Chapter 3 (Data Structures)

1. Role of Data Structures
2. Review: Contiguous vs. Linked Data Structures
3. Review: Stacks & Queues
4. Review: Dictionaries
5. Review: Hash Tables
6. Binary Search Trees
7. Balanced Binary Search Trees

Thoughts for the Day

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"Changing a data structure in a slow program can work the same way an organ transplant does in a sick patient"

Elementary Data Structures

Elementary data structures such as stacks, queues, lists, and heaps are the "off-the-shelf" components we build our algorithm from.

Abstraction v/s Implementation

There are two aspects to any data structure:

- The abstract operations which it supports.
- The implementation of these operations.

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- That we can describe the behavior of our data structures in terms of abstract operations is why we can use them without thinking.
- That there are different implementations of the same abstract operations enables us to optimize performance.

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- Contiguously-allocated structures are composed of single slabs of memory, and include arrays, matrices, heaps, and hash tables.
- Linked data structures are composed of multiple distinct chunks of memory bound together by *pointers*, and include lists, trees, and graph adjacency lists.

Arrays

An array is a structure of fixed-size data records such that each element can be efficiently located by its index or (equivalently) address.

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- Physical continuity (memory locality) between successive data accesses helps exploit the high-speed cache memory on modern computer architectures.

Dynamic Arrays

- Unfortunately we cannot adjust the size of simple arrays in the middle of a program's execution.
- Compensating by allocating extremely large arrays can waste a lot of space.
- With *dynamic arrays* we start with an array of size 1, and double its size each time we run out of space.
- How many times will we double for n elements?

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Dynamic Array Worksheet

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If half the elements move once, a quarter of the elements twice, and so on, the total number of movements M is given by .....

Analysis

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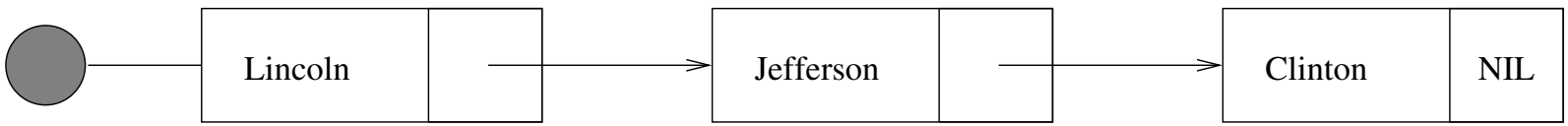
Thus each of the n elements move an average of only twice, and the total work of managing the dynamic array is $O(n)$ (same as a simple array)!

Pointers and Linked Structures

- Pointers represent the address of a location in memory.
- A cell-phone number can be thought of as a pointer to its owner as they move about the planet.
- In C/C++, `*p` denotes the item pointed to by `p`, and `&x` denotes the address (i.e. pointer) of a particular variable `x`.
- A special `NULL` pointer value is used to denote structure-terminating or unassigned pointers.

Linked List Structures

```
typedef struct list {
        item_type item;
        struct list *next;
} list;
```



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Why $O(n)$ and $\Theta(1)$ above??

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Dynamic memory allocation provides us with flexibility on how and where we use our limited storage resources.

Comic Break (pointers)

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Lines in banks are based on queues, while a pile of dishes is a stack.

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Above are primary operations.

Auxiliary Operations (need ordered sets)

- *Min(S), Max(S)* – Returns the element of the totally ordered set S which has the smallest (largest) key.
- *Successor(S,x), Predecessor(S,x)* – Given an element x whose key is from a totally ordered set S, returns the next largest (smallest) element in S, or NIL if x is the maximum (minimum) element.

