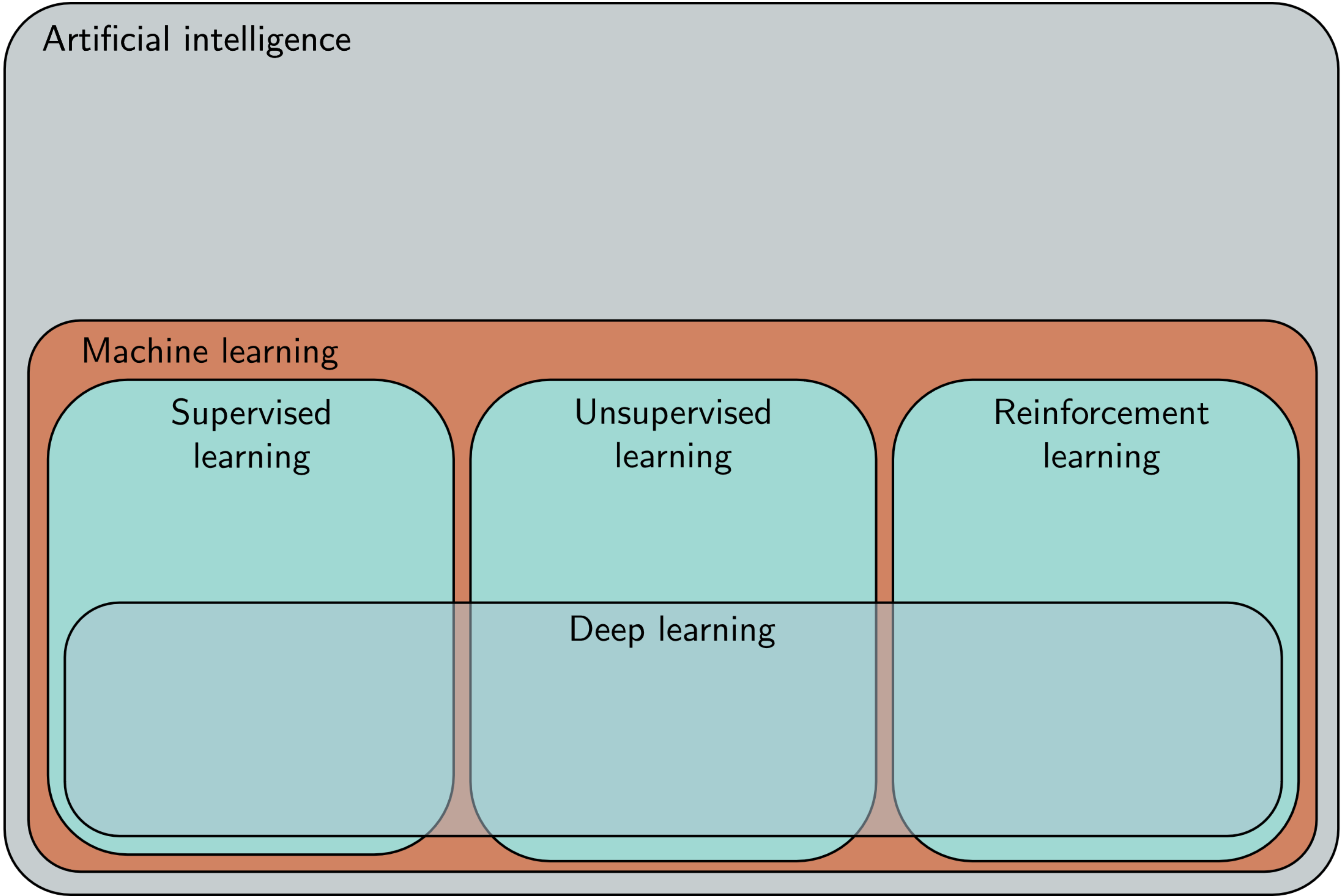
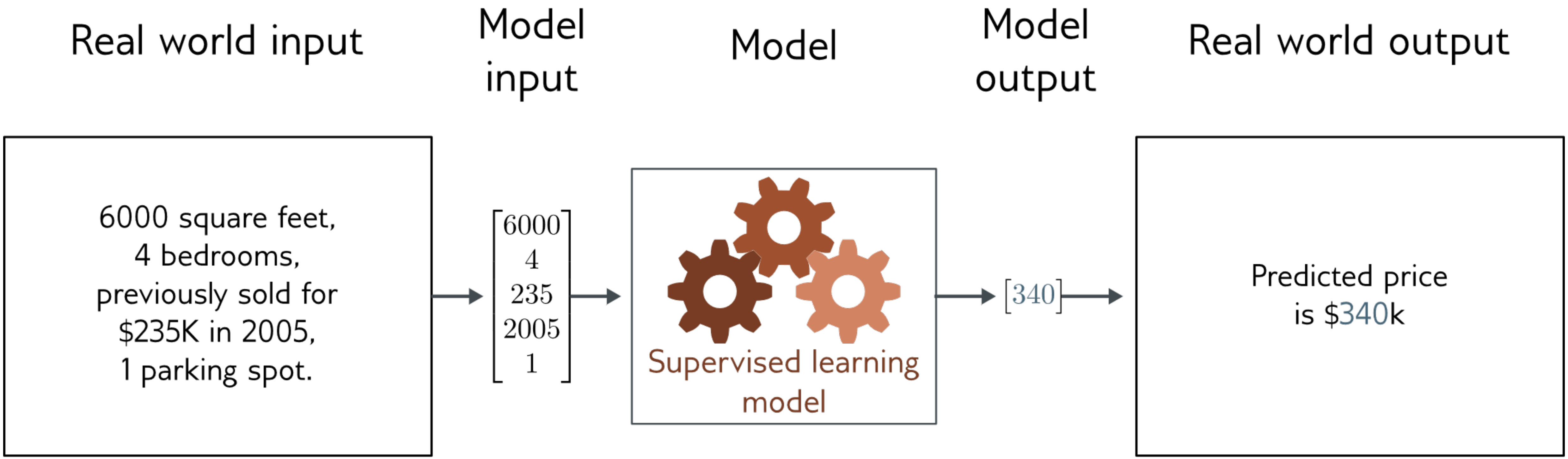
Supervised Learning

Abdul Samad

Adopted from Prof. Simon Prince


Artificial intelligence
Machine learning
Supervised learning
Unsupervised learning
Reinforcement learning
Deep learning

Regression



- Univariate regression problem (one output, real value)

Supervised learning

- Overview
- Notation
  - Model
  - Loss function
  - Training
  - Testing
- 1D Linear regression example
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- Where are we going?

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Supervised learning overview

- Supervised learning model = mapping from one or more inputs to one or more outputs
- Model is a mathematical equation
- Computing the inputs from the outputs = inference

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- Example:
  - Input is age and milage of secondhand Toyota Prius
  - Output is estimated price of car

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- Parameters affect outcome of equation

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- Training a model = finding parameters that predict outputs "well" from inputs for a training dataset of input/output pairs

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Notation:

- Input:

$$\mathbf{x}$$

- Output:

$$\mathbf{y}$$

- Model:

$$\mathbf{y} = \mathbf{f}[\mathbf{x}]$$

Variables always Roman letters

Normal = scalar
Bold = vector
Capital Bold = matrix

Functions always square brackets

Normal = returns scalar
Bold = returns vector
Capital Bold = returns matrix

Notation example:

- Input:

$$\mathbf{x} = \begin{bmatrix} \text{age} \\ \text{mileage} \end{bmatrix}$$

Structured or tabular data

- Output:

$$y = \begin{bmatrix} \text{price} \end{bmatrix}$$

- Model:

$$y = \mathrm{f}[\mathbf{x}]$$

Model

- Parameters:

$$\phi$$

Parameters always Greek letters

- Model :

$$\mathbf{y} = \mathbf{f}[\mathbf{x}, \phi]$$

Loss function

- Training dataset of I pairs of input/output examples:

$$\{\mathbf{x}_i, \mathbf{y}_i\}_{i=1}^{I}$$

- Loss function or cost function measures how bad model is:

$$L\Big[\underbrace{\phi, \mathrm{f}[\mathbf{x}, \phi]}_{\text{model}}, \underbrace{\{\mathbf{x}_i, \mathbf{y}_i\}_{i=1}^{I}}_{\text{train data}}\Big]$$

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or for short:

$$L[\phi]$$

Returns a scalar that is smaller when model maps inputs to outputs better

Training

- Loss function:

$$L[\phi]$$

Returns a scalar that is smaller when model maps inputs to outputs better

- Find the parameters that minimize the loss:

$$\hat{\phi} = \underset{\phi}{\operatorname{argmin}} \Big[\mathrm{L}[\phi] \Big]$$

Testing

- To test the model, run on a separate test dataset of input / output pairs
- See how well it generalizes to new data

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Example: 1D Linear regression model

- Model:

$$y = \mathrm{f}[x, \boldsymbol{\phi}] \\ = \phi_0 + \phi_1 x$$

- Parameters

$$\boldsymbol{\phi} = \begin{bmatrix} \phi_0 \\ \phi_1 \end{bmatrix} \begin{array}{l} \longleftarrow \text{ y-offset} \\ \longleftarrow \text{ slope} \end{array}$$

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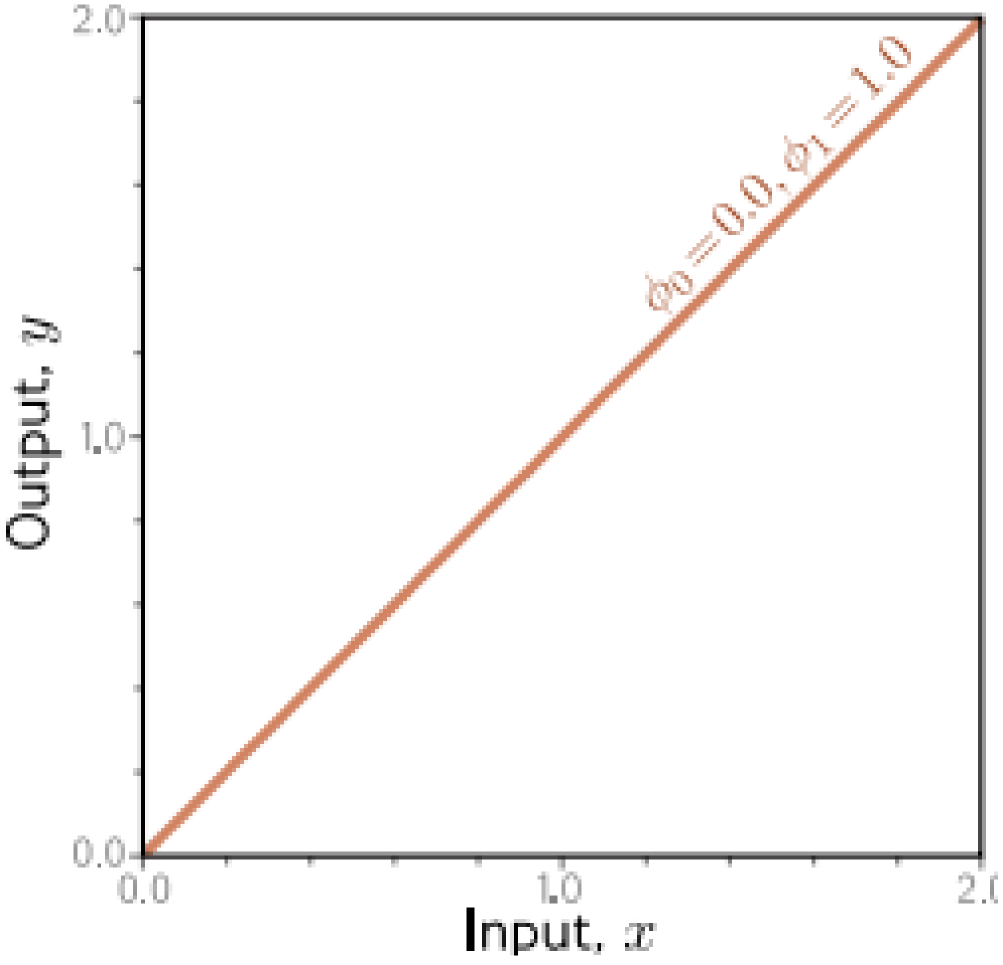
y-offset

slope



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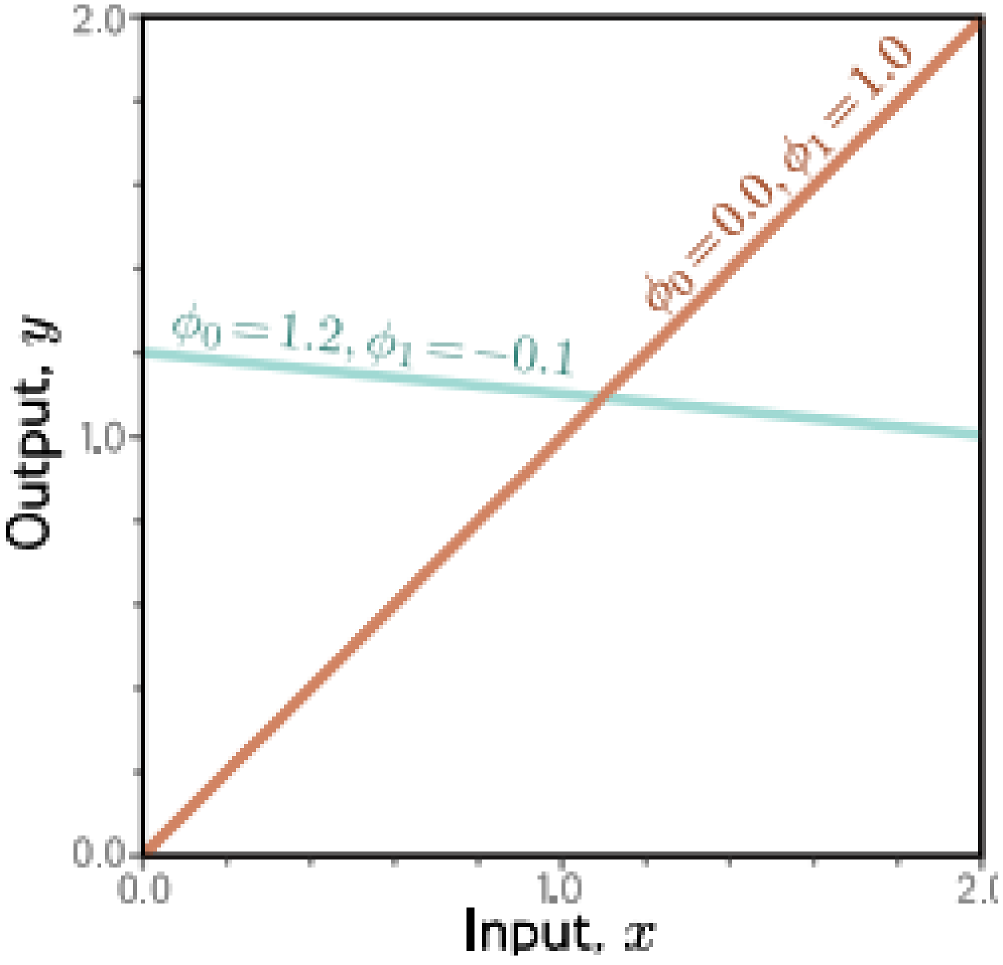
← slope



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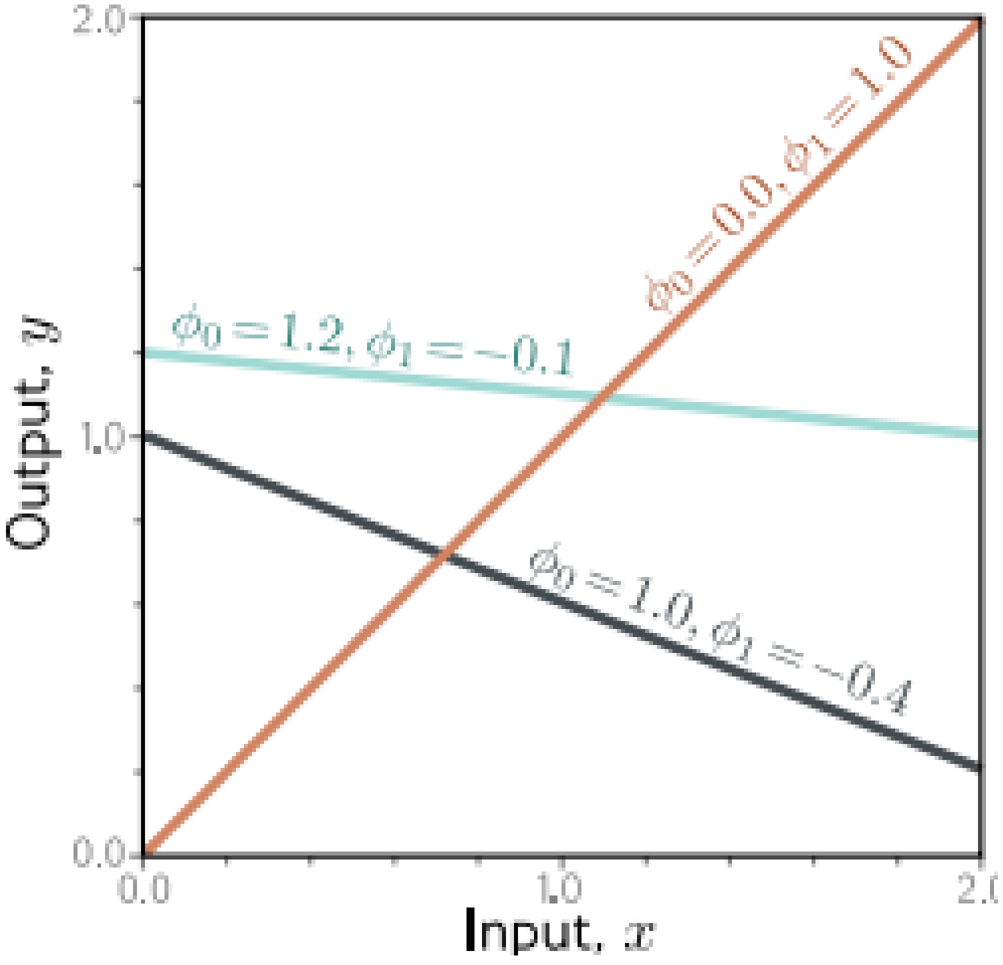
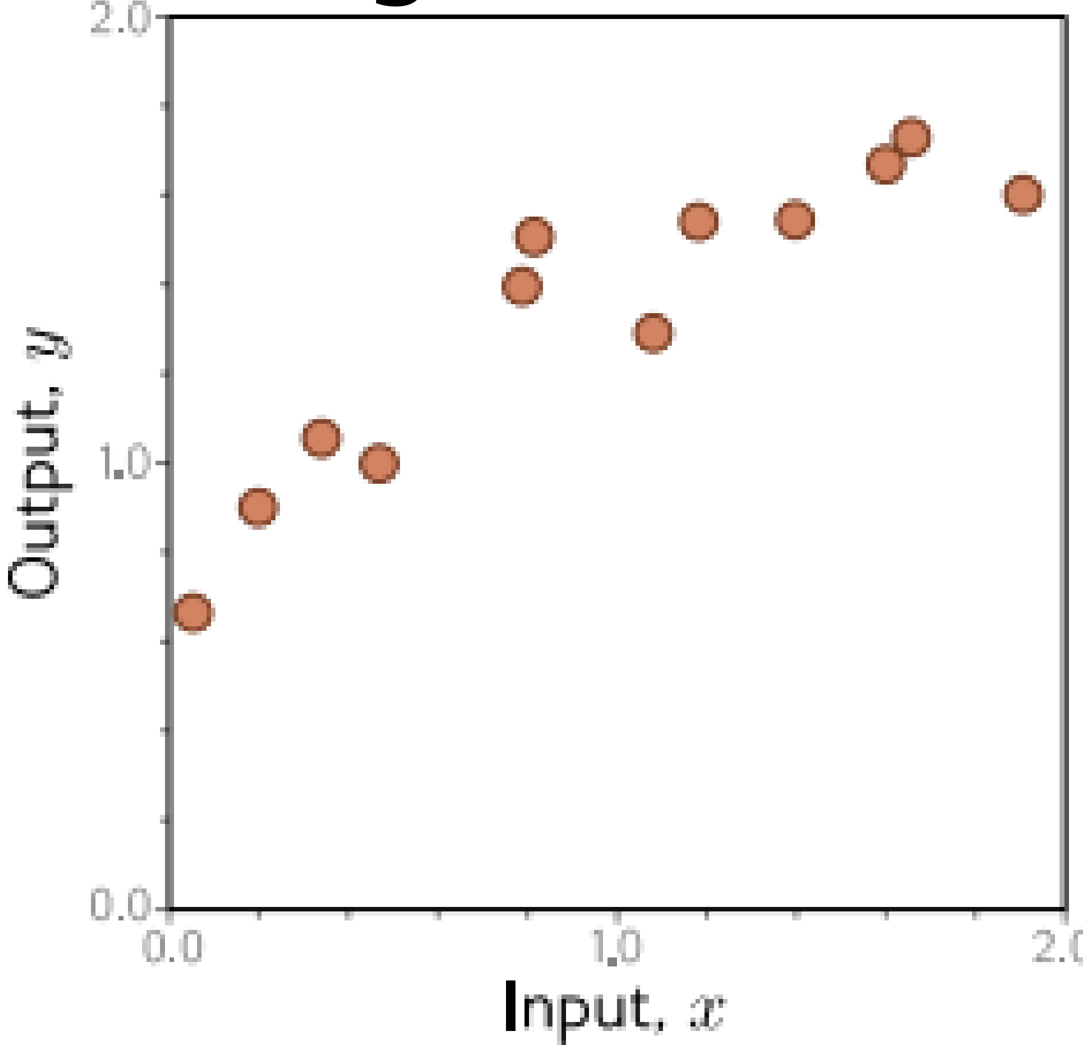


