Homework 3 Prob Stats

Ali Muhammad

aa07190

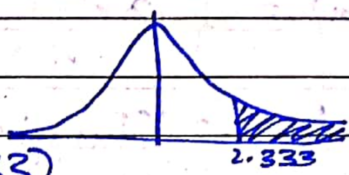
Q1) $H \sim N(65, 9)$ $\mu = 65, \sigma = 3, \sigma^2 = 9$

$P(H \geq 6ft)$ 1ft = 12 inches. 6ft = 72 inches.

$P(H \geq 72)$

$\Rightarrow P(Y \geq (72-65)/3)$

$P(Y \geq 2.333)$



$P(Y \geq 2.333) = 1 - P(Y \leq 2.333)$

$= 1 - [0.99010]$

$= 0.0099$

$\underline{P(H \geq 72) = 0.0099}$

Q2) $f(x) = \frac{1}{20}$ $25 \leq x \leq 45$

Uniform Continuous Random Variable

(a) $E[x] = \frac{25+45}{2} = 70/2 \Rightarrow E[x] = 35$

(b) $\sigma_x = \frac{b-a}{\sqrt{12}} = 45-25/\sqrt{12} = 5.77$

(c) Prob time until departure at most 30 = A

$P(A) = \int_{25}^{30} \frac{1}{20} \cdot dx = \frac{x}{20}\Big|_{25}^{30} = \frac{30}{20} - \frac{25}{20} = \frac{5}{20}$

$\underline{P(A) = 1/4}$

(d) A = Prob time until departure is between 30 & 40 minutes

$P(A) = \int_{30}^{40} \frac{1}{20} \cdot dx = \frac{x}{20}\Big|_{30}^{40} = \frac{40}{20} - \frac{30}{20} = \frac{10}{20}$

$\underline{P(A) = 1/2}$

Q3) Exponential RV $\mu = 15000$ kms $\rightarrow$ accident

Karachi $\rightarrow$ Khunjerab $= 2{,}100$

Prob you will make it back to Karachi

$E[X] = 1/\lambda = 15000 \Rightarrow \lambda = 1/15000$

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Karachi $\rightarrow$ Khunjerab $\rightarrow$ Karachi $= 2100 + 2100$
$= 4200$ km.

$\cancel{P(X \geq 4200) = \frac{1}{15000} e^{-\frac{1}{15000} \times 4200}}$

$\cancel{\Rightarrow P(X \geq 4200) =}$

$P(X \geq 4200) = e^{-\lambda x}$

$= e^{-\frac{1}{15000} \times 4200}$

$\underline{P(X \geq 4200) = 0.7558}$

Q4) (a) $f_{X,Y}(x,y) = xy^2/81$ $0 \leq x \leq 3$, $0 \leq y \leq 3$

For Independence, $f_{X,Y}(x,y) = f_X(x) f_Y(y)$.

$f_X(x) = \int_0^3 \frac{xy^2}{81} \cdot dy = \frac{xy^3}{81 \times 3} \Big|_0^3 = \frac{x(3)^3}{81 \times 3} - 0$

$= \frac{9x}{81} = \frac{x}{9}$

$f_Y(y) = \int_0^3 \frac{xy^2}{81} \cdot dx = \frac{x^2y^2}{81 \times 2} \Big|_0^3 = \frac{3^2 \times y^2}{81 \times 2} - 0$

$= \frac{9}{81} \times y^2/2 = y^2/18$

$f_X(x) f_Y(y) = \frac{x}{9} \times \frac{y^2}{18} = xy^2/162.$

Since $f_{X,Y}(x,y) \neq f_X(x) f_Y(y)$,
they are not independent.

(b) $f_{X,Y}(x,y) = x + cy^3 \quad 0 \le x \le 1, \; 0 \le y \le 1$

i) $c = ?$ $\quad 1 = \int_0^1 \int_0^1 x + cy^3 \cdot dx \cdot dy$

$1 = \int_0^1 \frac{x^2}{2} + cy^3 x \Big|_0^1 \cdot dy$

$1 = \int_0^1 \frac{1}{2} + cy^3 \cdot dy$

$1 = \frac{y}{2} + \frac{1}{4} cy^4 \Big|_0^1 \Rightarrow 1 = \frac{1}{2} + \frac{c}{4}$

