Habib University
Course Code: EE 468/CE 468: Mobile Robotics
Course Title: Mobile Robotics
Instructor name: Dr. Basit Memon
Examination: Quiz 1 Exam Date: October 4, 2023
Total Marks: 100 Duration: 30 minutes

Instructions

1. You can refer to course slides, your notes, homework solutions, or any of the books included in the course syllabus. You're not permitted to specifically search for responses to any of the exam questions online. Where appropriate, you can cite the slides and don't have to redo what has already been done.

2. You are welcome to utilize MATLAB in whichever way you find useful. You can submit the response to a question in the form of a MATLAB live script as well, but you cannot use numerical methods where question explicitly asks you to employ analytical methods.

3. The exam will be administered under HU student code of conduct (see Chapter 3 of `https://habibuniversity.sharepoint.com/sites/Student/code-of-conduct`). You may not talk, discuss, compare, copy from, or consult with any other human on this planet (except me) on questions or your responses during the exam.

4. Make sure that you show your work (intermediate steps). Most of the points for any question will be given based on the followed process.

5. Kindly use a black/blue pen to write your hand-written solutions, so that they are legible.

6. The questions or their associated points are not arranged by complexity or their time demand.

Questions

Problem 1
CLO1-C3
50 points

Consider a mobile robot exploring an underwater ecosystem. The robot is moving in a 2D plane underwater, mapping the ocean currents. The robot is equipped with a vector sensor that measures the direction and magnitude of water flow (Q) in its own frame of reference. The robot's mission is to study the ocean currents and understand their patterns for scientific research.

At $t = 0$, the robot is located at coordinates $(x, y) = (2, 3)$ meters on the ocean floor. The robot is moving with a linear velocity vector $(4m/s, 2m/s)$ and angular velocity $0.1 rad/s$ about the z-axis, expressed in the global coordinates, relative to the global frame of reference. The robot has an initial orientation $\theta = 30°$ with respect to the global frame. The vector sensor is mounted at coordinates $(-1, 1)$ meters relative to the robot's local frame of reference. It measures the water flow vector through an instrument placed directly below the origin of the sensor frame. The is currently providing measurement $Q = (2m/s, 3m/s)$ in the sensor's local frame of reference, which is the velocity of water at the tip of the sensor instrument. **Determine the water flow, Q, with respect to the global frame of reference at $t = 0$.** You're welcome to make justifiable assumptions about missing information.

Solution 1 Let's note down the information available to us using our convention:

$$
\begin{aligned}
{}^w r_v^w &= (2, 3, 0) \\
{}^w v_v^w &= (4, 2, 0) \\
{}^w \omega_v^w &= (0, 0, 0.1) \\
{}^v r_s^v &= (-1, 1, 0) \\
{}^s v_o^s &= (2, 3, 0),
\end{aligned}
$$

where w is the global frame, v is the robot frame, s is the sensor frame, and o is the frame at the tip of the instrument where water flow is being measured. Let's assume that the orientation of the sensor frame and robot frame are aligned. We're interested in finding v_o^w. We can use transport theorem to obtain an expression for this:

$$
\begin{aligned}
r_o^w &= r_v^w + r_s^v + r_o^s \\
v_o^w &= v_v^w + v_s^v + \omega_v^w \times r_s^v + v_o^s + \omega_s^w \times r_o^s
\end{aligned}
$$

As the sensor frame is fixed with respect to robot frame, $v_s^v = 0$. Also, $\omega_v^w = \omega_s^w$ and r_o^s only has a z component. Now, let's express the previous equation entirely in the coordinates of the robot frame:

$$
\begin{aligned}
{}^v v_o^w &= {}^v v_v^w + {}^v \omega_v^w \times {}^v r_s^v + {}^v v_o^s + {}^v \omega_v^w \times {}^v r_o^s \\
&= R_w^v \, {}^w v_v^w + R_w^v \, {}^w \omega_v^w \times {}^v r_s^v + R_s^v \, {}^s v_o^s + R_w^v \, {}^w \omega_v^w \times R_s^v \, {}^s r_o^s
\end{aligned}
$$

