Linear Algebra
Homework 1

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Chapter 1 : Linear Equations and Matrices

Ex Set 1.5 Elementary Matrices and a Method for finding A^{-1}

2. Find a row operation that will restore the given matrix to an identity matrix

(a) $\begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & -\frac{1}{7} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Solution:

3. Consider the matrices:

$$A = \begin{bmatrix} 3 & 4 & 1 \\ 2 & -7 & -1 \\ 8 & 1 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 8 & 1 & 5 \\ 2 & -7 & -1 \\ 3 & 4 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 3 & 4 & 1 \\ 2 & -7 & -1 \\ 2 & -7 & 3 \end{bmatrix}$$

Find the elementary matrices $E_1, E_2, E_3,$ and E_4 such that
(a) $E_1A = B$ (b) $E_2B = A$ (c) $E_3A = C$ (d) $E_4C = A$

Solution:

9. Find the inverse of the following 4×4 matrices, where k_1, k_2, k_3, k_4 and k are all nonzero

(a) $\begin{bmatrix} k_1 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 \\ 0 & 0 & k_3 & 0 \\ 0 & 0 & 0 & k_4 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 0 & 0 & k_1 \\ 0 & 0 & k_2 & 0 \\ 0 & k_3 & 0 & 0 \\ k_4 & 0 & 0 & 0 \end{bmatrix}$

Solution:

10. Consider the matrix $A = \begin{bmatrix} 1 & 0 \\ -5 & 2 \end{bmatrix}$
(a) Find the elementary matrices E_1 and E_2 such that $E_2 E_1 A = I$
(b) Write A^{-1} as a product of two elementary matrices.
(c) Write A as a product of two elementary matrices.

Solution:

16. Show that $A = \begin{bmatrix} 0 & a & 0 & 0 & 0 \\ b & 0 & c & 0 & 0 \\ 0 & d & 0 & e & 0 \\ 0 & 0 & f & 0 & g \\ 0 & 0 & 0 & h & 0 \end{bmatrix}$ is not invertible for any values of the entries.

[*Hint: The row containing b must become the first row after ERO have been carried out and the one with g must be the last row. Why? this caused problems when trying to reduce the middle column to only one nonzero entry.*]

Solution: