Homework-1 Solution

Q1. Convert the following numbers with the indicated bases to decimal:

Q1) a) $(124)_5$

$\Rightarrow 1\times5^2 + 2\times5^1 + 4\times5^0 = \boxed{39}$

b) $(397)_{12}$

$\Rightarrow 3\times12^2 + 9\times12^1 + 7\times12^0 = 547$

c) $(534.276)_8$

$\Rightarrow 5\times8^2 + 3\times8^1 + 4\times8^0 + 2\times8^{-1} + 7\times8^{-2} + 6\times8^{-3} =$

$\boxed{348.3710938}$

d) $(0.234)_6$

$\Rightarrow 0\times6^0 + 2\times6^{-1} + 3\times6^{-2} + 4\times6^{-3} \Rightarrow \boxed{0.4351851852}$

Q2. Convert the following number

Q. Convert the following numbers.

a) $(715)_8$ to base-13

$\rightarrow 7\times8^2+1\times8^1\times5\times8^0 = 461.$

$\rightarrow (715)_8 = (296)_{13}.$

13	461	
13	35	6
13	2	9
	0	2

b) $(34.41)_6$ to octal.

$\rightarrow 3\times6^1+4\times6^0+4\times6^{-1}+1\times6^{-2}$

$\hookrightarrow 22.69444444$

$\Rightarrow (34.41)_6 = (26.5434)_8.$

8	22	
8	2	6
	0	2

$\times 8$

$0.69444444 = 5+0.556$

$'' \times 8 = 4+0.444$

$'' \times 8 = 3+0.555$

$'' \times 8 = 4+0.444$

c) $(0.6775)_8$ to hexadecimal.

~~$\rightarrow 0\times8+6\times8+6\times7+$~~

$\rightarrow 0\times8^0+6\times8^{-1}+7\times8^{-2}+7\times8^{-3}+5\times8^{-4}$

$= 0.8742675781 \rightarrow$ (a)

$\rightarrow (0.6775)_8 = (0.DFD)_{16}$

(a) $\times 16 = 13+0.988..$

$'' \times 16 = 15+0.8125$

$'' \times 16 = 13$

d) $(110110.011)_2$ to decimal.

$\Rightarrow 1\times2^5+1\times2^4+0\times2^3+1\times2^2+1\times2^1+0\times2^0+0\times2^{-1}+1\times2^{-2}+1\times2^{-3}$

$\Rightarrow 54.375.$

Q3. A 12-bit register stores 100010010111. Determine what value is stored in decimal in the register, if the bits represent:

Date:

3) 12-bit

1000 10010111

a) $(1\times2^{11}) + (0\times2^{10}) + (0\times2^{9}) + (0\times2^{8}) + (1\times2^{7}) + (0\times2^{6}) + (0\times2^{5}) + (1\times2^{4}) + (0\times2^{3}) + (1\times2^{2}) + (1\times2^{1}) + (1\times2^{0})$

$= (2199)_{10}$

b) Signed number (2's complement)

1 0 0 0 1 0 0 1 0 1 1 1

0 1 1 1 0 1 1 0 1 0 0 0

+1

0 1 1 1 0 1 1 0 1 0 0 1

$(0\times2^{11}) + (1\times2^{10}) + (1\times2^{9}) + (1\times2^{8}) + (0\times2^{7}) + (1\times2^{6}) + (1\times2^{5}) + (0\times2^{4}) + (1\times2^{3}) + (0\times2^{2}) + (0\times2^{1}) + (1\times2^{0}) =$

$1024 + 512 + 256 + 64 + 32 + 8 + 1 = \boxed{-1897}$

Date:

c) Signed number 1's complement

1000 1001 0111

$\Rightarrow$ 0111 0110 1000

$\Rightarrow (0\times2^{11}) + (1\times2^{10}) + (1\times2^{9}) + (1\times2^{8}) + (0\times2^{7}) + (1\times2^{6}) + (1\times2^{5}) + (0\times2^{4}) + (1\times2^{3}) + (0\times2^{2}) + (0\times2^{1}) + (0\times2^{0})$

