

# Design and Analysis of Algorithms

## Practice Questions

**Q1)** Given a set  $N$  of integers and an integer  $y$ , determine if there exist two elements in  $N$  whose absolute difference is equal to  $y$  and also print those numbers. The algorithm should take  $O(n \lg n)$  time. Justify why your algorithm runs in  $O(n \lg n)$  time. [10 Marks]

e.g. Let  $N = \{3, 7, 2, 1, 4, 10\}$

$y = 1$

there are three pairs of elements in  $N$  whose absolute difference is 1

Pair 1 =  $|3 - 2| = |-1| = 1$

Pair 2 =  $|3 - 4| = |-1| = 1$

Pair 3 =  $|2 - 1| = 1$

**Q2)** Given two sorted arrays  $X[]$  and  $Y[]$  of sizes  $M$  and  $N$  where  $M \geq N$ , devise an algorithm to merge them into a new sorted array  $C[]$  using  $O(N \lg M)$  comparison operations. Suppose arrays  $M$  and  $N$  are indexed from 1 to  $M$  and from 1 to  $N$  respectively.

*Hint:* use binary search.

**Q3)** Consider an array of distinct numbers sorted in increasing order. The array has been rotated (clockwise)  $k$  number of times. Given such an array, find the value of  $k$ . The solution should be efficient and use divide and conquer approach.

**Examples:**

Input :  $arr[] = \{15, 18, 2, 3, 6, 12\}$

Output: 2

Explanation : Initial array must be  $\{2, 3, 6, 12, 15, 18\}$ . We get the given array after rotating the initial array twice.

Input :  $arr[] = \{7, 9, 11, 12, 5\}$

Output = 4

**Q4)** Given an array of integers which is initially increasing and then decreasing, find the maximum value in the array.

**Examples :**

Input:  $arr[] = \{8, 10, 20, 80, 100, 200, 400, 500, 3, 2, 1\}$

Output: 500

Input: arr[] = { 1, 3, 50, 10, 9, 7, 6 }

Output: 50

Corner case (No decreasing part)

Input: arr[] = { 10, 20, 30, 40, 50 }

Output: 50

Corner case (No increasing part)

Input: arr[] = { 120, 100, 80, 20, 0 }

Output: 120

**Q5)** Suppose you have an unsorted array A of colors *red*, *white* and *blue*. You want to sort this array so that all *reds* are before all *whites*, followed by all *blues*. Only operations available to you for this purpose are: equality comparison  $A[i] == c$  where  $c$  is one of the three colors, and  $\text{swap}(i, j)$  which swaps the colors at indices  $i$  and  $j$  in A. How to sort this array in  $O(n)$  worst case time and  $O(1)$  additional space. Assume that some satellite data is also there with these colors so counting the number of reds, whites and blues will not solve the problem.