Basic Mathematics Concepts

1.2 Mathematics Review

from

Mark Allen Weiss's Book

Exponents

$$x^A x^B = x^{A+B}$$

$$\frac{x^A}{x^B} = x^{A-B}$$

$$x^N + x^N = 2x^N \neq x^{2N}$$

$$2^N + 2^N = 2^{N+1}$$

Logarithms and properties

- In algorithm analysis we often use the notation "log n" without specifying the base

Binary logarithm $\lg n = \log_2 n$

Natural logarithm $\ln n = \log_e n$

$\lg^k n = (\lg n)^k$

$\lg \lg n = \lg(\lg n)$

$x^{\log_x y} = \mathrm{y}$

$\log x^y = y \log x$

$\log xy = \log x + \log y$

$\log \frac{x}{y} = \log x - \log y$

$a^{\log_b x} = x^{\log_b a}$

$\log_b x = \frac{\log_a x}{\log_a b}$

Series & Summations

- Arithmetic series:

$$\sum_{k=1}^{n} k = 1 + 2 + \ldots + n = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \text{ (for n >= 1)}$$

$$\sum_{i=1}^{n} a_i = \frac{n}{2}(a_1 + a_n)$$

- Geometric series:

$$\sum_{k=0}^{n} x^k = 1 + x + x^2 + \ldots + x^n = \frac{x^{n+1} - 1}{x - 1} (x \neq 1)$$

  - Special case: $|x| < 1$:

$$\sum_{k=0}^{\infty} x^k = \frac{1}{1 - x}$$

- Harmonic series:

$$\sum_{k=1}^{n} \frac{1}{k} = 1 + \frac{1}{2} + \ldots + \frac{1}{n} \approx \ln n$$

Common Summations

- Other important formulas:

$$\sum_{k=1}^{n} \lg k \approx n \lg n$$

$$\sum_{k=1}^{n} k^p = 1^p + 2^p + \ldots + n^p \approx \frac{1}{p+1} n^{p+1}$$

$$\sum_{k=0}^{n} k x^k = \frac{x - (n+1)x^{(n+1)} + n x^{(n+2)}}{(x-1)^2}$$