$\Rightarrow 1024 + 512 + 256 + 64 + 32 + 8 = -1896$

d) BCD

8 4 2 1

1000 1001 0111

897

Q4.

a. (05 marks) Fill in the following table and state your assumptions (4 marks)

Solution:

Actual	0	1	2	3	4	5	6	7	8	9	10	11	12
Coded	@	#	$	%	+	&	<	>	#@	##	#$	#%	#+

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b. (05 marks) Represent 43_{10} minutes in your coded notation (3 marks)

Solution:

Quotient	**Remainder**	**Code**	
43			
43/8 = 5	3	%	least significant
5/8 = 0	5	&	most significant

$(43)_{10}=(\&\%)_{coded}$

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c. (05 marks) In your number system what does ($@#) represents in decimal?

Solution:

$ = 2 , @ = 0 , # = 1

$(x)_{10}=(8^2)(2)+(8^1)(0)+(8^1)(1)x10$

$(x)_{10}=128+0+1x10=128+0+1$

$(x)_{10}=129_{10}$

Q5. Given equation:

(34+25) x 21 = 1481

Let ‘b’ be the base of number of system.

5) $(3\times b^1 + 4\times b^0 + 2\times b^1 + 5\times b^0)(2\times b^1 + 1\times b^0) = b^3 + 4b^2 + 8 + 1$

$(3b + 4 + 2b + 5)(2b + 1) = b^3 + 4b^2 + 8 + 1$

$6b^2 + 8b + 4b^2 + 10b + 3b + 4 + 2b\ 15$

$10b^2 + 23b + 9 = b^3 + 4b^2 + 8b + 1$

$6b^2 + 15b + 8 = b^3$

$0 = b^3 - 6b^2 - 15b - 8$

$b = 8, \; b = -1$

From above results, the solution 8 is positive but since the equation has figure 8 in it therefore the given equation cannot be correct in base-8.
N.B: If any student has worked on this solution and come-up with explanation equivalent to above will receive full marks.

(This question has been changed by the professor)

Correct Equation: $(34 + 25) \times 45 = 3425$

Solution: If this equation is written in arbitrary base say ‘r’

$(3 \times r + 4 \times 1 + 2 \times r + 5 \times 1) \times (4 \times r + 5 \times 1) = 3 \times r^3 + 4 \times r^2 + 2 \times r + 5 \times 1$

$(5r + 9) \times (4r + 5) = 3r^3 + 4r^2 + 2r + 5$

$20r^2 + 61r + 45 = 3r^3 + 4r^2 + 2r + 5$

$3r^3 - 16r^2 - 59r - 40 = 0$

Following are solutions of cubic equation: -1.667, -1,8

Since the base and hence number of fingers cannot be negative, so aliens must be using base-8 system.

Q6.

2) method 1

a)

2	786
2	393 - 0
2	196 - 1
2	98 - 0
2	49 - 0
2	24 - 1
2	12 - 0
2	6 - 0
2	3 - 0
	1 - 1

$(1100010010)_2$

method 2

16	786
16	49 - 2
	3 - 1

$(312)_{16}$

$(0011\ 0001\ 0010)_2$

Ans: 2nd method more faster since it takes less time and saves us from repeatedly dividing the numbers by base 2 as in method 1

(Ans may vary depending on the method used)

Q7.

a)

→	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
base-20:	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F

→	16	17	18	19.
base-20:	G	H	I	J

b) person B pays the principal amount of $(DHJ5F)_{20}$ along with the $(21DHH)_{20}$ 6-month interest and twice the $(5B3E)_{20}$ interest as the total interval is 8-months.

hence;

$$
\begin{array}{rccccc}
 & {}^{1} & {}^{2} & {}^{1} & {}^{3} & \\
 & D & H & J & 5 & F \\
 & 2 & 1 & D & H & H \\
 & & 5 & B & 3 & E \\
+ & & 5 & B & 3 & E \\
\hline
\text{Total amount :} & G & A & F & B & 0
\end{array}
$$

The total amount person B pays is $(GAFBO)_{20}$.

