Parallel and Distributed Computing
CS3006 (BDS-6A)
Lecture 15

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Previous Lecture

- Clusters
  - High Performance
- Grid Computing
- Cloud Computing
- Basic Communication Operations
  - One-to-all broadcast
  - All-to-one reduction

Basic Communication Operations

- Preliminaries
  - Exchanging data is a fundamental requirement for most of the parallel algorithms
  - $\boldsymbol{t_s + mt_w}$ - the simplified communication cost model :-
    - Over distributed memory infrastructure
    - Assuming cut-through routing
  - This chapter is about commonly used basic communication patterns over the different interconnections
    - We shall derive communication costs of these operations on different interconnections.

Basic Communication Operations

- **Assumptions for the Operations**
  - Interconnections support cut-through routing
  - Communication time between any pair of nodes in the network is the same (regardless of the number of intermediate nodes)
  - Links are bi-directional
    - The directly connected nodes can simultaneously send messages of *m words* without any congestion
  - Single-port communication model
    - A node can send on only one of its links at a time
    - A node can receive on only one of its links at a time
  - However, a node can receive a message while sending another message at the same time on the same or a different link.

Basic Communication Operations
(One-to-All Broadcast and All-to-One Reduction)

- **Linear Array or Ring**
  - Recursive Doubling Broadcast

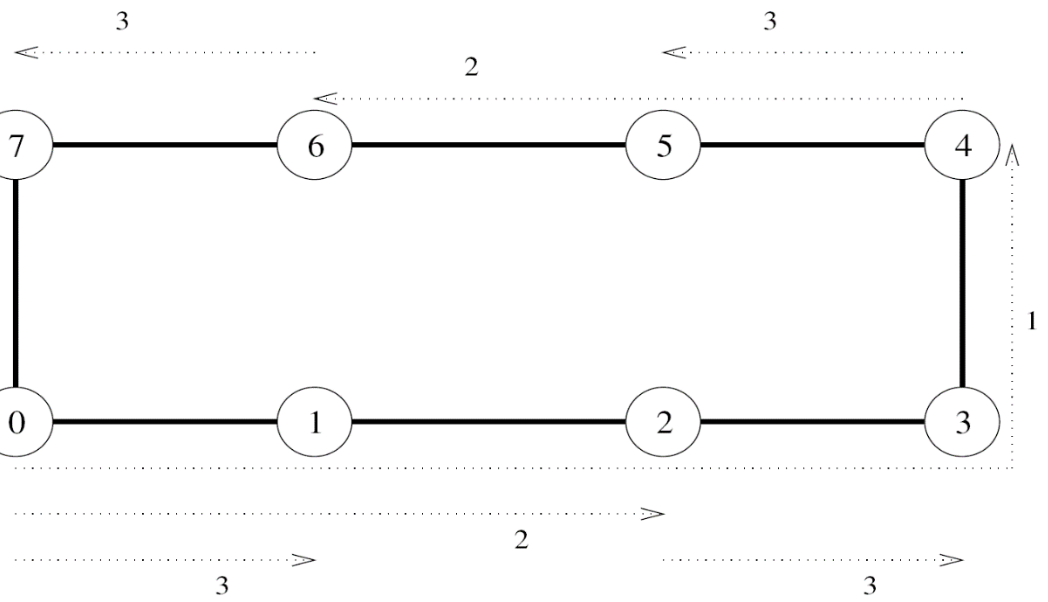


Figure 4.2 One-to-all broadcast on an eight-node ring. Node 0 is the source of the broadcast. Each message transfer step is shown by a numbered, dotted arrow from the source of the message to its destination. The number on an arrow indicates the time step during which the message is transferred.

Basic Communication Operations
(One-to-All Broadcast and All-to-One Reduction)

- **Linear Array or Ring**
  - Recursive Doubling Reduction

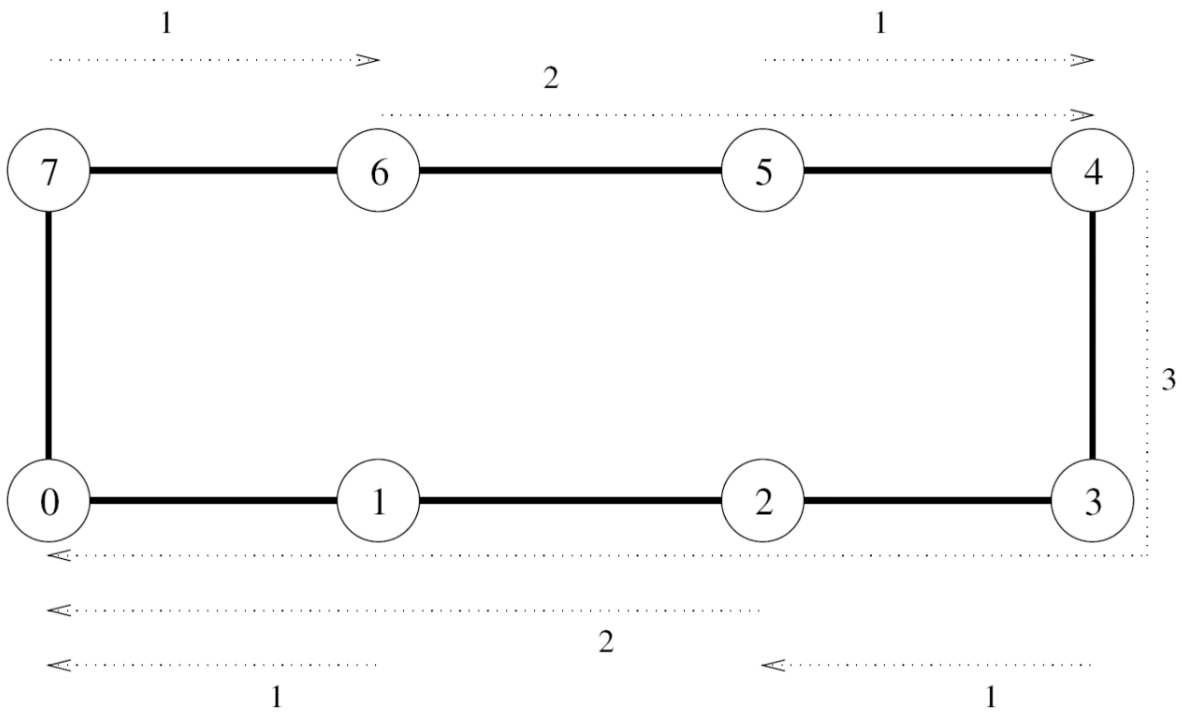


Figure 4.3 Reduction on an eight-node ring with node 0 as the destination of the reduction.

