

Breadth-first search

BFS algorithm

BFS(G, start)

Create new queue Q

Q.push(start)

dist[1..n] = $\{\infty, \dots, \infty\}$

dist[start] = 0

while Q is not empty

 u = Q.pop()

 for each node v adjacent to u

 if dist[v] = ∞ then

 dist[v] = dist[u] + 1

 Q.push(v)

Suppose we have a graph
G = (V,E) containing
|V| = n nodes and
|E| = m edges.

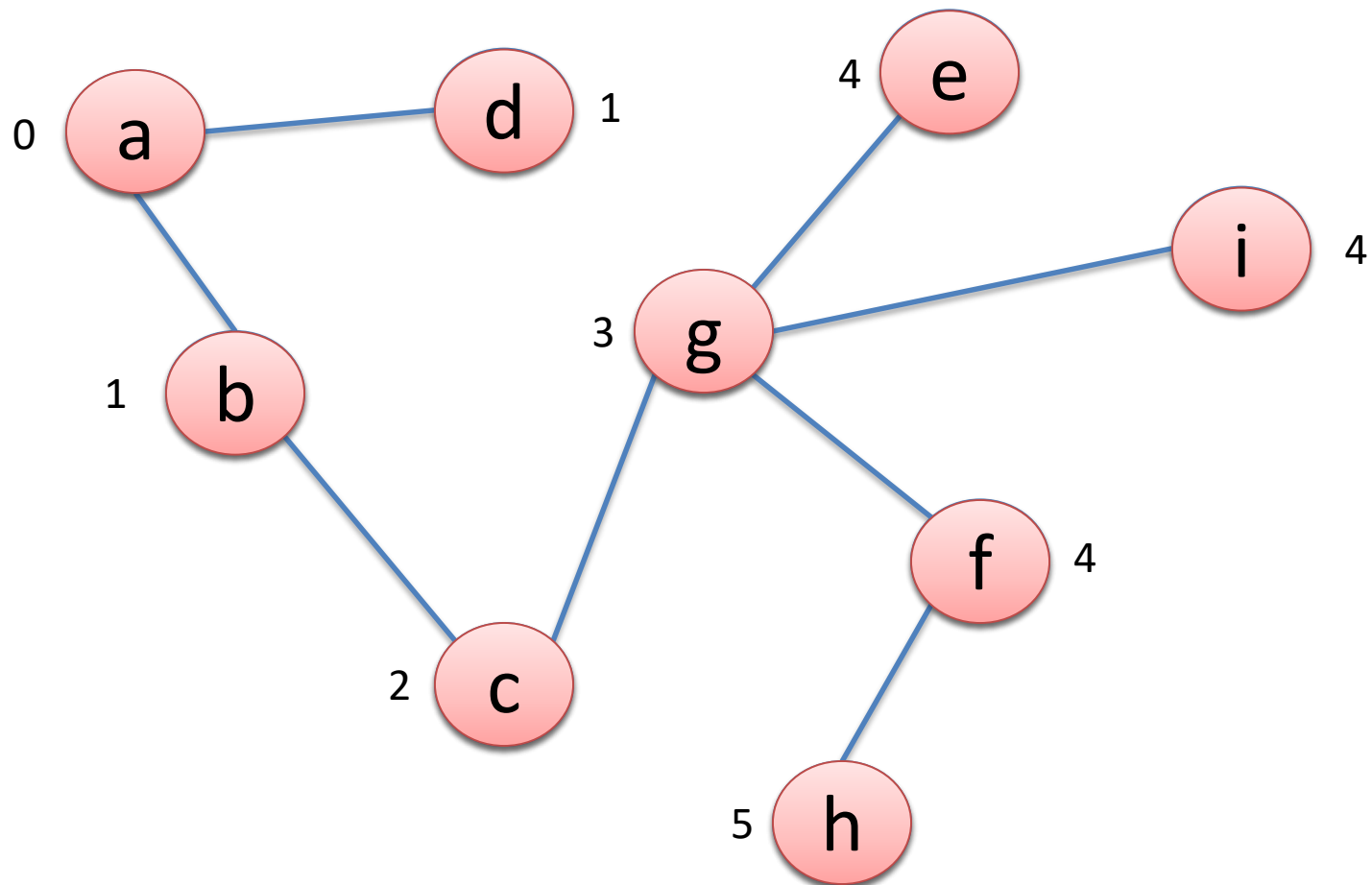
BFS takes **O(n+m)**
time and space.

BFS Application

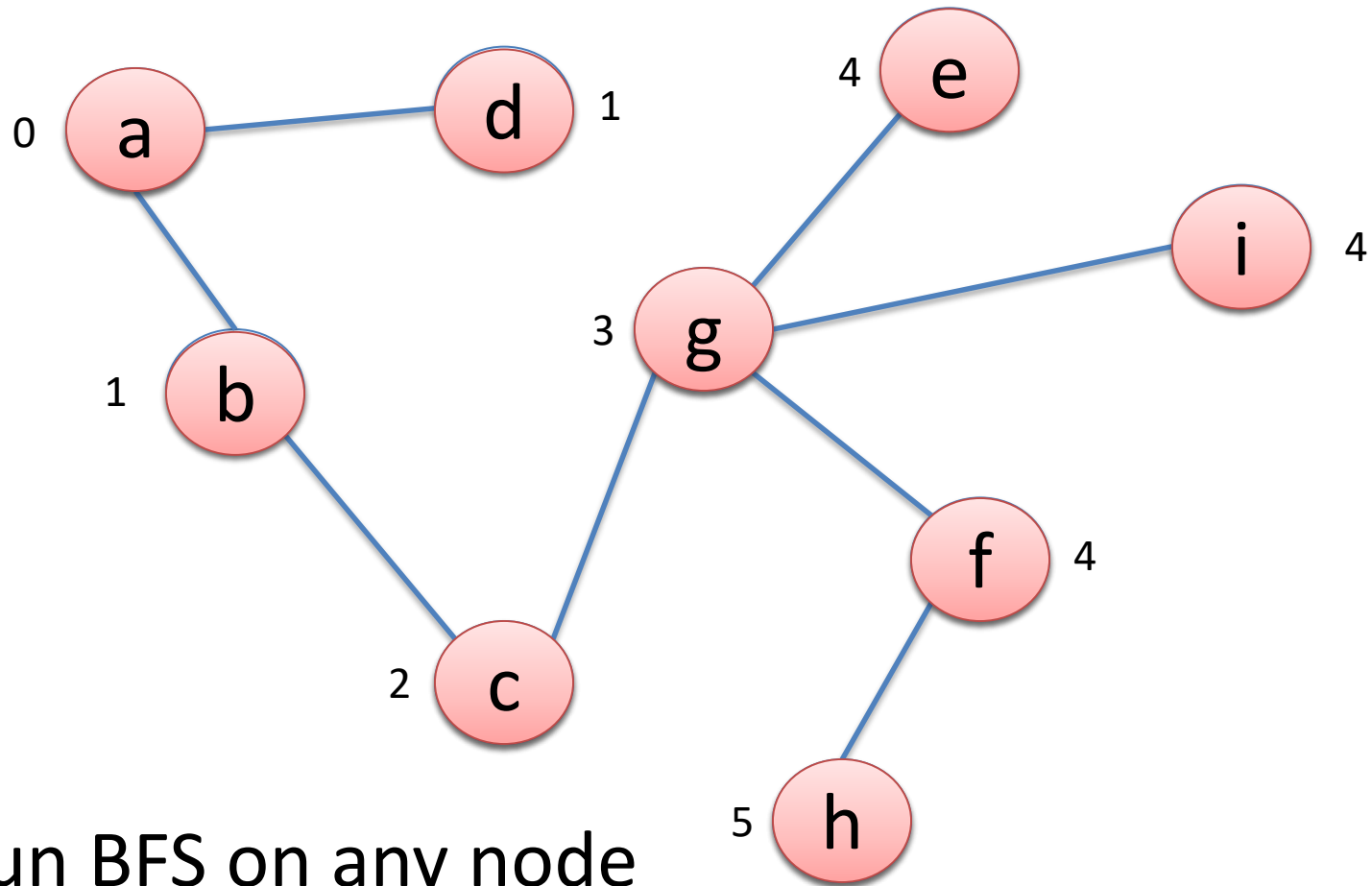
- Finding Shortest Paths
 - When each edge has same weight

Example: BFS starting at node a

- All weights initially set to infinity.

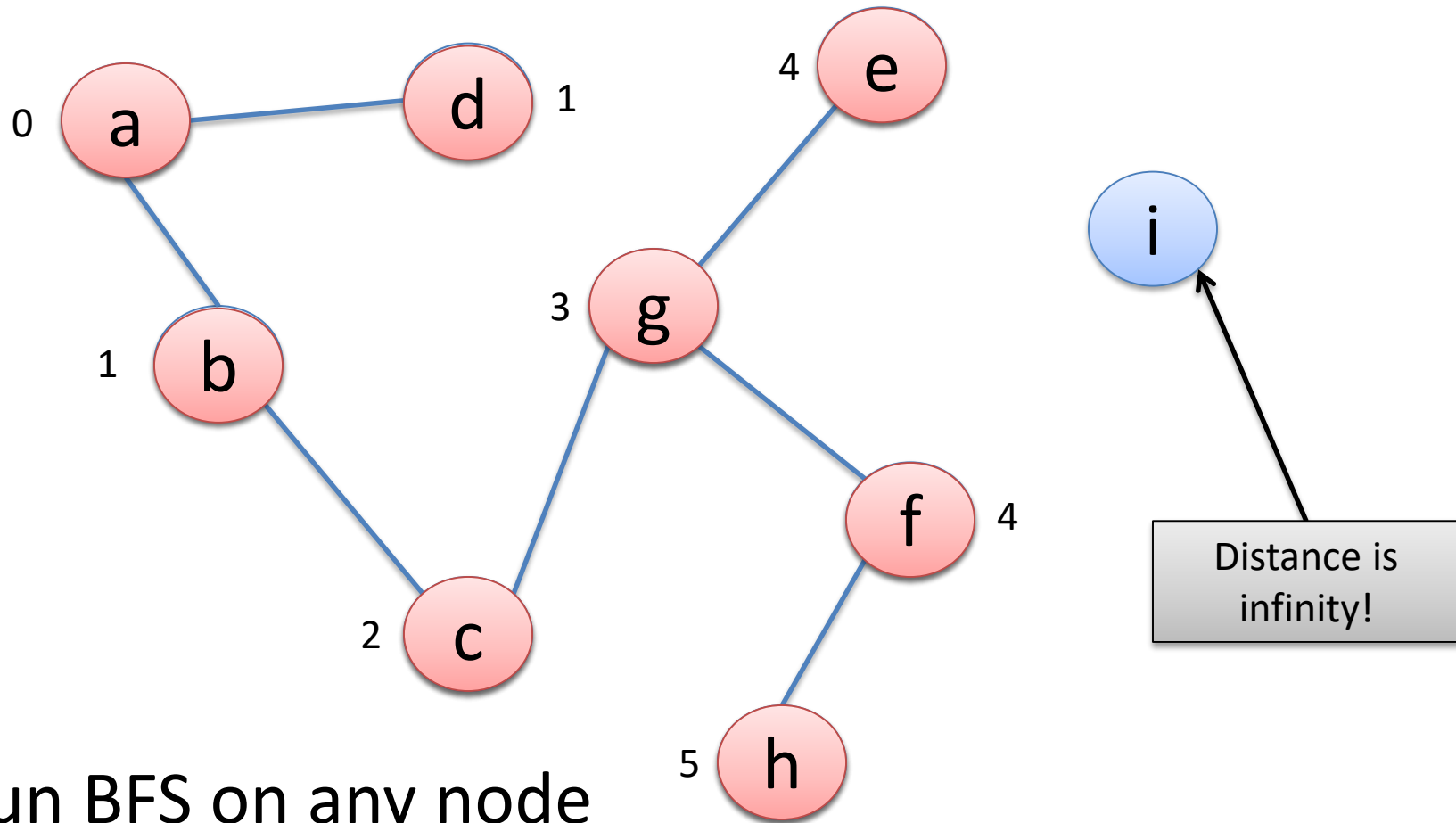


Application 1: checking connectedness



- Run BFS on any node
- If no node has distance infinity, it's **connected**!

Application 1: checking connectedness



- Run BFS on any node
- If a node has distance infinity, **disconnected!**

Application 2:

finding connected components

Algorithm:

initially label each node 0

$c := 1$

for each node $u = 1..n$ do

 if u is still labeled 0 then

 do a BFS starting at u

 give the label c to each node reached by the BFS

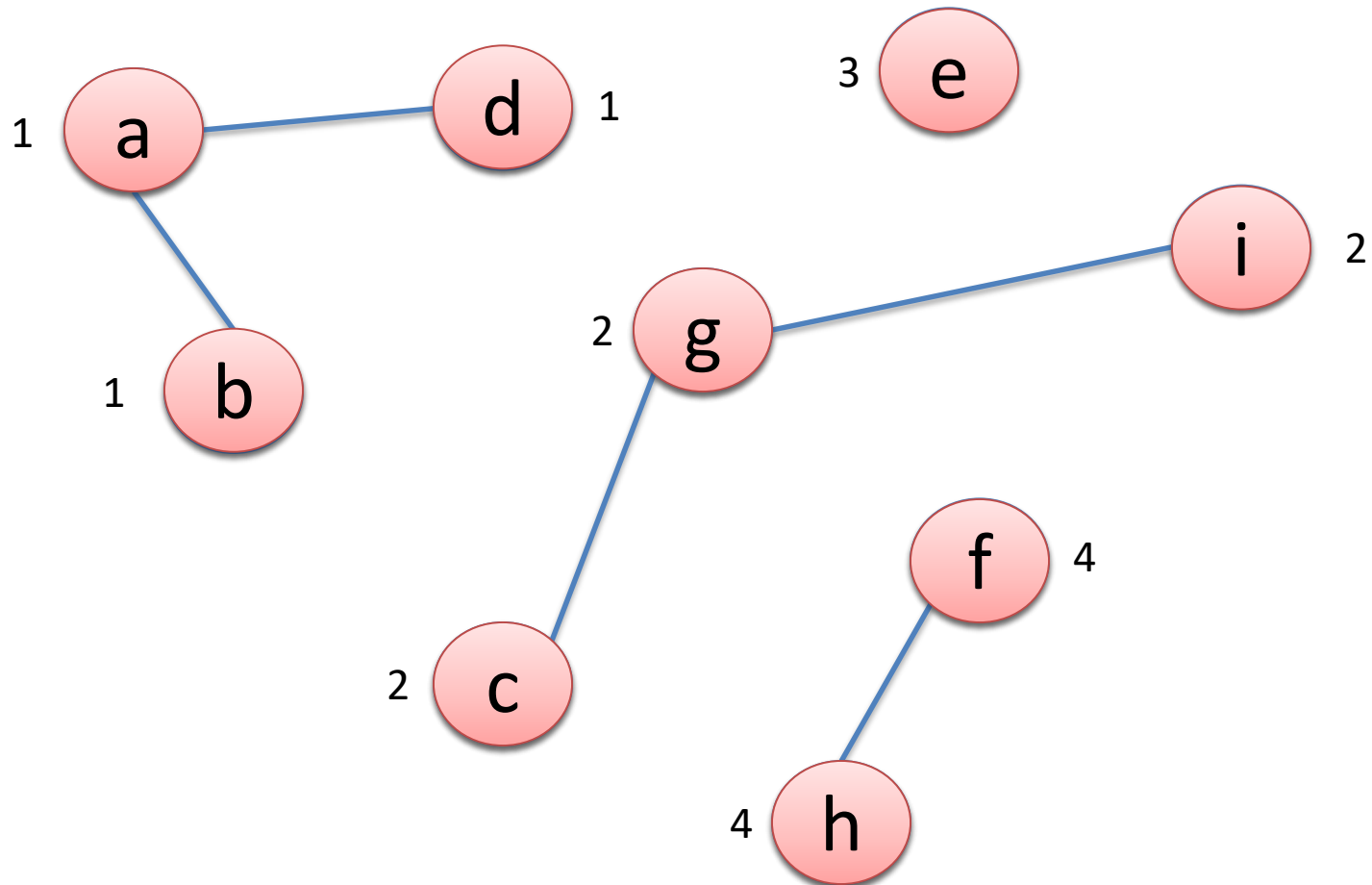
$c := c+1$

 end if

end for

Application 2: finding connected components

- Initially, each node has label 0



BFS algorithm with colors

BFS(G, start)

Create new queue Q

Q.push(start)

dist[1..n] = $\{\infty, \dots, \infty\}$

dist[start] = 0

colour[1..n] = {white, ..., white}

while Q is not empty

 u = Q.pop()

 colour[u] = black

 for each node v adjacent to u

 if colour[v] = white then

 Q.push(v)

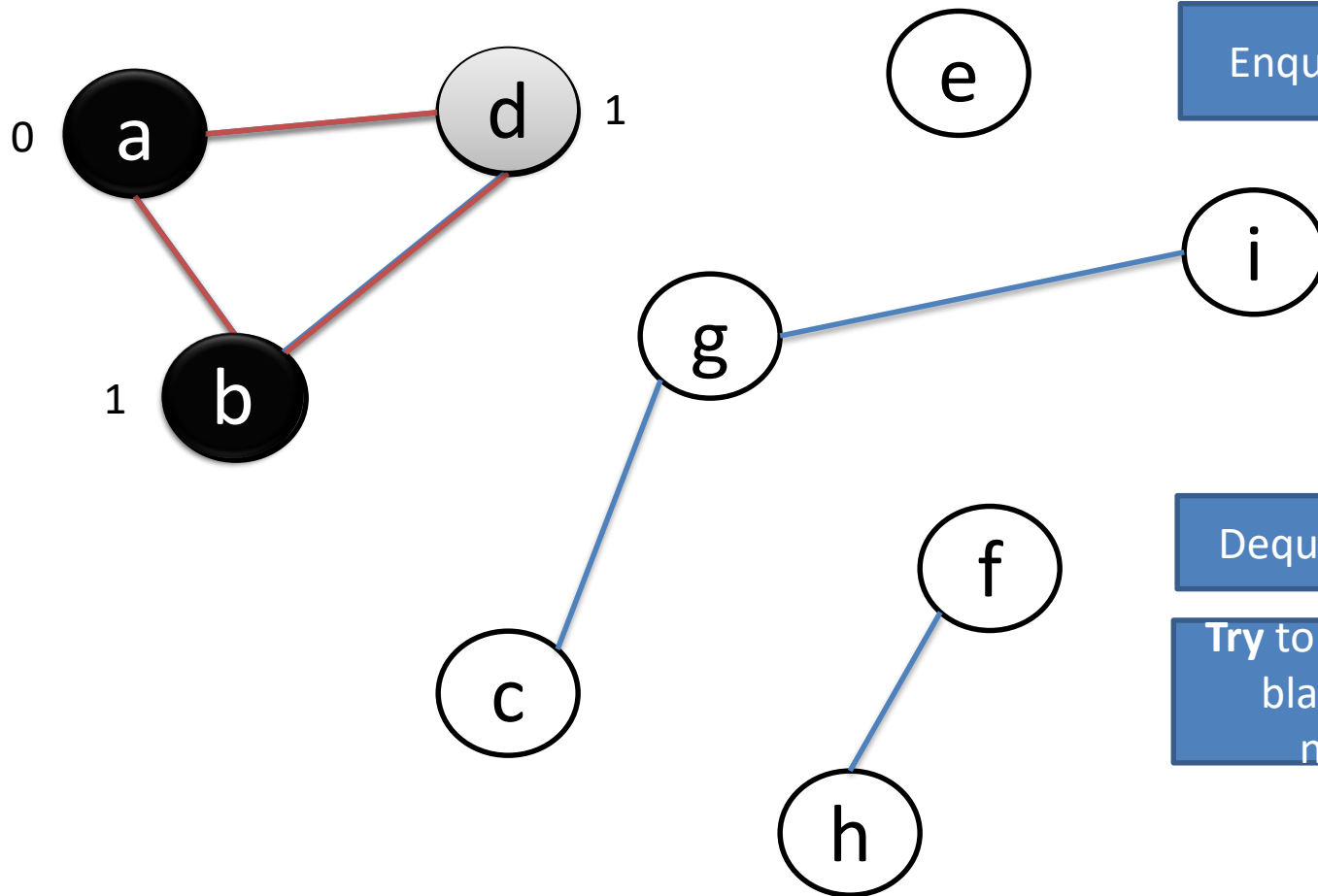
 dist[v] = dist[u] + 1

 colour[v] = gray

Suppose we have a graph
G = (V,E) containing
|V| = n nodes and
|E| = m edges.

BFS takes **O(n+m)**
time and space.

BFS with Colors



Enqueue node a

Dequeue node a

Enqueue node b

Enqueue node d

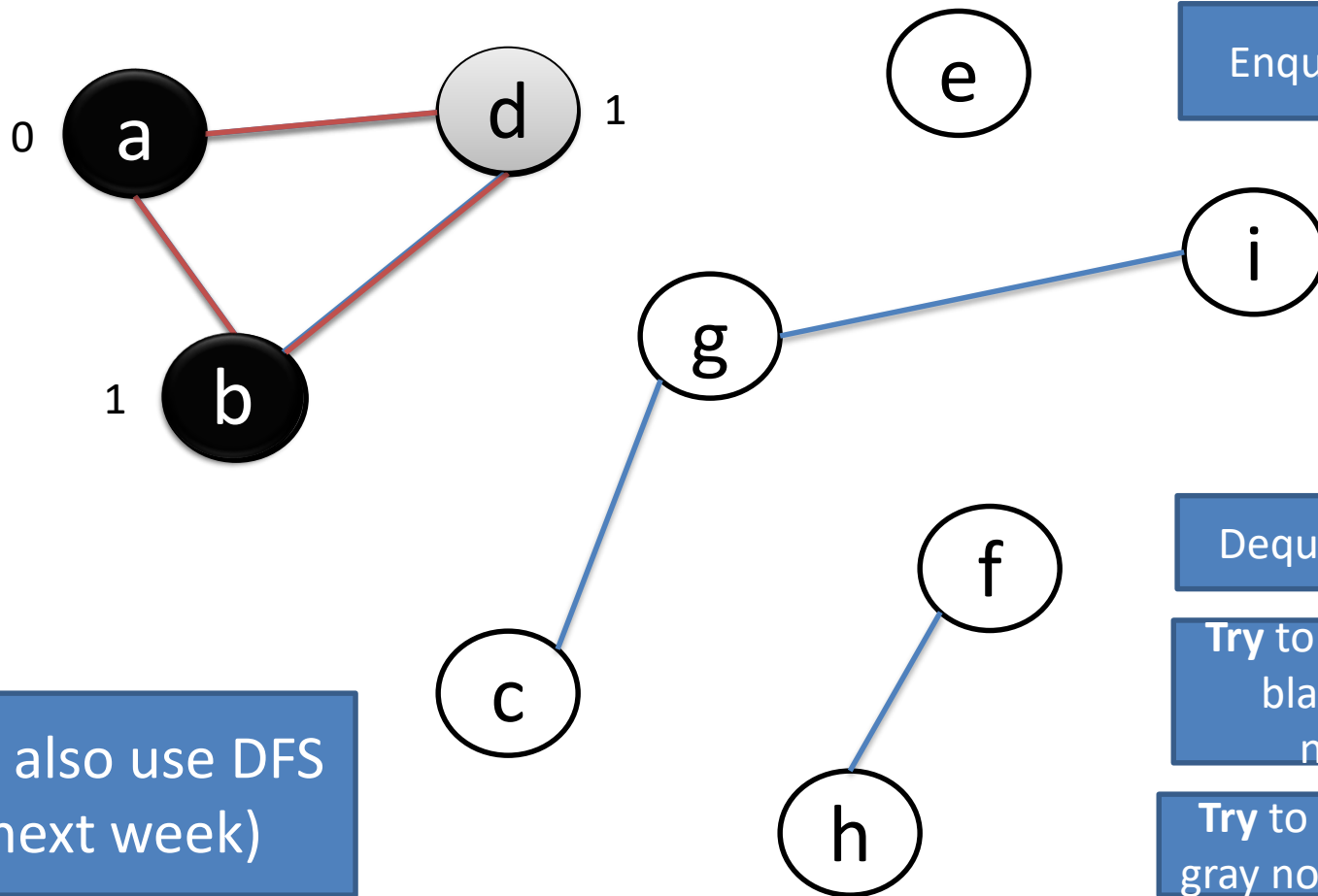
Dequeue node b

Try to enqueue a;
black, so do
nothing

Application 3: finding cycles

- Repeatedly do BFS from any unvisited node, until all nodes are visited.
- In any of these BFSs, if we see an edge that points to a **gray** node, then **there is a cycle!**

Application 3: finding cycles



Enqueue node a

Dequeue node a

Enqueue node b

Enqueue node d

Dequeue node b

Try to enqueue a;
black, so do
nothing

Try to enqueue d;
gray node, so cycle!

Can also use DFS
(next week)

Application 4: computing distance

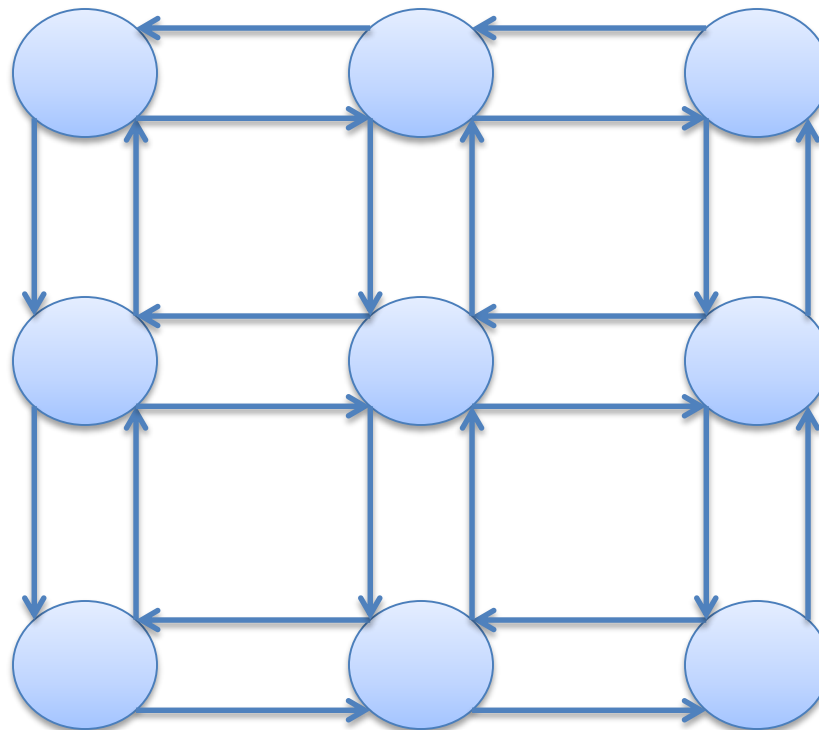
- Suppose we have an $m \times n$ grid of farms.
- Initially, one of the farms is on fire!
- At each hour, the fire spreads from each burning farm to each farm (going up, down, left and right).
- How long before all farms are on fire?

Application 4: computing distance

- How can we represent this problem as a graph problem?
 - Nodes are farms.
 - There is an edge between two farms if the farms are adjacent (next to one another).

Application 4: computing distance

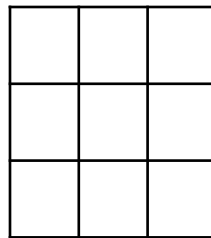
- How should we store this graph in memory?
- One possibility (for a 3x3 grid of farms):



Wastes lots
of memory!

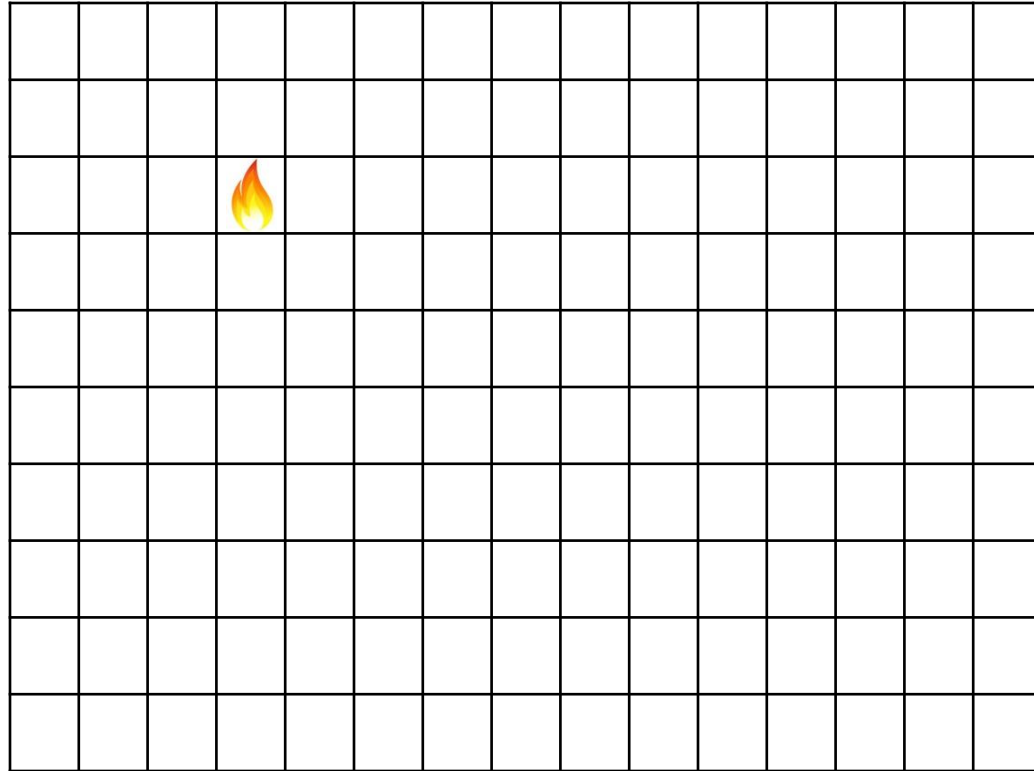
Application 4: computing distance

- How should we store this graph in memory?
- A more memory efficient way:
 - Store any data associated with the farms in a 3x3 array (one element for each farm)



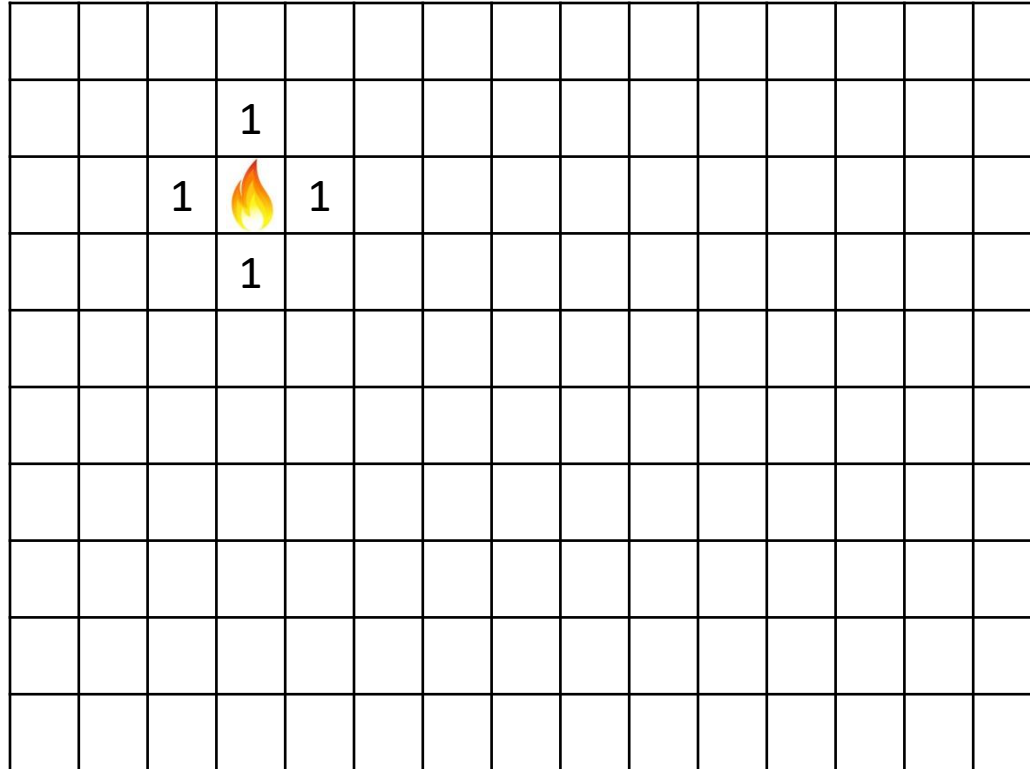
- Two farms are adjacent if their array elements are adjacent in the 3x3 array.

Application 4: computing distance (for a 15 x 10 grid of farms)



Solution: run BFS starting from the fire to compute the “distance” (actually time) to each farm.

Application 4: computing distance



Application 4: computing distance

[illegible]

Application 4: computing distance

[illegible]

Application 4: computing distance

[illegible]

Application 4: computing distance

[illegible]

Application 4: computing distance

[illegible]


Application 4: computing distance

[illegible]


Application 4: computing distance

[illegible]


Application 4: computing distance

5	4	3	2	3	4	5	6	7	8	9	10			
4	3	2	1	2	3	4	5	6	7	8	9	10		
3	2	1		1	2	3	4	5	6	7	8	9	10	
4	3	2	1	2	3	4	5	6	7	8	9	10		
5	4	3	2	3	4	5	6	7	8	9	10			
6	5	4	3	4	5	6	7	8	9	10				
7	6	5	4	5	6	7	8	9	10					
8	7	6	5	6	7	8	9	10						
9	8	7	6	7	8	9	10							
10	9	8	7	8	9	10								


Application 4: computing distance

5	4	3	2	3	4	5	6	7	8	9	10	11		
4	3	2	1	2	3	4	5	6	7	8	9	10	11	
3	2	1		1	2	3	4	5	6	7	8	9	10	11
4	3	2	1	2	3	4	5	6	7	8	9	10	11	
5	4	3	2	3	4	5	6	7	8	9	10	11		
6	5	4	3	4	5	6	7	8	9	10	11			
7	6	5	4	5	6	7	8	9	10	11				
8	7	6	5	6	7	8	9	10	11					
9	8	7	6	7	8	9	10	11						
10	9	8	7	8	9	10	11							


Application 4: computing distance

5	4	3	2	3	4	5	6	7	8	9	10	11	12	
4	3	2	1	2	3	4	5	6	7	8	9	10	11	12
3	2	1		1	2	3	4	5	6	7	8	9	10	11
4	3	2	1	2	3	4	5	6	7	8	9	10	11	12
5	4	3	2	3	4	5	6	7	8	9	10	11	12	
6	5	4	3	4	5	6	7	8	9	10	11	12		
7	6	5	4	5	6	7	8	9	10	11	12			
8	7	6	5	6	7	8	9	10	11	12				
9	8	7	6	7	8	9	10	11	12					
10	9	8	7	8	9	10	11	12						


Application 4: computing distance

5	4	3	2	3	4	5	6	7	8	9	10	11	12	13
4	3	2	1	2	3	4	5	6	7	8	9	10	11	12
3	2	1		1	2	3	4	5	6	7	8	9	10	11
4	3	2	1	2	3	4	5	6	7	8	9	10	11	12
5	4	3	2	3	4	5	6	7	8	9	10	11	12	13
6	5	4	3	4	5	6	7	8	9	10	11	12	13	
7	6	5	4	5	6	7	8	9	10	11	12	13		
8	7	6	5	6	7	8	9	10	11	12	13			
9	8	7	6	7	8	9	10	11	12	13				
10	9	8	7	8	9	10	11	12	13					


Application 4: computing distance

5	4	3	2	3	4	5	6	7	8	9	10	11	12	13
4	3	2	1	2	3	4	5	6	7	8	9	10	11	12
3	2	1		1	2	3	4	5	6	7	8	9	10	11
4	3	2	1	2	3	4	5	6	7	8	9	10	11	12
5	4	3	2	3	4	5	6	7	8	9	10	11	12	13
6	5	4	3	4	5	6	7	8	9	10	11	12	13	14
7	6	5	4	5	6	7	8	9	10	11	12	13	14	
8	7	6	5	6	7	8	9	10	11	12	13	14		
9	8	7	6	7	8	9	10	11	12	13	14			
10	9	8	7	8	9	10	11	12	13	14				


Application 4: computing distance

5	4	3	2	3	4	5	6	7	8	9	10	11	12	13
4	3	2	1	2	3	4	5	6	7	8	9	10	11	12
3	2	1		1	2	3	4	5	6	7	8	9	10	11
4	3	2	1	2	3	4	5	6	7	8	9	10	11	12
5	4	3	2	3	4	5	6	7	8	9	10	11	12	13
6	5	4	3	4	5	6	7	8	9	10	11	12	13	14
7	6	5	4	5	6	7	8	9	10	11	12	13	14	15
8	7	6	5	6	7	8	9	10	11	12	13	14	15	
9	8	7	6	7	8	9	10	11	12	13	14	15		
10	9	8	7	8	9	10	11	12	13	14	15			


Application 4: computing distance

5	4	3	2	3	4	5	6	7	8	9	10	11	12	13
4	3	2	1	2	3	4	5	6	7	8	9	10	11	12
3	2	1		1	2	3	4	5	6	7	8	9	10	11
4	3	2	1	2	3	4	5	6	7	8	9	10	11	12
5	4	3	2	3	4	5	6	7	8	9	10	11	12	13
6	5	4	3	4	5	6	7	8	9	10	11	12	13	14
7	6	5	4	5	6	7	8	9	10	11	12	13	14	15
8	7	6	5	6	7	8	9	10	11	12	13	14	15	16
9	8	7	6	7	8	9	10	11	12	13	14	15	16	
10	9	8	7	8	9	10	11	12	13	14	15	16		

Application 4: computing distance

5	4	3	2	3	4	5	6	7	8	9	10	11	12	13
4	3	2	1	2	3	4	5	6	7	8	9	10	11	12
3	2	1		1	2	3	4	5	6	7	8	9	10	11
4	3	2	1	2	3	4	5	6	7	8	9	10	11	12
5	4	3	2	3	4	5	6	7	8	9	10	11	12	13
6	5	4	3	4	5	6	7	8	9	10	11	12	13	14
7	6	5	4	5	6	7	8	9	10	11	12	13	14	15
8	7	6	5	6	7	8	9	10	11	12	13	14	15	16
9	8	7	6	7	8	9	10	11	12	13	14	15	16	17
10	9	8	7	8	9	10	11	12	13	14	15	16	17	

Application 4: computing distance

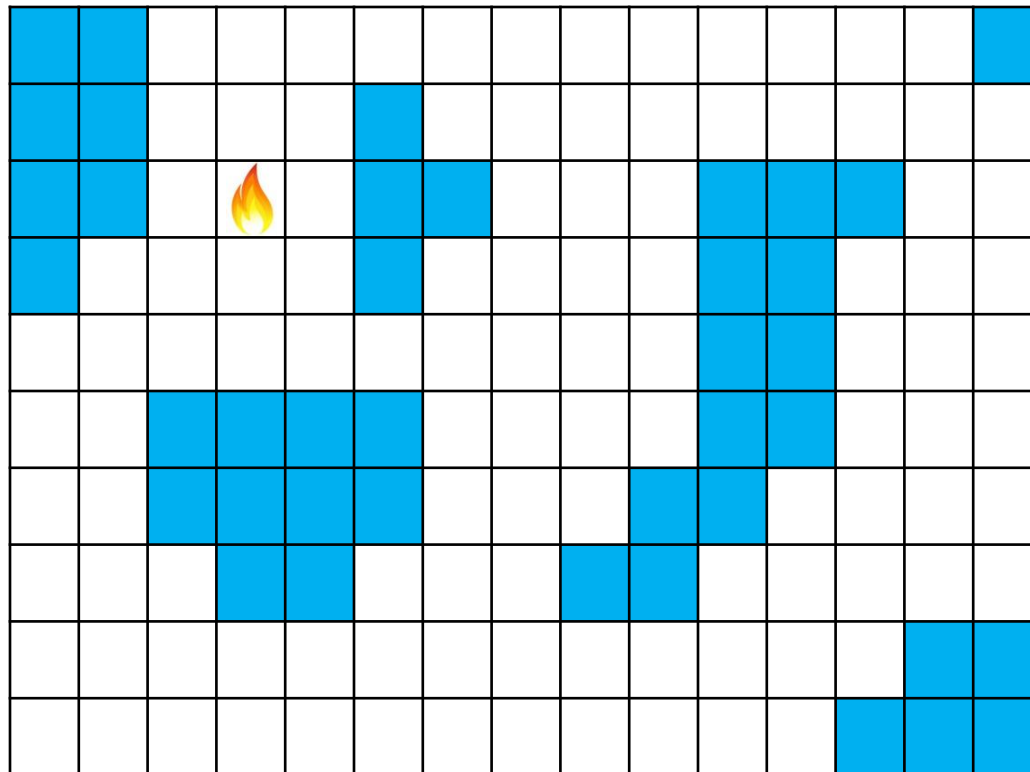
5	4	3	2	3	4	5	6	7	8	9	10	11	12	13
4	3	2	1	2	3	4	5	6	7	8	9	10	11	12
3	2	1		1	2	3	4	5	6	7	8	9	10	11
4	3	2	1	2	3	4	5	6	7	8	9	10	11	12
5	4	3	2	3	4	5	6	7	8	9	10	11	12	13
6	5	4	3	4	5	6	7	8	9	10	11	12	13	14
7	6	5	4	5	6	7	8	9	10	11	12	13	14	15
8	7	6	5	6	7	8	9	10	11	12	13	14	15	16
9	8	7	6	7	8	9	10	11	12	13	14	15	16	17
10	9	8	7	8	9	10	11	12	13	14	15	16	17	18

18 hours until all
farms are on fire!

Application 4: **modification 1**

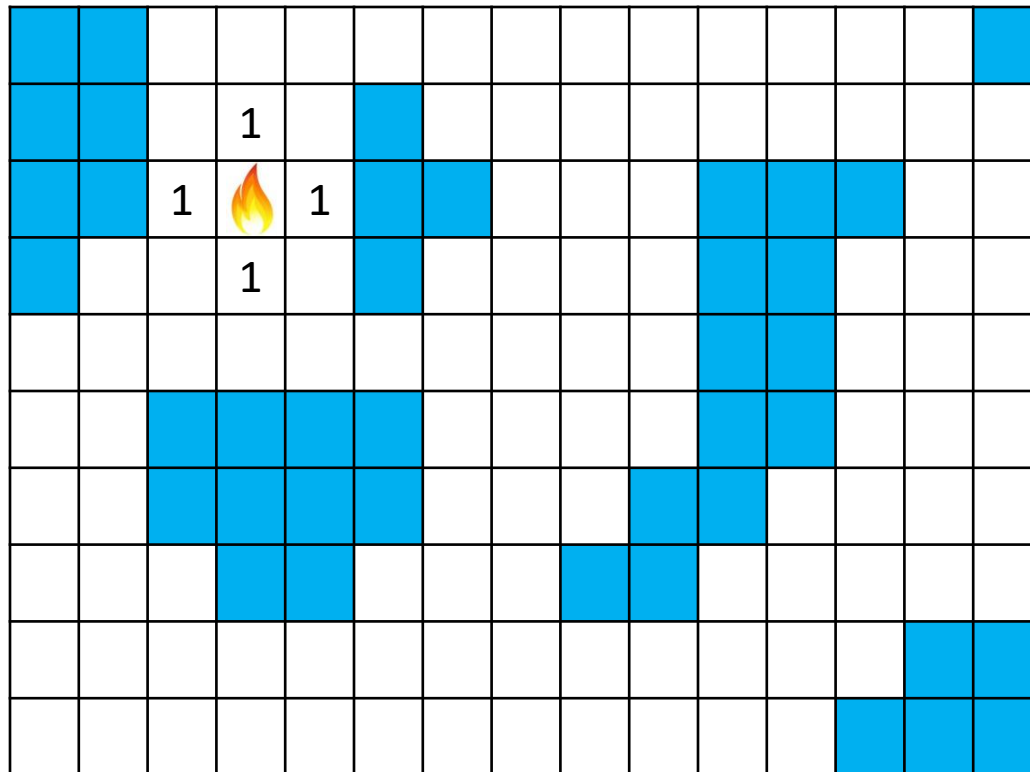
- What if there are lakes in the grid, where fire cannot pass?

Just don't let BFS visit the lakes!



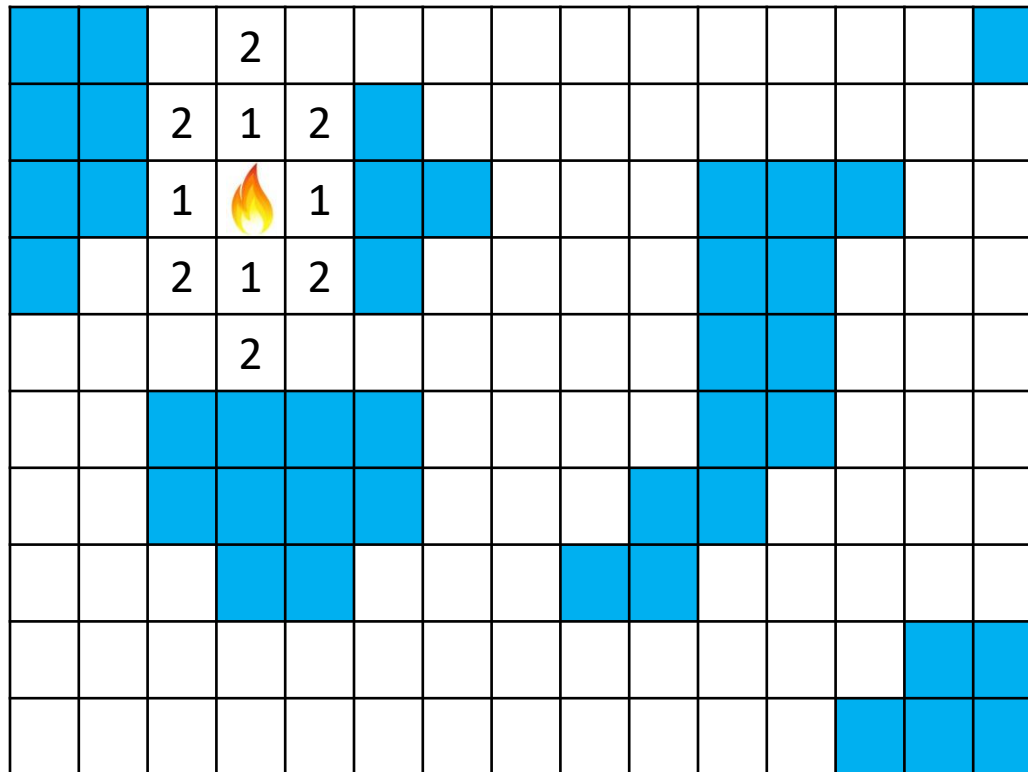
Application 4: **modification 1**

- What if there are lakes in the grid, where fire cannot pass?



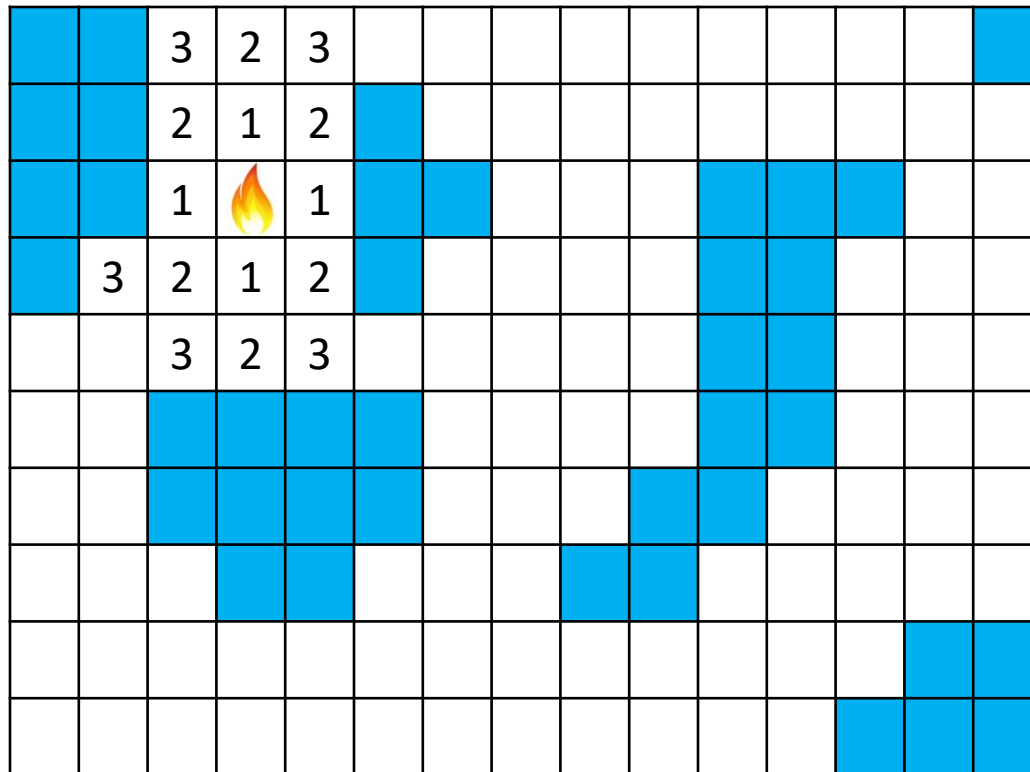
Application 4: **modification 1**

- What if there are lakes in the grid, where fire cannot pass?



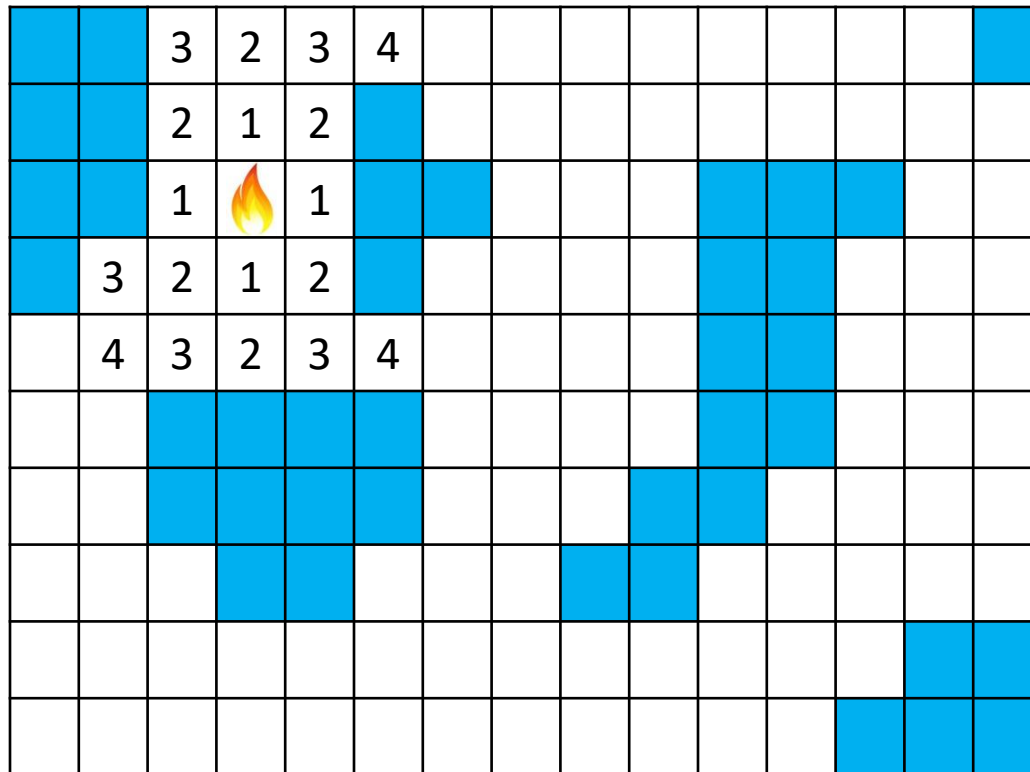
Application 4: modification 1

- What if there are lakes in the grid, where fire cannot pass?



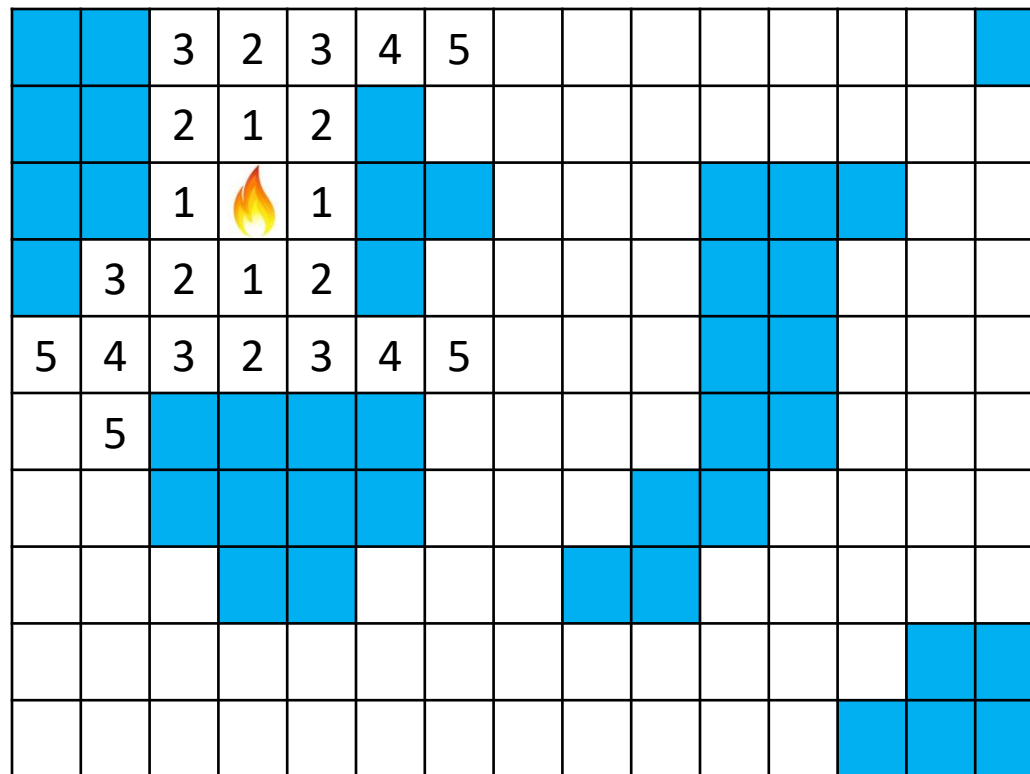
Application 4: modification 1

- What if there are lakes in the grid, where fire cannot pass?



Application 4: modification 1

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Application 4: modification 1

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[illegible]

Application 4: modification 1

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[illegible]

Application 4: modification 1

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[illegible]


Application 4: modification 1

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[illegible]


Application 4: **modification 1**

- What if there are lakes in the grid, where fire cannot pass?

		3	2	3	4	5	6	7	8	9	10			
		2	1	2		6	7	8	9	10				
		1		1			8	9	10					
	3	2	1	2		6	7	8	9					
5	4	3	2	3	4	5	6	7	8					
6	5					6	7	8	9					
7	6					7	8	9						
8	7	8			9	8	9							
9	8	9	10		10	9	10							
10	9	10				10								


Application 4: **modification 1**

- What if there are lakes in the grid, where fire cannot pass?

		3	2	3	4	5	6	7	8	9	10	11		
		2	1	2		6	7	8	9	10	11			
		1		1			8	9	10					
	3	2	1	2		6	7	8	9					
5	4	3	2	3	4	5	6	7	8					
6	5					6	7	8	9					
7	6					7	8	9						
8	7	8			9	8	9							
9	8	9	10	11	10	9	10	11						
10	9	10	11		11	10	11							


Application 4: **modification 1**

- What if there are lakes in the grid, where fire cannot pass?

		3	2	3	4	5	6	7	8	9	10	11	12	
		2	1	2		6	7	8	9	10	11	12		
		1		1			8	9	10					
	3	2	1	2		6	7	8	9					
5	4	3	2	3	4	5	6	7	8					
6	5					6	7	8	9					
7	6					7	8	9						
8	7	8			9	8	9							
9	8	9	10	11	10	9	10	11	12					
10	9	10	11	12	11	10	11	12						


Application 4: **modification 1**

- What if there are lakes in the grid, where fire cannot pass?

		3	2	3	4	5	6	7	8	9	10	11	12	
		2	1	2		6	7	8	9	10	11	12	13	
		1		1			8	9	10					
	3	2	1	2		6	7	8	9					
5	4	3	2	3	4	5	6	7	8					
6	5					6	7	8	9					
7	6					7	8	9						
8	7	8			9	8	9							
9	8	9	10	11	10	9	10	11	12	13				
10	9	10	11	12	11	10	11	12	13					


Application 4: **modification 1**

- What if there are lakes in the grid, where fire cannot pass?

		3	2	3	4	5	6	7	8	9	10	11	12	
		2	1	2		6	7	8	9	10	11	12	13	14
		1		1			8	9	10				14	
	3	2	1	2		6	7	8	9					
5	4	3	2	3	4	5	6	7	8					
6	5					6	7	8	9					
7	6					7	8	9						
8	7	8			9	8	9			14				
9	8	9	10	11	10	9	10	11	12	13	14			
10	9	10	11	12	11	10	11	12	13	14				


Application 4: **modification 1**

- What if there are lakes in the grid, where fire cannot pass?

		3	2	3	4	5	6	7	8	9	10	11	12	
		2	1	2		6	7	8	9	10	11	12	13	14
		1		1			8	9	10				14	15
	3	2	1	2		6	7	8	9				15	
5	4	3	2	3	4	5	6	7	8					
6	5					6	7	8	9					
7	6					7	8	9						
8	7	8			9	8	9			14	15			
9	8	9	10	11	10	9	10	11	12	13	14	15		
10	9	10	11	12	11	10	11	12	13	14	15			


Application 4: **modification 1**

- What if there are lakes in the grid, where fire cannot pass?

		3	2	3	4	5	6	7	8	9	10	11	12	
		2	1	2		6	7	8	9	10	11	12	13	14
		1		1			8	9	10				14	15
	3	2	1	2		6	7	8	9			16	15	16
5	4	3	2	3	4	5	6	7	8				16	
6	5					6	7	8	9					
7	6					7	8	9			16			
8	7	8			9	8	9			14	15	16		
9	8	9	10	11	10	9	10	11	12	13	14	15		
10	9	10	11	12	11	10	11	12	13	14	15			


Application 4: **modification 1**

- What if there are lakes in the grid, where fire cannot pass?

		3	2	3	4	5	6	7	8	9	10	11	12	
		2	1	2		6	7	8	9	10	11	12	13	14
		1		1			8	9	10				14	15
	3	2	1	2		6	7	8	9			16	15	16
5	4	3	2	3	4	5	6	7	8			17	16	17
6	5					6	7	8	9				17	
7	6					7	8	9			16	17		
8	7	8			9	8	9			14	15	16	17	
9	8	9	10	11	10	9	10	11	12	13	14	15		
10	9	10	11	12	11	10	11	12	13	14	15			


Application 4: **modification 1**

- What if there are lakes in the grid, where fire cannot pass?

		3	2	3	4	5	6	7	8	9	10	11	12	
		2	1	2		6	7	8	9	10	11	12	13	14
		1		1			8	9	10				14	15
	3	2	1	2		6	7	8	9			16	15	16
5	4	3	2	3	4	5	6	7	8			17	16	17
6	5					6	7	8	9			18	17	18
7	6					7	8	9			16	17	18	
8	7	8			9	8	9			14	15	16	17	18
9	8	9	10	11	10	9	10	11	12	13	14	15		
10	9	10	11	12	11	10	11	12	13	14	15			

Application 4: modification 1

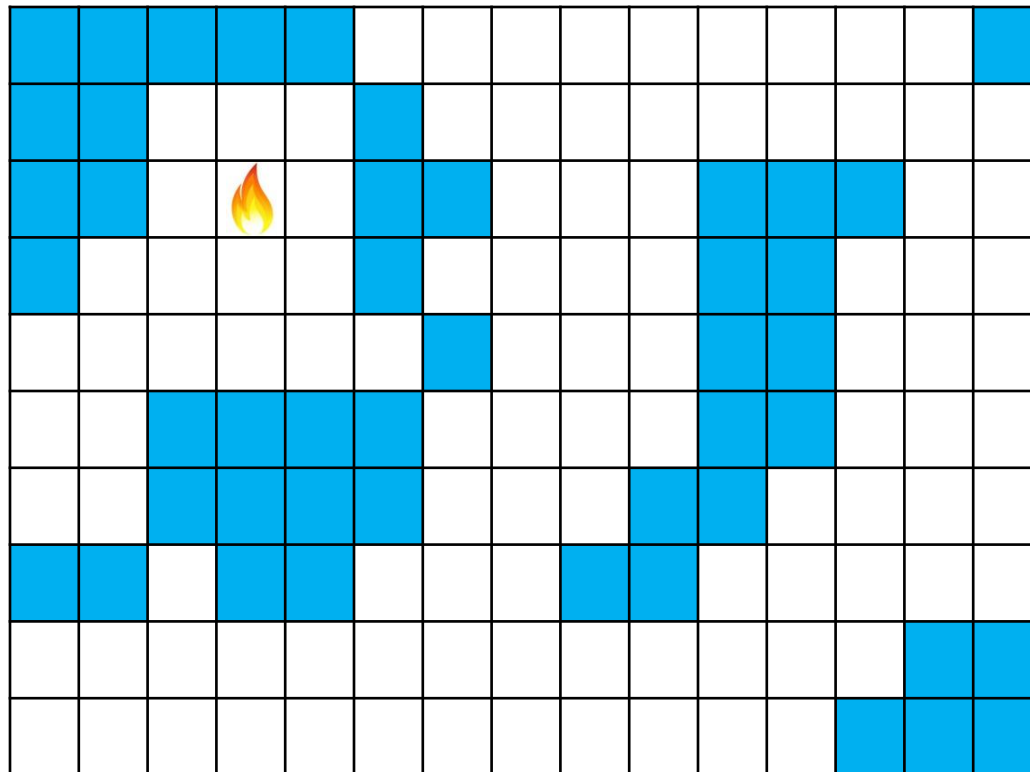
- What if there are lakes in the grid, where fire cannot pass?

