Linear Algebra Spring 23
Proofs

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Chapter 1 : Linear Equations and Matrices

1. If $A_1, A_2, ..., A_n$ are invertible matrices of the same size, then $(A_1A_2...A_n)^{-1} = A_n^{-1}A_{n-1}^{-1}...A_2^{-1}A_1^{-1}$. [Prove using induction]

Solution:

2. (a) If A is an ivnertible matrix, then $(A^{-1})^{-1} = A$

(b) Prove that $(A^n)^{-1} = (A^{-1})^n$ for $n = 0, 1, 2, ...$ [Prove using induction]

[Hint: $A^n = AA...A$ n times]

Solution:

3. (a) If A, B are matrices s.t. AB is defined, then $(AB)^T = B^TA^T$.

(b) Prove that the transpose of a product of any number of matrices is equal to the product of their transposes in the reverse order i.e. $(A_1A_2...A_n)^T = A_n^T...A_2^TA_1^T$ [Prove using induction]

Solution:

4. If a system of equations has a unique solution, then Gaussian Elimination will find it. [*Hint: Think induction - if we can prove it for one variable, and then prove it for k variables, then we can prove it for* $(k + 1)$ *variables.*]

Solution:

5. Prove that the reduced row echelon form is always unique [Qns needs to be worded properly, will do that]

Solution: