Habib University - City Campus
Instructors: Aeyaz Jamil Keyani
Course: MATH 307 Mathematical Foundations and Reasoning
Examination: Quiz 3 – Spring 2025
Exam Date: Thursday, February 6, 2025
Exam Time: 10:05 – 10:20

Total Marks: 10 Marks

Duration: 15 Minutes

1. $\boxed{\text{3 points}}$ Let A be a set by using the axiom of power set show that there exists a set $P(A)$ such that $x \in P(A) \iff x \subseteq A$.

Solution: For set A, the set $P(A)$ can be constructed as follows:

$$P(A) = \{y \mid y = f^{-1}(\{1\}) \text{ for } f : X \to \{0,1\} \in \{0,1\}^X\}$$

Now we show that $y \in P(A) \iff y \subseteq A$. Suppose $y \subseteq A$ then we have a function $f : X \to \{0,1\} \in \{0,1\}^X$ such that:

$$f(x) = \begin{cases} 1 & \text{if } x \in y \\ 0 & \text{if } x \notin y \end{cases}$$

So $f^{-1}(\{1\}) = y$ then and we have that $y \in P(A)$.

Conversely, suppose that $y \in P(A)$ then there exists a function f in $\{0,1\}^X$ such that $f^{-1}(\{1\}) = y$, as f is a function from X to $\{0,1\}$ by definition of inverse image we have that $y \subseteq A$.

So we have that $y \in P(A) \iff y \subseteq A$.

$\square$

2. $\boxed{\text{7 points}}$ Let X, Y, Z be some sets and let $f : X \to Y$, $f' : X \to Y$, $g : Y \to Z$, and $g' : Y \to Z$ be functions.

(a) Show that if $g \circ f = g \circ f'$ and g is an injection then $f = f'$.

Solution: Suppose we have that $g \circ f = g \circ f'$ and g is an injection. For the sake of contradiction suppose that $f \neq f'$ then there is a $x \in X$ such that $f(x) \neq f'(x)$. As g is an injection then $f(x) \neq f'(x) \implies g(f(x)) \neq g(f'(x))$ (from the definition of injection). And therefore f must be equal to f'

$\square$

(b) Show that if $g' \circ f = g \circ f$ and f is a surjection then $g = g'$.

Solution: Suppose we have that $g \circ f = g' \circ f$ and f is a surjection. For the sake of contradiction suppose that $g \neq g'$ then there is a $y \in Y$ such that $g(y) \neq g'(y)$. As f is a surjection there exits some $x \in X$ such that $f(x) = y$ then we have that $g(f(x)) \neq g(f(x'))$, which contradicts our assumption that $g \circ f = g' \circ f$. And therefore g must be equal to g'

$\square$