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  aka Next/Previous

Dictionary Implementations

1. Unsorted Array
2. Sorted Array
3. Singly-Linked Unsorted List
4. Doubly-Linked Unsorted List
5. Singly-Linked Sorted List
6. Doubly-Linked Sorted List
7. Binary Search Tree
8. Balanced Binary Search Tree
9. Hash Tables

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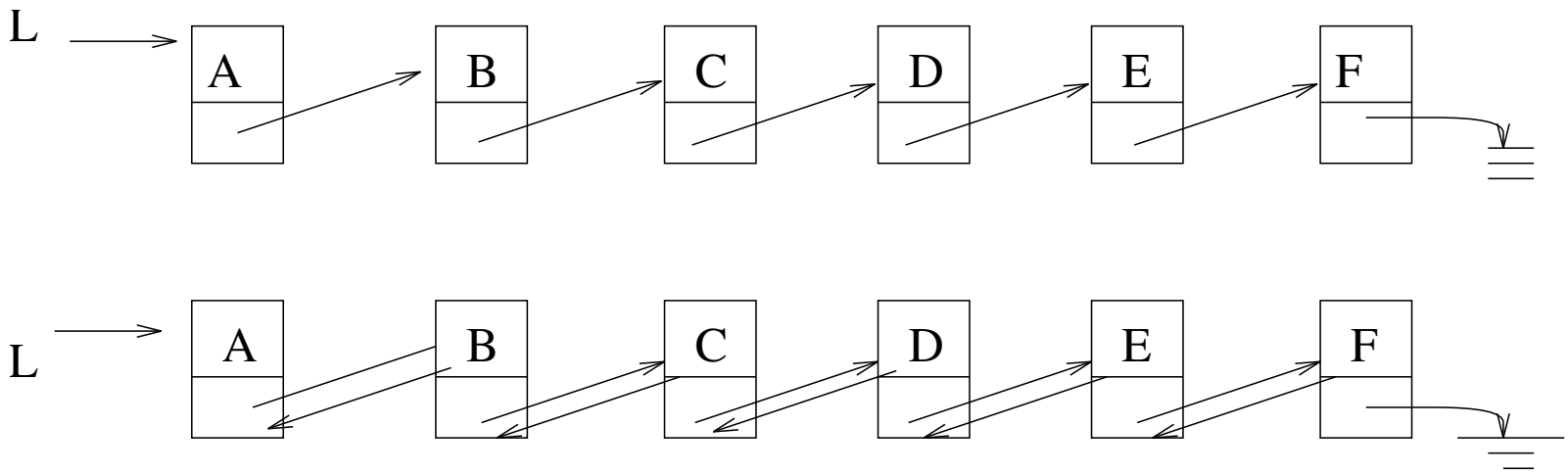
- Successor(S,x), Predecessor(S,x) - Add or subtract 1 from pointer, $\Theta(1)$

Pointer Based Implementation

We can maintain a dictionary in either a singly or doubly linked list.



Doubly Linked Lists

- We gain extra flexibility on predecessor queries at a cost of doubling the number of pointers by using doubly-linked lists.
- The extra big-Oh costs of doubly-linked lists is zero.

Linked List Complexities

Op	Singly unsorted	Double unsorted	Singly sorted	Doubly sorted
Search	$O(n)$	$O(n)$	$O(n)$	$O(n)$
Insert	$\Theta(1)$	$\Theta(1)$	$O(n)$	$O(n)$
Delete	$O(n)$*	$\Theta(1)$	$O(n)$*	$\Theta(1)$
Succ/Pred	$O(n)$	$O(n)$	$\Theta(1)/O(n)$	$\Theta(1)$
Min/Max	$O(n)$	$O(n)$	$\Theta(1)$	$\Theta(1)$

Array Complexities (from earlier)

Op	Unsorted	Sorted
Search	$O(n)$	$O(\log n)$
Insert	$\Theta(1)$	$O(n)$
Delete	$\Theta(1)$	$O(n)$
Succ/Pred	$O(n)$	$\Theta(1)$
Min/Max	$O(n)$	$\Theta(1)$

DS Operation Expectations: Discussion

What complexity is considered reasonable for a data structure operation and why?

