Lab_6_aa07190

November 8, 2024

1 CS 316 : Introduction to Deep Learning - Fall 2024

1.1 Lab 06 : Gradient Descent

1.1.1 Dr. Abdul Samad

2 Instructions

1. Make a copy of this notebook on google colab at start of the lab.
2. Please rename your notebook as *Lab_6_aa1234.ipynb* before starting the lab. Notebooks which do not follow appropriate naming convention will not be graded.
3. You have to submit this lab during the lab timings. You are allowed to submit till 11:59 PM on the day of your lab with a 30% penalty. No submissions will be accepted afterwards.
4. Use of AI is strictly prohibited. Anyone caught using Any AI tool during lab or while grading will be immediately reported to OCVS without any further discussion.
5. At the end of the lab, download the notebook (ipynb extension file) and upload it on canvas as a file. Submitting link to notebook or any other file will not be accepted.
6. Each task has points assigned to it. Total Lab is of 100 points.
7. Use of for loops is strictly prohibited.
8. For every theoretical question, there is a separate cell given at the end. You have to write your answer there.
9. If you have any questions, please feel free to reach out to the course instructor or RA.

2.1 Task Overview

In this lab we will explore Gradient Descent. Work through the cells below, running each cell in turn. In various places you will see the words "TO DO". Follow the instructions at these places and make predictions about what is going to happen or write code to complete the functions.

Let's start with importing Libraries first

[1]:
```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib import cm
from matplotlib.colors import ListedColormap
```

```python
import math
```

[2]:
```python
# Define the Rectified Linear Unit (ReLU) function
def ReLU(preactivation):
  activation = preactivation.clip(0.0)
  return activation

# Define a shallow neural network
def shallow_nn(x, beta_0, omega_0, beta_1, omega_1):
    # Make sure that input data is (1 x n_data) array
    n_data = x.size
    x = np.reshape(x,(1,n_data))

    # This runs the network for ALL of the inputs, x at once so we can draw␣
 ↪graph
    h1 = ReLU(np.matmul(beta_0,np.ones((1,n_data))) + np.matmul(omega_0,x))
    y = np.matmul(beta_1,np.ones((1,n_data))) + np.matmul(omega_1,h1)
    return y
```

3 Updating Parameters

Whenever we have calculated the loss, we want to give feedback to our model about hyperparameters and update them. This involves 2 variables. Look at the equation given below.

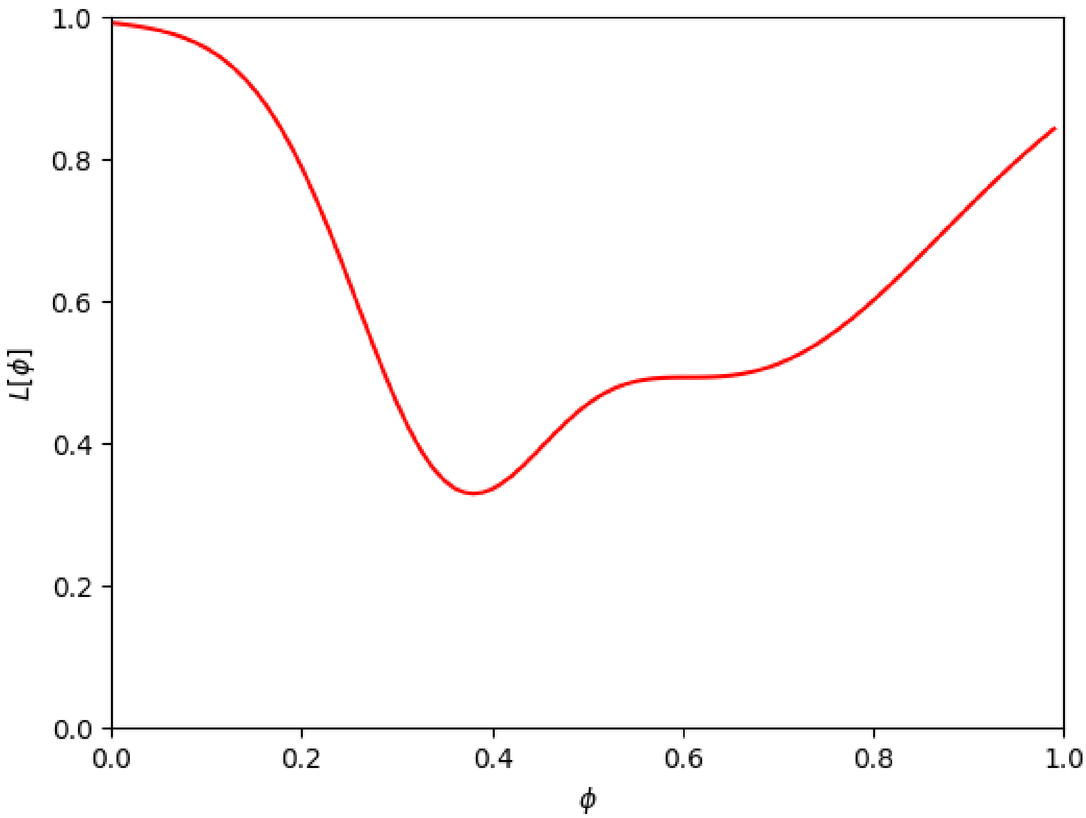
Here, represents hyperparameters, represents learning rate and $\frac{\partial L}{\partial}$ represents Gradient Descent. Let's try finding out and $\frac{\partial L}{\partial}$ in this notebook.

4 Line search

[3]:
```python
# Let's create a simple 1D function
def loss_function(phi):
  return 1- 0.5 * np.exp(-(phi-0.65)*(phi-0.65)/0.1) - 0.45 *np.exp(-(phi-0.
 ↪35)*(phi-0.35)/0.02)

def draw_function(loss_function,a=None, b=None, c=None, d=None):
  # Plot the function
  phi_plot = np.arange(0,1,0.01);
  fig,ax = plt.subplots()
  ax.plot(phi_plot,loss_function(phi_plot),'r-')
  ax.set_xlim(0,1); ax.set_ylim(0,1)
  ax.set_xlabel(r'$\phi$'); ax.set_ylabel(r'$L[\phi]$')
  if a is not None and b is not None and c is not None and d is not None:
      plt.axvspan(a, d, facecolor='k', alpha=0.2)
      ax.plot([a,a],[0,1],'b-')
      ax.plot([b,b],[0,1],'b-')
      ax.plot([c,c],[0,1],'b-')
```

```
    ax.plot([d,d],[0,1],'b-')
  plt.show()
```

[4]:
```
# Draw this function
draw_function(loss_function)
```



5 Q1: Task [40 Points]

In this task we will create a line search procedure to find the minimum in the range 0,1. Purpose of line search will be covered in class and why it works. There are multiple code segments which you will have to complete in this code. Whereever TODO is written in comments, you need to complete the that section till you find End Solution comment.

We will be implementing multiple rules in line search. Firstly, go throught the code block given below and then read these rules and implement them.

Rule 1:

If the loss at point a is less than the loss at points b, c, and d then halve the distance from a to points b, c, and d. Then add a to all of them.

Rule 2:

If the loss at point b is less than the loss at point c then:

```
point d becomes point c, and
point b becomes 1/3 of difference of (new d and a) which is then summed with a
point c becomes 2/3 of difference of (new d and a) which is then summed with a.
```

Rule 3:

If the loss at point c is less than the loss at point b then

```
point a becomes point b, and
point b becomes 1/3 of difference of (new d and a) which is then summed with a
point c becomes 2/3 of difference of (new d and a) which is then summed with a.
```

Final Solution:

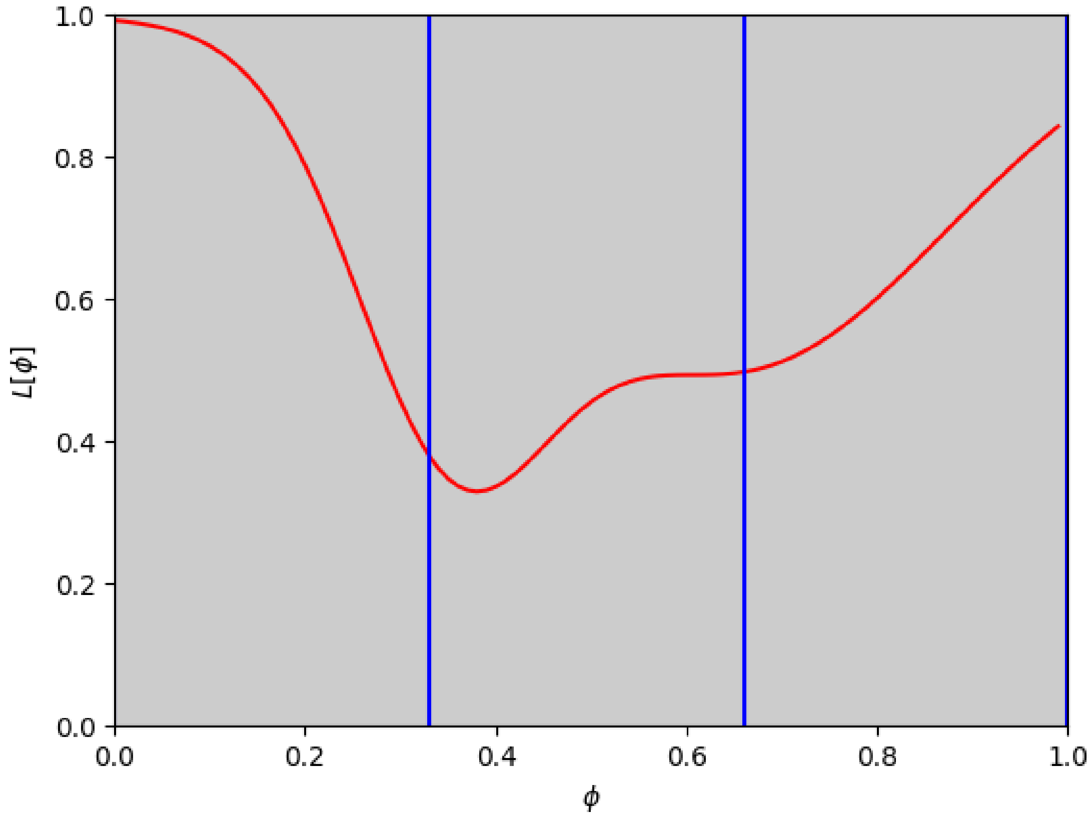
Update soln variable with final solution which is average of b and c.

[5]:
```
def line_search(loss_function, thresh=.0001, max_iter = 10, draw_flag = False):

    # Initialize four points along the range we are going to search
    a = 0
    b = 0.33
    c = 0.66
    d = 1.0
    n_iter = 0

    # While we haven't found the minimum closely enough
    while np.abs(b-c) > thresh and n_iter < max_iter:
        # Increment iteration counter (just to prevent an infinite loop)
        n_iter = n_iter+1

        # Calculate all four points
        lossa = loss_function(a)
        lossb = loss_function(b)
        lossc = loss_function(c)
        lossd = loss_function(d)

        if draw_flag:
          draw_function(loss_function, a,b,c,d)

        print('Iter %d, a=%3.3f, b=%3.3f, c=%3.3f, d=%3.3f'%(n_iter, a,b,c,d))


        # TODO
        # Implement Rule 1 Here
        # Begin Solution
        if lossa < lossb and lossa < lossc and lossa < lossd:
           #halve distances from point a to b, c, and d and add a to all of them
            b = (b-a)/2 + a; c = (c-a)/2 + a; d = (d-a)/2 + a
```

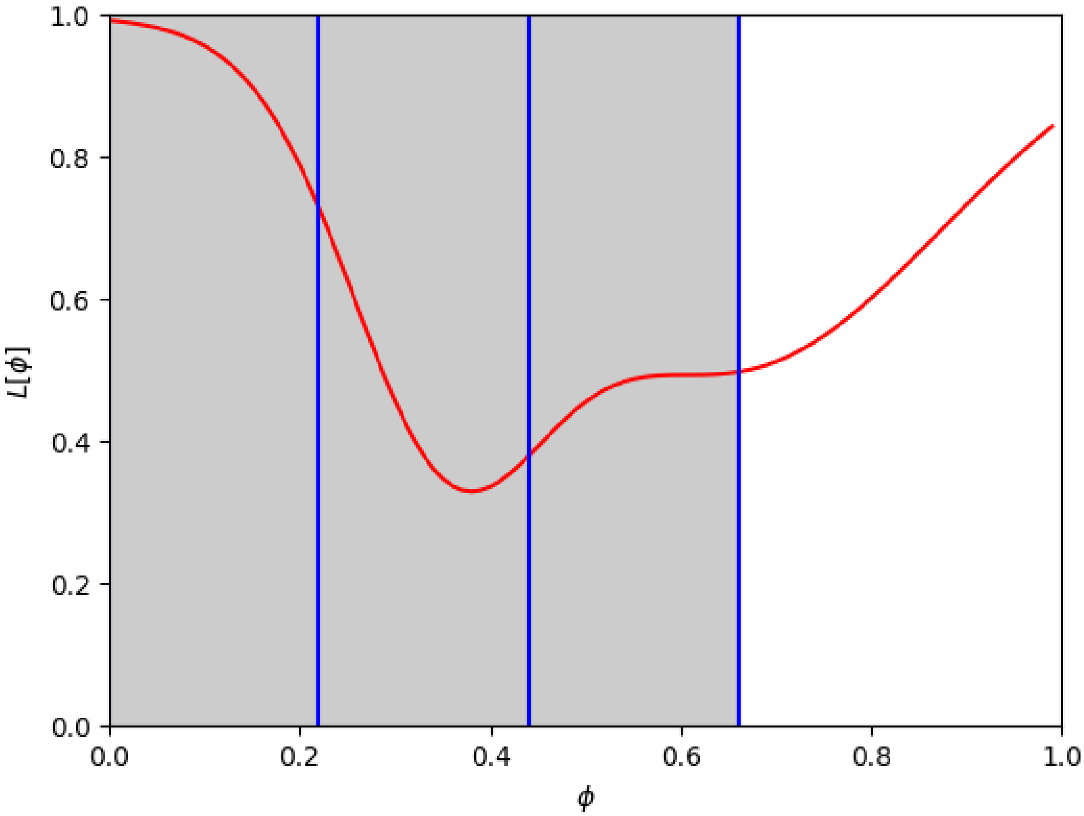
```
        # End Solution
        # TODO
        # Implement Rule 2 Here
        # Begin Solution
        elif lossb < lossc:
            d = c; b = (d-a)/3 + a; c = 2*(d-a)/3 + a
        # End Solution
        # TODO
        # Implement Rule 3 Here
        # Begin Solution
        elif lossc < lossb:
            a = b; b = (d-a)/3 + a; c = 2*(d-a)/3 + a
        # End Solution
    # TODO
    # Final Solution
    # Begin Solution
    soln = (b+c)/2
    # End Solution
    return soln
```

[6]:

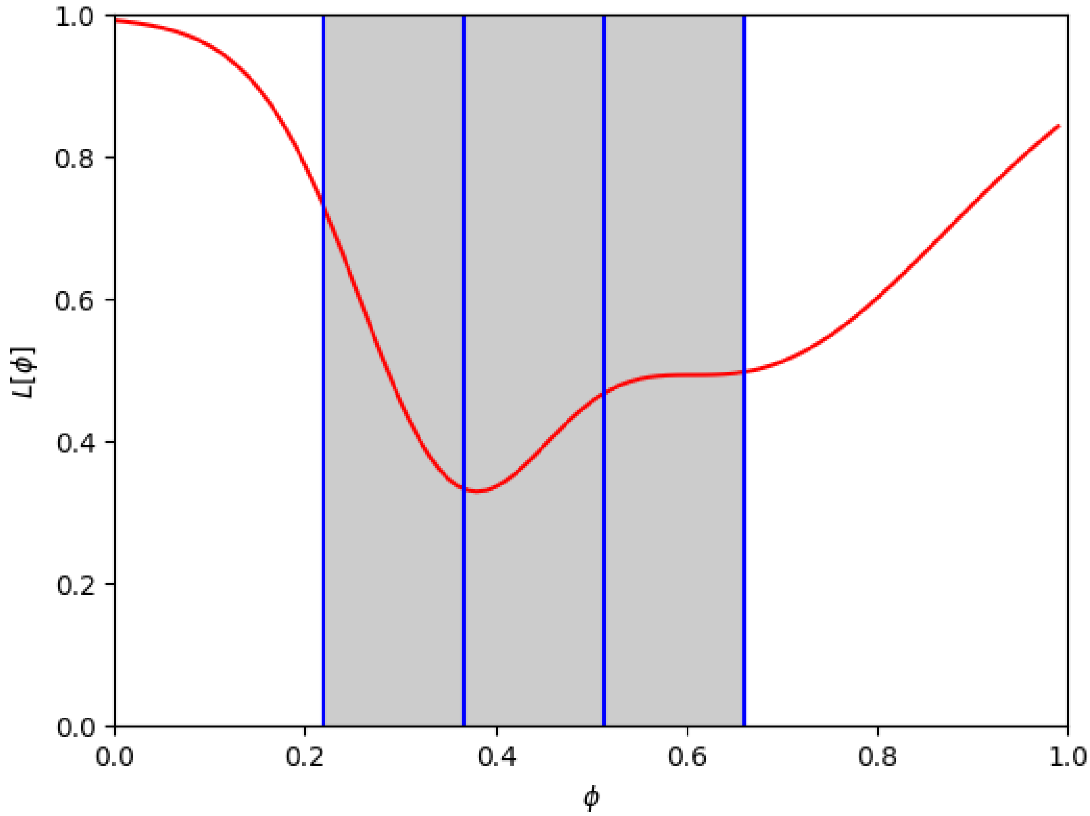
```
#Do Not Edit
#Use this Code for Testing
soln = line_search(loss_function, draw_flag=True)
print('Soln = %3.3f, loss = %3.3f'%(soln,loss_function(soln)))

assert np.round(soln,3) ==0.383
assert np.round(loss_function(soln),3) ==0.329
```



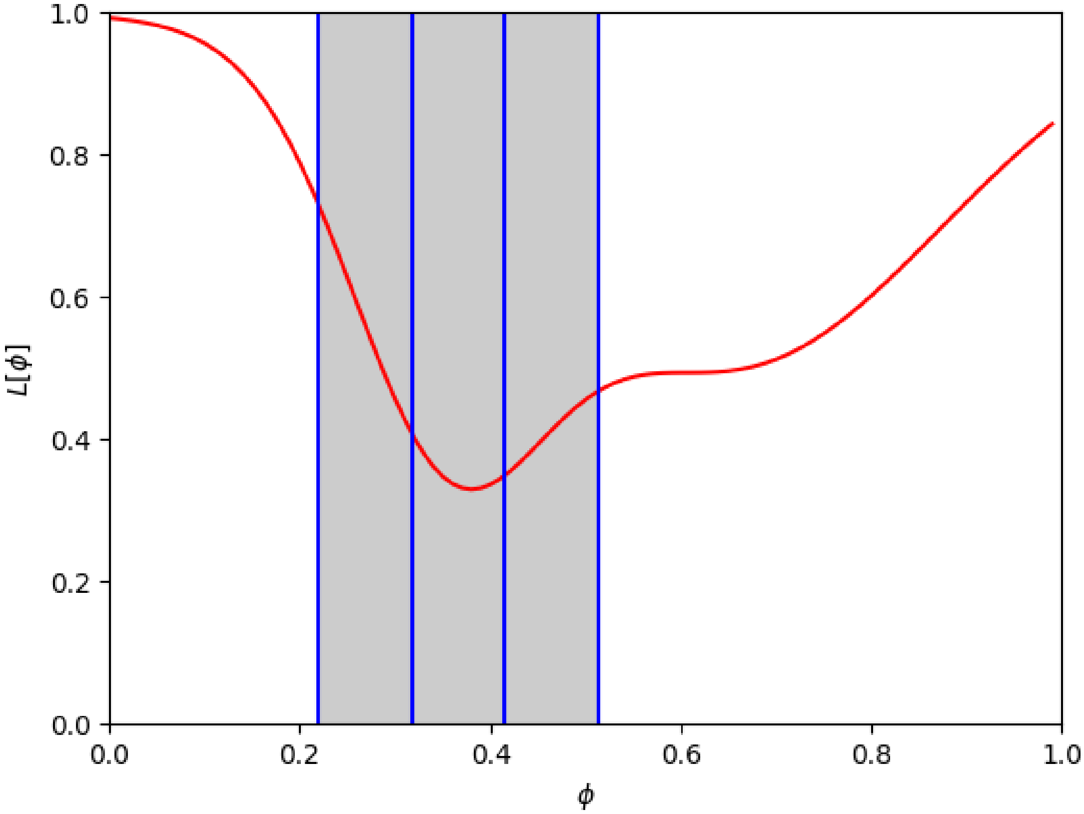

```
Iter 1, a=0.000, b=0.330, c=0.660, d=1.000
```