Example: 1D Linear regression training data



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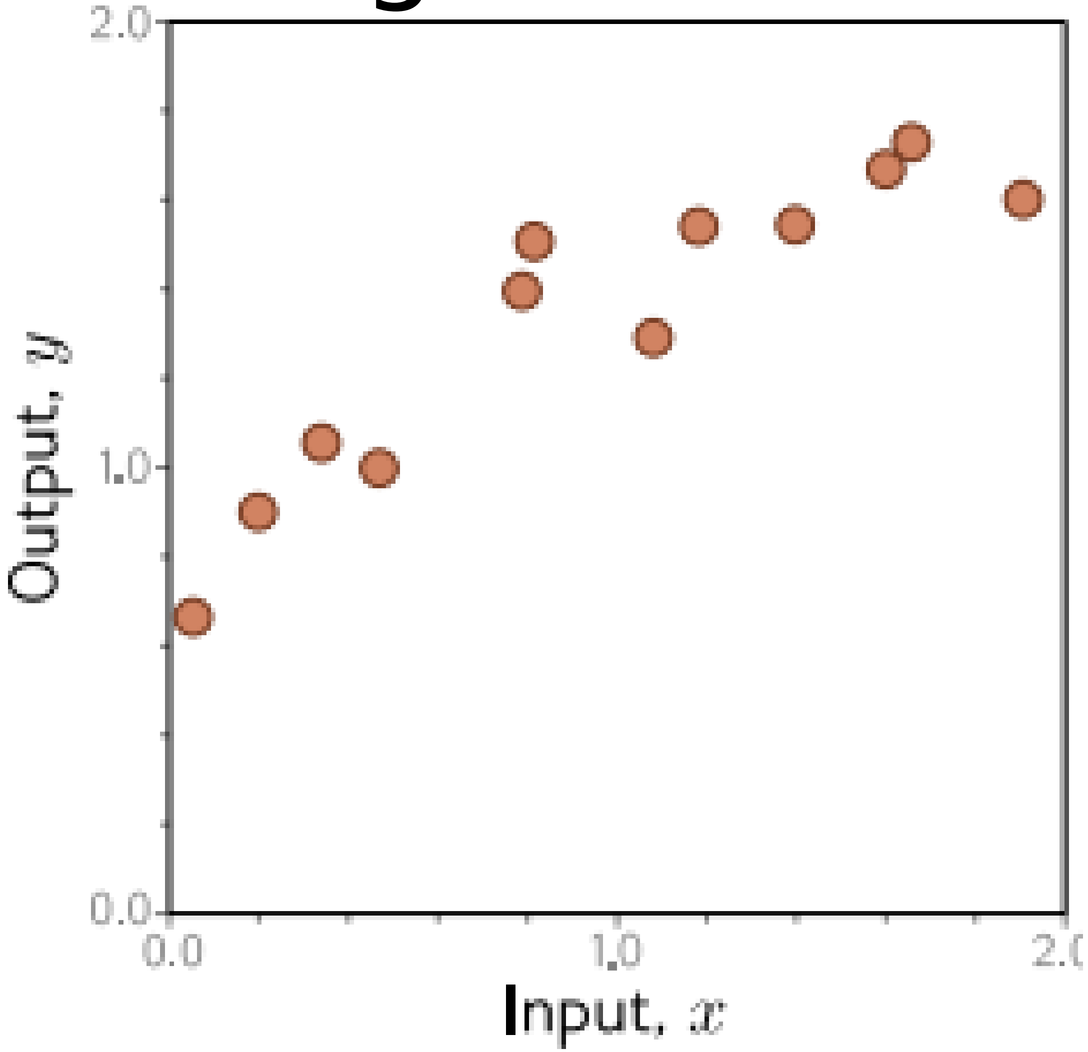
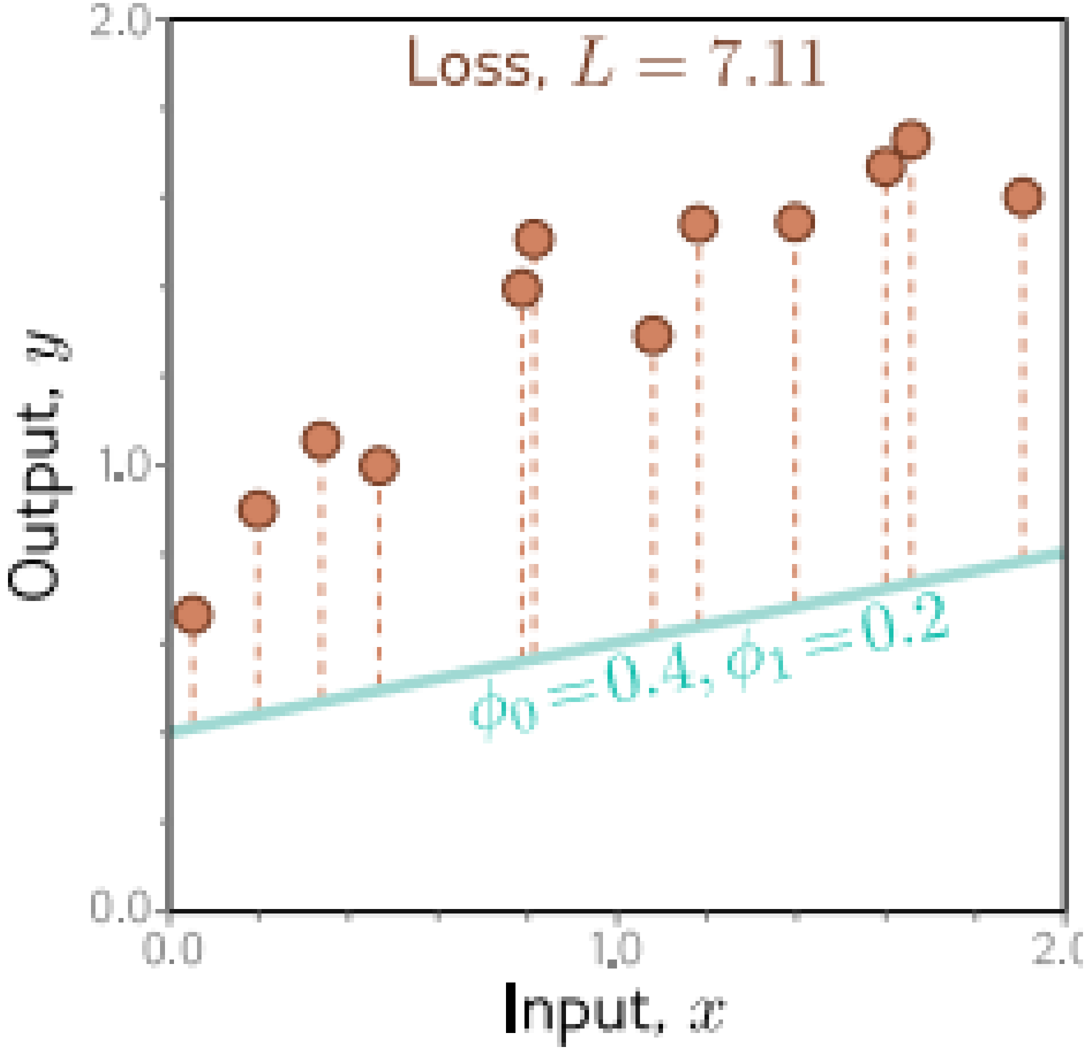


Loss function:

$$
\begin{aligned}
L[\phi] &= \sum_{i=1}^{I} (\mathrm{f}[x_i, \phi] - y_i)^2 \\
&= \sum_{i=1}^{I} (\phi_0 + \phi_1 x_i - y_i)^2
\end{aligned}
$$

"Least squares loss function"