- $\Theta(1)$
- $O(n)$
- $O(\log n)$

Binary Search Trees

Binary search trees support all dictionary ops.

Op	Binary Search tree
Search	$O(h)$
Insert	$O(h)$
Delete	$O(h)$
Succ/Pred	$O(h)$
Min/Max	$O(h)$

h denotes the height of the tree and is $\Theta(\log n)$ on the average and $\Theta(n)$ in the worst case.

Representation of BST

- Uses pointers.
- Node contains
  - left & right (child) ptrs.
  - key field.
  - parent ptr.
- The parent link is optional, since we can store the ptrs on a stack on the way down.

Definition of BST

- Rooted[2] binary tree.
- Let x be any node in the BST.
- If y is a node in x's left subtree, then $y \rightarrow key \leq x \rightarrow key$.
- If y is a node in x's right subtree, then $y \rightarrow key \geq x \rightarrow key$.

[2] what's an unrooted tree?

Inorder Traversal of a BST

An inorder traversal of a BST prints all the keys in sorted order.

INORDER(x)

if ($x \neq 0$)

INORDER($x \rightarrow left$);

cout $<<$ $x \rightarrow key$;

INORDER($x \rightarrow right$);

Searching a BST

SEARCH(x,k)

if $(x == 0$ **or** $k == x \rightarrow key)$
then return x;
if $(k < x \rightarrow key)$
then return SEARCH$(x \rightarrow left, k)$;
else return SEARCH$(x \rightarrow right, k)$;

Searching in a BST: how much?

The algorithm works because both the left and right subtrees of a binary search tree *are* binary search trees – recursive structure, recursive algorithm. This takes time proportional to the height of the tree, $O(h)$.

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- How do you get to an ancestor of a node?
- $O(h)$ complexity.

Insertion

- Do a binary tree search to find where it should be, then replace the termination NIL ptr with the new item.
- Need to maintain a trailing pointer in the code.
- Insertion takes time proportional to the height of the tree, $O(h)$.

Insertion Code

INSERT(T,z)

$y = 0;$
$x = T.root;$
while $(x! = 0)$ **do**
$y = x$
if $(z \rightarrow key < x \rightarrow key)$
then $x = x \rightarrow left$
else $x = x \rightarrow right$

Insertion Code continued

$z \rightarrow p = y$
if $(y == 0)$ $T.root = z;$
elseif $(z \rightarrow key < y \rightarrow key)$
then $y \rightarrow left = z;$
else $y \rightarrow right = z;$

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    3. z has two children. (Bit harder)
        1. Let y be z's successor.
        2. Replace z's contents with y' contents.
        3. Splice y out (y has at most one child. Why?).

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The best height we could hope to get is $\lg n$, if the tree was perfectly balanced, since

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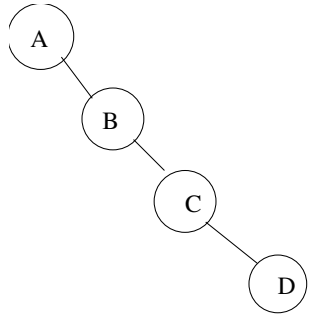
But if we get unlucky with our order of insertion or deletion, we could get linear height!

