Exam 2 – 11^{th} Nov (syllabus from 27 Sep onwards)
Quiz 5 – 4^{th} Nov

$P \subseteq NP \subseteq PSPACE$

puzzles — 2 player games

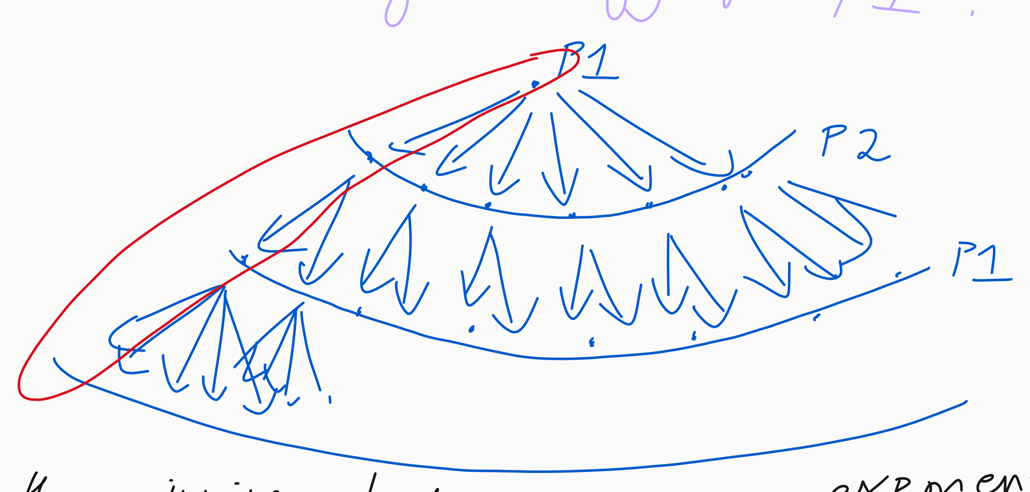
↳ PSPACE Completeness
TQBF

2-player perfect info. game:

Players alternate their turns, i.e. P1 then P2 then P1 ...
Game board is equally visible to both.
Is there a winning strategy for a player?

1-player game: winning strategy? polynomial length certificate
e.g. Sudoku:
↳ solved sudoku board

Chess: winning strategy for P1?



the winning strategy appears exponential in length

QBF: Suppose there are 2 players, E and A
such that E makes the first move,
and no matter what move A makes,
E can make another move.
Ultimately E wins by making a move that leaves no more moves for A.

i.e. $\exists x_1 \forall x_2 \exists x_3 \forall x_4 \ldots \varphi$ is true
$\Leftrightarrow$ Player E has a winning strategy

2 player games.

- QBF Game — Alternating quantifiers
- TQBF — Equivalent computationally
No condition on quantifiers

$TQBF \leq_p QBF\text{-}Game$

$\varphi \in TQBF \rightarrow f(\varphi) \in$ QBF-Game

↳ Hence PSPACE-Complete

2 player Game $\overset{p?}{\leftarrow}$ QBF – Game

$P \subseteq NP \subseteq PSPACE$

3SAT $\in$ LINSPACE = SPACE(n)
↳ certificate → amount of space

$\boxed{L \subseteq NL \subseteq P} \subseteq NP \subseteq PSPACE$

$L = SPACE(\log n)$ — Sublinear

$NL = NSPACE(\log n)$

Even = { x : x has an even number of ones }

| 1 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 |

$\in L$

MULT = $\{(n, m, nm) \mid m, n \in \mathbb{N}\} \in L$

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def is_mult (n, m, nm):
    return (n*m == nm)
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1. write n on the tape,
then add n to it,
then add n to it,
⋮
until n is added m times.
check whether this number is equal to nm

n = log   m = log   nm = log

linear space

$$
\begin{array}{r}
123 \\
\times\ 15 \\
\hline
+\ 1\ 2\ 3\ \ \\
6\ 1\ 5 \\
\hline
1\ 8\ 4\ 5
\end{array}
$$

1845

$\in L$

$L \subseteq P$ — $SPACE(f) \subseteq TIME(2^{O(f)})$

configuration graph

$L \subseteq NL$

PATH = { (G, s, t) : Graph G has a path from s to t }

$\in NL$

* Nondeterministically follow all paths that start from s.
If any path reaches t, accept.

$\boxed{L \bigcirc NL}$

$P \overset{?}{=} NP$