Weekly Challenge 04: Regular Expressions

CS 212 Nature of Computation
Habib University

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1. Closures

Given a language, L, and the definitions below, prove or disprove the given claim.

Definition 1 (Kleene closure). $L^* = \{u_1u_2u_3 \dots u_n \mid \text{ each } u_i \in L, n \geq 0\}$

Definition 2 (Positive closure). $L^+ = \{u_1u_2u_3 \dots u_n \mid \text{ each } u_i \in L, n \geq 1\}$

Claim 1. $(L^+)^* = (L^*)^+$

Solution: Claim: $(L^+)^* = (L^*)^+$

1) $(L^+)^* \subseteq (L^*)^+$

Consider any arbitrary string s in $(L^+)^*$. s is composed of zero or more stirngs from L^+, which includes the empty string ε by the definition of Kleene Closure. Then s can be represented as a concatenation of other strings (substrings)

$$s = s_1s_2s_3...s_n \text{ where each } s_i \in L^+, \text{ and } n \geq 0$$

Now for any s_i, s_i must exist in L^* because L^* includes all strings (zero or more) from L^+. So each s_i also exists in L^*, including the empty string as by the definition, L^* also contains zero strings which amounts to the empty string ε. Then $s = s_1s_2s_3...s_n \in (L^*)^+, \ \forall s \in (L^+)^*$ since the Positive Closure will be a concatenation of all strings from L^* which will include ε since ε is a member of L^*. Hence, for any arbitrary string s in $(L^+)^*$, $s \in (L^*)^+$, which implies that $(L^+)^* \subseteq (L^*)^+$.

2) $(L^*)^+ \subseteq (L^+)^*$

Consider any arbitrary string s in $(L^*)^+$. s is composed of one or more strings from L^*, which includes the empty string ε due to the Kleene Closure. Then s can be represented as a concatenation of other strings (substrings)

$$s = s_1s_2s_3...s_n \text{ where each } s_i \in L^*, \text{ and } n \geq 1$$

Now for any s_i which is a component of s, if $s_i \neq \varepsilon$, then $s_i \in L^+$ and subsequently in $(L^+)^*$. However, due to the application of Kleene Closure on L^+, the resulting language will have ε as a member, hence $s_i \in (L^+)^*$. Then each $s_i \in L^+$ where $s_i \neq \varepsilon$, however, $\varepsilon \in (L^+)^*$. Therefore, $s = s_1s_2s_3...s_n \in (L^+)^*, \ \forall s \in (L^*)^+$ since its a concatenation of possible strings from L^+ including the empty string by the definition. Hence, for any arbitrary string s in $(L^*)^+$, $s \in (L^+)^*$, which implies that $(L^*)^+ \subseteq (L^+)^*$.

Since $(L^+)^* \subseteq (L^*)^+$ and $(L^*)^+ \subseteq (L^+)^*$, then $(L^+)^* = (L^*)^+$.

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