# [Exam 3]
# Fall 2023: CS 313: Computational Complexity Theory

Due: 6:30 pm, Saturday, December 16, 2023. Total Marks: 30

This exam copy contains 7 questions, of which **any 5 must be attempted** for full credit. Make sure to indicate the question number accurately on your answer booklet, and cross off any answers that you do not want to be considered for grading. Only the first 5 uncrossed answers will be graded. All questions and parts carry equal points.

**Question 1** [6 points]

A deterministic machine is a special case of a non-deterministic machine. Copy the statement below to your answer booklet, and instead of the blanks, insert "$=$" where equality is known, and "$\subseteq$", which means equal or contained, otherwise, and write very brief explanations to support your choice in each case. In total, you should give six brief explanations.

**L_____NL_____coNL_____P_____NP_____PSPACE_____NPSPACE**

**Question 2** [6 points]

A language $L$ is in the class **DP** if and only if there are two languages $L_1 \in$ **NP** and $L_2 \in$ **coNP** such that $L = L_1 \cap L_2$.

SAT-UNSAT: Given two Boolean expressions $\phi$ and $\phi'$ (both in 3CNF). Is it true that $\phi$ is satisfiable and $\phi'$ is not?

Show that SAT-UNSAT is **DP-Complete**.

**Question 3** [6 points]

For a complexity class $C$, let co-$C = \{\overline{L} \mid L \in C\}$ and say that $C$ is closed under complementation whenever $C =$ co-$C$. Argue as to whether the following statements are true, false, or unknown.

(a) All deterministic time complexity classes are closed under complementation.

(b) All nondeterministic time complexity classes are closed under complementation.

**Question 4** [6 points]

Consider the following two decision problems:

$\mathbf{REACH}$ – the problem of deciding, given a directed graph $G$ and two vertices $a$ and $b$ in $G$, whether there is a path in $G$ from $a$ to $b$.

$\mathbf{UREACH}$ – the problem of deciding, given an undirected graph $G$ and two vertices $a$ and $b$ in $G$, whether there is a path in $G$ from $a$ to $b$.

It is known that REACH is **NL-Complete** (under logarithmic-space reductions) and that UREACH is in the complexity class **L**.

Based on the above information, for each of the following statements, state whether it is true, false, or unknown. In the case it is true or false, give full justification for your answer. In the case it is unknown, there is no need to justify.

(a) REACH $\leq_L$ UREACH, i.e., REACH is reducible in logarithmic-space to UREACH.

(b) UREACH $\leq_L$ REACH.

(c) UREACH is in **P**.

**Question 5** [6 points]

Let $S \subseteq \mathbb{N}$ be a set of numbers. We write BIN-S for the set of binary strings (i.e., strings in $\{0,1\}^*$) $x$ such that $x$ is a binary representation of a number in $S$. We also write UN-S for the set $\{a^k | k \in S\}$ where the notation $a^k$ means the string consisting of $k$ repetitions of the symbol $a$.

Prove that BIN-S $\in$ **SPACE**$(n)$ if and only if UN-S $\in$ **L**.

**Question 6** [6 points]

The language CKT-SAT consists of all (strings representing) circuits that produce a single bit of output and that have a satisfying assignment. That is, a string representing an $n$-input circuit, $C$, is in CKT-SAT iff there exists $u \in \{0,1\}^n$ such that $C(u) = 1$.

Show that CKT-SAT is **NP-Complete**.

**Question 7** [6 points]

(a) Describe a decidable language in **P/poly** that is not in **P**.

(b) Describe an undecidable language in **P/poly** that is not in **P**.