Let's first find the rotation matrices:

$$R_v^w = \begin{bmatrix} \cos(30°) & -\sin(30°) & 0 \\ \sin(30°) & \cos(30°) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow R_w^v = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_s^v = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Substitute them in the velocity expression:

$$\begin{aligned} {}^v v_o^w &= R_w^v \; {}^w v_v^w + R_w^v \; {}^w \omega_v^w \times {}^v r_s^v + R_s^v \; {}^s v_o^s + R_w^v \; {}^w \omega_v^w \times R_s^v \; {}^s r_o^s \\ &= \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0.1 \end{bmatrix} \times \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} \\ &\quad + \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0.1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ z \end{bmatrix} \\ &= \begin{bmatrix} 2\sqrt{3} \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -0.1 \\ -0.1 \\ 0 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} 2\sqrt{3} + 1.9 \\ 3.9 \\ 0 \end{bmatrix} \end{aligned}$$

Problem 2
CLO1-C3

50 points

Consider the differential-drive robot shown in Figure 1, where the circle at the top is a caster wheel. We desire the velocity of the caster wheel to be v_c as indicated in the figure. Compute

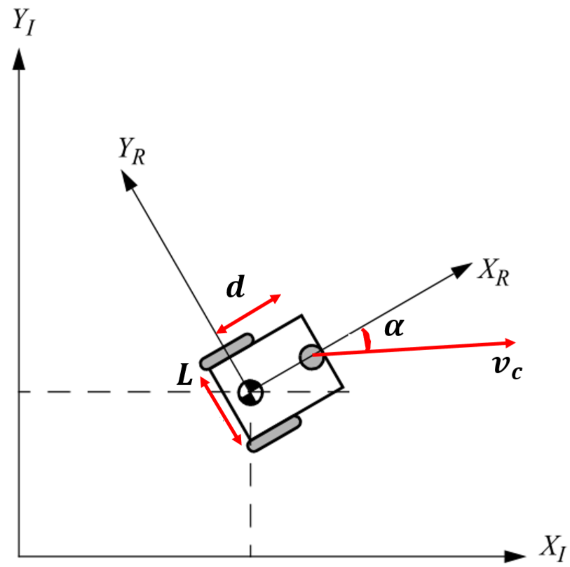

Figure 1: A differential-drive robot

the left wheel and right wheel velocities, v_l and v_r, required to achieve this objective. Assume $L = 0.4m$, $d = 0.3m$, $\alpha = 45°$, and $\|v_c\| = 0.1m/s$.

Solution 2 We've already derived a kinematic model for the differential-drive robot in class (*see slide 34, slide deck 04-kinematics*):

$$v_r = v_x + \frac{\omega L}{2}$$
$$v_l = v_x - \frac{\omega L}{2},$$

where v_x is the velocity of the origin of the robot frame along the direction X_R and ω is angular velocity of the robot with respect to the global frame.

We need to relate these velocities, v_x and ω, to the velocity of the castor wheel. This can be done through an application of the transport theorem:

$$v_c^I = v_R^I + \omega_R^I \times r_c^R.$$

Let's express all quantities in the coordinates of the robot frame, R:

$$
\begin{aligned}
{}^{R}v_c^{I} &= \begin{bmatrix} v_x \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \omega \end{bmatrix} \times \begin{bmatrix} d \\ 0 \\ 0 \end{bmatrix} \\
\Rightarrow \begin{bmatrix} \|v_c\| \cos\alpha \\ -\|v_c\| \sin\alpha \\ 0 \end{bmatrix} &= \begin{bmatrix} v_x \\ d\omega \\ 0 \end{bmatrix} \\
\Rightarrow v_r &= \|v_c\| \cos\alpha - \frac{\|v_c\| L \sin\alpha}{2d} = 0.0236 \\
v_l &= \|v_c\| \cos\alpha + \frac{\|v_c\| L \sin\alpha}{2d} = 0.1179
\end{aligned}
$$