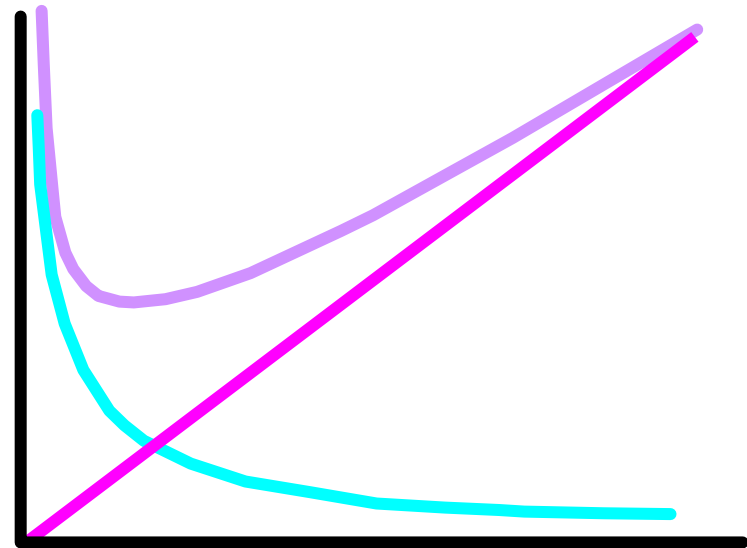


# ***Inventory Management***

## ***Chapter 9***



# *Inventory at **WAL-MART***

- Making sure the shelves are stocked with tens of thousands of items at their 5,379 stores in 10 countries is no small matter for inventory managers at Wal-Mart.
- Knowing what is in stock, in what quantity, and where it is being held, is critical to effective inventory management.
- With inventories in excess of \$29 billion, Wal-Mart is aware of the benefits from improved inventory management.
- They know that effective inventory management must include the entire supply chain.
- The firm is implementing radio frequency identification (RFID) technology in its supply chain.
- When passed within 15' of a reader, the chip activates, and its unique product identifier code is transmitted to an inventory control system.



# ***Inventory Management***

- **Inventory management** is the planning and controlling of inventories in order to meet the competitive priorities of the organization.
  - Effective inventory management is essential for realizing the full potential of any value chain.
- Inventory management requires information about expected demands, amounts on hand and amounts on order for every item stocked at all locations.
  - The appropriate timing and size of the reorder quantities must also be determined.



# ***Inventory Basics***

- **Inventory is created** when the receipt of materials, parts, or finished goods exceeds their disbursement.
- **Inventory is depleted** when their disbursement exceeds their receipt.
- An inventory manager's job is to balance the advantages and disadvantages of both low and high inventories.
  - Both have associated cost characteristics.



# ***Pressures for Low Inventories***

- **Inventory holding cost** is the sum of the cost of capital and the variable costs of keeping items on hand, such as storage and handling, taxes, insurance, and shrinkage.
  - **Cost of Capital** is the opportunity cost of investing in an asset relative to the expected return on assets of similar risk.
  - **Storage and Handling** arise from moving in and out of a storage facility plus the rental cost and/or opportunity cost of that space.
  - **Taxes, Insurance, and Shrinkage**: More taxes are paid and insurance costs are higher if end-of-the-year inventories are high. Shrinkage comes from theft, obsolescence and deterioration.



# ***Pressures for High Inventories***

- **Customer Service:** Reduces the potential for stockouts and backorders.
- **Ordering Cost:** The cost of preparing a purchase order for a supplier or a production order for the shop.
- **Setup Cost:** The cost involved in changing over a machine to produce a different item.
- **Labor and Equipment:** Creating more inventory can increase workforce productivity and facility utilization.
- **Transportation Costs:** Costs can be reduced.
- **Quantity Discount:** A drop in the price per unit when an order is sufficiently large.



# *Types of Inventory*

- **Cycle Inventory:** The portion of total inventory that varies directly with lot size ( $Q$ ).

$$\text{Average cycle inventory} = \frac{Q}{2}$$

- **Lot Sizing:** The determination of how frequently and in what quantity to order inventory.
- **Safety Stock Inventory:** Surplus inventory that a company holds to protect against uncertainties in demand, lead time and supply changes.



# *Types of Inventory*

- **Anticipation Inventory** is used to absorb uneven rates of demand or supply, which businesses often face.
- **Pipeline Inventory**: Inventory moving from point to point in the materials flow system.

$$\text{Pipeline inventory} = \bar{D}_L = dL$$

$\bar{D}_L$  is the average demand for the item per period ( $d$ ) times the number of periods in the item's lead time ( $L$ ).





# Estimating Inventory Levels

## Example 12.1

A plant makes monthly shipments of electric drills to a wholesaler in average lot sizes of **280** drills. The wholesaler's average demand is **70** drills a week. Lead time is **3** weeks. The wholesaler must pay for the inventory from the moment the plant makes a shipment. If the wholesaler is willing to increase its purchase quantity to **350** units, the plant will guarantee a lead time of **2** weeks. What is the effect on cycle and pipeline inventories?

$$\text{Average cycle inventory} = \frac{Q}{2} = \frac{280}{2} = 140 \text{ drills}$$

$$\text{Pipeline inventory} = \bar{D}_L = dL = 70(3) = 210 \text{ drills}$$

Under new proposal, the average lot size becomes **350** and lead time of **2** weeks. Average demand remains at 70 drills a week.

$$\text{Average cycle inventory} = \frac{Q}{2} = \frac{350}{2} = 175 \text{ drills}$$

$$\text{Pipeline inventory} = \bar{D}_L = dL = 70(2) = 140 \text{ drills}$$



# ***Application 12.1***

Management has decided to establish three Distribution Centers in its supply chain, located in different regions of the country, to save on transportation costs. For one of the products, the average weekly demand at **each** DC will be 50 units. The product is valued at \$650 per unit. Average shipment sizes into each DC will be 350 units per trip. The average lead time will be two weeks. Each DC will carry one week's supply as safety stock, since the demand during the lead time sometimes exceeds its average of 100 units ( $50 \text{ units/wk} \times 2 \text{ wk}$ ). Anticipation inventory should be negligible.

How many dollars, on the average, of **cycle inventory** will be held at **each** DC?

How many dollars of **safety stock** will be held at **each** DC?



# ***Application 12.1***

## ***continued***

How many dollars of **pipeline inventory**, on the average, will be in transit for each DC?

$$\text{Average pipeline inventory} = \bar{D}_L = dL$$

How much inventory, on the average, will be held at each DC?

$$\text{Average cycle inventory} = \frac{Q + 0}{2} = \frac{Q}{2}$$

Which type of inventory is your first candidate for reduction?



# ***Reducing Cycle Inventory***

- The primary tactic (lever) for reducing cycle inventory is to reduce lot size.
- This can be devastating if other changes are not made, so two secondary levers can be used:
  1. *Streamline the methods for placing orders and making setups* in order to reduce ordering and setup costs and allow  $Q$  to be reduced.
  2. *Increase repeatability* in order to eliminate the need for changeovers.
- **Repeatability** is the degree to which the same work can be done again.



# Reducing Safety Stock Inventory

- The primary lever to reduce safety stock inventory is to place orders closer to the time they must be received. However, this approach can lead to unacceptable customer service.
- Four secondary levers can be used in this case:
  1. *Improve demand forecasts* so that fewer surprises come from customers.
  2. *Cut the lead times* of purchased or produced items to reduce demand uncertainty.
  3. *Reduce supply uncertainties*. Share production plans with suppliers. Surprises from unexpected scrap or rework can be reduced by improving manufacturing processes. Preventive maintenance can minimize unexpected downtime caused by equipment failure.
  4. *Rely more on equipment and labor buffers*, such as capacity cushions and cross-trained workers.



# ***Reducing Anticipation Inventory***

- The primary lever to reduce anticipation inventory is simply to match demand rate with production rate.
- Secondary levers can be used to even out customer demand in one of the following ways:
  1. *Add new products with different demand cycles* so that a peak in the demand for one product compensates for the seasonal low for another.
  2. *Provide off-season promotional campaigns.*
  3. *Offer seasonal pricing plans.*



# *Reducing Pipeline Inventory*

- The primary lever for reducing pipeline inventory is to reduce the lead time.
- Two secondary levers can help managers cut lead times:
  1. *Find more responsive suppliers and select new carriers* for shipments between stocking locations or improve materials handling within the plant.
  2. *Decrease lot size,  $Q$* , at least in those cases where the lead time depends on the lot size. Smaller jobs generally require less time to complete.



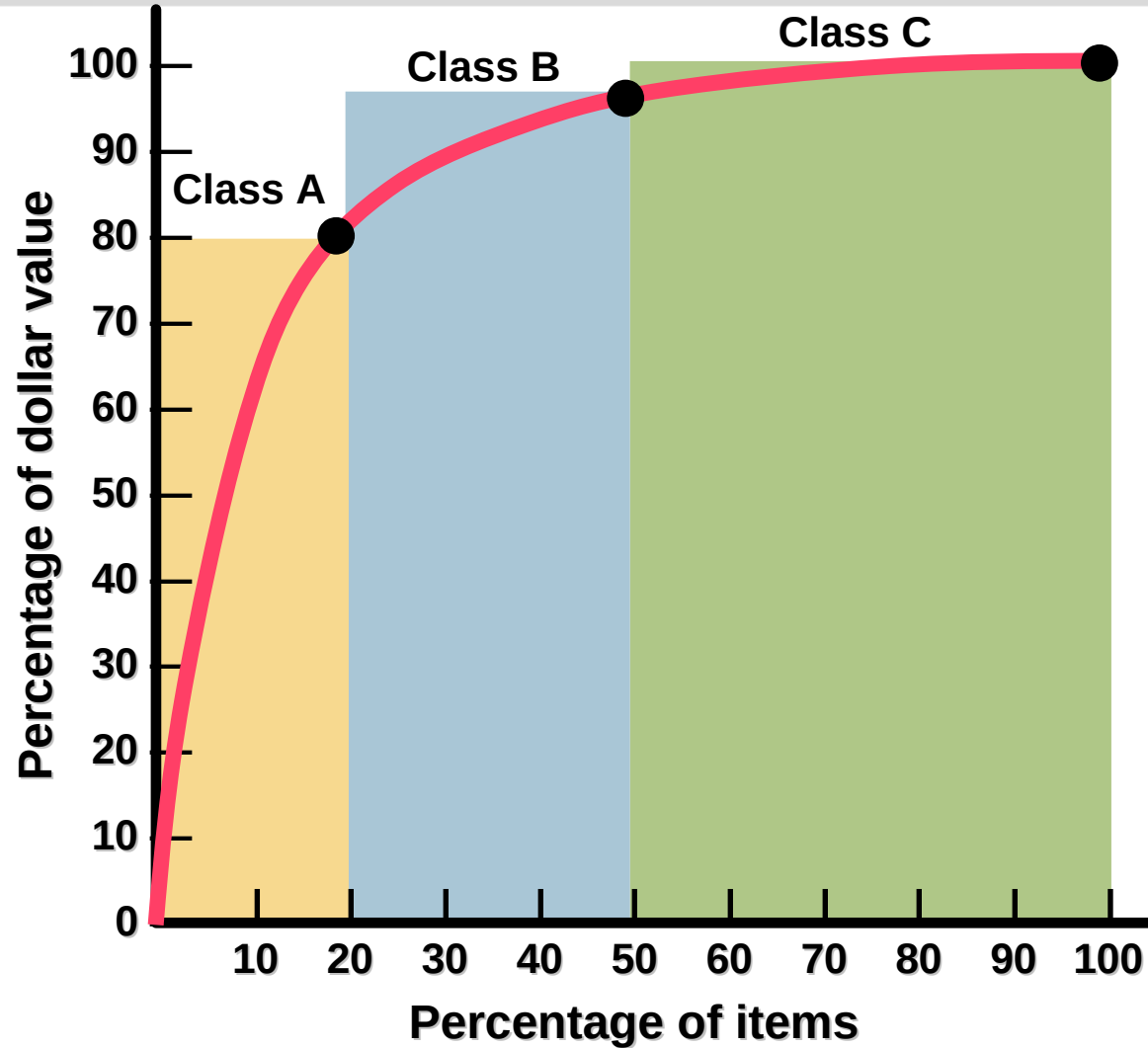
## *Identifying Critical Inventory Items*

- Thousands of items are held in inventory by a typical organization, but only a small % of them deserves management's closest attention and tightest control.
- **ABC analysis:** The process of dividing items into three classes, according to their dollar usage, so that managers can focus on items that have the highest dollar value.





# *ABC Analysis*

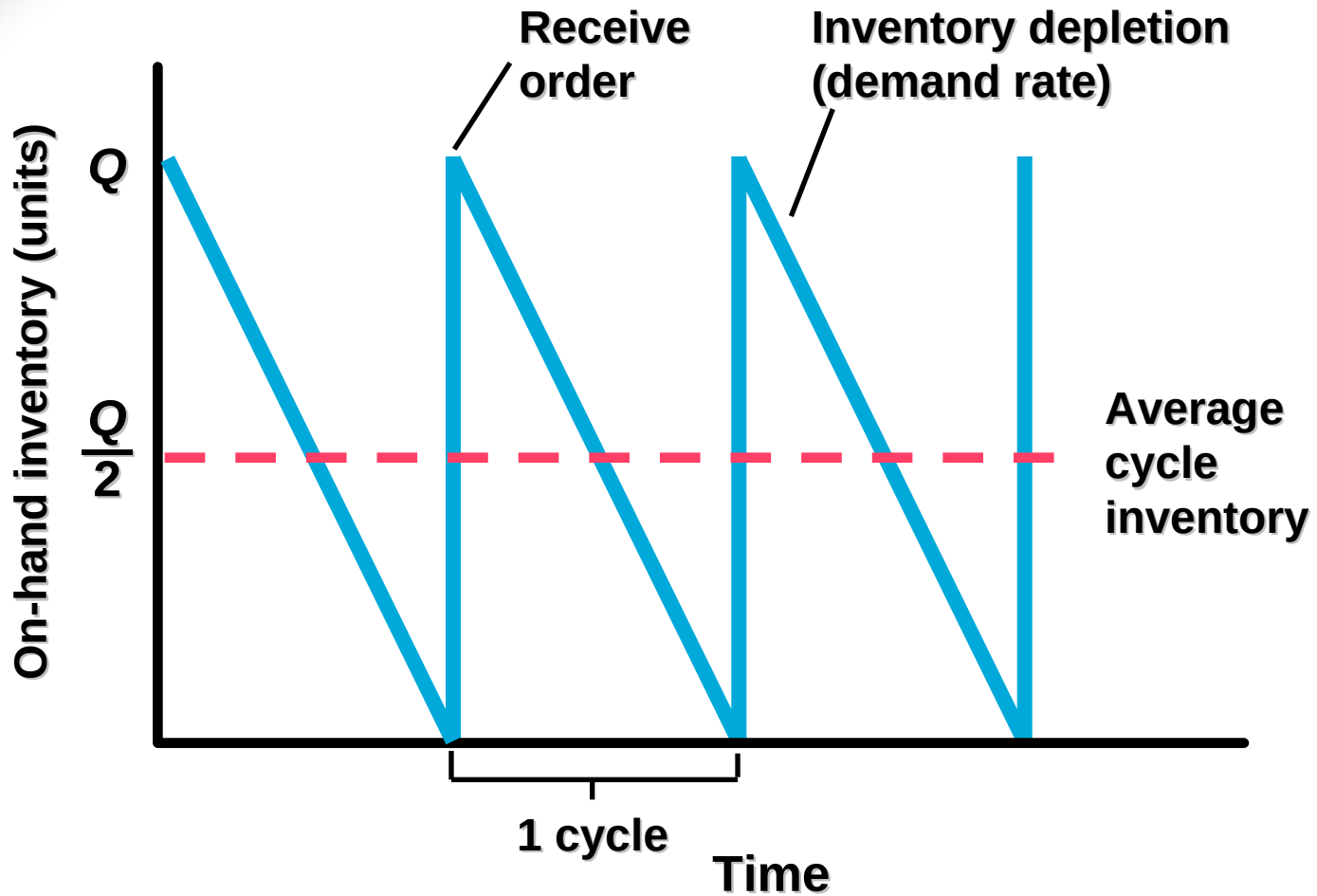


# ***Economic Order Quantity***

- **Economic Order Quantity (EOQ)** is the lot size that minimizes total annual inventory holding and ordering costs.
- Assumptions of EOQ
  1. The demand rate is constant and known with certainty.
  2. There are no constraints on lot size.
  3. The only relevant costs are holding costs and ordering/setup costs.
  4. Decisions for items can be made independently of other items.
  5. Lead time is constant and known with certainty.

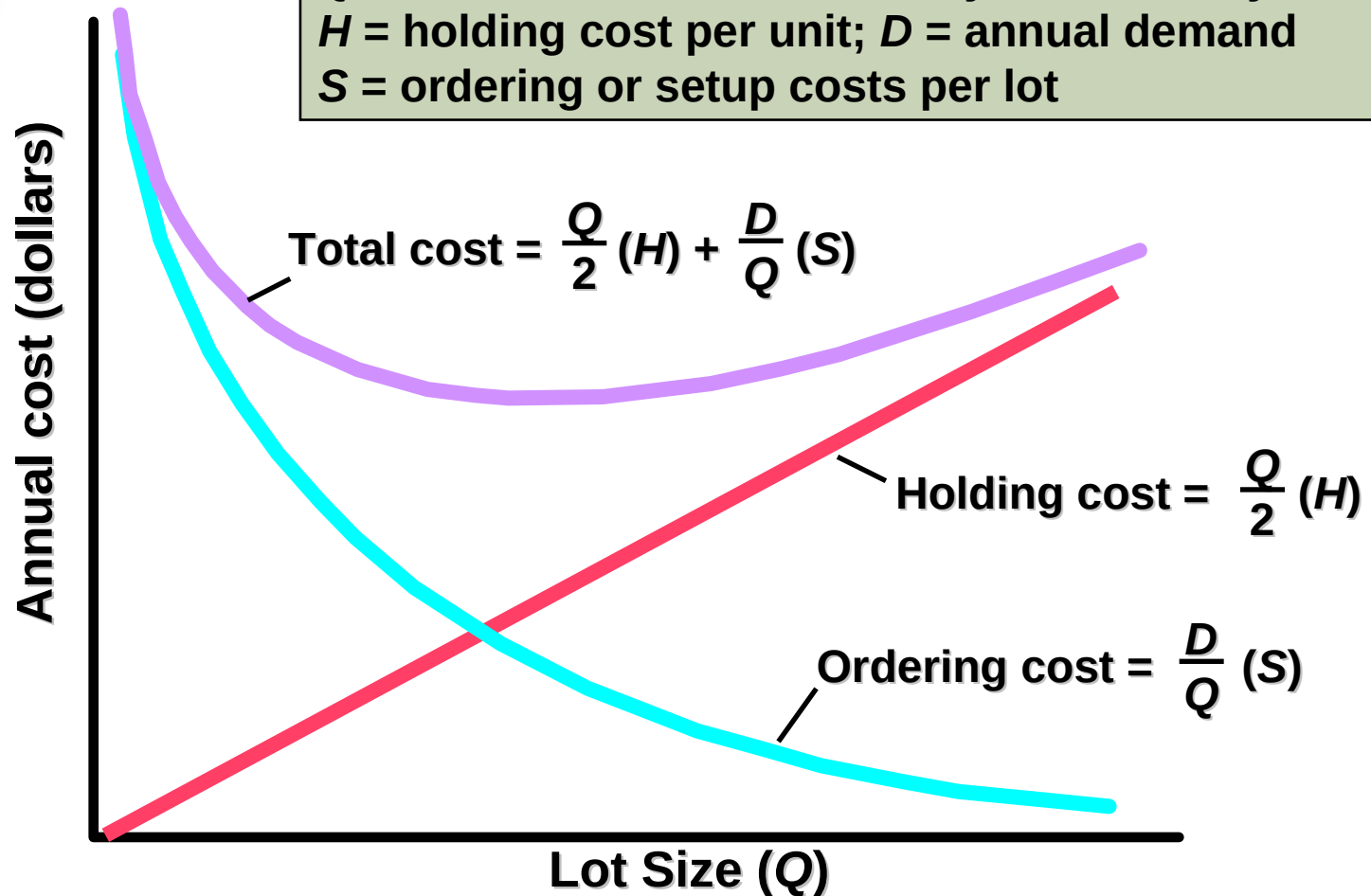


# Cycle-Inventory Levels



# Total Annual Cycle-Inventory Costs

$Q$  = lot size;  $C$  = total annual cycle-inventory cost  
 $H$  = holding cost per unit;  $D$  = annual demand  
 $S$  = ordering or setup costs per lot



# Costing out a Lot Sizing Policy

## Example 12.2

### *Museum of Natural History Gift Shop:*

- Bird feeder sales are **18** units per week, and the supplier charges **\$60** per unit. The cost of placing an order (**S**) with the supplier is **\$45**.
- Annual holding cost ( $H$ ) is **25%** of a feeder's value, based on operations 52 weeks per year.
- Management chose a **390**-unit lot size (**Q**) so that new orders could be placed less frequently.
- What is the annual cycle-inventory cost ( $C$ ) of the current policy of using a 390-unit lot size?



# Costing out a Lot Sizing Policy

## Example 12.2

### Museum of Natural History Gift Shop:

- What is the annual cycle-inventory cost (C) of the current policy of using a 390-unit lot size?

$$D = (18 \text{ /week})(52 \text{ weeks}) = 936 \text{ units}$$

$$H = 0.25 (\$60/\text{unit}) = \$15$$

$$C = \frac{Q}{2} (H) + \frac{D}{Q} (S) = \frac{390}{2} (15) + \frac{936}{390} (45)$$

$$C = \$2925 + \$108 = \$3033$$



# ***Lot Sizing at the Museum of Natural History Gift Shop***

$D = 936$  units;  $H = \$15$ ;  $S = \$45$ ;  $Q = 468$  units;  $C = ?$

Would a lot size of 468 be better?

$$C = \frac{Q}{2} (H) + \frac{D}{Q} (S) = \frac{468}{2} (15) + \frac{936}{468} (45)$$

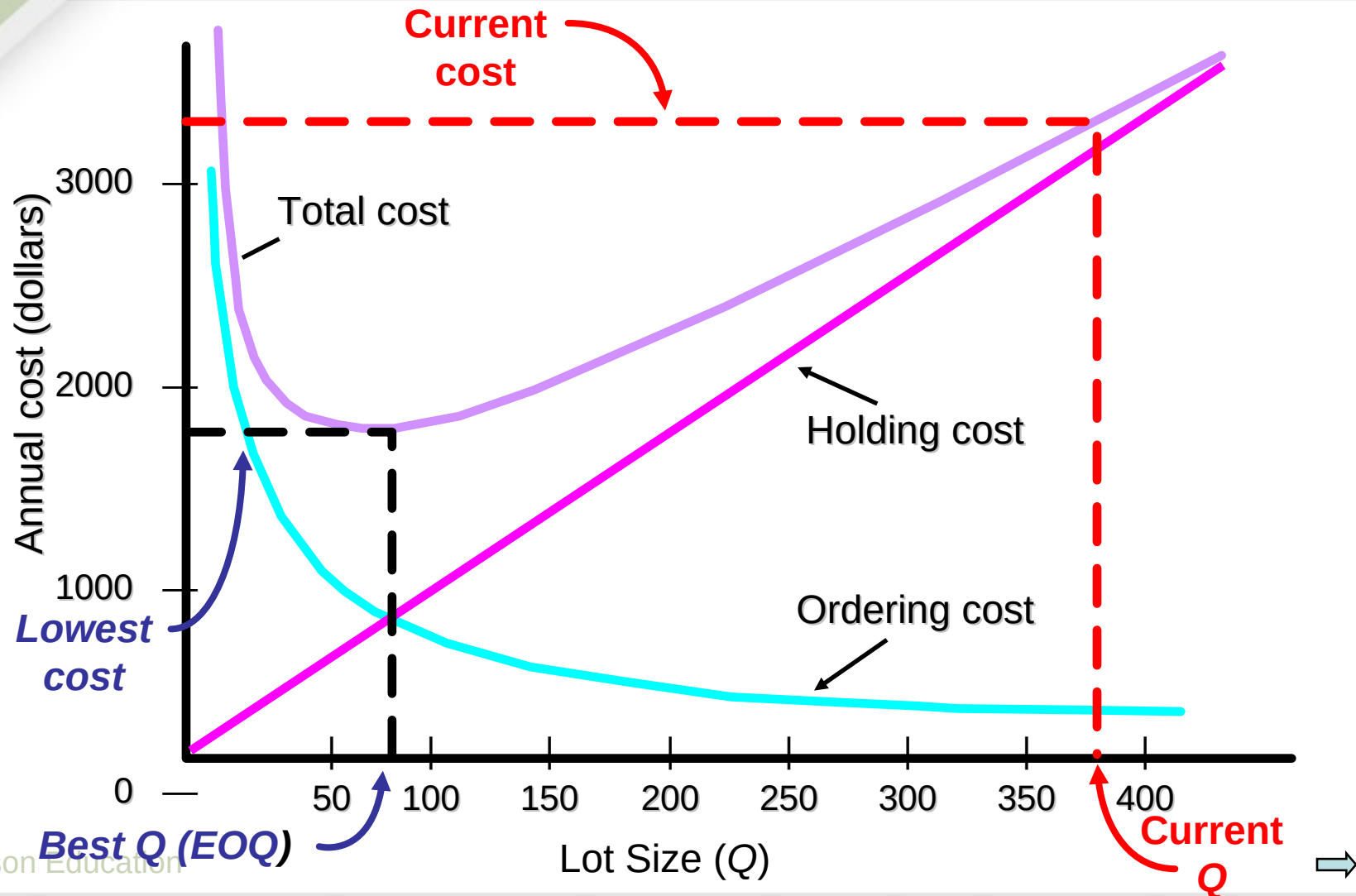
$$C = \$3510 + \$90 = \$3600$$

$Q = 468$  is a more expensive option.