Basic Communication Operations
(One-to-All Broadcast and All-to-One Reduction)

- Matrix-Vector Multiplication (An Application)

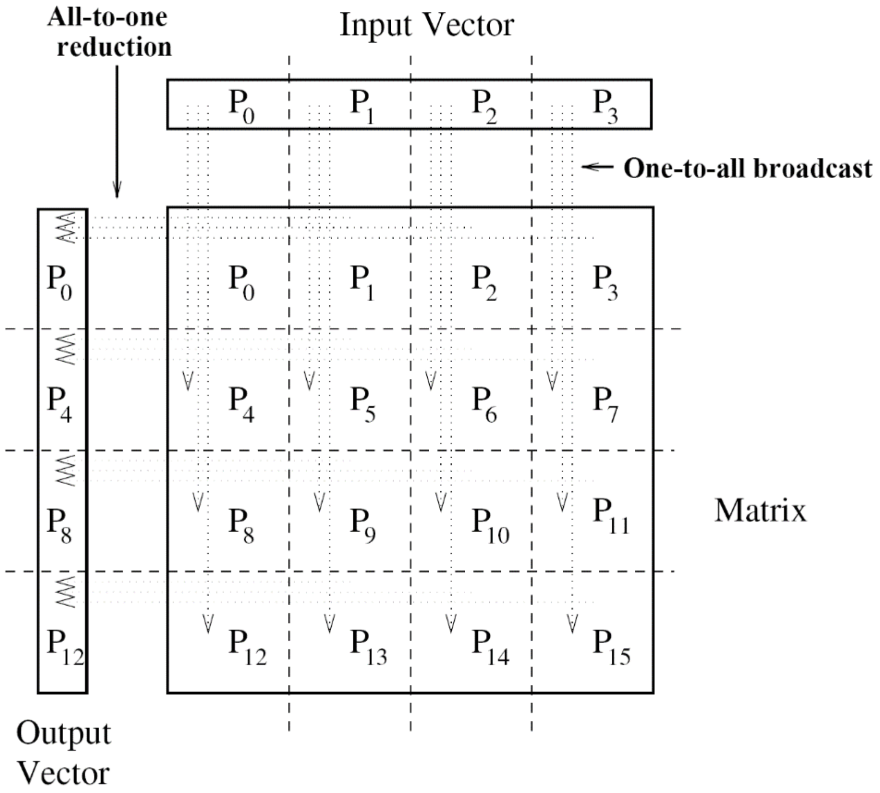


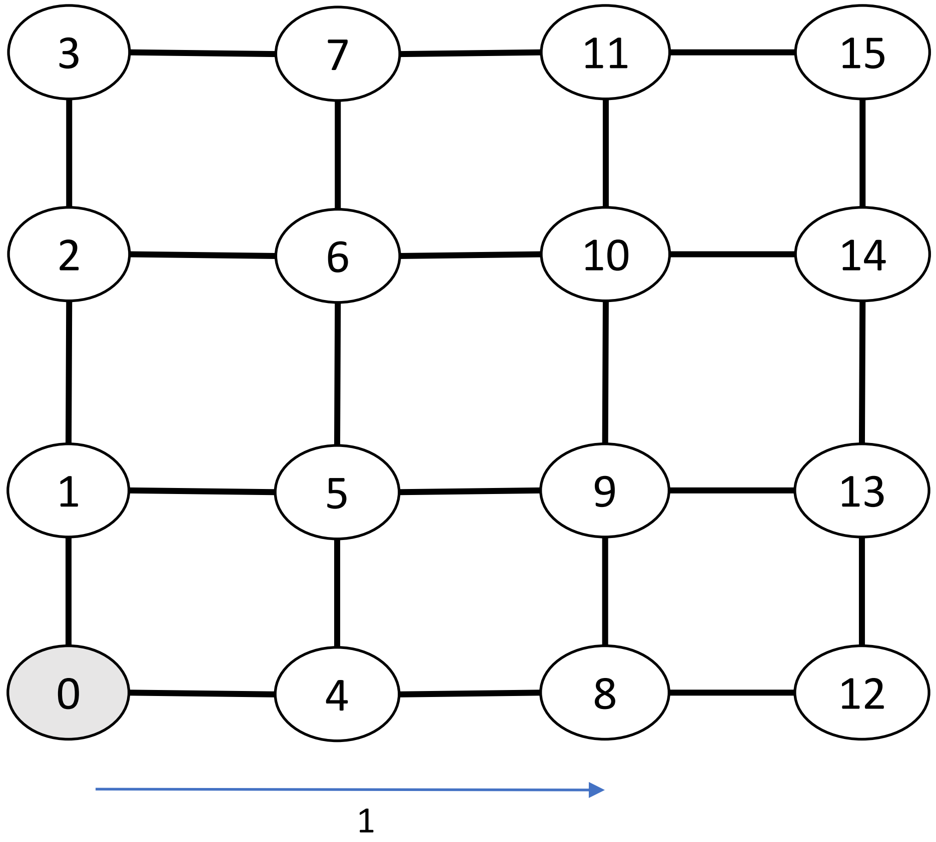
Figure 4.4 One-to-all broadcast and all-to-one reduction in the multiplication of a 4×4 matrix with a 4×1 vector.

