Ali Mushtaq FA21-BEE-023

Assignment #1
Multi-Cal

Q1:

a) $A(4,4,1)$
$B(-4,3,-4)$
$C(4,-1,-2)$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= \begin{bmatrix} -4 \\ 3 \\ -4 \end{bmatrix} - \begin{bmatrix} 4 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} -8 \\ 7 \\ -5 \end{bmatrix} = -8\hat{i} + 7\hat{j} - 5\hat{k}$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$

$= \begin{bmatrix} 4 \\ -1 \\ -2 \end{bmatrix} - \begin{bmatrix} 4 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ -3 \end{bmatrix} = 3\hat{j} - 3\hat{k}$

$n = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} i & j & k \\ -8 & 7 & -5 \\ 0 & 3 & -3 \end{vmatrix}$

$= -6\hat{i} + 24\hat{j} - 24\hat{k}$

$= i + 4j + 4k$

$d = a.n = (4i - 4j + k)(i + 4j + 4k)$

$= 4 - 16 + 4$

$= -8$

Equation of plane:

$r.n = d$

$r.(i + 4j + 4k) = -8$

$(xi + yj + zk).(i + 4j + 4k) = -8$

$x + 4y + 4z = -8$

$x + 4y + 4z + 8 = 0$

b)

Perp distance $= \dfrac{d}{|n|} = \dfrac{8}{\sqrt{1^2 + 4^2 + 4^2}}$

$= \dfrac{8}{33} = 1.39$

c)

line OO = $r = O + \lambda b$

$$r = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ +3 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} 2\lambda \\ 3\lambda \\ -3\lambda \end{pmatrix} + \begin{pmatrix} 1 \\ 4 \\ 4 \end{pmatrix} = -8$$

$$2\lambda + 12\lambda - 12\lambda = -8$$

$$\frac{\cancel{2}\lambda}{\cancel{2}} = \frac{-8}{8}$$

$$\lambda = -4$$

$$r \begin{pmatrix} 2(-4) \\ 3(-4) \\ -3(-4) \end{pmatrix} = (-8, -12, 12) \text{ Ans.}$$

Q2.

$A = (7i + 4j - k) \quad B(11i + 3j) +$

$C(i + 6j + 3k)$

$D = (2i + 7j + \lambda k)$

$AB = 4i - 1j + k$

$CD = 0i - j + (3 - \lambda) k$

$L_1 = \text{line } AB = OA + \lambda B$

$L_2 = OC + \mu CD$

$$L_1 = \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 4 \\ -1 \\ 1 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} 2 \\ 6 \\ -3 \end{bmatrix} + 4 \begin{bmatrix} 0 \\ -1 \\ 3-\lambda \end{bmatrix}$$

$$D = \frac{|b_1 \times b_2| \cdot (a_2 - a_1)}{|b_1 \times b_2|}$$

$$b_1 \times b_2 \begin{vmatrix} i & j & k \\ 4 & -1 & 1 \\ 0 & -1 & 3-\lambda \end{vmatrix}$$

$b_1 \times b_2 = \sqrt{17\lambda^2 - 26\lambda + 169}$

$$a_1 - a_2 = \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} - \begin{bmatrix} 2 \\ 6 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ -2 \\ 2 \end{bmatrix}$$

$$3 = \frac{(-3 + \lambda) 5 + 2 (12 - 4\lambda) - 2(4)}{\sqrt{17\lambda^2 - 26\lambda + 169}}$$

$9(17\lambda^2 - 26\lambda + 169) = (-15 + 5\lambda + 24 - 8\lambda - 8)^2$

$\lambda = 49\lambda^2 + 1 - 24\lambda$

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b) AB × AD

$$\begin{vmatrix} i & j & k \\ 4 & -1 & 1 \\ 5 & 3 & 2 \end{vmatrix}$$

$= -5i - 13j + 7k$

$= -5(2) - 13(7)s + 7(-1)t$

Eqn $= -10 - 91s - 7t$

For plane $= \vec{r_2}$, when $\lambda = 4$

$$= \begin{vmatrix} i & j & k \\ 4 & - & 11 \\ 4 & -3 & 5 \end{vmatrix}$$

$= -8i - 15j + 17k$

Eqn $= -8(x-11) - 15(y-3) + 17(z-0)$

$= 8x + 15y - 17z - 133$

c)

$\theta = \cos^{-1}\left(\dfrac{318}{\sqrt{25+169+49}\ \sqrt{64+225+289}}\right)$

$\theta = \cos^{-1}\left(\dfrac{318}{(15.58)(24.04)}\right)$

$\theta = \cos^{-1}\left(\dfrac{318}{374.56}\right)$

$\theta = 31.89°$ Ans.