Example: 1D Linear regression loss function



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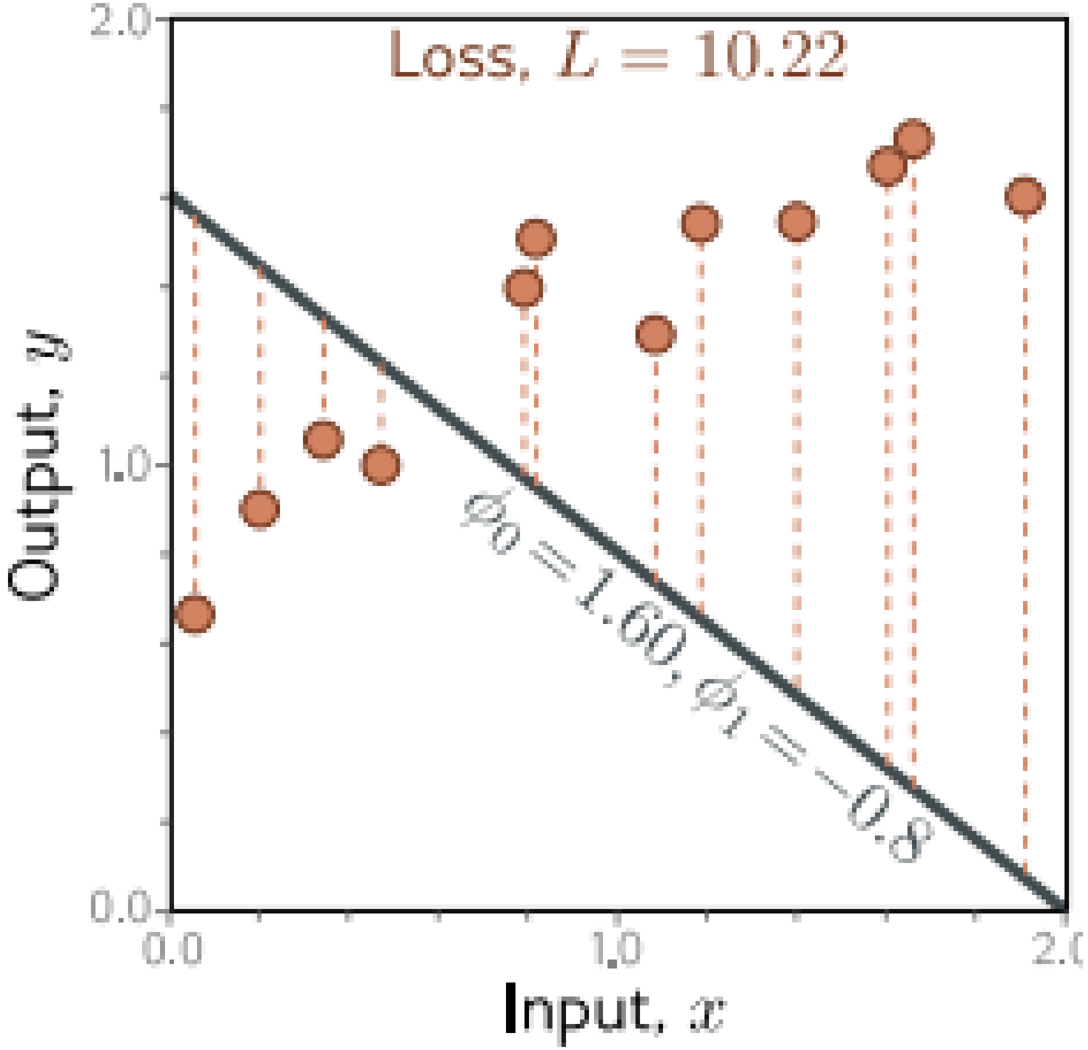

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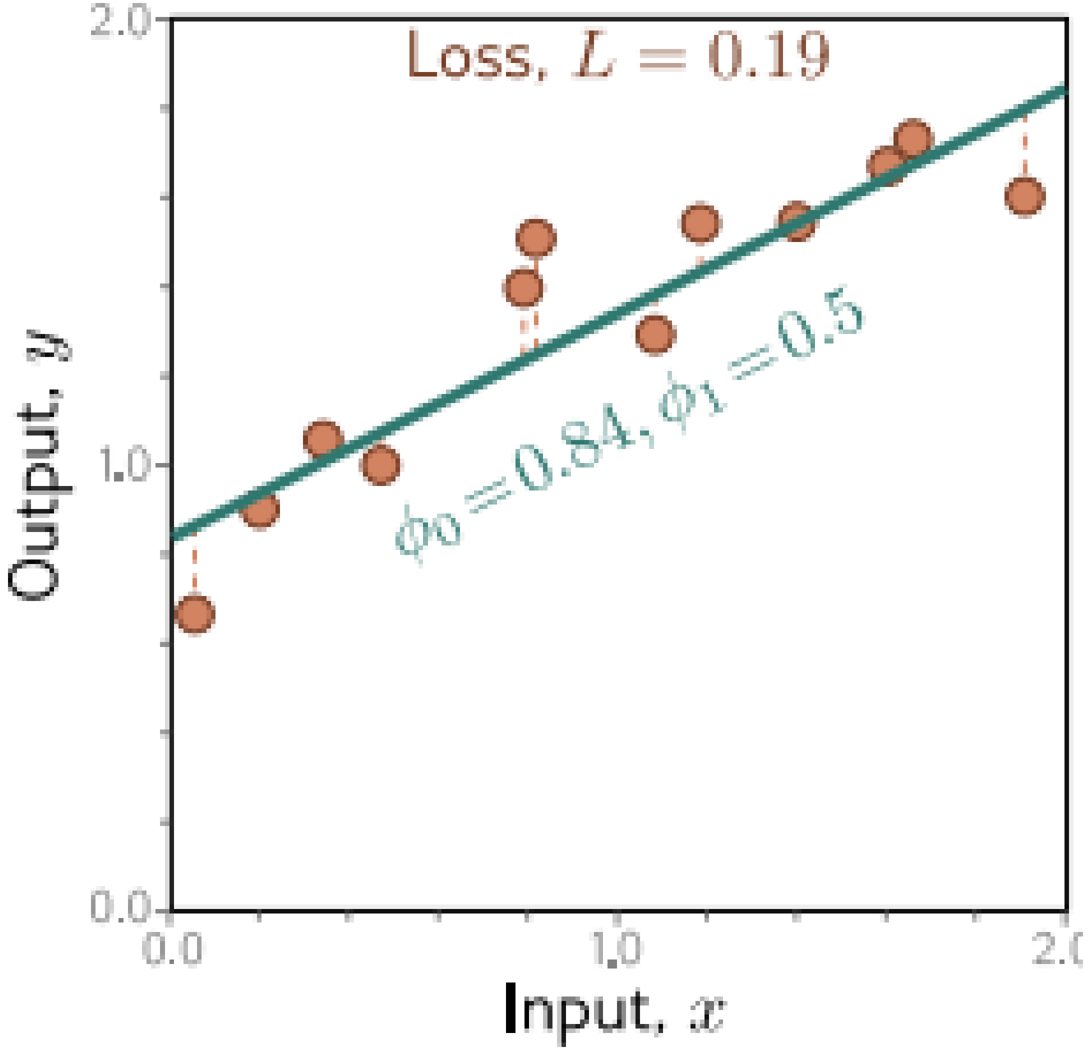

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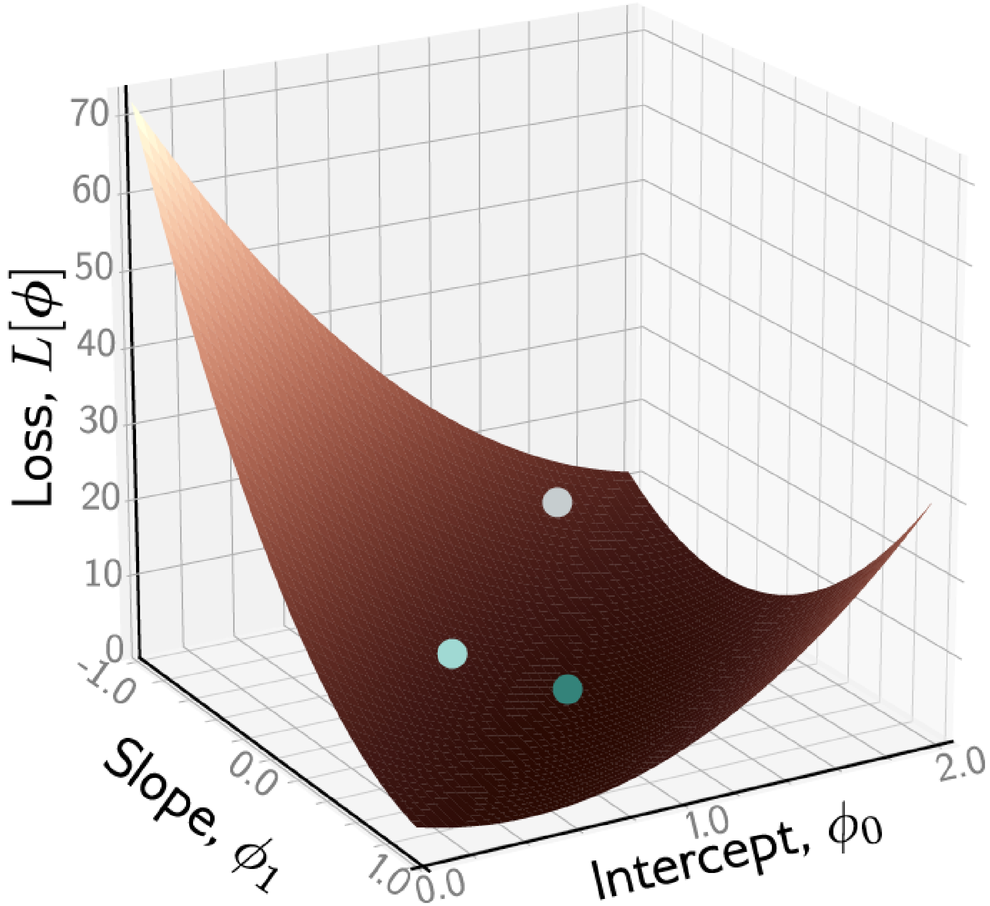

