Counting sort

- Counting sort assumes that each of the n input elements is an integer in the range 0 to k. that is n is the number of elements and k is the highest value element.
- Consider the input set : 4, 1, 3, 4, 3. Then n=5 and k=4
- Counting sort determines for each input element x, the number of elements less than x. And it uses this information to place element x directly into its position in the output array. For example if there exits 17 elements less that x then x is placed into the 18^{th} position into the output array.
- The algorithm uses three array:

  Input Array: **A[1..*n*]** store input data where A[j] $\in$ {1, 2, 3, …, *k*}

  Output Array: **B[1..*n*]** finally store the sorted data

  Temporary Array: **C[1..*k*]** store data temporarily

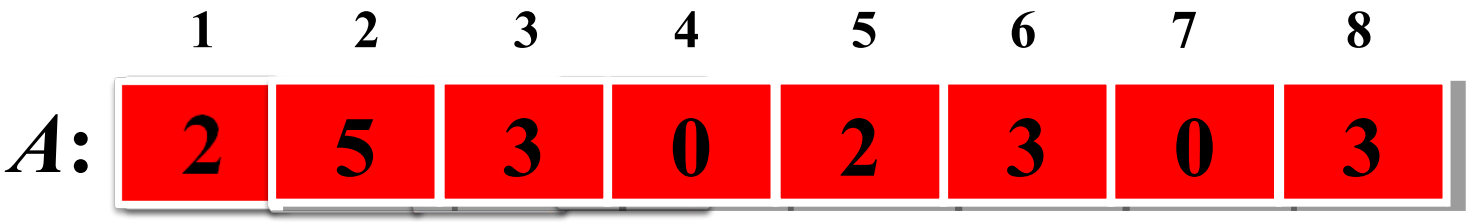
Counting Sort

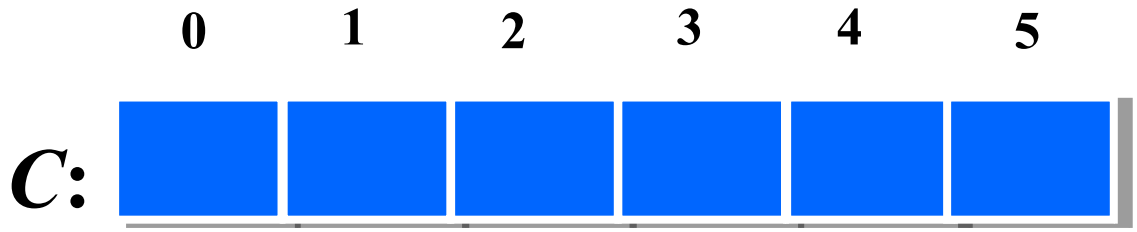
```
1.  Counting-Sort(A, B, k)
2.  Let C[0…..k] be a new array
3.  for i=0 to k
4.        C[i]= 0;
5.  for j=1 to A.length  or n
6.        C[ A[j] ] = C[ A[j] ]  + 1;
7.  for i=1 to k
8.        C[i] = C[i] + C[i-1];
9.  for j=n or A.length down to 1
10.       B[  C[ A[j] ]  ] = A[j];
11.       C[ A[j] ] = C[ A[j] ]  - 1;
```

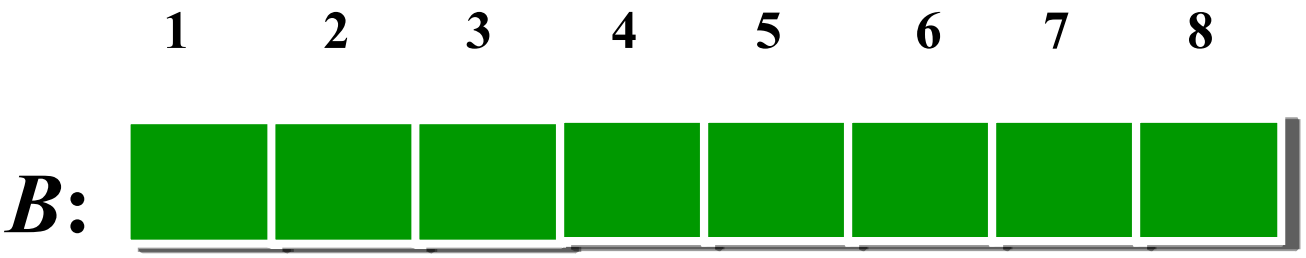
Counting Sort

```
1.  Counting-Sort(A, B, k)
2.  Let C[0…..k] be a new array
3.  for i=0 to k                                  [Loop 1]
4.        C[i]= 0;
5.  for j=1 to A.length( or n)                    [Loop 2]
6.        C[ A[j] ] = C[ A[j] ]  + 1;
7.  for i=1 to k                                  [Loop 3]
8.        C[i] = C[i] + C[i-1];
9.  for j=n or A.length down to 1                 [Loop 4]
10.       B[  C[ A[j] ]  ] = A[j];
11.       C[ A[j] ] = C[ A[j] ]  - 1;
```

Counting-sort example

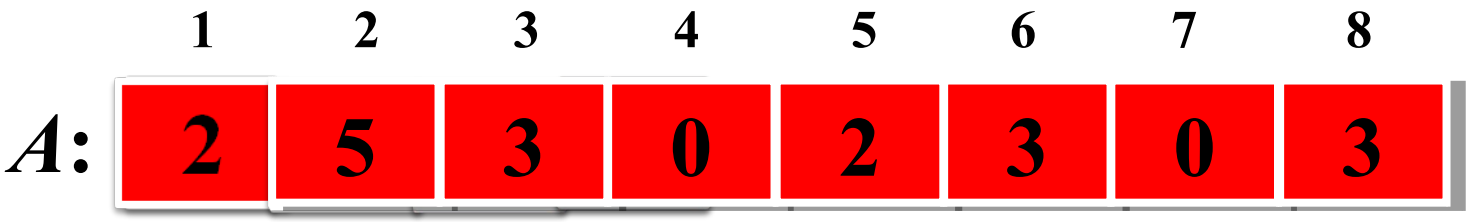


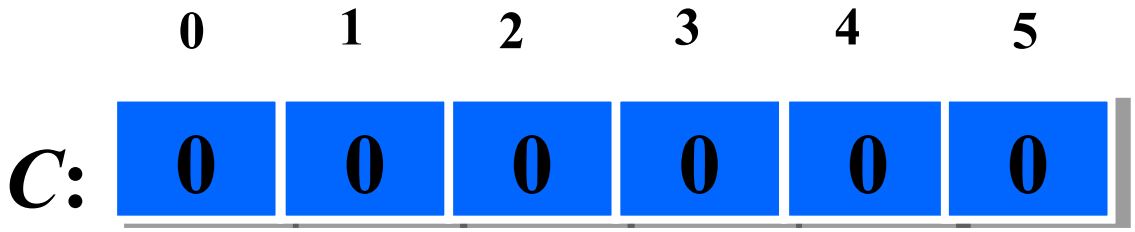


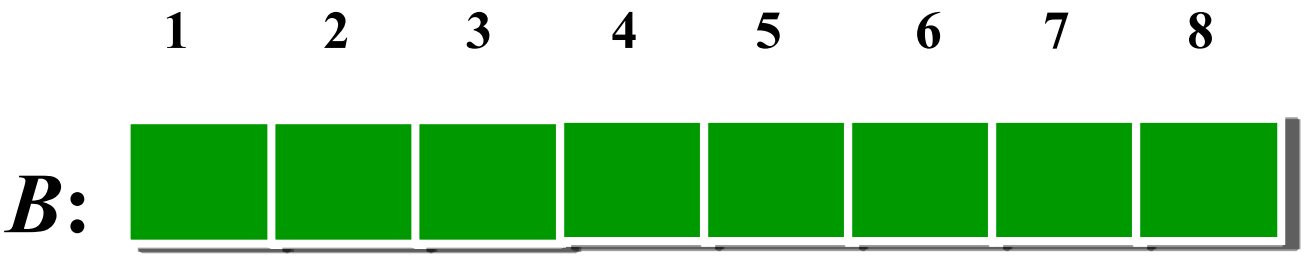


Executing Loop 1

```
3.  for i=0 to k
4.      C[i]= 0;
```

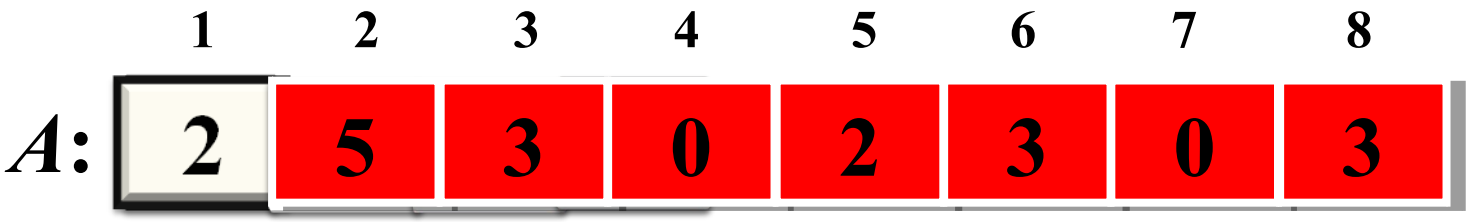


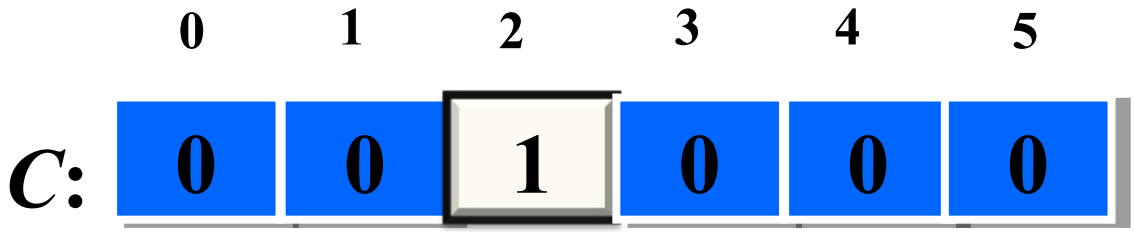


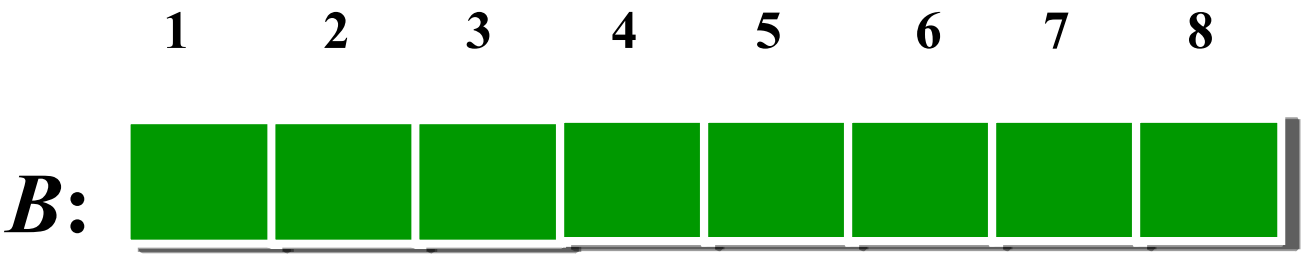


Executing Loop 2

```
5.  for j=1 to A.length or n
6.      C[ A[j] ] = C[ A[j] ] + 1;
```







Executing Loop 2

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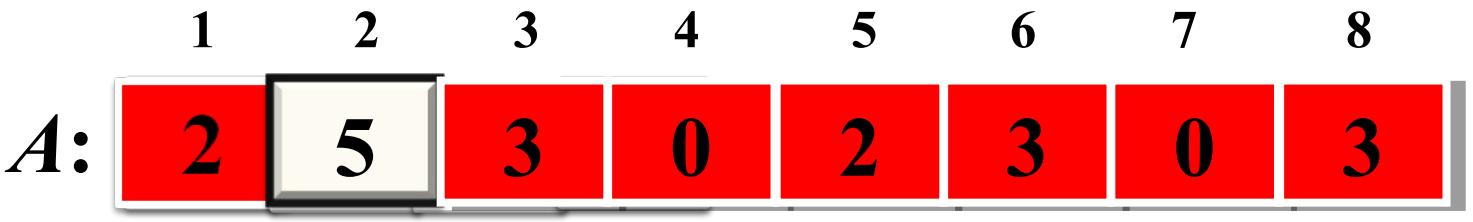