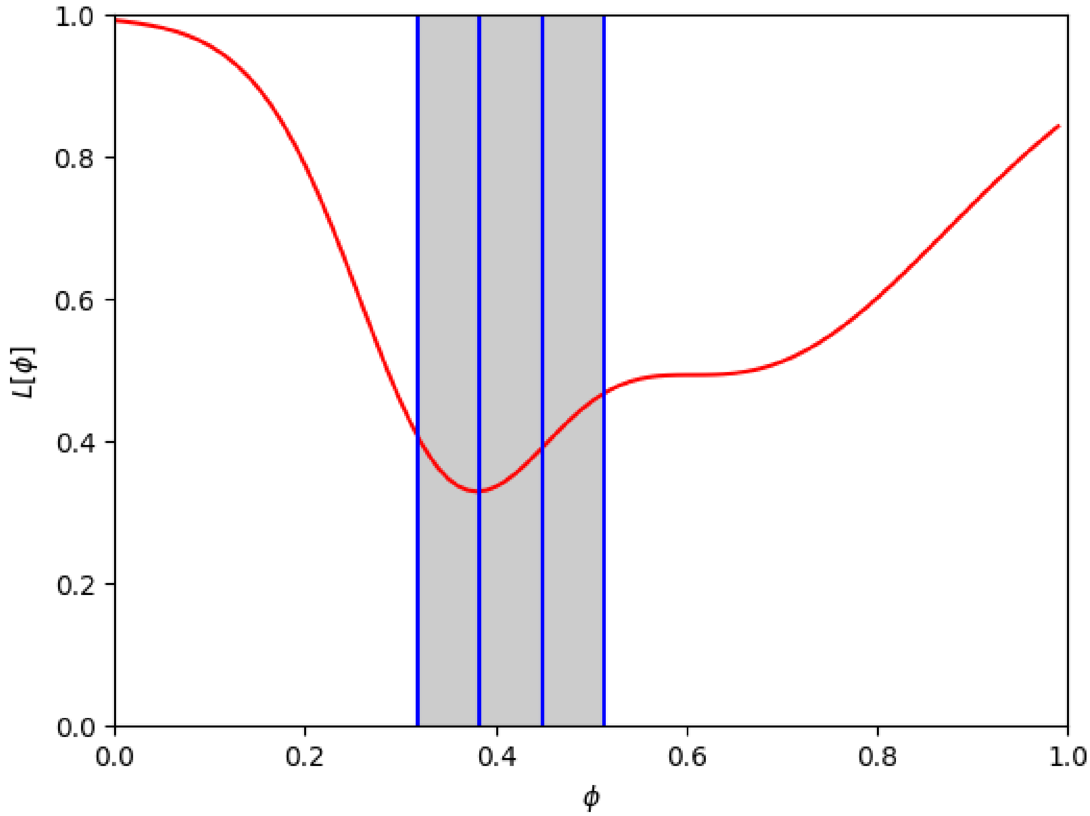

```
Iter 2, a=0.000, b=0.220, c=0.440, d=0.660
```



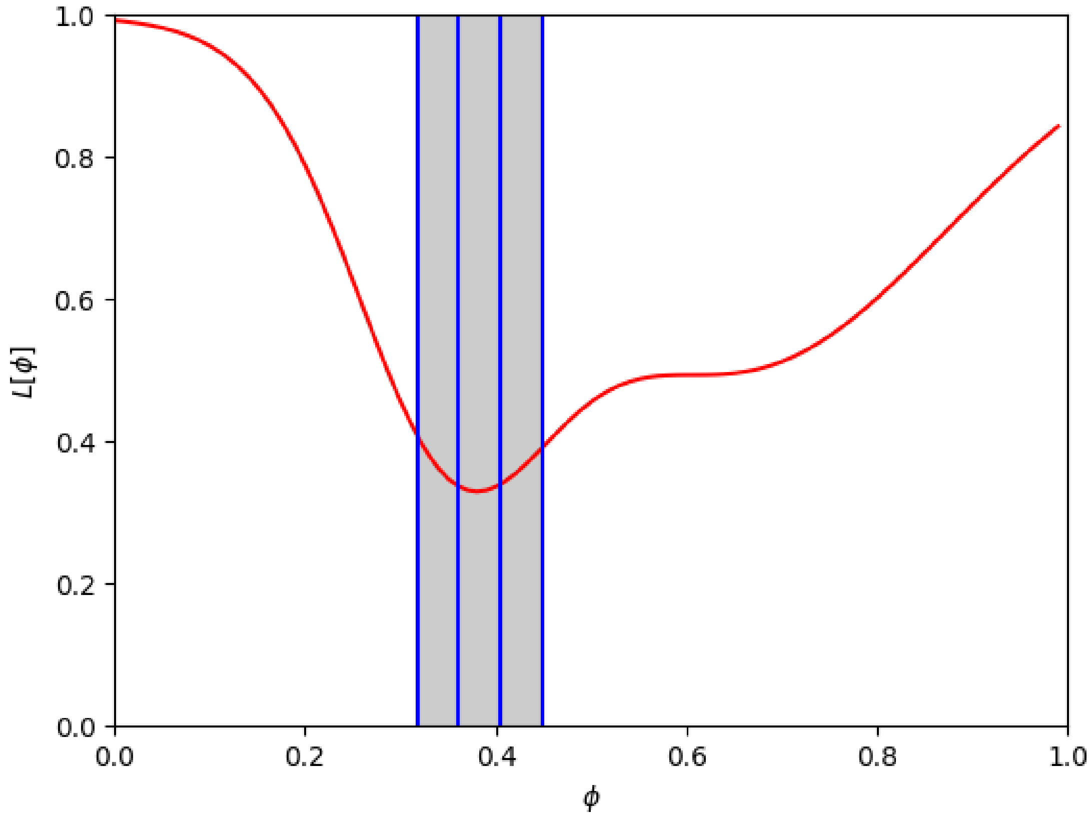

```
Iter 3, a=0.220, b=0.367, c=0.513, d=0.660
```



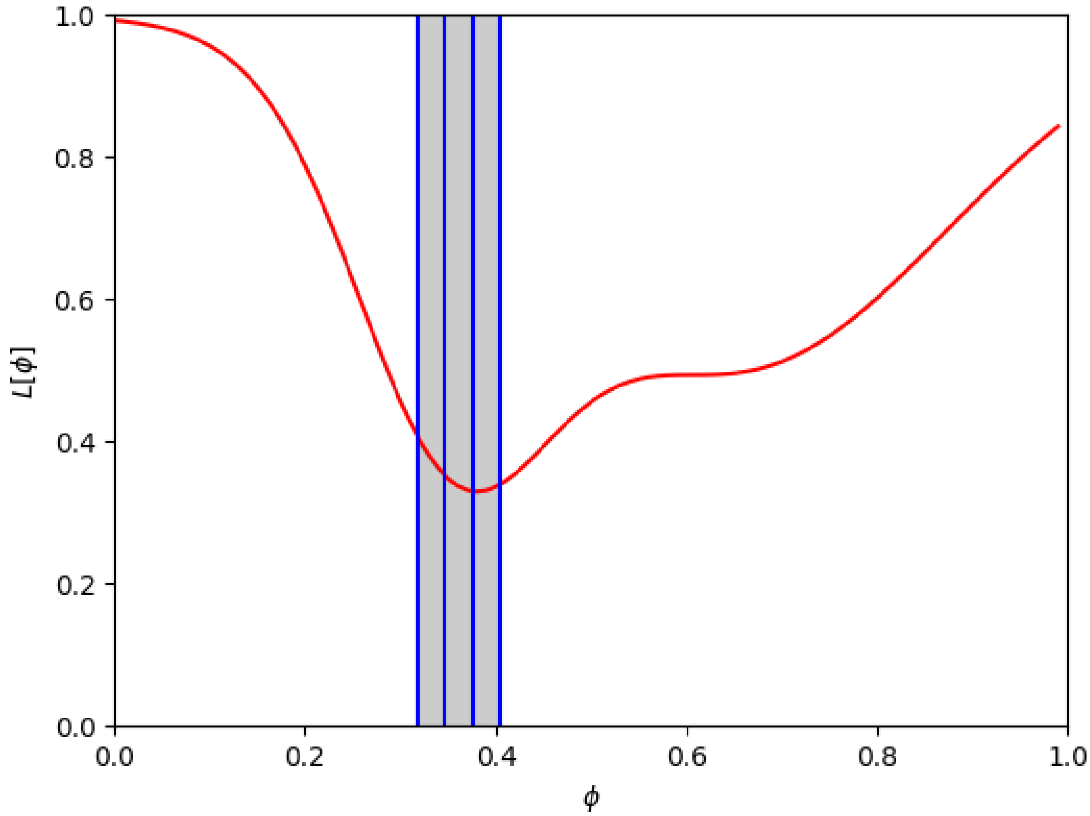

```
Iter 4, a=0.220, b=0.318, c=0.416, d=0.513
```



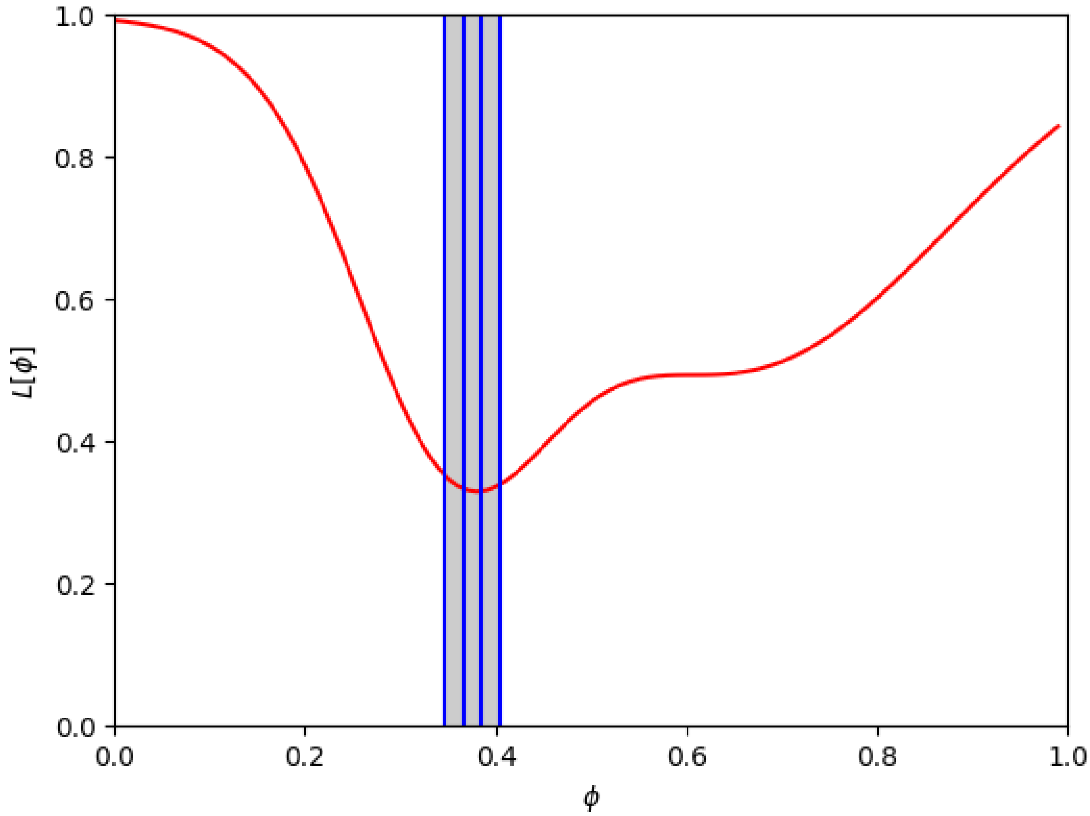

```
Iter 5, a=0.318, b=0.383, c=0.448, d=0.513
```