Basic Communication Operations (One-to-All Broadcast and All-to-One Reduction)

- **Mesh**
  - We can regard each row and column of a square mesh of *p* nodes as a linear array of nodes
  - Communication algorithms on the mesh are simple extensions of their linear array counterparts
  - **Broadcast and Reduction**
    - Two step breakdown:
      - The operation is performed along one by treating the row as linear array
      - Then all the columns are treated similarly

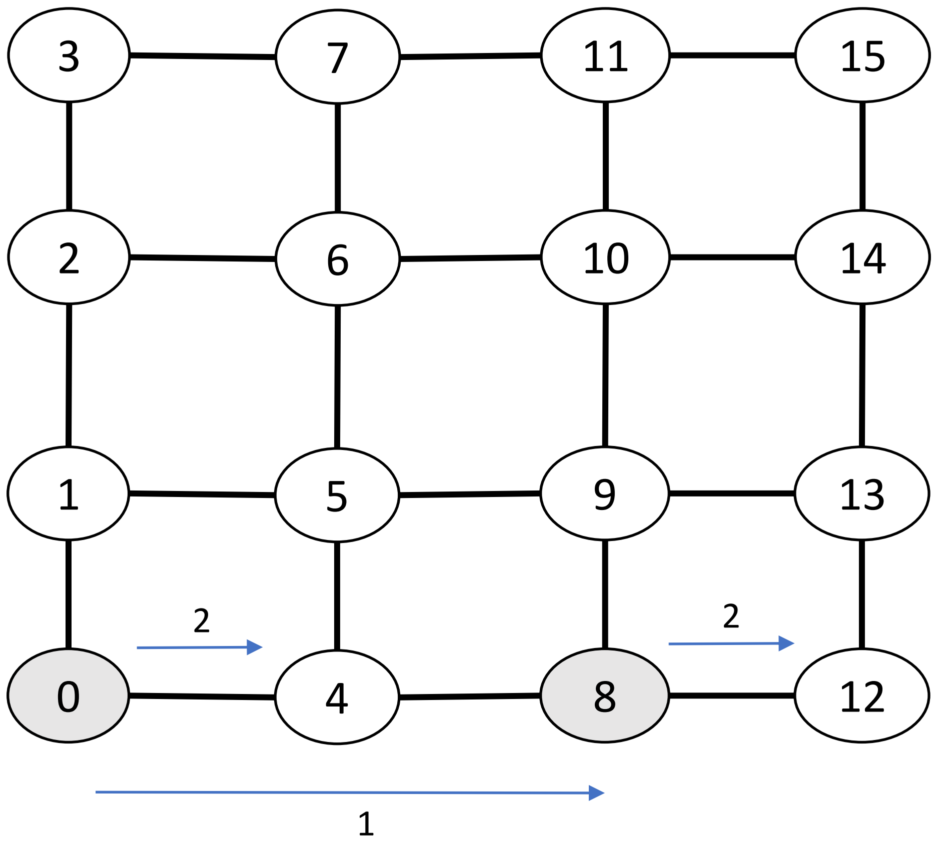
Basic Communication Operations (One-to-All Broadcast and All-to-One Reduction)

- **Mesh (broadcast and reduction)**



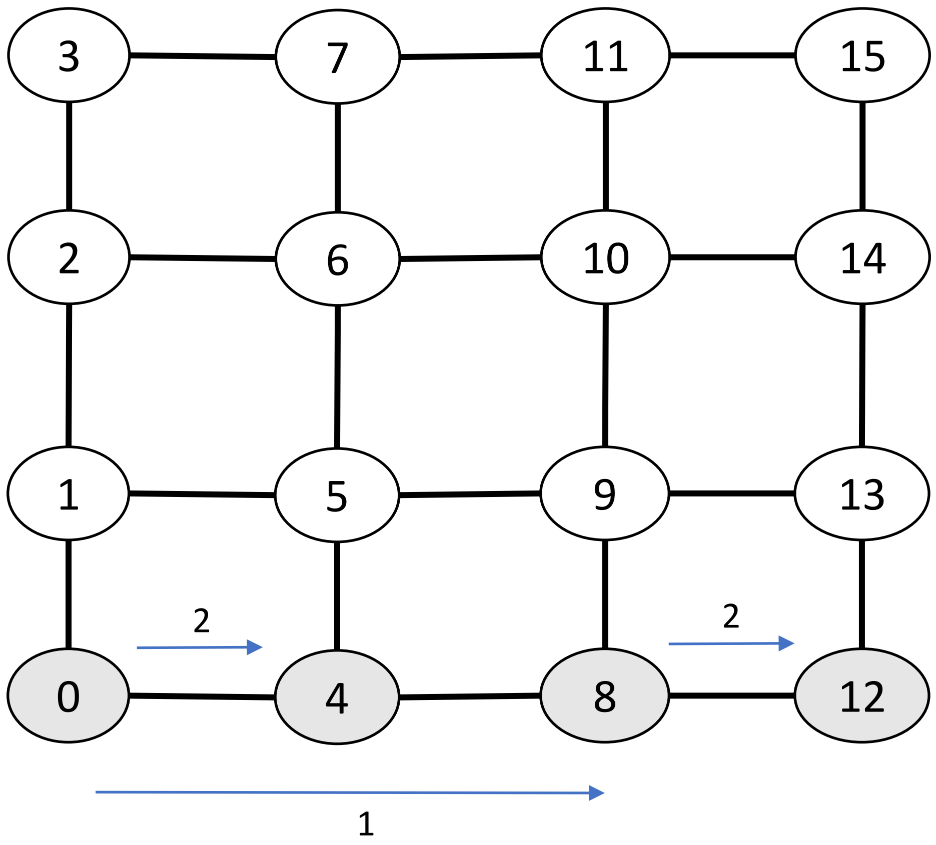
Basic Communication Operations (One-to-All Broadcast and All-to-One Reduction)

