Using Packages

Week 3 SEL Activity 4

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December 14, 2023

1 MGF of Bernoulli RV

MGF of Discrete RVs is given as,

$$M_X(t) = \sum_{x \in X} e^{tx} p_X(x)$$

For $X \sim \mathrm{Ber}(;p)$,

$$\begin{aligned} M_X(t) &= e^{t(0)}(1-p) + e^{t(1)}p \\ M_X(t) &= 1 - p + pe^t \end{aligned} \tag{1}$$

2 Mean and Variance of Bernoulli RV

First and second moment of X would be,

$$\mu_1 = \left.\frac{\partial M_X(t)}{\partial t}\right|_{t=0} = \left.pe^t\right|_{t=0} = p$$

$$\mu_2 = \left.\frac{\partial^2 M_X(t)}{\partial t^2}\right|_{t=0} = \left.\frac{\partial}{\partial t}\left(pe^t\right)\right|_{t=0} = \left.pe^t\right|_{t=0} = p$$

The mean and variance is,

$$\mu_X = \mu_1 = p$$

$$\sigma_X^2 = \mu_2 - \mu_1^2 = p - p^2 = p(1-p)$$

3 Mean and Variance of Binomial RV

Let $Y \sim \mathrm{B}(;p,n)$. Y can be written as the sum of multiple Bernoulli RV.

$$Y = \sum_{i=1}^{n} X_i$$

where $X_i \sim \mathrm{Ber}(;p)$ for $1 \leq i \leq n$. According to the theorem mentioned in the question.

$$M_Y(t) = \prod_{i=1}^{n} M_{X_i}(t)$$

We have calculated the MGF of Bernoulli RV.

$$M_Y(t) = \prod_{i=1}^{n} \left(1 - p + pe^t\right) = \left(1 - p + pe^t\right)^n$$

First and second partial differentail would be,

$$\partial_t M_Y(t) = npe^t \left(1 - p + pe^t\right)^{n-1}$$

$$\partial_t^2 M_Y(t) = npe^t \left(1 - p + pe^t\right)^{n-1} + n(n-1)p^2 e^{2t} \left(1 - p + pe^t\right)^{n-2}$$

$$\partial_t^2 M_Y(t) = npe^t \left(1 - p + pe^t\right)^{n-2} \left(1 - p + npe^t\right)$$

Moments will be,

$$\mu_1 = \partial_t M_Y(t)\big|_{t=0} = np$$

$$\mu_2 = \partial_t^2 M_Y(t)\big|_{t=0} = np(1 - p + np)$$

Mean and Varaince would be,

$$\mu_Y = \mu_1 = np$$

$$\sigma_Y^2 = \mu_2 - \mu_1^2 = np(1 - p + np) - n^2 p^2 = np(1 - p)$$