Linear regression with one variable

Gradient Descent

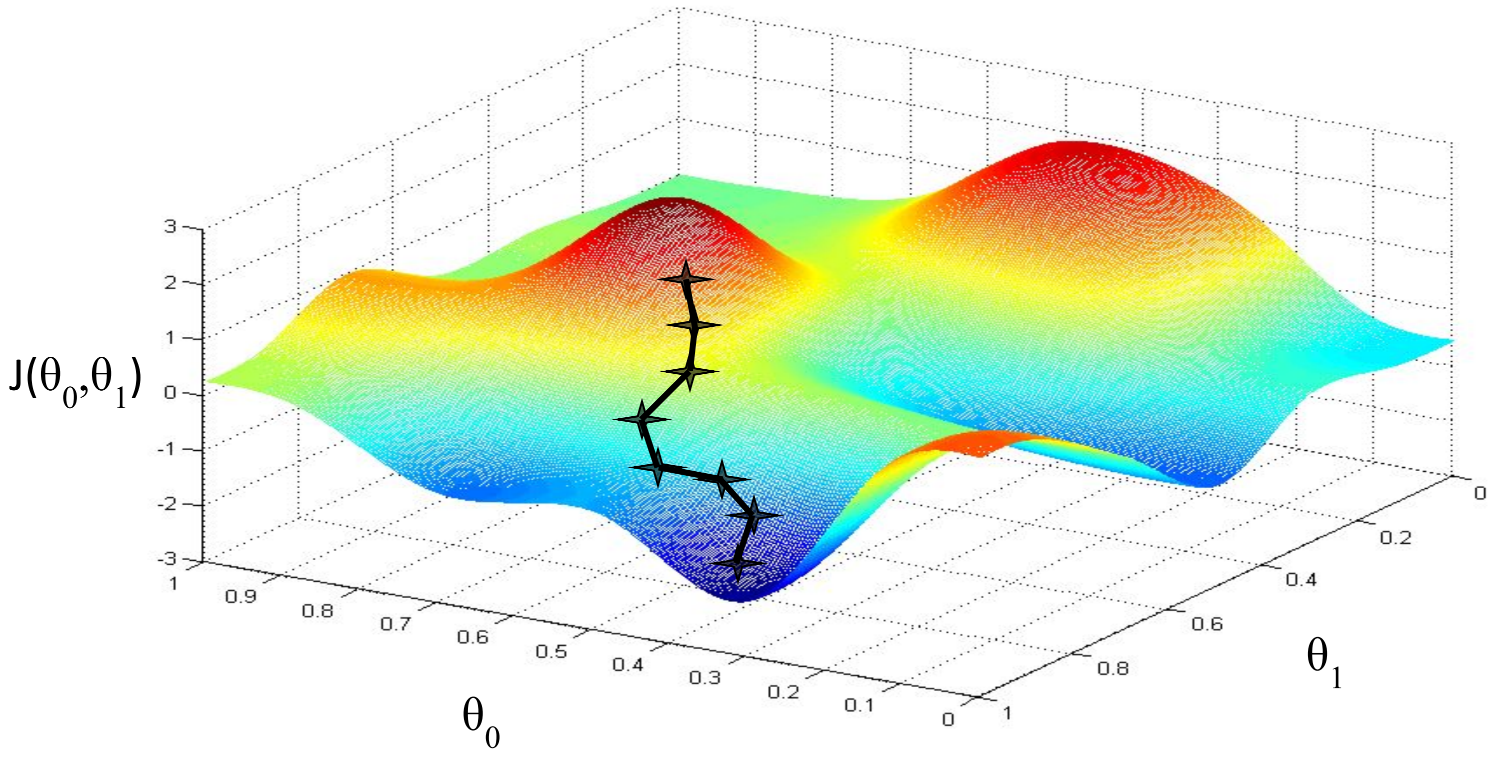
Have some function $J(\theta_0, \theta_1)$

Want $\min_{\theta_0, \theta_1} J(\theta_0, \theta_1)$

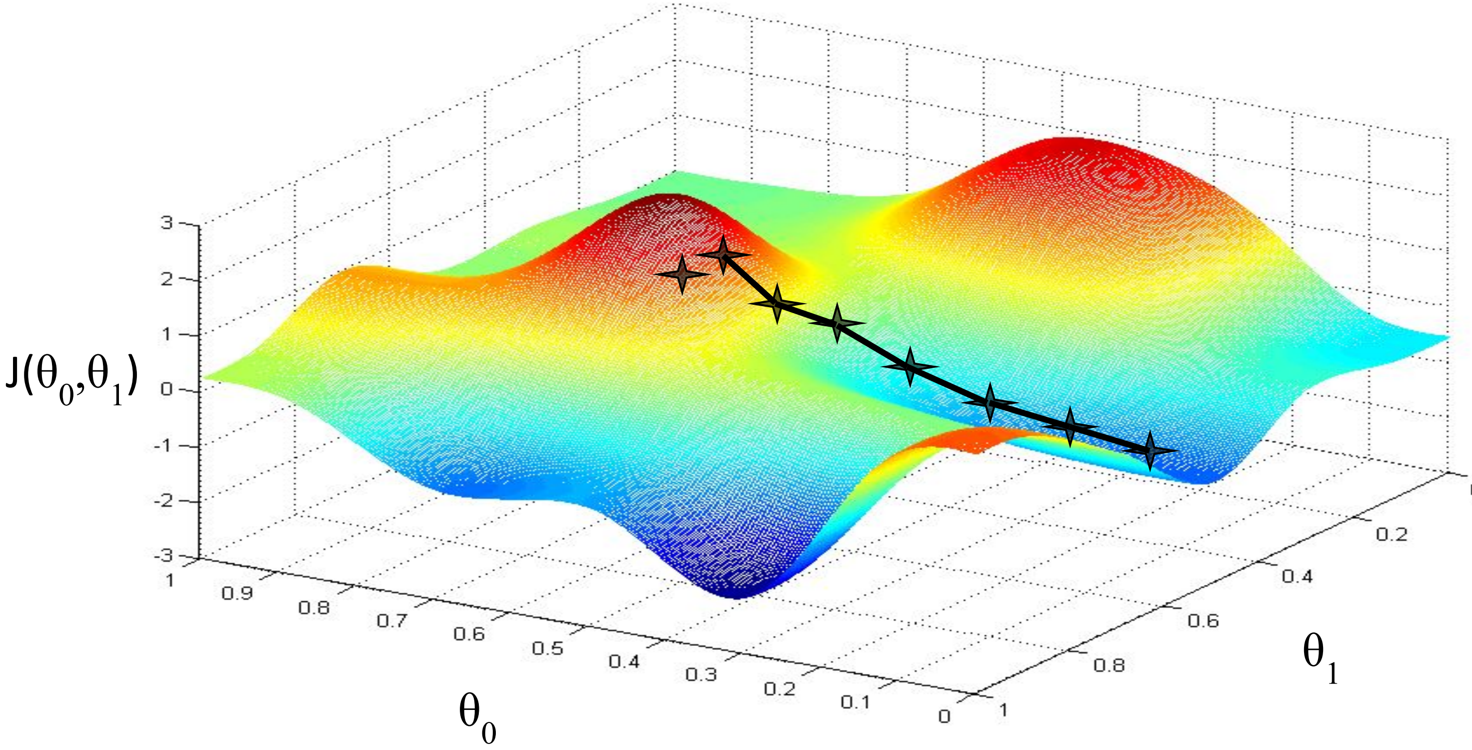
Outline:

- Start with some θ_0, θ_1
- Keep changing θ_0, θ_1 to reduce $J(\theta_0, \theta_1)$

  until we hopefully end up at a minimum



J(θ0,θ1)
3
2
1
0
-1
-2
-3
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
θ0
0
0.2
0.4
0.6
0.8
1
θ1



$J(\theta_0,\theta_1)$
3
2
1
0
-1
-2
-3
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
θ_0
0
0.2
0.4
0.6
0.8
1
θ_1