- **Mesh (broadcast and reduction)**



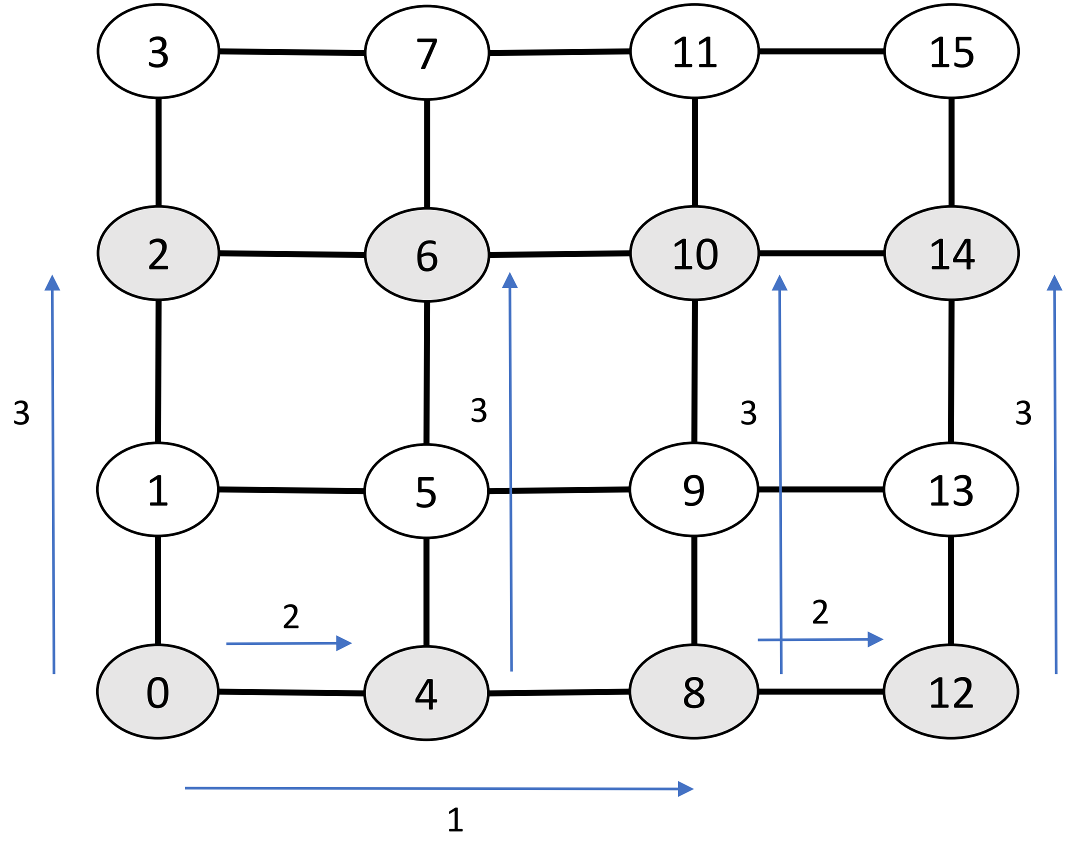
Basic Communication Operations (One-to-All Broadcast and All-to-One Reduction)

- **Mesh (broadcast and reduction)**



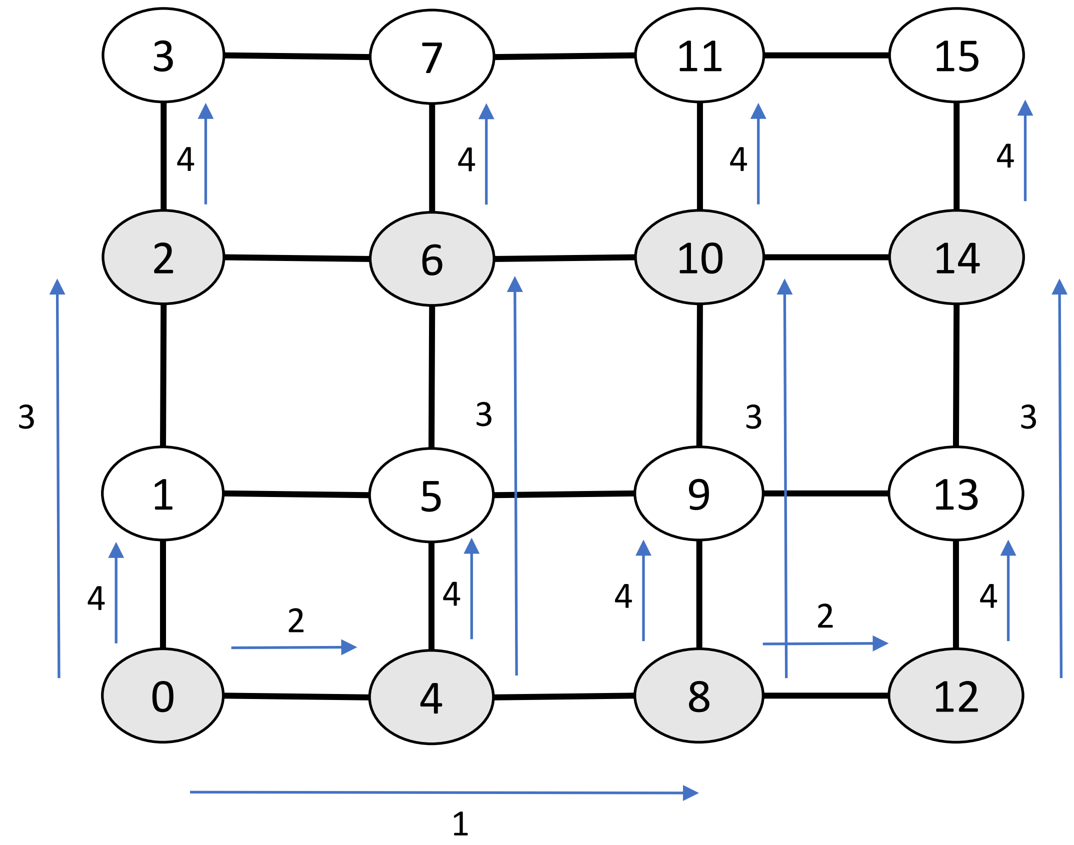
Basic Communication Operations
(One-to-All Broadcast and All-to-One Reduction)

