

Linear Partition Worksheet

Define $M[n, k]$ to be the minimum possible cost over all partitions of (s_1, \dots, s_n) into k ranges.

1. Consider the input (100, 200, 300, 400, 500, 600, 700) with $k = 3$. What is $M[7, 3]$?
(Hint: It should be possible to answer this by visual inspection.)

$$100 \quad 200 \quad 300 \quad 400 \quad | \quad 500 \quad 600 \quad | \quad 700$$

$\min = 1100$

2. What are $M[1, 2]$, $M[2, 2]$, $M[3, 2]$, $M[4, 2]$, $M[5, 2]$, $M[6, 2]$, and $M[7, 2]$? (again, use visual inspection).

$$M[1, 2] \quad 100 \quad | \quad \rightarrow \quad 100$$

$$M[2, 2] \quad 100 \quad | \quad 200 \quad \rightarrow \quad 200$$

$$M[3, 2] \quad 100 \quad 200 \quad | \quad 300 \quad \rightarrow \quad 300$$

$$M[4, 2] \quad 100 \quad 200 \quad 300 \quad | \quad 400 \quad \rightarrow \quad 400$$

$$M[5, 2] \quad 100 \quad 200 \quad 300 \quad | \quad 400 \quad 500 \quad \rightarrow \quad 900$$

$$M[6, 2] \quad 100 \quad 200 \quad 300 \quad 400 \quad | \quad 500 \quad 600 \quad \rightarrow \quad 1100$$

$$M[7, 2] \rightarrow 100 \quad 200 \quad 300 \quad 400 \quad 500 \quad | \quad 600 \quad 700 \rightarrow 1500$$

3. Can you write a formula for $M[7, 3]$ in terms of $M[1, 2]$, $M[2, 2]$, $M[3, 2]$, $M[4, 2]$, $M[5, 2]$, $M[6, 2]$, and $M[7, 2]$?

$$M[1, 2]: 100, 200 \xrightarrow{\max} 200$$

$$M[2, 2]: 200, 300 \xrightarrow{\max} 300$$

$$M[3, 2]: 300, 400 \xrightarrow{\max} 400$$

$$M[4, 2]: 400, 500 \xrightarrow{\max} 500$$

$$M[5, 2]: 500, 600 \xrightarrow{\max} 600$$

$$M[6, 2]: 600, 700 \xrightarrow{\max} 700$$

$$M[7, 2]: 700, 0 \xrightarrow{\max} 700$$

min

1100