Computational Intelligence

Unit # 2

Acknowledgement

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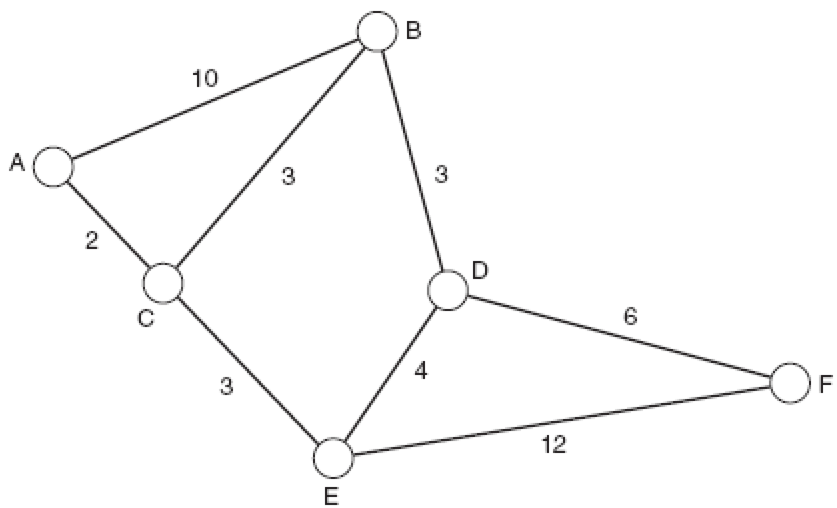
What is Optimization?

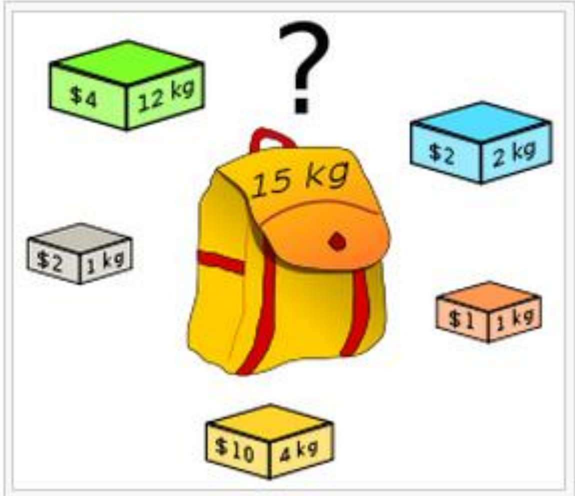
Travelling Salesman Problem

- Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city?



Knapsack Problem

- Given a set of items, each with a weight and a value, determine the number of each item to include in a collection so that the total weight is less than or equal to a given limit and the total value is as large as possible.



Profit Maximization

- An automobile manufacturer produces several kinds of cars. Each kind requires a certain amount of factory time per car to produce, and yields a certain profit per car. A certain amount of factory time has been scheduled for the next week, and it is desired to use all this time; but at least a certain number of each kind of car must be manufactured to meet dealer requirements.

