Lab_8_aa07190

November 8, 2024

1 CS 316 : Introduction to Deep Learning - Fall 2024

1.1 Lab 08 : Initialization and Tarde Off

1.1.1 Dr. Abdul Samad

2 Instructions

1. Make a copy of this notebook on google colab at start of the lab.
2. Please rename your notebook as *Lab_8_aa1234.ipynb* before starting the lab. Notebooks which do not follow appropriate naming convention will not be graded.
3. You have to submit this lab during the lab timings. You are allowed to submit till 11:59 PM on the day of your lab with a 30% penalty. No submissions will be accepted afterwards.
4. Use of AI is strictly prohibited. Anyone caught using Any AI tool during lab or while grading will be immediately reported to OCVS without any further discussion.
5. At the end of the lab, download the notebook (ipynb extension file) and upload it on canvas as a file. Submitting link to notebook or any other file will not be accepted.
6. Each task has points assigned to it. Total Lab is of 100 points.
7. Use of for loops is strictly prohibited.
8. For every theoretical question, there is a separate cell given at the end. You have to write your answer there.
9. If you have any questions, please feel free to reach out to the course instructor or RA.

2.1 Task Overview

In this lab we will first look at how initialization of hyperparamters affects the output of the model. Then we will look at Bias-Variance Trade Off. This Lab is going to be short. Work through the cells below, running each cell in turn. In various places you will see the words "TODO". Follow the instructions at these places and make predictions about what is going to happen or write code to complete the functions.

Let's start with importing Libraries first

[1]:
```
import numpy as np
import matplotlib.pyplot as plt
```

```python
from matplotlib import cm
from matplotlib.colors import ListedColormap
import math
```

3 Initialization

First let's define a neural network. We'll just choose the weights and biases randomly for now

[2]:
```python
def init_params(K, D, sigma_sq_omega):
  # Set seed so we always get the same random numbers
  np.random.seed(0)

  # Input layer
  D_i = 1
  # Output layer
  D_o = 1

  # Make empty lists
  all_weights = [None] * (K+1)
  all_biases = [None] * (K+1)

  # Create input and output layers
  all_weights[0] = np.random.normal(size=(D, D_i))*np.sqrt(sigma_sq_omega)
  all_weights[-1] = np.random.normal(size=(D_o, D)) * np.sqrt(sigma_sq_omega)
  all_biases[0] = np.zeros((D,1))
  all_biases[-1]= np.zeros((D_o,1))

  # Create intermediate layers
  for layer in range(1,K):
    all_weights[layer] = np.random.normal(size=(D,D))*np.sqrt(sigma_sq_omega)
    all_biases[layer] = np.zeros((D,1))

  return all_weights, all_biases
```

[3]:
```python
# Define the Rectified Linear Unit (ReLU) function
def ReLU(preactivation):
  activation = preactivation.clip(0.0)
  return activation
```

[4]:
```python
def compute_network_output(net_input, all_weights, all_biases):

  # Retrieve number of layers
  K = len(all_weights)-1

  # We'll store the pre-activations at each layer in a list "all_f"
  # and the activations in a second list "all_h".
  all_f = [None] * (K+1)
```

```
  all_h = [None] * (K+1)

  #For convenience, we'll set
  # all_h[0] to be the input, and all_f[K] will be the output
  all_h[0] = net_input

  # Run through the layers, calculating all_f[0...K-1] and all_h[1...K]
  for layer in range(K):
      # Update preactivations and activations at this layer according to eqn 7.5
      all_f[layer] = all_biases[layer] + np.matmul(all_weights[layer],␣
 ↪all_h[layer])
      all_h[layer+1] = ReLU(all_f[layer])

  # Compute the output from the last hidden layer
  all_f[K] = all_biases[K] + np.matmul(all_weights[K], all_h[K])

  # Retrieve the output
  net_output = all_f[K]

  return net_output, all_f, all_h
```

Now let's investigate how the size of the outputs vary as we change the initialization variance:

[5]:
```
# Number of layers
K = 5
# Number of neurons per layer
D = 8
# Input layer
D_i = 1
# Output layer
D_o = 1
# Set variance of initial weights to 1
sigma_sq_omega = 1.0
# Initialize parameters
all_weights, all_biases = init_params(K,D,sigma_sq_omega)

n_data = 1000
data_in = np.random.normal(size=(1,n_data))
net_output, all_f, all_h = compute_network_output(data_in, all_weights,␣
 ↪all_biases)

for layer in range(1,K+1):
  print("Layer %d, std of hidden units = %3.3f"%(layer, np.std(all_h[layer])))
```

```
Layer 1, std of hidden units = 0.811
Layer 2, std of hidden units = 1.472
Layer 3, std of hidden units = 4.547
Layer 4, std of hidden units = 8.896
```

```
Layer 5, std of hidden units = 10.106
```

4 Q1: TODO [20 Points]

You can see that the values of the hidden units are increasing on average (the variance is across all hidden units at the layer and the 1000 training examples).

Change this to 50 layers with 80 hidden units per layer. Then experiment with sigma_sq_omega to try to stop the variance of the forward computation exploding.

[20]:
```python
# TODO
# Do your experimentation in this tab only

# Number of layers
K_50 = 55
# Number of neurons per layer
D_80 = 80
# Input layer
D_i = 1
# Output layer
D_o = 1
# Set variance of initial weights to 1
sigma_sq_omega = 0.03
# Initialize parameters
all_weights, all_biases = init_params(K_50,D_80,sigma_sq_omega)

n_data = 1000
data_in = np.random.normal(size=(1,n_data))
net_output, all_f, all_h = compute_network_output(data_in, all_weights,
 ↪all_biases)

for layer in range(1,K_50+1):
  print("Layer %d, std of hidden units = %3.3f"%(layer, np.std(all_h[layer])))

# END TODO
```

```
Layer 1, std of hidden units = 0.112
Layer 2, std of hidden units = 0.097
Layer 3, std of hidden units = 0.114
Layer 4, std of hidden units = 0.151
Layer 5, std of hidden units = 0.183
Layer 6, std of hidden units = 0.177
Layer 7, std of hidden units = 0.181
Layer 8, std of hidden units = 0.205
Layer 9, std of hidden units = 0.195
Layer 10, std of hidden units = 0.186
Layer 11, std of hidden units = 0.215
Layer 12, std of hidden units = 0.207
```

```
Layer 13, std of hidden units = 0.221
Layer 14, std of hidden units = 0.243
Layer 15, std of hidden units = 0.261
Layer 16, std of hidden units = 0.293
Layer 17, std of hidden units = 0.342
Layer 18, std of hidden units = 0.402
Layer 19, std of hidden units = 0.432
Layer 20, std of hidden units = 0.614
Layer 21, std of hidden units = 0.567
Layer 22, std of hidden units = 0.623
Layer 23, std of hidden units = 0.719
Layer 24, std of hidden units = 0.900
Layer 25, std of hidden units = 1.079
Layer 26, std of hidden units = 1.121
Layer 27, std of hidden units = 1.050
Layer 28, std of hidden units = 1.114
Layer 29, std of hidden units = 1.517
Layer 30, std of hidden units = 1.902
Layer 31, std of hidden units = 1.740
Layer 32, std of hidden units = 1.349
Layer 33, std of hidden units = 1.400
Layer 34, std of hidden units = 1.528
Layer 35, std of hidden units = 1.731
Layer 36, std of hidden units = 1.696
Layer 37, std of hidden units = 2.036
Layer 38, std of hidden units = 1.913
Layer 39, std of hidden units = 2.093
Layer 40, std of hidden units = 2.331
Layer 41, std of hidden units = 2.751
Layer 42, std of hidden units = 3.300
Layer 43, std of hidden units = 3.348
Layer 44, std of hidden units = 3.889
Layer 45, std of hidden units = 5.047
Layer 46, std of hidden units = 5.259
Layer 47, std of hidden units = 6.105
Layer 48, std of hidden units = 5.640
Layer 49, std of hidden units = 5.716
Layer 50, std of hidden units = 6.637
Layer 51, std of hidden units = 8.194
Layer 52, std of hidden units = 8.022
Layer 53, std of hidden units = 8.311
Layer 54, std of hidden units = 9.306
Layer 55, std of hidden units = 10.993
```

Now let's define a loss function. We'll just use the least squares loss function. We'll also write a function to compute dloss_doutput

[21]:
```python
def least_squares_loss(net_output, y):
  return np.sum((net_output-y) * (net_output-y))

def d_loss_d_output(net_output, y):
    return 2*(net_output -y);
```

Here's the code for the backward pass

[22]:
```python
# We'll need the indicator function
def indicator_function(x):
  x_in = np.array(x)
  x_in[x_in>=0] = 1
  x_in[x_in<0] = 0
  return x_in

# Main backward pass routine
def backward_pass(all_weights, all_biases, all_f, all_h, y):
  # Retrieve number of layers
  K = len(all_weights) - 1

  # We'll store the derivatives dl_dweights and dl_dbiases in lists as well
  all_dl_dweights = [None] * (K+1)
  all_dl_dbiases = [None] * (K+1)
  # And we'll store the derivatives of the loss with respect to the activation␣
↪and preactivations in lists
  all_dl_df = [None] * (K+1)
  all_dl_dh = [None] * (K+1)
  # Again for convenience we'll stick with the convention that all_h[0] is the␣
↪net input and all_f[k] in the net output

  # Compute derivatives of net output with respect to loss
  all_dl_df[K] = np.array(d_loss_d_output(all_f[K],y))

  # Now work backwards through the network
  for layer in range(K,-1,-1):
    # Calculate the derivatives of biases at layer from all_dl_df[K]. (eq 7.13,␣
↪line 1)
    all_dl_dbiases[layer] = np.array(all_dl_df[layer])
    # Calculate the derivatives of weight at layer from all_dl_df[K] and␣
↪all_h[K] (eq 7.13, line 2)
    all_dl_dweights[layer] = np.matmul(all_dl_df[layer], all_h[layer].
↪transpose())

    # Calculate the derivatives of activations from weight and derivatives of␣
↪next preactivations (eq 7.13, line 3 second part)
    all_dl_dh[layer] = np.matmul(all_weights[layer].transpose(),␣
↪all_dl_df[layer])
```

```
    # Calculate the derivatives of the pre-activation f with respect to␣
↪activation h (eq 7.13, line 3, first part)
    if layer > 0:
      all_dl_df[layer-1] = indicator_function(all_f[layer-1]) * all_dl_dh[layer]

  return all_dl_dweights, all_dl_dbiases, all_dl_dh, all_dl_df
```

Now let's look at what happens to the magnitude of the gradients on the way back.

[23]:
```
# Number of layers
K = 5
# Number of neurons per layer
D = 8
# Input layer
D_i = 1
# Output layer
D_o = 1
# Set variance of initial weights to 1
sigma_sq_omega = 1.0
# Initialize parameters
all_weights, all_biases = init_params(K,D,sigma_sq_omega)

# For simplicity we'll just consider the gradients of the weights and biases␣
↪between the first and last hidden layer
n_data = 100
aggregate_dl_df = [None] * (K+1)
for layer in range(1,K):
  # These 3D arrays will store the gradients for every data point
  aggregate_dl_df[layer] = np.zeros((D,n_data))


# We'll have to compute the derivatives of the parameters for each data point␣
↪separately
for c_data in range(n_data):
  data_in = np.random.normal(size=(1,1))
  y = np.zeros((1,1))
  net_output, all_f, all_h = compute_network_output(data_in, all_weights,␣
↪all_biases)
  all_dl_dweights, all_dl_dbiases, all_dl_dh, all_dl_df =␣
↪backward_pass(all_weights, all_biases, all_f, all_h, y)
  for layer in range(1,K):
    aggregate_dl_df[layer][:,c_data] = np.squeeze(all_dl_df[layer])

for layer in range(1,K):
  print("Layer %d, std of dl_dh = %3.3f"%(layer, np.std(aggregate_dl_df[layer].
↪ravel())))
```

```
Layer 1, std of dl_dh = 446.654
```

```
Layer 2, std of dl_dh = 340.657
Layer 3, std of dl_dh = 109.132
Layer 4, std of dl_dh = 56.472
```

5 Q2: TODO [20 Points]

You can see that the gradients of the hidden units are increasing on average (the standard deviation is across all hidden units at the layer and the 100 training examples).

Change this to 50 layers with 80 hidden units per layer. Then experiment with sigma_sq_omega to try to stop the variance of the gradients.

[33]:
```
# TODO
# Do your experimentation in this tab only



# Number of layers
K_50_2 = 50
# Number of neurons per layer
D_80_2 = 80
# Input layer
D_i = 1
# Output layer
D_o = 1
# Set variance of initial weights to 1
sigma_sq_omega = 0.03
# Initialize parameters
all_weights, all_biases = init_params(K_50_2,D_80_2,sigma_sq_omega)

# For simplicity we'll just consider the gradients of the weights and biases␣
 ↪between the first and last hidden layer
n_data = 100
aggregate_dl_df = [None] * (K_50_2+1)
for layer in range(1,K_50_2):
  # These 3D arrays will store the gradients for every data point
  aggregate_dl_df[layer] = np.zeros((D_80_2,n_data))


# We'll have to compute the derivatives of the parameters for each data point␣
 ↪separately
for c_data in range(n_data):
  data_in = np.random.normal(size=(1,1))
  y = np.zeros((1,1))
  net_output, all_f, all_h = compute_network_output(data_in, all_weights,␣
 ↪all_biases)
  all_dl_dweights, all_dl_dbiases, all_dl_dh, all_dl_df =␣
 ↪backward_pass(all_weights, all_biases, all_f, all_h, y)
```

```
  for layer in range(1,K_50_2):
    aggregate_dl_df[layer][:,c_data] = np.squeeze(all_dl_df[layer])

for layer in range(1,K_50_2):
  print("Layer %d, std of dl_dh = %3.3f"%(layer, np.std(aggregate_dl_df[layer].
 ↪ravel())))

# END TODO
```

```
Layer 1, std of dl_dh = 277.407
Layer 2, std of dl_dh = 267.048
Layer 3, std of dl_dh = 278.201
Layer 4, std of dl_dh = 248.859
Layer 5, std of dl_dh = 211.970
Layer 6, std of dl_dh = 201.306
Layer 7, std of dl_dh = 192.209
Layer 8, std of dl_dh = 157.027
Layer 9, std of dl_dh = 143.825
Layer 10, std of dl_dh = 149.252
Layer 11, std of dl_dh = 138.974
Layer 12, std of dl_dh = 127.547
Layer 13, std of dl_dh = 106.351
Layer 14, std of dl_dh = 100.830
Layer 15, std of dl_dh = 96.427
Layer 16, std of dl_dh = 80.132
Layer 17, std of dl_dh = 62.544
Layer 18, std of dl_dh = 57.883
Layer 19, std of dl_dh = 47.348
Layer 20, std of dl_dh = 53.106
Layer 21, std of dl_dh = 44.569
Layer 22, std of dl_dh = 36.050
Layer 23, std of dl_dh = 34.043
Layer 24, std of dl_dh = 27.309
Layer 25, std of dl_dh = 25.572
Layer 26, std of dl_dh = 32.532
Layer 27, std of dl_dh = 24.597
Layer 28, std of dl_dh = 24.874
Layer 29, std of dl_dh = 24.549
Layer 30, std of dl_dh = 24.231
Layer 31, std of dl_dh = 22.683
Layer 32, std of dl_dh = 20.011
Layer 33, std of dl_dh = 19.320
Layer 34, std of dl_dh = 19.856
Layer 35, std of dl_dh = 16.891
Layer 36, std of dl_dh = 15.619
Layer 37, std of dl_dh = 12.557
Layer 38, std of dl_dh = 11.054
Layer 39, std of dl_dh = 9.027
```

```
Layer 40, std of dl_dh = 9.209
Layer 41, std of dl_dh = 8.328
Layer 42, std of dl_dh = 9.168
Layer 43, std of dl_dh = 9.171
Layer 44, std of dl_dh = 9.173
Layer 45, std of dl_dh = 9.036
Layer 46, std of dl_dh = 7.908
Layer 47, std of dl_dh = 7.366
Layer 48, std of dl_dh = 6.169
Layer 49, std of dl_dh = 4.783
```

6 Bias-Variance Trade-Off

There are three possible sources of error, which are known as noise, bias, and variance respectively. For a fixed-size training dataset, the variance term typically increases as the model capacity increases (by model capacity it means hyperparamters i.e. number of layers and hidden units). Consequently, increasing the model capacity does not necessarily reduce the test error. This is known as the bias-variance trade-off.

[34]:
```
# The true function that we are trying to estimate, defined on [0,1]
def true_function(x):
    y = np.exp(np.sin(x*(2*3.1413)))
    return y
```

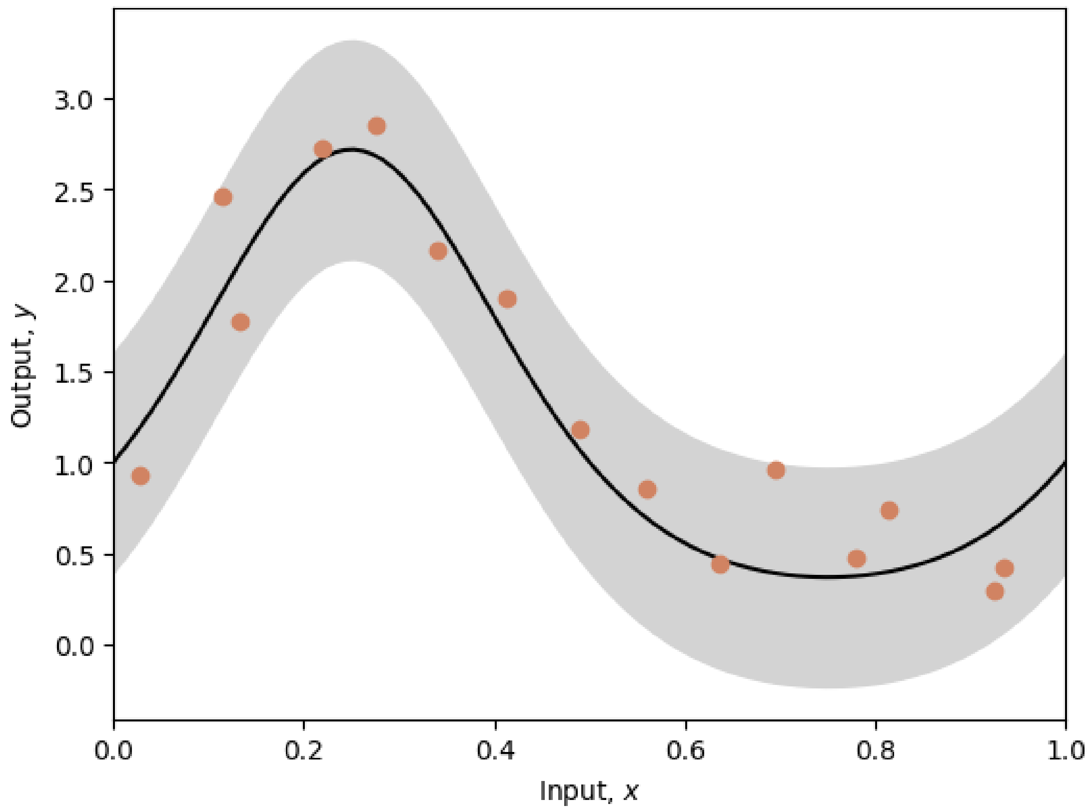
[35]:
```
# Generate some data points with or without noise
def generate_data(n_data, sigma_y=0.3):
    # Generate x values quasi uniformly
    x = np.ones(n_data)
    for i in range(n_data):
        x[i] = np.random.uniform(i/n_data, (i+1)/n_data, 1)

    # y value from running through function and adding noise
    y = np.ones(n_data)
    for i in range(n_data):
        y[i] = true_function(x[i])
        y[i] += np.random.normal(0, sigma_y, 1)
    return x,y
```

[36]:
```
# Draw the fitted function, together with uncertainty used to generate points
def plot_function(x_func, y_func, x_data=None,y_data=None, x_model = None,␣
 ↪y_model =None, sigma_func = None, sigma_model=None):

    fig,ax = plt.subplots()
    ax.plot(x_func, y_func, 'k-')
    if sigma_func is not None:
      ax.fill_between(x_func, y_func-2*sigma_func, y_func+2*sigma_func,␣
 ↪color='lightgray')
```

```
    if x_data is not None:
        ax.plot(x_data, y_data, 'o', color='#d18362')

    if x_model is not None:
        ax.plot(x_model, y_model, '-', color='#7fe7de')

    if sigma_model is not None:
      ax.fill_between(x_model, y_model-2*sigma_model, y_model+2*sigma_model,␣
↪color='lightgray')

    ax.set_xlim(0,1)
    ax.set_xlabel('Input, $x$')
    ax.set_ylabel('Output, $y$')
    plt.show()
```

[37]:
```
# Generate true function
x_func = np.linspace(0, 1.0, 100)
y_func = true_function(x_func);

# Generate some data points
np.random.seed(1)
sigma_func = 0.3
n_data = 15
x_data,y_data = generate_data(n_data, sigma_func)

# Plot the functinon, data and uncertainty
plot_function(x_func, y_func, x_data, y_data, sigma_func=sigma_func)
```

```
/tmp/ipykernel_6630/1040896329.py:6: DeprecationWarning: Conversion of an array
with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you
extract a single element from your array before performing this operation.
(Deprecated NumPy 1.25.)
  x[i] = np.random.uniform(i/n_data, (i+1)/n_data, 1)
/tmp/ipykernel_6630/1040896329.py:12: DeprecationWarning: Conversion of an array
with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you
extract a single element from your array before performing this operation.
(Deprecated NumPy 1.25.)
  y[i] += np.random.normal(0, sigma_y, 1)
```



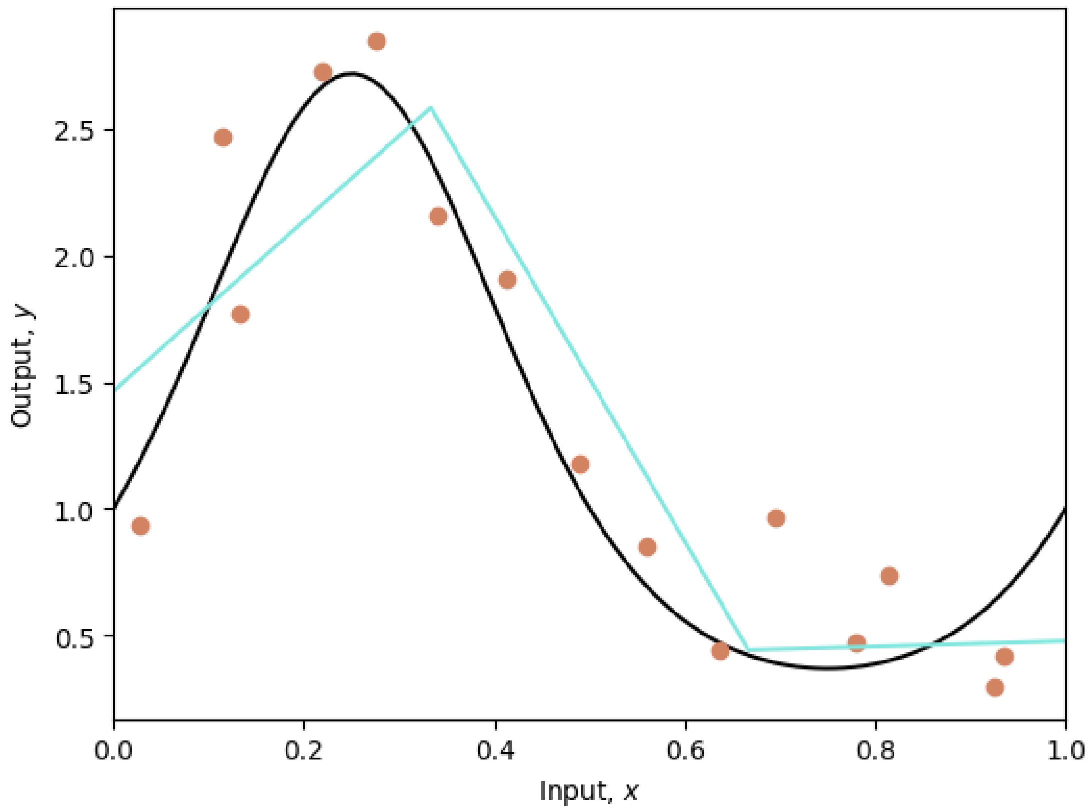
This figure will be used for below given cell.

[38]:
```
# Define model -- beta is a scalar and omega has size n_hidden,1
def network(x, beta, omega):
    # Retrieve number of hidden units
    n_hidden = omega.shape[0]

    y = np.zeros_like(x)
    for c_hidden in range(n_hidden):
        # Evaluate activations based on shifted lines (figure 1, b-d)
        line_vals =  x  - c_hidden/n_hidden
        h =  line_vals * (line_vals > 0)
        # Weight activations by omega parameters and sum
        y = y + omega[c_hidden] * h
    # Add bias, beta
    y = y + beta

    return y
```

Below given function gives you best possible parameters for beta and omega. hence, you can assume that your model is already trained well. You can utlize time outside the lab to figure out how it is doing that. Maybe some of the functions used here can help you later.

[39]:
```python
# This fits the n_hidden+1 parameters (see figure 1) in closed form.
# If you have studied linear algebra, then you will know it is a least
# squares solution of the form (A^TA)^-1A^Tb.  If you don't recognize that,
# then just take it on trust that this gives you the best possible solution.
def fit_model_closed_form(x,y,n_hidden):
  n_data = len(x)
  A = np.ones((n_data, n_hidden+1))
  for i in range(n_data):
      for j in range(1,n_hidden+1):
          A[i,j] = x[i]-(j-1)/n_hidden
          if A[i,j] < 0:
              A[i,j] = 0;

  beta_omega = np.linalg.lstsq(A, y, rcond=None)[0]

  beta = beta_omega[0]
  omega = beta_omega[1:]

  return beta, omega
```

[40]:
```python
# Closed form solution
beta, omega = fit_model_closed_form(x_data,y_data,n_hidden=3)

# Get prediction for model across graph range
x_model = np.linspace(0,1,100);
y_model = network(x_model, beta, omega)

# Draw the function and the model
plot_function(x_func, y_func, x_data,y_data, x_model, y_model)
```



7 Q3: TODO [30 Points]

In the function given below, complete the code by following these steps.

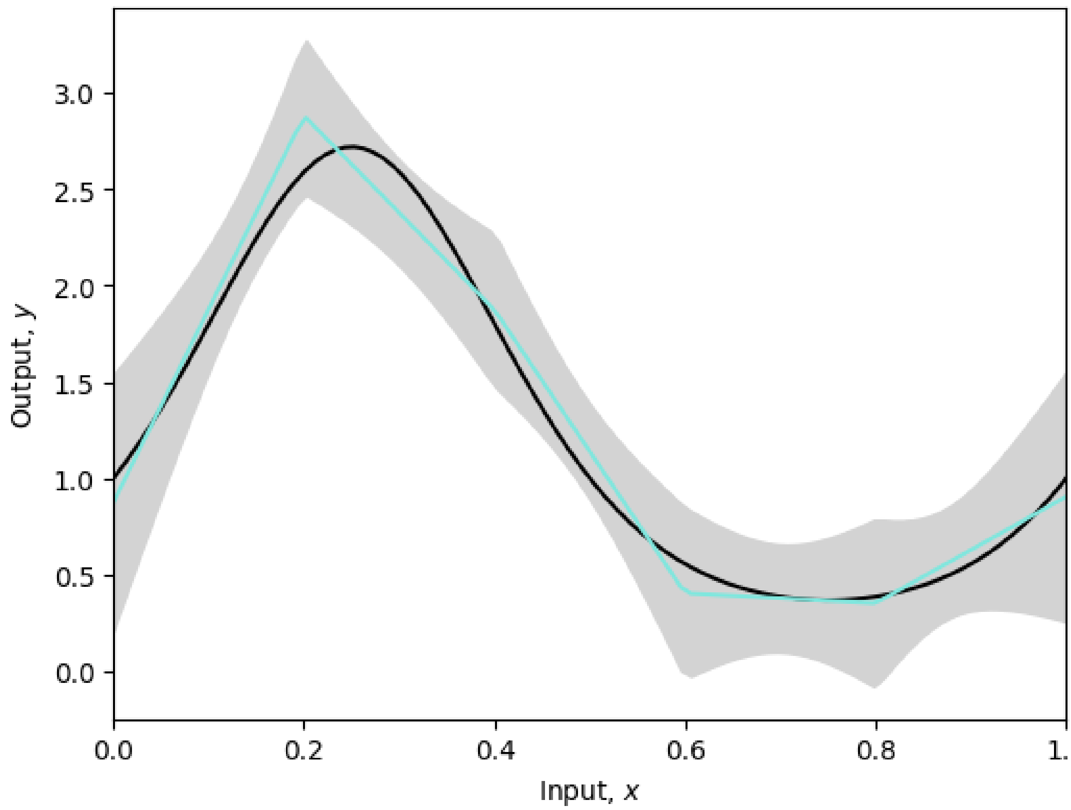
```
Generate  x_data,y_data pairs using generate data function with standard deviation sigma_func a
Fit the model using fit_model_closed_form function
Run the fitted model on x_model and store it in y_model variable
```

[41]:
```
# Run the model many times with different datasets and return the mean and␣
 ↪variance
def get_model_mean_variance(n_data, n_datasets, n_hidden, sigma_func):

  # Create array that stores model results in rows
  y_model_all = np.zeros((n_datasets, x_model.shape[0]))

  for c_dataset in range(n_datasets):
    # TODO
    x_data, y_data = generate_data(n_data, sigma_func)
    beta_update, omega_update = fit_model_closed_form(x_data, y_data, n_hidden)
    y_model = network(x_model, beta_update, omega_update)
```

```
    # END TODO

    # Store the model results
    y_model_all[c_dataset,:] = y_model

  # Get mean and standard deviation of model
  mean_model = np.mean(y_model_all,axis=0)
  std_model = np.std(y_model_all,axis=0)

  # Return the mean and standard deviation of the fitted model
  return mean_model, std_model
```

