# Quiz 15A: The class NP

CS 212 Nature of Computation

Habib University — Fall 2023

Total Marks: 10 | Date: November 29, 2023
Duration: 15 minutes | Time: 830–845h

**Student ID**: _____

**Student Name**: _____

1. (10 points) Show that the class NP is closed under union.

**Solution:** We prove the closure by constructing a non-deterministic polynomial-time decider for the union of two languages in NP.

*Proof.* Consider the languages $L_1, L_2 \in \text{NP}$ and let $N_1$ and $N_2$ be their respective non-deterministic polynomial-time deciders.
Construct $N$ to decide $L_1 \cup L_2$ as follows.
On input $w$:

1. Simulate $N_1$ on $w$.
2. If $N_1$ accepts, *accept*.
3. If $N_1$ rejects,
   (a) Simulate $N_2$ on $w$.
   (b) If $N_2$ accepts, *accept*; if $N_2$ rejects, *reject*.

$N$ utilizes $N_1$ and $N_2$ so is non-deterministic.
Step 1 runs in polynomial time, Step 2 in constant time, and Step 3 altogether in polynomial time.
$N$ halts in all cases. □