Chapter 3 of `https://habibuniversity.sharepoint.com/sites/Student/code-of-conduct`). You may not talk, discuss, compare, copy from, or consult with any other human on this planet (except me) on questions or your responses during the exam.
4. Make sure that you show your work (intermediate steps). Most of the points for any question will be given based on the followed process.
5. Kindly use a black/blue pen to write your hand-written solutions, so that they are legible.
6. The questions or their associated points are not arranged by complexity or time consumption.

Questions

Problem 1
10 points

If your car had four Swedish wheels, which of the following, one or more, would be true? Justify.

(a) Car will be able to move sideways.

(b) Parallel parking would be much easier.

(c) We would have trouble driving quickly on curved roads.

(d) All of the above.

Solution 1

(a) Because of the omniwheels, the car is able to move in any direction instantaneously, including sideways. Its differential dof is 2.

(b) Since the car can move sideways, we can directly insert our car between two parked cars and parallel parking becomes much easier.

(c) While driving fast on a curved road, the car will experience a large centripetal force. In a regular car, this force is countered passively by the lateral motion constraint on standard wheels. This has to be actively counteracted in the case of omniwheels, which means that driving on a curved road will be more difficult with swedish wheels. See [1, p87].

Problem 2
10 points

Suppose x-axis of right-handed frame is along the length of your phone, pointing from bottom to top. Say the z-axis of the frame is pointed from right to left. Given it is a right-handed frame, which direction is the y-axis pointed?

Solution 2

According to the right hand rule, the y-axis should be pointed into the phone.

Problem 3
40 points

Suppose a differential drive robot has wheels of differing diameters. The left wheel has diameter 2 and the right wheel has diameter 3. The distance of either wheel from mid-point of the line connecting them is 5. The robot is currently positioned at $(1, 1)$ and $\theta = pi/4$. The robot spins both wheels at an angular speed of 6.

(a) Compute the robot's instantaneous linear and angular velocities in the global reference frame.

(b) Compute the robot's position after 1 second. State the method you're employing or any assumptions that you have made.

Solution 3

(a) Adapting the equations derived for a differential drive robot in class (see slide 27, deck 04) for the case of differing wheel diameters, we have:

$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\phi}(t) \end{bmatrix} = \begin{bmatrix} \frac{r_L}{2}\cos\phi & \frac{r_R}{2}\cos\phi \\ \frac{r_L}{2}\sin\phi & \frac{r_R}{2}\sin\phi \\ -\frac{r_L}{L} & \frac{r_R}{L} \end{bmatrix} \begin{bmatrix} \omega_L \\ \omega_R \end{bmatrix}$$

As suggested in the question, $\omega_L = \omega_R = 6$, $r_L = 2/2 = 1$, $r_R = 3/2 = 1.5$, $L = 5(2) = 10$, and $\phi = \pi/4$.

$$\begin{aligned} \begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\phi}(t) \end{bmatrix} &= \begin{bmatrix} \frac{1}{2}\cos\pi/4 & \frac{3}{4}\cos\pi/4 \\ \frac{1}{2}\sin\pi/4 & \frac{3}{4}\sin\pi/4 \\ -\frac{1}{10} & \frac{3}{20} \end{bmatrix} \begin{bmatrix} 6 \\ 6 \end{bmatrix} \\ &= \begin{bmatrix} 5.3 \\ 5.3 \\ 0.3 \end{bmatrix} \end{aligned}$$

(b) Since we have found the equations for velocities in the previous part, we can obtain the positions by integration. We can directly integrate the previous equations, if possible, or use one of the methods for forward integration discussed in class, e.g. Euler, Trapezoidal, etc.

For instance, employing Euler integration:

$$\begin{aligned} x(t+1) &= x(t) + \dot{x}(t)\Delta t \\ &= 1 + (5.3)(1) = 6.3 \end{aligned}$$

Similarly,

$$\begin{aligned} y(t+1) &= y(t) + \dot{y}(t)\Delta t \\ &= 1 + (5.3)(1) = 6.3 \end{aligned}$$

It is also possible here to directly integrate the expressions for velocities, which is not always possible. Assuming that current time is $t = 0$ wlog, we have the following based on the expressions from the previous part:

$$
\begin{aligned}
\phi(t) &= \int_0^t \frac{\omega_R r_R - \omega_L r_L}{L} dt \\
&= 0.3t + \pi/4 \\
x(t) &= x(0) + \int_0^t \frac{\omega_R r_R + \omega_L r_L}{L} \cos\phi(t) dt \\
&= x(0) + 7.5 \int_0^t \cos(\pi/4 + 0.3t) dt \\
&= x(0) + 7.5 \int_0^t [\cos(\pi/4)\cos(0.3t) - \sin(\pi/4)\sin(0.3t)] dt \\
&= x(0) + 7.5\cos(\pi/4) \int_0^t \cos(0.3t) dt - 7.5\sin(\pi/4) \int_0^t \sin(0.3t) dt \\
&= x(0) + \frac{5.3}{0.3}[\sin(0.3t) + \cos(0.3t) - 1]
\end{aligned}
$$

So,

$$x(1) = 1 + (5.3)(0.83)$$

Problem 4
40 points

Determine open-loop control functions, i.e. $\omega_L(t)$ and $\omega_R(t)$, to move the above differential-drive robot on a circular reference trajectory of radius 5, i.e. $x(t) = 5\cos t$ and $y(t) = 5\sin t$. State any assumptions that you make.

Solution 4

We're provided a trajectory for the robot, so we can determine the required open-loop robot velocity and angular velocity from this trajectory. The requirement is that the trajectory is at least twice differentiable, which is the case here. Based on slides 39-40, in slide deck 04,

$$
\begin{aligned}
v(t) &= \sqrt{\dot{x}^2(t) + \dot{y}^2(t)} = \sqrt{25\sin^2 t + 25\cos^2 t} = 5 \\
\omega(t) &= \frac{\dot{x}(t)\ddot{y}(t) - \dot{y}(t)\ddot{x}(t)}{\dot{x}^2(t) + \dot{y}^2(t)} \\
&= \frac{(-5\sin t)(-5\sin t) - (5\cos t)(-5\cos t)}{25} = 1
\end{aligned}
$$

Note that these expressions provide v and ω in the robot frame. We now need to find the wheel velocities. We can use the following expressions, describing kinematics of differential drive robot:

$$
\begin{aligned}
v_R &= \frac{2v + \omega L}{2} = \frac{2(5) + 1(10)}{2} = 10 \\
v_L &= \frac{2v - \omega L}{2} = \frac{2(5) - 1(10)}{2} = 0 \\
\omega_R &= \frac{v_R}{1.5} = 6.67 \text{ rad/s} \\
\omega_L &= \frac{v_L}{1} = 0 \text{ rad/s}
\end{aligned}
$$

References

[1] Roland Siegwart, Illah R Nourbakhsh, and Davide Scaramuzza. Autonomous mobile robots. *A Bradford Book*, 15, 2011.