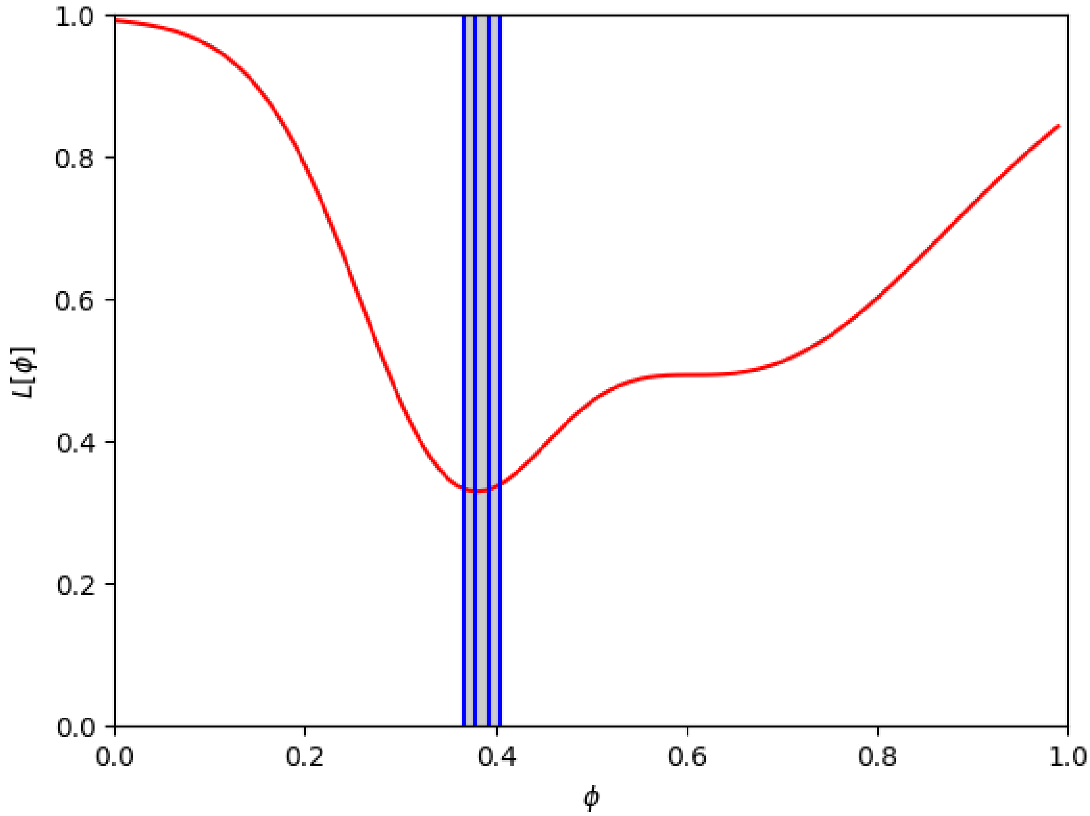

```
Iter 6, a=0.318, b=0.361, c=0.405, d=0.448
```



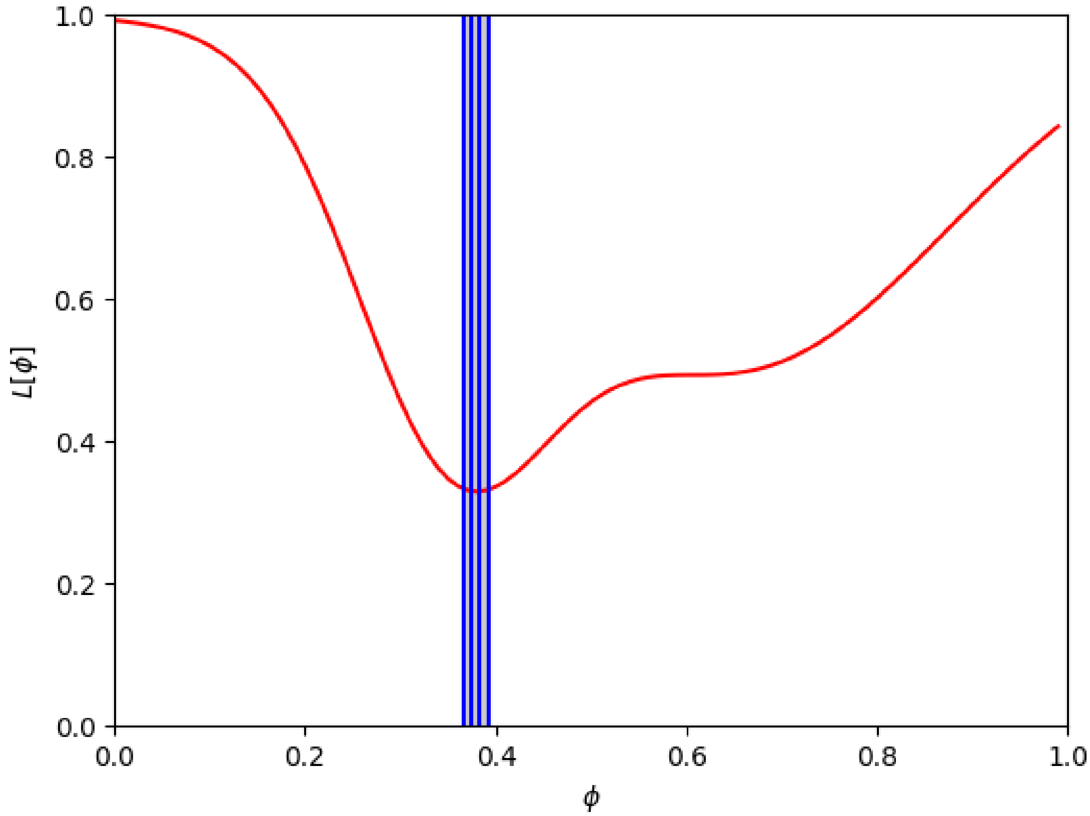

```
Iter 7, a=0.318, b=0.347, c=0.376, d=0.405
```




```
Iter 8, a=0.347, b=0.366, c=0.385, d=0.405
```




```
Iter 9, a=0.366, b=0.379, c=0.392, d=0.405
```




```
Iter 10, a=0.366, b=0.375, c=0.383, d=0.392
Soln = 0.383, loss = 0.329
```

6 Gradient descent

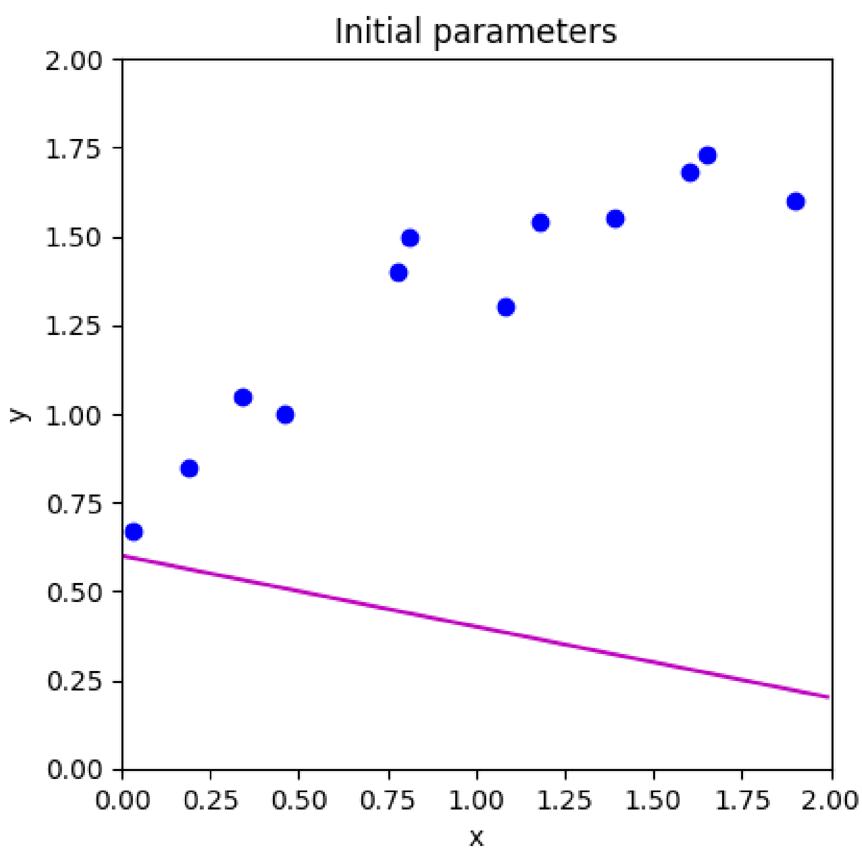
[7]:
```
# Let's create our training data 12 pairs {x_i, y_i}
# We'll try to fit the straight line model to these data
data = np.array([[0.03,0.19,0.34,0.46,0.78,0.81,1.08,1.18,1.39,1.60,1.65,1.90],
                 [0.67,0.85,1.05,1.00,1.40,1.50,1.30,1.54,1.55,1.68,1.73,1.60]])
```

[8]:
```
# Let's define our model -- just a straight line with intercept phi[0] and␣
 ↪slope phi[1]
def model(phi,x):
  y_pred = phi[0]+phi[1] * x
  return y_pred
```

[9]:
```
# Draw model
def draw_model(data,model,phi,title=None):
  x_model = np.arange(0,2,0.01)
  y_model = model(phi,x_model)
```

```
fix, ax = plt.subplots()
ax.plot(data[0,:],data[1,:],'bo')
ax.plot(x_model,y_model,'m-')
ax.set_xlim([0,2]);ax.set_ylim([0,2])
ax.set_xlabel('x'); ax.set_ylabel('y')
ax.set_aspect('equal')
if title is not None:
  ax.set_title(title)
plt.show()
```

[10]:
```
# Initialize the parameters to some arbitrary values and draw the model
phi = np.zeros((2,1))
phi[0] = 0.6      # Intercept
phi[1] = -0.2      # Slope
draw_model(data,model,phi, "Initial parameters")
```



Now let's compute the sum of squares loss for the training data

[11]:
```python
def compute_loss(data_x, data_y, model, phi):
  pred_y = model(phi,data_x)
  loss = np.sum(np.square(pred_y-data_y))

  return loss
```

