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Multi-Cal

Assignment 2

Q1:

a)

$$\lim_{(x,y)(2,1)} \frac{x^2-2xy}{x^2-4y^2}$$

$x^2=4y^2$

$x=4y$

$y=x/4$

$$\frac{x^2/16 - \cancel{2}(x)(x/\cancel{4}_2)}{x^2-\cancel{4}\left(\frac{x^2}{\cancel{16}_4}\right)}$$

$$\frac{\frac{x^2}{16}-\frac{x^2}{2}}{x^2-\frac{x^2}{4}} = \frac{(x^2-8x^2)4}{4(16)(4x^2-x^2)}$$

$$= \frac{-7x^2}{4(3x^2)} = -7/12 \text{ Ans}$$

b) $$\lim_{(x,y)\to(0,0)} \frac{x-4y}{6y+7x}$$

$6y=7x$

$y=7x/6$

$$\frac{x-4(7x/6)}{7x+7x}$$

$$\frac{x-28x/6}{14x}$$

$$\frac{6x-28x}{6(14x)} = \frac{-11}{52}$$

c) $\lim_{(x,y)\to(0,0)} \frac{x^2-y^6}{xy^3}$

$y=mx$

$\frac{x^2-m^6x^6}{xm^3x^3}$

$\frac{x^2(1-m^6x^3)}{x\,m^3x^3}$

$\frac{\cancel{x^2}(1-m^6x^3)}{x^4 m^3}$

$\frac{1-m^6x^3}{m^3x^3}$

d) $\lim_{(x,y,z)\to(-1,0,4)} \frac{x^3-4ze^{2y}}{6x+2y-3z}$

$= \frac{(-1)^3-4(0)e^3}{6(-1)+2(0)-3(4)}$

$= \frac{-1-0}{-6+2-12}$

$= \frac{1}{16}$

Q2:

a) $f(x,y)=\cos(x/y)$ in $v(3,-4)$

$f\nabla = \frac{\partial}{\partial x}(\cos x/y)\,i + \frac{\partial}{\partial y}(\cos x/y\,j)$

$= \frac{-1}{y}\sin(x/y) + x(-\sin x/y)\frac{-1}{y^2}\,j$

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$\frac{-1}{y}\sin\left(\frac{x}{y}\right)i + \frac{x}{y}\sin(x/y)j$

$\frac{3i-4j}{\sqrt{9+16}} = \frac{3}{\sqrt{25}}i - \frac{4}{\sqrt{25}}j$

$D_{\vec{v}}f = \frac{-3}{5y}\sin\left(\frac{x}{y}\right) + \frac{4x}{5y^2}\sin(x/y)$

$D_{\vec{u}}f = \frac{1}{5y}\sin\left(\frac{x}{y}\right)\left(\frac{4x}{y} - 3\right)$

b) $F(x,y,z) = x^2y^3 - 4xz$, $\vec{v} = (-1,2,0)$

$\nabla F = (2y^3xi + (-4zk) + 3y^2x^2j - 4zk$

$\vec{v} = \frac{-i + 2j + 0k}{\sqrt{1+4}}$

$= \frac{-1}{\sqrt{5}}i + \frac{2}{\sqrt{5}}j + 0k$

$D_{\vec{v}}F = \frac{1}{\sqrt{5}}\left(2yx - 4z\right) + \frac{2}{\sqrt{5}}\left(3y^2x^2\right) + 0$

Question 3:

$\nabla f = 4 - y^2e^{3xz}(3z)i - 2ye^{3xz}j - y^2e^{3xz}\cdot 3xk$

$\nabla f\Big|_{(3,-1,0)} = 4 - (-1)^2e^{3(3)(0)}(2)(3)i - 2(-1)e^{3(3)(0)}j - (-1)^2e^{3(3)(0)}\cdot 3(3)$

$= (4-0)i - 2j - 9k$

$V = (-1,4,2)$

$v = \frac{-1i + 4j + 2k}{\sqrt{1+16+4}} = \frac{-1}{\sqrt{21}}i + \frac{4}{\sqrt{21}}j + \frac{2k}{\sqrt{21}}k$

$\frac{-1}{\sqrt{21}}(4) - \frac{2(4)}{\sqrt{21}} - \frac{9(2)}{\sqrt{21}}$

$\frac{-32}{\sqrt{21}}$ Ans.