Other similar problems

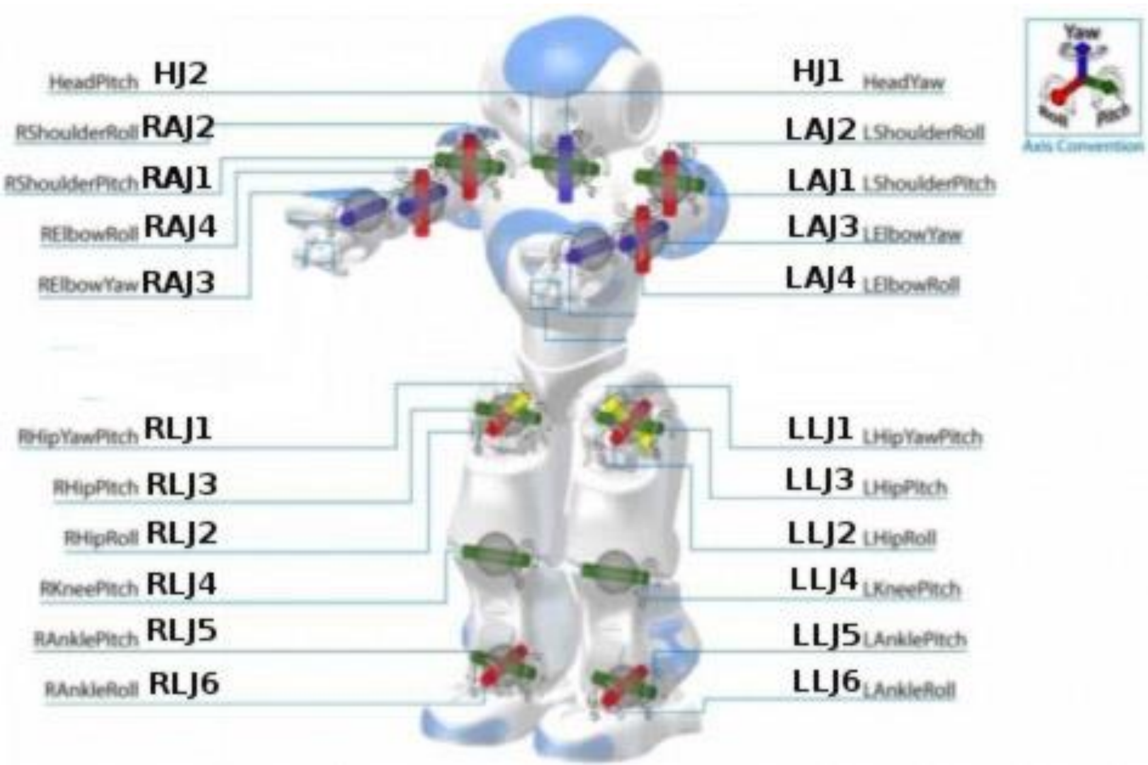
- Resource Allocations
- Scheduling Problems
- Shelf Placement
- Graph Coloring
- VLSI Design
- …….
- And many more combinatorial optimization problems

Combinatorial optimization

- **Combinatorial optimization** is a topic that consists of finding an optimal object from a finite set of objects. In many such problems, exhaustive search is not tractable.

