Midterm Examination

CS 212 L3 Nature of Computation
Habib University
Oct 4, Fall 2023

25 points

Sample Solution

Instructions:

1. You may consult any offline resources.
2. The questions in this exam rely on argumentation. Make sure to provide sound justifications that are both precise and concise. Simply stating in answer will earn very few marks.
3. Use of unfair means, including collaboration, attempts at collaboration, and copying, violates academic honesty and will be met with disciplinary action.
4. Attempt all problems.

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viel Spaß und viel Glück!

Short Problems

1. Provide brief answers and/or supporting justifications for the following as applicable.

(a) $\boxed{\text{2 points}}$ What is the relation between the number of states of an NFA and the number of states of its corresponding DFA?

> **Solution:** If the NFA has n states and the corresponding DFA has m states, then $m \leq 2^n$, depending on the number of unreachable states in the DFA.

(b) $\boxed{\text{2 points}}$ Is the following statement True or False? All subsets of a regular language are regular.

> **Solution:** We prove the statement False using a counterexample from class.
>
> *Proof.* Consider $L_1 = \{0^n 1^n\}$ and $L_2 = \{0, 1\}^*$.
> $L_1 \subseteq L_2$
> L_1 is not regular but L_2 is. □

(c) $\boxed{\text{2 points}}$ We are given languages A, B, and C such that $A \cup B = C$, and B and C are regular languages. What can we say about A?

> **Solution:** Nothing conclusive can be deduced about A. To illustrate, consider $B = C = \Sigma^*$. The given properties will hold irrespective of any properties of A.

(d) $\boxed{\text{2 points}}$ Argue about the regularity of the language that contains all the strings over its alphabet whose length is even and less than 100.

> **Solution:** This language is finite, hence regular.

(e) $\boxed{\text{2 points}}$ Argue about the relationship between the following classes of languages: `RL`, the class of regular languages; `RL'`, the class of languages that are not regular; and `CFL`, the class of context-free languages.

> **Solution:** $RL \subset CFL$; `RL` is a proper subset of `CFL`.
> $(RL \cap RL' = \emptyset) \wedge (RL \cup RL' = \mathbb{U})$; `RL` and `RL'` are disjoint and together cover the set of all languages.
> $CFL \subset RL'$; `CFL` is a proper subset of `RL'`, there are non-regular languages that are not context-free.

Sample Solution

Long Problems

2. $\boxed{\text{5 points}}$ Prove or disprove the claim: If L is a language, then L^* is closed under concatenation.

Solution: For L^* to be closed under concatenation, given any strings u, v in L^*, the string $u \circ v$ must also be in L^*. We provide a direct proof.

Proof. Consider $u, v \in L^*$.
Then $u = u_1 \circ u_2 \circ u_3 \circ \ldots \circ u_m$ where each $u_i \in L$ and $m \geq 0$.
And $v = v_1 \circ v_2 \circ v_3 \circ \ldots \circ v_n$ where each $v_i \in L$ and $n \geq 0$.
Then $u \circ v = u_i \circ u_2 \circ \ldots \circ u_m \circ v_i \circ v_2 \circ \ldots \circ v_n$.
$\therefore u \circ v \in L^*$. $\square$

3. For each of the following languages over $\Sigma = \{0, 1\}$, argue about its membership in `RL`, `RL'`, and `CFL` as defined above.

   (a) $\boxed{\text{5 points}}$ all strings in which the total number of 0s is a non-negative multiple of 3.

   Solution: This language, L, has the regular expression: $(1^*01^*01^*01^*)^*$.
   Therefore L is regular.
   Therefore $L \in RL, L \in CFL, L \notin RL'$.

   (b) $\boxed{\text{5 points}}$ all strings of the form $x\#y$ where $x, y \in \Sigma^*$ and x and y have equal lengths.

   Solution: This language, L, can be proven non-regular using the pumping lemma, e.g. by considering the string $s = 0^p\#0^p$.
   L can be proven to be context-free by constructing a PDA that recognizes it. The PDA starts by pushing \$ to the stack as a marker for the empty stack. It then pushes to the stack for every input symbol until # is encoutnered. Thereafter, it pops the stack for every input symbol. If the top of the stack is \$ at the end of the input, the PDA accepts.
   Therefore $L \notin RL, L \in CFL, L \in RL'$.

Sample Solution