[Exam 1] Name: ID:
Fall 2023: CS 313: Computational Complexity Theory

Due: 4:45 pm, Thursday, November 16, 2023. Total Marks: 24

This exam copy contains 3 pages, including this one.

Question 1 [12 points]

For each part, provide brief explanations and/or proofs.

1. We have defined a relation $\leq_p$ among languages. This relation is reflexive (i.e. $L \leq_p L'$ for all languages) and transitive (i.e. if $L \leq_p L'$ and $L' \leq_p L''$ then $L \leq_p L''$). Why is it not symmetric, namely, why is it that $L \leq_p L'$ need not imply $L' \leq_p L$?

2. Show that **NP** is closed under union.

3. Why is every **NP-Hard** language not decidable by a Turing Machine?

4. Show that if **P** = **NP** then **NP** $\subset$ **EXP**, where $\subset$ denotes the proper subset relation.

Question 2 [6 points]

Show that the language

FCLIQUE $= \{ (G, k, F) \mid$ Undirected graph G has a clique[1] of size k, such that no vertex from the forbidden set[2] F is in the clique $\}$.

is **NP-Complete**.

[1] a set of vertices such that every two distinct vertices in are adjacent

[2] the forbidden set contains vertices

Question 3 [6 points]

Show that **NP** = **coNP** *iff* **CLIQUE** and **TAUTOLOGY** are polynomial time reducible to one another.