Problem 1. **[30 points]**

Define following terms in **your own words**: [Kindly provide *short answers* not longer than few lines.]

(a) Turing decidability.

(b) Halting problem.

(c) NP-completeness.

(d) Nondeterministic Turing machine.

(e) Uncountable set.

(f) Polynomial time reduction.

Problem 2. **[20 points]**

State whether following statements are True or False also provide correct reasoning. [Kindly provide *short answers* not longer than few lines.]

(a) The complement of every Turing decidable language is Turing-decidable.

(b) CLIQUE $\leq_P$ VERTEX-COVER.

(c) $\overline{A_{\text{TM}}}$, the complement of the acceptance problem for Turing machines, is known to be **NP**.

(d) For every multi-tape nondeterministic Turing machine there exists a single tape deterministic Turing machine.

(e) A polynomial-time solution to some known problem in **NP**-complete implies $\mathbf{P} = \mathbf{NP}$.

Problem 3. **[10 points]**

Design a Turing machine M to decide the following language L:

$$L = \{ww^{\text{rev}} : w \in \{a, b\}^* \text{ where } w^{\text{rev}} \text{ is the reverse of the string } w\ \}.$$

For example, ϵ, *bbaabb*, *bbbb* are in L, whereas *aaa*, *baba* are not in L.

Problem 4. **[10 points]**

If A is a Turing-recognizable language and $B \subseteq A$, is B necessarily a Turing-recognizable language? (Prove or disprove.)

Problem 5. **[10 points]**

Prove that the language S defined below is in **NP**.

$$S = \{\langle x_1, x_2, \ldots, x_n, k\rangle : \text{there exists a set } \{c_1, \ldots, c_n\ \} \text{ of coefficients where each } c_i \in \{1, 2\} \text{ and } \sum_{1 \leq i \leq n} c_i x_i = k\}$$

Problem 6. **[10 points]**

Is $R_{\text{TM}} = \{\langle M, w\rangle : M \text{ is a TM that does not accept } w\}$ Turing-decidable?

Problem 7. **[10 points]**

Prove the following statement:

If B is **NP***-complete and $B \in \mathbf{P}$, then $\mathbf{P} = \mathbf{NP}$.*