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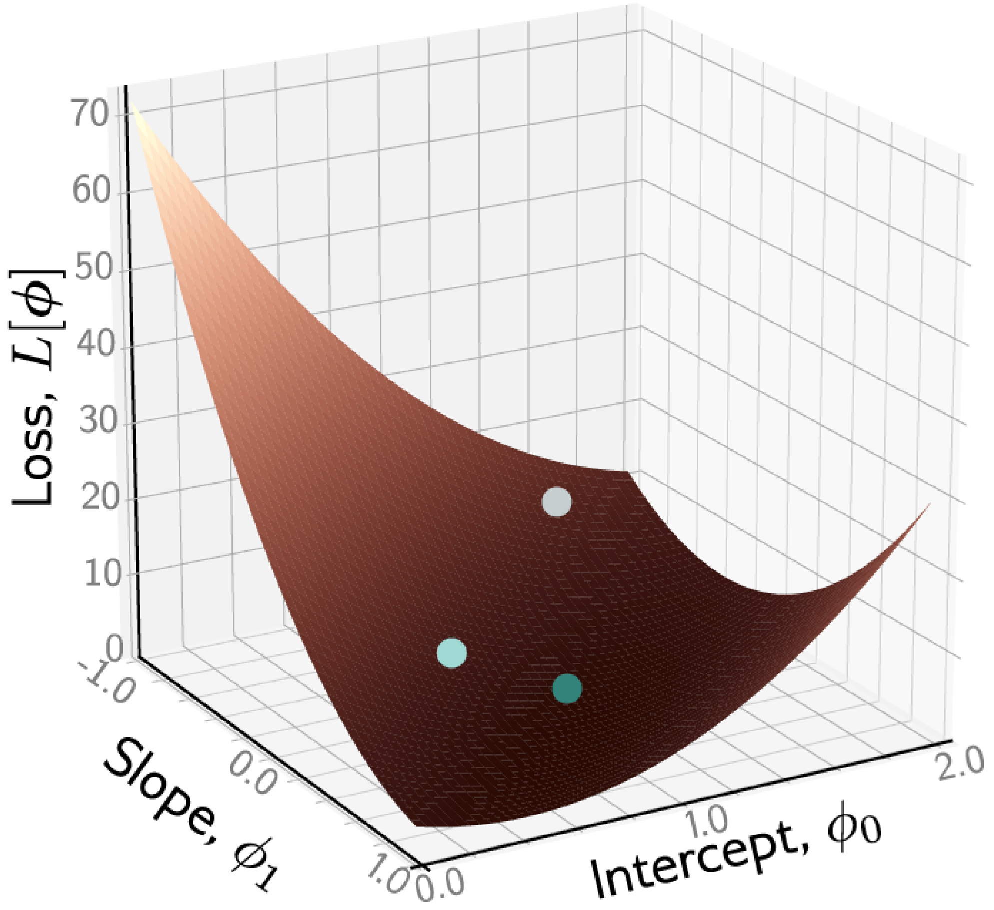

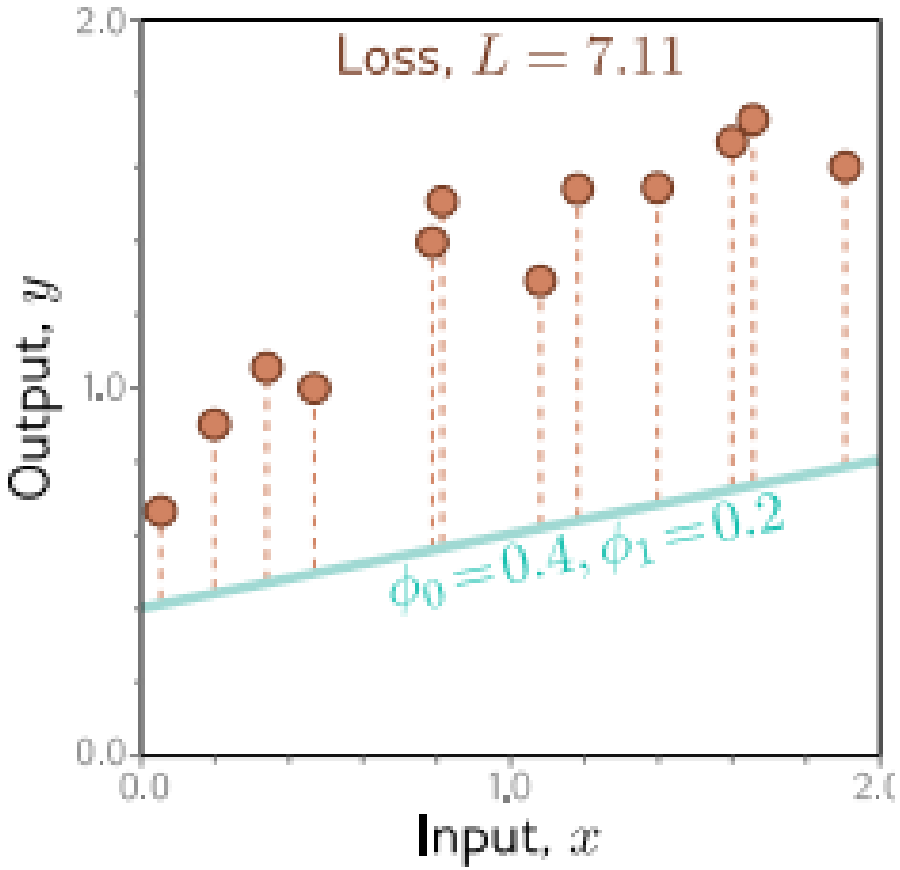

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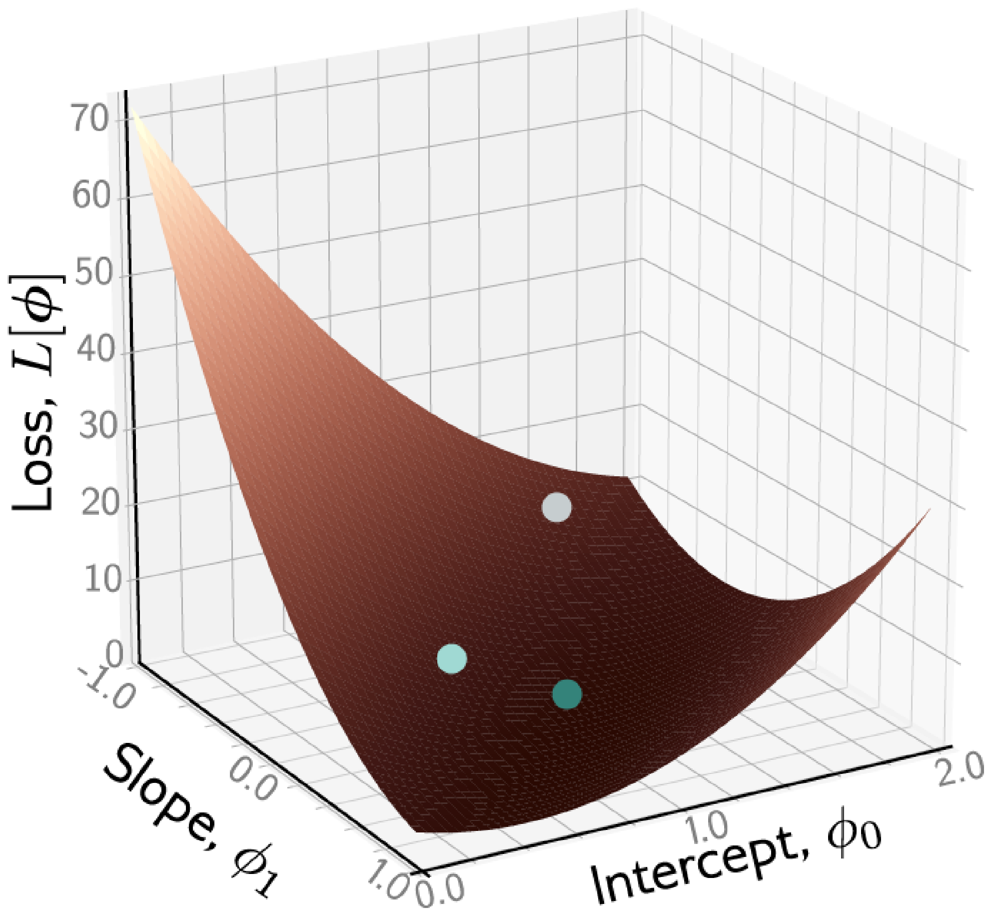