Now let's plot the whole loss function

[12]:
```python
def draw_loss_function(compute_loss, data,  model, phi_iters = None):
  # Define pretty colormap
  my_colormap_vals_hex =('2a0902', '2b0a03', '2c0b04', '2d0c05', '2e0c06',␣
↪'2f0d07', '300d08', '310e09', '320f0a', '330f0b', '34100b', '35110c',␣
↪'36110d', '37120e', '38120f', '39130f', '3a1410', '3b1411', '3c1511',␣
↪'3d1612', '3e1613', '3f1713', '401714', '411814', '421915', '431915',␣
↪'451a16', '461b16', '471b17', '481c17', '491d18', '4a1d18', '4b1e19',␣
↪'4c1f19', '4d1f1a', '4e201b', '50211b', '51211c', '52221c', '53231d',␣
↪'54231d', '55241e', '56251e', '57261f', '58261f', '592720', '5b2821',␣
↪'5c2821', '5d2922', '5e2a22', '5f2b23', '602b23', '612c24', '622d25',␣
↪'632e25', '652e26', '662f26', '673027', '683027', '693128', '6a3229',␣
↪'6b3329', '6c342a', '6d342a', '6f352b', '70362c', '71372c', '72372d',␣
↪'73382e', '74392e', '753a2f', '763a2f', '773b30', '783c31', '7a3d31',␣
↪'7b3e32', '7c3e33', '7d3f33', '7e4034', '7f4134', '804235', '814236',␣
↪'824336', '834437', '854538', '864638', '874739', '88473a', '89483a',␣
↪'8a493b', '8b4a3c', '8c4b3c', '8d4c3d', '8e4c3e', '8f4d3f', '904e3f',␣
↪'924f40', '935041', '945141', '955242', '965343', '975343', '985444',␣
↪'995545', '9a5646', '9b5746', '9c5847', '9d5948', '9e5a49', '9f5a49',␣
↪'a05b4a', 'a15c4b', 'a35d4b', 'a45e4c', 'a55f4d', 'a6604e', 'a7614e',␣
↪'a8624f', 'a96350', 'aa6451', 'ab6552', 'ac6552', 'ad6653', 'ae6754',␣
↪'af6855', 'b06955', 'b16a56', 'b26b57', 'b36c58', 'b46d59', 'b56e59',␣
↪'b66f5a', 'b7705b', 'b8715c', 'b9725d', 'ba735d', 'bb745e', 'bc755f',␣
↪'bd7660', 'be7761', 'bf7862', 'c07962', 'c17a63', 'c27b64', 'c27c65',␣
↪'c37d66', 'c47e67', 'c57f68', 'c68068', 'c78169', 'c8826a', 'c9836b',␣
↪'ca846c', 'cb856d', 'cc866e', 'cd876f', 'ce886f', 'ce8970', 'cf8a71',␣
↪'d08b72', 'd18c73', 'd28d74', 'd38e75', 'd48f76', 'd59077', 'd59178',␣
↪'d69279', 'd7937a', 'd8957b', 'd9967b', 'da977c', 'da987d', 'db997e',␣
↪'dc9a7f', 'dd9b80', 'de9c81', 'de9d82', 'df9e83', 'e09f84', 'e1a185',␣
↪'e2a286', 'e2a387', 'e3a488', 'e4a589', 'e5a68a', 'e5a78b', 'e6a88c',␣
↪'e7aa8d', 'e7ab8e', 'e8ac8f', 'e9ad90', 'eaae91', 'eaaf92', 'ebb093',␣
↪'ecb295', 'ecb396', 'edb497', 'eeb598', 'eeb699', 'efb79a', 'efb99b',␣
↪'f0ba9c', 'f1bb9d', 'f1bc9e', 'f2bd9f', 'f2bfa1', 'f3c0a2', 'f3c1a3',␣
↪'f4c2a4', 'f5c3a5', 'f5c5a6', 'f6c6a7', 'f6c7a8', 'f7c8aa', 'f7c9ab',␣
↪'f8cbac', 'f8ccad', 'f8cdae', 'f9ceb0', 'f9d0b1', 'fad1b2', 'fad2b3',␣
↪'fbd3b4', 'fbd5b6', 'fbd6b7', 'fcd7b8', 'fcd8b9', 'fcdaba', 'fddbbc',␣
↪'fddcbd', 'fddebe', 'fddfbf', 'fee0c1', 'fee1c2', 'fee3c3', 'fee4c5',␣
↪'ffe5c6', 'ffe7c7', 'ffe8c9', 'ffe9ca', 'ffebcb', 'ffeccd', 'ffedce',␣
↪'ffefcf', 'fff0d1', 'fff2d2', 'fff3d3', 'fff4d5', 'fff6d6', 'fff7d8',␣
↪'fff8d9', 'fffada', 'fffbdc', 'fffcdd', 'fffedf', 'ffffe0')
```

```
my_colormap_vals_dec = np.array([int(element,base=16) for element in␣
↪my_colormap_vals_hex])
r = np.floor(my_colormap_vals_dec/(256*256))
g = np.floor((my_colormap_vals_dec - r *256 *256)/256)
b = np.floor(my_colormap_vals_dec - r * 256 *256 - g * 256)
my_colormap = ListedColormap(np.vstack((r,g,b)).transpose()/255.0)

# Make grid of intercept/slope values to plot
intercepts_mesh, slopes_mesh = np.meshgrid(np.arange(0.0,2.0,0.02), np.
↪arange(-1.0,1.0,0.002))
loss_mesh = np.zeros_like(slopes_mesh)
# Compute loss for every set of parameters
for idslope, slope in np.ndenumerate(slopes_mesh):
   loss_mesh[idslope] = compute_loss(data[0,:], data[1,:], model, np.
↪array([[intercepts_mesh[idslope]], [slope]]))

fig,ax = plt.subplots()
fig.set_size_inches(8,8)
ax.contourf(intercepts_mesh,slopes_mesh,loss_mesh,256,cmap=my_colormap)
ax.contour(intercepts_mesh,slopes_mesh,loss_mesh,40,colors=['#80808080'])
if phi_iters is not None:
  ax.plot(phi_iters[0,:], phi_iters[1,:],'go-')
ax.set_ylim([1,-1])
ax.set_xlabel('Intercept $\phi_{0}$'); ax.set_ylabel('Slope, $\phi_{1}$')
plt.show()
```

7 Q2: TODO [40 points]

Now let's compute the gradient vector for a given set of parameters:

$$\frac{\partial L}{\partial \phi} = \begin{bmatrix} \frac{\partial L}{\partial \phi_0} \\ \frac{\partial L}{\partial \phi_1} \end{bmatrix}. \tag{1}$$

You have already seen sum of square loss expression. Use that to find out the expression for the derivative with respect to phi0 and phi1. You will have to submit picture of that derivative along with this notebook. Based on that derivative you just calculated, write the code for calculating change in loss with respect to phi0 and change in loss with respect to phi1.

[14]:
```
def compute_gradient(data_x, data_y, phi):
    # Begin Solution
    y_pred = model(phi,data_x)
    error = y_pred - data_y

    dl_dphi0 = 2*np.sum(error)
    dl_dphi1 = 2*np.sum(error*data_x)
    # End Solution
```

```
    return np.array([[dl_dphi0],[dl_dphi1]])
```

[15]:
```
#Do Not Edit
#Use this code for testing
# Compute the gradient using your function
gradient = compute_gradient(data[0,:],data[1,:], phi)
print("Your gradients: (%3.2f,%3.2f)"%(gradient[0],gradient[1]))
# Approximate the gradients with finite differences
delta = 0.0001
dl_dphi0_est = (compute_loss(data[0,:],data[1,:],model,phi+np.
 ↪array([[delta],[0]])) - \
                    compute_loss(data[0,:],data[1,:],model,phi))/delta
dl_dphi1_est = (compute_loss(data[0,:],data[1,:],model,phi+np.
 ↪array([[0],[delta]])) - \
                    compute_loss(data[0,:],data[1,:],model,phi))/delta
print("Approx gradients: (%3.2f,%3.2f)"%(dl_dphi0_est,dl_dphi1_est))
# There might be small differences in the last significant figure because␣
 ↪finite gradients is an approximation

assert np.abs(dl_dphi0_est-gradient[0]) < 0.002
assert np.abs(dl_dphi1_est-gradient[1]) < 0.002
```

```
Your gradients: (-21.90,-26.84)
Approx gradients: (-21.90,-26.84)
```

```
/tmp/ipykernel_4374/2541310509.py:5: DeprecationWarning: Conversion of an array
with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you
extract a single element from your array before performing this operation.
(Deprecated NumPy 1.25.)
  print("Your gradients: (%3.2f,%3.2f)"%(gradient[0],gradient[1]))
```

Now we are ready to perform gradient descent. We'll need to use our line search routine from notebook 6.1, which I've reproduced here plus the helper function loss_function_1D that maps the search along the negative gradient direction in 2D space to a 1D problem (distance along this direction)

[15]:
```
def loss_function_1D(dist_prop, data, model, phi_start, search_direction):
  # Return the loss after moving this far
  return compute_loss(data[0,:], data[1,:], model, phi_start+ search_direction␣
 ↪* dist_prop)

def line_search(data, phi, gradient, thresh=.00001, max_dist = 0.1, max_iter =␣
 ↪15, verbose=False):
    # Initialize four points along the range we are going to search
    a = 0
    b = 0.33 * max_dist
    c = 0.66 * max_dist
    d = 1.0 * max_dist
```

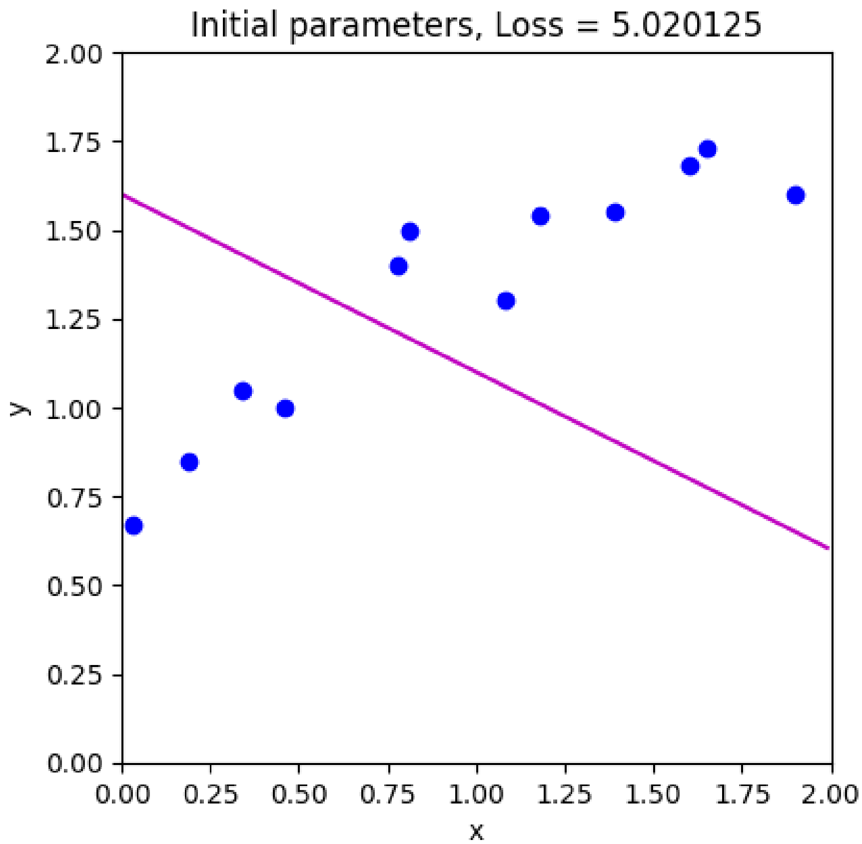
```python
  n_iter = 0

  # While we haven't found the minimum closely enough
  while np.abs(b-c) > thresh and n_iter < max_iter:
      # Increment iteration counter (just to prevent an infinite loop)
      n_iter = n_iter+1
      # Calculate all four points
      lossa = loss_function_1D(a, data, model, phi,gradient)
      lossb = loss_function_1D(b, data, model, phi,gradient)
      lossc = loss_function_1D(c, data, model, phi,gradient)
      lossd = loss_function_1D(d, data, model, phi,gradient)

      if verbose:
        print('Iter %d, a=%3.3f, b=%3.3f, c=%3.3f, d=%3.3f'%(n_iter, a,b,c,d))
        print('a %f, b%f, c%f, d%f'%(lossa,lossb,lossc,lossd))

      # Rule #1 If point A is less than points B, C, and D then halve␣
↪distance from A to points B,C, and D
      if np.argmin((lossa,lossb,lossc,lossd))==0:
        b = a+ (b-a)/2
        c = a+ (c-a)/2
        d = a+ (d-a)/2
        continue;

      # Rule #2 If point b is less than point c then
      #                     point d becomes point c, and
      #                     point b becomes 1/3 between a and new d
      #                     point c becomes 2/3 between a and new d
      if lossb < lossc:
        d = c
        b = a+ (d-a)/3
        c = a+ 2*(d-a)/3
        continue

      # Rule #2 If point c is less than point b then
      #                     point a becomes point b, and
      #                     point b becomes 1/3 between new a and d
      #                     point c becomes 2/3 between new a and d
      a = b
      b = a+ (d-a)/3
      c = a+ 2*(d-a)/3

  # Return average of two middle points
  return (b+c)/2.0
```

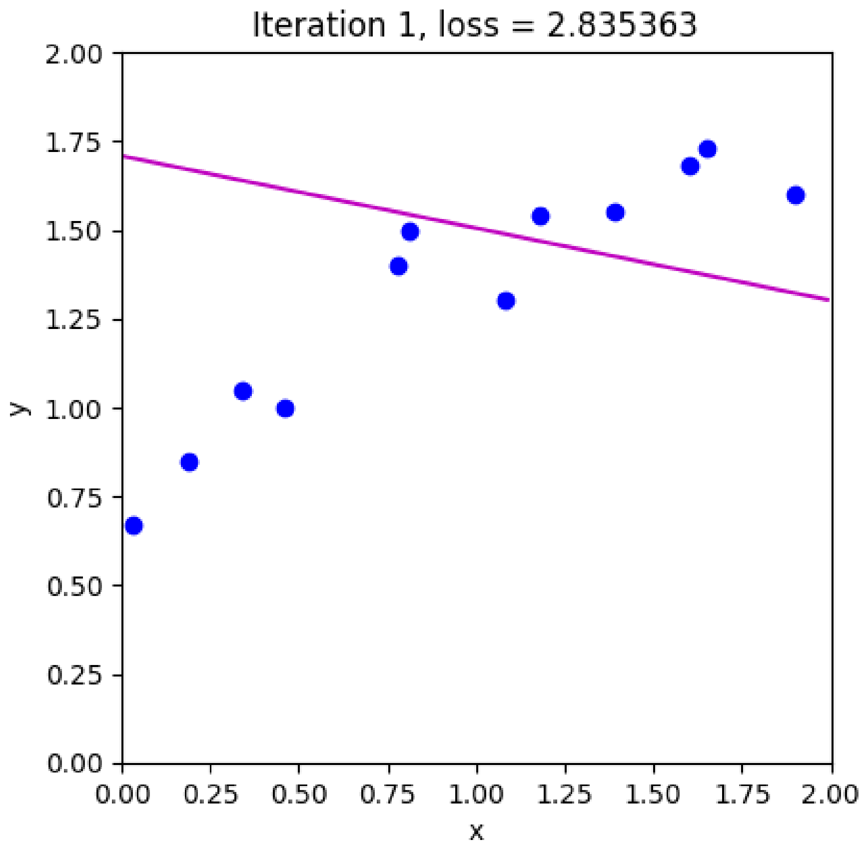
8 Q5: TODO [20 Points]

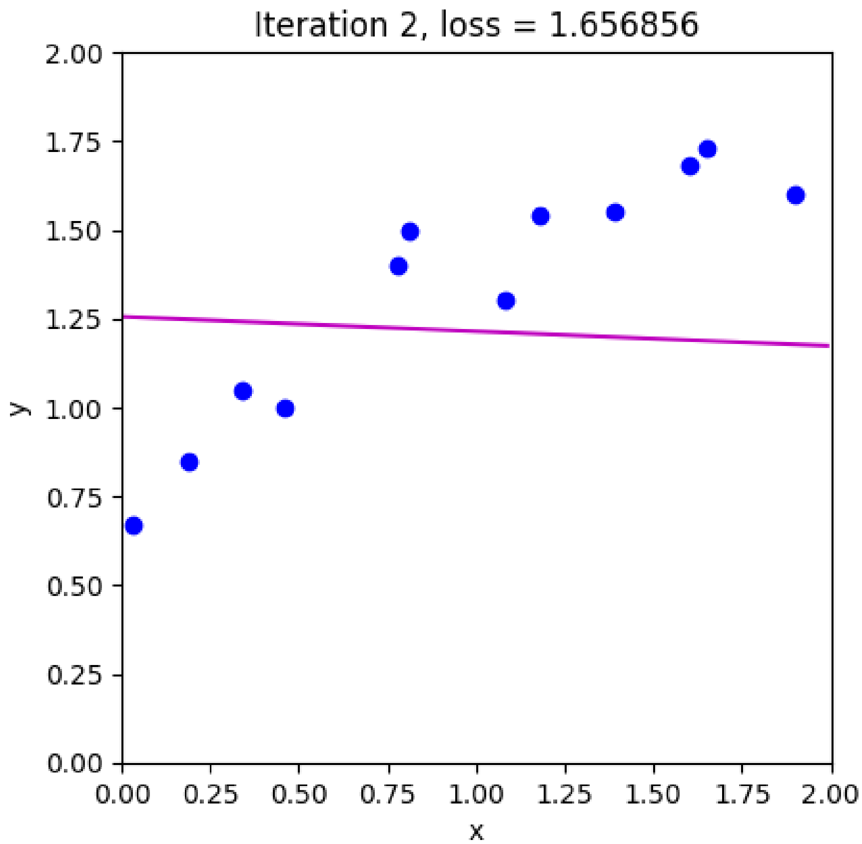
Update Phi with the gradient descent step Compute the gradient (using compute gradient function above) Find the best step size alpha using line search function (above) – use negative gradient as going downhill Update the parameters phi based on the gradient and the step size alpha.

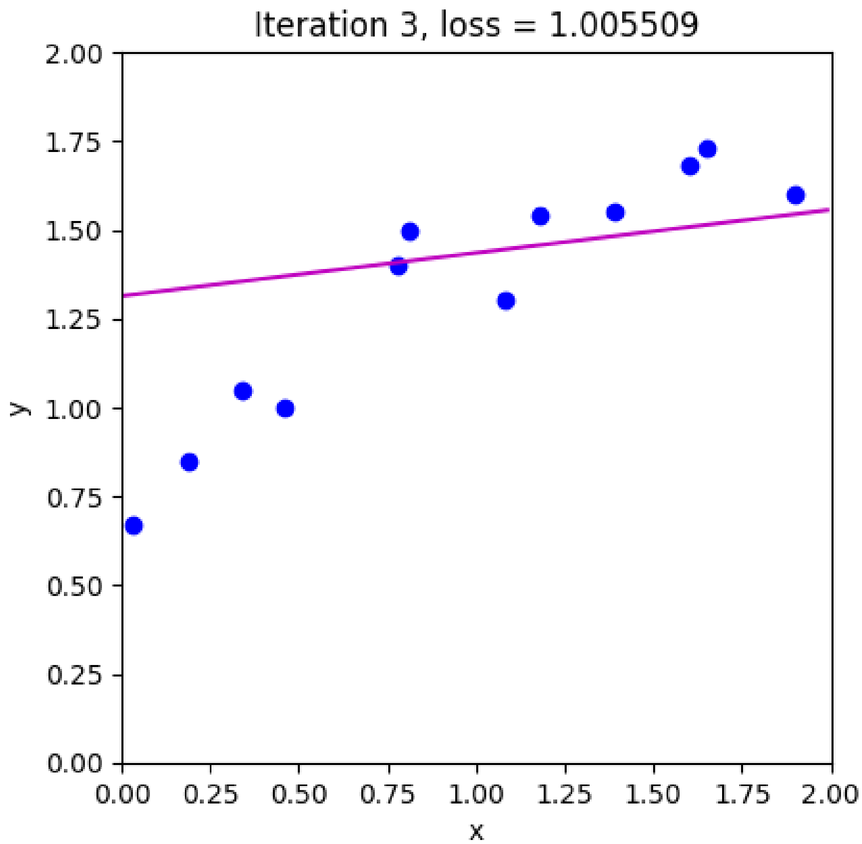
[16]:
```
def gradient_descent_step(phi, data):
  # Being Solution
  data_x = data[0,:]; data_y = data[1,:]

  gradient = compute_gradient(data_x, data_y, phi)
  alpha = line_search(data, phi, -gradient)
  phi = phi - alpha * gradient
  # End Solution
  return phi
```