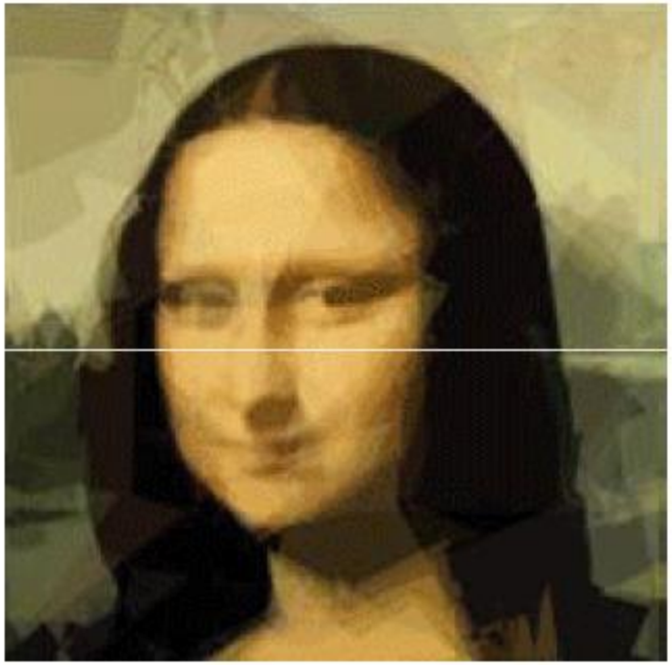
Evolving Bipedal Robot Walk

Nao Body



http://www.youtube.com/watch?v=3cj-UQN6rj0

Evolutionary Art: Mona Lisa Evolution

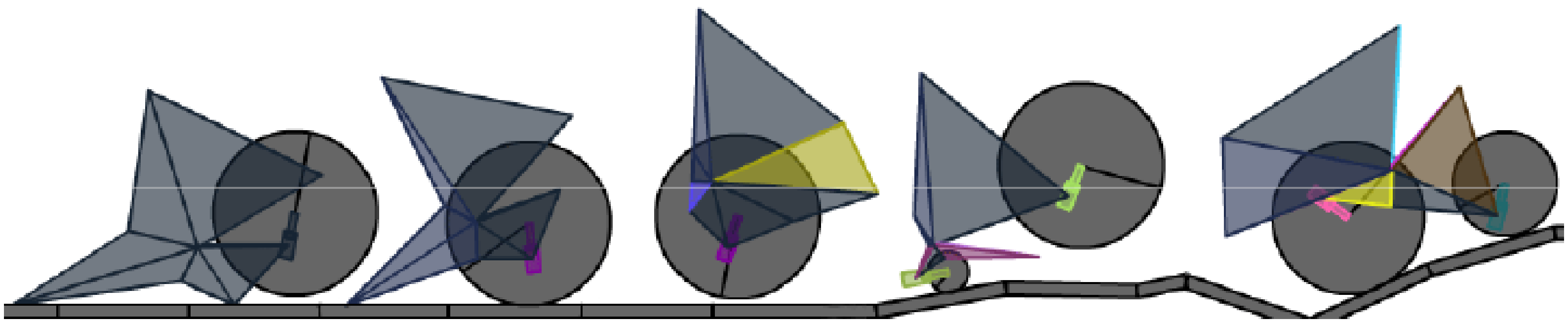


Problem: paint a replica of the Mona Lisa using only 50 semi transparent polygons

[Genetic Programming: Evolution of Mona Lisa – Roger Johansson Blog]

Structure Design – Car Evolution

Design a car using polygons and wheels which able to run on a terrain.