		3	2	3	4	5	6	7	8	9	10	11	12	
		2	1	2		6	7	8	9	10	11	12	13	14
		1		1			8	9	10				14	15
	3	2	1	2		6	7	8	9			16	15	16
5	4	3	2	3	4	5	6	7	8			17	16	17
6	5					6	7	8	9			18	17	18
7	6					7	8	9			16	17	18	19
8	7	8			9	8	9			14	15	16	17	18
9	8	9	10	11	10	9	10	11	12	13	14	15		
10	9	10	11	12	11	10	11	12	13	14	15			

19 hours until all farms are on fire!

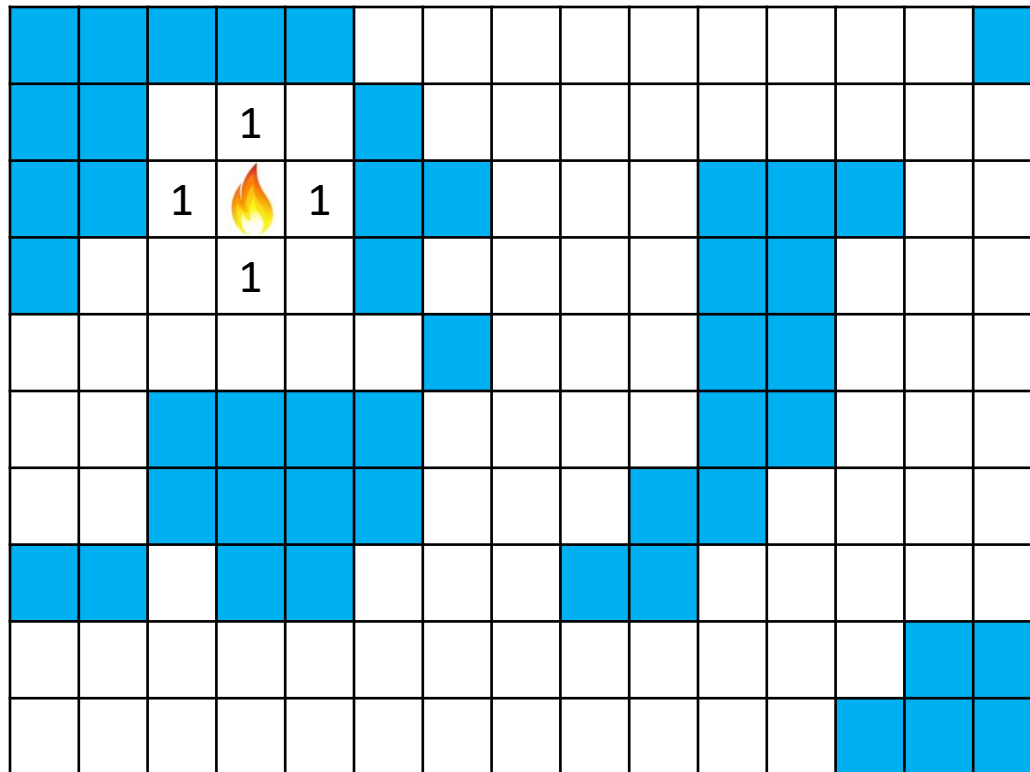
Application 4: **modification 1**

- Will the fire always burn everything?



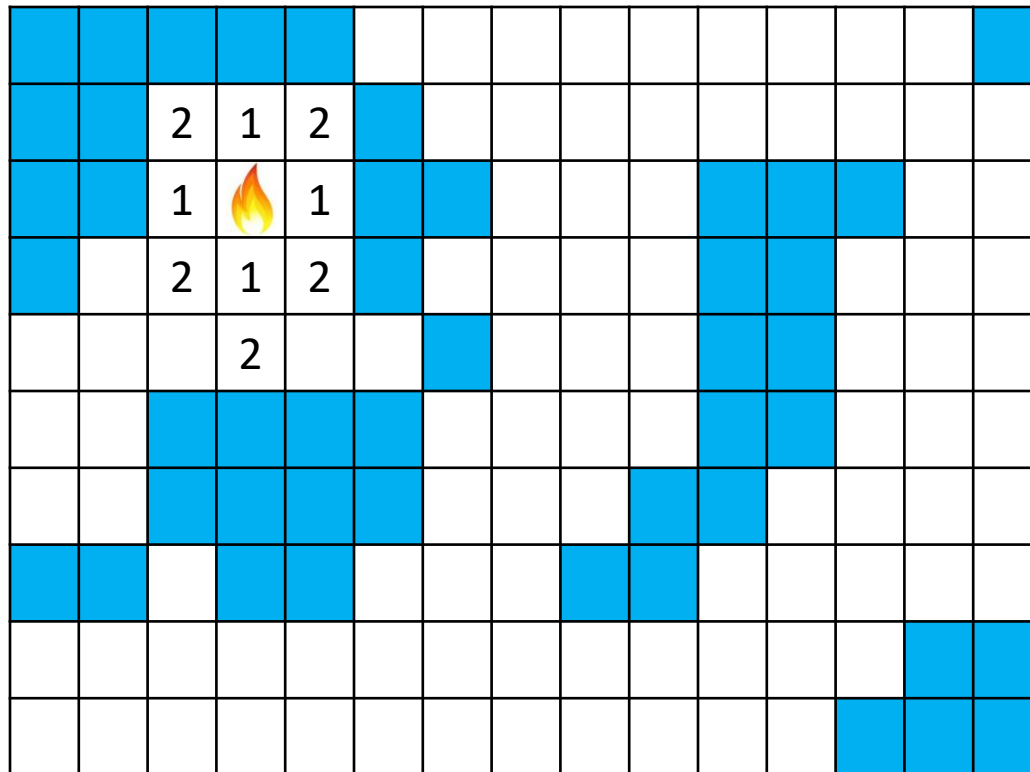
Application 4: **modification 1**

- Will the fire always burn everything?



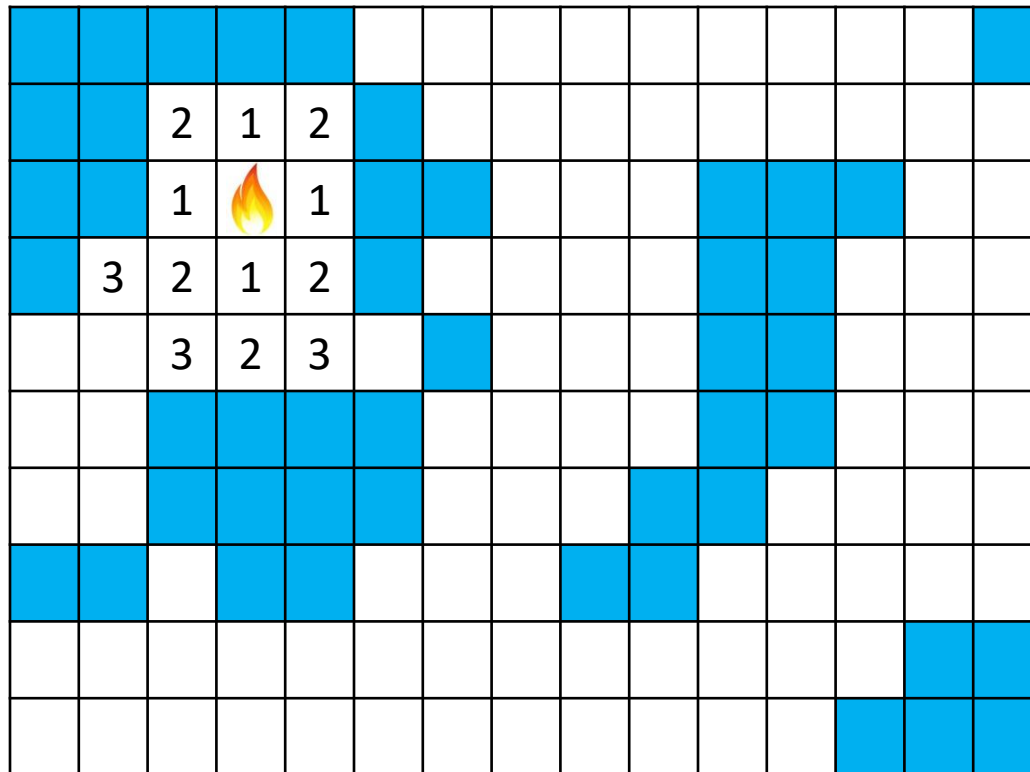
Application 4: **modification 1**

- Will the fire always burn everything?



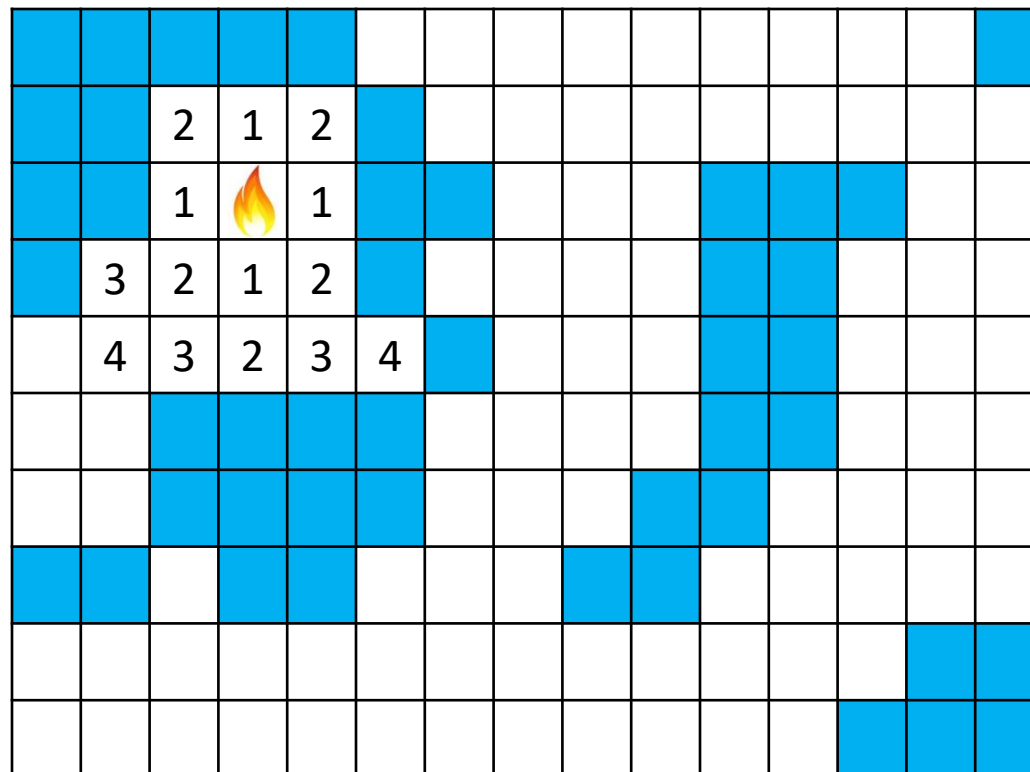
Application 4: **modification 1**

- Will the fire always burn everything?



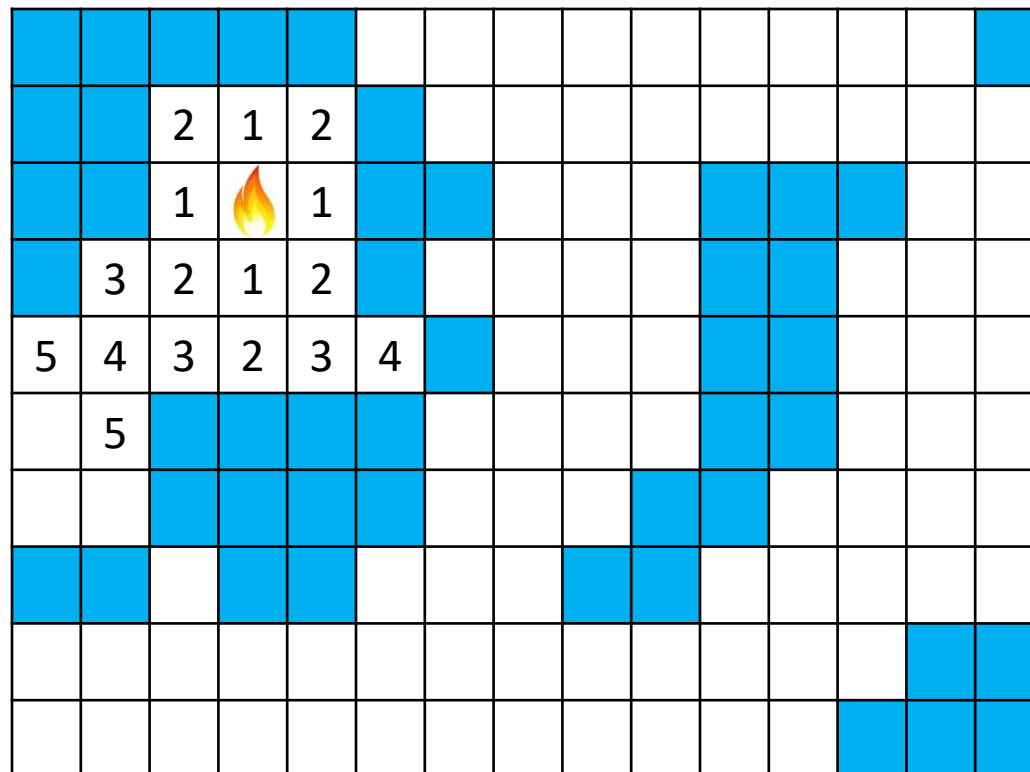
Application 4: **modification 1**

- Will the fire always burn everything?



