Homework 1

Assigned on September 4, 2023
Due on September 18, 2023

Maximum Points: 100

Learning Outcomes

After this homework, you should be able to:

(a) assign frames and obtain expressions for physical quantities in different frames;

(b) obtain kinematic model of a wheeled robot.

Instructions

- The homework assignment can be attempted in groups of two.
- Each group will register themselves as a group on Canvas under People.
- The homework submission on Canvas will be set up for group submission, so each group needs to make only one submission.
- **If it appears that a group member has not contributed to a homework assignment, then each member will be graded individually.**

Tasks

Problem 1
CLO-1/C-2
15 points

In terms of the $\hat{x}_w$, $\hat{y}_w$, $\hat{z}_w$ coordinates of a fixed world frame $\{w\}$, the frame $\{a\}$ has its $\hat{x}_a$-axis pointing in the direction $(0, 0, 1)$ and its $\hat{y}_a$-axis pointing in the direction $(1, 0, 0)$, and the frame $\{b\}$ has its $\hat{x}_b$-axis pointing in the direction $(1, 0, 0)$ and its $\hat{y}_b$-axis pointing in the direction $(0, 0, 1)$. All frames are stationary.

(a) Draw by hand the three frames, at different locations so that they are easy to see.

(b) Write down the rotation matrices R^s_a and R^s_b.

(c) Given R^s_b, find $\left(R^s_b\right)^1$ without using a matrix inverse? Verify its correctness using your drawing.

(d) Given R^s_a and R^s_b, how do you calculate R^a_b?

(e) Use R^s_b to change the representation of the point ${}^b p = (1, 2, 3)$ (which is in $\{b\}$ coordinates) to $\{s\}$ coordinates.

(f) Choose a point p represented by ${}^s p = (1, 2, 3)$ in $\{s\}$ coordinates. Calculate $p = R^s_b \, {}^s p$ and $p'' = R^b_s \, {}^s p$. For each operation, should the result be interpreted as changing coordinates (from the $\{s\}$ frame to $\{b\}$) without moving the point p or as moving the location of the point without changing the reference frame of the representation?

(g) An angular velocity ω is represented in $\{s\}$ as ${}^s\omega = (3, 2, 1)$. What is its representation in $\{a\}$?

Problem 2
CLO-1/C-3

20 points

An inertial measurement unit (IMU) is a popular sensor employed by a number of mobile robots for inertial navigation. It is comprised of at least two sensors - a 3-axis accelerometer and a 3-axis gyroscope [1]. Linear acceleration of an object can be determined from an accelerometer's measurements[1] and the gyroscope measures angular velocities. Historically, these sensors were mounted on mechanical structures, e.g. gimbals, isolating the sensors from the motion of the vehicle. In modern system, however, the sensors are *strapped down* on the vehicle body. The laws of physics are such that the sensors can still only measure quantities with respect to an inertial reference frame, so the effects of the motion of the vehicle and motion of the earth have to be mathematically adjusted from the measurement data.

Highly sensitive IMUs can detect the rotation of the earth as well. Assume that you have access to such an IMU. A possible frame assignment for such an IMU is shown in Figure 1. Frame $\{i\}$ in Figure 1 is an inertial frame of reference at the center of earth. Frame $\{t\}$ is a frame located at the earth surface and incorporates the rotation of the earth. Frames $\{v\}$ and $\{s\}$ are the vehicle and sensor frames respectively. The IMU sensor located at the origin of $\{s\}$ is measuring ω^i_s and a^i_s, where ω and a are angular velocity and acceleration respectively. Find an expression for ${}^v a^t_v$, i.e. the acceleration of the robot with respect to earth surface written in the body coordinates.

[1] I intentionally didn't say that an accelerometer measures acceleration, because it does not.

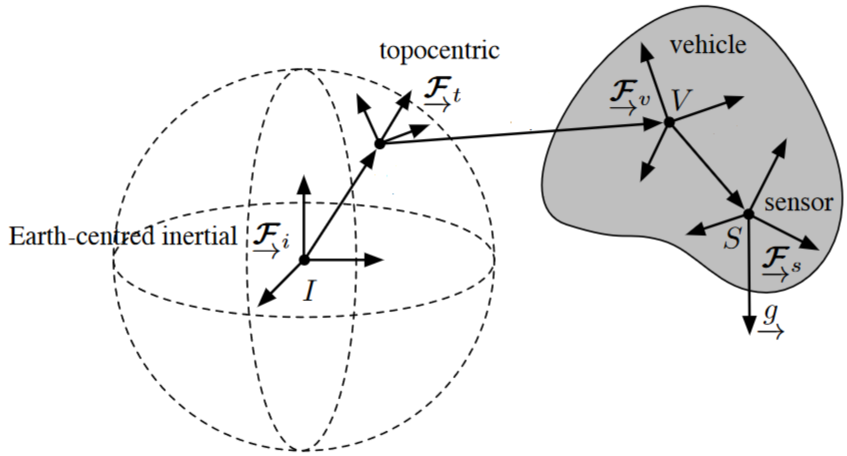

Figure 1: Frame assignment for an IMU [2]

Problem 3
CLO-1/C-3

30 points

In this task, you're going to develop a kinematic model for CROMSCI, a climbing robot, illustrated in Figure 2, **using the equation for the standard steerable wheel**. This robot