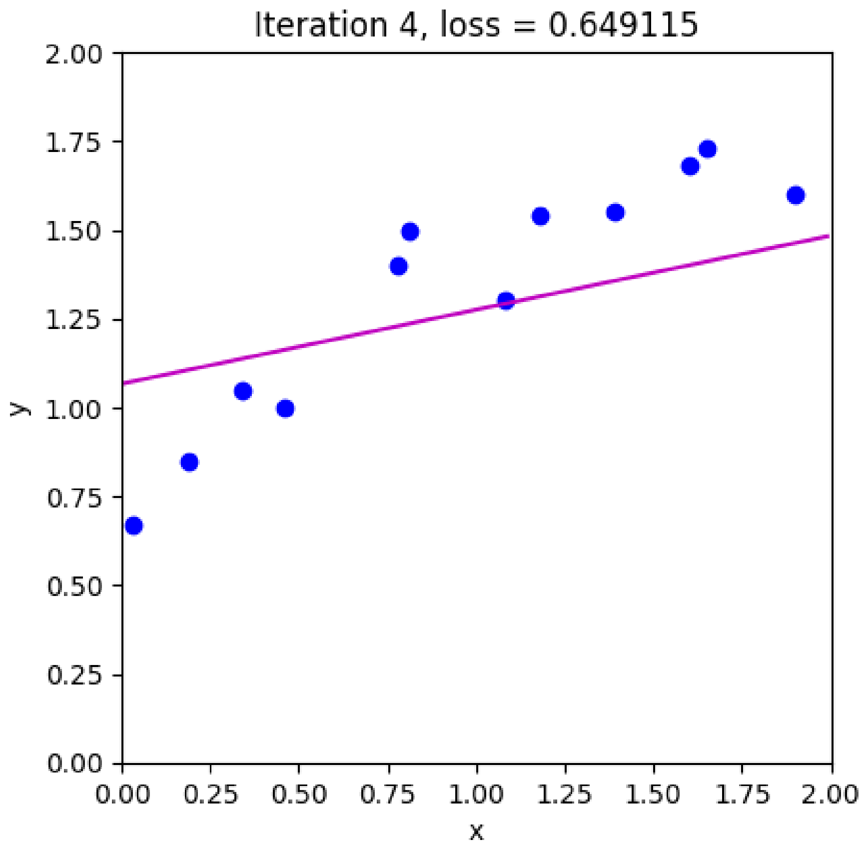
[17]:
```
# Initialize the parameters and draw the model
n_steps = 10
phi_all = np.zeros((2,n_steps+1))
phi_all[0,0] = 1.6
phi_all[1,0] = -0.5

# Measure loss and draw initial model
loss =  compute_loss(data[0,:], data[1,:], model, phi_all[:,0:1])
draw_model(data,model,phi_all[:,0:1], "Initial parameters, Loss = %f"%(loss))

# Repeatedly take gradient descent steps
for c_step in range (n_steps):
  # Do gradient descent step
  phi_all[:,c_step+1:c_step+2] = gradient_descent_step(phi_all[:,c_step:
 ↪c_step+1],data)
  # Measure loss and draw model
  loss =  compute_loss(data[0,:], data[1,:], model, phi_all[:,c_step+1:
 ↪c_step+2])
  draw_model(data,model,phi_all[:,c_step+1], "Iteration %d, loss =␣
 ↪%f"%(c_step+1,loss))

# Draw the trajectory on the loss function
draw_loss_function(compute_loss, data, model,phi_all)
```

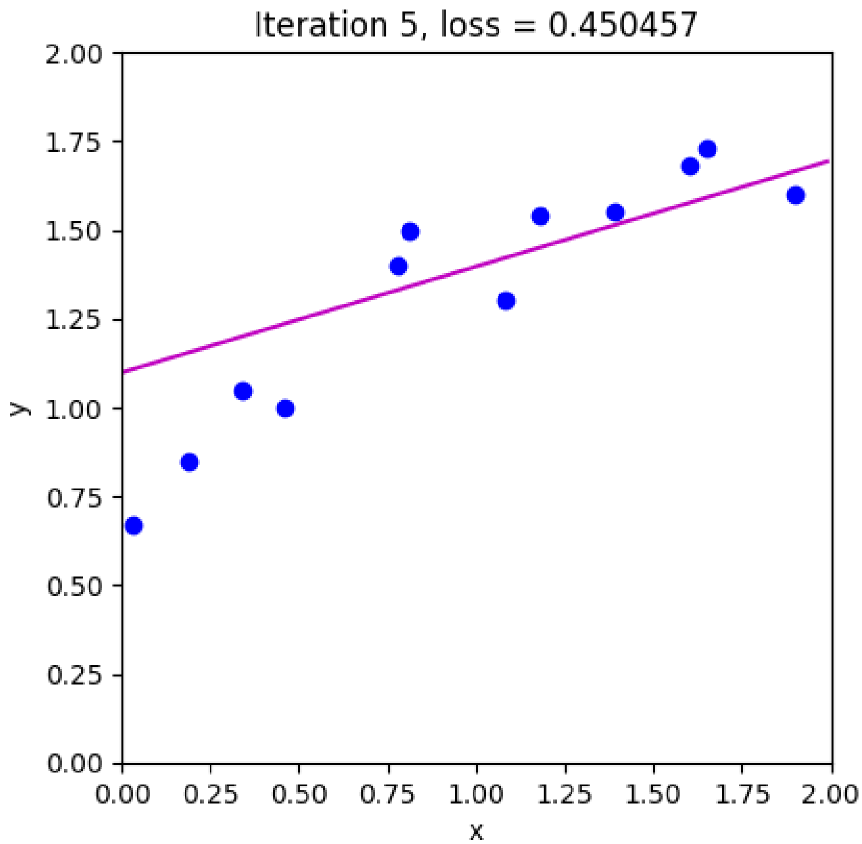

Initial parameters, Loss = 5.020125
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
y
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
x

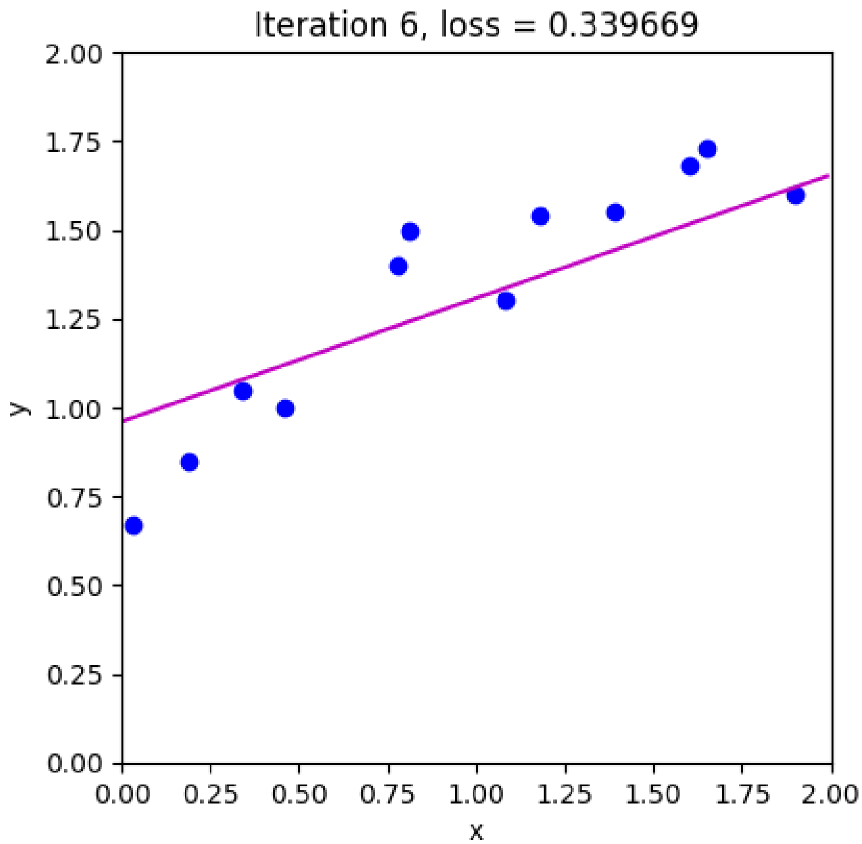

Iteration 1, loss = 2.835363
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
y
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
x

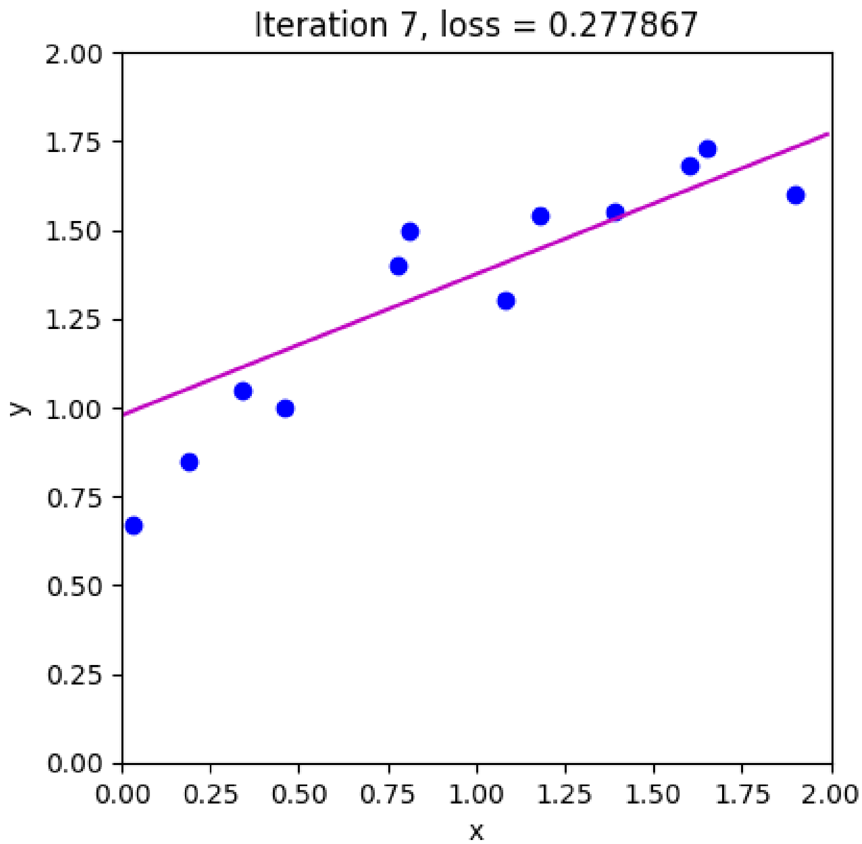

Iteration 2, loss = 1.656856
y
x
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
0.00 0.25 0.50 0.75 1.00 1.25 1.50 1.75 2.00


Iteration 3, loss = 1.005509
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
y
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
x