Worst Case Height

insert(A); insert(B)
insert(C); insert(D)
$\vdots$

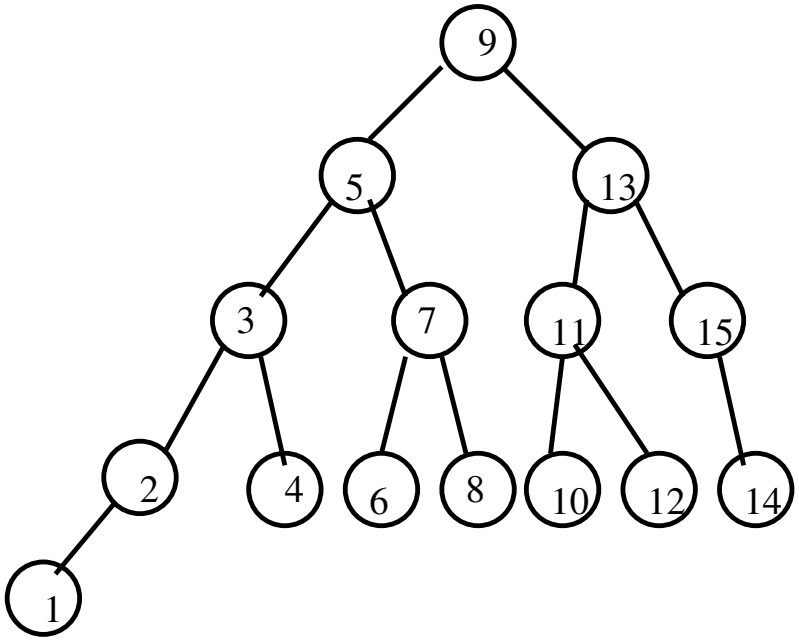


Tree Insertion Analysis

In fact, binary search trees constructed with random insertion orders *on average* have $\Theta(\lg n)$ height.
The worst case is linear, however.

Perfectly Balanced Trees

Perfectly balanced trees are high-maintenance: if we insert the key 1, we must move every single node in the tree to rebalance it, taking $\Theta(n)$ time.



Balanced Search Trees

Therefore, when we talk about "balanced" trees, we mean trees whose height is $O(\log n)$, so all dictionary operations (insert, delete, search, min/max, successor/predecessor) take $O(\lg n)$ time in the worst case.

