Hiring Problem

CS-6th

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Problem definition

- Suppose you decide to use an employment agency.
- The employment agency sends you one candidate each day.
- You are committed to having, at all times, the best possible person for the job.
- Therefore, you decide that, after interviewing each applicant, if that applicant is better qualified than the current office assistant, you will fire the current office assistant and hire the new applicant.

- Write down an algorithm for the Hiring Problem.

Algorithm

```
HIRE-ASSISTANT(n)
best = 0            // candidate 0 is a least-qualified dummy candidate
for i = 1 to n
    interview candidate i
    if candidate i is better than candidate best
        best = i
        hire candidate i
```

Cost Analysis

- Interviewing has a low cost, say c_i , whereas hiring is expensive, costing c_h.
- Interview Cost=$c_i n$
- Hiring Cost=$c_h m$, where m is the number of candidates hired.
- Total cost: $O(c_i n + c_h m)$

- Worst Case?

Worse Case

- $O(c_h n)$
- Best Case?
- $O(c_i n)$

Average Case

- We will consider all the permutations and take the average

Randomized Algorithm

- In order to develop a randomized algorithm for the hiring problem, you need greater control over the order in which you'll interview the candidates.
- The employment agency sends you a list of the n candidates in advance.
- On each day, you choose, randomly, which candidate to interview.

Expected Time Complexity

Indicator variable

$$\mathrm{I}\{A\} = \begin{cases} 1 & \text{if } A \text{ occurs}, \\ 0 & \text{if } A \text{ does not occur}. \end{cases}$$

$$
\begin{aligned}
X_i &= \mathrm{I}\{\text{candidate } i \text{ is hired}\} \\
&= \begin{cases} 1 & \text{if candidate } i \text{ is hired}\ , \\ 0 & \text{if candidate } i \text{ is not hired}\ , \end{cases}
\end{aligned}
$$

and

$$
X = X_1 + X_2 + \cdots + X_n \ .
$$

- $X=\sum_{i=1}^{n} Xi$
- $E[X]=E[\sum_{i=1}^{n} Xi]$
- $E[X]= \sum_{i=1}^{n} E[Xi]$
- $E[X]= \sum_{i=1}^{n} pr(Xi)$
- $E[X]= \sum_{i=1}^{n} \frac{1}{i}$
- $E[X]=\lg n+O(1)$
- Total Cost=$O(c_h \lg n)$

Taking the expectation on both sides.

Informally, the expected value is the arithmetic mean of the possible values a random variable can take, weighted by the probability of those outcomes (weighted average).

Candidate i has a probability of 1/i of being better qualified than candidates 1 through i-1 and thus a probability of 1/i of being hired.