The best lot size (EOQ) is the lowest point on the total annual cost curve!



# Lot Sizing at the Museum of Natural History Gift Shop





# Computing the EOQ

## Example 12.3

### Bird Feeders:

$$EOQ = \sqrt{\frac{2DS}{H}}$$

$D$  = annual demand

$S$  = ordering or setup costs per lot

$H$  = holding costs per unit

$$\begin{aligned} D &= 936 \text{ units} \\ H &= \$15 \\ S &= \$45 \end{aligned}$$

$$EOQ = \sqrt{\frac{2(936)45}{15}} = 74.94 \text{ or } 75 \text{ units}$$

$$C = \frac{Q}{2} (H) + \frac{D}{Q} (S)$$

$$C = \frac{75}{2} (15) + \frac{936}{75} (45)$$

$$C = \$1,124.10$$



# Computing EOQ using the Excel Solver

## Parameters

Current Lot Size (Q)	390
Demand (D)	936
Order Cost (S)	\$45
Unit Holding Cost (H)	\$15

Economic Order Quantity	75
-------------------------	----

## Annual Costs

Orders per Year	2.4
Annual Ordering Cost	\$108.00
Annual Holding Cost	\$2,925.00
Annual Inventory Cost	\$3,033.00

## Annual Costs based on EOQ

Orders per Year	12.48
Annual Ordering Cost	\$561.60
Annual Holding Cost	\$562.50
Annual Inventory Cost	\$1,124.10



# ***Time Between Orders***

- **Time between orders (TBO)** is the average elapsed time between receiving (or placing) replenishment orders of  $Q$  units for a particular lot size.

$$\text{TBO}_{\text{EOQ}} = \frac{\text{EOQ}}{D}$$

**Example 12.3** continued:

- For the birdfeeder example, using an EOQ of 75 units.  $\text{TBO}_{\text{EOQ}} = \frac{\text{EOQ}}{D} = 75/936 = 0.080$  year

$$\text{TBO}_{\text{EOQ}} = (75/936)(12) = 0.96 \text{ months}$$

$$\text{TBO}_{\text{EOQ}} = (75/936)(52) = 4.17 \text{ weeks}$$

$$\text{TBO}_{\text{EOQ}} = (75/936)(365) = 29.25 \text{ days}$$



## ***Application 12.2***

Suppose that you are reviewing the inventory policies on an \$80 item stocked at a hardware store. The current policy is to replenish inventory by ordering in lots of 360 units.

Additional information is:

$D = 60$  units per week, or 3120 units per year

$S = \$30$  per order

$H = 25\%$  of selling price, or \$20 per unit per year

a. What is the EOQ?

$$EOQ = \sqrt{\frac{2DS}{H}} = .$$



# Application 12.2

## continued

- b. What is the total annual cost of the current policy ( $Q = 360$ ), and how does it compare with the cost with using the EOQ?

Current policy	EOQ policy
$Q =$	$Q =$
$C =$	$C =$
$C =$	$C =$
$C =$	$C =$

- c. What is the time between orders (TBO) for the current policy and the EOQ policy, expressed in weeks?



# ***Understanding the Effect of Changes***

- **A Change in the Demand Rate ( $D$ ):** When demand rises, the lot size also rises, but more slowly than actual demand.
- **A Change in the Setup Costs ( $S$ ):** Increasing  $S$  increases the EOQ and, consequently, the average cycle inventory.
- **A Change in the Holding Costs ( $H$ ):** EOQ declines when  $H$  increases.
- **Errors in Estimating  $D$ ,  $H$ , and  $S$ :** Total cost is fairly insensitive to errors, even when the estimates are wrong by a large margin. The reasons are that errors tend to cancel each other out and that the square root reduces the effect of the error.



# ***Inventory Control Systems***

- Inventory control systems tell us how much to order and when to place the order.
  - **Independent demand items**: Items for which demand is influenced by market conditions and is not related to the inventory decisions for any other item held in stock.
- **Continuous review (Q) systems** (Reorder point systems ROP) are designed to track the remaining inventory of an item each time a withdrawal is made to determine whether it is time to reorder.
- **Periodic review (P) systems** (Fixed Interval Reorder systems) in which an item's inventory position is reviewed periodically rather than continuously.



# ***Inventory Control Systems***

- **Inventory position (IP)** is the measurement of an item's ability to satisfy future demand.

$$IP = OH + SR - BO$$

- **Scheduled receipts (SR)** or **Open orders** are orders that have been placed but have not yet been received.
- **Reorder point (R)** is the predetermined minimum level that an inventory position must reach before a fixed order quantity  $Q$  of the item is ordered.



## ***Application 12.3***

The on-hand inventory is only 10 units, and the reorder point,  $R$ , is 100. There are no backorders and one open order for 200 units.

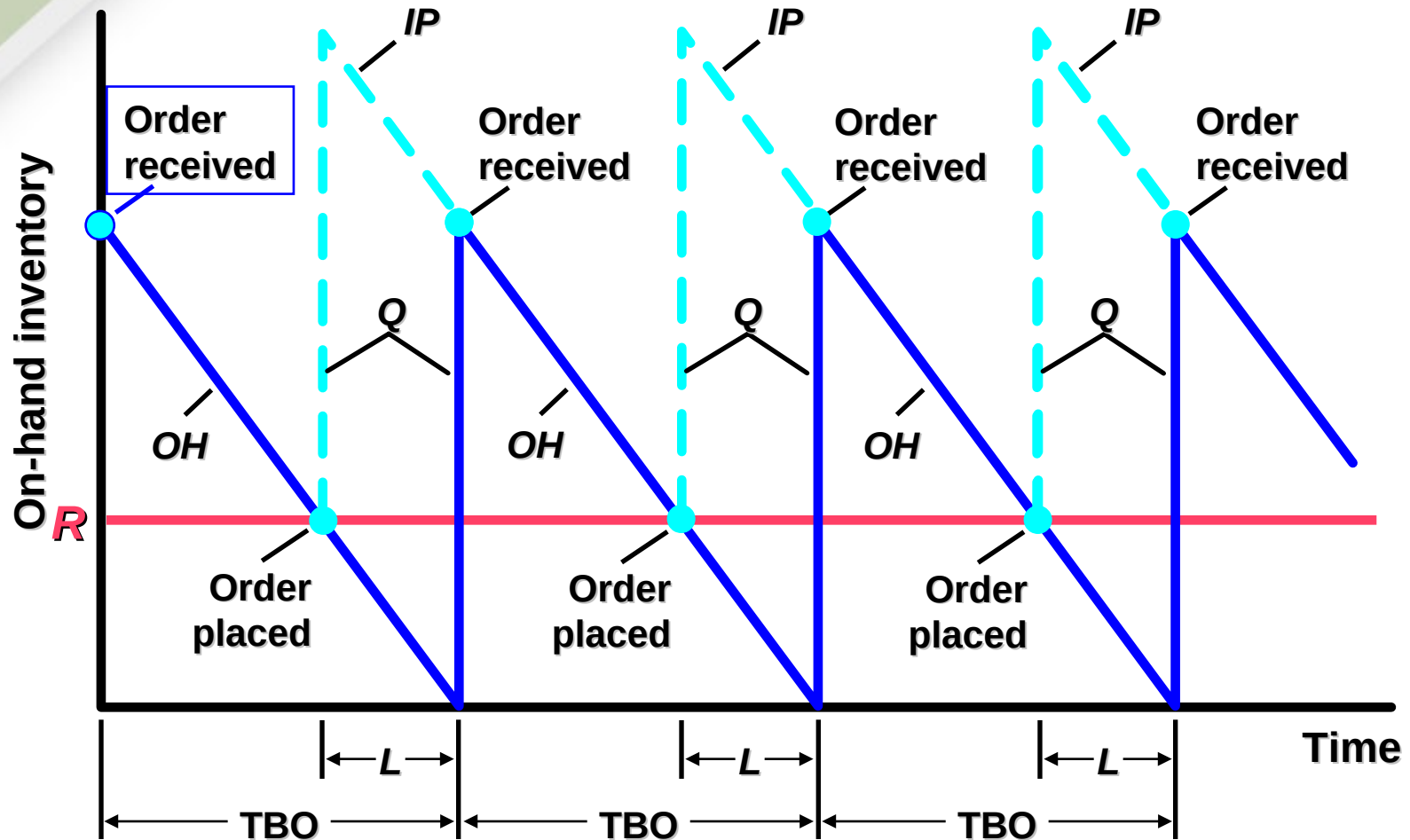
Should a new order be placed?

Decision:



# Continuous Review

$Q$  systems when demand & lead time are constant and certain.



# Determining Whether to Place an Order

## Example 12.4

Demand for chicken soup is always **25** cases a day and lead time is always **4** days. Chicken soup was just restocked, leaving an on-hand inventory of **10** cases. No backorders currently exist. There is an open order for **200** cases. *What is the **inventory position**? Should a new order be placed?*

**$R$  = Average demand during lead time**

$$= (\mathbf{25})(\mathbf{4}) = 100 \text{ cases}$$

**$IP = OH + SR - BO$**

$$= \mathbf{10} + \mathbf{200} - 0 = 210 \text{ cases}$$

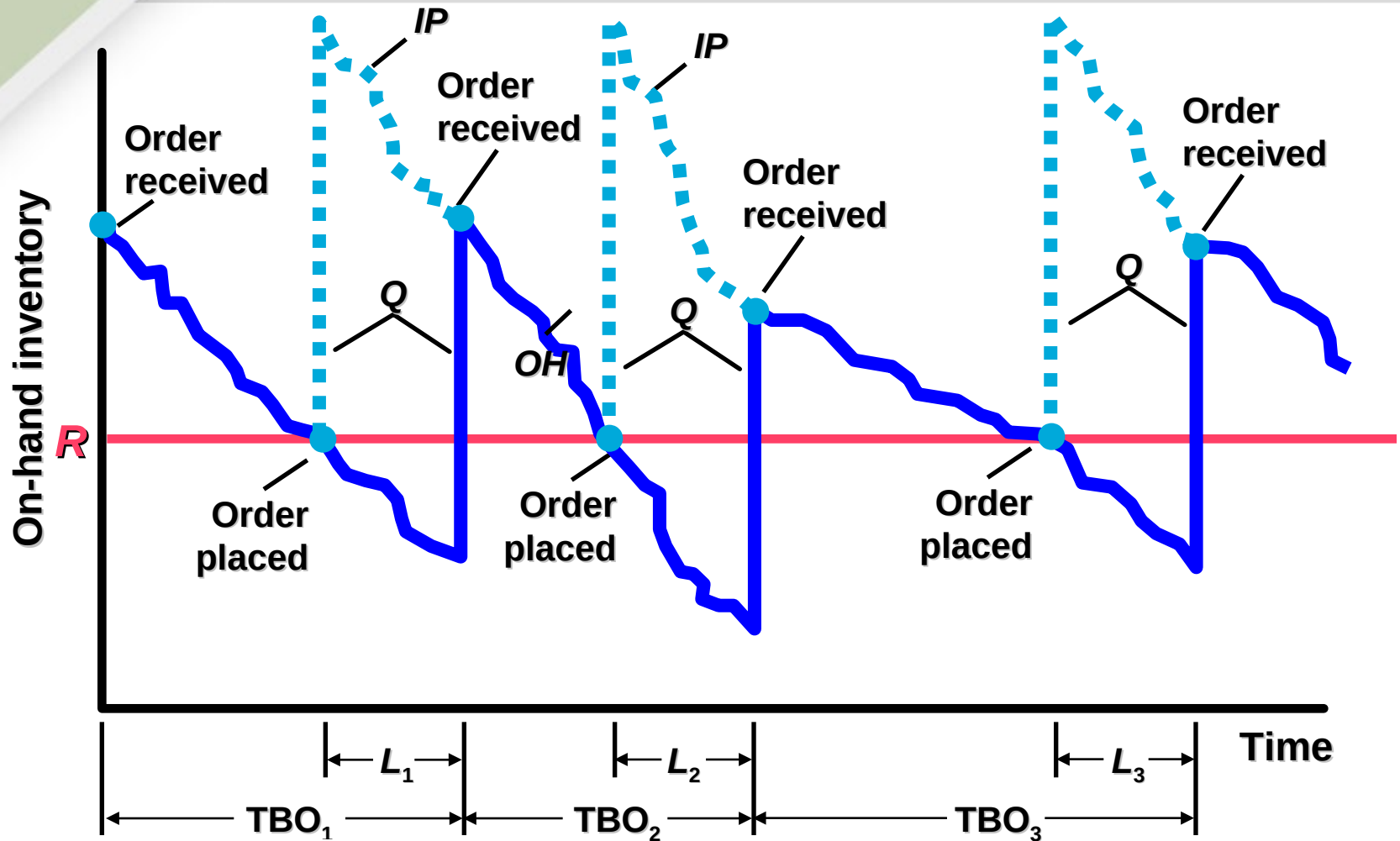
**IP** = Inventory Position  
**OH** = On-hand Inventory  
**SR** = Scheduled receipts  
**BO** = Back ordered

Since IP exceeds  $R$  ( $210 > 100$ ), do not reorder. An SR is pending.



# Continuous Review

*Q system when demand is uncertain.*



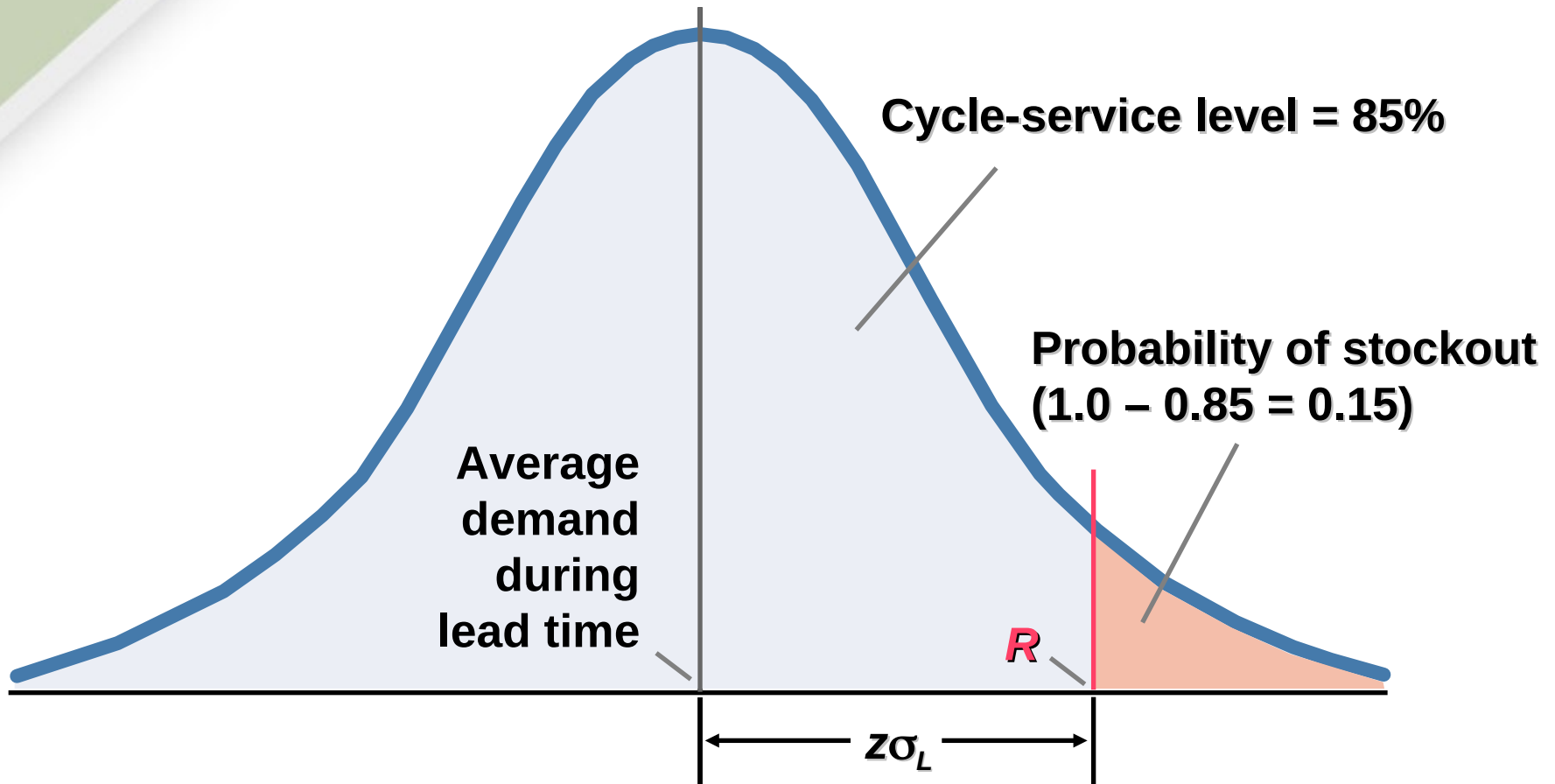
# *Choosing an Appropriate Service-Level Policy*

- **Service level (Cycle-service level):** The desired probability of not running out of stock in any one ordering cycle, which begins at the time an order is placed and ends when it arrives.
- **Protection interval:** The period over which safety stock must protect the user from running out.
- **Safety stock =  $z\sigma_L$** 
  - $z$**  = The number of standard deviations needed for a given cycle-service level.
  - $\sigma_L$**  = The standard deviation of demand during the lead time probability distribution.



# ***Finding Safety Stock***

***With a normal Probability Distribution  
for an 85% Cycle-Service Level***



# ***Finding Safety Stock and R***

## ***Example 12.5***

Records show that the demand for dishwasher detergent during the lead time is normally distributed, with an average of **250** boxes and  $\sigma_L = \mathbf{22}$ . What safety stock should be carried for a 99 percent cycle-service level? What is R?

$$\begin{aligned}\text{Safety stock} &= z\sigma_L \\ &= 2.33(\mathbf{22}) = 51.3 \\ &= \mathbf{51 \text{ boxes}}\end{aligned}$$

$$\begin{aligned}\text{Reorder point} &= \bar{D}_L + \text{SS} \\ &= \mathbf{250} + \mathbf{51} \\ &= \mathbf{301 \text{ boxes}}\end{aligned}$$

2.33 is the number of standard deviations,  $z$ , to the right of average demand during the lead time that places 99% of the area under the curve to the left of that point.



## Application 12.4

Suppose that the demand during lead time is normally distributed with an average of 85 and  $\sigma_L = 40$ .

Find the safety stock, and reorder point  $R$ , for a **95 percent** cycle-service level.

$$\text{Safety stock} = z\sigma_L =$$

$$R = \text{Average demand during lead time} + \text{Safety stock}$$

$$R =$$

Find the safety stock, and reorder point  $R$ , for an **85 percent** cycle-service level.

$$\text{Safety stock} =$$

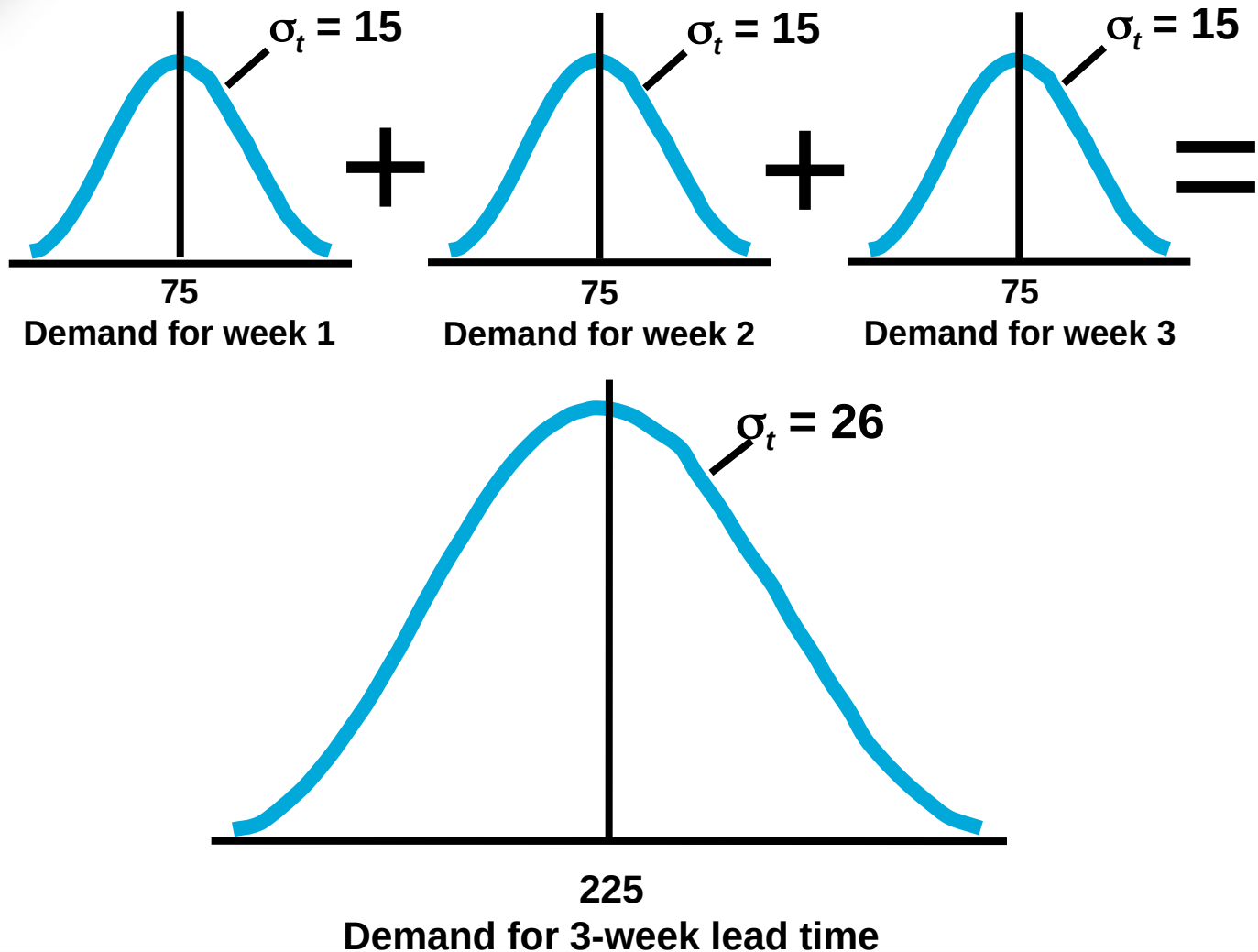
$$R = \text{Average demand during lead time} + \text{Safety stock}$$

$$R =$$





# ***Development of Demand Distributions for the Lead Time***



# ***Finding Safety Stock and R***

## ***Example 12.6***

Suppose that the average demand for bird feeders is 18 units per week with a standard deviation of 5 units. The lead time is constant at 2 weeks. **Determine the safety stock and reorder point for a 90 percent cycle-service level. What is the total cost of the Q system?** ( $t = 1$  week;  $d = 18$  units per week;  $L = 2$  weeks)

Demand distribution for lead time must be developed:

$$\sigma_L = \sigma_t \sqrt{L} = 5 \sqrt{2} = 7.1$$

$$\text{Safety stock} = Z\sigma_L = 1.28(7.1) = 9.1 \text{ or } \mathbf{9 \text{ units}}$$

$$\text{Reorder point} = dL + \text{safety stock} = 2(18) + 9 = \mathbf{45 \text{ units}}$$

$$C = \frac{75}{2}(\$15) + \frac{936}{75}(\$45) + 9(\$15)$$

$$\mathbf{C = \$562.50 + \$561.60 + \$135 = \$1259.10}$$



# Selecting Reorder Point

## *When Demand & Lead Time Are Uncertain*

PROBABILITY DISTRIBUTION FOR LEAD TIME	
Lead Time (weeks)	Probability for Lead Time
1	0.05
2	0.25
3	0.40
4	0.25
5	0.05

PROBABILITY DISTRIBUTION FOR DEMAND	
Demand (units per week)	Probability of Demand
24	0.15
28	0.20
32	0.30
36	0.20
40	0.15



# Simulating Demand (Solved Problem 8)

Microsoft Excel - Q System Simulator

File Edit View Insert Format Tools Data Window Help

Type a question for help

85% Arial 10 B I U

Reply with Changes... End Review...

