Linear Algebra
Homework 2 part ii

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# Chapter 1 : Linear Equations and Matrices

## Ex Set 1.5 Elementary Matrices and a Method for finding $A^{-1}$

**2.** Find a row operation that will restore the given matrix to an identity matrix

(a) $\begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & -\frac{1}{7} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

**Solution:**

**3.** Consider the matrices:

$$A = \begin{bmatrix} 3 & 4 & 1 \\ 2 & -7 & -1 \\ 8 & 1 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 8 & 1 & 5 \\ 2 & -7 & -1 \\ 3 & 4 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 3 & 4 & 1 \\ 2 & -7 & -1 \\ 2 & -7 & 3 \end{bmatrix}$$

Find the elementary matrices $E_1, E_2, E_3,$ and $E_4$ such that
(a) $E_1A = B$ (b) $E_2B = A$ (c) $E_3A = C$ (d) $E_4C = A$

**Solution:**

**9.** Find the inverse of the following $4 \times 4$ matrices, where $k_1, k_2, k_3, k_4$ and $k$ are all nonzero

(a) $\begin{bmatrix} k_1 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 \\ 0 & 0 & k_3 & 0 \\ 0 & 0 & 0 & k_4 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 0 & 0 & k_1 \\ 0 & 0 & k_2 & 0 \\ 0 & k_3 & 0 & 0 \\ k_4 & 0 & 0 & 0 \end{bmatrix}$

**Solution:**

**10.** Consider the matrix $A = \begin{bmatrix} 1 & 0 \\ -5 & 2 \end{bmatrix}$

(a) Find the elementary matrices $E_1$ and $E_2$ such that $E_2 E_1 A = I$
(b) Write $A^{-1}$ as a product of two elementary matrices.
(c) Write $A$ as a product of two elementary matrices.

**Solution:**

**16.** Show that $A = \begin{bmatrix} 0 & a & 0 & 0 & 0 \\ b & 0 & c & 0 & 0 \\ 0 & d & 0 & e & 0 \\ 0 & 0 & f & 0 & g \\ 0 & 0 & 0 & h & 0 \end{bmatrix}$ is not invertible for any values of the entries.

[*Hint: The row containing* $b$ *must become the first row after ERO have been carried out and the one with* $g$ *must be the last row. Why? this caused problems when trying to reduce the middle column to only one nonzero entry.*]

**Solution:**