Habib University
Course Code: EE 468/CE 468: Mobile Robotics
Course Title: Mobile Robotics
Instructor name: Dr. Basit Memon
Examination: Quiz 4 Exam Date: December 4, 2023
Total Marks: 100 Duration: 30 minutes

## Instructions

1. This quiz is open book/notes. Use of internet is not permitted.
2. You are welcome to write computer programs to assist your computation.
3. The exam will be administered under HU student code of conduct (see Chapter 3 of `https://habibuniversity.sharepoint.com/sites/Student/code-of-conduct`).
4. Make sure that you show your work (intermediate steps). Most of the points for any question will be given based on the followed process.
5. Kindly use a black/blue pen to write your hand-written solutions, so that they are legible.

## Questions

**Problem 1**
**CLO2-C4**

**100 points**

Consider two omnidirectional point robots navigating in an environment containing a single point landmark. It can be assumed that the motion of the robot in the $x$ and $y$ directions can be controlled independently. The motion of the robot is not error-free. Each robot is equipped with a sensor that can measure (i) the relative distance between itself and the other robot; (ii) the relative distance between itself and the landmark. As usual, assume that the sensors are noisy. Furthermore, assume that the sensor can distinguish between the other robot and the landmark, and that it can always see both. Suppose that two robots are communicating with each other and sharing their measurements and control signals with each other at each time step, i.e. at time $k$, robot A shares its $z_k^A$ and $u_k^A$, the control commands it sent, with robot B; at the same time, robot B shares $z_k^B$ and $u_k^B$ with robot A. Build an EKF-based algorithm for robot A, which uses both robots' measurements and controls, to simultaneously localize both the robots and the landmark, given an initial estimate of their positions.

(a) What should be the state vector to be estimated in this case?

(b) The motion models of both robots will be identical. What is the motion model?

(c) What is the measurement model?

(d) Provide the complete SLAM algorithm for this case.

## Solution 1

(a) Let $(x_A, y_A)$, $(x_B, y_B)$, and $(x_l, y_l)$ be the locations of robot A, robot B, and the landmark respectively. Then, the state vector can be extended to be:

$$\begin{bmatrix} x_A \\ y_A \\ x_B \\ y_B \\ x_l \\ y_l \end{bmatrix}.$$

(b) Since the motion in all directions can be controlled independently, our motion model can be:

$$x_{k+1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} x_k + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_{A,x} \\ u_{A,y} \\ u_{B,x} \\ u_{B,y} \end{bmatrix} + \begin{bmatrix} w_{A,x} \\ w_{A,y} \\ w_{B,x} \\ w_{B,y} \\ 0 \\ 0 \end{bmatrix}$$

$$x_{k+1} = Ax_k + Bu_k + w_k,$$

where $u_k$ is the control vector and $w_k$ is the noise vector with corresponding covariance matrix $R$.

(c) Each robot gets measurement $z$, where

$$\begin{aligned} z_k^A &= h(x_k) + v_k^A \\ &= \begin{bmatrix} \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2} \\ \sqrt{(x_A - x_l)^2 + (y_A - y_l)^2} \end{bmatrix} + v_k^A, \end{aligned}$$

where $v_k^A$ is the noise term. Similarly,

$$z_k^B = \begin{bmatrix} \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2} \\ \sqrt{(x_B - x_l)^2 + (y_B - y_l)^2} \end{bmatrix} + v_k^B,$$

The combined measurement accessible to each robot is:

$$
\begin{aligned}
z_k &= \begin{bmatrix} z_k^A \\ z_k^B \end{bmatrix} + \begin{bmatrix} v_k^A \\ v_k^B \end{bmatrix} \\
&= \begin{bmatrix} \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2} \\ \sqrt{(x_A - x_I)^2 + (y_A - y_I)^2} \\ \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2} \\ \sqrt{(x_B - x_I)^2 + (y_B - y_I)^2} \end{bmatrix} + \begin{bmatrix} v_k^A \\ v_k^B \end{bmatrix}
\end{aligned}
$$

The corresponding noise covariance matrix is $Q_k$.

(d) Since measurement model is nonlinear, let's first derive the measurement Jacobian before stating the EKF SLAM steps:

$$
H_k = \begin{bmatrix}
\frac{\bar{x}_A - \bar{x}_B}{d_{AB}} & \frac{\bar{y}_A - \bar{y}_B}{d_{AB}} & \frac{\bar{x}_B - \bar{x}_A}{d_{AB}} & \frac{\bar{y}_B - \bar{y}_A}{d_{AB}} & 0 & 0 \\
\frac{\bar{x}_A - \bar{x}_I}{d_{AI}} & \frac{\bar{y}_A - \bar{y}_I}{d_{AI}} & 0 & 0 & \frac{\bar{x}_I - \bar{x}_A}{d_{AI}} & \frac{\bar{y}_I - \bar{y}_A}{d_{AI}} \\
\frac{\bar{x}_A - \bar{x}_B}{d_{AB}} & \frac{\bar{y}_A - \bar{y}_B}{d_{AB}} & \frac{\bar{x}_B - \bar{x}_A}{d_{AB}} & \frac{\bar{y}_B - \bar{y}_A}{d_{AB}} & 0 & 0 \\
\frac{\bar{x}_B - \bar{x}_I}{d_{BI}} & \frac{\bar{y}_B - \bar{y}_I}{d_{BI}} & 0 & 0 & \frac{\bar{x}_I - \bar{x}_B}{d_{BI}} & \frac{\bar{y}_I - \bar{y}_B}{d_{BI}}
\end{bmatrix},
$$

and,

$$
\begin{aligned}
d_{AB} &= \sqrt{(\bar{x}_A - \bar{x}_B)^2 + (\bar{y}_A - \bar{y}_B)^2} \\
d_{AI} &= \sqrt{(\bar{x}_A - \bar{x}_I)^2 + (\bar{y}_A - \bar{y}_I)^2} \\
d_{BI} &= \sqrt{(\bar{x}_B - \bar{x}_I)^2 + (\bar{y}_B - \bar{y}_I)^2}
\end{aligned}
$$

The EKF SLAM steps are:

(a) $\bar{x}_k = A\hat{x}_{k-1} + Bu_k$

(b) $\bar{\Sigma}_k = A\Sigma_{k-1}A^T + R$

(c) $K_k = \bar{\Sigma}_k H_k^T \left(H_k \bar{\Sigma}_k H_k^T + Q\right)^{-1}$

(d) $\hat{x}_k = \bar{x}_k + K_k[z_k - h(\bar{x}_k)]$

(e) $\Sigma_k = (I - K_k H_k)\bar{\Sigma}_k$