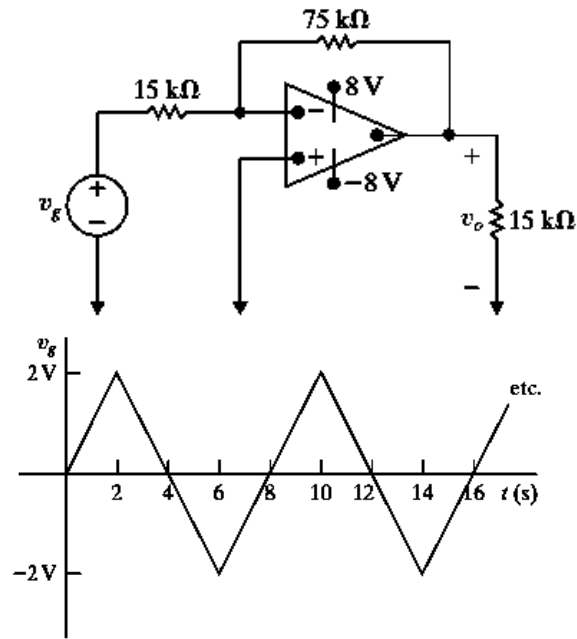


## جواب سوالات 9 Homework

-۲



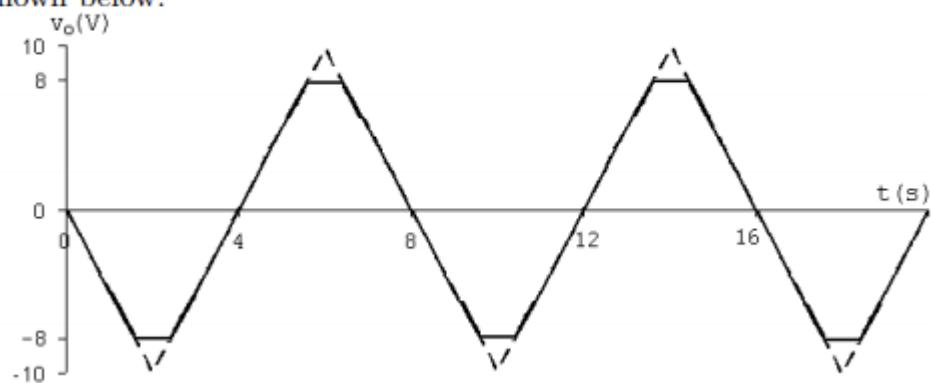
It follows directly from the circuit that  $v_o = -(75/15)v_g = -5v_g$   
 From the plot of  $v_g$  we have  $v_g = 0$ ,  $t < 0$

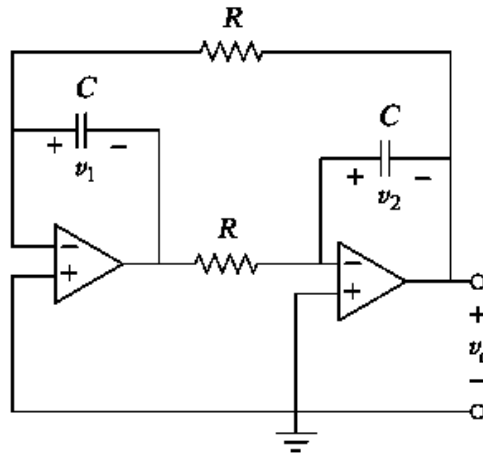
$$\begin{aligned} v_g &= t & 0 \leq t \leq 2 \\ v_g &= 4 - t & 2 \leq t \leq 6 \\ v_g &= t - 8 & 6 \leq t \leq 10 \\ v_g &= 12 - t & 10 \leq t \leq 14 \\ v_g &= t - 16 & 14 \leq t \leq 18, \text{ etc.} \end{aligned}$$

Therefore

$$\begin{aligned} v_o &= -5t & 0 \leq t \leq 2 \\ v_o &= 5t - 20 & 2 \leq t \leq 6 \\ v_o &= 40 - 5t & 6 \leq t \leq 10 \\ v_o &= 5t - 60 & 10 \leq t \leq 14 \\ v_o &= 80 - 5t & 14 \leq t \leq 18, \text{ etc.} \end{aligned}$$

These expressions for  $v_o$  are valid as long as the op amp is not saturated. Since the peak values of  $v_o$  are  $\pm 9$ , the output is clipped at  $\pm 9$ . The plot is shown below.