- **Mesh (broadcast and reduction)**



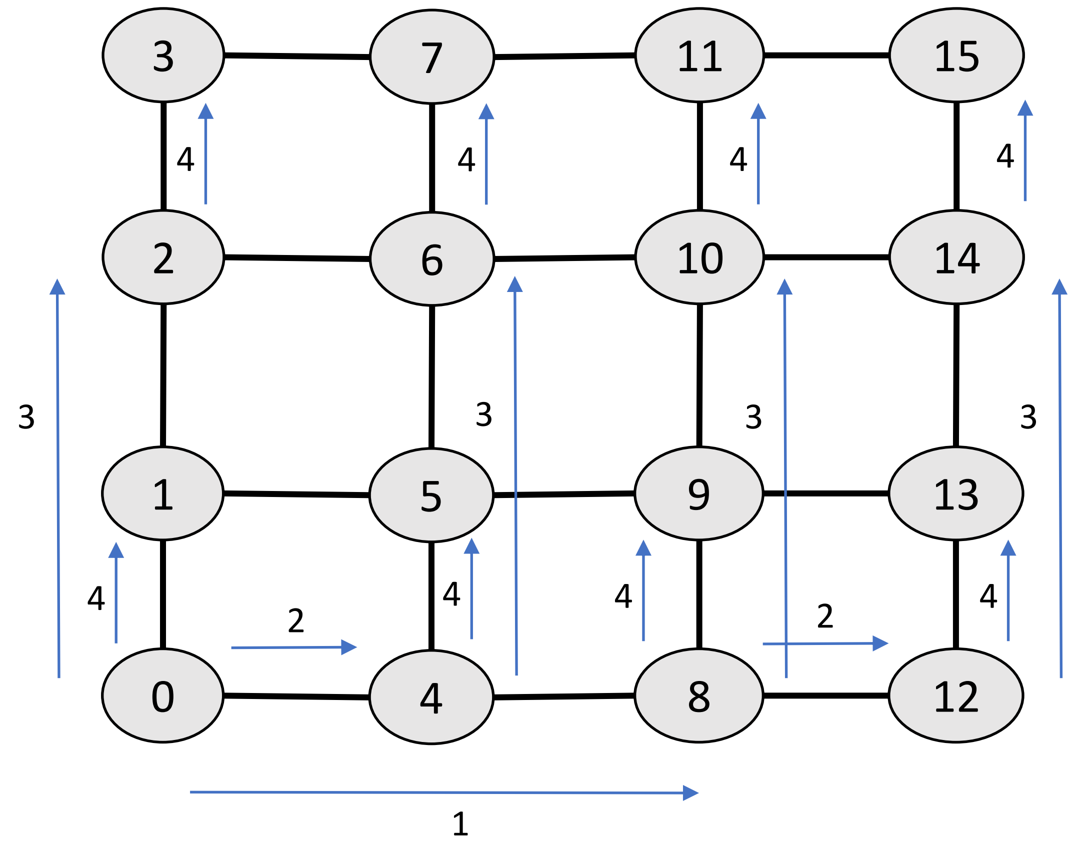
Basic Communication Operations (One-to-All Broadcast and All-to-One Reduction)

- **Mesh (broadcast and reduction)**



Basic Communication Operations (One-to-All Broadcast and All-to-One Reduction)

- **Mesh (broadcast and reduction)**



Basic Communication Operations (One-to-All Broadcast and All-to-One Reduction)

- **Mesh (broadcast and reduction)**

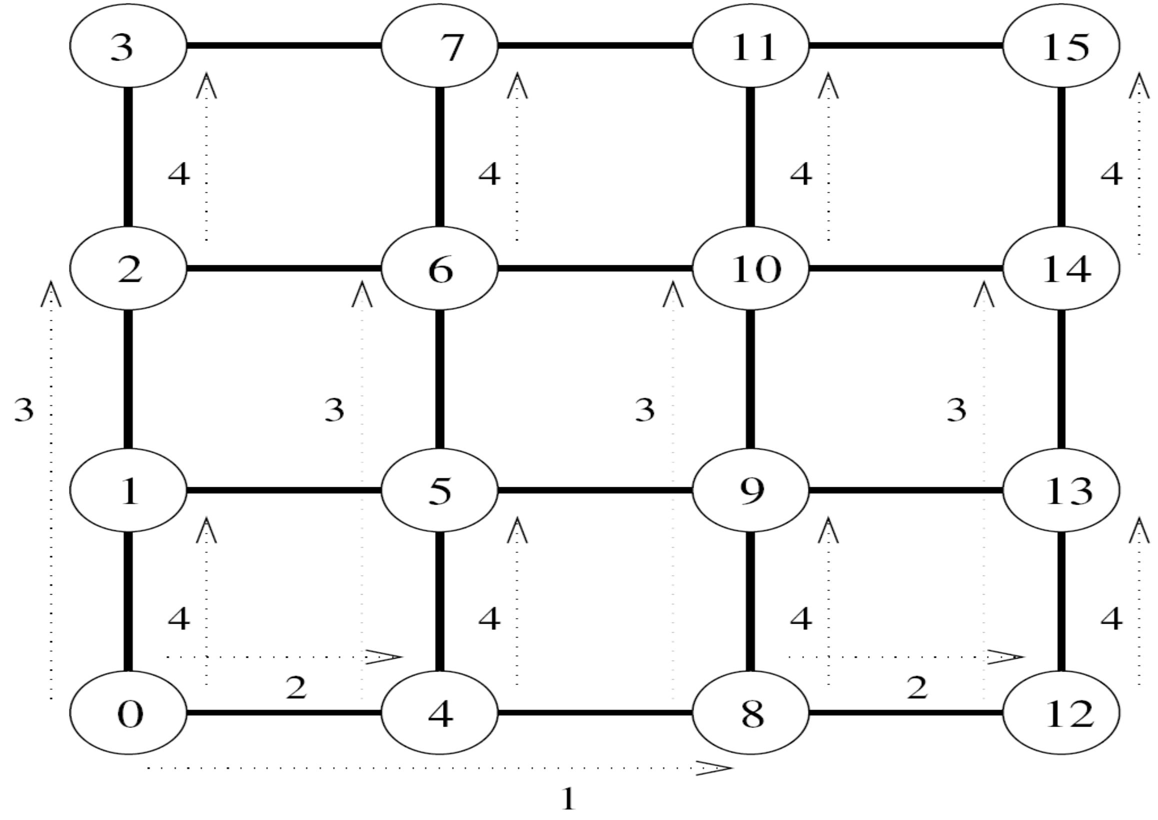


Figure 4.5 One-to-all broadcast on a 16-node mesh.