Q3:

$L_1 = ti + j \quad -2i - j$

$L_2 = j + tk \quad -2j + k$

Shortest distance b/w line is $\sqrt{21}$

$\vec{r_1} = OA + \lambda AB$

$\vec{r_2} = OA + \mu AB$

$$l_1 = r_1 = \begin{bmatrix} t \\ 1 \\ 0 \end{bmatrix} + \lambda \begin{pmatrix} -2 \\ -1 \\ 0 \end{pmatrix}$$

$$l_2 = \vec{r_2} = \begin{bmatrix} 0 \\ 1 \\ t \end{bmatrix} + \mu \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$$

$$D = \frac{(b_2 \times b_1) \cdot (a_2 - a_1)}{|b_1 \times b_2|}$$

$$b_1 \times b_2 = \begin{vmatrix} i & j & k \\ -2 & 1 & 0 \\ 0 & -2 & 1 \end{vmatrix}$$

$= i + j2 + 4k$

$a_2 - a_1 = -ti + kt$

$$\sqrt{21} = \frac{(i + j2 + 4k) \cdot (-ti + 0j + kt)}{\sqrt{21}}$$

$21 = -t + 4t$

$t = 7$

c) $5x - 6y + 7x = 0$

$n_1 = line = (0, 2, 1)$

$n_2 = \pi = \quad = (5, -6, 7)$

$\theta = \cos^{-1}\left(\frac{0 + 12 + 7}{\sqrt{4+1}\ \sqrt{25+16+49}}\right)$

$\theta = \cos^{-1}\left(\frac{11}{23 \cdot 43}\right)$

$\theta = \quad 35 \cdot 813^\circ$

d) $\pi_1\ n = \begin{bmatrix} 7 \\ 1 \\ 0 \end{bmatrix}, \quad \pi_2 \begin{bmatrix} 5 \\ -6 \\ 7 \end{bmatrix}$

$\theta = \cos^{-1}\left(\frac{35 - 6 + 0}{\sqrt{49+1}\ \sqrt{25+36+49}}\right)$

$\theta = \cos^{-1}\left(\frac{29}{\sqrt{50}\ \sqrt{110}}\right)$

$\theta = 67.09^\circ$

Q5:

a) $M.P = \left(\frac{-6-2}{2}, \frac{-3-1}{2}\right)$

$= -4, -2$

$(x-h)^2 + (y-k)^2 = r^2$

$(x+4)^2 + (y+2)^2 = 5 \qquad (-2+4)^2 + (-1+2)^2 = r^2$

$r^2 = 5$

b)

let y be center point.

$(4)^2 + (0-b)^2 = r^2$

$16 + b^2 = r^2$

at point $(0, 2)$

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$(0)^2 + (2-b)^2 = r^2$

$4 + b^2 - 4b = r^2$

$16 + b^2 = 4 + b - 4b$

$12 = -4b$

$b = -3$

$16 + 9 = r^2$

$r = 5$

c) $y^2 = 100x$

$y^2 = 4ax$

$4ax = 100x$

$a = 25$

$x = -a$

$-25 = x$

d) $x^2 = 24y$

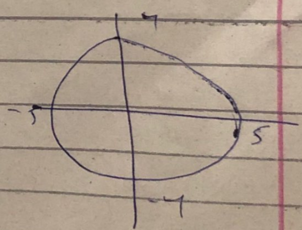
$x^2 = 24y$

$4ay = 24y$

$a = 6$

$x = -6$

e) $\left(\frac{x^2}{25}\right) + \left(\frac{y}{16}\right)^2 = 1$



$\left(\frac{x^2}{a}\right) + \left(\frac{y}{b}\right)^2 = 1$

$a = 5, \quad b = 4$

$c = \sqrt{a^2 - b^2}$

$c = \sqrt{25 - 16} = \pm 3$

$F_1 = (-3, 0), \quad F_2 = (3, 0)$

lenght $= 2a$

$= 2(5)$

$= 10$ A.

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f)

major axis = 10
minor axis = 8

$2a = 10$

$a = 5$

$2b = 8$

$b = 4$

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

$$\left(\frac{x}{5}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$$

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