Gradient descent algorithm

$$\text{repeat until convergence } \{$$
$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) \qquad (\text{for } j = 0 \text{ and } j = 1)$$
$$\}$$

Correct: Simultaneous update

$$\text{temp0} := \theta_0 - \alpha \tfrac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$$
$$\text{temp1} := \theta_1 - \alpha \tfrac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$$
$$\theta_0 := \text{temp0}$$
$$\theta_1 := \text{temp1}$$

Incorrect:

$$\text{temp0} := \theta_0 - \alpha \tfrac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$$
$$\theta_0 := \text{temp0}$$
$$\text{temp1} := \theta_1 - \alpha \tfrac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$$
$$\theta_1 := \text{temp1}$$

Linear regression with one variable

Gradient Descent Intuition

Gradient descent algorithm

$$\text{repeat until convergence } \{$$

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) \qquad \text{(simultaneously update } j = 0 \text{ and } j = 1\text{)}$$

$$\}$$

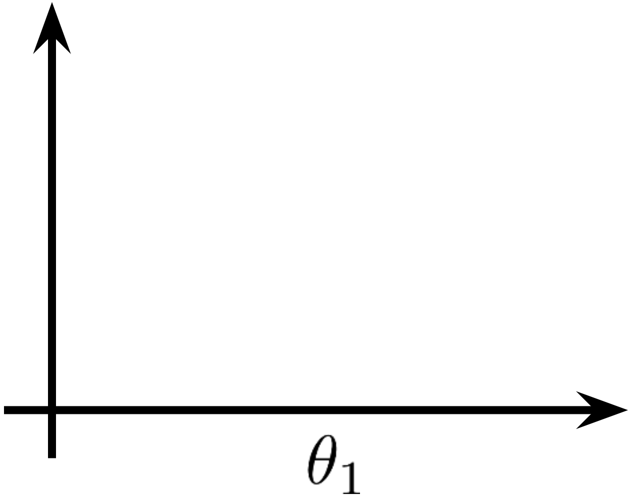
$$\theta_1 := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_1)$$

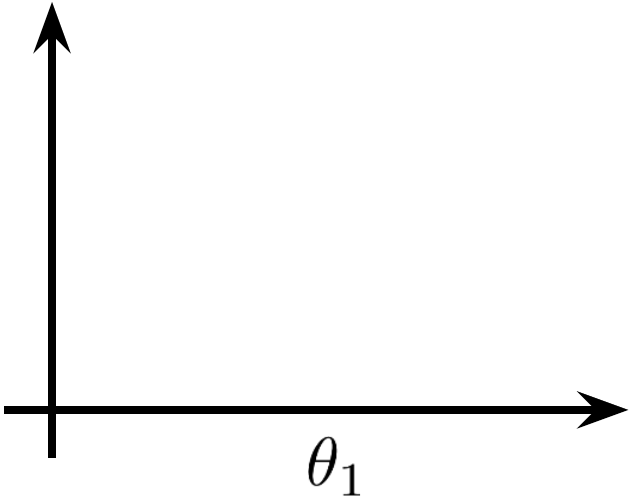
What happens If α is too small?

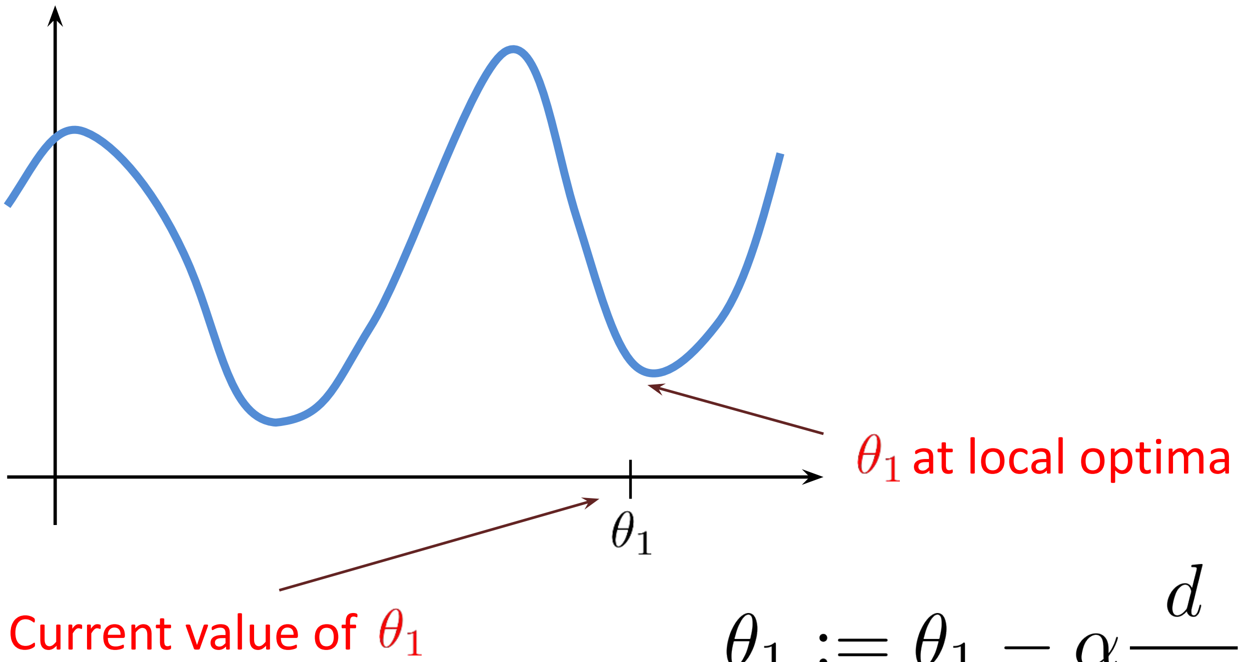
Gradient descent can be slow.

What if α is too large?

Gradient descent can overshoot the minimum. It may fail to converge, or even diverge.





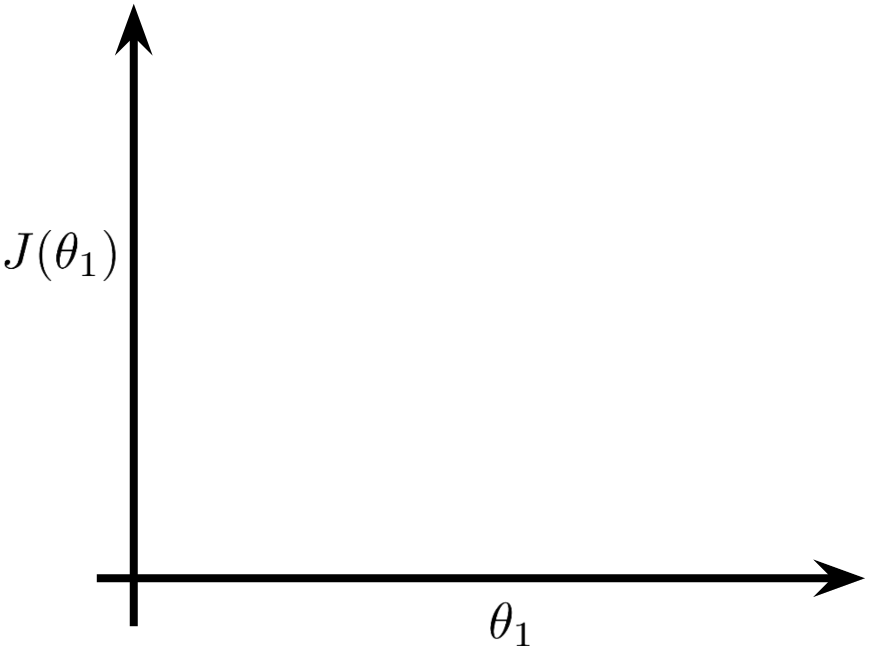


$$\theta_1 := \theta_1 - \alpha \frac{d}{d\theta_1} J(\theta_1)$$

Gradient descent can converge to a local minimum, even with the learning rate α fixed.

$$\theta_1 := \theta_1 - \alpha \frac{d}{d\theta_1} J(\theta_1)$$

As we approach a local minimum, gradient descent will automatically take smaller steps. So, no need to decrease α over time.



Linear regression
with one variable

Gradient Descent
for
Linear Regression

Gradient descent algorithm

$$
\begin{aligned}
&\text{repeat until convergence } \{ \\
&\quad \theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) \\
&\qquad (\text{for } j = 1 \text{ and } j = 0) \\
&\}
\end{aligned}
$$

Linear Regression Model

$$h_\theta(x) = \theta_0 + \theta_1 x$$

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

$$\frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) = \quad = \frac{\partial}{\partial j} \left(\frac{1}{2m} \sum_{i=1}^{m} \left(\theta_0 - \theta_1 x^{(i)} - y^{(i)} \right)^2 \right.$$

$$j = 0 : \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1) =$$

$$j = 1 : \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1) =$$

$$j = 0 : \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1) =$$

$$= \frac{1}{\cancel{2}m} \times \cancel{2} \sum_{i=1}^{m} \left((\theta_0 + \theta_1 x^{(i)}) - y^{(i)} \right)^{1} \cdot \frac{\partial}{\partial \theta_0} \left(\theta_0 + \theta_1 x^{(i)} - y^{(i)} \right)$$

$$= \frac{1}{m} \sum_{i=1}^{m} \left(\theta_0 + \theta_1 x^{(i)} - y^{(i)} \right) \cdot \left(\frac{\partial}{\partial \theta_0} \theta_0 + \frac{\partial}{\partial \theta_0} \theta_1 x^{(i)} - \frac{\partial}{\partial \theta_0} y^{(i)} \right)$$

$$= \frac{1}{m} \sum_{i=1}^{m} \left(\theta_0 + \theta_1 x^{(i)} - y^{(i)} \right) \cdot 1$$

$$j = 1 : \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1) =$$

$$= \frac{1}{2m} \times 2 \sum_{i=1}^{m} \left(\theta_0 + \theta_1 x^{(i)} - y^{(i)} \right)^1 \cdot \frac{\partial}{\partial \theta_1} \left(\theta_0 + \theta_1 x^{(i)} - y^{(i)} \right)$$

$$= \frac{1}{m} \sum_{i=1}^{m} \left(\theta_0 + \theta_1 x^{(i)} \right) \cdot \left(\frac{\partial}{\partial \theta_1} \theta_0 + \frac{\partial}{\partial \theta_1} \theta_1 x^{(i)} - \frac{\partial}{\partial \theta_1} y^{(i)} \right)$$

$$= \frac{1}{m} \sum_{i=1}^{m} \left(\theta_0 + \theta_1 x^{(i)} \right) \cdot \left(0 + x^{(i)} - 0 \right)$$

$$= \frac{1}{m} \sum_{i=1}^{m} \left(\theta_0 + \theta_1 x^{(i)} - y^{(i)} \right) \cdot x^{(i)}$$