Basic Communication Operations (One-to-All Broadcast and All-to-One Reduction)

- Balanced Binary Tree

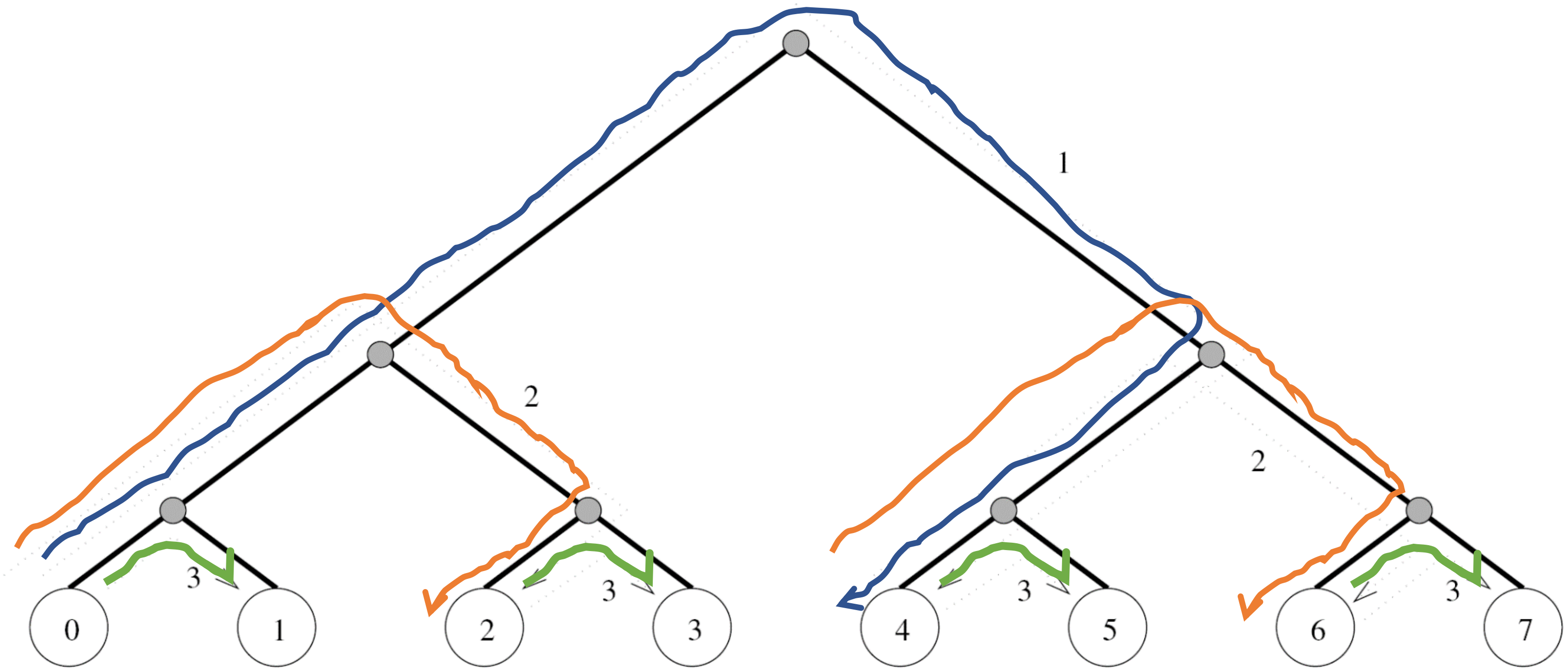


Figure 4.7 One-to-all broadcast on an eight-node tree.

Basic Communication Operations (One-to-All Broadcast and All-to-One Reduction)

Hypercube

- Broadcast
  - Source node first sends data to one node in the highest dimension
  - The communication successively proceeds along lower dimensions in the subsequent steps
  - The algorithm is the same as used for linear array
    - But here, changing order of dimension does not congest the network

Basic Communication Operations (One-to-All Broadcast and All-to-One Reduction)

Hypercube

- Broadcast

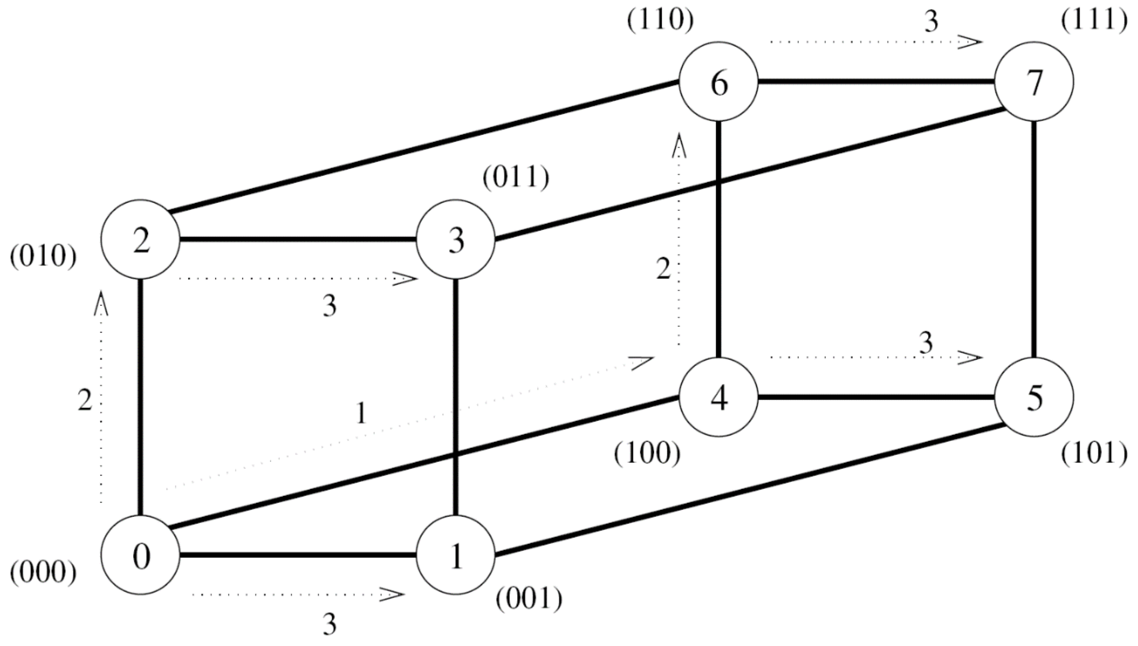


Figure 4.6 One-to-all broadcast on a three-dimensional hypercube. The binary representations of node labels are shown in parentheses.