Note:- Stepwise calculation/addition can also be accepted.
→ Conversion to decimal & reconversion to base 20 is also acceptable.

c)

$$
\begin{array}{rccccc}
 & & {}^{1} & {}^{1} & {}^{2} & \\
 & 2 & 1 & D & H & H \\
 & & 5 & B & 3 & E \\
+ & & 5 & B & 3 & E \\
\hline
 & 2 & C & G & 5 & 5
\end{array}
$$

Profit = $(2CG55)_{20}$.

Conversion :-

$$2 \times 20^4 + 12 \times 20^3 + 16 \times 20^2 + 5 \times 20^1 + 5 \times 20^0$$

$$\Rightarrow (422505)_{10}.$$

Q8. Convert the last three alphabets of your first name and first three alphabets of your last name to **hexadecimal** system, according to the given table. (3x3)

a	b	c	d	e	f	g	h	i	j	k	l	m
61	62	63	64	65	66	67	68	69	6A	6B	6C	6D

n	o	p	q	r	s	t	u	v	w	x	y	z
6E	6F	70	71	72	73	74	75	76	77	78	79	7A

For example, your name is "Hassan Ali", you'll convert "san" from Hassan and "ali" from Ali as follows. (Use lowercase letters)

"san" ➔ 73616E

"ali" ➔ 616C69

(Let the first number be X1 and the last number be X2)

a) Using 16's Compliment method, perform the following:

X1 - X2

6 1 6 C 6 9 ➔ 16's Complement = 9E9397

7 3 6 1 6 E

9 E 9 3 9 7

----

1 1 1 F 5 0 5 Ans

b) Using 15's Compliment method, perform the following:

X2 – X1

7 3 6 1 6 E ➔ 15's Complement = 8C9E91

6 1 6 C 6 9

8 C 9 E 9 1

E E 0 A F A Ans

c) Convert both X1 and X2 to radix 8 and find their sum

73616E → 011 100 110 110 000 101 101 110 → 34660556

616C69 → 011 000 010 110 110 001 101 001 → 30266151

3 4 6 6 0 5 5 6

3 0 2 6 6 1 5 1

----

6 5 1 4 6 7 2 7 Ans

Q9. Using the r's compliment method, where r is the radix of the numbers mentioned, find the difference between the following: (2x4)

a) $(11043)_8$ and $(5214)_8$

1 1 0 4 3 4643

7 2 5 6 4 → Verified 2700

--------------- ---------------

1 0 3 6 2 7 (1 discarded) 1943

b) $(10111.001)_2$ and $(01101.001)_2$

1 0 1 1 1. 0 0 1 23.125

1 0 0 1 0. 1 1 1 → Verified 13.125 ----------

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1 0 1 0 1 0. 0 0 0 (1 discarded) 10.000

Q10

(a) 579 − 286.

579 = 0101 0111 1001

286 = 0010 1000 0110

10's complement for 286 = 714

714 = 0111 0001 0100

Adding 579 and 714.

$$
\begin{array}{rrrr}
 & 0101 & 0111 & 1001 \\
+ & 0111 & 0001 & 0100 \\
\hline
 & 1100 & 1000 & 1101 \\
+ & 0110 & & 0110 \\
\hline
\cancel{1} & 0010 & 1001 & 0011 \\
\hline
\end{array}
$$

carry discarded

∴ 0010 1001 0011

or

∴ $\boxed{293}$

b) 865 + 1365

865 = 1000 0110 0101

1365 = 0001 0011 0101 0110

$$
\begin{array}{rrrrr}
\rightarrow & & 1000 & 0110 & 0101 \\
 & 0001 & 0011 & 0101 & 0110 \\
\hline
 & 0001 & 1011 & 1011 & 1011 \\
 & & 0110 & 0110 & 0110 \\
\hline
 & 0010 & 0010 & 0010 & 0001 \\
\hline
\end{array}
$$

∴ 0010 0010 0010 0001

or

$\boxed{2221}$

(c) 78.54 − 32.91.

78.54 = 0111 1000 . 0101 0100

32.91 = 0011 0010 . 1001 0001

10's complement of 32.91 = 67.09

67.09 = 0110 0111 . 0000 1001

adding 78.54 and 67.09:

```
   0111 1000 . 0101 0100
 + 0110 0111 . 0000 1001
 -----------------------
   1101 1111 . 0101 1101
   0110 0110        0110
 -----------------------
 1 0100 0101 . 0110 0011
```

carry discarded.

