Complexity Theory Quiz 03

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TQBF = True Quantified Boolean Formula.
~~a~~ every variable is quantified using $\forall$, or $\exists$ at the begining of the sentence.

TQBF is coNP Hard.

We need to show that all problems ~~can~~ in coNP can be reduced to TQBF in poly time.
[→ Not required i think]

~~* Firstly, we show TQBF is in coNP. This can easily be done through a verifier V that given a certificate ~~cert~~ verifies whether a TQBF formula φ results in true or false. Since a TQBF formula would have n variables, then this can be verified in poly time.~~

* Next we show that a known coNP-complete problem can be reduced to TQBF in poly time. We can choose TAUTOLOGY for this, which is coNP-complete, & show it can be reduced to TQBF.

Then given a boolean formula φ for TAUT, such that $\varphi(x_1, x_2 ---, x_n)$ where $\{x_1, x_2 --- x_n\}$ are free variables, we transform φ into ϕ which universally quantifies all the free variables of φ.

~~Then the fully quantified formula ψ is the TQBF instance.~~

We reduce as so:

~~* if φ is a~~

That is, we create: ~~ψ_2 $\forall x_1 \forall x_2 \cdots \forall x_n$~~

$$\psi = \forall x_1 \forall x_2 \cdots \forall x_n \, \varphi(x_1, x_2, \ldots, x_n)$$

This formula ψ is the fully quantified TQBF formula.

The reduction works as so:

* if φ is a tautology, then ψ will evaluate to true, because no matter what values we assign to the variables, φ will always be true. Thus, $\psi \in$ TQBF.
* if φ is not a tautology, then there exists some truth assignment that makes φ false. Then ψ evaluates to false because the universal quantifier $\forall$ requires the formula to be true for every assignment. Since there is at least one assignment making φ false, $\psi \notin$ TQBF.

The reduction is polytime since it simply adds the universal quantifier to the boolean formula. This is polytime.

Thus, TAUTOLOGY is reducible to TQBF, & since TAUTOLOGY is coNP-complete, we conclude that TQBF is coNP-Hard.