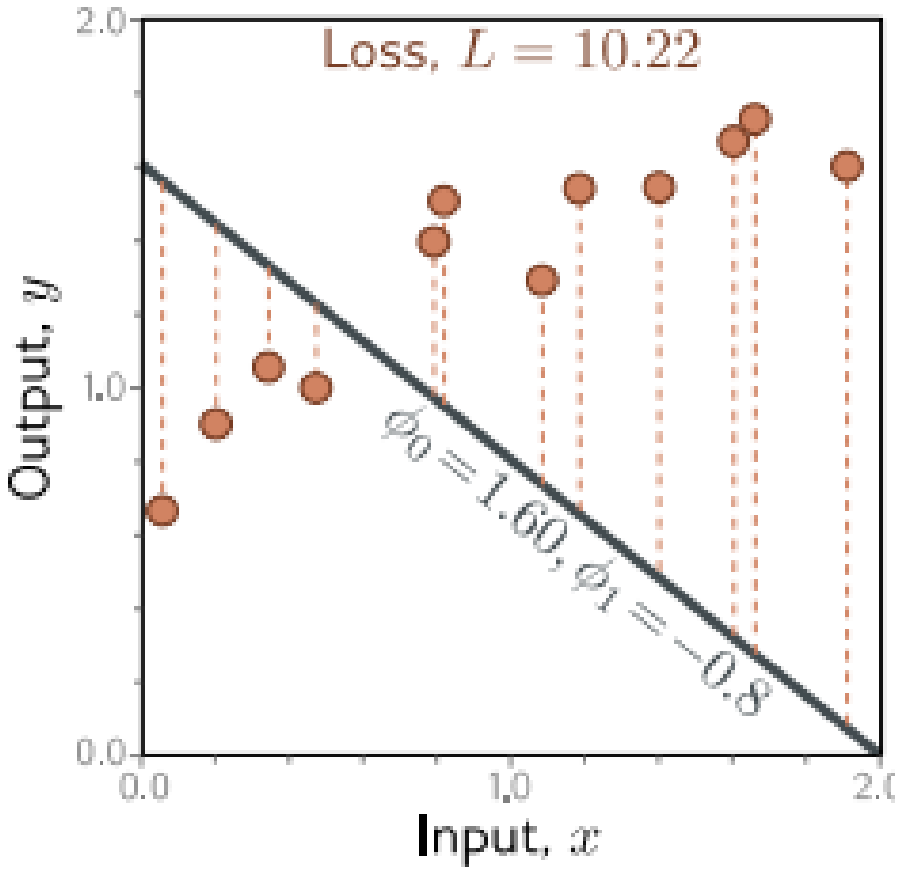

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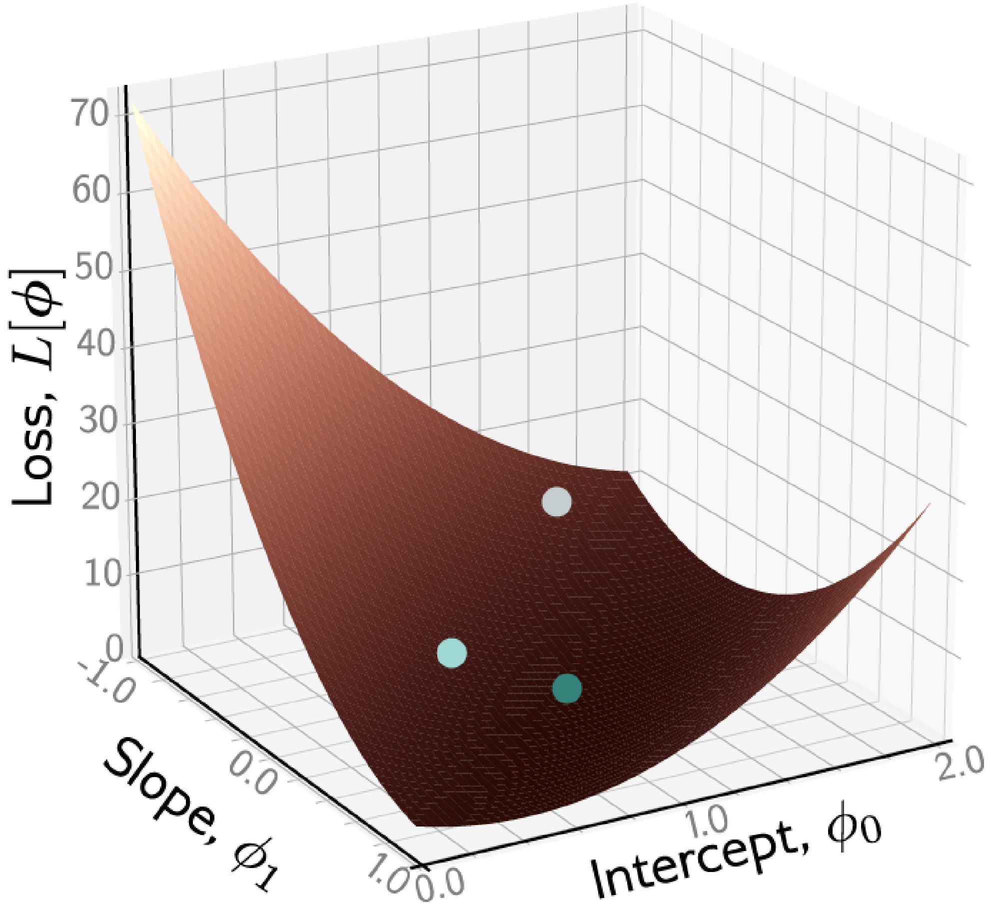

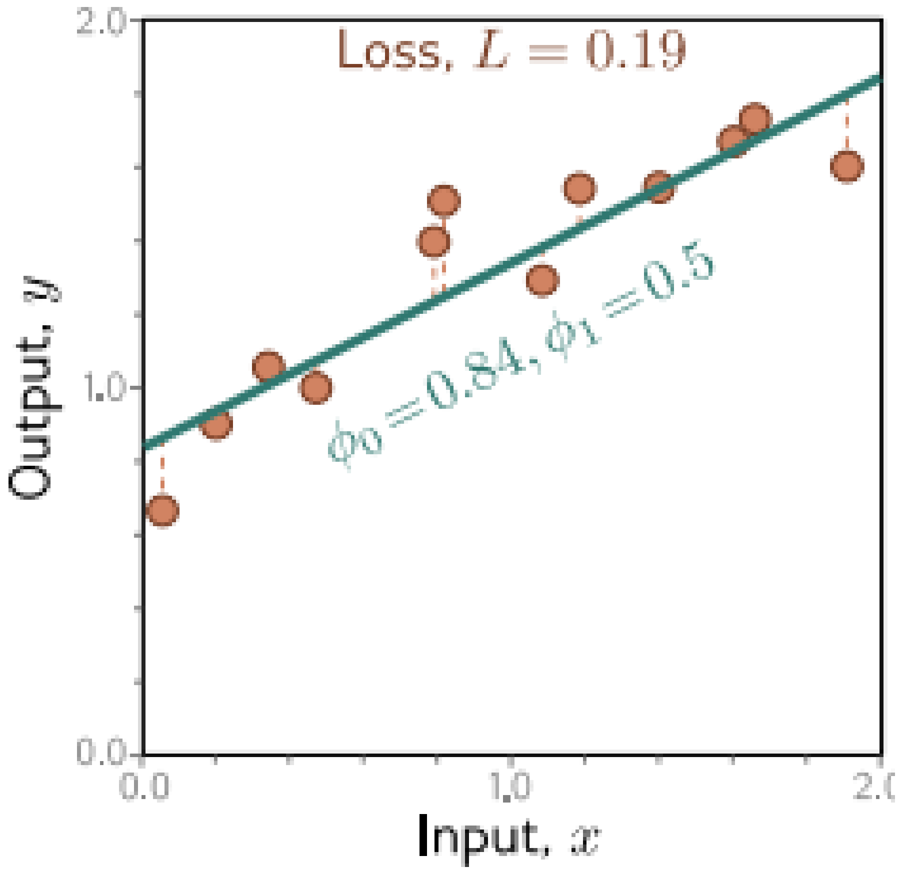

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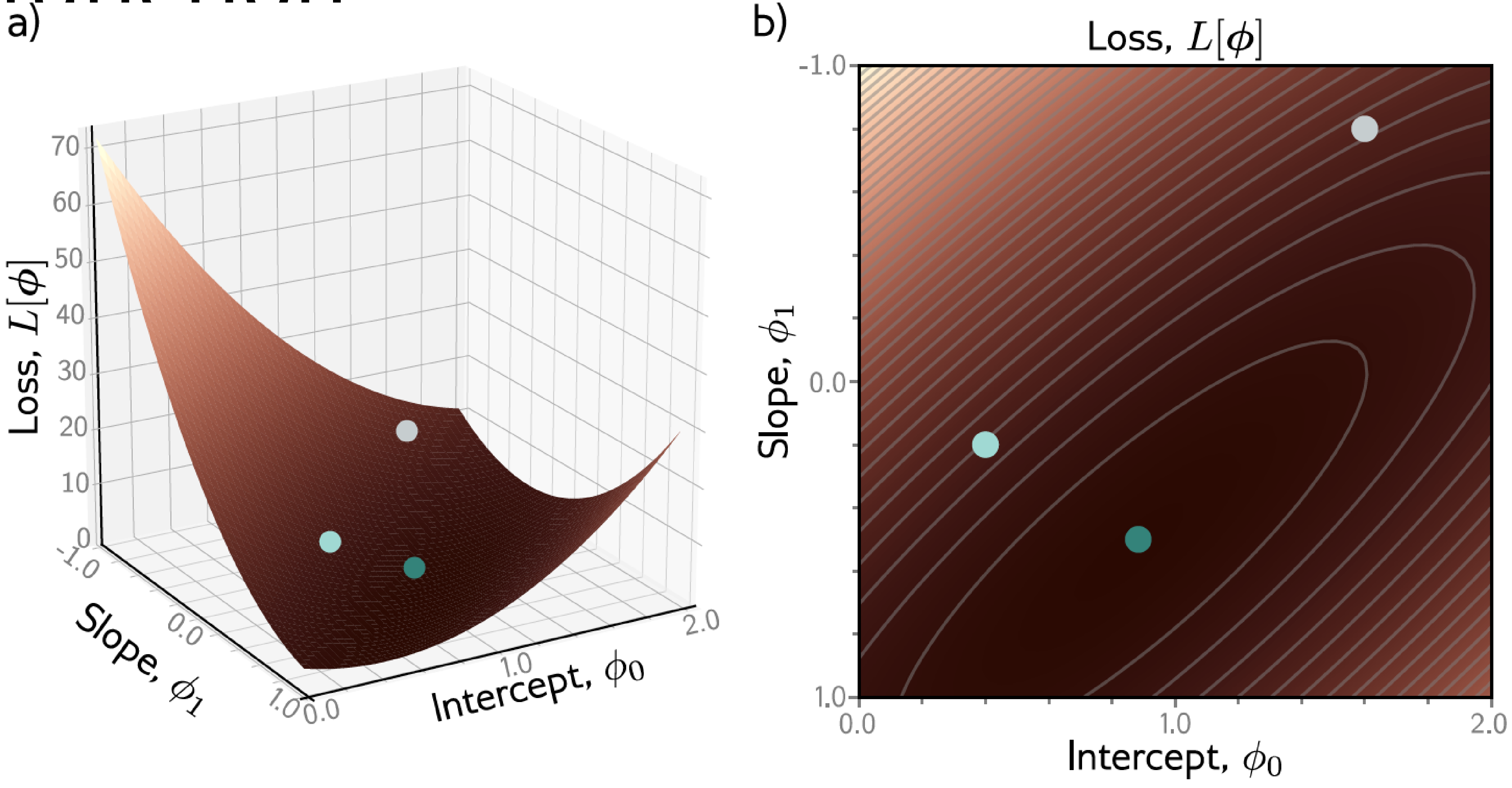
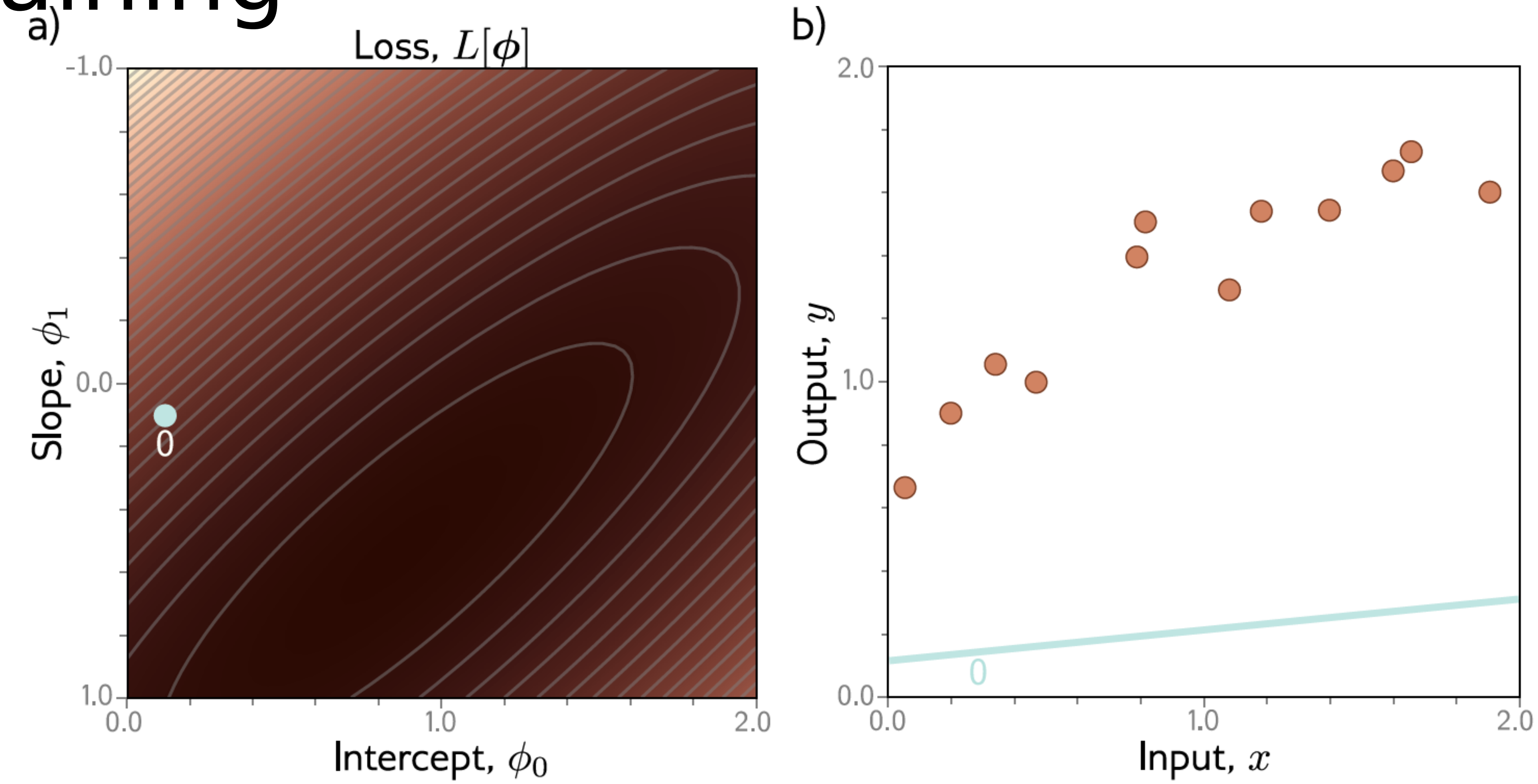