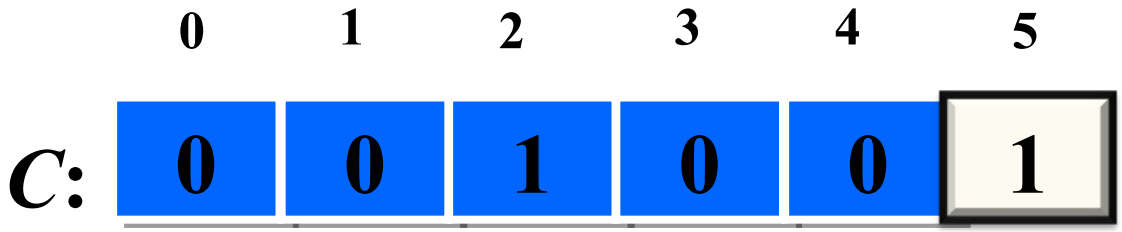

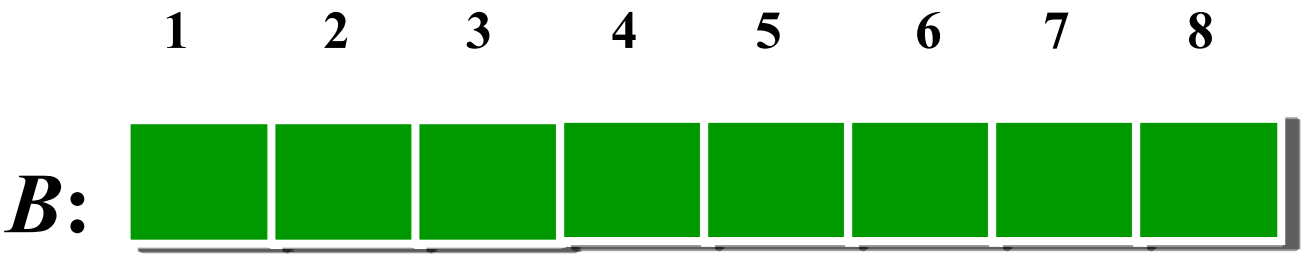

Executing Loop 2

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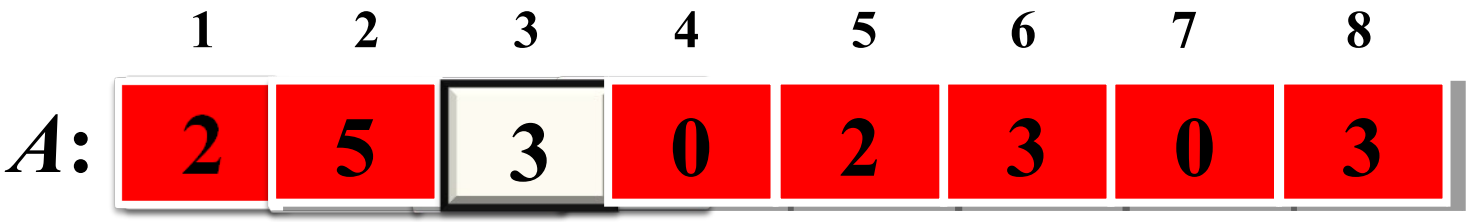

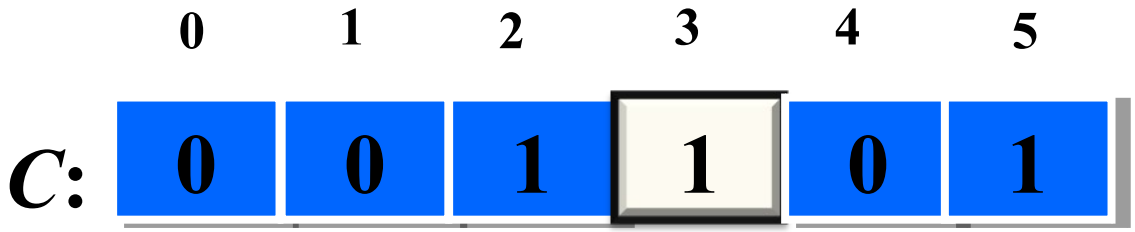

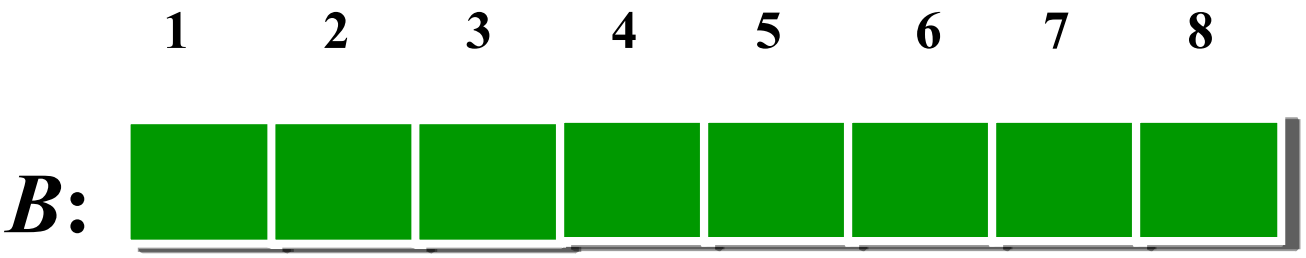

Executing Loop 2

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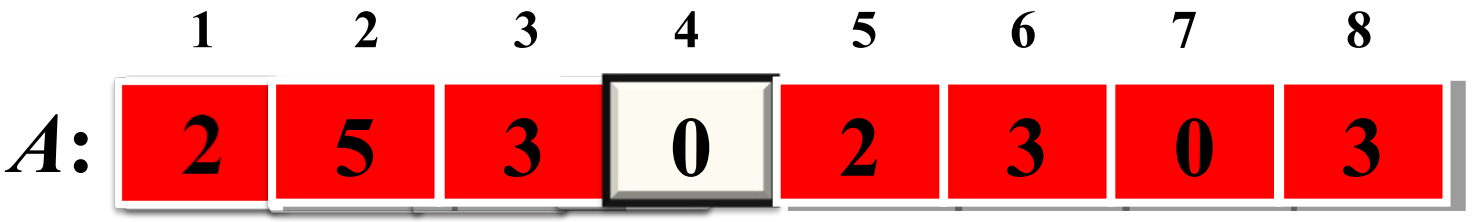

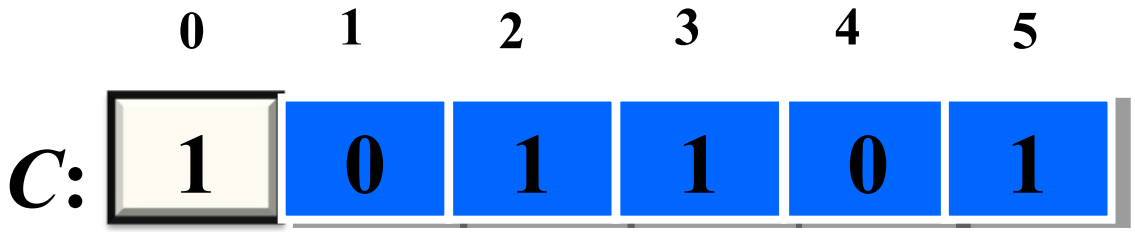

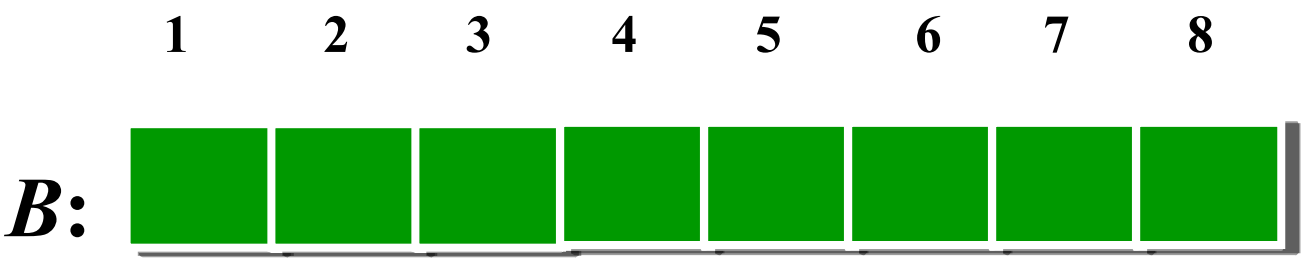

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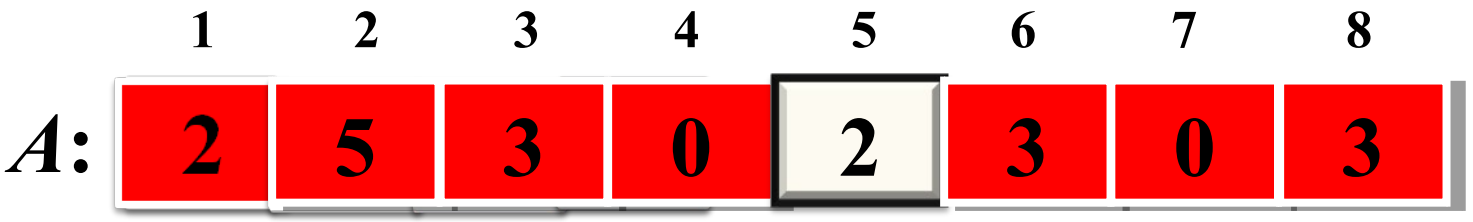

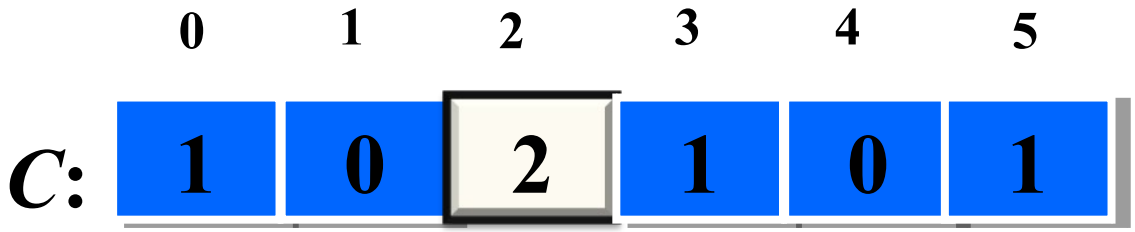

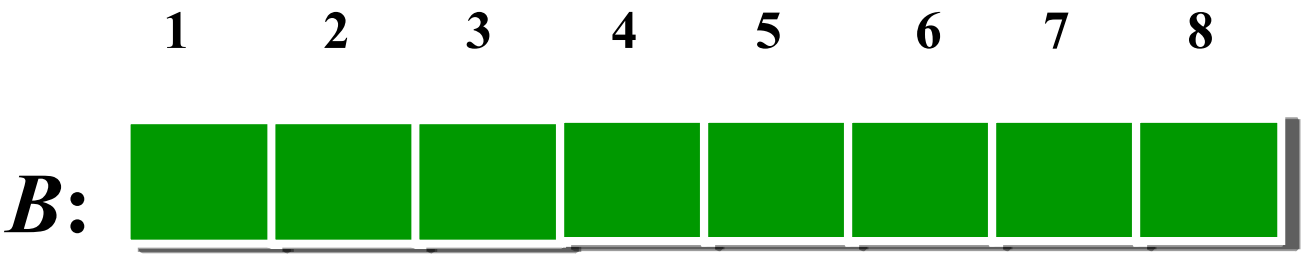

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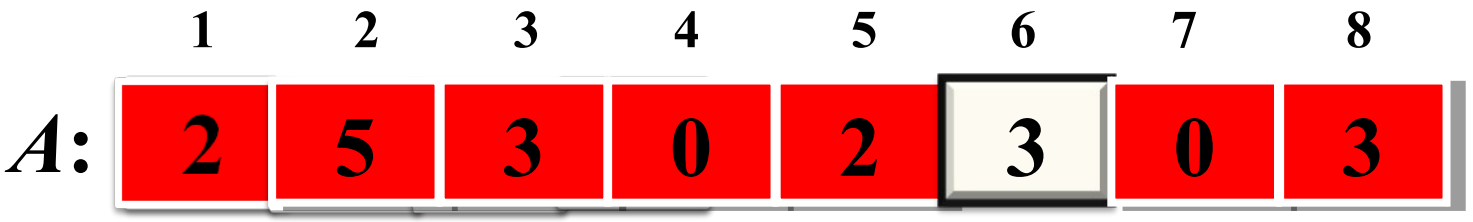

