Linear Algebra
Homework 1

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Chapter 1 : Linear Equations and Matrices

Ex Set 1.4 : Inverses; Rules of Matrix Arithmetic

11. Find the inverse of $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

Solution: Determinant of the matrix $= (\cos\theta \times \cos\theta) - (-\sin\theta \times \sin\theta)$
Det $= \cos^2\theta + \sin^2\theta \implies$ Det $= 1$
Adjoint $= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$
Inverse $= \frac{\text{Adjoint}}{\text{Determinant}} \implies$ Inverse $= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

13. Consider the matrix

$$A = \begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & a_{nn} \end{bmatrix}$$

where $a_{11}a_{22}\cdots a_{nn} \neq 0$. Show that A is invertible, and find its inverse.

Solution:

15. (a) Show that a matrix with a row of zeroes cannot have an inverse.

(b) Show that a matrix with a column of zeroes cannot have an inverse.

Solution:

16. Is the sum of two invertible matrices necessarily invertible?

Solution:

17. Let A and B be square matrices such that $AB = 0$. Show that if A is invertible, then $B = 0$.

Solution:

29. (a) Show that if A is invertible and $AB = AC$, then $B = C$.

(b) Explain why part (a) and Example 3 (from the book) do not contradict one another.

Solution: