Quiz 13B: Mapping Reduction

CS 212 Nature of Computation

Habib University — Fall 2023

Total Marks: 10 | Date: November 15, 2023
Duration: 15 minutes | Time: 830–845h

Student ID: _____

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1. (10 points) Let us extend mapping reduction to arbitrary sets.

> A set A is *mapping reducible* to a set B, written $A \leq_m B$, if there is a computable function f, where for every u,
>
> $$u \in A \iff f(u) \in B.$$
>
> The function f is called the *reduction* from A to B.

Prove or disprove the following claim.

Claim 1. *The following function, $f : P(\Sigma^*) \to P(\Sigma^*)$, is a reduction from the class of context-free languages to the class of recursively enumerable languages.*

$$f(L) = L.$$

Solution: We disprove the claim by showing that f does not meet the required condition for a reduction. For ease of description, we denote the class of context-free languages as CFL and the class of recursively enumerable languages as RE.

Proof. For f to be a reduction from CFL to RE, the following must hold.

$$\forall L \, (L \in CFL \iff f(L) \in RE).$$

This further reduces to 2 cases, one of which we show through a counterexample to not hold.

Case: $\forall L \, (L \notin CFL \implies f(L) \notin RE)$
That is, $\forall L \, (L \notin CFL \implies L \notin RE)$
We know that $CFL \subset RE$.
Consider a language, C, such that $C \in RE$ and $C \notin CFL$.
$L = C$ is a counterexample to the claim. □