Linear Algebra Spring 23
Assignment 2 — Lecture 5

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Chapter 1 : Linear Equations and Matrices

Q1. (a) Under what conditions $AB = BA$ where $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$

(b) If A is a matrix then $A^r A^s = A^{r+s}$ $[\forall r, s \in \mathbb{Z}^+]$.
Is this result true for negative integers also? Justify your answer

(c) If $A = \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}$ and $c = \begin{bmatrix} 2 & 5 \\ 3 & 4 \end{bmatrix}$. Then $AB = BC = \begin{bmatrix} 3 & 4 \\ 6 & 8 \end{bmatrix}$ but $B \neq C$.
Why?

Solution:

Q2. Using the technique of forming a block matrix $\left[A \,\vdots\, I\right]$ and performing EROS such that $\left[A \,\vdots\, I\right] \xrightarrow{EROS} \left[I \,\vdots\, A^{-1}\right]$. Find the inverse of the following where A is given by

(a) $\begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 & -4 \\ -1 & -1 & 5 \\ 2 & 7 & -3 \end{bmatrix}$

Solution:

Q3. Solve the following system of equations by reducing them to Echelon form (Guassian Elimination method)

$x + y + 2z = 9$
$2x + 4y - 3z = 1$
$3x + 6y - 5z = 0$

Solution:

Q4. Solve the following system by Gauss-Jordan elimination (reduced row Echelon form)
$2x_1 + 2x_2 + 2x_3 = 0$
$-2x_1 + 5x_2 + 2x_3 = 1$
$8x_1 + x_2 + 4x_3 = -1$

Solution:

Q5. Reduce $\begin{bmatrix} 2 & 1 & 3 \\ 0 & -2 & 7 \\ 3 & 4 & 5 \end{bmatrix}$ to reduced row echelon form without introducing any fractions.

Solution:

Q6. Find two different row Echelon forms of $\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$

Solution:

Q7. Exercise set 1.3 Q25

Solution: Done in Homework 3 i

Q8. Exercise set 1.3 Q18 and 19

Solution: Done in Homework 3 i