[42]:
```
# Let's generate N random data sets, fit the model N times and look the mean
 ↪and variance
n_datasets = 100
n_data = 15
sigma_func = 0.3
n_hidden = 5

# Get mean and variance of fitted model
np.random.seed(1)
# Based on the parameters given, this function will generate 100 datasets where
 ↪each dataset will have 15
# data points and 5 hidden units. It then returns mean of those models and std
 ↪deviation as well.
mean_model, std_model = get_model_mean_variance(n_data, n_datasets, n_hidden,
 ↪sigma_func) ;

# Plot the results
plot_function(x_func, y_func, x_model=x_model, y_model=mean_model,
 ↪sigma_model=std_model)
```

```
/tmp/ipykernel_6630/1040896329.py:6: DeprecationWarning: Conversion of an array
with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you
extract a single element from your array before performing this operation.
(Deprecated NumPy 1.25.)
  x[i] = np.random.uniform(i/n_data, (i+1)/n_data, 1)
/tmp/ipykernel_6630/1040896329.py:12: DeprecationWarning: Conversion of an array
with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you
extract a single element from your array before performing this operation.
(Deprecated NumPy 1.25.)
  y[i] += np.random.normal(0, sigma_y, 1)
```



If code for previous task is correct then your graph will look like this.

8 Q4: TODO [10 Points]

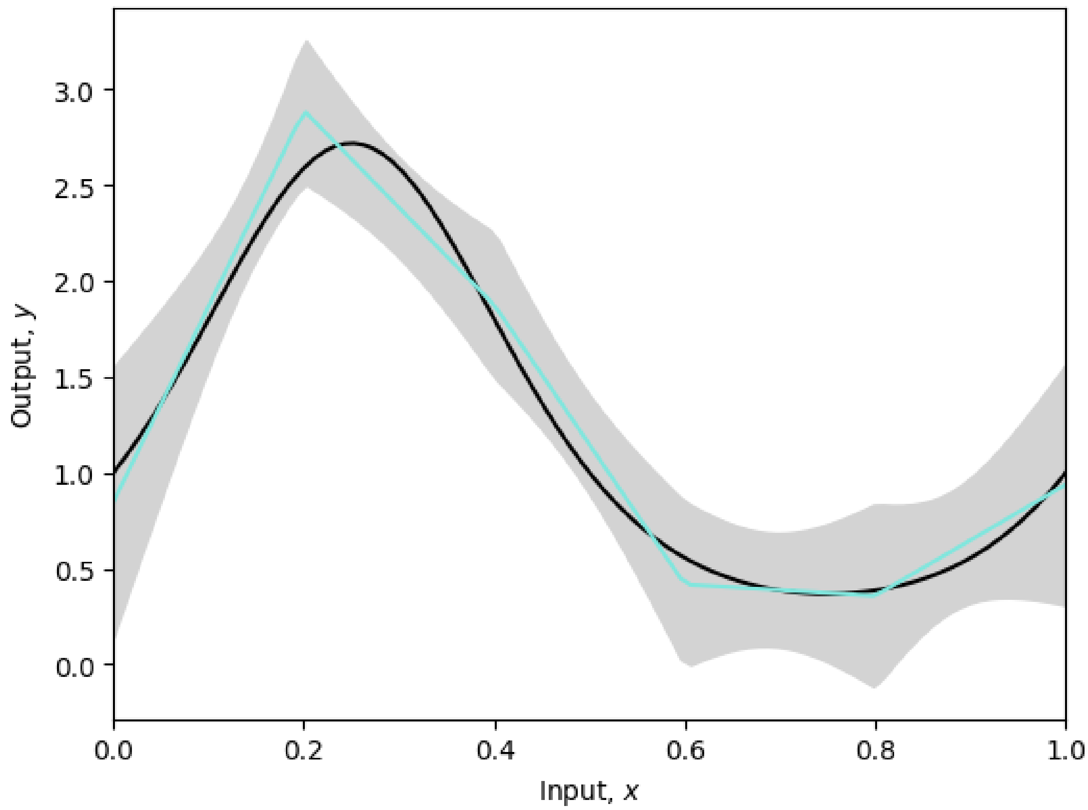
Experiment with changing the number of data points and the number of hidden variables in the model. Get a feeling for what happens in terms of the bias (squared deviation between cyan and black lines) and the variance (gray region) as we manipulate these quantities.

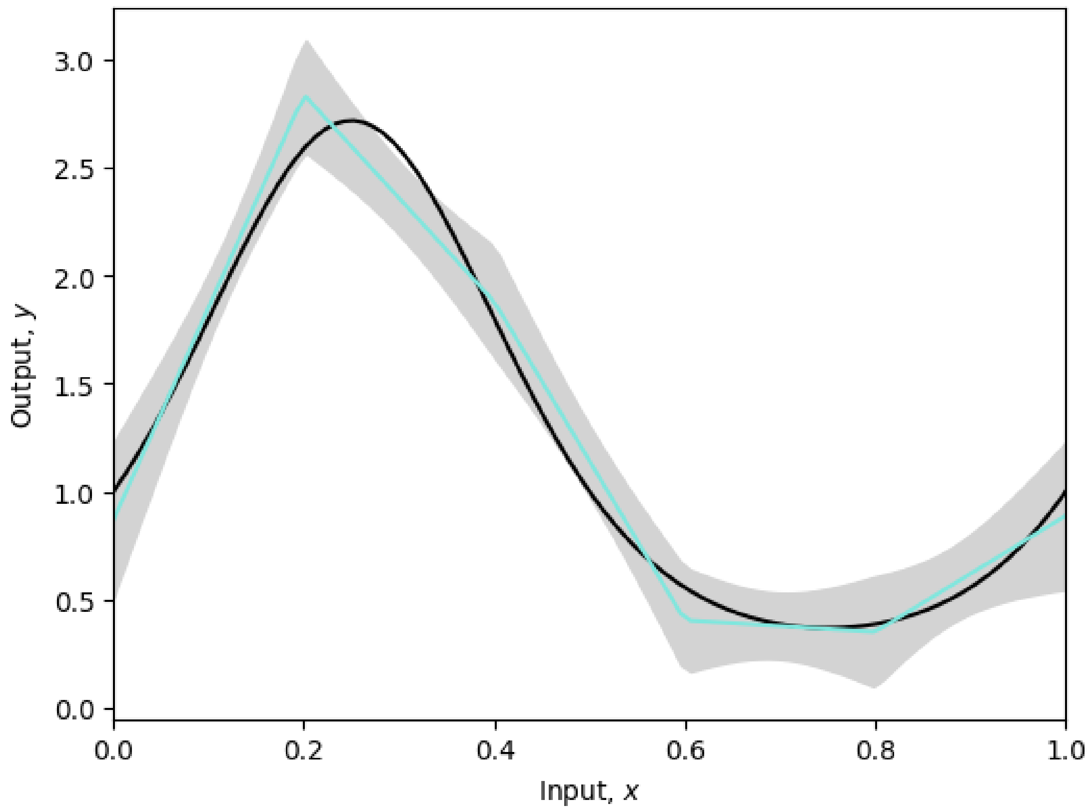
[58]:
```
# TODO
# Do your experimentation in this tab only
# Let's generate N random data sets, fit the model N times and look the mean␣
↪and variance
# Change datasets from 100 to 50
n_datasets = 50
n_data = 15
sigma_func = 0.3
n_hidden = 5

# Get mean and variance of fitted model
np.random.seed(1)
```