Q4:

a) $f(x,y) = \sqrt{x^2+y^3}$ at $(-2,+3)$

$\nabla f = \frac{1}{2}(x^2+y^3)^{-1/2}(2x)i + \frac{1}{2}(x^2+y^3)^{-1/2}(3y^2)j$

$\nabla f(-2,+3) = (4+9)^{-1/2}(2)i + \frac{1}{2}(4+9)^{-1/2}(3+9)j$

$= \frac{-2}{\sqrt{13}} i + \frac{27}{2\sqrt{13}} j$

b) $f(x,y,z) = e^{2x}\cos(y-2z)$ at $(4,-2,0)$

$\nabla f = (e^{2x} \cdot 2\cos(y-2z))i + e^{2x}(-\sin(y-2x))(1)j + e^{2x}(-\sin(y-2x))(-2)k$

$\nabla f(4,-2,0) = e^{4x}(2\cos(-2-0))i + e^{4x}(-\sin(-2))j + e^{4x}(-\sin(-2))(-2)$

$= e^{4x}(-2\cos 2i - \sin(-2)j + 2\sin(-2)k)$.

Q5:

a) $F = x^2yi - (z^3 - 3x)j + 4y^2k)$

$\nabla = 2xyi - 9j + 0$

Div $F = \nabla f \cdot f$

$(2xyi)(x^2yi - (z^3-3x)j + 4y^2k)$

Div $f = 2x^3y^2 + 0$

Curve $= \nabla f \times f$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & +(2z^3)(1) & 4y^2 \end{vmatrix}$$

$(8y + 3z^2)\, i - 0 j + (2x^2) k$ Ans.

$F = (2x + 2z^2) i + \frac{x^3}{2} y^2 j - (2 \cdot 7x) k$

div $= \nabla \cdot f$

$\left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k\right) \cdot \left(2x + 2z^2 + \frac{x^3}{2} y^3 j - (2 \cdot 7x) k\right)$

$= 2 + \frac{2x^3}{2} y + (1)$

$= 2 + \frac{2x^3 y}{2} - 1$

Curve $= \nabla f \times f$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x + 2z^2 & \frac{x^3y^2}{z} & -(2 - 7x) \end{vmatrix}$$

$i\left(0 + \frac{x^3y^2}{z^2}\right) - j(7 - 4z) + k(3x^2y^2 - 0)$

$\frac{x^3y^2}{z^2} i - (7 - 4z) j + \left(\frac{3x^2y^2}{z}\right) k$

$\frac{dy}{dx} = \frac{d}{d} \sin(x^2) = \cos x^2 \cdot 2x$

$= 2x \cos x^2$

$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$

$= (4x^2y^3 - 2)(2x \cos x^2)$

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c) x^2y

$2xy^4 + \cancel{4x^2y^3\frac{dy}{dx}} = 4\cos(xy)$

$2xy^4 + 4x^2y^3\frac{dy}{dx}$

$4x^2y^3\frac{dy}{dx} = y\cos(xy) - 2xy^4$

Q7:

a) $z = \frac{x^2 - w}{y^4}$, $x = t^3 + 7$

$y = \cos(2t)$, $w = 4t$

$\frac{dz}{dt} = \frac{dz}{dx}\cdot\frac{dx}{dt} + \frac{dz}{y}\cdot\frac{dy}{dt} + \frac{dz}{dw}\cdot\frac{dw}{dt}$

$\frac{dz}{dt} = 3t^2$, $\frac{dz}{dy} = (x^2 - w)\cdot -4y^{-5}$

$\frac{dz}{dy} = \frac{-4(x^2 - w)}{y^5}$

$\frac{dx}{dt} = \frac{2x}{y^4}$, $\frac{\partial z}{\partial y} = (x^2 - w)y^4$

$\frac{dy}{dt} = \frac{d}{dt}\cos 2t = -\sin 2t \cdot 2$

$= -2\sin 2t$

$\frac{\partial z}{\partial w} = \frac{-1}{y^4}$ $\frac{dw}{dt} = 4$

$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial z}{dy}\cdot\frac{dy}{dt} + \frac{\partial z}{\partial w}\cdot\frac{dw}{dt}$