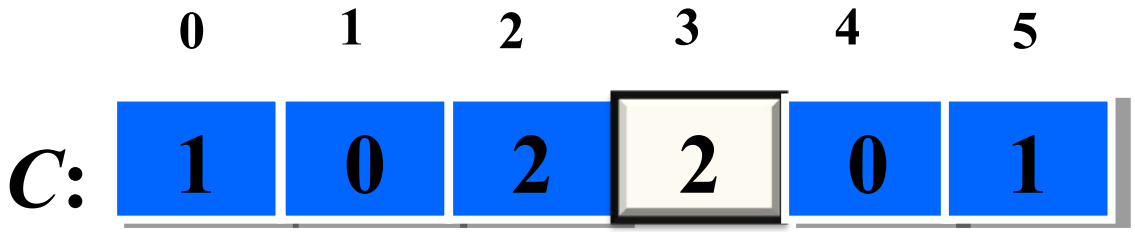

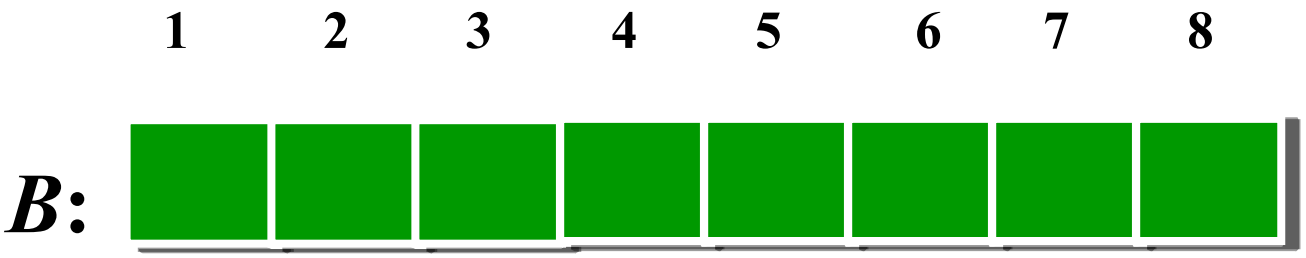

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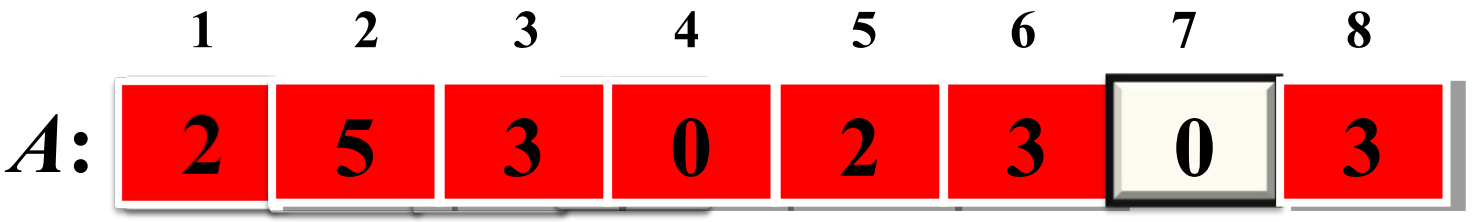

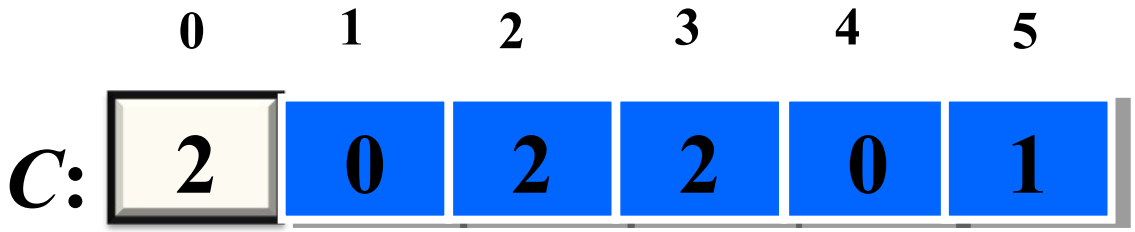

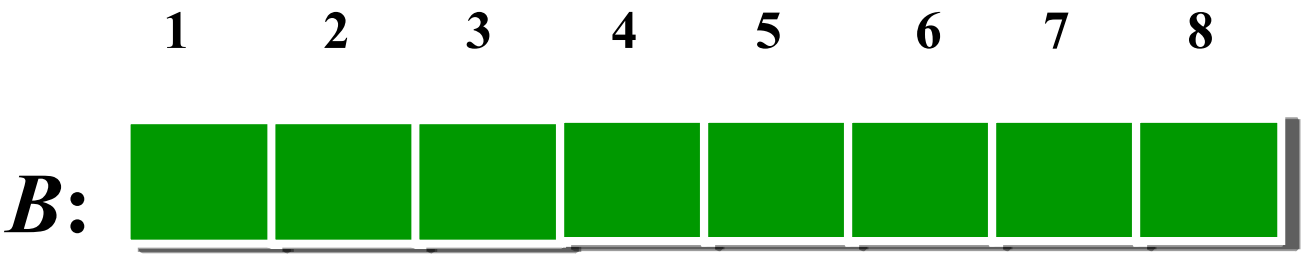

Executing Loop 2

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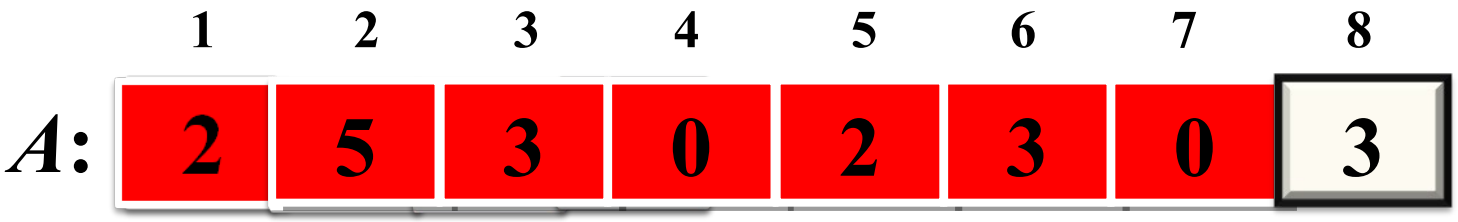
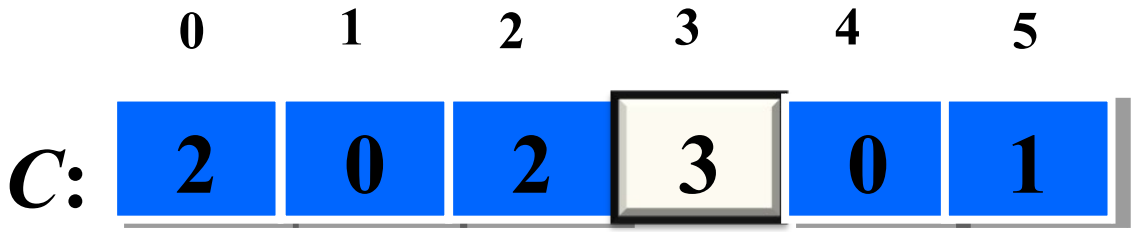
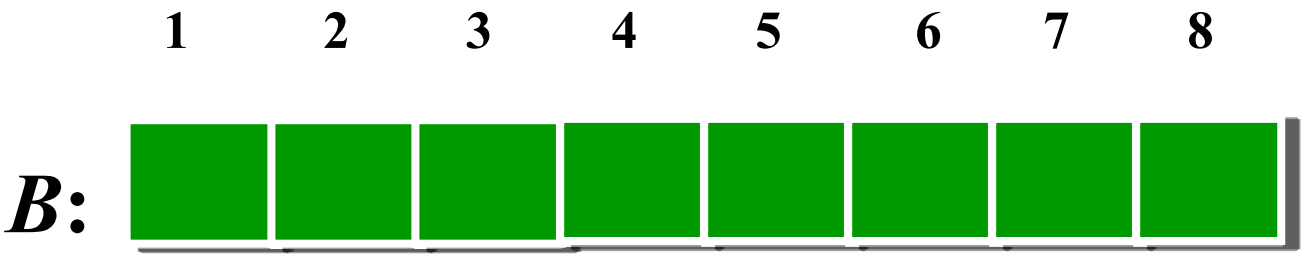
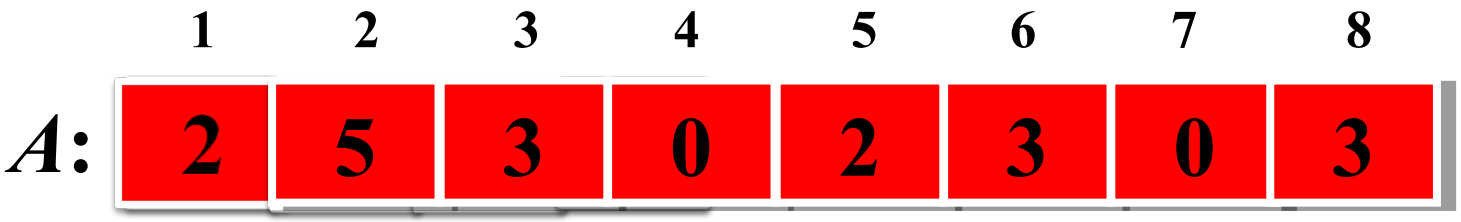
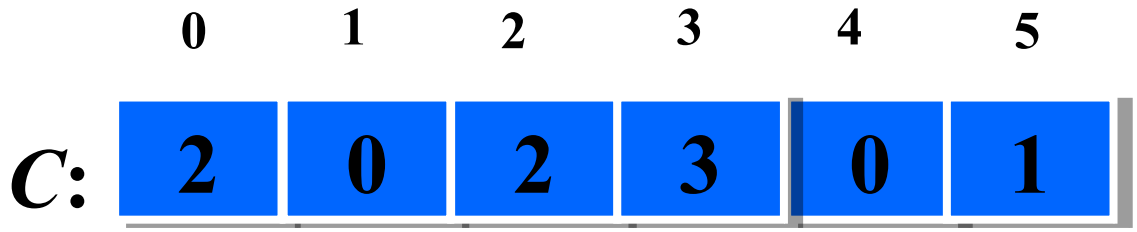