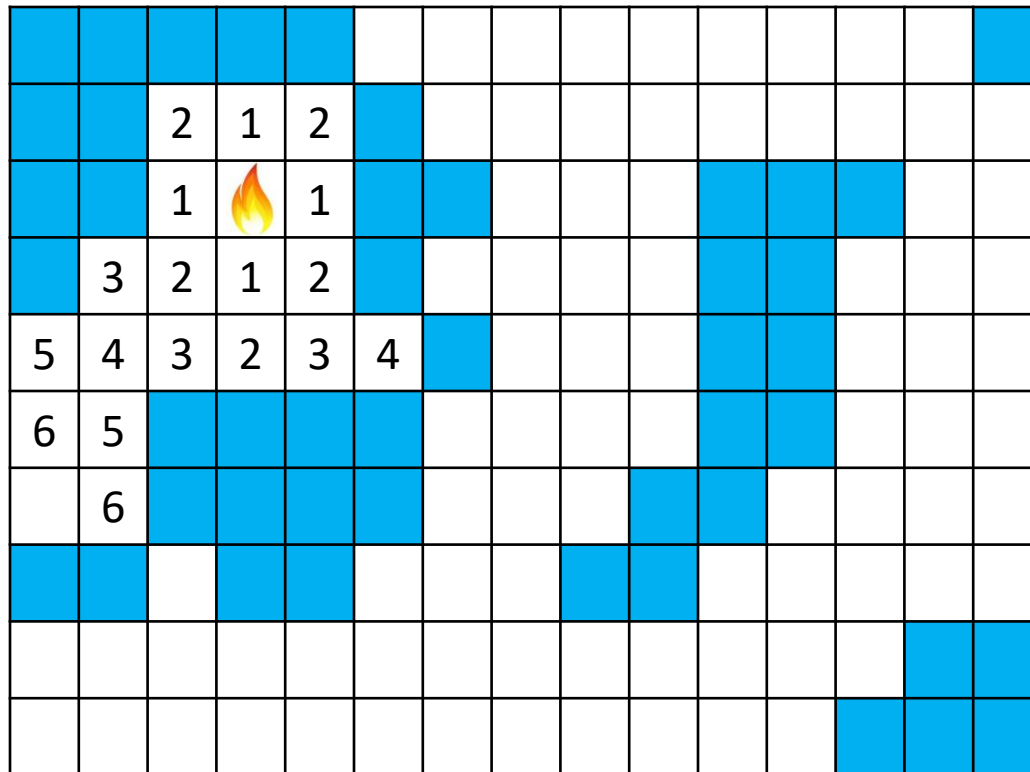
Application 4: modification 1

- Will the fire always burn everything?



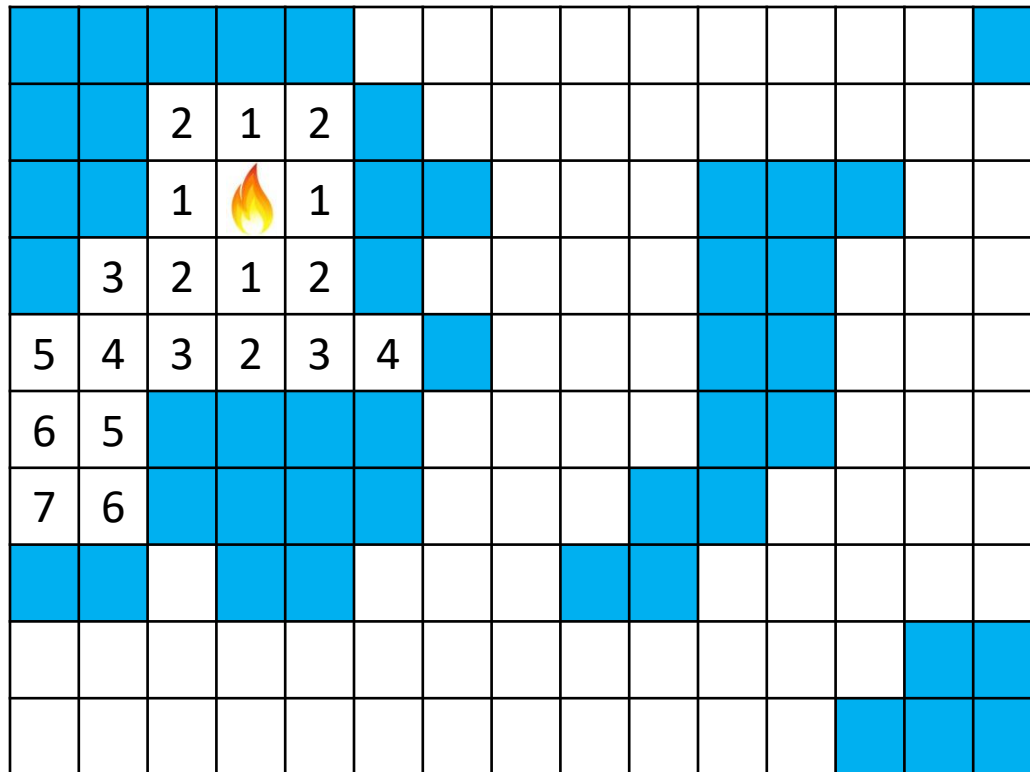
Application 4: modification 1

- Will the fire always burn everything?



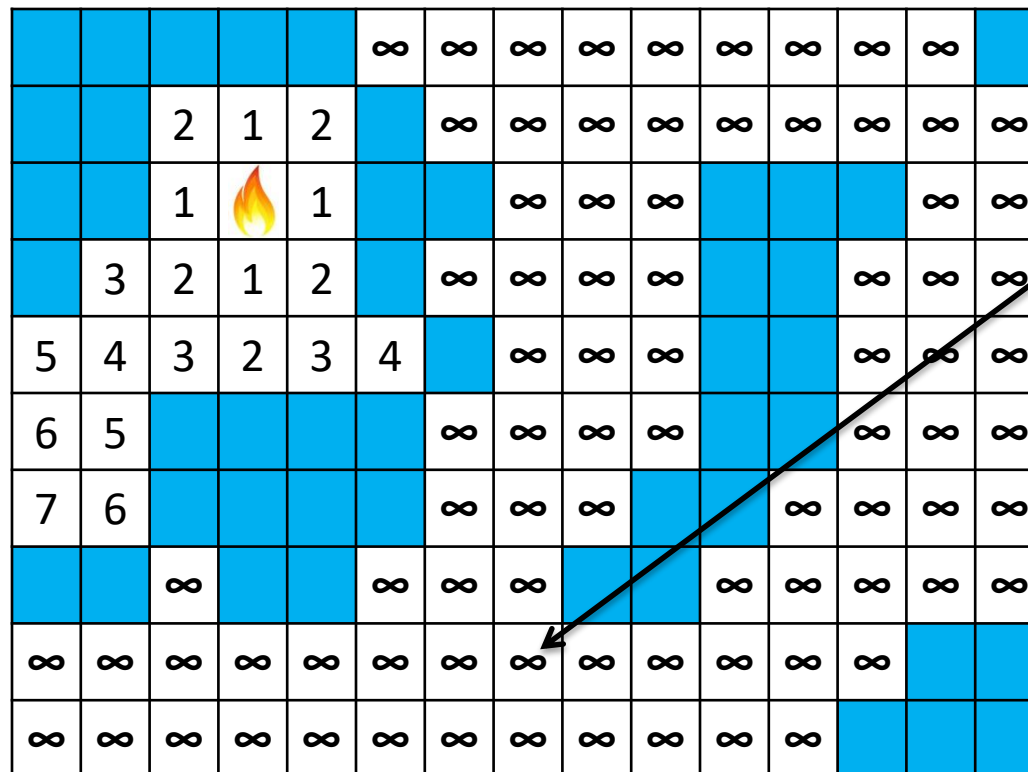
Application 4: modification 1

- Will the fire always burn everything?



Application 4: modification 1

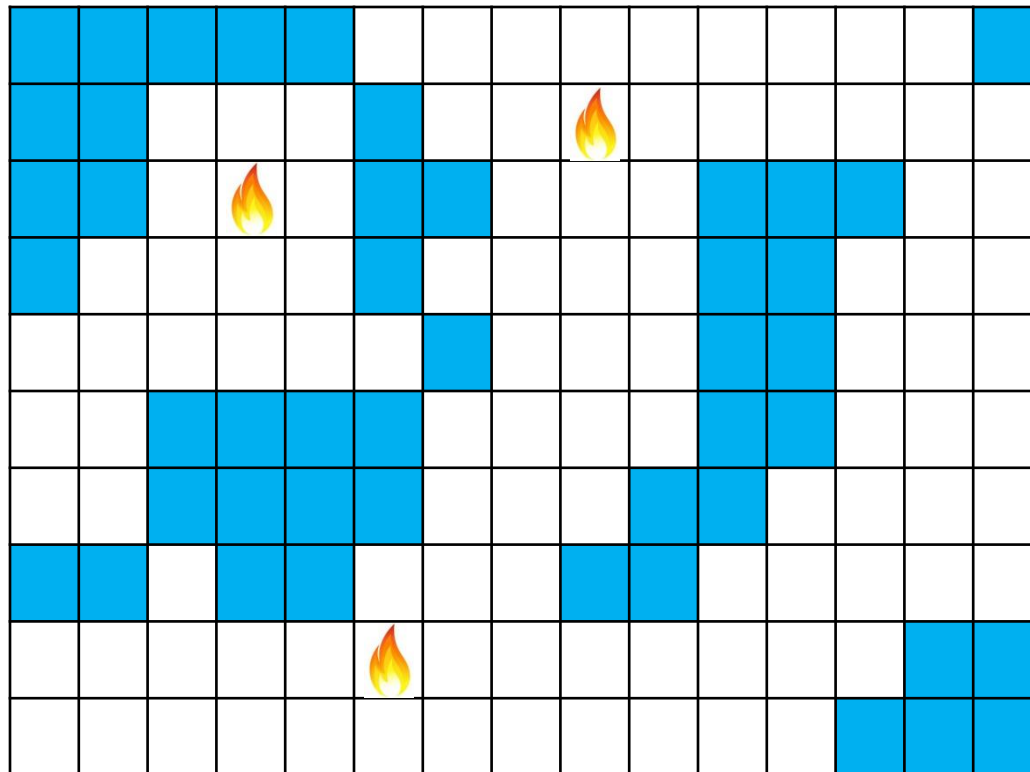
- Will the fire always burn everything?



All of these
farms are safe!