Example: 1D Linear regression training



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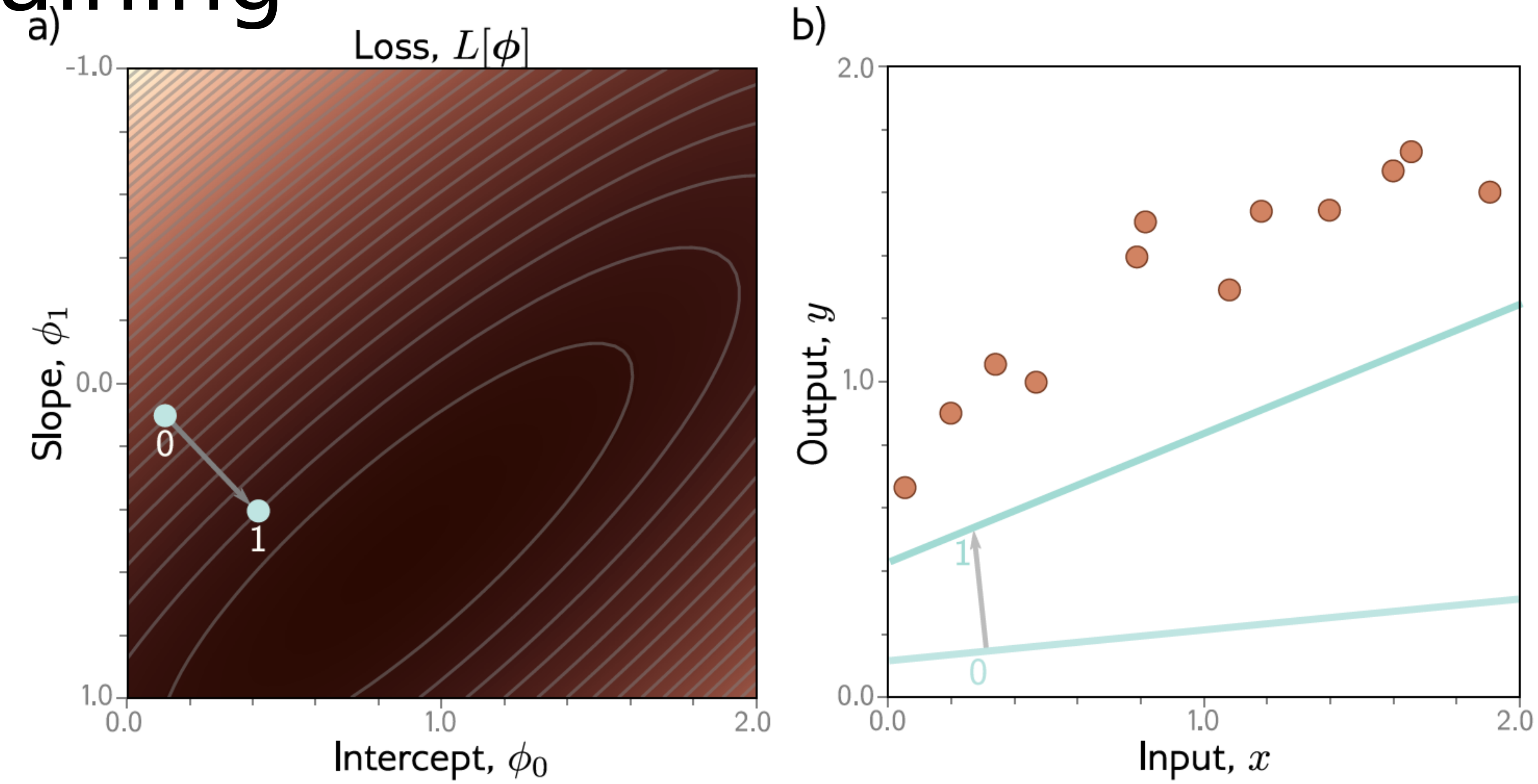

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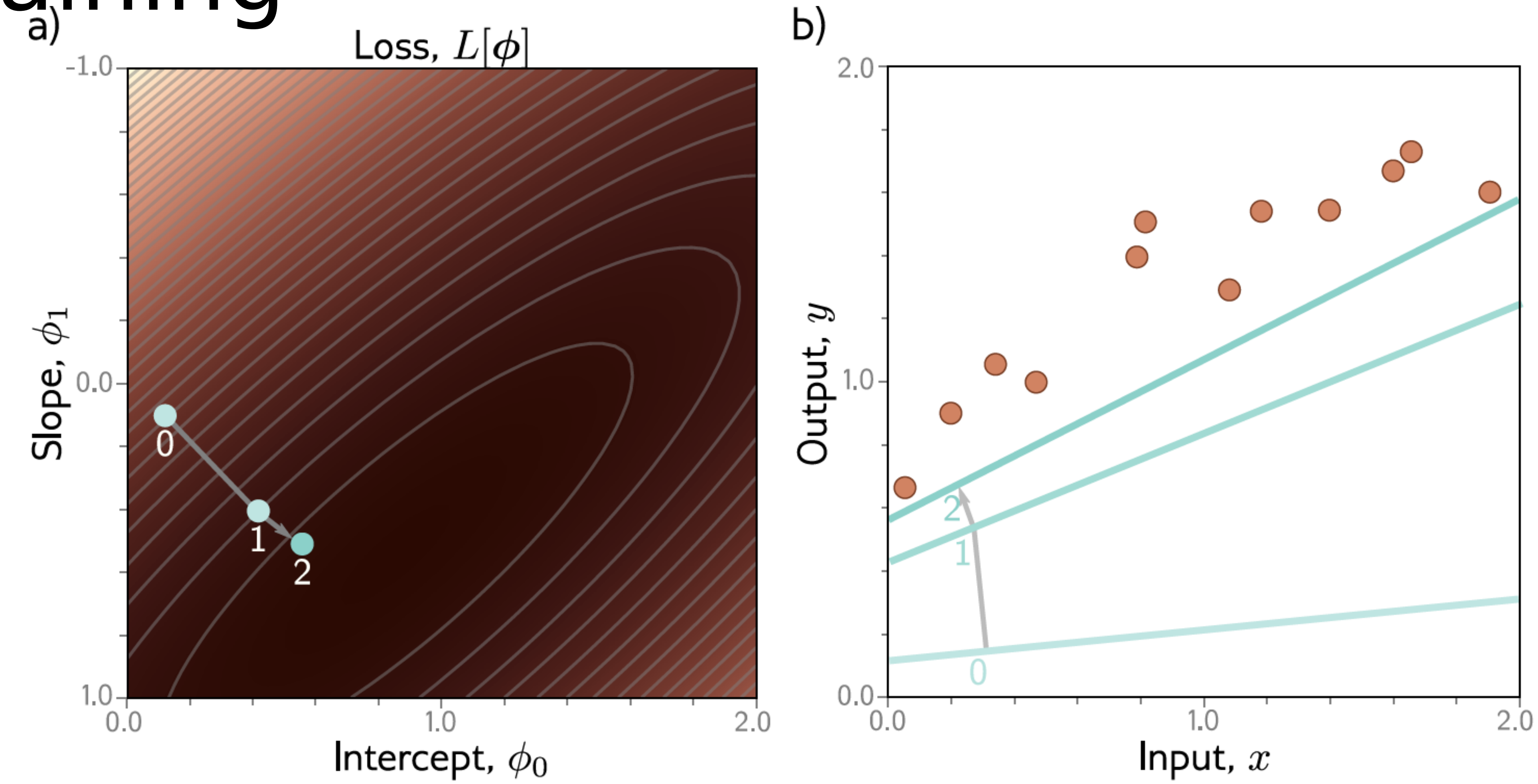

