Data Representation

Introduction

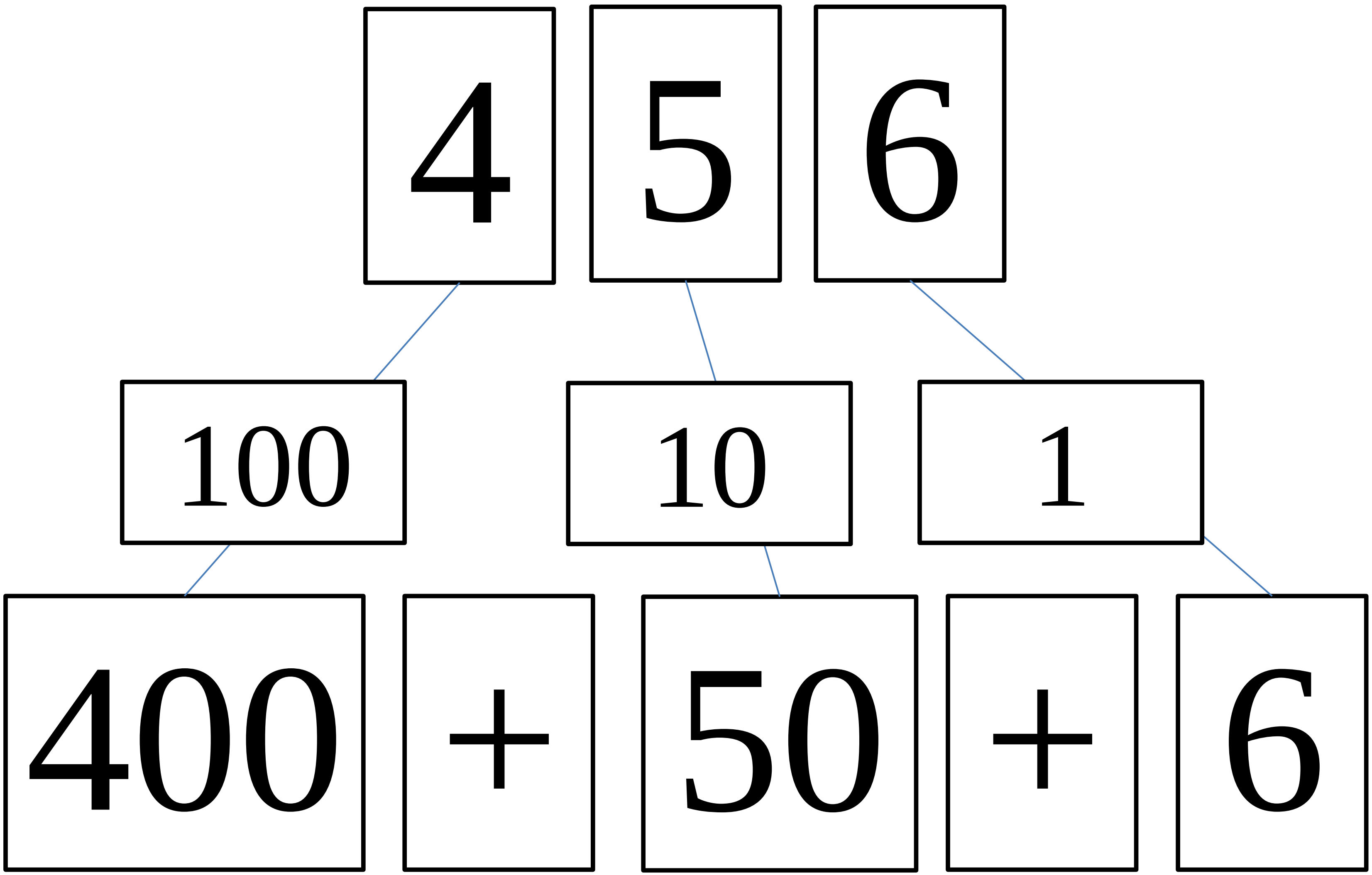
- Most operating systems and programming languages assume that experienced users are familiar with how information is stored in a computer. Without this knowledge, nothing but very basic use of a computer is possible. It would be also impossible to understand many of the fundamental design concepts of digital computers.

Introduction(cont.)

How is data represented in a computer?

Computer systems operate using two voltage levels (usually 0v and +5v). With two such levels, we can represent exactly two different values, zero and one. All kinds of information processed by the computer are expressed using only these two values.

Numbering systems: Decimal System



Numbering Systems: Decimal Numbering System (cont.)

Decimal Numbering

0 – 1 – 2 – 3 – 4 – 5 – 6 – 7 – 8 – 9

10 digits

Weight: 10^0 - 10^1 - 10^2 - 10^3 - 10^4 - …
Weight: 1 - 10 – 100 - 1000 - 10000 - ...