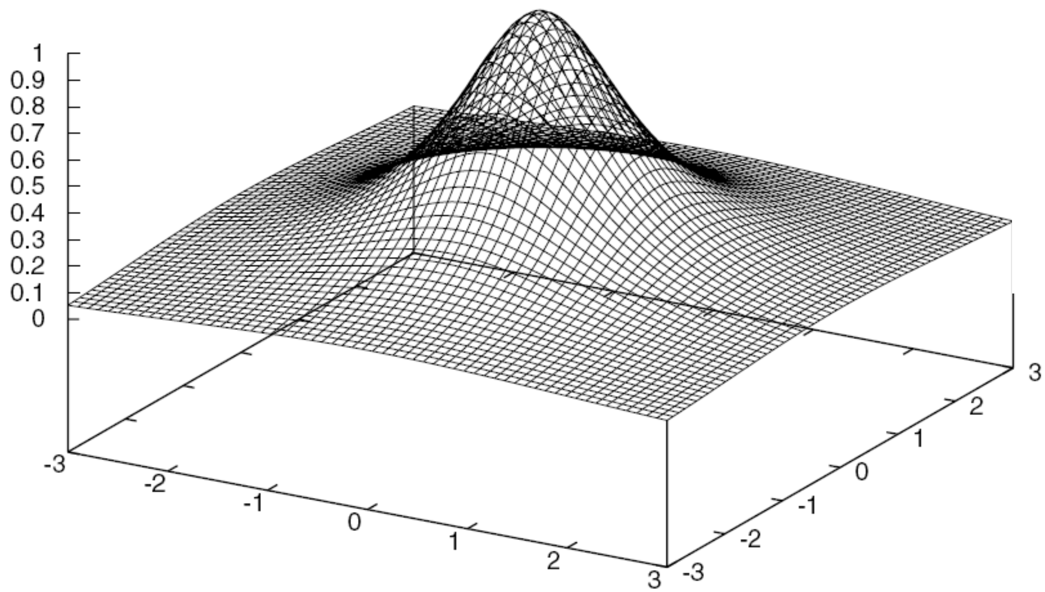
http://boxcar2d.com/about.html

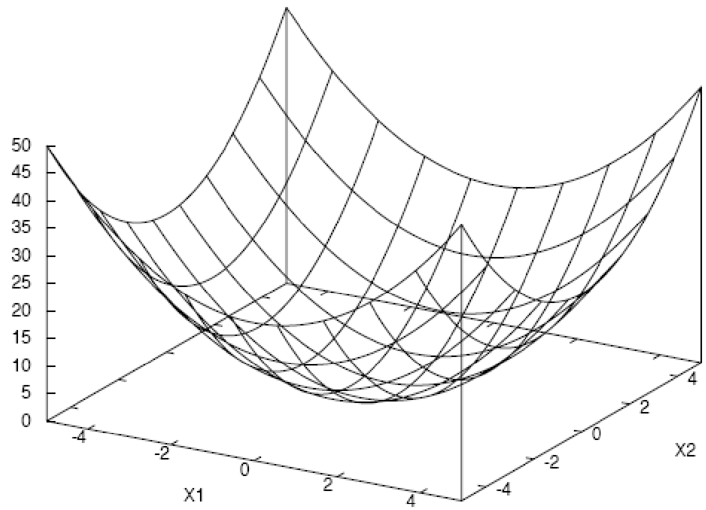
[BoxCar2D. Self Evolving computer at work. (youtube.com)]()

Search vs. Optimization

- The difference between optimization algorithms and search algorithms is that when performing a search algorithm, we know the element x_i we are looking for and just want to find its position in a structured set.
- In global optimization algorithms on the other hand we do not even know the characteristics of the x_i beforehand and are only given some criteria which describe if a given configuration is good or not.

Simple Functions





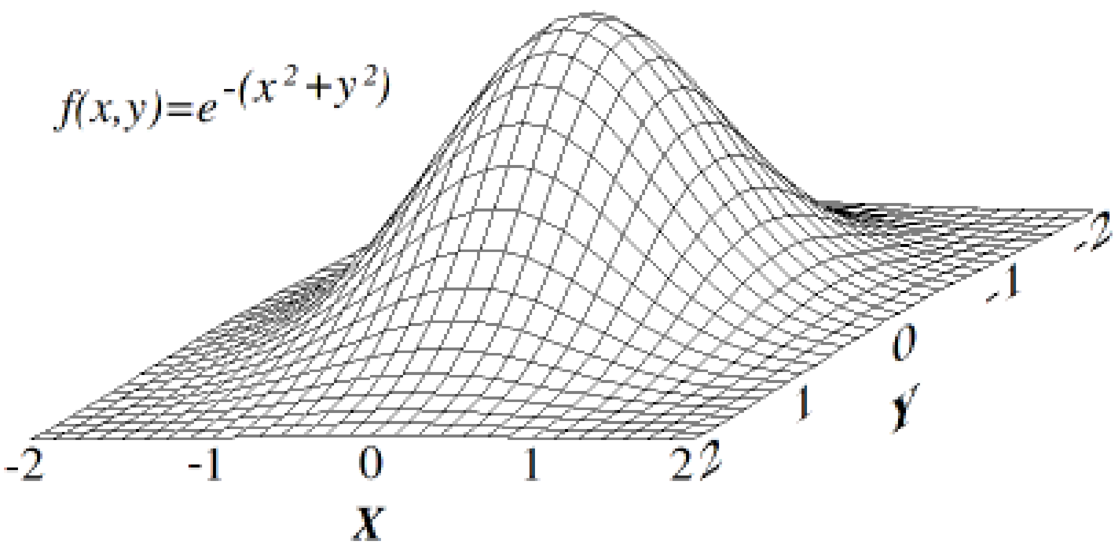
Hill Climbing

- General Idea:
  - Start wherever
  - Always choose the best neighbor
  - If no neighbors have better scores than current, quit

Why can this be a terrible idea?

Local vs Global Optimum?

