SOLUTION

Habib University CS/PHYS 314/300: Quantum Computing [Quiz 01] Fall Semester 2024

Name: _____ ID: _____ Section: L1

For each of the following questions choose the correct answer. Write your answer choice in the space provided at the end of this document.

1. Which of the following is a valid qubit?

   A. $0.7|0\rangle$ B. $\frac{1+i}{\sqrt{2}}|0\rangle + \frac{1-i}{\sqrt{2}}|1\rangle$ C. $\frac{1}{\sqrt{2}}|0\rangle + \frac{1+i}{2}|1\rangle$

   D. $\frac{1+2i}{3}|0\rangle - \frac{2}{9}|1\rangle$ E. $(2+i)|1\rangle$

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2. Given $|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$, and $|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$, what is the value of their inner product $\langle +|-\rangle$:

   A. $\sin 90°$ B. $\sin 45°$ C. $\tan 90°$

   D. $\cos 90°$ E. $\cos 45°$

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3. Consider the following unitary operation R as shown below. Then $R^2 =$

$$\begin{pmatrix} \frac{1+i}{2} & \frac{1-i}{2} \\ \frac{1-i}{2} & \frac{1+i}{2} \end{pmatrix}.$$

   A. X B. Y C. Z

   D. H E. I

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4. Consider an operator Q expressed as an outer product: $U = |0\rangle\langle 0| - |1\rangle\langle 1|$. Which of the following operators is it?

   A. X B. Y C. Z

   D. H E. I

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5. Suppose we know that a given qubit q is one of the following qubits $|\psi 1\rangle = \frac{\sqrt{3}}{2}|0\rangle - \frac{1}{2}|1\rangle$, and $|\psi 2\rangle = i|1\rangle$. Upon measuring q we get a certain output. Which of the following statements is correct?

   A. If the outcome is 0, we are certain (with probability 100%) that q is $|\psi 2\rangle$.

   B. If the outcome is 0, we are certain (with probability 100%) that q is $|\psi 1\rangle$.

   C. If the outcome is 1, we are certain (with probability 100%) that q is $|\psi 2\rangle$.

   D. If the outcome is 1, we are certain (with probability 100%) that q is $|\psi 1\rangle$.

   E. None of the above

6. Given the following vector on a Bloch Sphere, identify which one of the following vectors represents the given vector. [Note that the vector is on the Y-Z plane, and its angle with the positive Y-Axis is π/6 radians]:

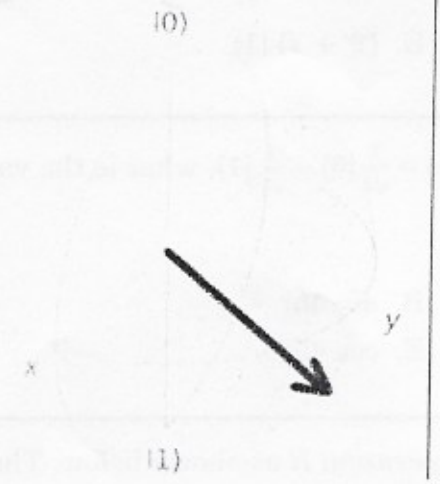
A. $\frac{\sqrt{3}}{2}|0\rangle - \frac{1}{2}i|1\rangle$

B. $\frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}i|1\rangle$

C. $\frac{\sqrt{3}}{2}|0\rangle - \frac{1}{2}i|1\rangle$

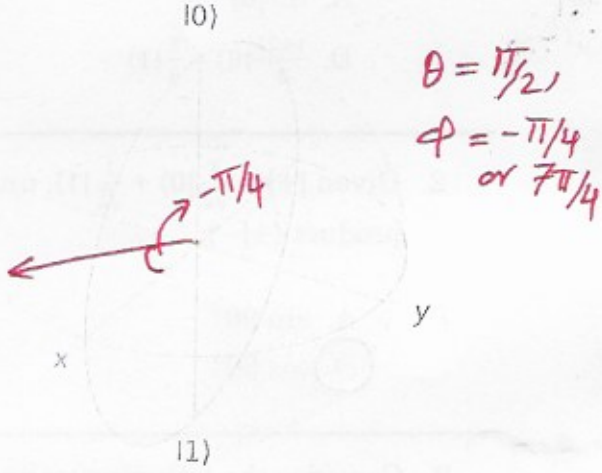
D. $\frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}i|1\rangle$

E. $\frac{1}{2}|0\rangle - \frac{\sqrt{3}}{2}i|1\rangle$



7. Plot the given qubit on the Bloch Sphere. In particular identify the angles θ and ϕ as well.

$$\frac{1}{\sqrt{2}}|0\rangle + \frac{1-i}{2}|1\rangle$$



8. Given an arbitrary qubit on a Bloch Sphere, i.e. $|\psi 1\rangle = \alpha|0\rangle + \beta|1\rangle$, what is the effect on its plot if the operator Z is applied to it? Explain with an example.

It rotates the qubit by 180° (π radians) along the Z-Axis. i.e., a phase flip along Z-Axis.

If $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, $Z|\psi\rangle = \alpha|0\rangle + e^{2\pi}\beta|1\rangle$.

For MCQs, write answers here:	1	2	3	4	5	6
	C	D	A	C	B	D

For your reference,

$$\mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

$$H = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \qquad P_\theta = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$$