

Quiz 13C: Mapping Reduction

CS 212 Nature of Computation

Habib University — Fall 2023

Total Marks: 10
Duration: 15 minutes

Date: November 15, 2023
Time: 830–845h

Student ID: _____

Student Name: _____

1. (10 points) Let us extend mapping reduction to arbitrary sets.

A set A is *mapping reducible* to a set B , written $A \leq_m B$, if there is a computable function, f , where for every u ,

$$u \in A \iff f(u) \in B.$$

The function f is called the *reduction* from A to B .

Prove or disprove the following claim.

Claim 1. *The following function, $f : \mathbb{Z} \rightarrow \mathbb{Z}$, is a reduction from \mathbb{N} to \mathbb{Z} .*

$$f(x) = x + 1.$$

Solution: We disprove the claim by showing that f does not meet the required condition for a reduction.

Proof. For f to be a reduction from \mathbb{N} to \mathbb{Z} , the following must hold.

$$\forall x (x \in \mathbb{N} \iff f(x) \in \mathbb{Z}).$$

This further reduces to 2 cases, one of which we show through a counterexample to not hold.

Case: $\forall x (x \notin \mathbb{N} \implies f(x) \notin \mathbb{Z})$

That is, $\forall x (x \notin \mathbb{N} \implies (x + 1) \notin \mathbb{Z})$

A counterexample is $x = -5$. □