Note: In the previous solution, question 26, 27 28's solution not correct. So, it is updated in this version.

Exercise Set 5.1 Solution

Question 06

The set of all pairs of real numbers of the form (x, y) , where $x \geq 0$, with the standard operations on R^2.
Solution: It does not satisfy the axiom 5 of vector space. See the previous solution.

Question 11

The set of all real-valued functions f defined everywhere on the real line and such that $f(1) = 0$, with the operations defined in Example 4.
Solution: It satisfies all axioms of vector space, one can consider $f(x) = x - 1$ which satisfies $f(1) = 0$. See the previous solution.

Question 26

It was shown in Exercise 14 above that the set of polynomials of degree 1 or less is a vector space under the operations stated in that exercise. Is the set of polynomials whose degree is exactly 1 a vector space under those operations? Explain your reasoning.
Solution: It does not contain zero vector hence it is not vector space, i,e. $x - x = 0$

Question 27

Consider the set whose only element is the *moon*. Is this set a vector space under the operations $moon + moon = moon$ and $k(moon) = moon$ for every real number k? Explain your reasoning.
Solution: Yes, *moon* has to be the zero element because $moon + moon = moon$ means that '*moon*' fulfills the definition of the additive identity.

Question 28

Do you think that it is possible to have a vector space with exactly two distinct vectors in it? Explain your reasoning
Solution: No, the fact that all scalar multiples are included means that it has infinite vectors.