Gradient descent algorithm

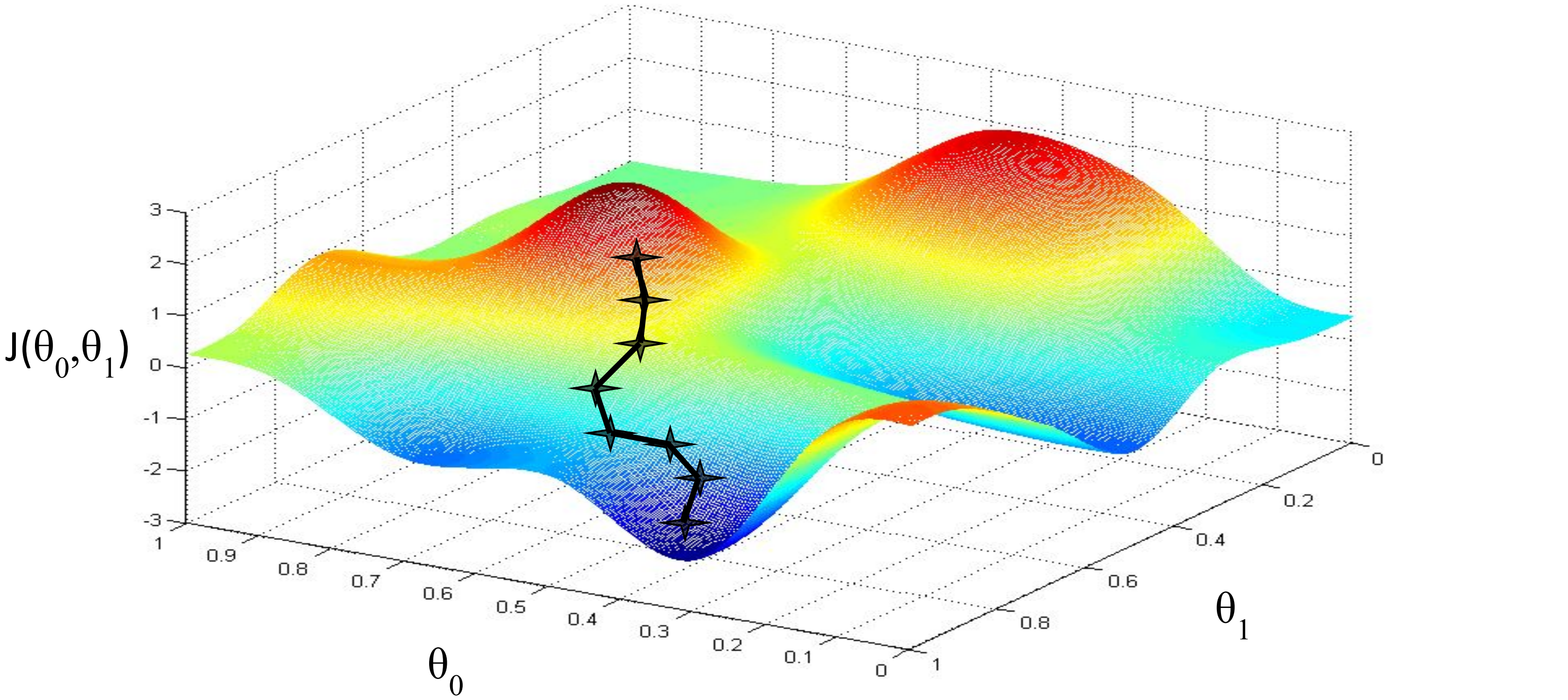
repeat until convergence {

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)$$

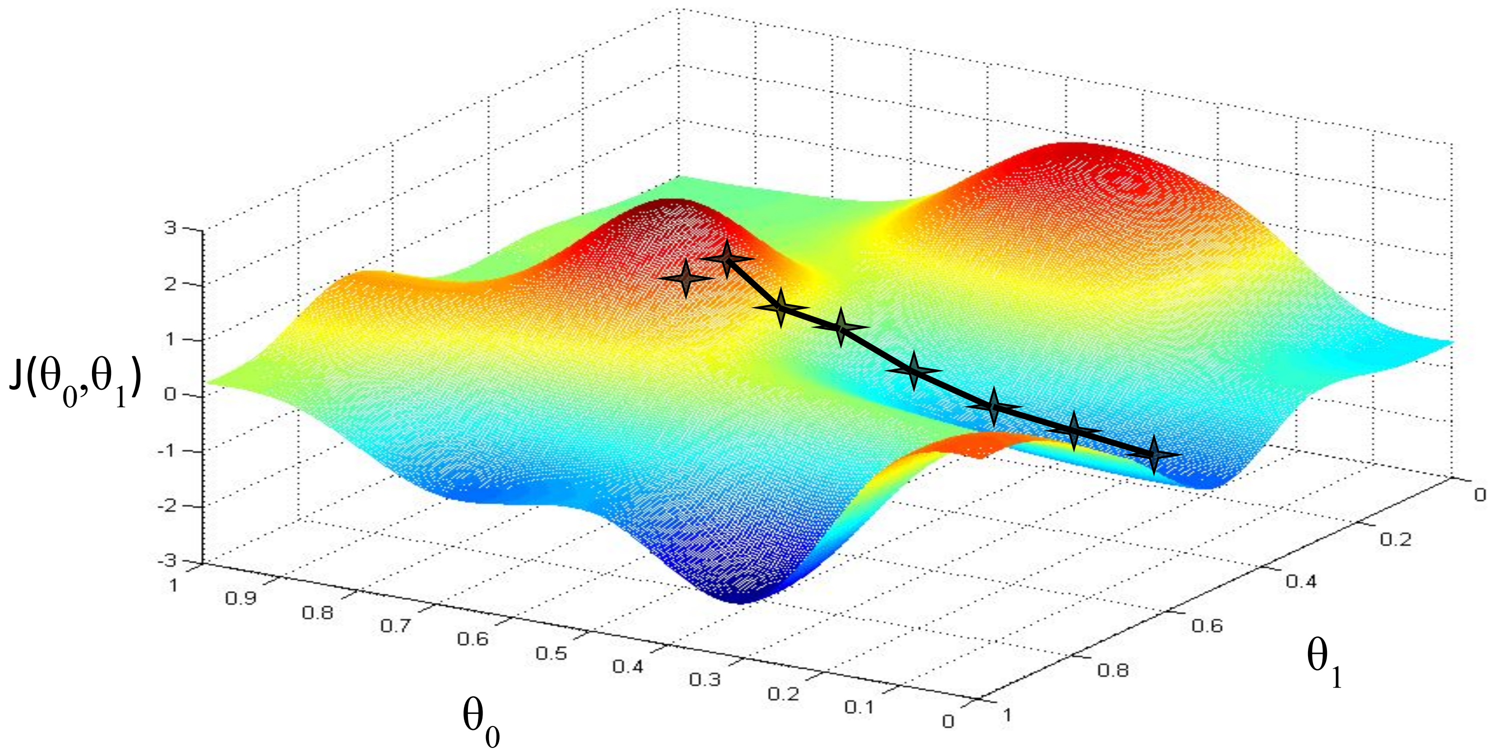
$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right) \cdot x^{(i)}$$

}

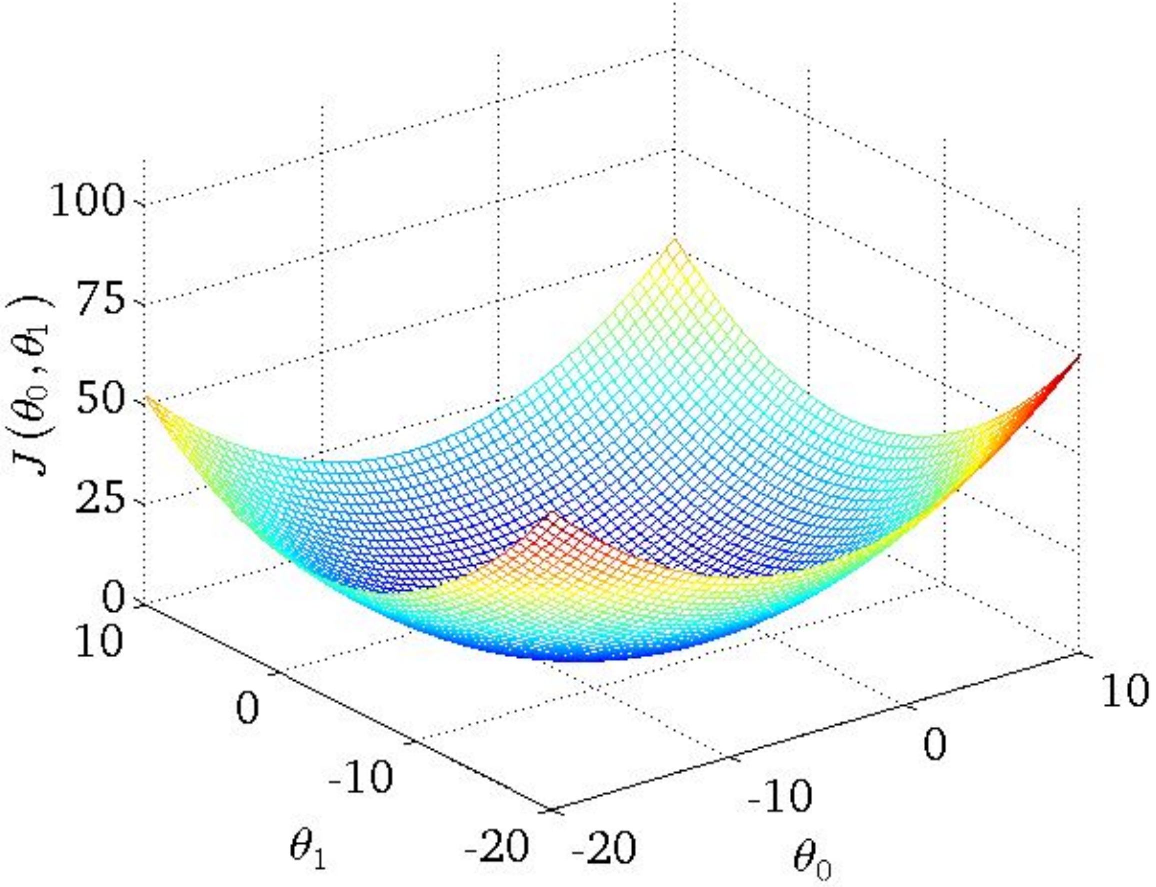
update θ_0 and θ_1 simultaneously



$J(\theta_0,\theta_1)$
3
2
1
0
-1
-2
-3
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
θ_0
0
0.2
0.4
0.6
0.8
1
θ_1



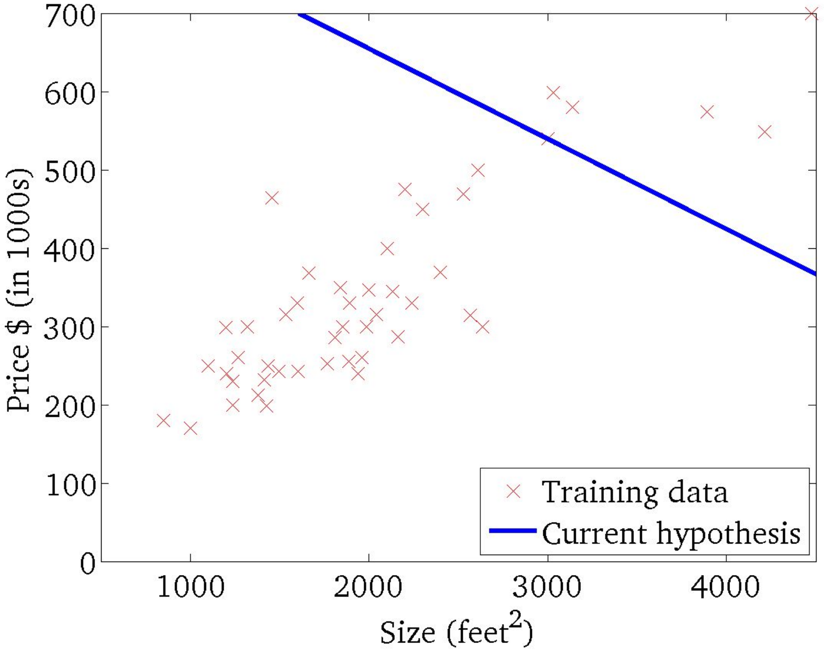
$J(\theta_0,\theta_1)$
3
2
1
0
-1
-2
-3
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
θ_0
0
0.2
0.4
0.6
0.8
1
θ_1



100
75
50
25
0
$J(\theta_0, \theta_1)$
10
0
-10
-20
θ_1
-20
-10
0
10
θ_0

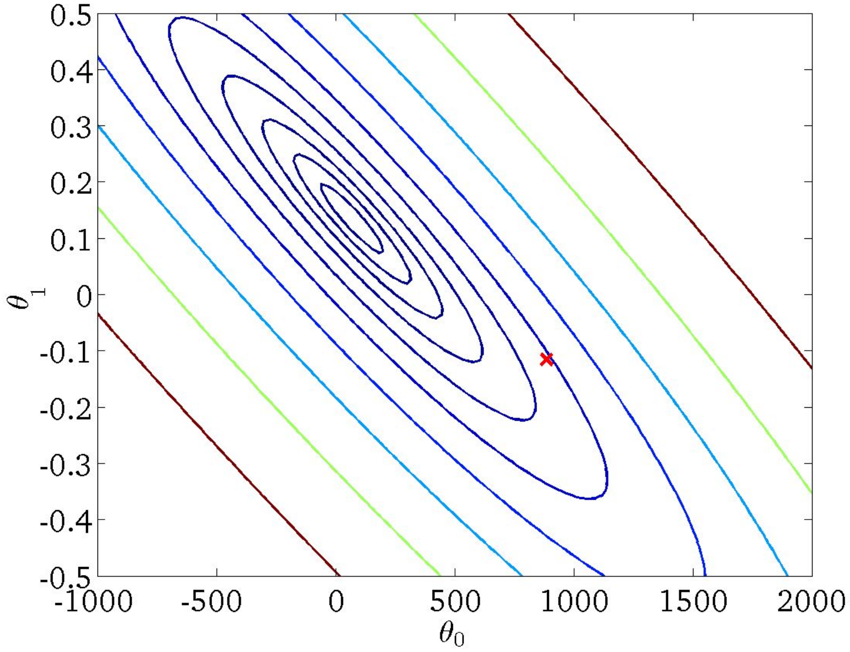
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



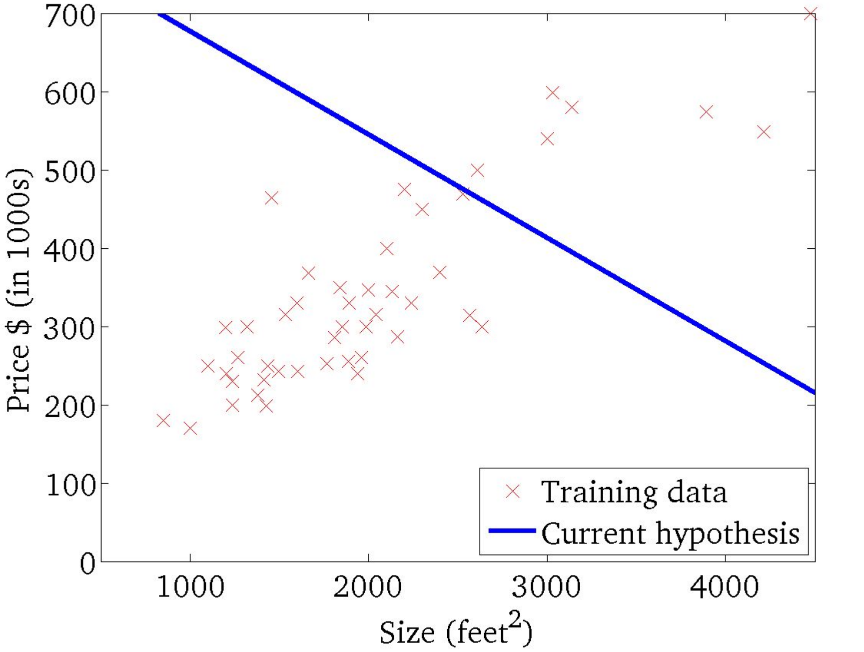
$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



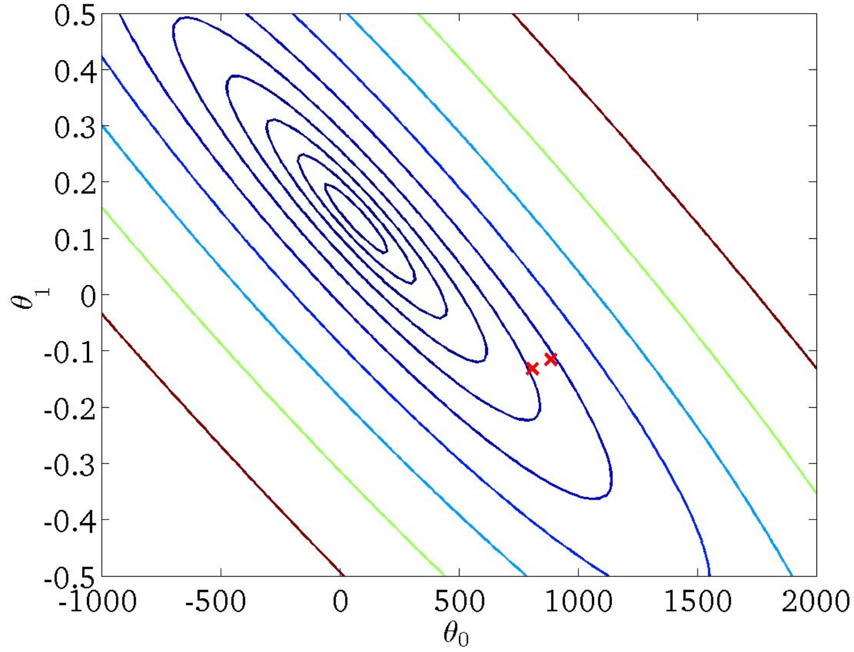
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



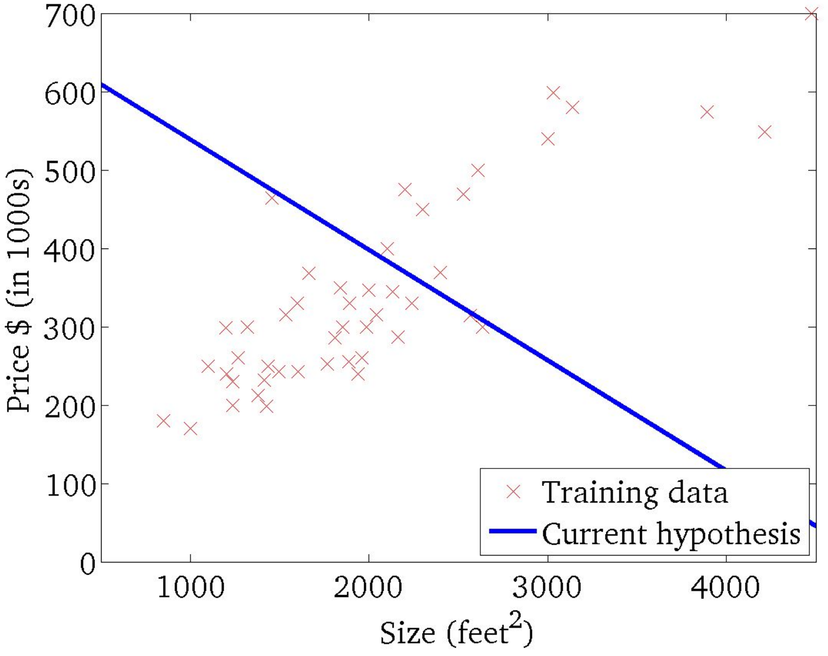
$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



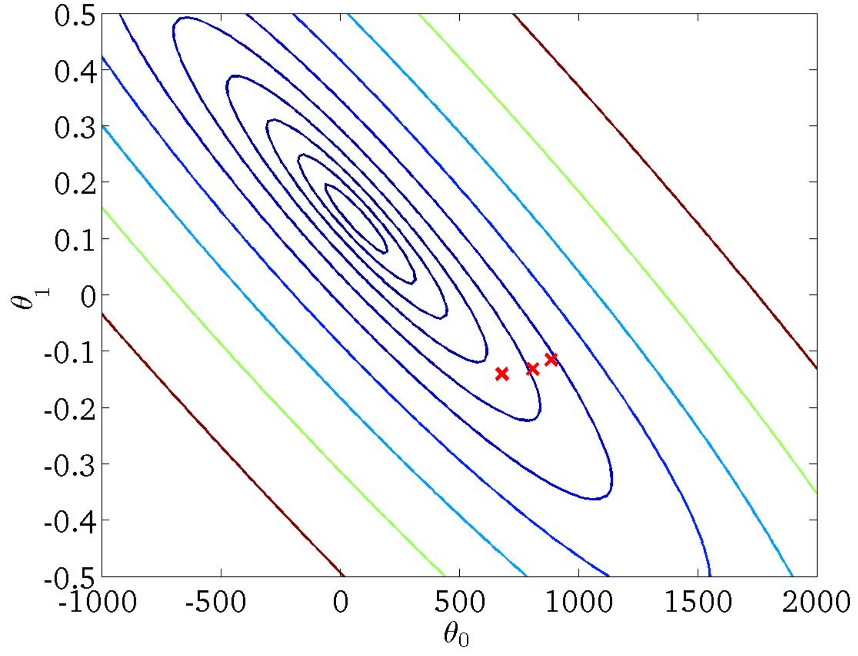
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



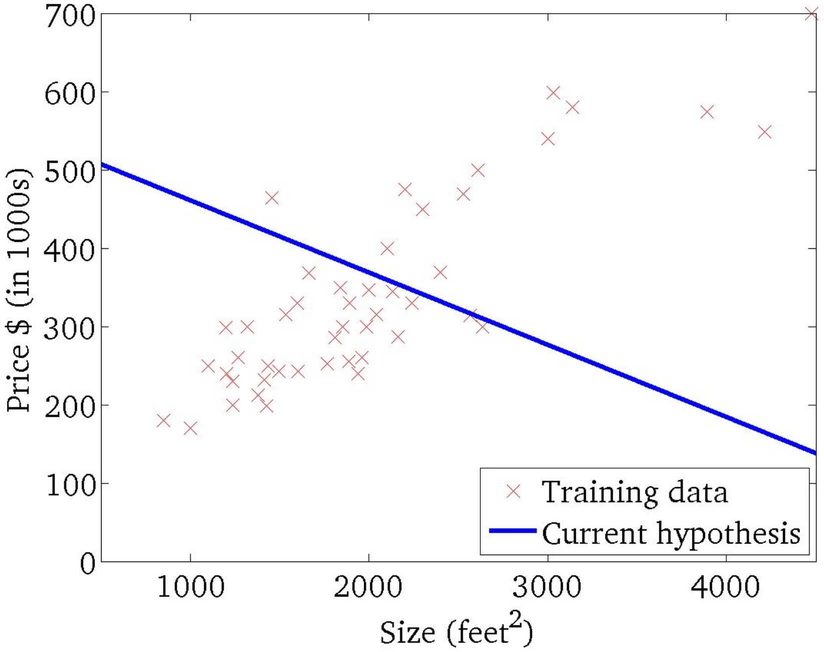
$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



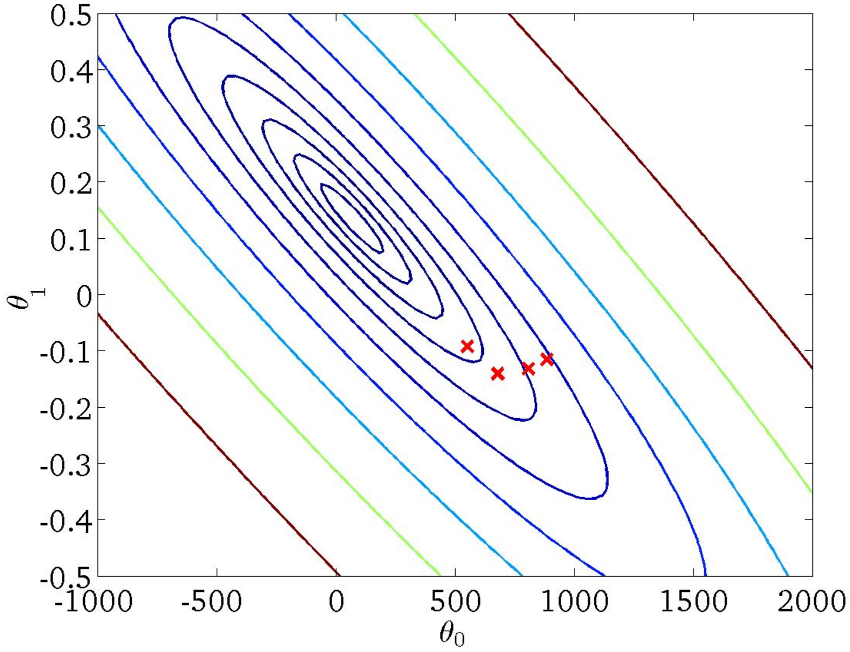
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



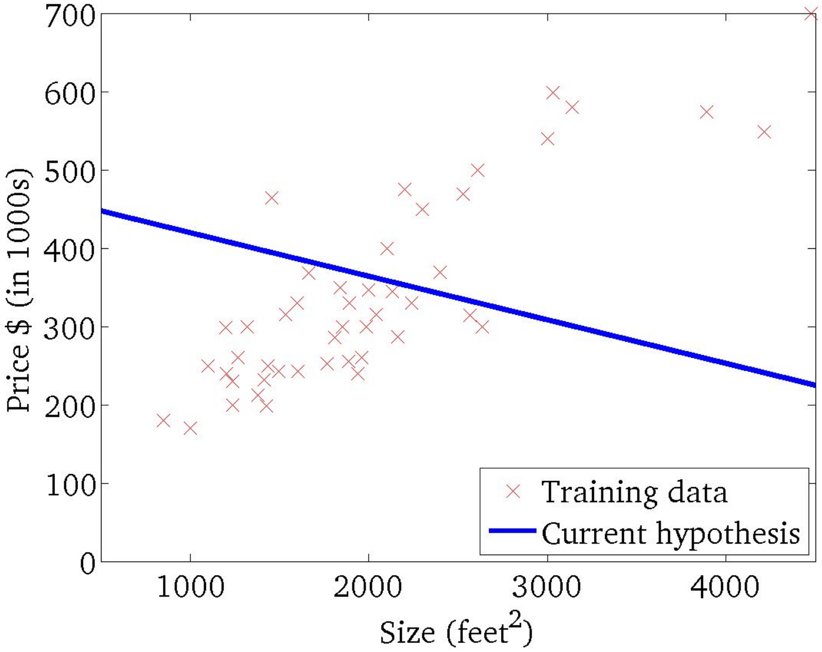
$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



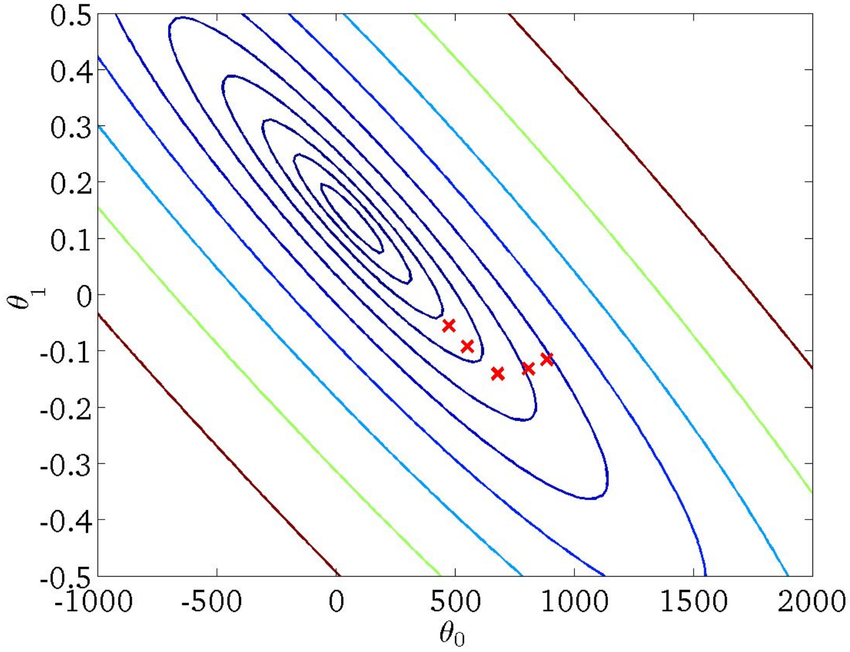
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



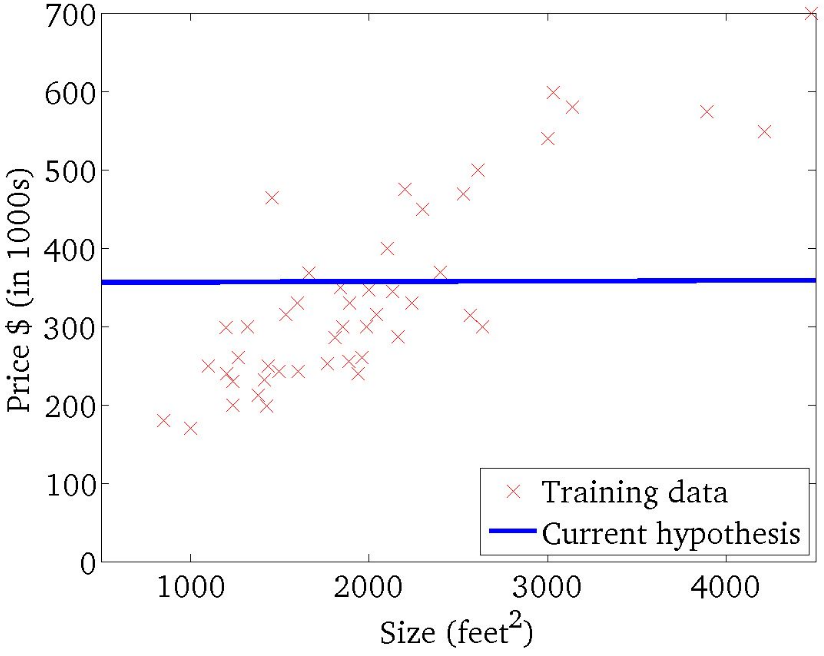
$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



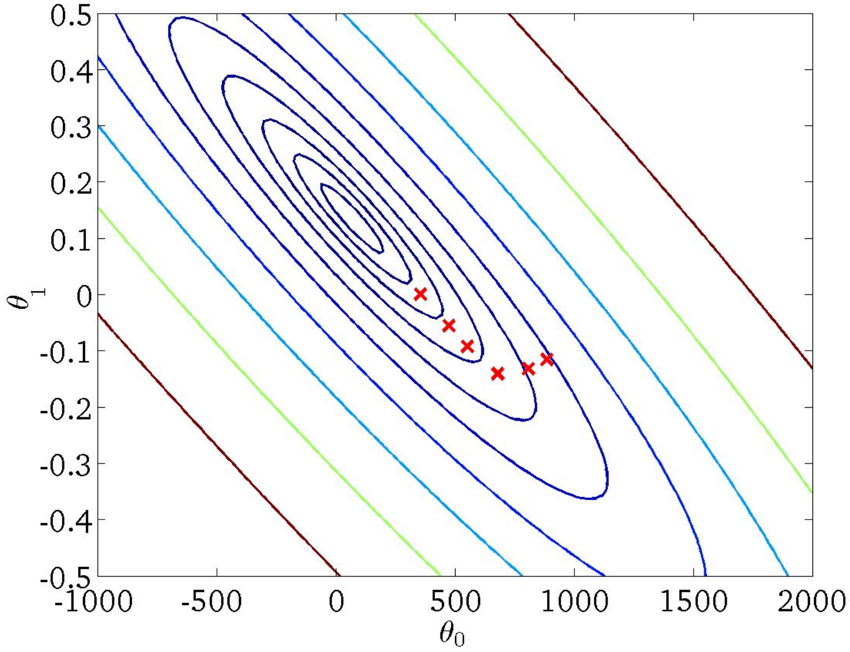
$$h_\theta(x)$$

(for fixed θ_0, θ_1, this is a function of x)



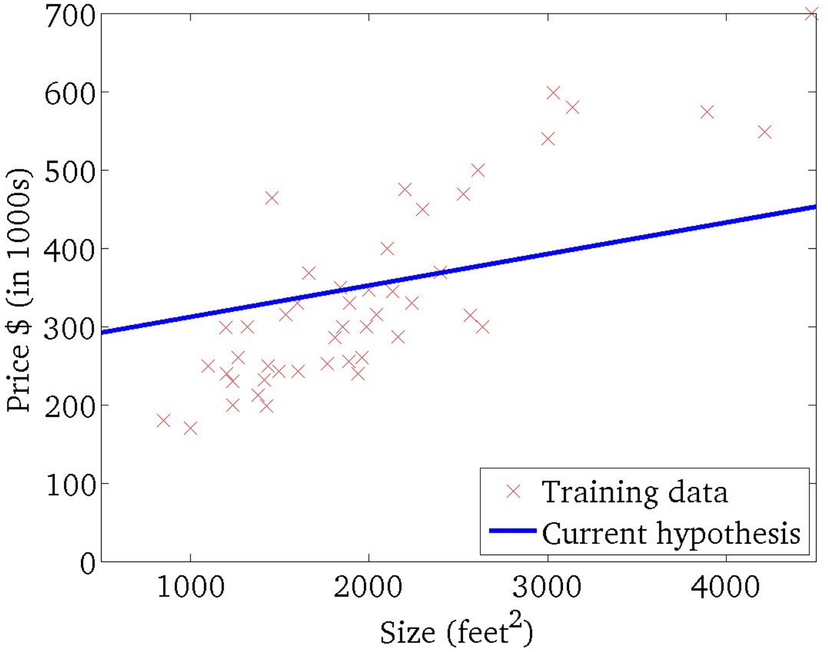
$$J(\theta_0, \theta_1)$$

(function of the parameters θ_0, θ_1)



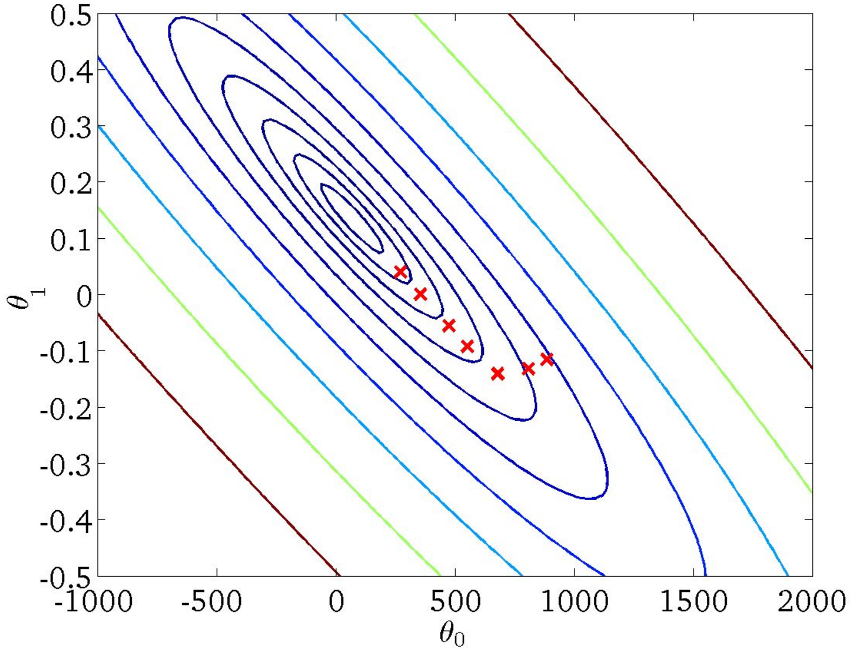
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



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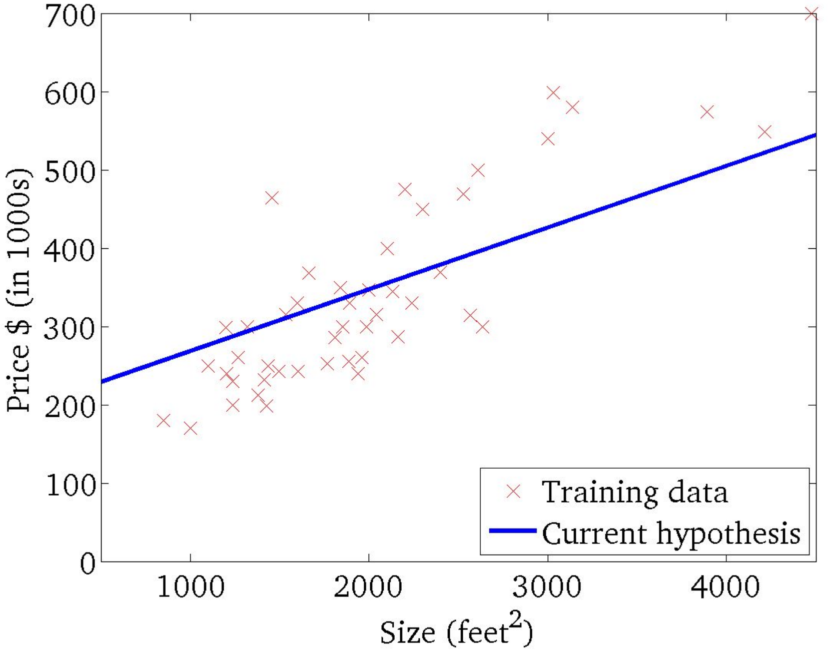
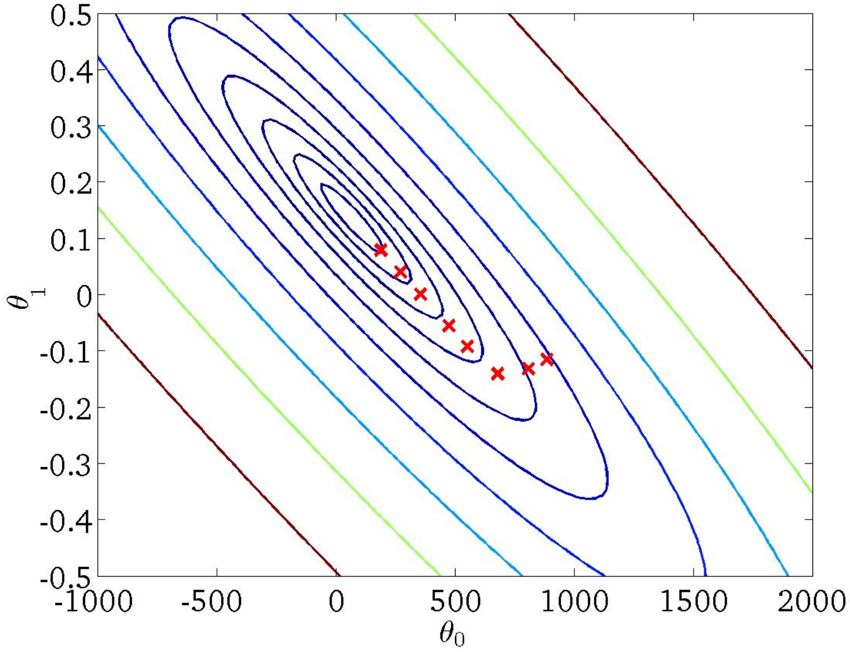


$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



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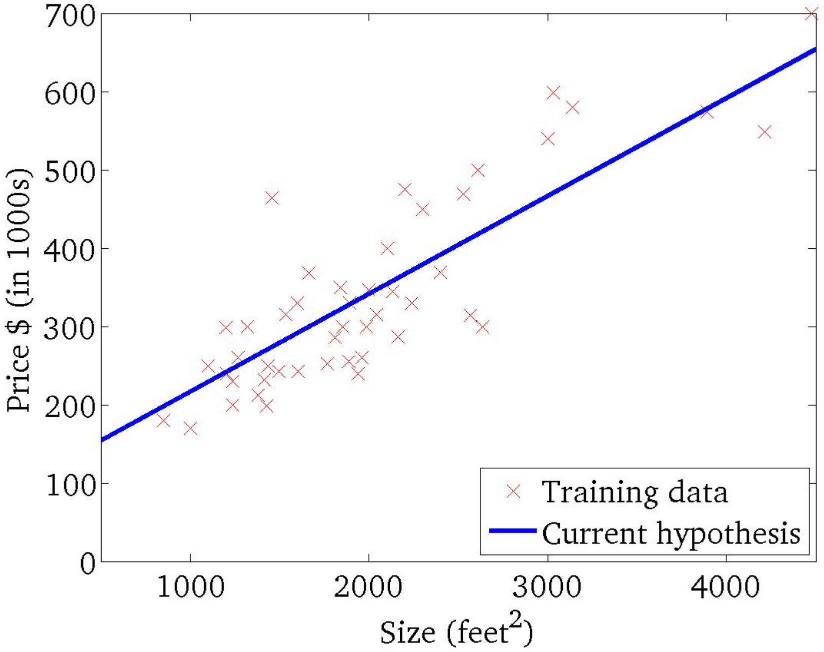
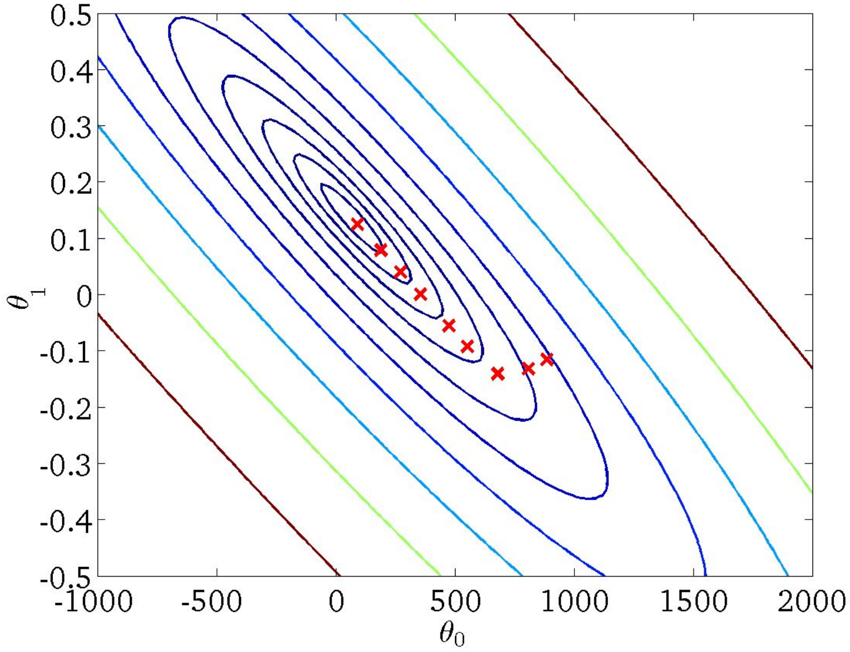


$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



“Batch” Gradient Descent

“Batch”: Each step of gradient descent uses all the training examples.

What’s Next

Two extensions:

1. In $\min J(\theta_0, \theta_1)$, solve for θ_0, θ_1 exactly, without needing iterative algorithm (gradient descent).
2. Learn with larger number of features.

Size (feet^2)	**Price (\$1000)**
2104	460
1416	232
1534	315
852	178

Two extensions:

1. In $\min J(\theta_0, \theta_1)$, solve for θ_0, θ_1 exactly, without needing iterative algorithm (gradient descent).
2. Learn with larger number of features.

Size (feet^2)	Number of bedrooms	Number of floors	Age of home (years)	Price ($1000)
2104	5	1	45	460
1416	3	2	40	232
1534	3	2	30	315
852	2	1	36	178

Linear Algebra

Notation and set of the things you can do with matrices and vectors.

Matrix:

$$X = \begin{bmatrix} 2104 & 5 & 1 & 45 \\ 1416 & 3 & 2 & 40 \\ 1534 & 3 & 2 & 30 \\ 852 & 2 & 1 & 36 \end{bmatrix}$$

Vector:

$$y = \begin{bmatrix} 460 \\ 232 \\ 315 \\ 172 \end{bmatrix}$$