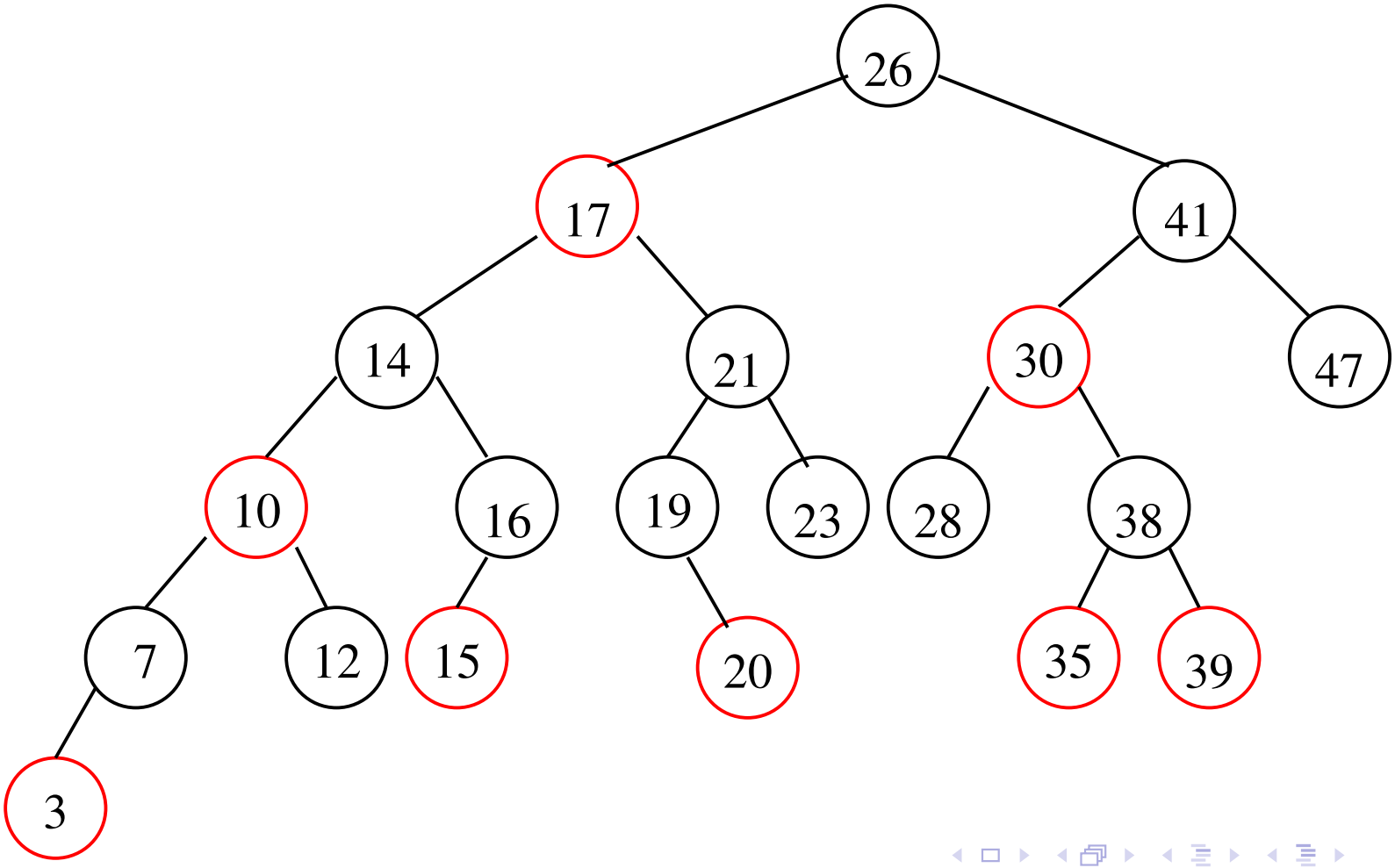
Red-Black Trees not in book

A bst is a red-black tree if it satisfies the following properties.
(1) Every node is either red or black.
(2) If a node is red, then both of its children are black.
(3) Every path from a node to a leaf contains the same number of black nodes.
(4) The root is always black.

Theorem

An RB tree with n nodes has $h \leq 2\log_2(n+1)$.

Example



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Rotation Worksheet

- Goal: understand rotation template.

Demo

- Goal: to see how red black trees self-adjust during insertion.

Skip Lists, Splay Trees

- Paris Kanellakis award
- Data Structures Handbook (Page 15, 173 (dictionary structures), 195 (balanced search trees), 235 (splay trees), 305 (B Trees)).

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Hash Functions

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Ideal hash function:

1. Is cheap to evaluate
2. Element is equally likely to hash into any of the m slots.
3. independently of other elements.

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One way is by $h(k) = k \mod m$