$\Rightarrow 1 = \frac{c+2}{4} \Rightarrow c + 2 = 4$

$\Rightarrow \underline{c = 2}$

ii) for Independence $f_{X,Y}(x,y) = f_X(x) f_Y(y)$

$f_X(x) = \int_0^1 x + 2y^3 \cdot dy$

$= xy + \frac{1}{2} y^4 \Big|_0^1 = x + \frac{1}{2} - 0$

$f_X(x) = x + 1/2$

$f_Y(y) = \int_0^1 x + 2y^3 \, dx$

$= \frac{x^2}{2} + 2xy^3 \Big|_0^1 = \frac{1}{2} + 2y^3 - 0$

$f_Y(y) = \frac{1}{2} + 2y^3$

$f_X(x) f_Y(y) = (x + \frac{1}{2})(\frac{1}{2} + 2y^3)$

$= \frac{x}{2} + 2xy^3 + \frac{1}{4} + y^3$

$\therefore f_{X,Y}(x,y) \neq f_X(x) f_Y(y)$

So they are not independent

Q5) $X \sim U[0,2]$ $\quad Y \sim U[0,2]$

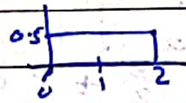
(a) $f_{X,Y}(x,y)$

$f_{X,Y}(x,y) = f_X(x) f_Y(y)$

$f_X(x)$:

~~$f_Y(y)$~~:

$f_Y(y)$:



$f_{X,Y}(x,y) = f_X(x) f_Y(y)$

$= 0.5 \times 0.5 = \underline{0.25}$

$\underline{f_{X,Y}(x,y) = 0.25}$

(b) $F_{X,Y}(1.5, 1.5) \Rightarrow x \leq 1.5, \; y \leq 1.5$

$F_{X,Y}(1.5, 1.5) = \int_0^{1.5}\int_0^{1.5} 0.25 \, dx dy$

$F_{X,Y}(1.5, 1.5) = \int_0^{1.5} 0.25x \Big|_0^{1.5} dy$

$= \int_0^{1.5} 0.25(1.5) \, dy =$

$= 0.375 y \Big|_0^{1.5}$

$= 0.375(1.5) - 0 = 0.5625$

$\underline{F_{X,Y}(1.5, 1.5) = 0.5625}$

(c) $f_{Y|X}(y \mid 0.5)$

$f_{Y|X}(y \mid 0.5) = f_Y(y)$ $\quad \therefore$ independent

$\underline{f_{Y|X}(y \mid 0.5) = 0.5}$

(d) $f_{X|Y}(0.5 \mid 1) = f_X(x)$ $\quad \therefore$ independent

$= \underline{0.5}$

(e) $E[X \mid Y > 1.5]$ $\quad \therefore$ X & Y independent, mean of X is not influenced by Y. Hence mean of X remains same.

$E[X \mid Y > 1.5] = E[X] = \frac{0+2}{2} = \frac{2}{2}$

$\Rightarrow \underline{E[X] = 1}$

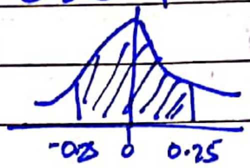
ALBA

Q6) (a) $Y = N + S$ $\quad N \sim N(0, 1)$ independent of S

$A = \{-0.25 \le N \le 0.25\}$

$E[N \mid A]$

$E[N] = 0$. Since A is the event that N takes values between -0.25 & 0.25, then the ~~[illegible]~~ resulting region is :



from which it can be clearly inferred that ~~since both sides (right & left) from the areas are~~ the mean has not changed.

Thus $E[N \mid A] = 0$ by symmetry.

(b) Average odometer reading of all red cars = 55000 km.

From the given information, the expected values is the odometer reading given that all cars were red.

Then we can form two random variables X & Y where

X = Odometer reading of a car

Y = Color of a car.

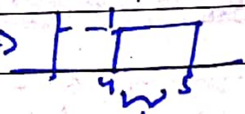
Then $E[X \mid Y = \text{red}] = 55000$ km.