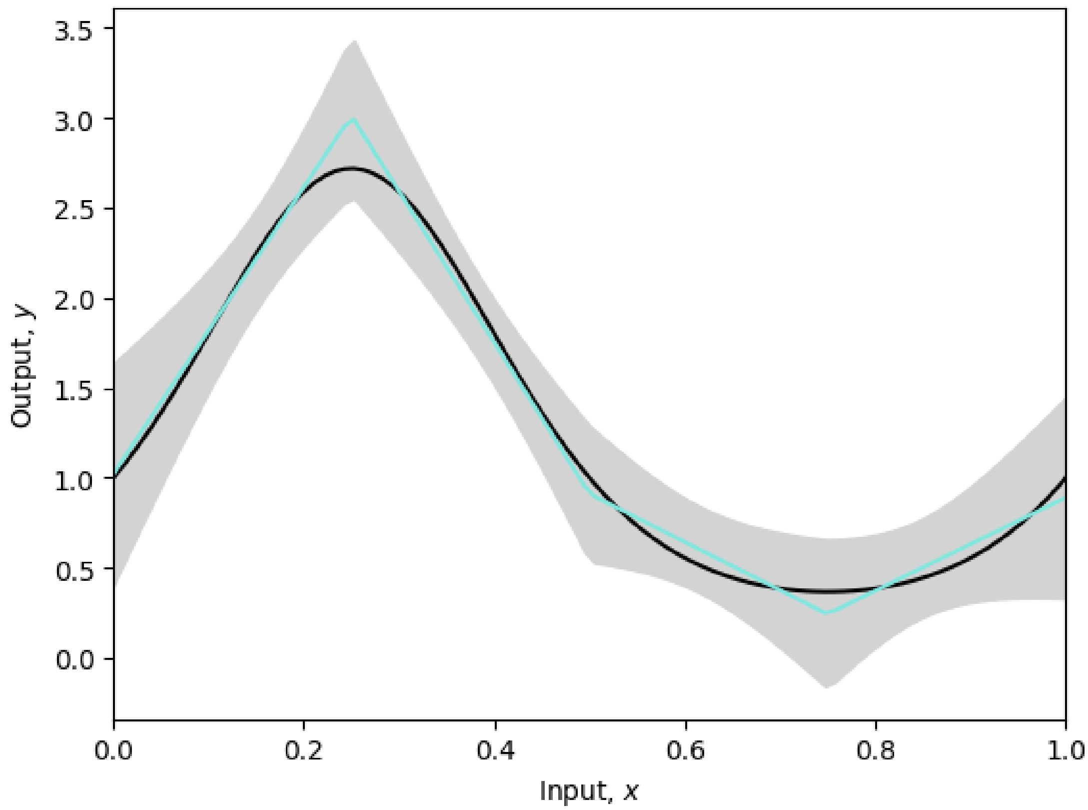
```
# Based on the parameters given, this function will generate 100 datasets where␣
↪each dataset will have 15
# data points and 5 hidden units. It then returns mean of those models and std␣
↪deviation as well.
mean_model, std_model = get_model_mean_variance(n_data, n_datasets, n_hidden,␣
↪sigma_func) ;

# Plot the results
plot_function(x_func, y_func, x_model=x_model, y_model=mean_model,␣
↪sigma_model=std_model)

#-----------------------------------------------------------------------------------------------------

# Let's generate N random data sets, fit the model N times and look the mean␣
↪and variance
# Chang n_data from 15 to 30
n_datasets = 100
n_data = 50
sigma_func = 0.3
n_hidden = 5

# Get mean and variance of fitted model
np.random.seed(1)
# Based on the parameters given, this function will generate 100 datasets where␣
↪each dataset will have 15
# data points and 5 hidden units. It then returns mean of those models and std␣
↪deviation as well.
mean_model, std_model = get_model_mean_variance(n_data, n_datasets, n_hidden,␣
↪sigma_func) ;

# Plot the results
plot_function(x_func, y_func, x_model=x_model, y_model=mean_model,␣
↪sigma_model=std_model)

#-----------------------------------------------------------------------------------------------------

# Let's generate N random data sets, fit the model N times and look the mean␣
↪and variance
# Change n_hidden from 5 to increasing and decreasing values
n_datasets = 100
n_data = 15
sigma_func = 0.3
n_hidden = 4

# Get mean and variance of fitted model
np.random.seed(1)
```

```python
# Based on the parameters given, this function will generate 100 datasets where␣
↪each dataset will have 15
# data points and 5 hidden units. It then returns mean of those models and std␣
↪deviation as well.
mean_model, std_model = get_model_mean_variance(n_data, n_datasets, n_hidden,␣
↪sigma_func) ;

# Plot the results
plot_function(x_func, y_func, x_model=x_model, y_model=mean_model,␣
↪sigma_model=std_model)



# END TODO
```

```
/tmp/ipykernel_6630/1040896329.py:6: DeprecationWarning: Conversion of an array
with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you
extract a single element from your array before performing this operation.
(Deprecated NumPy 1.25.)
  x[i] = np.random.uniform(i/n_data, (i+1)/n_data, 1)
/tmp/ipykernel_6630/1040896329.py:12: DeprecationWarning: Conversion of an array
with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you
extract a single element from your array before performing this operation.
(Deprecated NumPy 1.25.)
  y[i] += np.random.normal(0, sigma_y, 1)
```