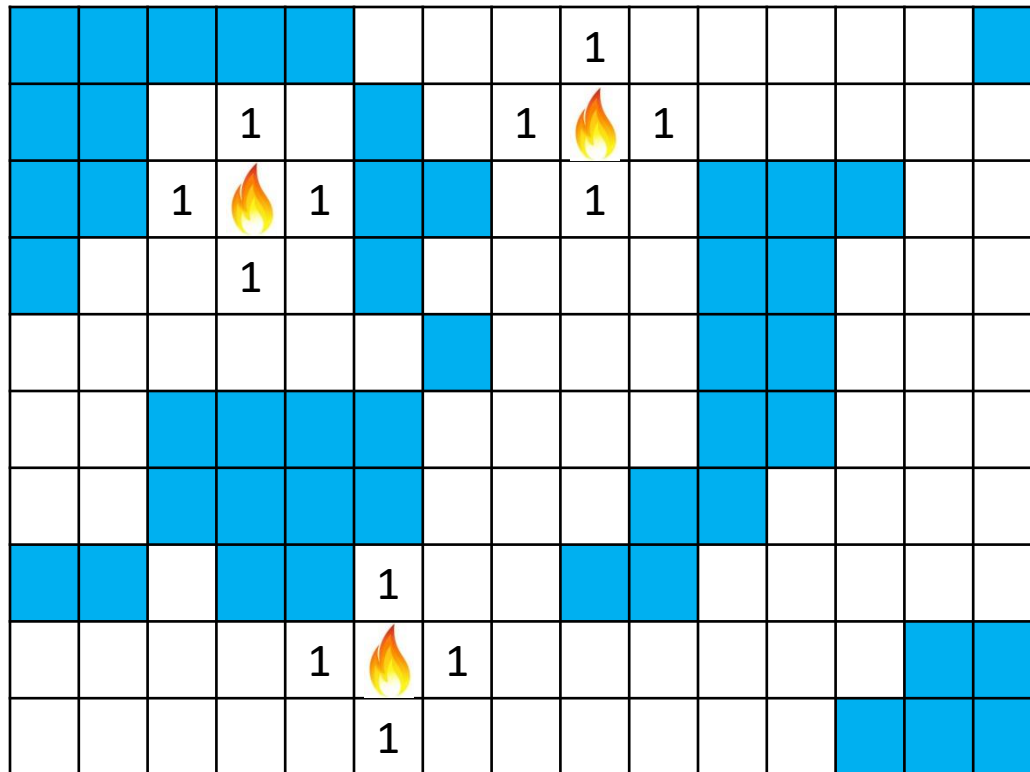
Application 4: **modification 2**

- What if multiple fires start at the same time?
- Just place **all** fires in the initial BFS queue!



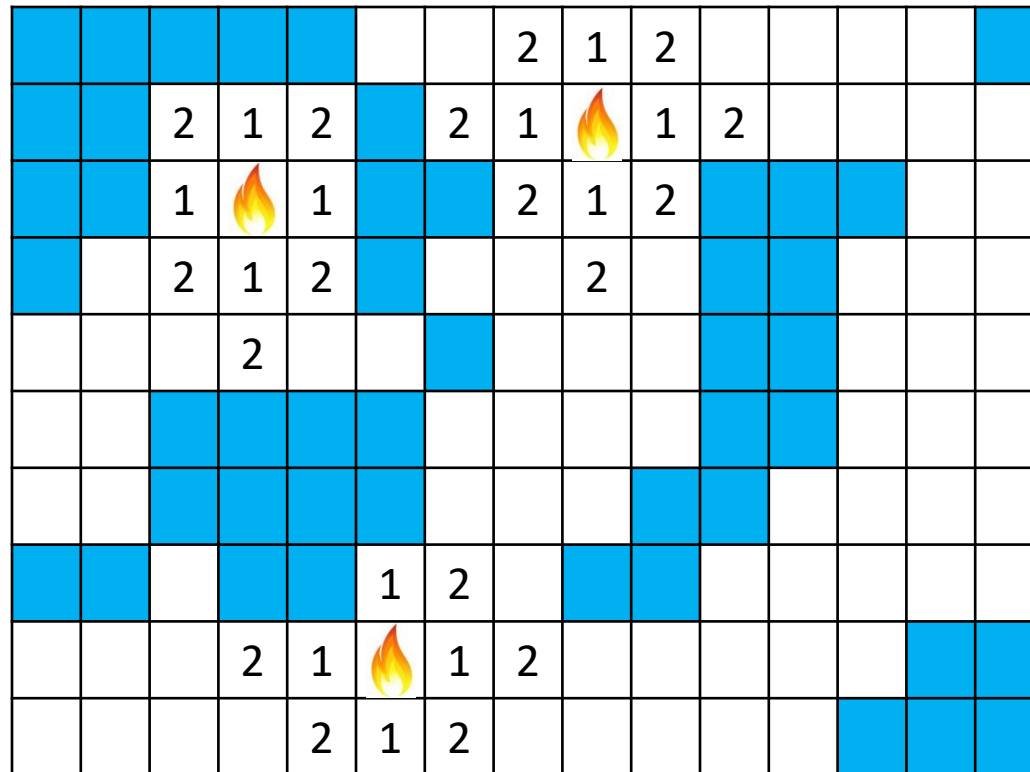
Application 4: **modification 2**

- What if multiple fires start at the same time?
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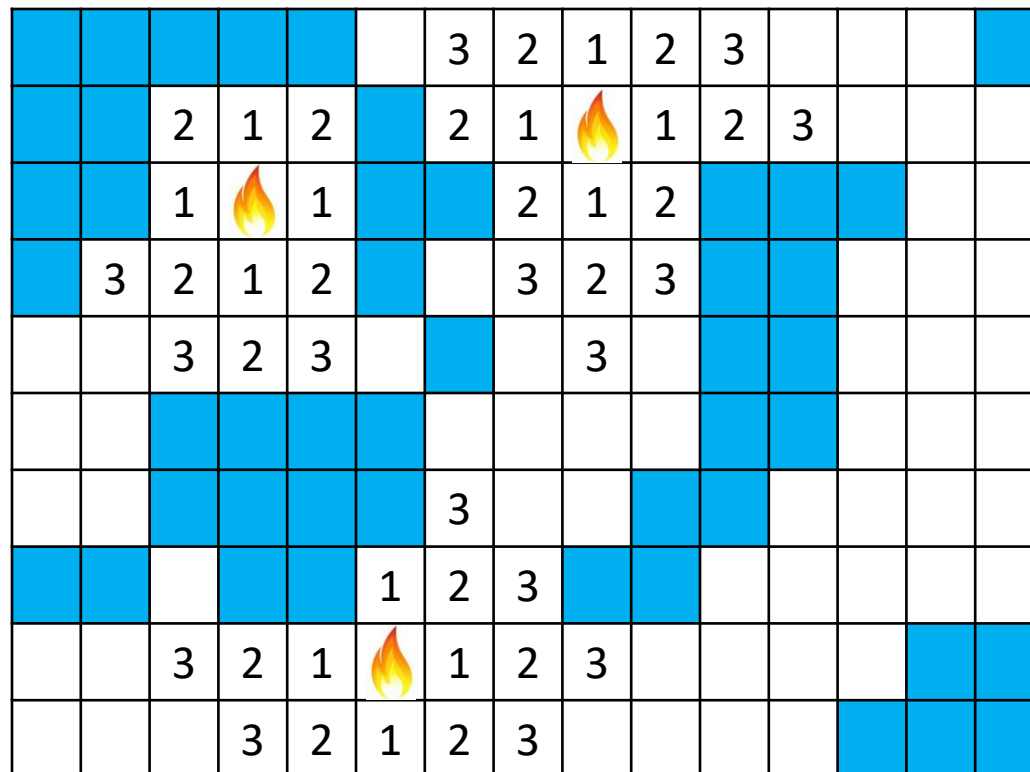
Application 4: modification 2

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Application 4: **modification 2**

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Application 4: **modification 2**

- What if multiple fires start at the same time?
- Just place **all** fires in the initial BFS queue!

					4	3	2	1	2	3	4			
		2	1	2		2	1		1	2	3	4		
		1		1			2	1	2					
	3	2	1	2		4	3	2	3					
	4	3	2	3	4		4	3	4					
						4		4						
						3	4							
		4			1	2	3							
	4	3	2	1		1	2	3	4					
		4	3	2	1	2	3	4						




Application 4: **modification 2**

- What if multiple fires start at the same time?
- Just place **all** fires in the initial BFS queue!

					4	3	2	1	2	3	4	5		
		2	1	2		2	1		1	2	3	4	5	
		1		1			2	1	2					
	3	2	1	2		4	3	2	3					
5	4	3	2	3	4		4	3	4					
	5					4	5	4	5					
						3	4	5						
		4			1	2	3							
5	4	3	2	1		1	2	3	4	5				
	5	4	3	2	1	2	3	4	5					




Application 4: **modification 2**

- What if multiple fires start at the same time?
- Just place **all** fires in the initial BFS queue!

					4	3	2	1	2	3	4	5	6	
		2	1	2		2	1		1	2	3	4	5	6
		1		1			2	1	2				6	
	3	2	1	2		4	3	2	3					
5	4	3	2	3	4		4	3	4					
6	5					4	5	4	5					
	6					3	4	5						
		4			1	2	3			6				
5	4	3	2	1		1	2	3	4	5	6			
6	5	4	3	2	1	2	3	4	5	6				




Application 4: **modification 2**

- What if multiple fires start at the same time?
- Just place **all** fires in the initial BFS queue!

					4	3	2	1	2	3	4	5	6	
		2	1	2		2	1		1	2	3	4	5	6
		1		1			2	1	2				6	7
	3	2	1	2		4	3	2	3				7	
5	4	3	2	3	4		4	3	4					
6	5					4	5	4	5					
7	6					3	4	5						
		4			1	2	3			6	7			
5	4	3	2	1		1	2	3	4	5	6	7		
6	5	4	3	2	1	2	3	4	5	6	7			




Application 4: **modification 2**

- What if multiple fires start at the same time?
- Just place **all** fires in the initial BFS queue!

					4	3	2	1	2	3	4	5	6	
		2	1	2		2	1		1	2	3	4	5	6
		1		1			2	1	2				6	7
	3	2	1	2		4	3	2	3			8	7	8
5	4	3	2	3	4		4	3	4				8	
6	5					4	5	4	5					
7	6					3	4	5			8			
		4			1	2	3			6	7	8		
5	4	3	2	1		1	2	3	4	5	6	7		
6	5	4	3	2	1	2	3	4	5	6	7			




Application 4: **modification 2**

- What if multiple fires start at the same time?
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					4	3	2	1	2	3	4	5	6	
		2	1	2		2	1		1	2	3	4	5	6
		1		1			2	1	2				6	7
	3	2	1	2		4	3	2	3			8	7	8
5	4	3	2	3	4		4	3	4			9	8	9
6	5					4	5	4	5				9	
7	6					3	4	5			8	9		
		4			1	2	3			6	7	8	9	
5	4	3	2	1		1	2	3	4	5	6	7		
6	5	4	3	2	1	2	3	4	5	6	7			




Application 4: **modification 2**

- What if multiple fires start at the same time?
- Just place **all** fires in the initial BFS queue!

					4	3	2	1	2	3	4	5	6	
		2	1	2		2	1		1	2	3	4	5	6
		1		1			2	1	2				6	7
	3	2	1	2		4	3	2	3			8	7	8
5	4	3	2	3	4		4	3	4			9	8	9
6	5					4	5	4	5			10	9	10
7	6					3	4	5			8	9	10	
		4			1	2	3			6	7	8	9	10
5	4	3	2	1		1	2	3	4	5	6	7		
6	5	4	3	2	1	2	3	4	5	6	7			

Application 4: modification 2

- What if multiple fires start at the same time?
- Just place **all** fires in the initial BFS queue!

					4	3	2	1	2	3	4	5	6	
		2	1	2		2	1		1	2	3	4	5	6
		1		1			2	1	2				6	7
	3	2	1	2		4	3	2	3			8	7	8
5	4	3	2	3	4		4	3	4			9	8	9
6	5					4	5	4	5			10	9	10
7	6					3	4	5			8	9	10	11
		4			1	2	3			6	7	8	9	10
5	4	3	2	1		1	2	3	4	5	6	7		
6	5	4	3	2	1	2	3	4	5	6	7			

11 hours until all farms are on fire!



Application 4:

other modifications to think about

- What if fires start at different times?
- What if fires spread at different speeds?



A neat problem to think about

- It's the year 2241. You're in a cave and it's collapsing! A device tells you when each section of ceiling will cave in. Can you escape?

					19	16	14	12	16	13	20	21	19	
		27	24	20		20	10	9	18	20	17	16	17	18
		23	18	16			9	12	14				14	20
	19	20	16	13		7	7	9	11			16	24	20
6	7		12	14	20	5	8	12	9			25	22	15
5	6					14	9	14	10			25	23	21
1	1					15	16	11			19	26	19	25
		2			12	18	24			24	13	27	30	
7	8	13	17	17	13	19	22	20	21	23	20	12		
10	12	15	18	16	15	20	18	16	19	18	15			

A neat problem to think about

- It's the year 2241. You're in a cave and it's collapsing! A device tells you when each section of ceiling will cave in. Can you escape?

					19	16	14	12	16	13	20	21	19	
		27	24	20		20	10	7	18	20	17	16	17	18
		23	18	16			9	12	14				14	20
	19	20	16	13		7	7	9	11			16	24	20
6	7		12	14	20	5	8	12	9			25	22	15
5	6					14	9	14	10			25	23	21
1	1					15	16	11			19	26	19	25
		2			12	18	24			24	13	27	30	
7	8	13	17	17	13	19	22	20	21	23	20	12		
10	12	15	18	16	15	20	18	16	19	18	15			

Here's a sample
would you solve

Slide Credits

- Trevor Brown, University of Toronto