Solution

Name: ______________________________ ID: __________ Section: L1

This quiz is based on multiple choice questions as well as 1 problem solving questions.

For the MCQs, state the correct answer option for each question in the space provided.

Q .1: [15 marks] Consider the situation where we have two items indexed by $|0\rangle$ and $|1\rangle$. Let us suppose that we are "searching" for our required element (indexed by $|1\rangle$) within these two elements only and apply Grover's Search Algorithm.

(a) [3 marks] State what will be the Grover's Diffusion Operator as a 2x2 matrix.

It will be the X operator, i.e. $X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

(b) [3 marks] Starting with a uniform superposition, write the result of applying the oracle and the Grover Diffusion Operator on the qubit in the following table:

Input Qubit	Iteration	Operation	Resultant Qubit
$\frac{1}{\sqrt{2}}(\|0\rangle + \|1\rangle)$	1	Oracle 'O' (reverses sign of the required qubit)	$\frac{1}{\sqrt{2}}(\|0\rangle - \|1\rangle)$
$\frac{1}{\sqrt{2}}(\|0\rangle - \|1\rangle)$	1	Grover Diffusion Operator found in (a)	$\frac{1}{\sqrt{2}}(\|1\rangle - \|0\rangle) = -\frac{1}{\sqrt{2}}(\|0\rangle - \|1\rangle)$
$-\frac{1}{\sqrt{2}}(\|0\rangle - \|1\rangle)$	2	Oracle 'O' (reverses sign of the required qubit)	$-\frac{1}{\sqrt{2}}(\|0\rangle + \|1\rangle)$ **(Note: The global phase can be ignored, its not necessary)**
$-\frac{1}{\sqrt{2}}(\|0\rangle + \|1\rangle)$	2	Grover Diffusion Operator found in (a)	$-\frac{1}{\sqrt{2}}(\|0\rangle - \|1\rangle)$

(c) [3 marks] What are the probabilities of measuring the required state at the end of each iteration? State the probabilities here:

At the end of iteration	1	2
Probability of measuring $\|1\rangle$ is:	**1/2**	**1/2**

(d) [3 marks] What can you conclude from this analysis, i.e., is it a good idea to apply Grover's search algorithm when there is only one qubit (in terms of probability)? Give an 'Yes' or 'No' and Justify your answer.

No, since there is no effect on the amplitude; the Grover's Algorithm effectively acts as cyclic,
i.e. keeps repeating the same operations without affecting the probability.

(e) [3 marks] Suppose that you have '$N = 2^n$' qubits with the required qubits being half of them i.e., items indexed by N / 2 qubits are required items. Is the above analysis and conclusion applicable in this case, i.e, is it a good idea to apply Grover's Search Algorithm? Give a 'Yes' or 'No' answer and Justify your answer.

Yes, this is applicable; since by the analysis that we did for Grover's Algorithm, we can collapse all required states to one qubit, and all not required states to another. Effectively the system becomes a single qubit system, and the above analysis applies.

For MCQs, Write your answers here clearly.

1. **C**	2. **A**	3. **E**	4. **B**	5. **D**

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1. Grover's Search Algorithm for n elements has time complexity (in terms of number of queries to the Oracle):

   A. $O(n^2)$ B. $O(n)$ C. $O(\sqrt{n})$ D. $O(\log_2 n)$ E. $O(1)$

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2. In Grover's Search Algorithm we used the operator $2\,|s\rangle\langle s| - I_n$, where $|s\rangle$ is the uniform superposition over all n qubits and I_n is the identity.
   Consider the following statements:

   First, let us use the following notation:
   - $|0^n\rangle = |00000\ldots0\rangle$ (a qubit with n zeros tensored),
   - $H^{\otimes n}$ is the repeated application of the H (Hadamard) operator on n qubits.

   Since $H^{\otimes n}\,|0^n\rangle = |s\rangle$, (i.e. uniform superposition of n qubits) therefore,

   $$H^{\otimes n}(2\,|0^n\rangle\langle 0^n| - I_n)\,H^{\otimes n} = 2\,(H^{\otimes n}|0^n\rangle\langle 0^n|\,H^{\otimes n} - H^{\otimes n} I_n H^{\otimes n})$$
   $$= 2\,(|s\rangle\langle s| - H^{\otimes n} I_n H^{\otimes n})$$

   To complete the argument, what is equivalent to $H^{\otimes n} I_n H^{\otimes n}$?

   A. $H^{\otimes n} I_n H^{\otimes n} = I_n$
   B. $H^{\otimes n} I_n H^{\otimes n}$ is the uniform superposition over n states.
   C. $H^{\otimes n} I_n H^{\otimes n}$ is the quantum fourier transform over n states.
   D. $H^{\otimes n} I_n H^{\otimes n} = Z^{\otimes n}$
   E. $H^{\otimes n} I_n H^{\otimes n} = H^{\otimes 2n}$

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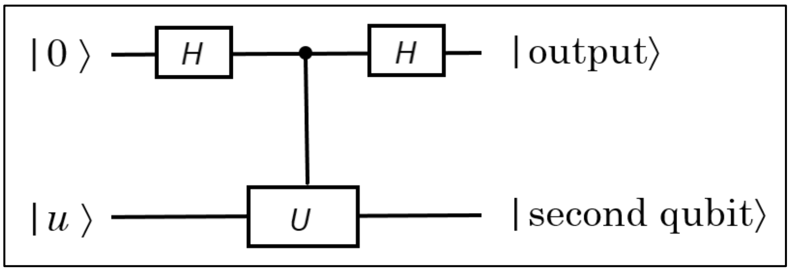
3. Consider the following QFT matrix:

   $$F_4 = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix}$$

   Which of the following is ***not*** a correct pair of eigenvalue and eigenvector for the F_4? (Note that these are *not* qubits, hence their norm may not be 1)

   A. $\lambda_1 = -1$, $v_1 = (-1, 1, 1, 1)$ B. $\lambda_2 = i$, $v_2 = (0, -1, 0, 1)$
   C. $\lambda_3 = 1$, $v_3 = (2, 1, 0, 1)$ D. $\lambda_4 = 1$, $v_1 = (1, 0, 1, 0)$
   E. None of these. [All of the above are correct eigenvalue/eigenvector pairs].

Question 4 refers to the following circuit:



4. Let the operator U act on the qubit $|u\rangle$ (which is its eigenvector) with the eigenvalue -1, i.e. $U|u\rangle = -|u\rangle$. What is the qubit "$|\text{output}\rangle$" in the above circuit?

   A. $|0\rangle$
   B. $|1\rangle$
   C. $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$
   D. $|u\rangle$
   E. $-|u\rangle$

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5. Which of the following statements is correct.

   A. The quantum fourier transform for a single qubit is the Z gate.
   B. The quantum fourier transform for a single qubit is the S gate.
   C. The quantum fourier transform for a single qubit is the X gate.
   D. The quantum fourier transform for a single qubit is the H gate.
   E. The quantum fourier transform for a single qubit is the T gate.

Quantum Gates Reference:

X	$\oplus$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	**H**	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
Y		$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$	**S**	$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$
Z		$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	**T**	$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$