∴ 0100 0101 . 0110 0011

or

$\boxed{45.63}$

Q4)(d) 0.4938 + 1.965

0.4938 = 0000 . 0100 1001 0011 1000

1.9650 = 0001 . 1001 0110 0101 0000

```
   0000 . 0100 1001 0011 1000
 + 0001 . 1001 0110 0101 0000
 ----------------------------
   0001 . 1101 1111 1000 1000
          0110 0110
 ----------------------------
   0010 . 0100 0101 1000 1000
```

∴ 0010 . 0100 0101 1000 1000

or

$\boxed{2.4588}$

Q11

a) $(+9)_{10} + (+12)_{10}$

01001

01100

10101 (overflow)

b) $(+12)_{10} - (+9)_{10}$

01100 - 01001 (taking 2's complement of the subtrahend)

0 1 1 0 0

1 0 1 1 1

----

1 0 0 0 1 1 discarding 1 we get +3 (no overflow)

c) $(-13)_{10} + (-12)_{10}$

10011 + 10100

1 0 0 1 1

1 0 1 0 0

----

1 0 0 1 1 1 (discarding 1) (overflow)

d) $(-13)_{10} + (-5)_{10}$

10011 + 11011

1 0 0 1 1

1 1 0 1 1 -

----

1 0 1 1 1 0 (overflow)

Q12.

12) 2's complement range

$\Rightarrow (-2^{8-1}, 2^{8-1}-1)$

$\Rightarrow (-128, 127)$

8-bit

positive numbers from 0 to 127

largest $(0\,1111111)_2$

$(127)_{10}$

$$\begin{array}{r|l}
2 & 127 \\ \hline
2 & 63 - 1 \\
2 & 31 - 1 \\
2 & 15 - 1 \\
2 & 7 - 1 \\
2 & 3 - 1 \\
 & 1 - 1
\end{array}$$

$(0\,1111111)_2$

$(7F)_{16}$

largest number

$$\left.\begin{array}{l} (0111\ 1111)_2 \\ (127)_{10} \\ (7F)_{16} \end{array}\right\}$$

Q13. Complete the following table using 8-bit binary numbers:

Decimal	Unsigned Numbers	Sign Magnitude	Signed 1's Complement	Signed 2's Complement
-0	0000 0000	1000 0000	1111 1111	*
-8	0000 1000	1000 1000	1111 0111	1111 1000
+40	0010 1000	0010 1000	0010 1000	0010 1000
-64	0100 0000	1100 0000	1011 1111	1100 0000
+127	0111 1111	0111 1111	0111 1111	0111 1111
-128	1000 0000	-	-	1000 0000

Note: Hyphen (-) in above table represents there is no representation for given number of bits.

* : -0 is same as +0 in 2's complement representation and is represented as 0000 0000

Q14.

Solution:

(a) Number of 1's in given number = 5 (Odd)

Therefore (even) parity bit = 1

(b) Number of 1's in given number = 3 (Odd)

Therefore (even) parity bit = 1

Parity bit is usually added to the left of given numbers.

for (a): 111011010

for (b): 110000101

Q15.

a) BCD = 1001 0101 0011 0111 = 9537_{10}

Excess-3 code can be obtained by adding three (0011) to BCD.

Excess-3 = 1100 1000 0110 1010

b)

$1001\ 0101\ 0011\ 0111_2$ = $1101\ 1111\ 1010\ 1100_{Gray}$

Steps:
Find gray code for given binary number:

Binary	Gray	Comments
1	1	MSB of Gray Code and Binary is same
0	1	Digit after MSB is XOR of MSB and digit right to MSB
0	0	Repeat till the end- Some examples of steps are highlighted.
1	1	
0	1	
1	1	
0	1	1 XOR 0 = 1
1	1	
0	1	
0	0	0 XOR 0 = 0
1	1	
1	0	
0	1	
1	1	0 XOR 1 = 1
1	0	
1	0	