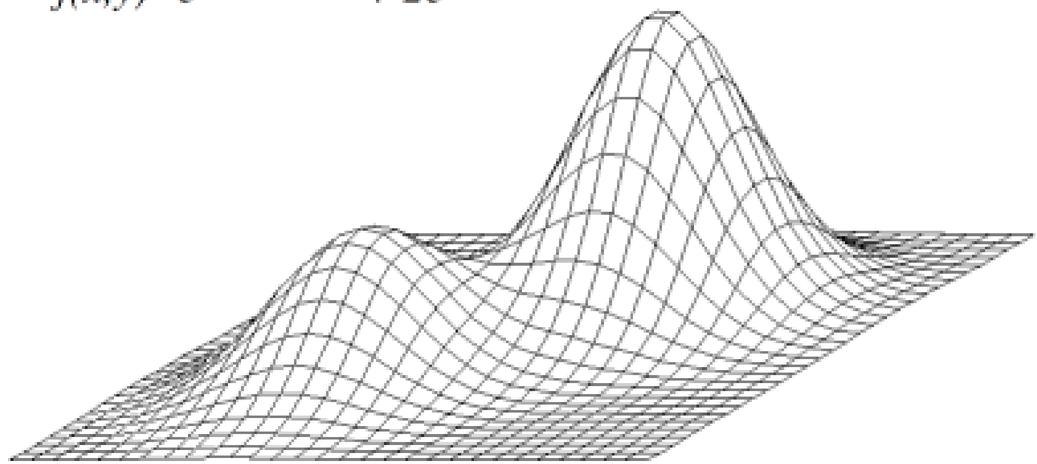
What is the optimum point?



Local vs Global Optimum?

What do you think about the optimum point now?

$$f(x,y)=e^{-(x^2+y^2)} + 2e^{-((x-1.7)^2+(y-1.7)^2)}$$



Greedy Algorithm
(Source: Wikipedia)

- Greedy algorithms can be characterized as being 'short sighted', and as 'non-recoverable'. They are ideal only for problems which have 'optimal substructure'. Despite this, greedy algorithms are best suited for simple problems.

Global Optimization

- Global optimization is the branch of applied mathematics and numerical analysis that deals with the optimization of single or maybe even multiple, possible conflicting, criteria.
- These criteria are expressed as a set of mathematical functions F = {f1, f2, . . . , fn}, the so-called objective functions.
- The result of the optimization process is the set of inputs for which these objective functions return optimal values.

Local Maximum and Minimum

Definition 2 (Local Maximum). A (local) maximum $\hat{x}_l \in X$ of an objective function $f : X \mapsto \mathbb{R}$ is an input element with $f(\hat{x}_l) \geq f(x)$ for all x neighboring $\hat{x}_l$.

If $X \subseteq \mathbb{R}$, we can write:

$$\hat{x}_l : \exists \varepsilon > 0 : f(\hat{x}_l) \geq f(x) \ \forall x \in X, |x - \hat{x}_l| < \varepsilon \tag{1.1}$$

Definition 3 (Local Minimum). A (local) minimum $\check{x}_l \in X$ of an objective function $f : X \mapsto \mathbb{R}$ is an input element with $f(\check{x}_l) \leq f(x)$ for all x neighboring $\check{x}_l$.

If $X \subseteq \mathbb{R}$, we can write:

$$\check{x}_l : \exists \varepsilon > 0 : f(\check{x}_l) \leq f(x) \ \forall x \in X, |x - \check{x}_l| < \varepsilon \tag{1.2}$$

Definition 4 (Local Optimum). An (local) optimum $x^\star_l \in X$ of an objective function $f : X \mapsto \mathbb{R}$ is either a local maximum or a local minimum (or both).