3.0
2.5
2.0
1.5
1.0
0.5
0.0
Output, y
0.0
0.2
0.4
0.6
0.8
1.0
Input, x


Output, y
Input, x
0.0
0.5
1.0
1.5
2.0
2.5
3.0
0.2
0.4
0.6
0.8



9 Q5: TODO [20 Points]

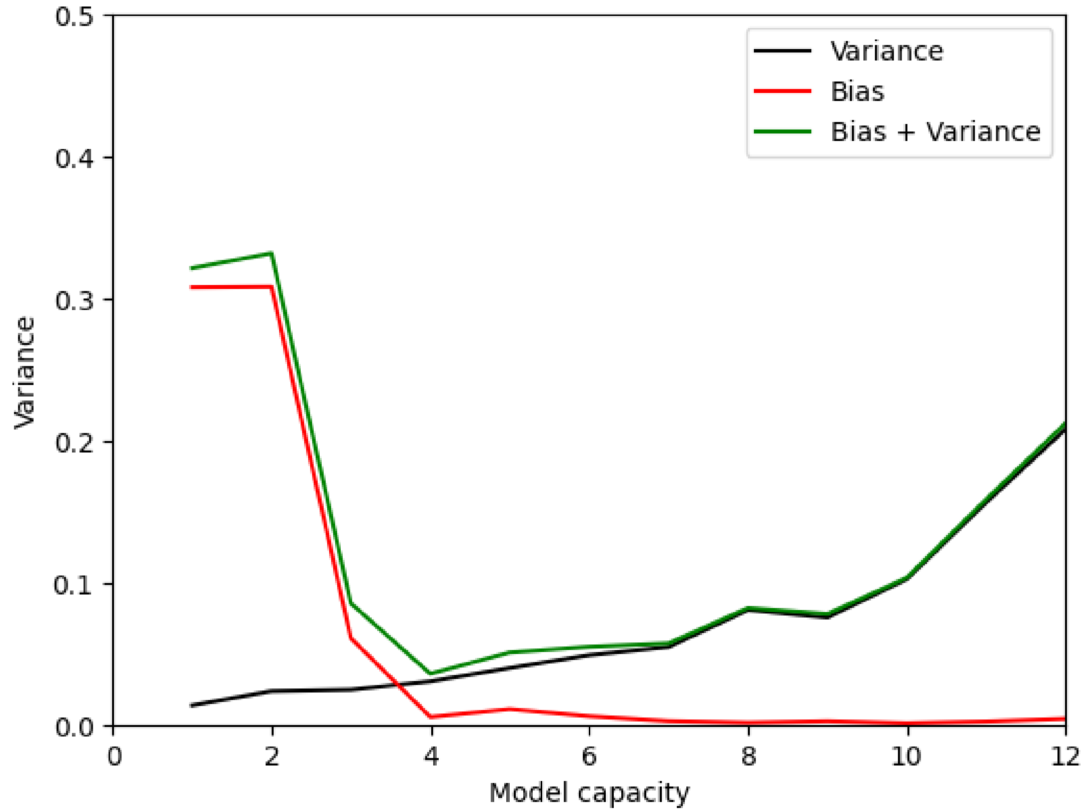
In the function given below, complete the code by Estimating bias and variance.

```
Compute variance by taking average of the model variance (as variance = std^2, we will take squ
Compute bias by taking average squared deviation of mean fitted model around true function (tru
stored in y_func variable. deviation here simply means how much it is far away from the other v
```

[59]:
```
# In previous case, we generated 100 datasets of 15 data points. We will do the␣
 ↪same except this time we
# will vary number of hidden neurons in them from 1-12 which was fixed to 5␣
 ↪previously.

# Plot the noise, bias and variance as a function of capacity
hidden_variables = [1,2,3,4,5,6,7,8,9,10,11,12]
bias = np.zeros((len(hidden_variables),1)) ;
variance = np.zeros((len(hidden_variables),1)) ;

n_datasets = 100
n_data = 15
sigma_func = 0.3
```

```python
# Set random seed so that we get the same result every time
np.random.seed(1)

for c_hidden in range(len(hidden_variables)):
  # Get mean and variance of fitted model
  mean_model, std_model = get_model_mean_variance(n_data, n_datasets,␣
 ↪hidden_variables[c_hidden], sigma_func) ;
  # TODO
  # compute variance by taking average of the model variance (as variance =␣
 ↪std^2)
  variance[c_hidden] = np.mean(std_model**2)

  # compute bias by taking average squared deviation of mean fitted model␣
 ↪around true function
  bias[c_hidden] = np.mean((mean_model - true_function(x_model))**2)

  # END TODO

# Plot the results
fig,ax = plt.subplots()
ax.plot(hidden_variables, variance, 'k-')
ax.plot(hidden_variables, bias, 'r-')
ax.plot(hidden_variables, variance+bias, 'g-')
ax.set_xlim(0,12)
ax.set_ylim(0,0.5)
ax.set_xlabel("Model capacity")
ax.set_ylabel("Variance")
ax.legend(['Variance', 'Bias', 'Bias + Variance'])
plt.show()
```

```
/tmp/ipykernel_6630/1040896329.py:6: DeprecationWarning: Conversion of an array
with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you
extract a single element from your array before performing this operation.
(Deprecated NumPy 1.25.)
  x[i] = np.random.uniform(i/n_data, (i+1)/n_data, 1)
/tmp/ipykernel_6630/1040896329.py:12: DeprecationWarning: Conversion of an array
with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you
extract a single element from your array before performing this operation.
(Deprecated NumPy 1.25.)
  y[i] += np.random.normal(0, sigma_y, 1)
```



If code for previous task is correct then your graph will look like this.