A more in-depth look

- Algorithms of these operations

Basic Communication Operations (One-to-All Broadcast and All-to-One Reduction)

```
procedure ONE_TO_ALL_BC(d, my_id, X)
begin
    mask := 2^d - 1;                        /* Set all d bits of mask to 1 */
    for i := d - 1 downto 0 do              /* Outer loop */
        mask := mask XOR 2^i;               /* Set bit i of mask to 0 */
        if (my_id AND mask) = 0 then        /* If lower i bits of my_id are 0 */
            if (my_id AND 2^i) = 0 then
                msg_destination := my_id XOR 2^i;
                send X to msg_destination;
            else
                msg_source := my_id XOR 2^i;
                receive X from msg_source;
            endelse;
        endif;
    endfor;
end ONE_TO_ALL_BC
```

Algorithm 4.1 One-to-all broadcast of a message X from node 0 of a d-dimensional p-node hypercube ($d = \log p$). AND and XOR are bitwise logical-and and exclusive-or operations, respectively.

Please read the book (chapter 4)

Basic Communication Operations (One-to-All Broadcast and All-to-One Reduction)

```
procedure GENERAL_ONE_TO_ALL_BC(d, my_id, source, X)
begin
    my_virtual_id := my_id XOR source;
    mask := 2^d - 1;
    for i := d - 1 downto 0 do      /* Outer loop */
        mask := mask XOR 2^i;   /* Set bit i of mask to 0 */
        if (my_virtual_id AND mask) = 0 then
            if (my_virtual_id AND 2^i) = 0 then
                virtual_dest := my_virtual_id XOR 2^i;
                send X to (virtual_dest XOR source);
    /* Convert virtual_dest to the label of the physical destination */
            else
                virtual_source := my_virtual_id XOR 2^i;
                receive X from (virtual_source XOR source);
    /* Convert virtual_source to the label of the physical source */
            endelse;
    endfor;
end GENERAL_ONE_TO_ALL_BC
```

Algorithm 4.2 One-to-all broadcast of a message X initiated by $source$ on a d-dimensional hypothetical hypercube. The AND and XOR operations are bitwise logical operations.

Please read the book (chapter 4)

Basic Communication Operations (One-to-All Broadcast and All-to-One Reduction)

```
procedure ALL_TO_ONE_REDUCE(d, my_id, m, X, sum)
begin
    for j := 0 to m - 1 do sum[j] := X[j];
    mask := 0;
    for i := 0 to d - 1 do
        /* Select nodes whose lower i bits are 0 */
        if (my_id AND mask) = 0 then
            if (my_id AND 2^i) ≠ 0 then
                msg_destination := my_id XOR 2^i;
                send sum to msg_destination;
            else
                msg_source := my_id XOR 2^i;
                receive X from msg_source;
                for j := 0 to m - 1 do
                    sum[j] := sum[j] + X[j];
            endelse;
        mask := mask XOR 2^i;    /* Set bit i of mask to 1 */
    endfor;
end ALL_TO_ONE_REDUCE
```

Please read the book (chapter 4)

Basic Communication Operations (One-to-All Broadcast and All-to-One Reduction)

- **Cost Estimation**
  - Broadcast needs **log(p)** point-to-point simple message transfer steps.
  - Message size of each transfer is **m**
  - Time for each of the transfers is: $\boldsymbol{t_s + mt_w}$

Hence cost for log(p) transfers→ T = $(\boldsymbol{t_s}+\boldsymbol{mt_w})\log \boldsymbol{p}$

All-to-All Broadcast and All-to-All Reduction

All-to-All Broadcast

- A generalization of one-to-all broadcast.
- Every process broadcasts an *m-word* message.
  - The broadcast-message for each of the processes can be *different* from others

All-to-All Reduction

- Dual of all-to-all broadcast
- Each node is the destination of an all-to-one reduction out of total P reductions.

All-to-All Broadcast and All-to-All Reduction

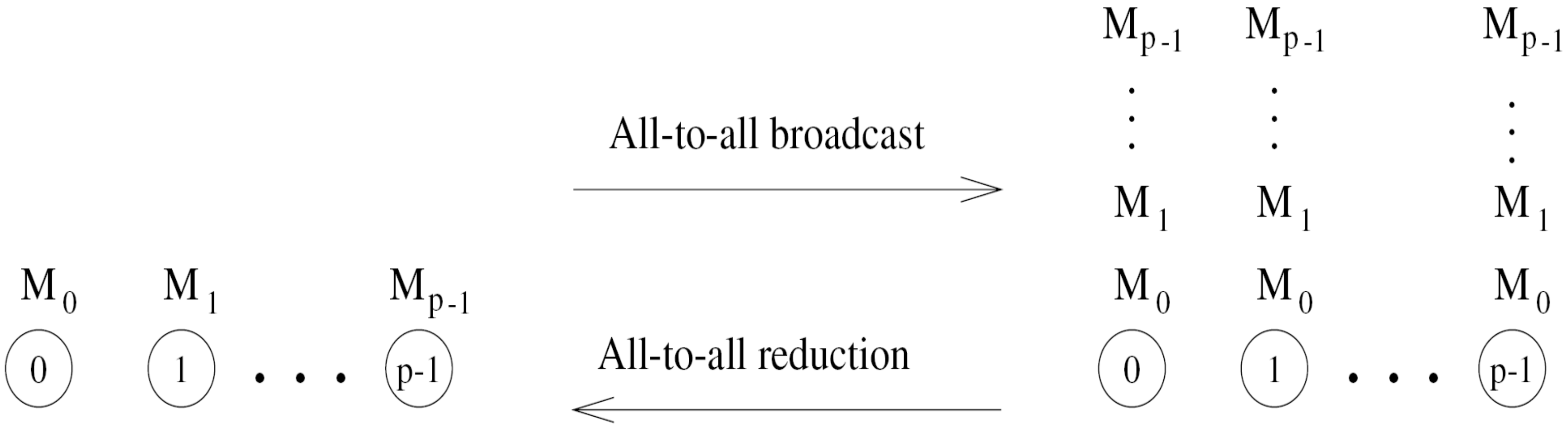


Figure 4.8 All-to-all broadcast and all-to-all reduction.