$= \frac{2x^2}{y^4}\cdot 3t^2 + \left(-\frac{4(x^2 - w)}{y^5}\right)\cdot -2\sin 2t$

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$+\left(\frac{-1}{y^4}\right)\cdot 4$

$= \frac{6x^2t^2}{y^4} + \frac{8(x^2-\omega)\sin 2t}{y^5} - \frac{y\omega}{y^4}$

b) $z = x^2y^4 - 2y \quad , y = \sin(x^2)$

$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$

$\frac{dz}{dy} = \frac{\partial}{\partial y}(x^2y^4 - 2y)$

$= (4y^3x^2 - 2)$

$= (4x^2y^3 - 2)$

$\frac{dy}{dx} = \frac{\partial}{\partial x}\sin(x^2)$

$= \cos x^2 \cdot 2x$

$= 2x\cos x^2$

$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y} \cdot \frac{dy}{\partial x}$

$= (4x^2y^3 - 2)(2x\cos^2 x)$

$= 8x^3y^3\cos x^2 - 4x\cos x^2$

c) $x^2y^4 - 3 = \sin(xy)$

$\frac{d}{dx}(x^2y^4 - 3) = \frac{d}{dx}(\sin(xy))$

$2xy^4 + x^2 4y^3 \frac{dy}{dx} = y\cos(xy)$

$2xy^4 + 4x^2y^3 \frac{dy}{dx} = y\cos(xy)$

$4x^2y^3 \frac{dy}{dx} = y\cos(xy) - 2xy^4$

$\frac{dy}{dx} = \cancel{y}\left[\frac{\cos(xy) - 2xy^3}{4x^2y^2}\right]$

$$\frac{dy}{dx} = \left\{ \frac{\cos(xy) - 2xy^3}{4x^2y} \right\} \text{ Ans.}$$

Q6:

$F = x^2yi -$

$$F = \left(4x^2 + \frac{3x^2y}{z^2}\right)i + \left(8xy + \frac{x^3}{z^2}\right)\hat{j} + \left(11 - \frac{2x^3y}{z^3}\right)\hat{k}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}, \quad \frac{\partial N}{\partial z} = \frac{\partial P}{\partial y}, \quad \frac{\partial M}{\partial z} = \frac{\partial P}{\partial x}$$

$$F = \left(4y^2 + \frac{3x^2y}{z^2}\right)i + \left(8xy + \frac{x^3}{z^3}\right)j +$$

$$\left(11 - \frac{2x^3y}{z^3}\right)\hat{k} = P$$

$$\frac{\partial M}{\partial y} = 8y + \frac{3x^2}{z}, \quad \frac{\partial N}{\partial x} = 8y - \frac{3x^2}{z^2}$$

$$\frac{\partial N}{\partial z} = x^3 z^{-2} = x^3(-2)z^{-3} = -\frac{2x^3}{z^3}$$

$$\frac{\partial P}{\partial y} = \frac{-2x^3}{\cancel{2}z^3}$$

$$\frac{\partial M}{\partial z} = 4x^2 + \frac{3x^2y}{z^2}$$

$$= 3x^2y(-2)z^{-3}$$

$$= \frac{-6x^2y}{z^3}$$

$$\frac{\partial P}{\partial x} = \frac{\partial}{\partial x}\left(11 - \frac{2x^2y}{z^3}\right) = \frac{-6x^2y}{z^3}$$

b) $\vec{F} = 6x\hat{i} + (2x-y^2)\hat{j} + (6z-x^3)\hat{k}$

$$\frac{\partial M}{\partial y} = \frac{\partial (6x)}{\partial y} = 0,$$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(2x-y^2) = 2$$

$$\frac{\partial N}{\partial z} = \frac{\partial}{\partial z}(2x-y^2) = 0,$$

$$\frac{\partial P}{\partial y} = \frac{\partial}{\partial y}(6z-x^3) = 0$$

$$\frac{\partial M}{\partial z} = \frac{\partial}{\partial z} 6x = 0$$

$$\frac{\partial P}{\partial x} = \frac{\partial}{\partial x}(6z-x^3) = -3x^2$$

So;

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}, \quad \frac{\partial N}{\partial z} = \frac{\partial P}{\partial y}; \quad \frac{\partial M}{\partial z} \neq \frac{\partial P}{\partial x}$$

It is not conservative