HABIB ID: NAME:

QUIZ 11 SOLUTION

VER A

Let *S* be a finite set of vectors in a finite-dimensional vector space *V*. If *S* spans *V* but is not already a basis for *V*, then *S* can be reduced to a basis for *V* by removing appropriate vectors from *S*.

Proof(a) If S is a set of vectors that spans V but is not a basis for V, then S' is a linearly dependent set. Thus some vector v in S' is expressible as a linear combination of the other vectors in S. By the Plus/Minus Theorem (Theorem 5.4.4b), we can remove v from S, and the resulting set will still span V. If S' is linearly independent, then is a basis for V, and we are done. If s' is linearly dependent, then we can remove some appropriate vector from S' to produce a set that still spans V. We can continue removing vectors in this way until we finally arrive at a set of vectors in S that is linearly independent and spans V. This subset of S is a basis for V.

VER B

Let *S* be a finite set of vectors in a finite-dimensional vector space *V*. If *S* is a linearly independent set that is not already a basis for *V*, then *S* can be enlarged to a basis for V by inserting appropriate vectors into *S*.

Proof(b) Suppose that dim(V) = n. If S' is a linearly independent set that is not already a basis for V, then S' fails to span V, and there is some vector v in V that is not in span(S). By the Plus/Minus Theorem (Theorem 5.4.4a), we can insert v into S, and the resulting set will still be linearly independent. If S’ spans V, then is a basis for V, and we are finished. If S’ does not span V, then we can insert an appropriate vector into to produce a set that is still linearly independent. We can continue inserting vectors in this way until we reach a set with n linearly independent vectors in V. This set will be a basis for V by Theorem 5.4.5.