Quiz 3D: NFAs

CS 212 Nature of Computation

Habib University — Fall 2023

Total Marks: 10	Date: September 6, 2023
Duration: 15 minutes	Time: 830–845h

Student ID: ____________________

Student Name: ____________________

1. Given the DFA $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ that recognizes the languages L_1, we construct the NFA $M_2 = (Q_2, \Sigma_\epsilon, \delta_2, q_2, F_2)$ as follows.

   1. $Q_2 = Q_1 \cup \{q_2\}$
   2. $F_2 = F_1 \cup \{q_2\}$
   3. $\delta_2(q_2, \epsilon) = \{q_1\}$
   4. $\forall q \in Q_1 \forall a \in \Sigma\ (\delta_2(q, a) = \{\delta_1(q, a)\})$
   5. $\forall q \in F_1\ (\delta_2(q, \epsilon) = \{q_2\})$

   (a) (7 points) Prove or disprove the following claim.
   Claim 1. $L(M_2) = L_1^*$

   (b) (3 points) If the above claim is true, what does it establish about the closure of the class of regular languages?

Solution:

(a) *Proof.* $L(M_2) = L_1^*$

We prove that each side is a subset of the other.

<u>Case 1</u>: $L(M_2) \subseteq L_1^*$
Assume $w \in L(M_2)$.
Then, there exists a state $q \in F_2$ such that reading w leaves M_2 in the state q.
Then, $q = q_2$ or $q \in F_1$.
Then, $w = \epsilon$ or $w = u_1 u_2 u_3 \dots u_n$ where each $u_i \in L_1$.
$\therefore w \in L_1^*$.

<u>Case 2</u>: $L_1^* \subseteq L(M_2)$
Assume $w \in L_1^*$.
Then, $w = \epsilon$ or $w = u_1 u_2 u_3 \dots u_n$ where each $u_i \in L_1$.
Then, from the construction, there exists a sequence of state transitions for w that leaves M_2 in q_2 or in a state $q \in F_1$.
$\therefore w \in L(M_2)$. □

(b) The truth of the claim establishes that the class of regular languages is closed under the star operation.