Linear Algebra – Math 205
Exercise Set of Lect 20
(SPRING 2023)
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Homework 5a: Exercise Set 6.1 Solution

Question 01(a)

Solution: One can note that $v_3 = v_1 + v_2$, hence this is not linearly dependent vector. It does not span $\mathbb{R}^3$.

Question 01(b)

Solution:

- True, non trivial solution exists.
- True, since W is a subspace of V and it has same number of bases as V.
- True, Every finite-dimensional inner product space has an orthonormal basis, which may be obtained from an arbitrary basis using the Gram–Schmidt process.
- True.
- True.
- True.

Question 02

Solution: Echelon form of a matrix $\begin{bmatrix} 1 & -2 & 0 & 0 & 3 \\ 2 & -5 & -3 & -2 & 6 \\ 0 & 5 & 15 & 10 & 0 \\ 2 & 6 & 18 & 8 & 6 \end{bmatrix}$ is $\begin{bmatrix} 1 & -2 & 0 & 0 & 3 \\ 0 & 1 & 3 & 2 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.
Column space of a matrix is $\{\{1, 2, 0, 2\}, \{-2, -5, 5, 6\}, \{0, 3, 15, 18\}\}$
Row space of $\{\{1, -2, 0, 0, 3\}, \{0, 1, 3, 2, 0\}, \{0, 0, 1, 1, 0\}\}$

Question 03

Solution: $\sum_{i=1}^{n} = w_i u_i v_i$

Question 04

Solution: $\langle u, v\rangle = 3u_1v_1 + 2u_2v_2 = 3v_1u_1 + 2v_2u_2 = \langle v, u\rangle$
$\langle (u + v), w\rangle = 3(u_1 + v_1)w_1 + 2(u_2 + v_2)w_2 = 3u_1w_1 + 2u_2w_2 + 3v_1w_1 + 2v_2w_2 = \langle u, w\rangle + \langle v, w\rangle$
$\langle ku, v\rangle = 3ku_1v_1 + 2ku_2v_2 = k(3u_1v_1 + 2u_2v_2) = k\,\langle u, v\rangle$
$\langle u, u\rangle = 3u_1u_1 + 2u_2u_2 = 3u_1^2 + 2u_2^2 \geq 0$

Question 17

Solution: (a) $||f|| = \langle f, f\rangle^{1/2} = \sqrt{\int_a^b f^2(x)dx}$
Find $||p||$ when $p = 1, x$, and x^2
$||p|| = \langle p, p\rangle^{1/2} = \sqrt{\int_a^b p^2(x)dx} = \sqrt{\int_{-1}^1 1^2 dx} = \sqrt{2}$
$||p|| = \langle p, p\rangle^{1/2} = \sqrt{\int_a^b p^2(x)dx} = \sqrt{\int_{-1}^1 x^2 dx} = \sqrt{2/3}$
$||p|| = \langle p, p\rangle^{1/2} = \sqrt{\int_a^b p^2(x)dx} = \sqrt{\int_{-1}^1 (x^2)^2 dx} = \sqrt{2/5}$
(b) See the solution.

Question 20

Solution: $||u+v||+||u-v|| = [(u+v).(u+v)]^{1/2}+[(u-v).(u-v)]^{1/2} = 2||u||^2+2||v||^2$

Question 27

Solution: See solution manual.

Question 28

Solution: $\langle f, g\rangle = \int_0^1 \cos(2\pi x)\sin(2\pi x)dx = \int_0^1 \frac{1}{2}\sin(4\pi x)dx = -\frac{1}{8\pi}\cos(4\pi x)|_0^1$
$\langle f, g\rangle = \int_0^1 xe^x dx = (xe^x - e^x)|_0^1$