Weekly Challenge 05: Regularity

CS 212 Nature of Computation
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1. Regular?

Prove or disprove the following claim.

Claim 1. The language, $L = \{w^i w^j \mid w \in \{0,1\}^*, 0 < i \leq j\}$, is regular.

Solution: The language $L = \{w^i w^j \mid w \in \{0,1\}^*, 0 < i \leq j\}$ consists of all possible concatenations of '0's and '1's repeated i times, and then j times where $0 < i \leq j$.

Assume that L is regular, then the pumping lemma should hold. Consider a string $s = 0^p 1 0^p 1$ where $w = 0^p 1, i = 1, j = 1$. Since $s \in L$, and $|s| \geq p$, the pumping pemma guarantees that s can be split into three pieces, $s = xyz$ such that:

1. for each $i \geq 0$, $xy^i z \in L$,
2. $|y| > 0$
3. $|xy| \leq p$

Condition 3 of the pumping lemma guarantees that y can only consist of 0s;

- y cannot be a 1, as that would imply $x = 0^p$, then $|xy| > p$, furthermore, then the first part of the string would have more 1s than the second part, which is a contradiction.
- By the same argument as above, y cannot be a combination of a 1 followed by some 0s either as that would imply $x = 0^p$, then $|xy| > p$

Then $xy = 0^p$ and $y = 0^m$, and $x = 0^{p-m}$ Pumping y into the string,

$$xyyz = 0^p 0^m 1 0^p 1$$
$$xy^2 z = 0^{p+m} 1 0^p 1$$

This shows that the first part of the string will contain more number of 0s than the later half since $p + m > p$, and since the later half still has only p 0s, then $xy^2 z \notin L$, hence we arrive at a contradiction as the pumping lemma does not hold.

Hence proved that L is not regular.

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