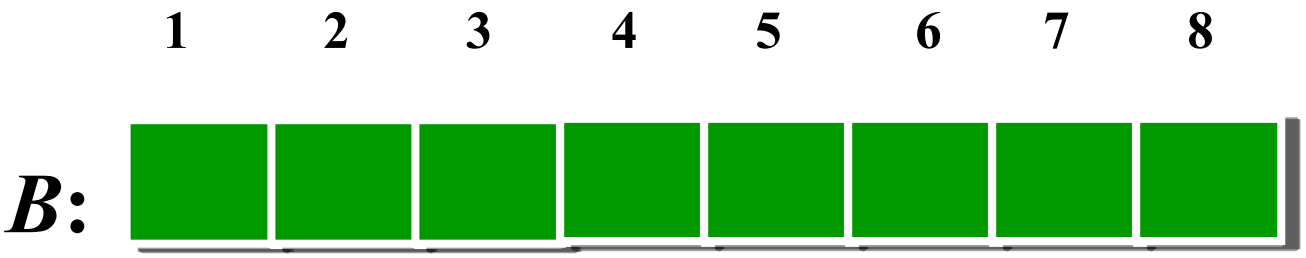




End of Loop 2

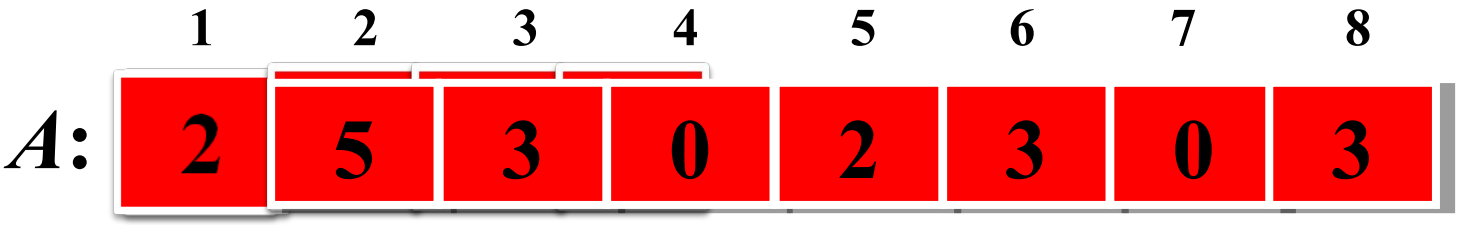


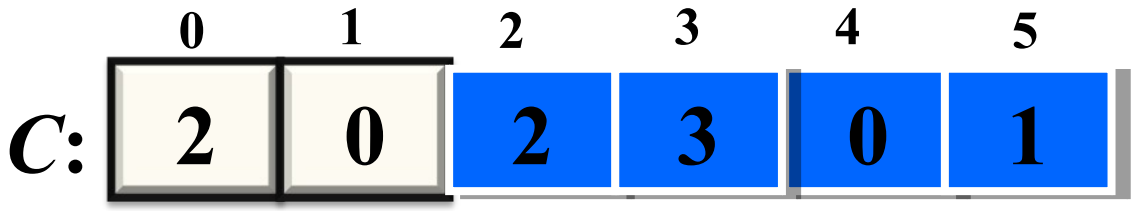


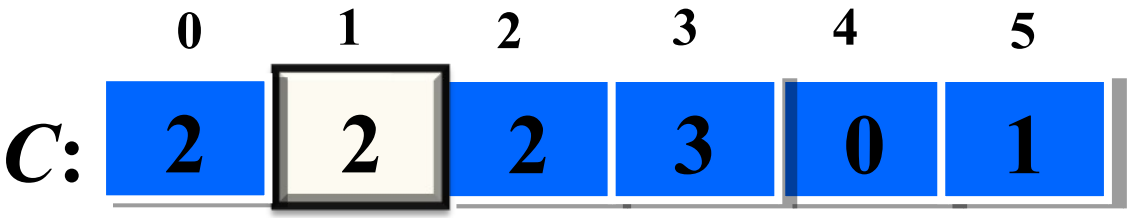


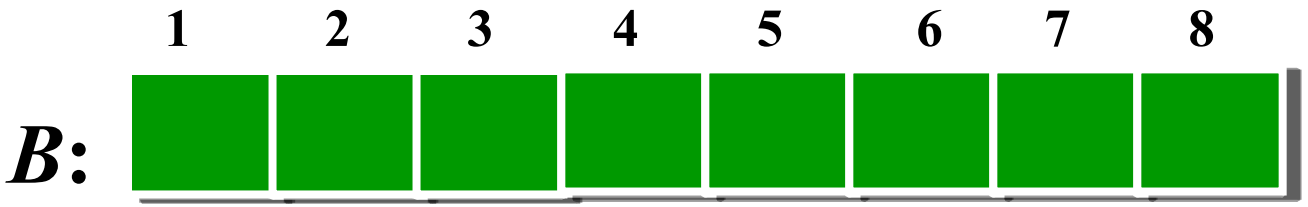
Executing Loop 3

```
7.  for i=1 to k
8.      C[i] = C[i] + C[i-1];
```









Executing Loop 3

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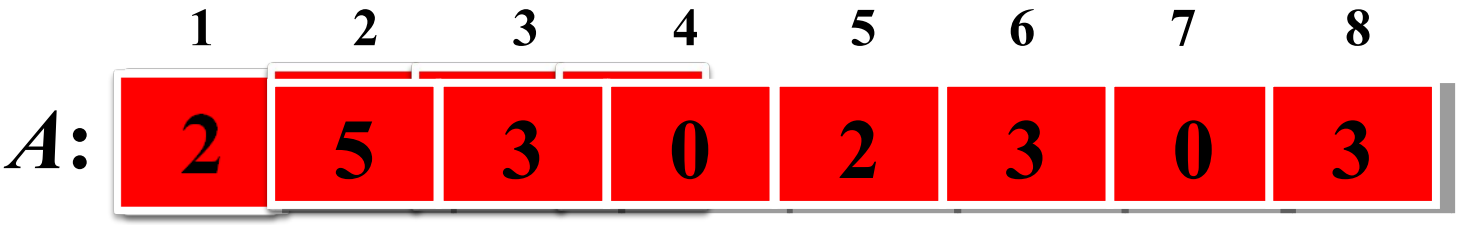

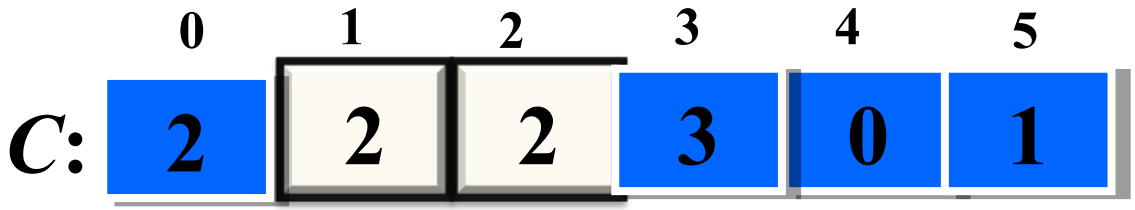

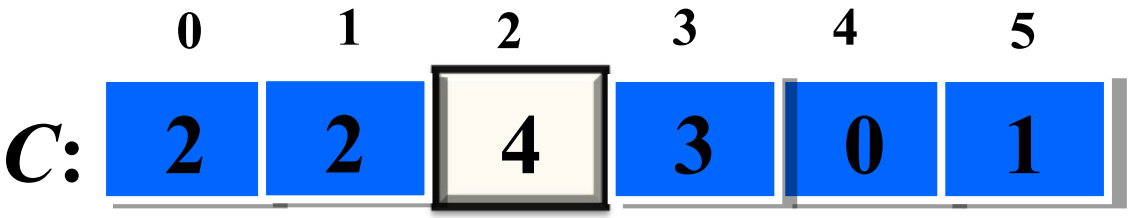

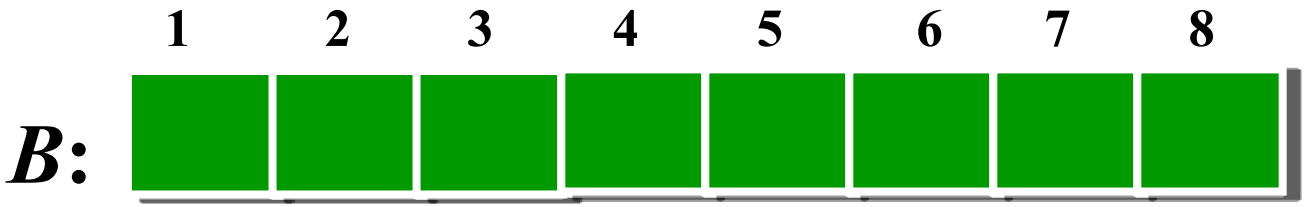

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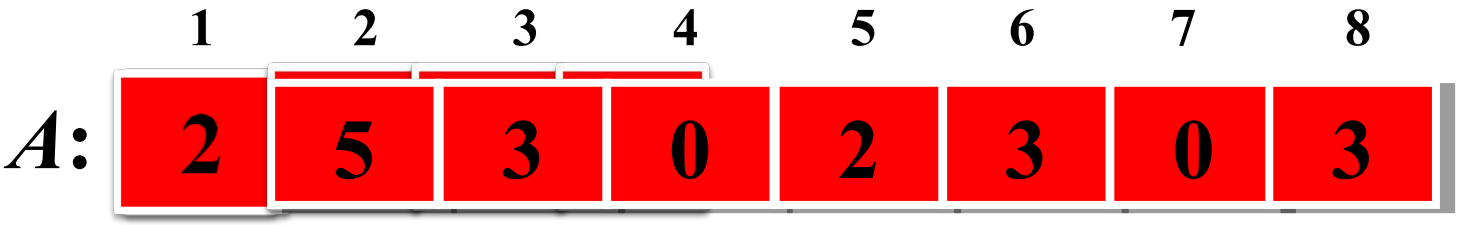

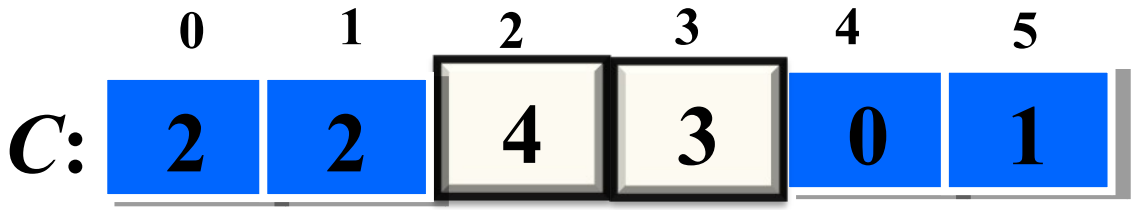

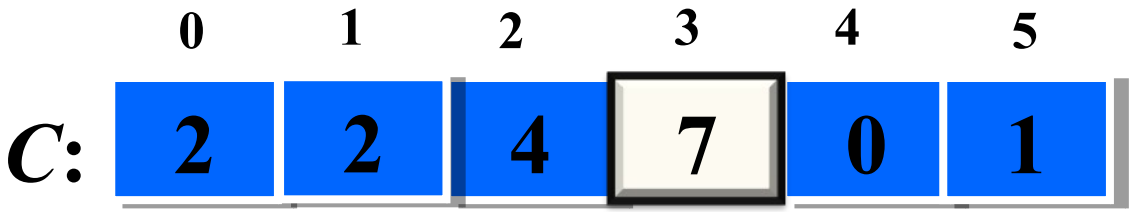

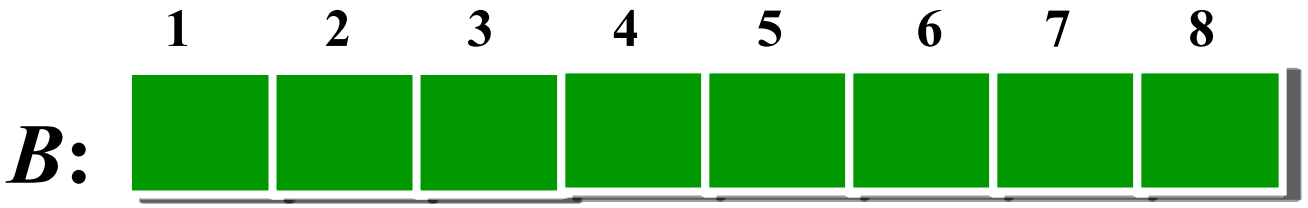

Executing Loop 3

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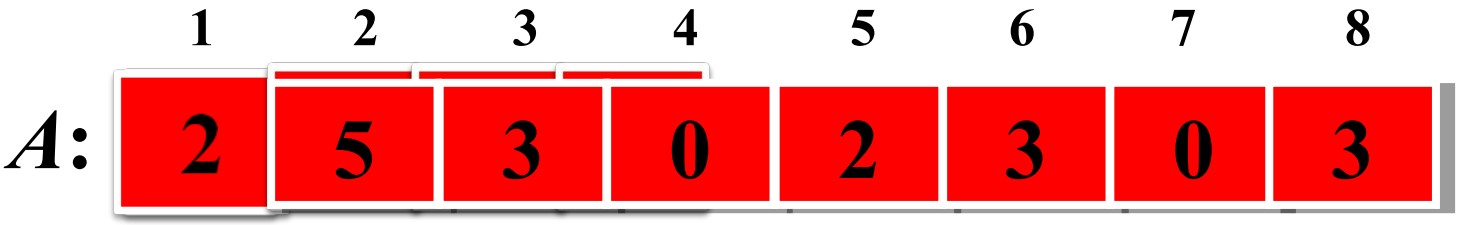