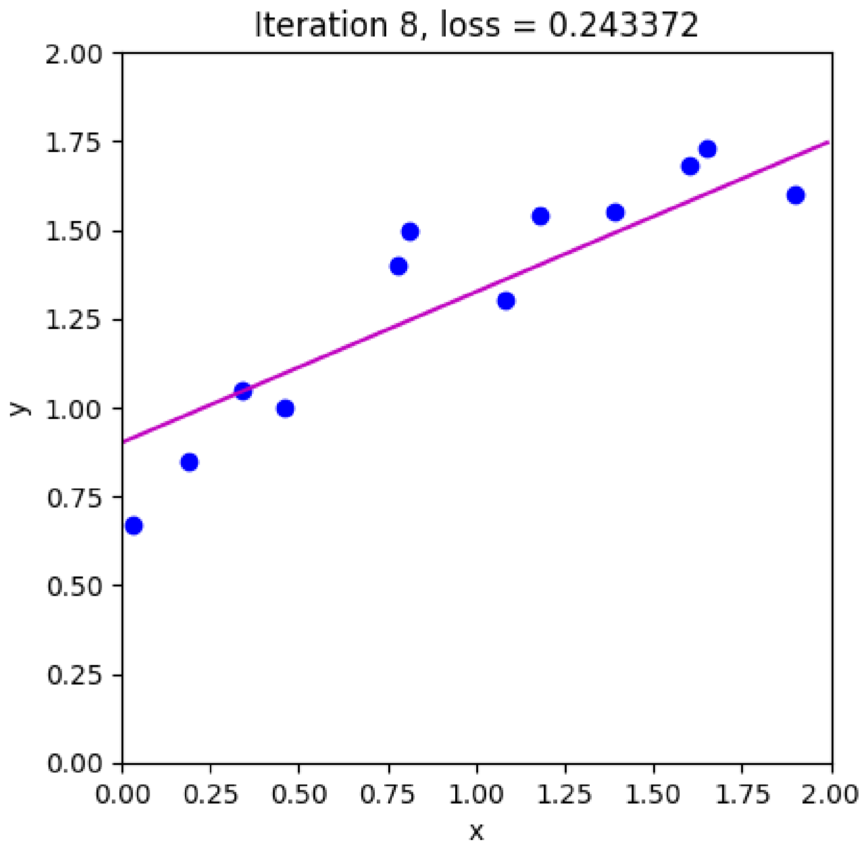

Iteration 4, loss = 0.649115
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
y
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
x

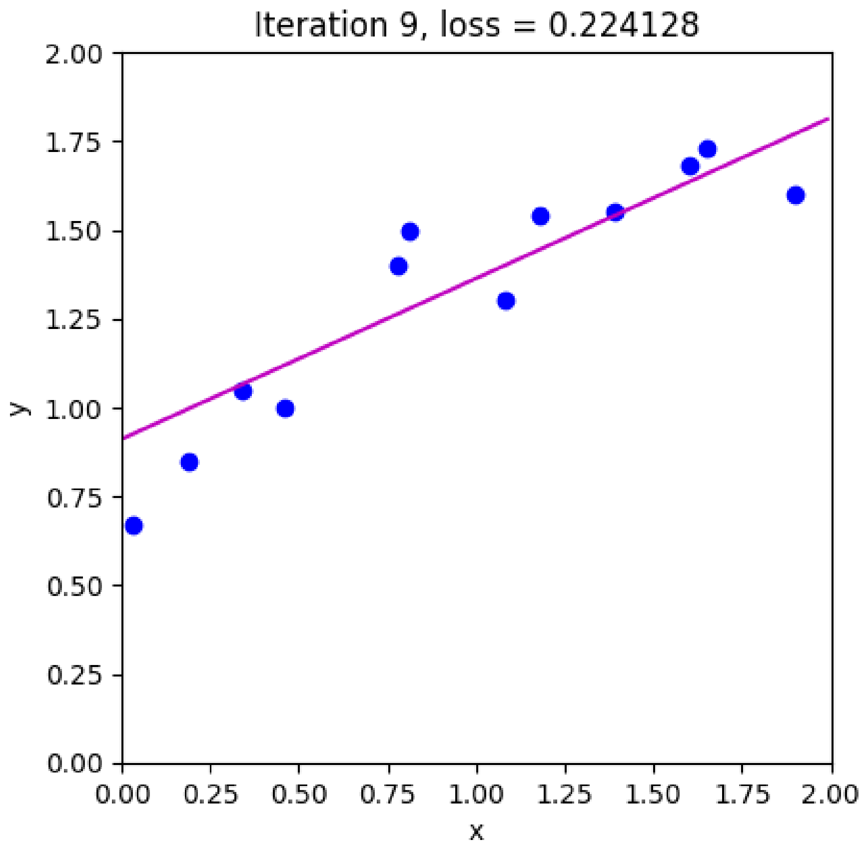

Iteration 5, loss = 0.450457
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
y
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
x

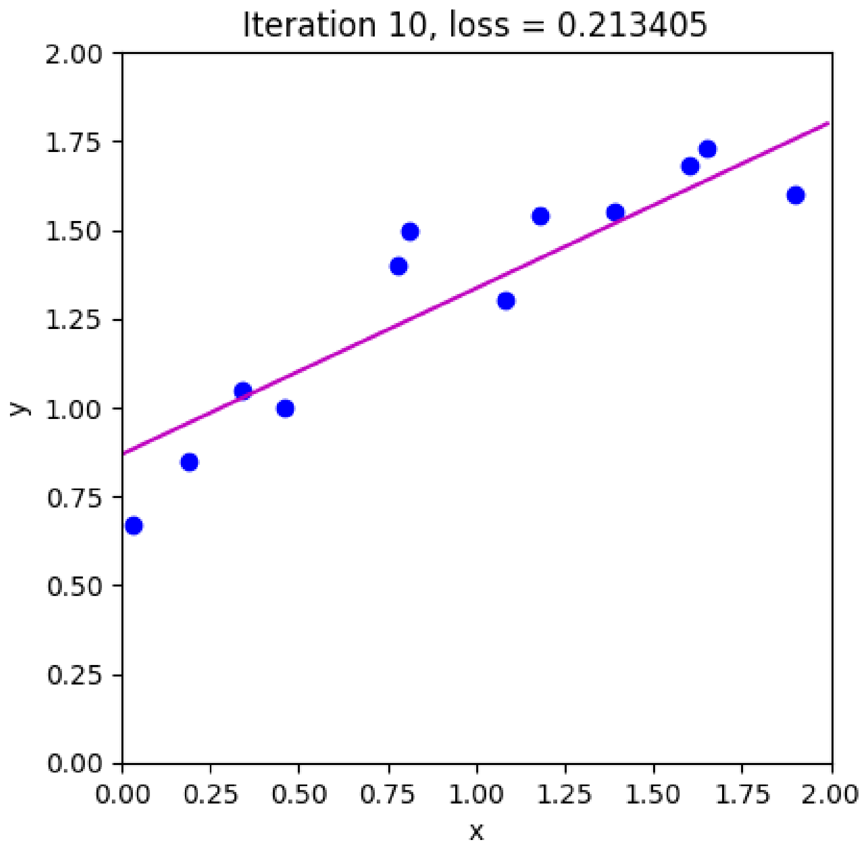

Iteration 6, loss = 0.339669
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
y
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
x

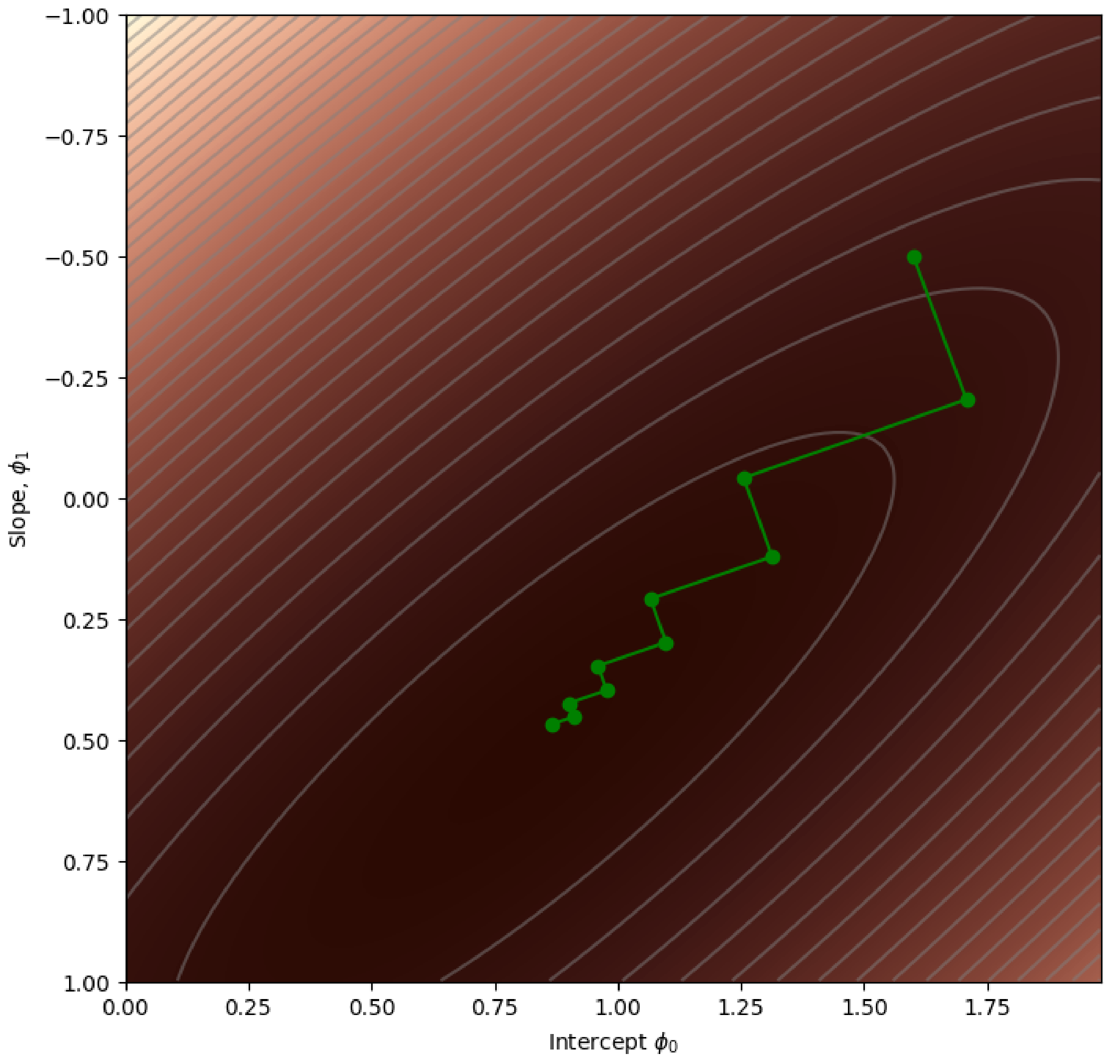


Iteration 7, loss = 0.277867
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
y
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
x


Iteration 8, loss = 0.243372
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
y
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
x


Iteration 9, loss = 0.224128
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
y
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
x


Iteration 10, loss = 0.213405
y
x
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00



8.1 If your answer to previous task is correct then last 2 graphs would look like this