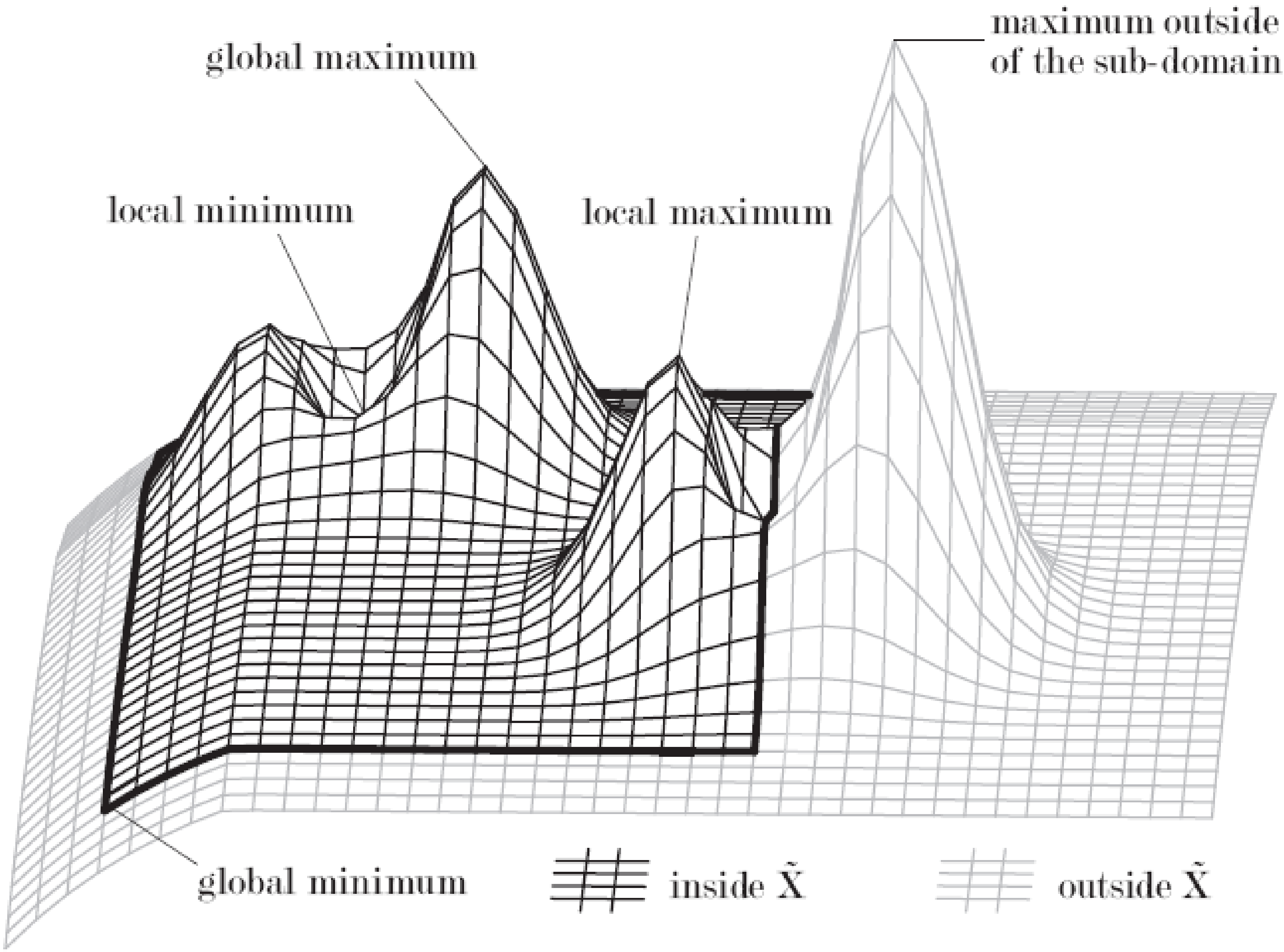
Global Maximum and Minimum

Definition 5 (Global Maximum). A global maximum $\hat{x} \in X$ of an objective function $f : X \mapsto \mathbb{R}$ is an input element with $f(\hat{x}) \geq f(x) \forall x \in X$.

Definition 6 (Global Minimum). A global minimum $\check{x} \in X$ of an objective function $f : X \mapsto \mathbb{R}$ is an input element with $f(\check{x}) \leq f(x) \forall x \in X$.

Definition 7 (Global Optimum). A global optimum $x^{\star} \in X$ of an objective function $f : X \mapsto \mathbb{R}$ is either a global maximum or a global minimum (or both).

Global and Local Optima



Continuous vs Combinatorial Optimization

Aspect	Continuous Optimization	Combinatorial Optimization
Decision Variables	Continuous	Discrete
Problem Structure	Smooth, continuous functions	Discrete, often combinatorial
Algorithms	Gradient-based, metaheuristics	Integer programming, greedy algorithms, etc.
Complexity	Analytically tractable, efficient convergence	Often computationally challenging, exponential solutions possible
Applications	Engineering, finance, machine learning	Logistics, scheduling, network design, etc.
Sensitivity to Initial Conditions	Crucial, especially for local methods	Important, discrete nature impacts exploration

Thanks