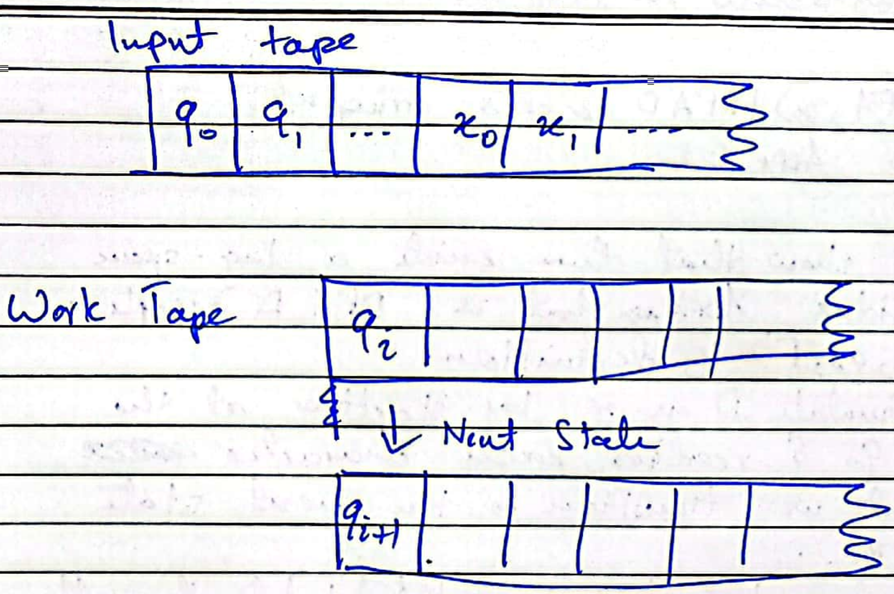
Complexity Theory Quiz 05

Q) $A_{DFA} = \{(DFA, x) \mid DFA\ D$ accepts string $x\}$.
Show $A_{DFA} \in L$.

* We need to show that there exists a log-space alg that decides A_{DFA} that is, DFA D accepts x.
$D = (Q, \Sigma, \delta, q_0, F)$ by definition.
* We can simulate D on x by starting at the start state q_0 & reading each character x_i in x, And we transition to the next state $q_{i+1} = \delta(q_i, x_i)$.
* ~~Now for each state, we can label the cells according to the index on the input tape starting from 0.~~
* We only need to keep track of the current state we are at on, & store it on our work tape. We can store the state, which would take only log space. If we store only the index of the state with respect to the cell, it would take $(\log n)$ space on the work tape.
* We can reuse that space each time we move onto a new state by ~~overwrit~~ overwriting on it. as we read each x_i & transition to a new state.
* Once we have completed our simulation, we simply check if the state we are currently on on our work tape exists in the set of Accept states F.
If yes, we accept, else we reject.

Since the whole simulation takes only $O(\log)$ space, to store the current space, it exists in L.
Thus, $A_{DFA} \in L$.

Hence proved!


Input tape
q_0
q_1
...
x_0
x_1
...
Work Tape
q_i
Next State
q_{i+1}