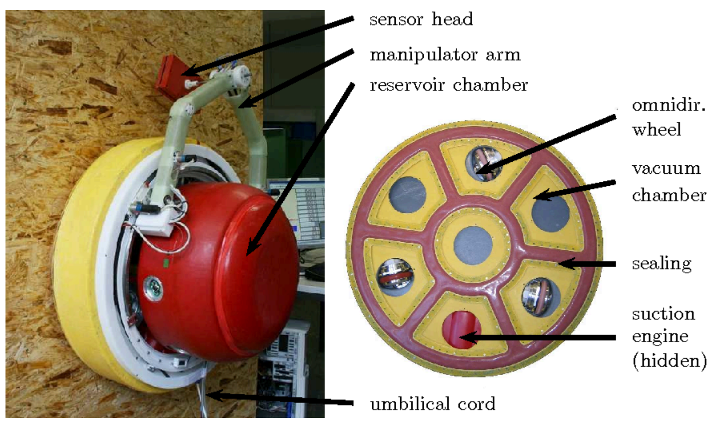

Figure 2: CROMSCI

has three wheels, each of which is both steerable and driven. The three wheels are arranged in a triangular configuration, as shown in Figure 3. This configuration allows the robot to be omnidirectional, i.e. it is able to move in any direction.

We'll set up our modeling in the following manner. The robot frame is to be placed at the center of the circular chassis. Let l_i be the line that runs through the center of a wheel and the kinematic center of the robot. Let α_i be the angle that the line l_i makes with x-axis of

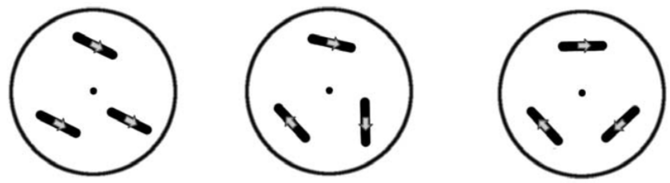

Figure 3: Arrangement of three wheels on Cromsci

the robot frame, as shown in Figure 4. Then, the three wheels on this robot are located at $\alpha_1 = 0°$, $\alpha_2 = 120°$, and $\alpha_3 = -120°$, and at a distance d from the center of the robot. The x-axis of the wheel frame or contact frame is along the rolling direction of each wheel respectively. Let β_i be the steering angle of wheel i, defined as the angle formed by the current wheel axis (after steering) with l_i.

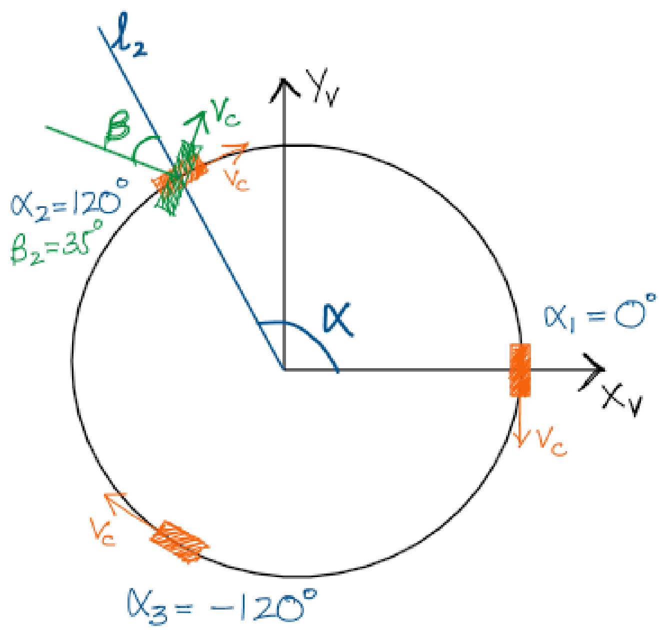

Figure 4: Modeling the Cromsci robot

Obtain the differential kinematic model for this robot. You can view the equations obtained in [3] for reference.

Problem 4
CLO-1/C-3

15 points

Alphabot, our pet differential drive robot is currently located at coordinates $(1, 1)$ in the world frame with heading or orientation $\phi = 90°$. The robot width or distance between the wheels is 1. The commands that you can send to the vehicle are of the format (v_l, v_r, t), where v_l and v_r are the left and right wheel velocities, and t is the duration for which the robot has to follow this command.

(a) What command will you send to the robot to make it move in a circle of radius 1 in the clockwise direction from its present position and return to its starting position?

(b) Assume that the robot is to be moved to the location $(1, 3)$ and its final orientation is to be $\phi = 90°$. If you were constrained to use the least number of commands, what would be your strategy to execute the motion described above?

(c) For the same destination described in the previous part, what would be your strategy if your objective was to minimize the path length between the starting and ending positions.

Problem 5
CLO-1/C-2

20 points

Answer the following questions individually:

(a) How many hours did each of you spend on this homework? Answer as accurately as you can, as this will be used to structure next year's class.

(b) Each group member is to specifically state their contribution in this homework assignment.

(c) Each group member is to provide a note of at least one paragraph or a concept map, highlighting their understanding of the topics covered in this assignment, and a list of muddiest points/open questions. This should not be a chronological account of our classes, but a representation of the concepts as they exist in your mental model, i.e. How have you linked it to your prior knowledge? How have you linked the course concepts to each other?

Don't forget to indicate your name with your respective paragraph.

References

[1] A. Kelly, *Mobile robotics: mathematics, models, and methods*. Cambridge University Press, 2013.

[2] T. D. Barfoot, *State estimation for robotics*. Cambridge University Press, 2017.

[3] K. Berns and E. Von Puttkamer, "Autonomous land vehicles," *Vieweg+ Teubner GWV Fachverlage GmbH, Wiesbaden*, 2009.

Grading:

To obtain maximal score for each question, make sure to elaborate and include all the steps.