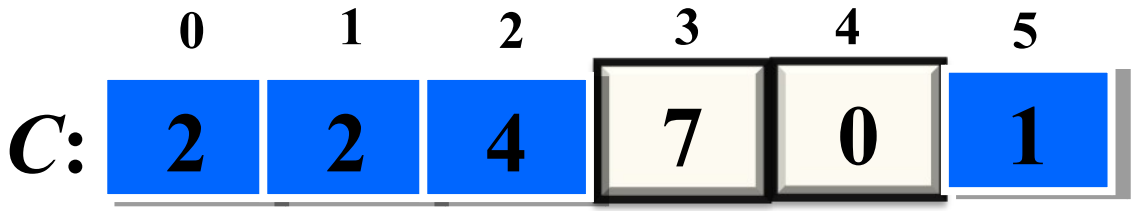

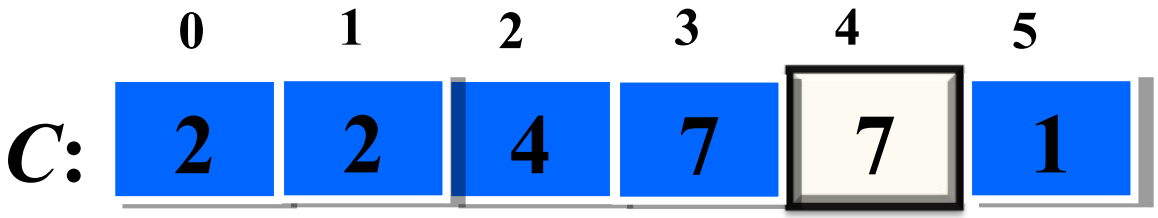

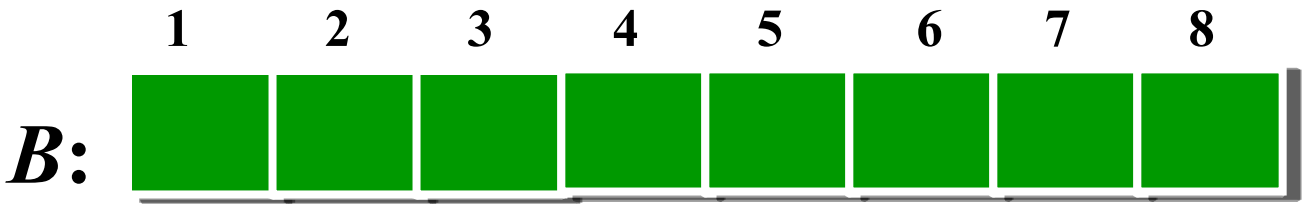

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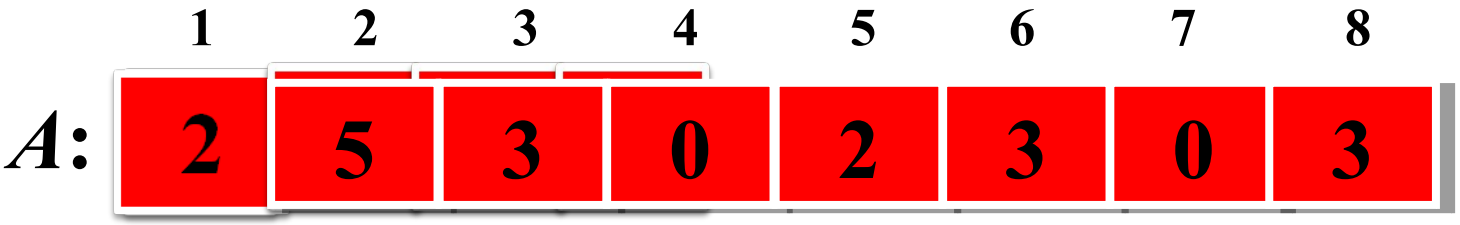
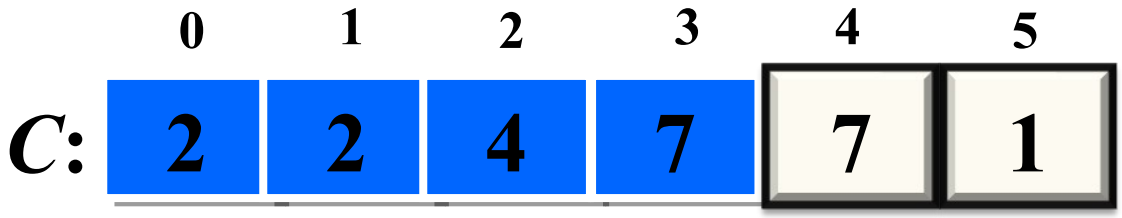
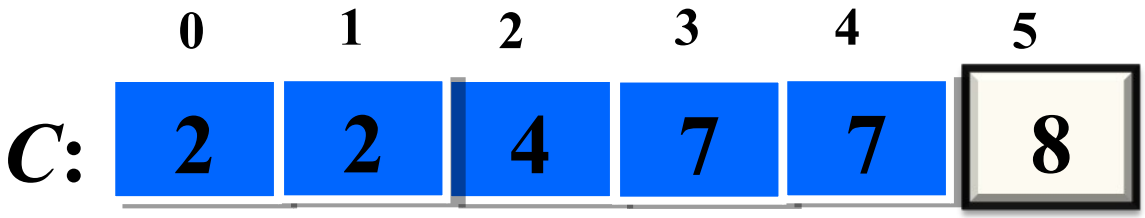
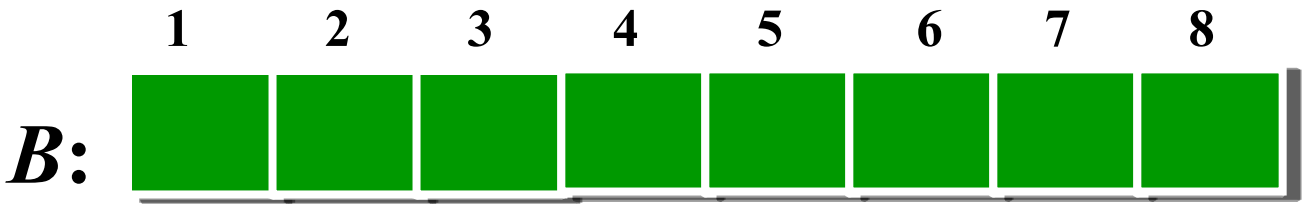
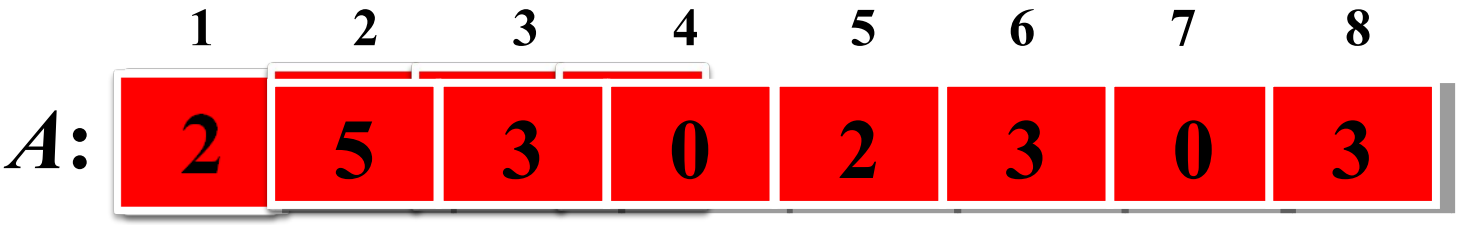


