Log Worksheet

1. If $\log_{100} x = y$, express $\log_{10} x^3$ in terms of y?

$$\log_{100} x = \frac{1}{۲} \log_{10} x = y \Rightarrow \log_{10} x = ۲y \Rightarrow \log_{10} x^{۳} = ۳ \log_{10} x = ۶y$$

2. Prove that $\log(n!) = O(n \log n)$.

$$\log(n!) = \log n + \log(n-1) + \dots + \log ۲ + \log 1 \leqslant n \log n$$

$c = 1$

$$\Rightarrow \log(n!) = O(n \log n)$$

3. Prove that $\log(n!) = \Omega(n \log n)$ (difficult).

$$n! \geqslant \left(\frac{n}{۲}\right)^{\frac{n}{۲}}$$

پایه : $n = 1 \rightarrow 1 \geqslant \left(\frac{1}{۲}\right)^{\frac{1}{۲}}$ ✓

فرض : $n = k \rightarrow k! \geqslant \left(\frac{k}{۲}\right)^{\frac{k}{۲}}$ ✓

حکم : $n = k+1 \rightarrow (k+1)! \geqslant \left(\frac{k+1}{۲}\right)^{\frac{k+1}{۲}}$

$$\times (k+1) \Rightarrow (k+1)! \geqslant \left(\frac{k}{۲}\right)^{\frac{k}{۲}} (k+1) \geqslant \left(\frac{k+1}{۲}\right)^{\frac{k+1}{۲}}$$ ✓

$$\Rightarrow \log(n!) \geqslant \log\left(\frac{n}{۲}\right)^{\frac{n}{۲}} = \frac{n}{۲} \log\left(\frac{n}{۲}\right) \Rightarrow \log n! = \Omega\left(\frac{n}{۲} \log \frac{n}{۲}\right)$$

$$\lim_{n \to \infty} \frac{n \log n}{\frac{n}{۲} \log \frac{n}{۲}} = ۲ \Rightarrow \frac{n}{۲} \log \frac{n}{۲} = \Omega(n \log n)$$

$$\Rightarrow \log n! = \Omega(n \log n)$$