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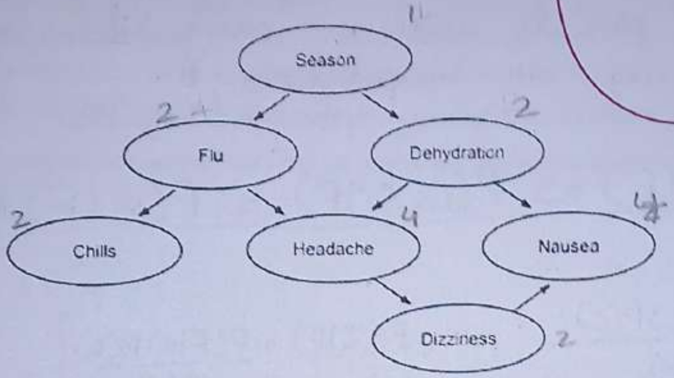
CS 452 – Probabilistic Graphical Models, Fall 2024

Quiz # 01

Total Points: 20

20/20

Q1 – [10 points] Given the network below:



a) How many probabilities do you need to know for a complete joint distribution (assuming that all nodes are binary)?

→ 7 nodes, all binary,

$2^7 - 1$

$[1 + 2 + 2 + 2 + 4 + 4 + 2 = 17]$

The total number of probabilities are: $2^7 - 1 = 127$

* My apologies, I realized what the qus was asking after reading the second part

b) How many probabilities (prior + conditional) do you need to specify for this Bayesian network?

The total number of probabilities are: 17

c) Apply chain rule to write the formula for computing joint probabilities for this Bayesian network?

$$P(S, F, Dgh, C, H, N, Di) = P(S)\,P(F|S)\,P(Deh|S)\,P(C|F)\,P(H|F, Deh) \times P(N|Deh, Di) \times P(Di|H)$$

d) How does Markov property simplify the computation of joint probabilities in a BN?

By the Markov property, a node is influenced ~~[illegible]~~ only by its previous state or only its immediate ~~[illegible]~~ parent(s). Therefore, we only need to know about the probability of a node given its immediate parents and not any other ancestors, thus minimizing our computation.

e) Which probability would you compute to verify that Chills ⊥ Season | Flu?

$$*P(C \perp S \mid F) \Rightarrow P(C, S \mid F) = P(C \mid F)\,P(S \mid F)$$

$$\left[P(C \mid F) = \frac{P(F \mid C)P(C)}{P(F)} \qquad P(S \mid F) = \frac{P(F \mid S)P(S)}{P(F)} \right]$$

f) Given sufficient data, Bayesian networks can easily be learned from past data. True/False? Reason.

False. Past Data tells us about the correlations between events, and not the cause-effect relations. Inferencing cause-effect relation is not always easy to learn from data which tells us about correlations.

g) The belief in a Bayesian network is propagated in a top-down manner i.e. from cause to effect. (True/False)? Reason.

It is propagated in a top-down manner, however, in some instances, having evidence about a child node influences our belief about its parents as well. Thus, it is not always the case.

Q2 – Consider the scenario below:

"Jason the Juggler"- Jason, the robot juggler, drops balls quite often when its battery is low. In previous trials, it has been determined that when its battery is low it will drop the ball 9 times out of 10. On the other hand, when its battery is not low, the chance that it drops a ball is much lower, about 1 in 100. The battery was recharged recently, so there is only a 5% chance that the battery is low. Another robot, Olga the observer, reports on whether or not Jason has dropped the ball. Unfortunately, Olga's vision system is somewhat unreliable. Though it accurately detects if the ball has been dropped, it sometimes (10%) mistakenly reports that the ball is dropped while it is not. Based on information from Olga, the task is to represent and draw inferences about whether the battery is low depending on how well Jason is juggling and what has Olga reported.

A) [05 points] Construct a Bayesian network to represent this scenario. Specify all CPTs.

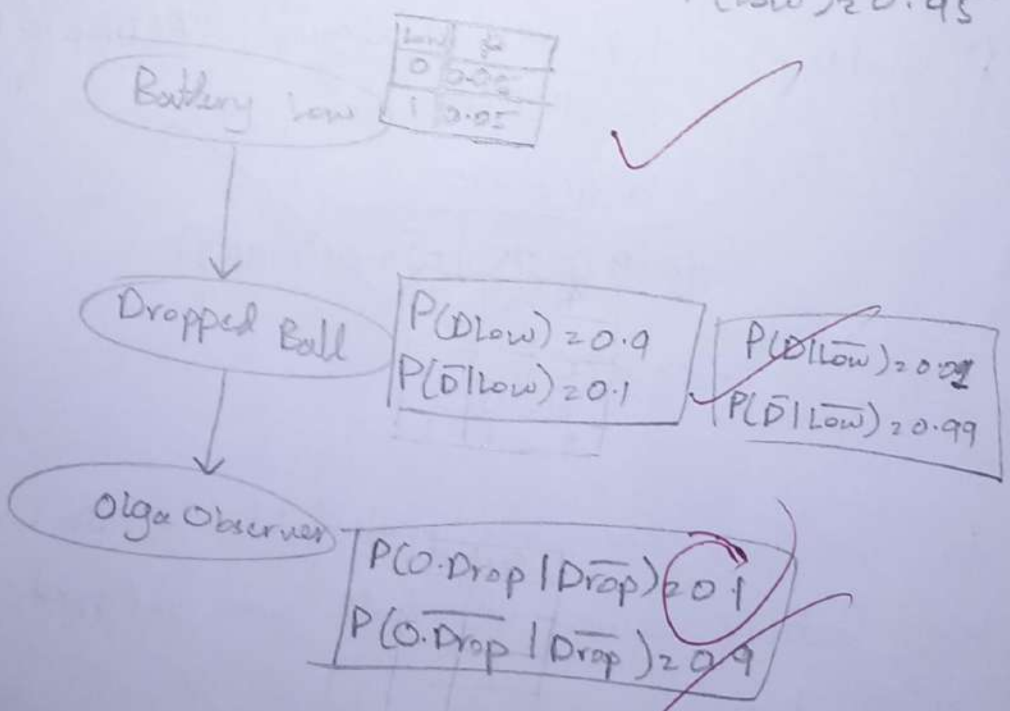
P(D | Low) = 0.9

P(D | $\overline{Low}$) = 0.01

P(O. Drop | $\overline{Drop}$) = 0.1

P(Low) = 0.05

P($\overline{Low}$) = 0.95



B) [05 points] Suppose that the ball has been dropped by the Jason. What effect does this have on your belief that the battery is low? Compute the posterior probability.

Since we know that the chance of dropping the ball given that the battery is low is quite high (9/10), then our belief in the low battery goes up if the ball has been dropped.

$$P(\text{Low}|\text{Drop}) = \frac{P(\text{Drop}|\text{Low})P(\text{Low})}{P(\text{Drop})}$$

$$P(\text{Drop}) = P(D|\text{Low})P(\text{Low}) + P(D|\overline{\text{Low}})P(\overline{\text{Low}})$$

$$= \frac{0.9(0.05)}{[0.9(0.05)] + [0.01(0.95)]}$$

$$= \underline{0.826}$$

As we can see our probability of having low battery given the ball was dropped is quite high.