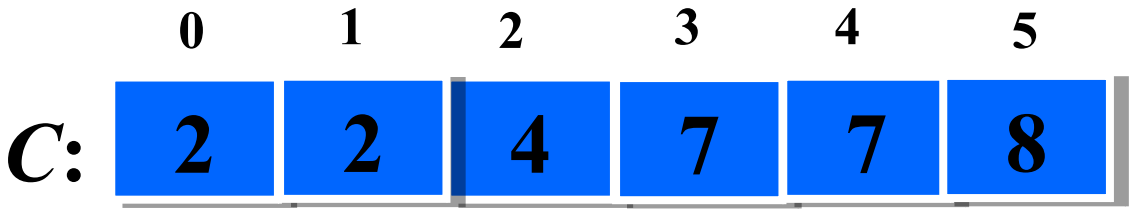


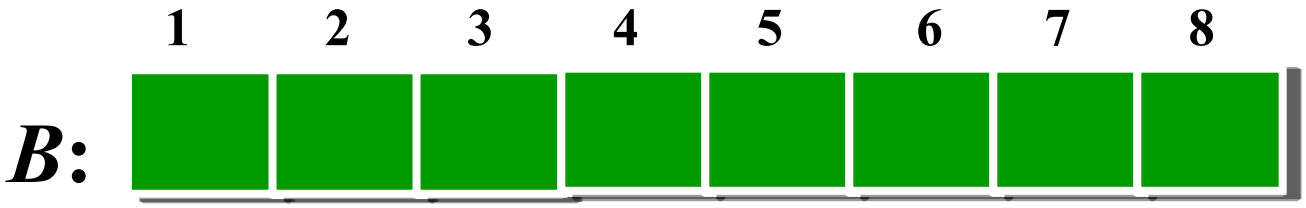




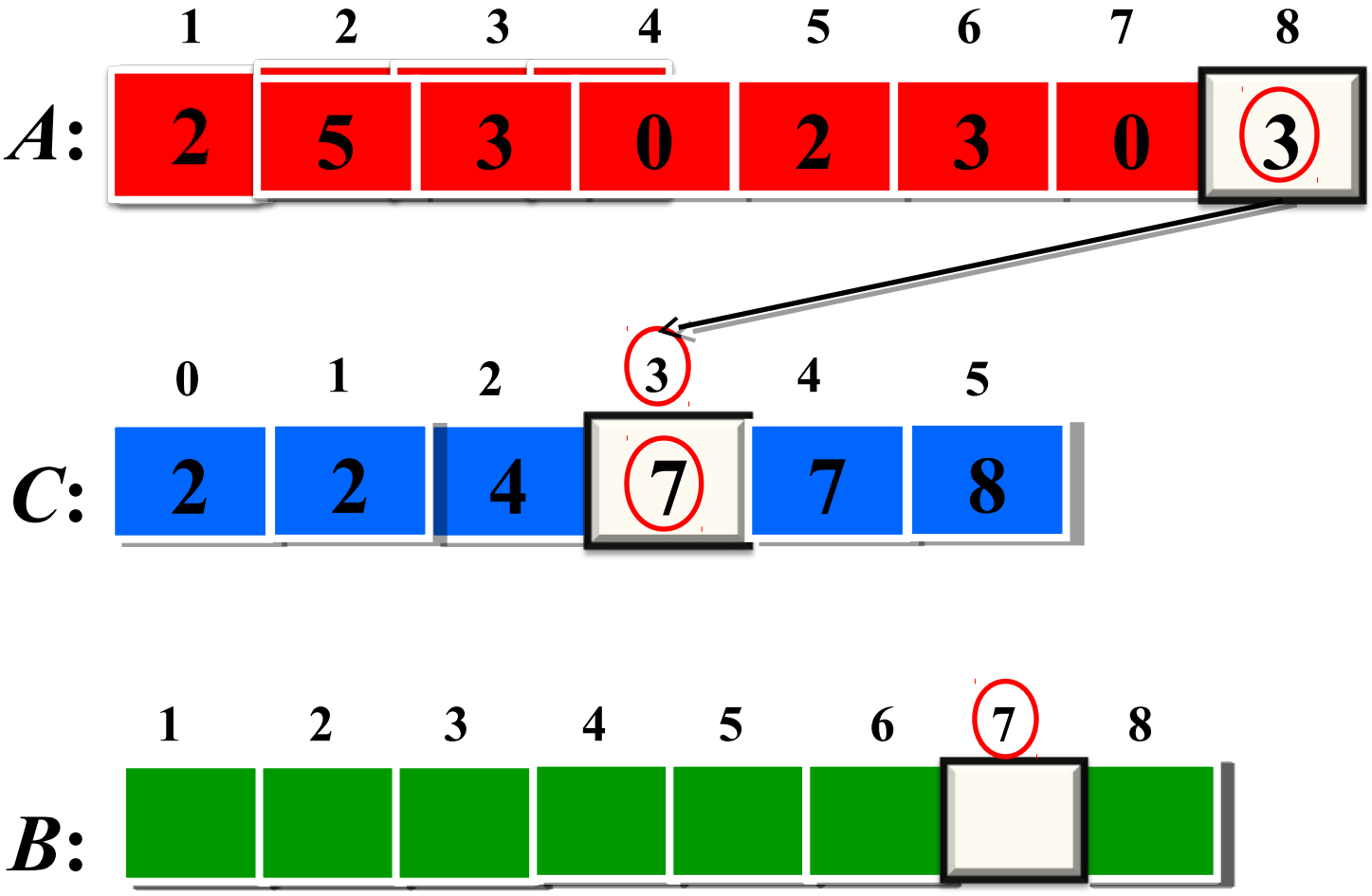
End of Loop 3





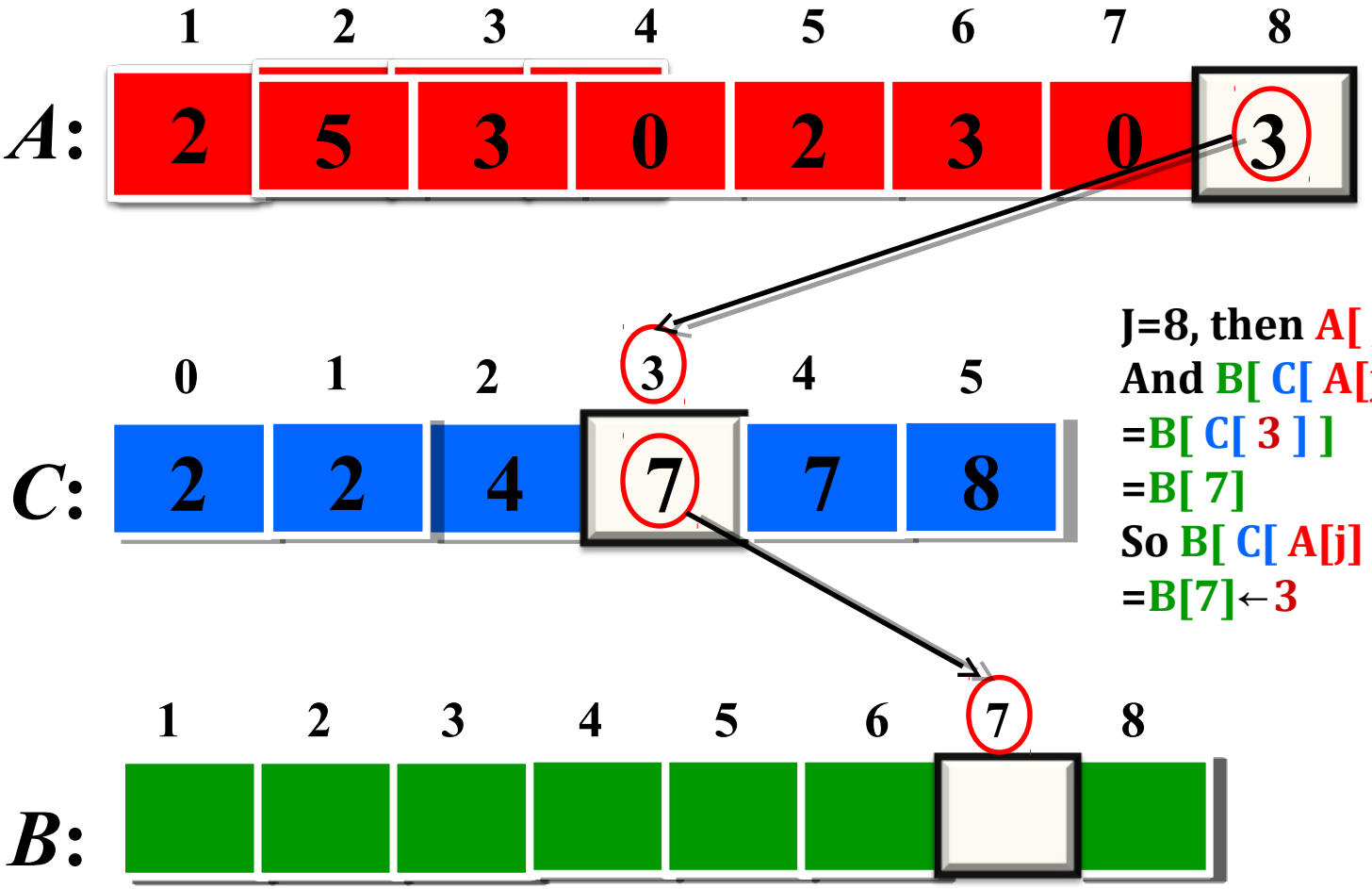


Executing Loop 4



Executing Loop 4

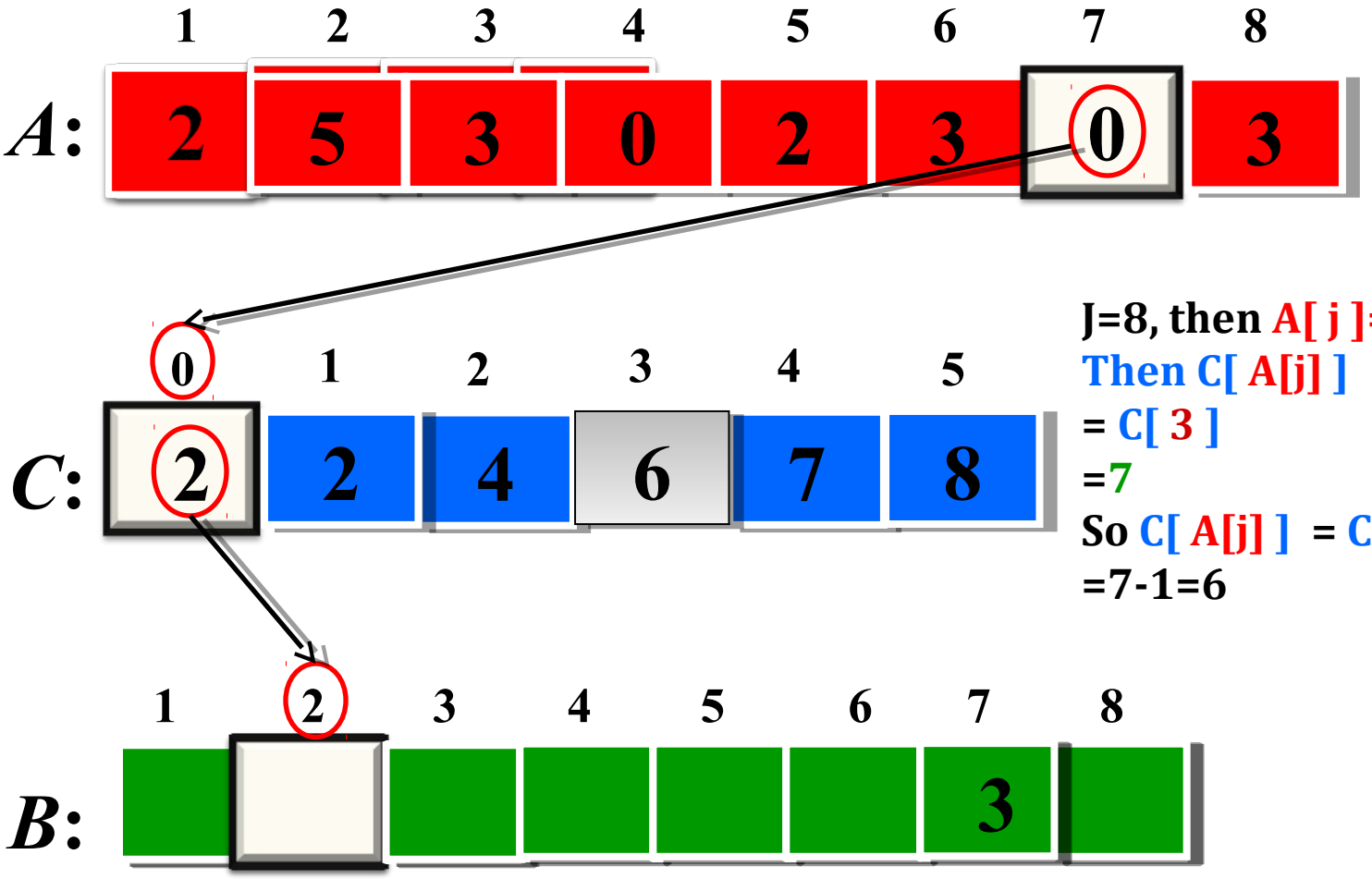
```
9.  for j=n or A.length down to 1
10.     B[ C[ A[j] ] ] = A[j];
11.     C[ A[j] ] = C[ A[j] ] - 1;
```



J=8, then A[j]=A[8]=3
And B[C[A[j]]]
=B[C[3]]
=B[7]
So B[C[A[j]]] ←A[j]
=B[7]←3

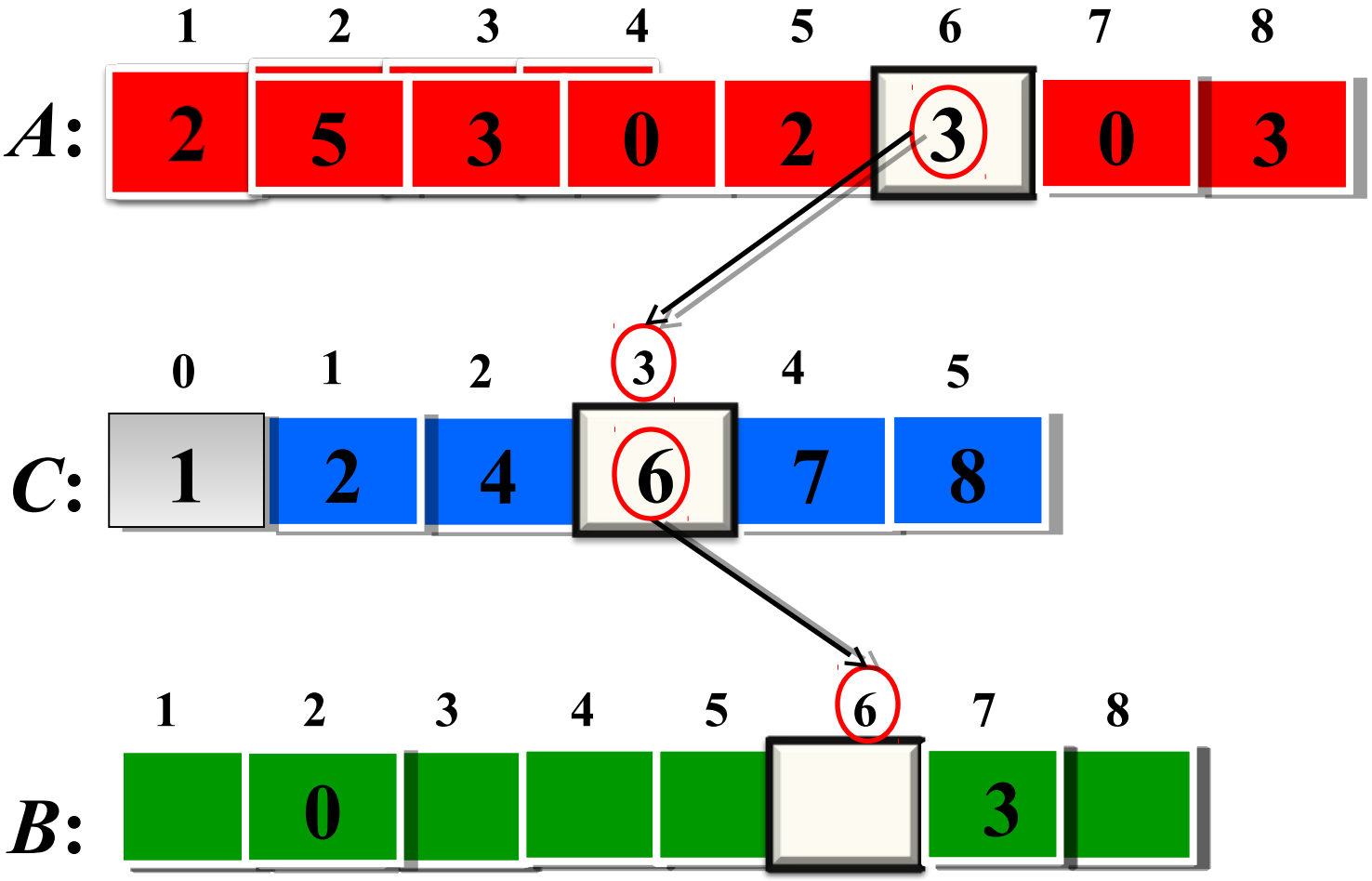
Executing Loop 4

```
9.  for j=n or A.length down to 1
10.     B[ C[ A[j] ] ] = A[j];
11.     C[ A[j] ] = C[ A[j] ] - 1;
```