Q7) Motorway : 4 to 5 hours $\qquad P(M) = 1/2$

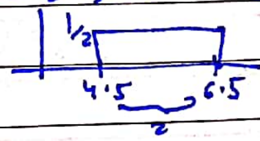
Not Motorway : 4.5 to 6.5 hours $\qquad P(M') = 1/2$

~~&~~ D = Ahsan drives to - - -

$f_{D|M}(d, m) \sim U[4, 5]$ =>

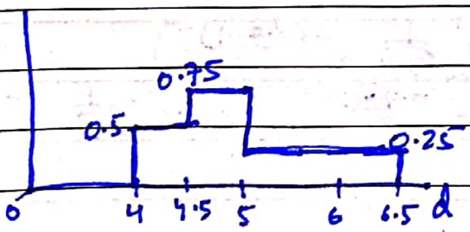


$f_{D|M'}(d, m') \sim U[4.5, 6.5]$ =>



=>

$f_D(d)$



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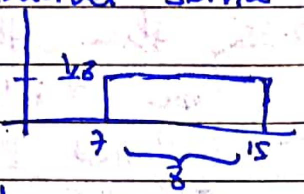
Q 8) Charter : $3/10$, 7Mbps – 15Mbps

Xfinity : $7/10$, 10Mbps – 20Mbps

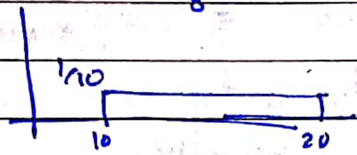
Probability served by Charter given that speed is 12Mbps

S = Speed. ISP = Internet Service Provider

$f_{S,ISP}(S, ISP=C)$



$f_{S,ISP}(S, ISP=X)$



$$P_{ISP|S}(ISP=C \mid 12) = \frac{P_{ISP}(C)\, f_{S|ISP}(12|C)}{P_S(12)}$$

$$P_{ISP|S}(ISP=C \mid 12) = \frac{3/10\,(1/8)}{3/10\,(1/8) + 7/10\,(1/10)}$$

$$= \underline{0.349}$$

Q9) $P(S=0)=0.4$ $\quad P(S=1)=0.6$ $\quad Y=N+S$

$N \sim N(0,1)$ $\quad P(S=1 \mid Y=0.7)$

$$P_{S|Y}(S=1 \mid Y=0.7) = \frac{P_S(1)\, f_{Y|S}(0.7|1)}{f_Y(0.7)}$$

$$f_{Y|S}(y|0) = \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} \quad [Y=N \sim N(0,1)]$$

$$f_{Y|S}(y|1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(y-1)^2}{2}} \quad [Y=N+1 \sim N(1,1)]$$

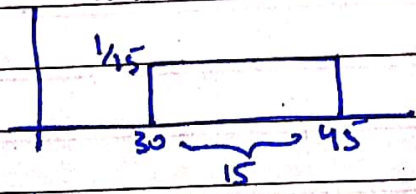
$$P(S=1 \mid Y=0.7) = \frac{0.6\left(\frac{1}{\sqrt{2\pi}} e^{-\frac{(0.7-1)^2}{2}}\right)}{0.6\left(\frac{1}{\sqrt{2\pi}} e^{-\frac{(0.7-1)^2}{2}}\right) + 0.4\left(\frac{1}{\sqrt{2\pi}} e^{-\frac{(0.7)^2}{2}}\right)}$$

$$= \underline{0.647}$$

Q10) Sunny : 30 - 45 minutes $\quad P(\text{Sunny}) = 0.75$

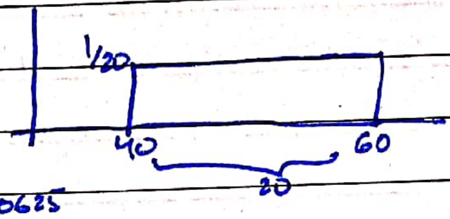
Raining : 40 - 60 minutes $\quad P(\text{Raining}) = 0.25$

(a) $f_{T|D}(t \mid \text{Sunny})$



$\frac{1}{15} \times \frac{3}{4} = \frac{1}{20} = 0.05$

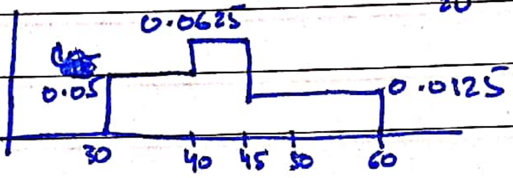
$f_{T|D}(t \mid \text{Raining})$



$\frac{1}{20} \times \frac{1}{4} = \frac{1}{80}$

$= 0.0125$

$f_T(t)$



(b) $$P(\text{Raining} \mid 42 \text{ minutes}) = \frac{P(\text{Raining})\, P(42 \text{ minutes} \mid \text{Raining})}{P(42 \text{ minutes})}$$

$$= \frac{0.25(1/20)}{0.25(1/20) + 0.75(1/15)}$$

$$= \underline{0.2}$$