Habib University
shaping futures

Linear Algebra – Math 205
Exercise Set of Lect 24 & 25
(SPRING 2023)
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Homework 6b: Linear Transformation

Question 27 (From Book)

Solution: (a) $(T_1 + T_2)(v + w) = T_1(v + w) + T_2(v + w)$

$$\begin{aligned}(T_1 + T_2)(v + w) &= T_1(v + w) + T_2(v + w)\\ &= T_1(v) + T_1(\omega) + T_2(v) + T_1(\omega)\\ &\therefore T_1 \& T_2 \xrightarrow{\text{Linear Transformation}}\\ &= (T_1 + T_2)v + (T_1 + T_2)\omega\end{aligned}$$

$$\begin{aligned}(T_1 + T_2)\alpha v &= T_1(\alpha v) + T_2(\alpha v)\\ &= \alpha T_1(v) + \alpha T_2(v)\\ &= \alpha\left(T_1(v) + T_2(v)\right)\\ &= \alpha\left(T_1 + T_2\right)(v)\end{aligned}$$

(b) $T_1(x, y) = (2y, 3x) \quad T_2(x, y) = (y, x)$

$$\begin{aligned}&\text{(b) } T_1(x, y) = (2y, 3x) \quad T_2(x, y) = (y, x)\\ &(T_1 + T_2)(x + y) = T_1(x, y) + T_2(x, y)\\ &= (2y, 3x) + (y, x)\\ &= (3y - 4x)\end{aligned}$$

Question 28 (From Book)

(a) $F(x_1 y) = (a_1 x + b_1 y_1, a_2 x + b_2 y)$
Solution:

$$\begin{aligned}F(\alpha(x_1) + x_2, \alpha(y_1) + y_2) &= \left(a_1(\alpha(x_1) + x_2) + b_1(\alpha(y_1) + y_2), a_2(\alpha(x_1) + x_2) + b_2(\alpha(y_1) + y_2)\right)\\ &= (a_1\alpha(x_1) b_1\alpha(y_1), a_2\alpha(x_1) + b_2\alpha(y_1)) + (a_1 x_2 + b_1 y_2, a_2 x_2 + b_2 y_2)\\ &= \alpha F(x_1, y_1) + F(x_2, y_2)\end{aligned}$$

b) $F(x_1 + x_2, y_1 + y_2) = \left(a_1(x_1 + x_2)^2 + b_1(y_1 + y_2)^2\right)$ It is nonlinear, one can verify by getting terms $2x_1 x_2$ and $2y_1 y_2$.

Question 29 (From Book)

Solution: Let any vector $v = \sum_{i=1}^{n} c_i v_i$

$$T(v) = T\left(\sum_i c_i v_i\right) = \sum c_i T(v_i) \qquad \because T \text{ is Linear}$$

since each v_i on T is 0

$$T(v) = \sum c_i 0 = 0$$

Question 30 (From Book)

Solution: Let any vector $v = \sum_{i=1}^{n} c_i v_i$

$$T(v) = T\left(\sum_i c_i v_i\right) = \sum c_i T(v_i) \qquad \because T \text{ is Linear}$$

since each v_i on T is v_i

$$T(v) = \sum c_i v_i = v$$

Q) 32 From Book

$$\begin{aligned} T(f+\alpha g)) &= 5(f(x)+\alpha g(x)) + 3\int_a^x (f(t)+\alpha g(t))dt \\ &= 5f(x) + \alpha(5g(x)) + 3\int_a^x f(t)dt + \alpha\left(3\int_a^x g(t)dt\right) \\ &= 5f(x) + 3\int_a^x f(t)dt + \alpha\left(5g(x) + 3\int_a^x (t)dt\right) \\ &= T(f) + \alpha T(g) \end{aligned}$$

Q5 From Lect. 25

Solution:

$$\begin{aligned} A(\underline{v_2}) &= \left(A\underline{u}_2 - \frac{(u_2 \cdot v_1)}{\|v_1\|^2}\underline{v_1}\right) = A\left(\underline{u}_2\right) - A\left(\frac{(v_2 \cdot v_1)}{\|v_1\|^2}\underline{v_1}\right) \\ &= \lambda\underline{u}_2 - \frac{(u_2 \cdot v_1)}{\|v_1\|^2}\lambda\underline{v_1} = \lambda\left(\underline{u}_2 - \frac{(v_2 \cdot v_1)}{\|v_1\|^2} - v_1\right) \\ A\left(\underline{v_2}\right) &= \lambda\underline{v_2} \end{aligned}$$

Q7 From Lect. 25

Solution: Kindly see the solution manual.