The Binary Numbering System

	Decimal(10)	Binary(2)
Number of digits	10	2
Symbols	0, 1, 2, 3, 4, 5, 6, 7, 8, 9	0, 1
Weights	10^0 , 10^1 , 10^2 , 10^3 , ...	2^0 , 2^1 , 2^2 , 2^3 , ...
	1, 10, 100, 1000, ...	1, 2, 4, 8, 16,....

The Binary Numbering System

Example $(1010)_{10}$, $(1010)_2$

1000	100	10	1	
1	0	1	0	1000+10
				one thousand and ten
8	4	2	1	
1	0	1	0	$8+2 \rightarrow 10$

The Binary Numbering System
Binary to Decimal(cont.)

$$(13)_{10} = (?)_2$$

$$13 = 8+4+1 \rightarrow (1101)_2$$

13/2	=	6	remainder 1
6/2	=	3	remainder 0
3/2	=	1	remainder 1
1/2	=	0	remainder 1

$(1\ 1\ 0\ 1)_2$

The Binary Numbering System
Binary $\leftrightarrow$ Decimal

$$(52)_{10} = (?)_2$$

$$52 = ?+?+..+? \rightarrow (?)_2$$

52/2	= 26	remainder 0	
26/2	= 13	remainder 0	
13/2	= 6	remainder 1	
6/2	= 3	remainder 0	
3/2	= 1	remainder 1	
1/2	= 0	remainder 1	

$(110100)_2$

check

1	1	0	1	0	0
32	16	8	4	2	1

$32+16+4 \rightarrow 52$

Hexadecimal Numbering System

	Decimal(10)	Binary(2)	Hexadecimal(16)
Number of digits	10	2	16
Symbols	0, 1, 2, 3, 4, 5, 6, 7, 8, 9	0, 1	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F
Weight	10^0 , 10^1 , 10^2, … 1, 10, 100, ….	2^0 , 2^1 , 2^2 , … 1, 2, 4, 8, ….	16^0 , 16^1 , 16^2 , … 1, 16, 256, ….

Hexadecimal Numbering System
Hexadecimal ↔ Decimal

$$(5378)_{10} = (?)_{16}$$

$$5378 = ?+?+..+? \rightarrow (?)_{16}$$

5378/16	= 336	remainder 2	
336 /16	= 21	remainder 0	
21/16	= 1	remainder 5	
1/16	= 0	remainder 1	

(1502)

check

1	5	0	2
16^3	16^2	16^1	16^0

$$4096 + 5*256 + 2*1 \rightarrow 5378$$

Hexadecimal Numbering System
Hexadecimal ↔ Decimal

$$(7739)_{10} = (?)_{16}$$

$$7739 = ?+?+..+? \rightarrow (?)_{16}$$

7739/16	= 483	remainder 11	→ B
483 /16	= 30	remainder 3	
30/16	= 1	remainder 14	→ E
1/16	= 0	remainder 1	

(1E3B)

check

1	E	3	B
16^3	16^2	16^1	16^0

4096 + 14*256 + 3*16+ 11*1 → 7739

Binary ↔ Hexadecimal

$(7739)_{10} = (1E3B)_{16} = (\ \ ?\ \ )_2$

$(1) \rightarrow (0001)$
$(E) \rightarrow (1110)$
$(3) \rightarrow (0011)$
$(B) \rightarrow (1011)$

$(1E3B)_{16} = (\underline{0001}\ \underline{1110}\ \underline{0011}\ \underline{1011})_2$

Decimal	Binary	Hexadecimal
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Check $\rightarrow (2^0 + 2^1 + 2^3) + (2^4 + 2^5) + (2^9 + 2^{10} + 2^{11}) + (2^{12})$
$= (1+2+8)+(16+32)+(512+1024+2048)+(4096) = 7739$

Non-integer number representation (Decimal)

345.678

100	10	1	0.1	0.01	0.001
10^{2}	10^{1}	10^{0}	10^{-1}	10^{-2}	10^{-3}

Non-integer number representation (Binary)

$$(101.101)_2$$

2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}
4	2	1	1/2	1/4	1/8

$$(101.101)_2 = (4+1+0.5+0.125)_{10} = (5.625)_{10}$$

Decimal addition

1

	8	4	7
	6	8	2
1	5	~~12~~	9
		2	

1529

Binary addition

weight	4	2	1
	1	0	1
	0	1	0
	1	1	1

Binary addition (cont.)

$$
\begin{array}{cccccc}
1 & 1 & & & & \\
 & 1 & 1 & 1 & \rightarrow & 7 \\
 & 1 & 1 & 0 & \rightarrow & 6 \\
\hline
1 & 1 & 0 & 1 & \rightarrow & 13 \\
\hline
\end{array}
$$

Hexadecimal addition (cont.)

$$
\begin{array}{ccc}
(5 & 8 & 4)_{16} \\
(7 & 1 & 9)_{16} \\
\hline
(12)_{10} & (9)_{10} & (13)_{10} \\
(C)_{16} & (9)_{16} & (D)_{16} \\
\hline
(C & 9 & D)_{16}
\end{array}
$$