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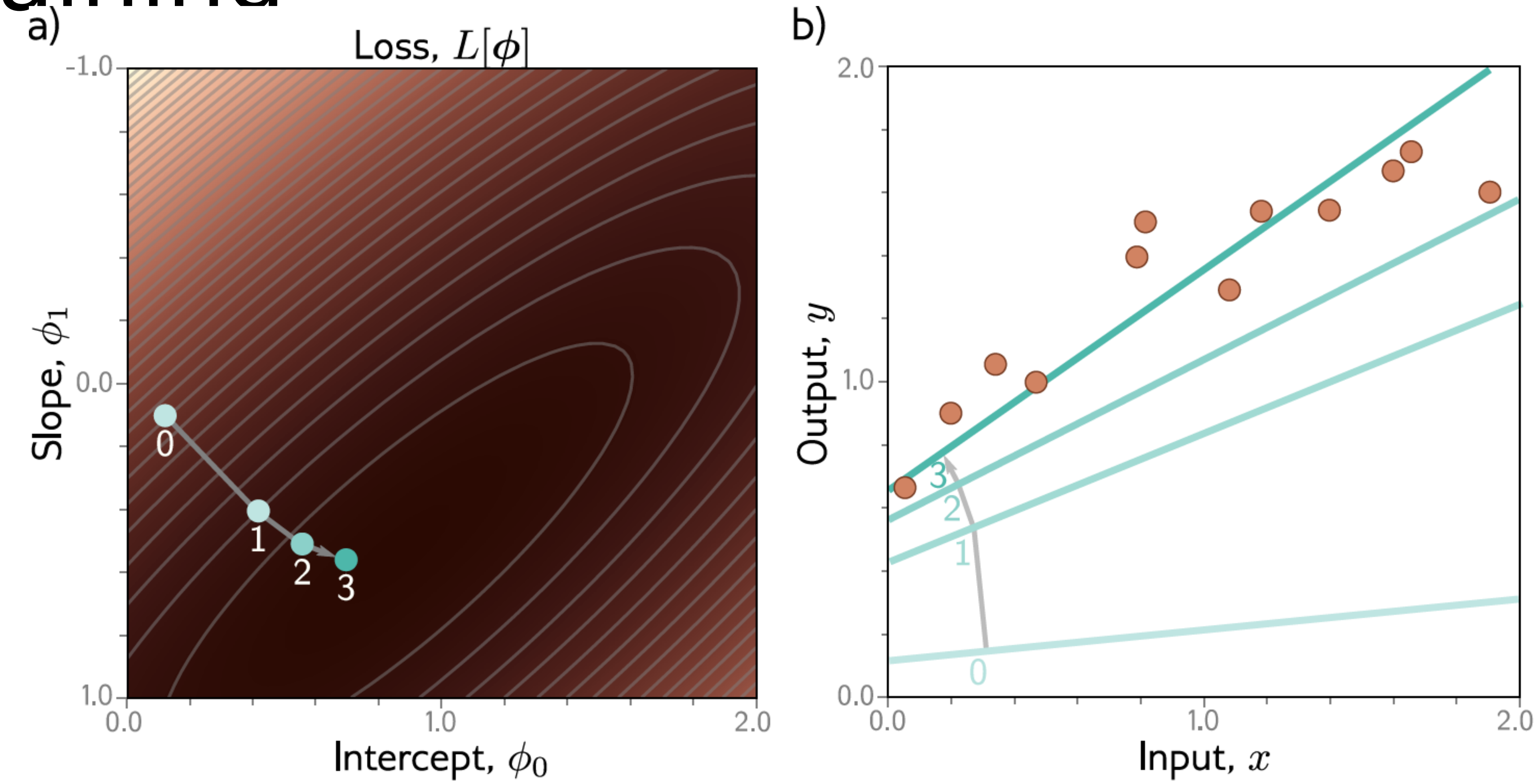

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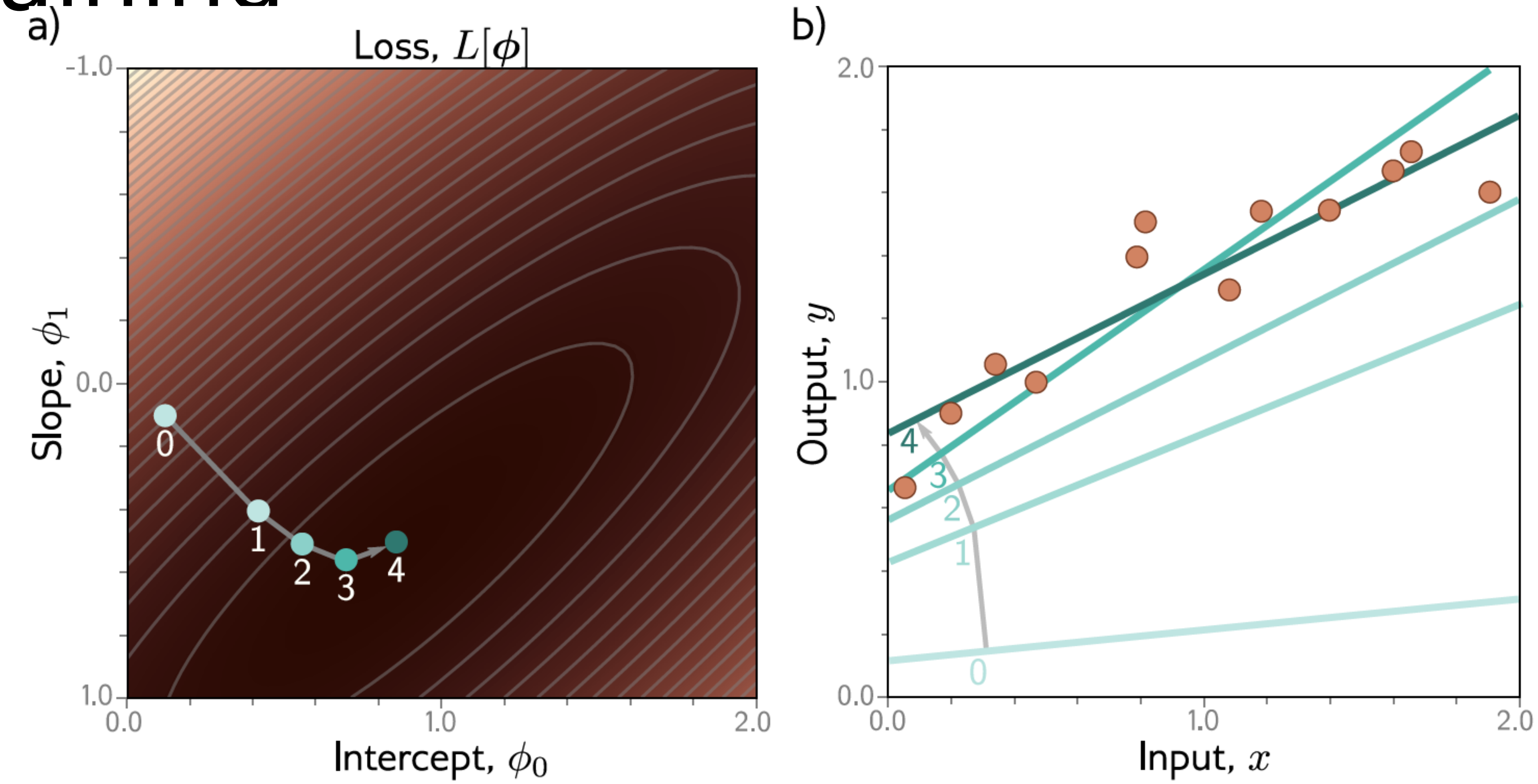
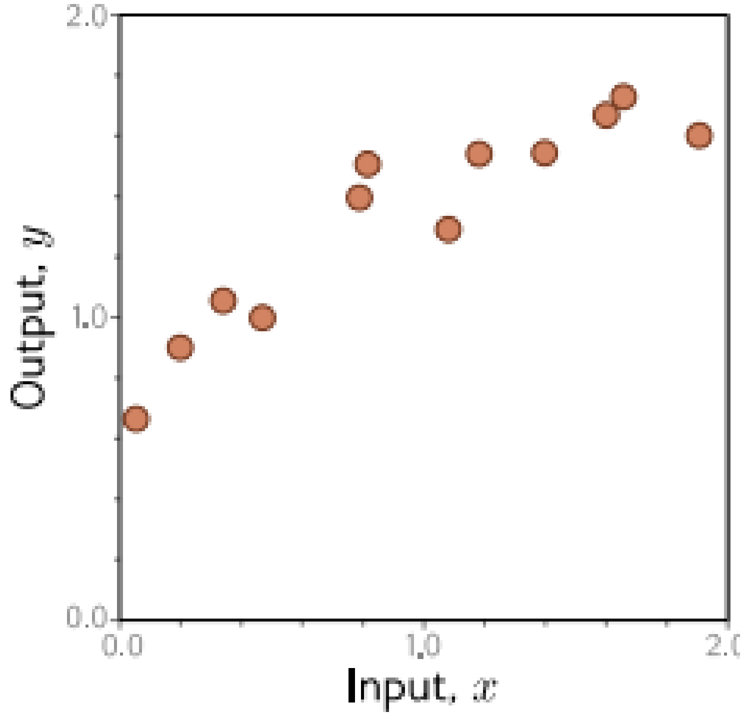


This technique is known as gradient descent

Example: 1D Linear regression testing

- Test with different set of paired input/output data
  - Measure performance
  - Degree to which this is same as training = generalization
- Might not generalize well because
  - Model too simple
  - Model too complex
    - fits to statistical peculiarities of data
    - this is known as overfitting



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- Shallow neural networks (a more flexible model)
- Deep neural networks (an even more flexible model)
- Loss functions (where did least squares come from?)
- How to train neural networks (gradient descent and variants)
- How to measure performance of neural networks (generalization)