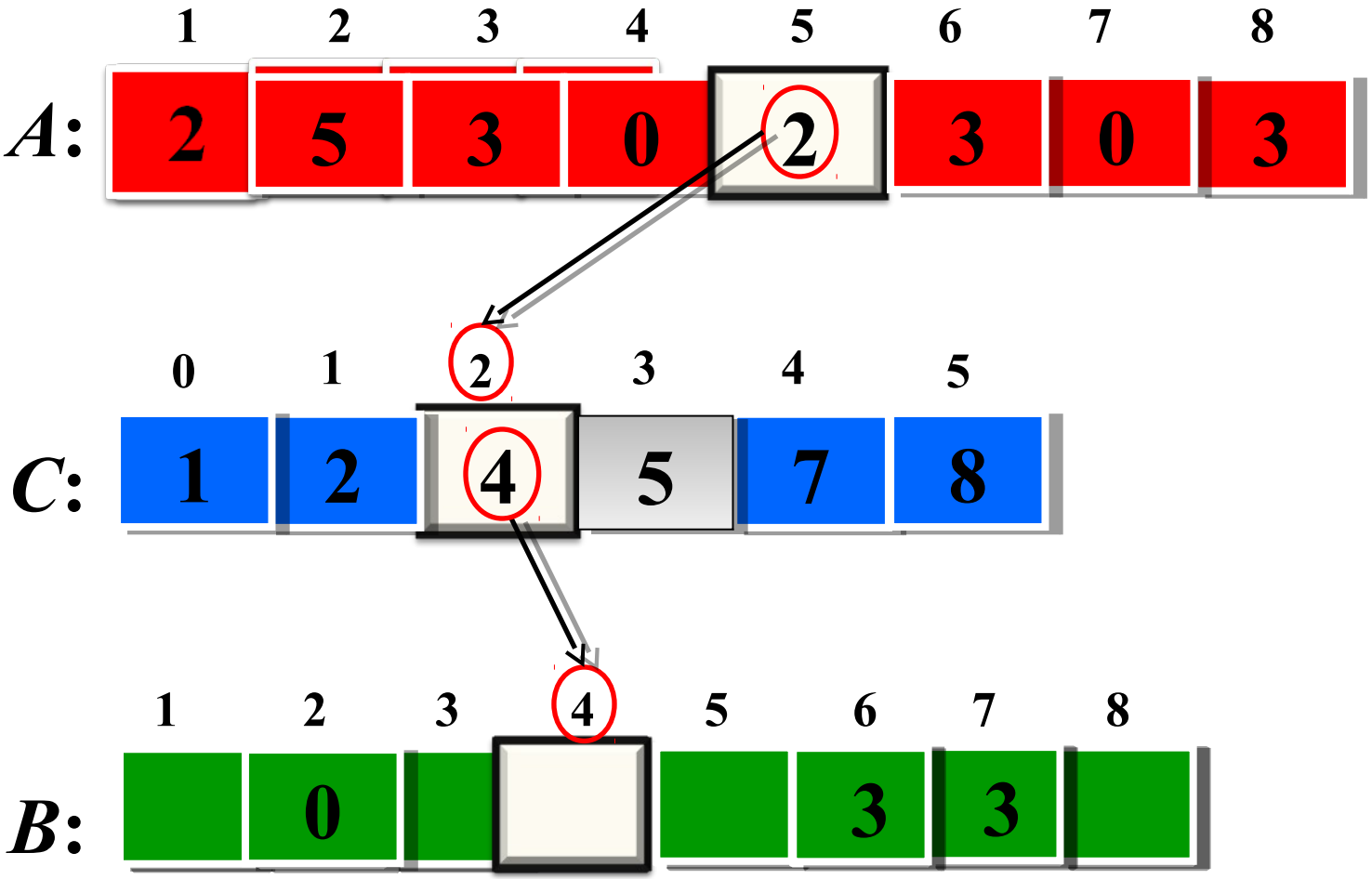


J=8, then A[j]=A[8]=3
Then C[A[j]]
= C[3]
=7
So C[A[j]] = C[A[j]] -1
=7-1=6

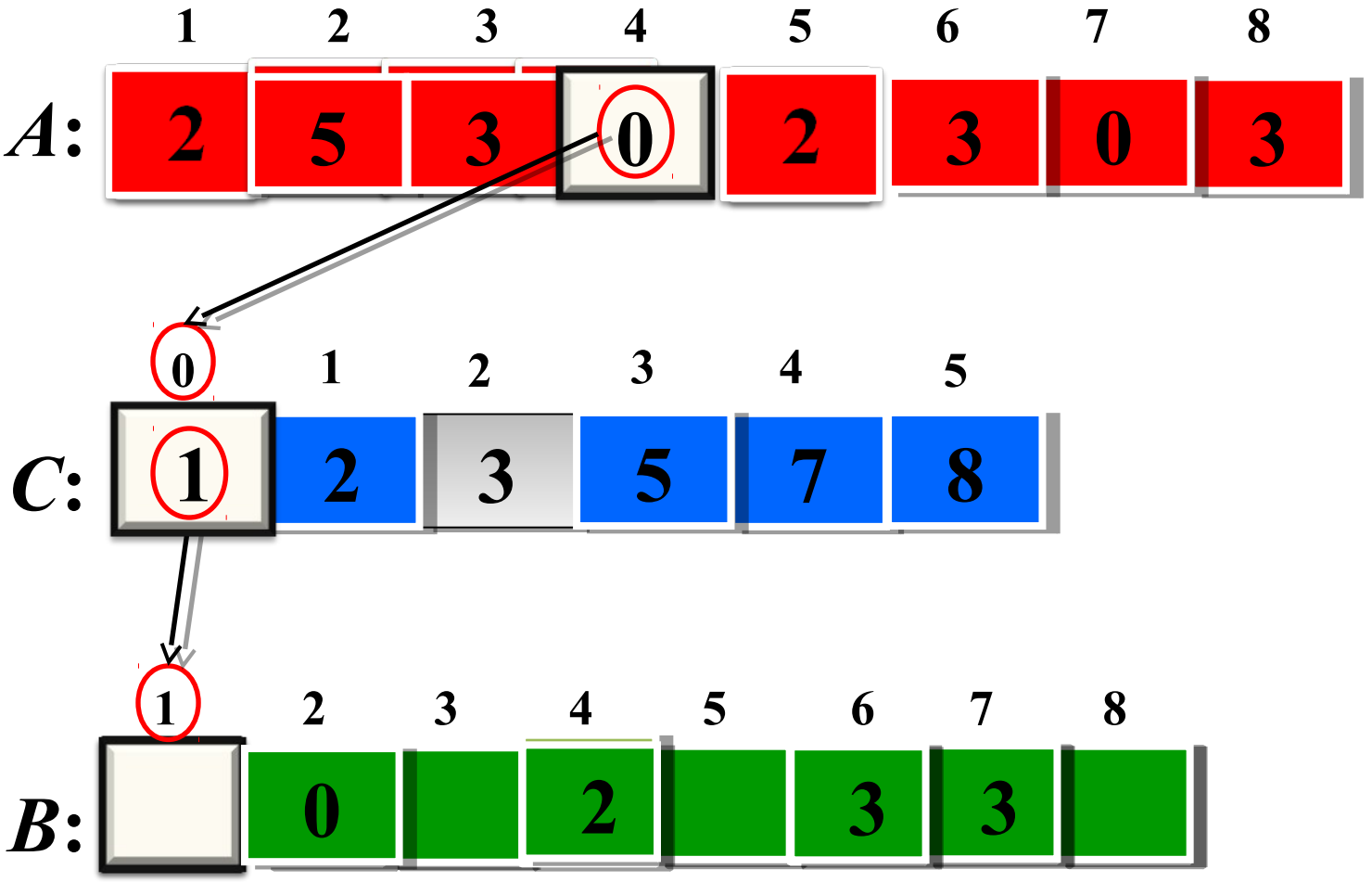
Executing Loop 4



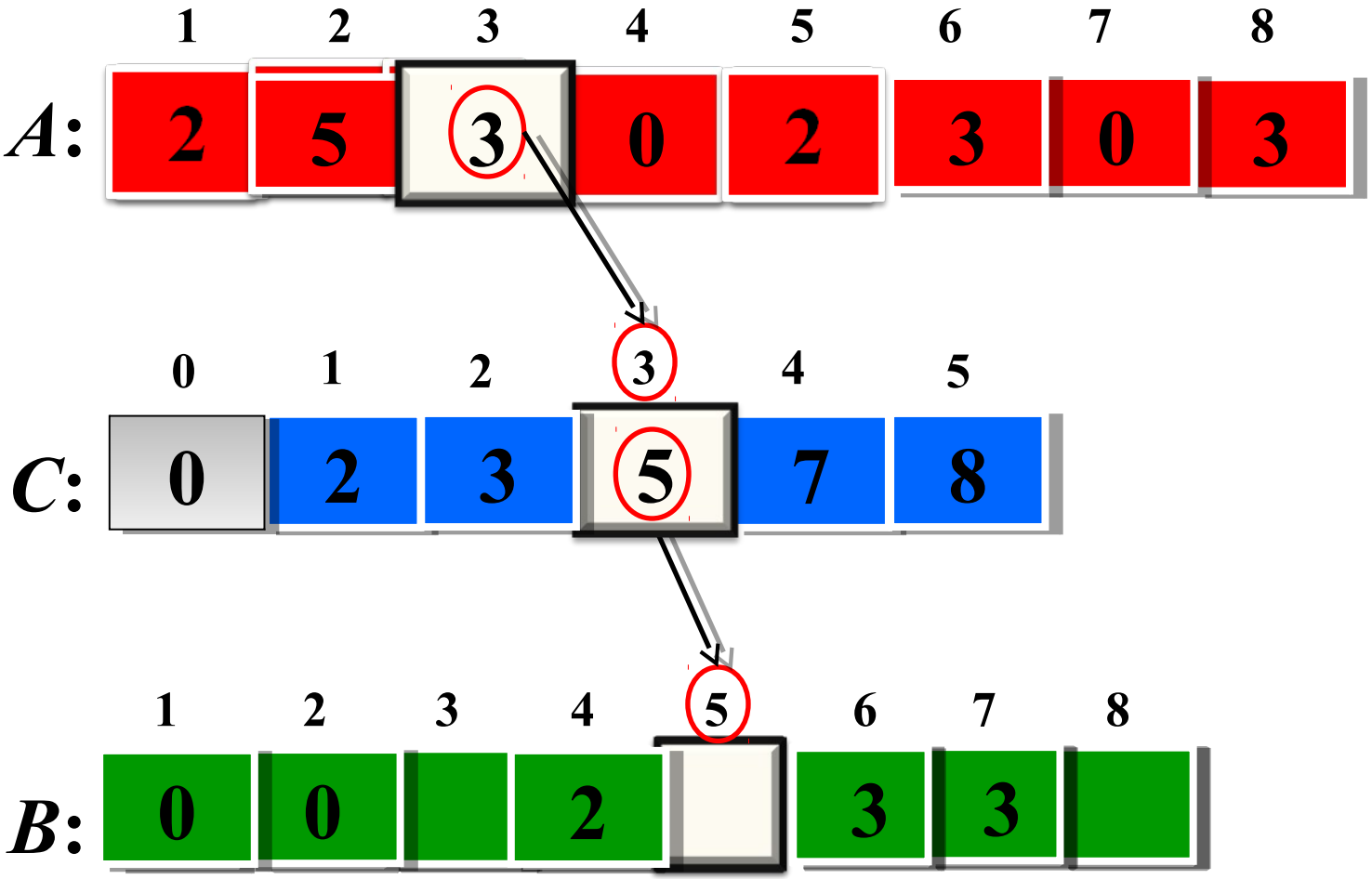
Executing Loop 4



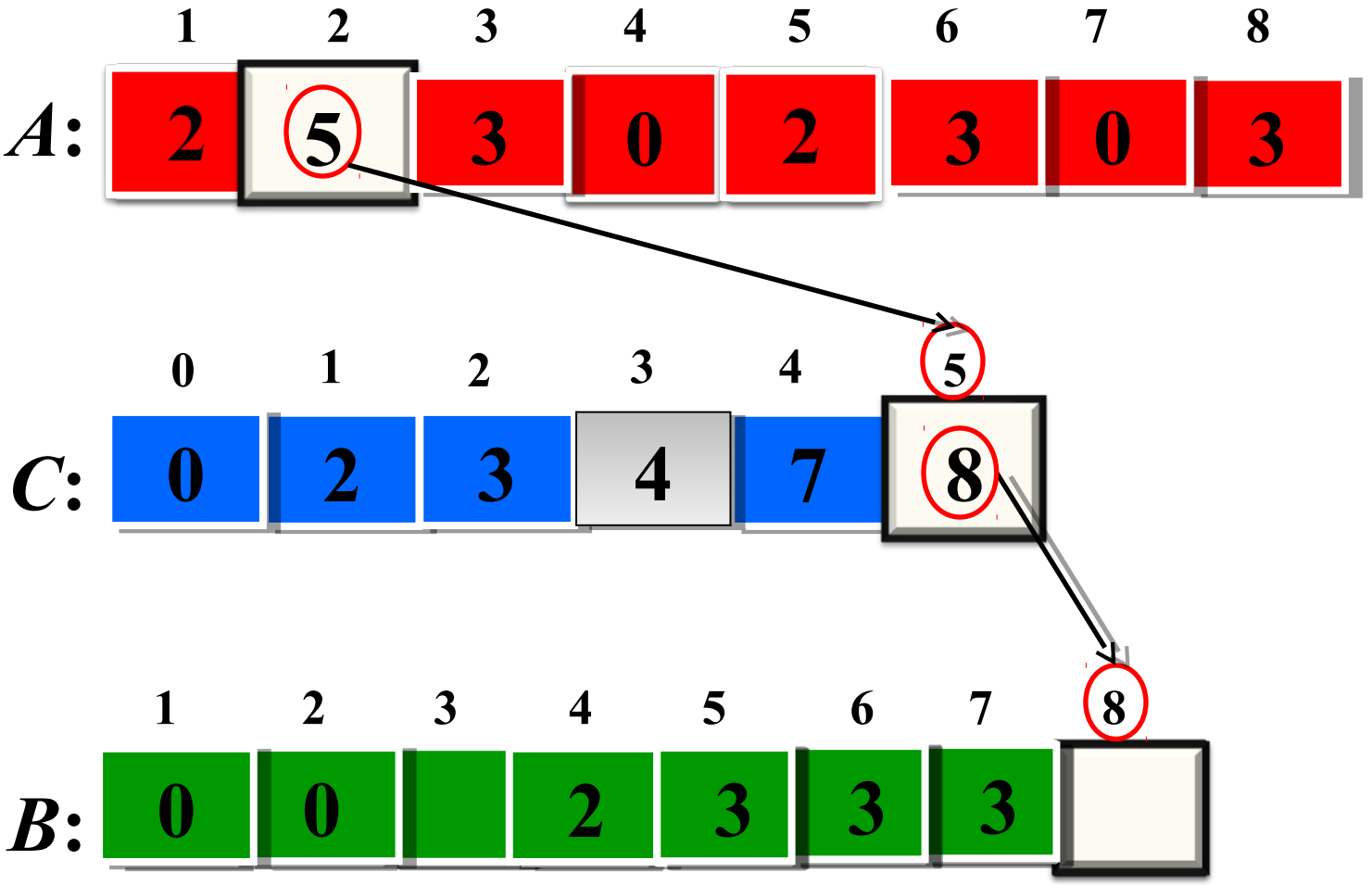
Executing Loop 4



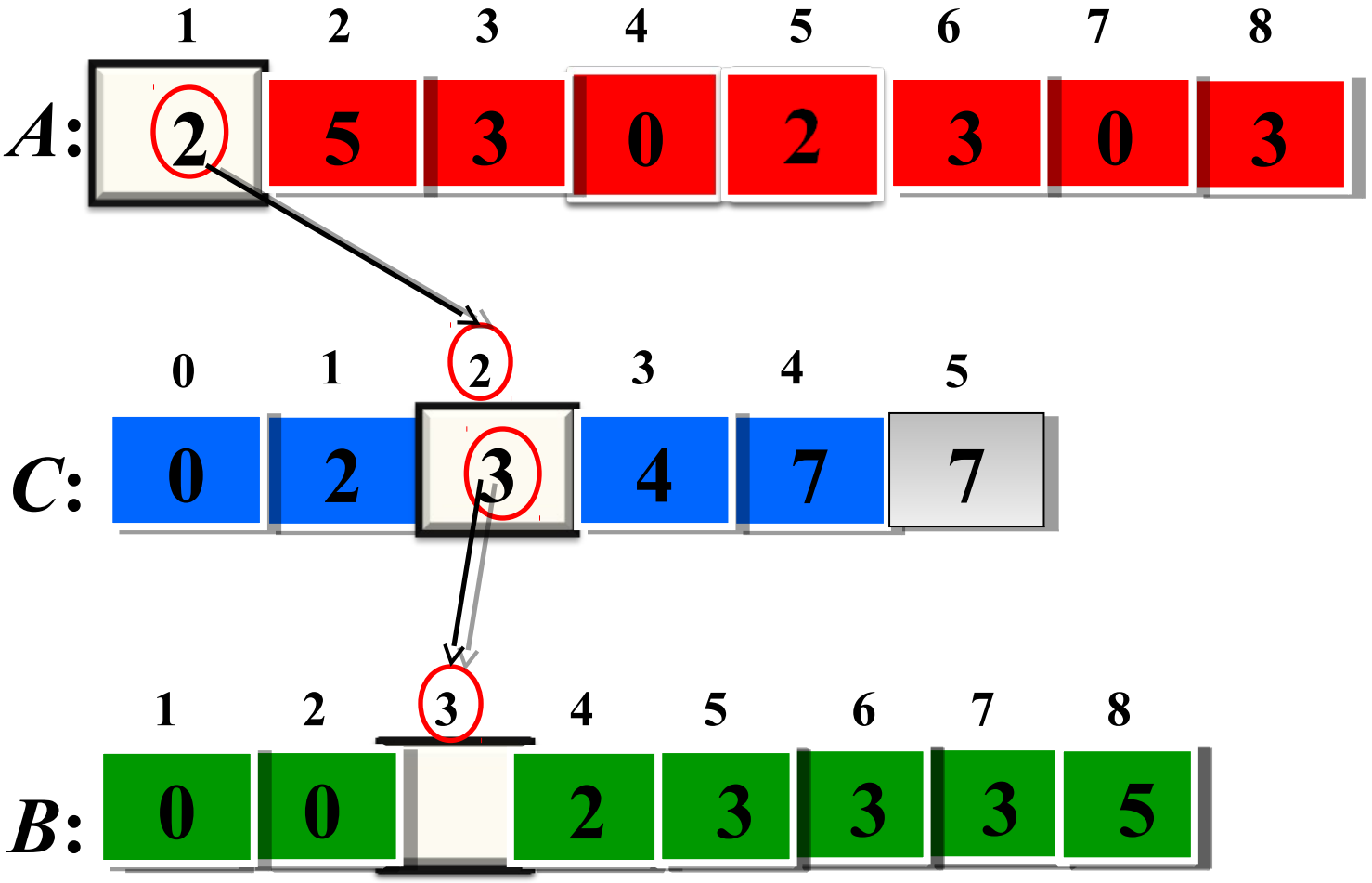
Executing Loop 4



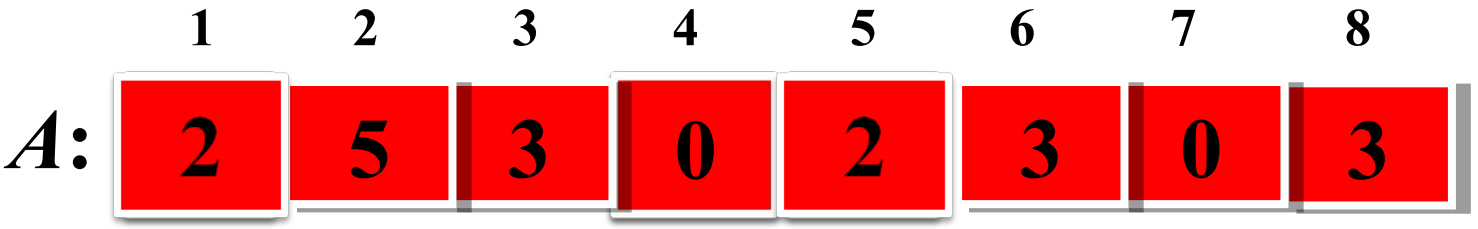
Executing Loop 4

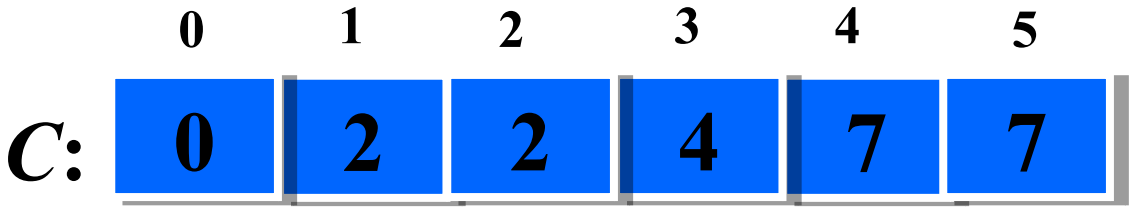


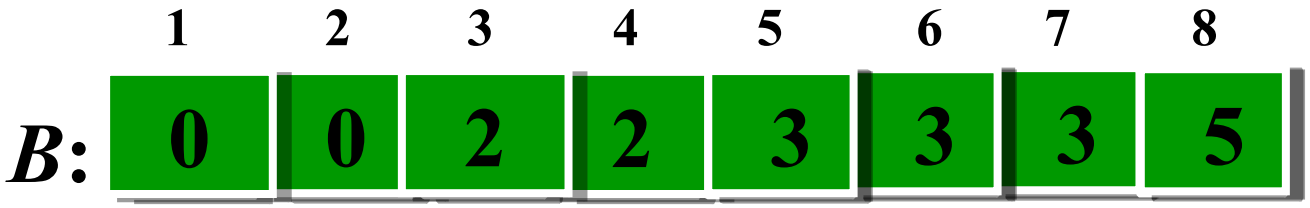
Executing Loop 4



End of Loop 4



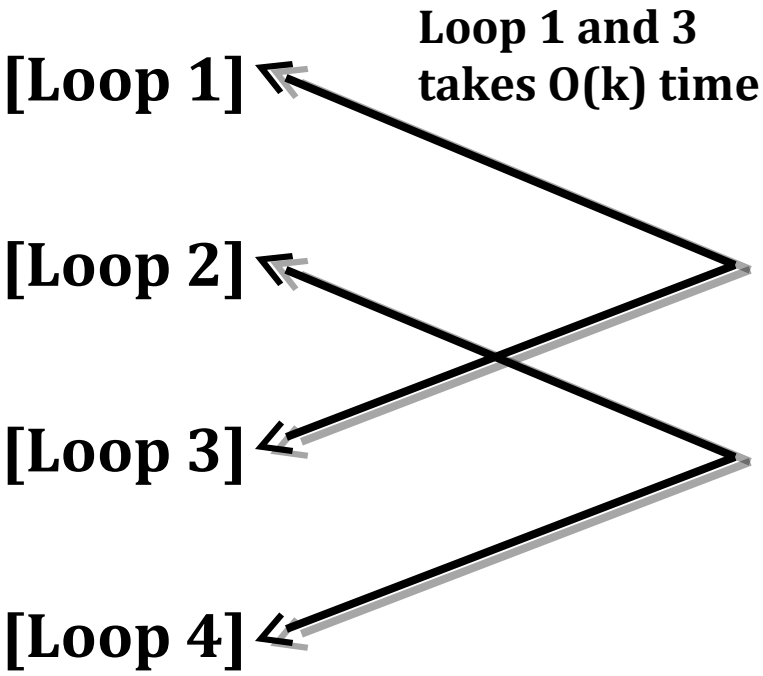




Sorted data in Array B

Time Complexity Analysis

```
1.  Counting-Sort(A, B, k)
2.  Let C[0…..k] be a new array
3.  for i=0 to k                                   [Loop 1]
4.        C[i]= 0;
5.  for j=1 to A.length  or n                      [Loop 2]
6.        C[ A[j] ] = C[ A[j] ]  + 1;
7.  for i=1 to k                                   [Loop 3]
8.        C[i] = C[i] + C[i-1];
9.  for j=n or A.length down to 1                  [Loop 4]
10.       B[  C[ A[j] ]  ] = A[j];
11.       C[ A[j] ] = C[ A[j] ]  - 1;
```



Time Complexity Analysis

- So the counting sort takes a total time of: $O(n + k)$
- Counting sort is called stable sort.
  - A sorting algorithm is ***stable*** when numbers with the same values appear in the output array in the same order as they do in the input array.

Counting Sort Review

- **Assumption:** input taken from **small** set of **numbers** of size k
- Basic idea:
  - Count number of elements less than you for each element.
  - This gives the position of that number – similar to selection sort.
- Pro's:
  - Fast
  - Asymptotically fast - $O(n+k)$
  - Simple to code
- Con's:
  - Doesn't sort in place.
  - Requires $O(n+k)$ extra storage.