Habib University - City Campus
Instructors: Aeyaz Jamil Keyani
Course: MATH 307 Mathematical Foundations and Reasoning
Examination: Quiz 4 – Spring 2025
Exam Date: Tuesday, February 18, 2025
Exam Time: 9:20 – 9:30

Total Marks: 10 Marks | Duration: 10 Minutes

Name: ______________________ Student ID: ________________ Section: ____________________

1. $\boxed{\text{5 points}}$ Let a and b be some integers. Show that exactly one of the statements $a > b$, $a < b$, or $a = b$ is true.

Solution: Let $a, b \in \mathbb{Z}$. First suppose $a = b$ then if $a > b$ we have a non zero natural number c such that $a = b + c$ and as $a = b$ by cancellation law we have that $c = 0$ and so we have that $a > b$ must be false. The same argument follows for $b > a$ so we have that if $a = b$ then $a > b$ and $b > a$ are both false.

Now suppose $a \neq b$ then lets consider the integer $a - b$. As $a \neq b$ we have that $a - b \neq 0$. So $n = a - b$ is either a positive natural number or negation of a positive natural number. If $n \in \mathbb{N}^+$ then we have that $a > b$, if n is not a positive natural number then $-n$ is a positive natural number and so $a - b = n \iff b - a = -n$ and as $-n \in \mathbb{N}^+$ we have that $b > a$. We know $a > b$ and $b > a$ can't both be true at the same time as $n \in \mathbb{N}^+$ and $-n \in \mathbb{N}^+$ can't both be true at the same time.

$\square$

2. $\boxed{\text{5 points}}$ Let x, y, z be rationals. Show that $x(y + z) = xy + xz$ and $(y + z)x = yx + zx$.

Solution: Let $x = a//b$, $y = c//d$ and $z = e//f$ then from definition of addition and multiplication on rationals we have

$$x(y + z) = a//b(c//d + e//f) = a//b(cf + ed//df) = a(cf + ed)//bdf$$

By distributivity of integers we have that

$$x(y + z) = a(cf + ed)//bdf = (acf + aed)//bdf$$

Again from definition of addition and multiplication on rationals we have that

$$x(y + z) = (acf + aed)//bdf = ac//bd + ae//bf = xy + xz$$

Similarly,

$$(y + z)s = (c//d + e//f)a//b = (cf + ed//df)a//b = (cf + ed)a//dbf$$

By distributivity of integers we have that

$$(y + z)x = (cf + ed)a//dbf = (cfa + eda)//dfb$$

Again from definition of addition and multiplication on rationals we have that

$$(y + z)x = (cfa + eda)//dfb = ca//bd + ea//bf = yx + zx$$

$\square$