O68 fx

	D	E	F	G	H	I	J	K	L	M	N	O	P
	Week	Beginning Inventory	Simulated Demand	Ending Inventory	Stockout Units	Place Order?	Simulated Lead Time	Weeks to Receive Order	Holding Cost	Ordering Cost	Stockout Cost	Total Cost	
14	1	170	36	134	0	Yes	4	4	\$ 304	\$ 25	\$ -	\$ 329	
15	2	134	32	102	0	No	-	3	\$ 236	\$ -	\$ -	\$ 236	
16	3	102	40	62	0	No	-	2	\$ 164	\$ -	\$ -	\$ 164	
17	4	62	28	34	0	No	-	1	\$ 96	\$ -	\$ -	\$ 96	
18	5	234	32	202	0	No	-	0	\$ 436	\$ -	\$ -	\$ 436	
19	6	202	40	162	0	No	-	-	\$ 364	\$ -	\$ -	\$ 364	
20	7	162	28	134	0	Yes	2	2	\$ 296	\$ 25	\$ -	\$ 321	
21	8	134	40	94	0	No	-	1	\$ 228	\$ -	\$ -	\$ 228	
22	9	294	32	262	0	No	-	0	\$ 556	\$ -	\$ -	\$ 556	
23	10	262	24	238	0	No	-	-	\$ 500	\$ -	\$ -	\$ 500	
24	11	238	32	206	0	No	-	-	\$ 444	\$ -	\$ -	\$ 444	
25	12	206	40	166	0	No	-	-	\$ 372	\$ -	\$ -	\$ 372	
26	13	166	24	142	0	No	-	-	\$ 308	\$ -	\$ -	\$ 308	
55	41	262	28	234	0	No	-	0	\$ 496	\$ -	\$ -	\$ 496	
56	42	234	40	194	0	No	-	-	\$ 428	\$ -	\$ -	\$ 428	
57	43	194	36	158	0	No	-	-	\$ 352	\$ -	\$ -	\$ 352	
58	44	158	36	122	0	Yes	3	3	\$ 280	\$ 25	\$ -	\$ 305	
59	45	122	36	86	0	No	-	2	\$ 208	\$ -	\$ -	\$ 208	
60	46	86	28	58	0	No	-	1	\$ 144	\$ -	\$ -	\$ 144	
61	47	258	36	222	0	No	-	0	\$ 480	\$ -	\$ -	\$ 480	
62	48	222	36	186	0	No	-	-	\$ 408	\$ -	\$ -	\$ 408	
63	49	186	40	146	0	No	-	-	\$ 332	\$ -	\$ -	\$ 332	
64	50	146	32	114	0	Yes	2	2	\$ 260	\$ 25	\$ -	\$ 285	
65	<b>Averages</b>	<b>167.12</b>	<b>33.12</b>	<b>134.00</b>	<b>0.00</b>		<b>3.00</b>		<b>\$301.12</b>	<b>\$4.50</b>	<b>\$0.00</b>	<b>\$305.62</b>	

# ***Application 12.5***

## ***Putting it all together for a Q System***

The Discount Appliance Store uses a continuous review system (Q system). One of the company's items has the following characteristics:

Demand = 10 units/wk (assume 52 weeks per year)

Ordering and setup cost ( $S$ ) = \$45/order

Holding cost ( $H$ ) = \$12/unit/year

Lead time ( $L$ ) = 3 weeks

Standard deviation in weekly demand = 8 units

Cycle-service level = 70%

What is the EOQ for this item?

$D =$

$$EOQ = \sqrt{\frac{2DS}{H}} = .$$



# ***Application 12.5***

## ***Putting it all together for a Q System***

What is the desired safety stock?

$$\sigma_L = \sigma_t \sqrt{L} \quad \sigma_L =$$

$$\text{Safety stock} = z \sigma_L =$$

What is the desired reorder point  $R$ ?

$$R = \text{Average demand during lead time} + \text{Safety stock}$$

$$R =$$

What is the total annual cost?

$$C =$$



# ***Application 12.5***

## ***Putting it all together for a Q System***

Suppose that the current policy is  $Q = 80$  and  $R = 150$ .

What will be the changes in average cycle inventory and safety stock if your EOQ and  $R$  values are implemented?

Reducing  $Q$  from      to

Cycle inventory reduction =

Safety stock reduction =

Reducing  $R$  from      to





# Selecting the Reorder Point When Both Demand and Lead Time Are Variable

In practice, it is often the case that both the demand and the lead time are variable. Unfortunately, the equations for the safety stock and reorder point become more complicated. In the model below we make two simplifying assumptions. First, the demand distribution and the lead time distribution are measured in the same time units. For example, both demand and lead time are measured in weeks. Second, demand and lead time are *independent*. That is, demand per week is not affected by the length of the lead time.

$$\text{Safety stock} = z\sigma_{dLT}$$

$$\begin{aligned} R &= (\text{Average weekly demand} \times \text{Average lead time in weeks}) + \text{Safety stock} \\ &= \bar{d}\bar{L} + \text{Safety stock} \end{aligned}$$

where

$\bar{d}$  = Average weekly or daily or monthly demand

$\bar{L}$  = Average weekly or daily or monthly lead time

$\sigma_d$  = Standard deviation of weekly or daily or monthly demand

$\sigma_{LT}$  = Standard deviation of the lead time, and

$$\sigma_{dLT} = \sqrt{\bar{L}\sigma_d^2 + \bar{d}^2\sigma_{LT}^2}$$

Now that we have determined the mean and standard deviation of the distribution of demand during lead time under these more complicated conditions, we can select the reorder point as we did before for the case where the lead time was constant.

[Go to Settings](#)



# Example

The Office Supply Shop estimates that the average demand for a popular ball-point pen is 12,000 pens per week with a standard deviation of 3,000 pens. The current inventory policy calls for replenishment orders of 156,000 pens. The average lead time from the distributor is 5 weeks, with a standard deviation of 2 weeks. If management wants a 95 percent cycle-service level, what should the reorder point be?

## SOLUTION

We have  $\bar{d} = 12,000$  pens,  $\sigma_d = 3,000$  pens,  $\bar{L} = 5$  weeks, and  $\sigma_{LT} = 2$  weeks.

$$\sigma_{dLT} = \sqrt{\bar{L}\sigma_d^2 + \bar{d}^2\sigma_{LT}^2} = \sqrt{(5)(3,000)^2 + (12,000)^2(2)^2} = 24,919.87 \text{ pens}$$

Consult the body of the Normal Distribution appendix for 0.9500, which corresponds to a 95 percent cycle-service level. That value falls exactly in the middle of the tabular values of 0.9495 (for a  $z$  value of 1.64) and 0.9505 (for a  $z$  value of 1.65). Consequently, we will use the more conservative value of 1.65. We calculate the safety stock and reorder point as follows:

$$\text{Safety stock} = z\sigma_{dLT} = (1.65)(24,919.87) = 41,117.79, \text{ or } 41,118 \text{ pens}$$

$$\text{Reorder point} = \bar{d}\bar{L} + \text{Safety stock} = (12,000)(5) + 41,118 = 101,118 \text{ pens}$$

## DECISION POINT

Whenever the stock of ball-point pens drops to 101,118, management should place another replenishment order of 156,000 pens to the distributor.

# ***Practice Problem***

Grey Wolf Lodge is a popular 500-room hotel in the North Woods. Managers need to keep close tabs on all room service items, including a special pine-scented bar soap. The daily demand for the soap is 275 bars, with a standard deviation of 30 bars. Ordering cost is \$10 and the inventory holding cost is \$0.30/bar/year. The lead time from the supplier is 5 days, with a standard deviation of 1 day. The lodge is open 365 days a year

- a. What is the economic order quantity for the bar of soap?
- b. What should the reorder point be for the bar of soap if management wants to have a 99 percent cycle-service level?
- c. What is the total annual cost for the bar of soap, assuming a Q system will be used?

# ***Approaches for Inventory Record Accuracy***

- Assign responsibility for reporting inventory transactions to specific employees.
- Secure inventory in locked storage areas.
- **Cycle counting**, an inventory control method, whereby storeroom personnel physically count a small percentage of the total number of items each day, correcting errors that they find, is used to frequently check records against physical inventory.
- Logic error checks on each transaction; reporting and fully investigating discrepancies.
- If inventory records prove to be accurate over several years' time, the annual physical count can be avoided. It is disruptive, adds no value to the products, and often introduces as many errors as it removes.