A naïve Broadcast method may be performing **P** one-to-all broadcasts. This will result **P(log(p)(t(s) + mt(w)))** communication time.
Solution?

All-to-All Broadcast and All-to-All Reduction

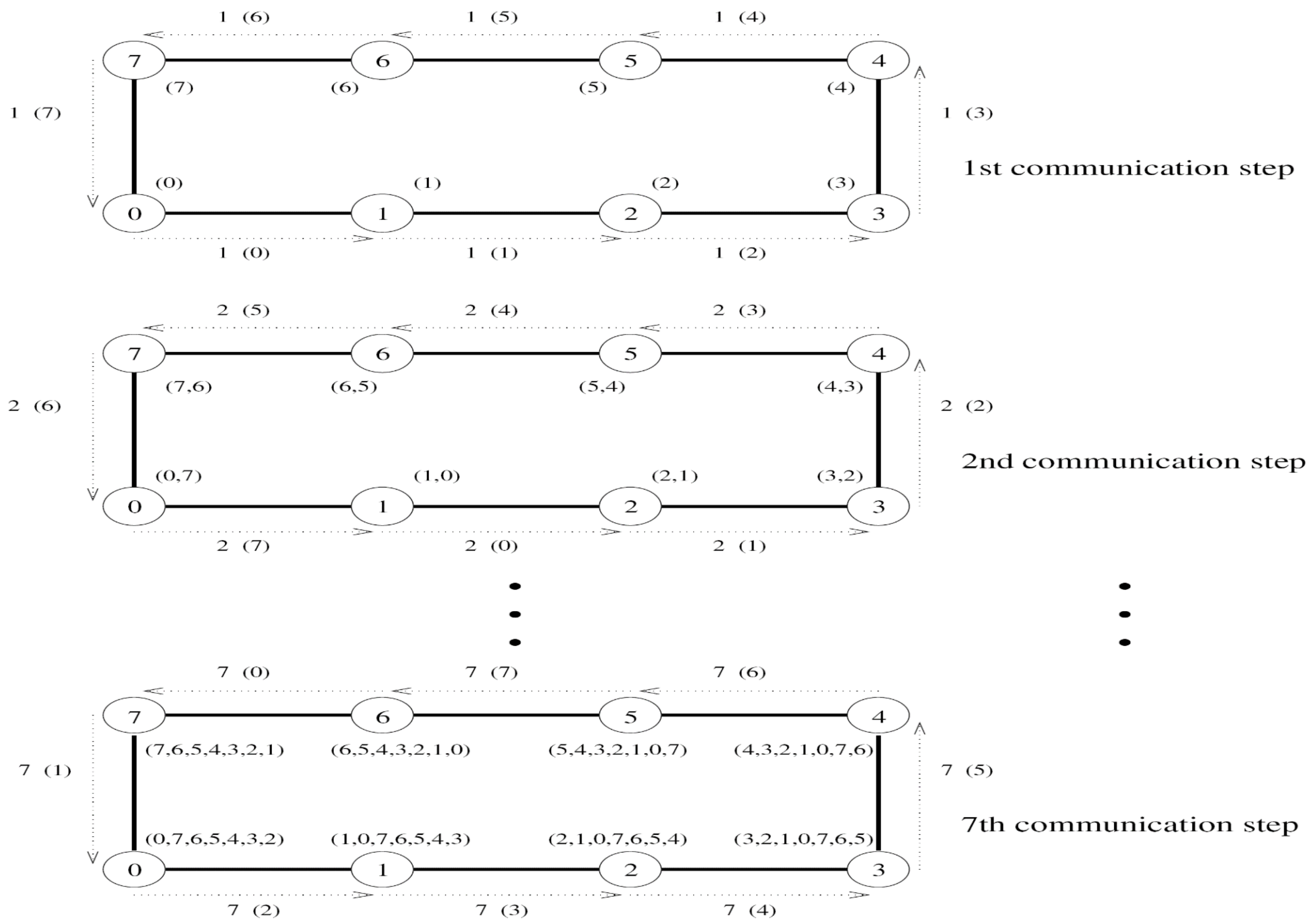


Figure 4.9 All-to-all broadcast on an eight-node ring. The label of each arrow shows the time step and, within parentheses, the label of the node that owned the current message being transferred before the beginning of the broadcast. The number(s) in parentheses next to each node are the labels of nodes from which data has been received prior to the current communication step. Only the first, second, and last communication steps are shown.

Linear Ring Broadcast

```
procedure ALL_TO_ALL_BC_RING(my_id, my_msg, p, result)
begin
    left := (my_id − 1) mod p;
    right := (my_id + 1) mod p;
    result := my_msg;
    msg := result;
    for i := 1 to p − 1 do
        send msg to right;
        receive msg from left;
        result := result ∪ msg;
    endfor;
end ALL_TO_ALL_BC_RING
```

Algorithm 4.4 All-to-all broadcast on a p-node ring.

All-to-All Reduction

Linear Array or Ring

- **Reduction**
  - Draw an All-to-All Broadcast on a P-node linear ring
  - Reverse the directions in each foreach of the step without changing message
  - After each communication step, combine messages having same broadcast destination with associative operator.

- **Now, Its your turn to draw?**
  - **Draw an All-to-All Broadcast on a 4-node linear ring**
  - **Reverse the directions and combine the results using 'SUM'**

Useful Links

- https://www.cs.unc.edu/~prins/Classes/633/Readings/Kumar-BasicCommunicationOperations.pdf
- https://phyweb.physics.nus.edu.sg/~phytaysc/cz4102_07/cz4102_le6.pdf
- http://www.math.nsysu.edu.tw/~lam/MPI/lecture/chap4_slides.pdf