Hexadecimal addition (cont.)

$$
\begin{array}{ccc}
 & 1 & \\
(5 & 8 & A)_{16} \\
(B & 4 & 9)_{16} \\
\hline
(16)_{10} & (13)_{10} & (19)_{10} \\
(10)_{16} & (D)_{16} & \cancel{(13)}_{16} \\
 & & (3)_{16} \\
\hline
\end{array}
$$

$$(10D3)_{16}$$

Negative Number Representation

- There are three formats for representing negative numbers in base-r system
  - Sign-magnitude
  - r’s complement
  - (r-1)’s complement

Negative Number Representation
Sign-magnitude

Sign-magnitude

This type uses one bit for the sign (0 = positive, 1=negative) and the remaining bits represent the magnitude of the number.

Decimal value	Positive	Negative
1	0 0 0 1	1 0 0 1
3	0 0 1 1	1 0 1 1
4	0 1 0 0	1 1 0 0
5	0 1 0 1	1 1 0 1

2's complement (Overview)

Given a number N in base r having n digits, the (r)'s complement of N is defined by = (r^n) -N.

For example if n= 4bits:

The 2's complement of $(0010)_2$ is $2^4 - 2 = 14 = (1110)_2$

The 2's complement of $(0110)_2$ is $2^4 - 6 = 10 = (1010)_2$

$2^4 - 6 = 16-6 =$

$= (1+ 15)-6$

$= 1+(15-6)= 1+(1111)_2 -(0110)_2= 1 + (1001)_2=(1010)_2$

2's complement (Ex)

4-bits

Subtract 5-3 $\rightarrow (0101)_2 - (0011)_2$

$\rightarrow (0101)_2 - (0011)_2 = (2)_{10} = (0010)$

2's complement of $(0011)_2 = 1 + (1100)_2 = (1101)_2$

$$
\begin{array}{ccccc}
 & 0 & 1 & 0 & 1 \\
 & 1 & 1 & 0 & 1 & + \\
\hline
\cancel{1} & 0 & 0 & 1 & 0
\end{array}
$$

$\rightarrow$ (0 0 1 0)

2's complement (Ex)

5-bits

Subtract 11-5 $\rightarrow (1011)_2 - (101)_2$

$\rightarrow (01011)_2 - (00101)_2 = (6)_{10} = (00110)$

2's complement of $(00101)_2 = 1 + (11010)_2 = (11011)_2$

$$
\begin{array}{cccccc}
 & 0 & 1 & 0 & 1 & 1 \\
 & 1 & 1 & 0 & 1 & 1 \\
\hline
\cancel{1} & 0 & 0 & 1 & 1 & 0
\end{array}
\quad +
$$

$\rightarrow (0 \quad 0 \quad 1 \quad 1 \quad 0)_2$

2's complement (Ex)

5-bits

Subtract 5-9 → $(00101)_2 - (01001)_2$

→ $(00101)_2 - (01001)_2 = -(4)_{10} = -(00100)$

2's complement of $(01001)_2 = 1 + (10110)_2 = (10111)_2$

$$
\begin{array}{cccccc}
 & 0 & 0 & 1 & 0 & 1 \\
 & 1 & 0 & 1 & 1 & 1 \quad + \\
\hline
\cancel{?} & 1 & 1 & 1 & 0 & 0
\end{array}
$$

2's complement of $(11100)_2 = 1 + (00011) = (00100) \rightarrow -(00100)$

2's complement (cont.)

Decimal	4bit (+ve)	4bit (-ve)
1	0001	1111
2	0010	1110
3	0011	1101
4	0100	1100
5	0101	1011
6	0110	1010
7	0111	1001

Weights of 2's complement (3 bit) are : -4 2 1

Weights of 2's complement (4 bit) are : -8 4 2 1

Weights of 2's complement (n bit) are : $-2^{n-1}\ 2^{n-2}\ 2^{n-3}\ \ldots\ 2^{1}\ 2^{0}$

Ex using 2's complement $(1010)_2$ is -8 +2 = -6

2's complement (cont.) (Fast method)

Example (using 2's complement)

$(1\ 0\ 0\ 1)_2 \rightarrow -8+1= -7$

$(0\ 1\ 0\ 1)_2 \rightarrow 4+1 = 5$

$(1\ 0\ 1)_2 \rightarrow -4+1 = -3$

$(0\ 1\ 1)_2 \rightarrow 1+2= 3$

2's complement (cont.) (Fast method- cont.)

Using 2's complement (4-bits)

Max → $(0111)_2 = +7$

Min → $(1000)_2 = -8$

Using 2's complement (3-bits)

Max → $(011)_2 = +3$

Min → $(100)_2 = -4$

Using 2's complement (n-bits)

Max → $+(2^{n-1})-1$

Min → $-(2^{n-1})$