# ***Solved Problem 1***

A distribution center's average weekly demand is 50 units for an item valued at \$650 per unit. Shipments from the warehouse average 350 units. Average lead time (including ordering delays and transit time) is 2 weeks. The distribution center operates 52 weeks per year & carries a 1-week supply as safety stock and no anticipation inventory. **What is the average aggregate inventory being held by the distribution center?**



# Solution

Type of Inventory	Calculation of Average Inventory Quantity
Cycle	$\frac{Q}{2} = \frac{350}{2} = 175$ units
Safety stock	1-week supply = 50 units
Anticipation	None
Pipeline	$dL = (50 \text{ units/week})(2 \text{ weeks}) = 100$ units
	<i>Average aggregate inventory</i> = 325 units

## Solved Problem 2

Booker's Book Bindery divides inventory items into 3 classes, according to their dollar usage. Calculate the usage values of the following inventory items and determine which is most likely to be classified as an A item.

Part Number	Description	Quantity Used per Year		Unit Value(\$)		Annual Dollar Usage(\$)
1	Boxes	500	×	3.00	=	1,500
2	Cardboard (square feet)	18,000	×	0.02	=	360
3	Cover stock	10,000	×	0.75	=	7,500
4	Glue (gallons)	75	×	40.00	=	3,000
5	Inside covers	20,000	×	0.05	=	1,000
6	Reinforcing tape (meters)	3,000	×	0.15	=	450
7	Signatures	150,000	×	0.45	=	67,500
<i>Total</i>						81,310

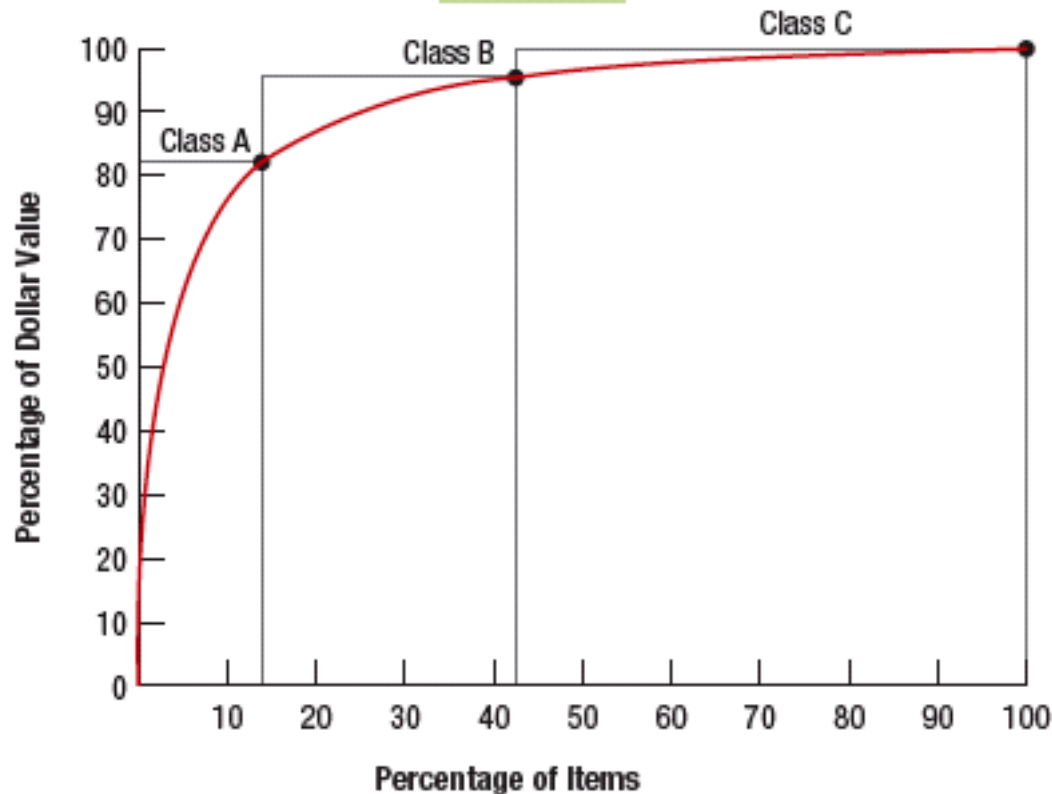


# Solved Problem 2 continued

Part #	Description	Qty Used/Year	Value	Dollar Usage	Pct of Total	Cumulative % of Dollar Value	Cumulative % of Item	Class
7	Signatures	150,000	\$0.45	\$67,500	83.0%	83.0%	14.3%	A
3	Cover stock	10,000	\$0.75	\$7,500	9.2%	92.2%	28.6%	B
4	Glue	75	\$40.00	\$3,000	3.7%	95.9%	42.9%	B
1	Boxes	500	\$3.00	\$1,500	1.8%	97.8%	57.1%	C
5	Inside covers	20,000	\$0.05	\$1,000	1.2%	99.0%	71.4%	C
6	Reinforcing tape	3,000	\$0.15	\$450	0.6%	99.6%	85.7%	C
2	Cardboard	18,000	\$0.02	\$360	0.4%	100.0%	100.0%	C

Total

\$81,310



## ***Solved Problem 3***

EOQ, is 75 units when annual demand,  $D$ , is 936 units/year, setup cost,  $S$ , is \$45, and holding cost,  $H$ , is \$15/unit/year. If we mistakenly estimate inventory holding cost to be \$30/unit/year, what is the new order quantity,  $Q$ , if  $D = 936$  units/year,  $S = \$45$ , and  $H = \$30$ /unit/year? What is the change in order quantity, expressed as a percentage of the EOQ (75 units)?





# ***Solution***

The new order quantity is

$$EOQ = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(936)(\$45)}{\$30}} = \sqrt{2,808} = 52.99 \text{ or } 53 \text{ units}$$

The change in percentage is

$$\left( \frac{53 - 75}{75} \right) (100) = -29.33 \text{ percent}$$

# Problem 4

A regional distributor purchases discontinued appliances from various suppliers and then sells them on demand to retailers in the region. The distributor operates 5 days per week, 52 weeks per year. Only when it is open for business can orders be received. The following data are estimated for a counter-top mixer:

Average daily demand ( $\bar{d}$ ) = 100 mixers

Standard deviation of daily demand ( $\sigma_d$ ) = 30 mixers

Lead time ( $L$ ) = 3 days

Holding cost ( $H$ ) = \$9.40/unit/year

Ordering cost ( $S$ ) = \$35/order

Cycle-service level = 92 percent

The distributor uses a continuous review  $Q$  system.

- a. What order quantity  $Q$ , and reorder point,  $R$ , should be used?
- b. What is the total annual cost of the system?
- c. If on-hand inventory is 40 units, one open order for 440 mixers is pending, and no backorders exist, should a new order be placed?

# Solution

a. Annual demand is

$$D = (5 \text{ days/week})(52 \text{ weeks/year})(100 \text{ mixers/day}) = 26,000 \text{ mixers/year}$$

The order quantity is

$$\text{EOQ} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(26,000)(\$35)}{\$9.40}} = \sqrt{193,167} = 440.02, \text{ or } 440 \text{ mixers}$$

The standard deviation of the distribution of demand during lead time is

$$\sigma_{dLT} = \sigma_d \sqrt{L} = 30 \sqrt{3} = 51.96$$

A 92 percent cycle-service level corresponds to  $z = 1.41$  (see the Normal Distribution appendix). Therefore,

$$\text{Safety stock} = z\sigma_{dLT} = 1.41(51.96 \text{ mixers}) = 73.26, \text{ or } 73 \text{ mixers}$$

$$\text{Average demand during the lead time} = \bar{d}L = 100(3) = 300 \text{ mixers}$$

$$\begin{aligned} \text{Reorder point } R &= \text{Average demand during the lead time} + \text{Safety stock} \\ &= 300 \text{ mixers} + 73 \text{ mixers} = 373 \text{ mixers} \end{aligned}$$

With a continuous review system,  $Q = 440$  and  $R = 373$ .

# Solution

- b.** The total annual cost for the  $Q$  systems is

$$C = \frac{Q}{2}(H) + \frac{D}{Q}(S) + (H)(\text{Safety stock})$$

$$C = \frac{440}{2}(\$9.40) + \frac{26,000}{440}(35) + (\$9.40)(73) = \$4,822.38$$

- c.** Inventory position = On-hand inventory + Scheduled receipts – Backorders

$$IP = OH + SR - BO = 40 + 440 - 0 = 480 \text{ mixers}$$

Because IP (480) exceeds  $R$  (373), do not place a new order.