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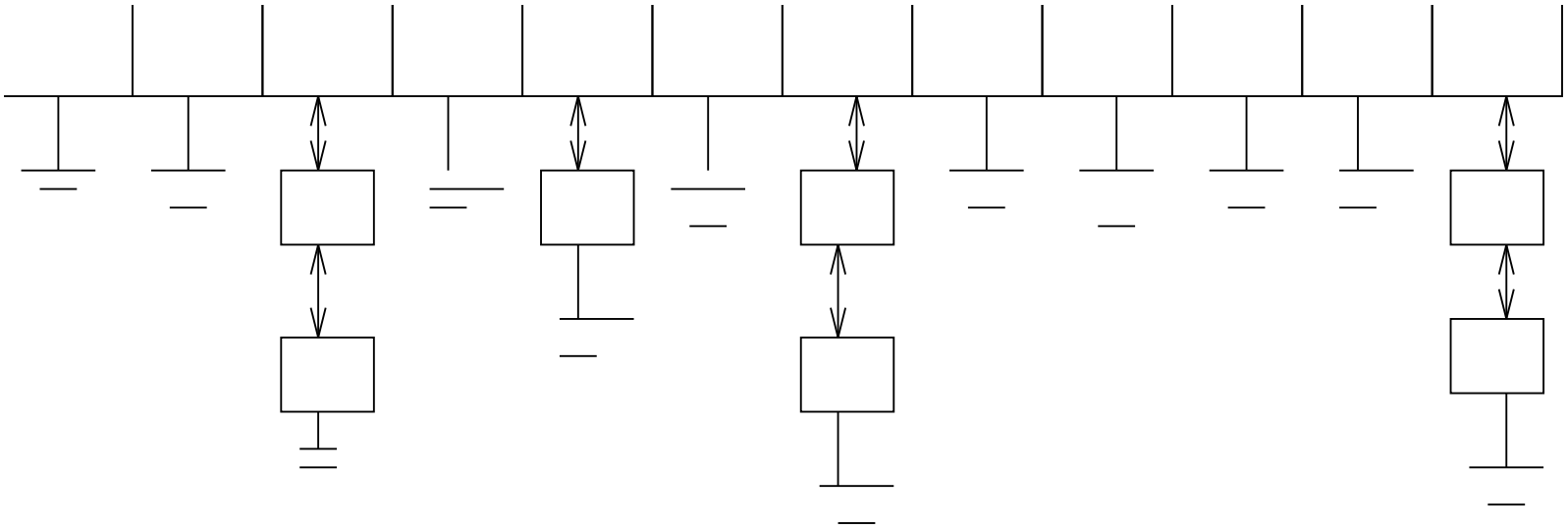
When $m = 365$, this drops below $1/2$ when $n = 23$ and sinks to almost 0 when $n \geq 50$.

Birthday Paradox Example

Collision Resolution by Chaining

The easiest approach is to let each element in the hash table be a pointer to a list of keys.



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- Chaining is easy, but devotes a considerable amount of memory to pointers, which could instead be used to make the table larger.

Open Addressing

We can dispense with pointers by using an implicit reference derived from a simple function:

1	2	3	4	5	6	7	8	9	10	11
		X		X	X		X	X		

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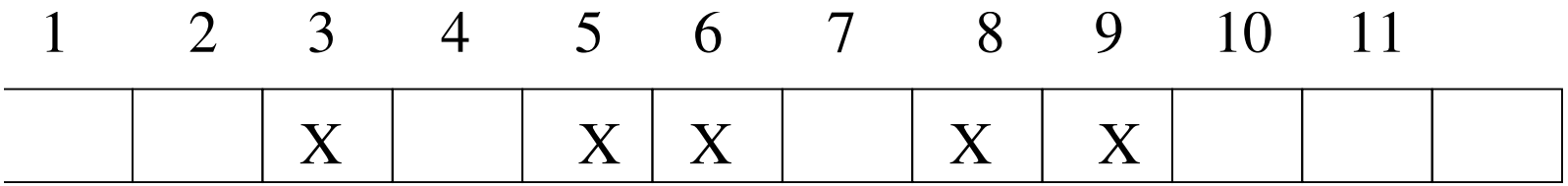

If the space we want to use is filled, we can examine the remaining locations:

1. Sequentially $h, h+1, h+2, \ldots$
2. Linearly $h, h+k, h+2k, h+3k, \ldots$
3. Quadratically $h, h+1^2, h+2^2, h+3^2 \ldots$

Open Addressing

- The reason for using a more complicated scheme is to avoid long runs from similarly hashed keys.
- Deletion in an open addressing scheme is ugly.

Performance on Dictionary Operations

With either chaining or open addressing:

- Search - $O(1)$ expected, $O(n)$ worst case
- Insert - $O(1)$ expected, $O(n)$ worst case
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Pragmatically, a hash table is often the best data structure to maintain a dictionary.
However, we will not use it to analyze algorithms since the worst-case time is unpredictable.

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Why?
Hashing gives you a short but distinctive representation of a larger document.

Applications

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- Randomized algorithms.