At the input of the first op amp,

$$(v_o - 0)/R = Cdv_1/dt \quad (1)$$

At the input of the second op amp,

$$(-v_1 - 0)/R = Cdv_2/dt \quad (2)$$

Let us now examine our constraints. Since the input terminals are essentially at ground, then we have the following,

$$v_o = -v_2 \text{ or } v_2 = -v_o \quad (3)$$

Combining (1), (2), and (3), eliminating  $v_1$  and  $v_2$  we get,

$$\frac{d^2 v_o}{dt^2} - \left( \frac{1}{R^2 C^2} \right) v_o = \frac{d^2 v_o}{dt^2} - 100 v_o = 0$$

$$\text{Which leads to } s^2 - 100 = 0$$

Clearly this produces roots of  $-10$  and  $+10$ .

And, we obtain,

$$v_o(t) = (Ae^{+10t} + Be^{-10t})V$$

At  $t = 0$ ,  $v_o(0+) = -v_2(0+) = 0 = A + B$ , thus  $B = -A$

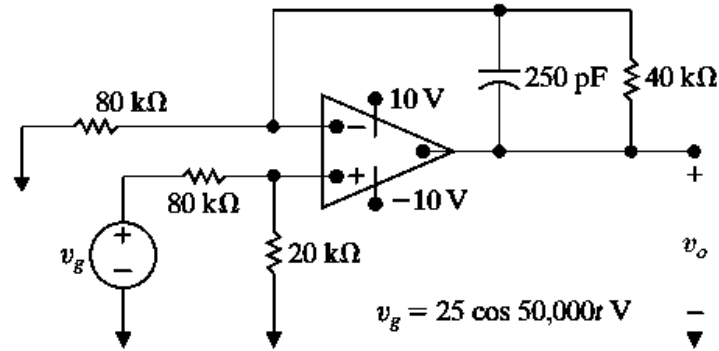
This leads to  $v_o(t) = (Ae^{+10t} - Ae^{-10t})V$ . Now we can use  $v_1(0+) = 2V$ .

From (2),  $v_1 = -RCdv_2/dt = 0.1dv_o/dt = 0.1(10Ae^{+10t} + 10Ae^{-10t})$

$$v_1(0+) = 2 = 0.1(20A) = 2A \text{ or } A = 1$$

$$\text{Thus, } v_o(t) = \underline{(e^{+10t} - e^{-10t})V}$$

It should be noted that this circuit is unstable (clearly one of the poles lies in the right-half-plane).



[a]  $V_g = 25/\underline{0^\circ}\text{ V}$

$$V_p = \frac{20}{100}V_g = 5/\underline{0^\circ}; \quad V_n = V_p = 5/\underline{0^\circ}\text{ V}$$

$$\frac{5}{80,000} + \frac{5 - V_o}{Z_p} = 0$$

$$Z_p = -j80,000 \parallel 40,000 = 32,000 - j16,000\ \Omega$$

$$V_o = \frac{5Z_p}{80,000} + 5 = 7 - j = 7.07/\underline{-8.13^\circ}$$

$$v_o = 7.07 \cos(50,000t - 8.13^\circ)\text{ V}$$

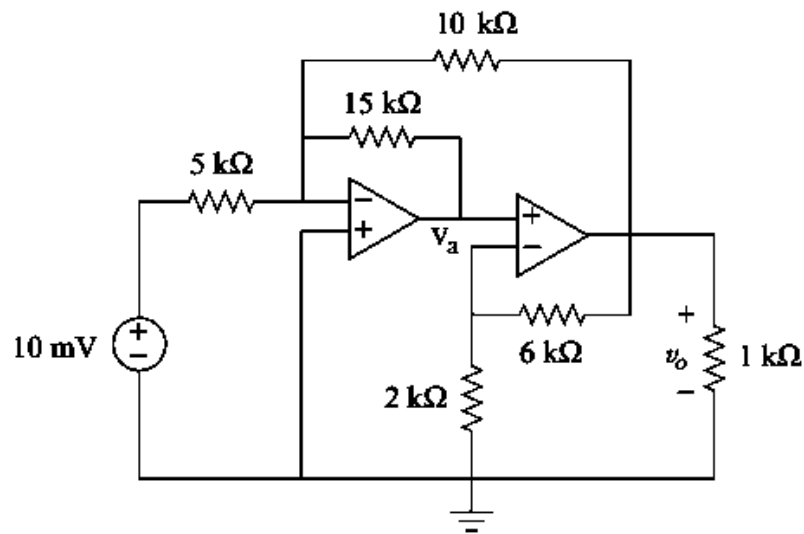
[b]  $V_p = 0.2V_m/\underline{0^\circ}; \quad V_n = V_p = 0.2V_m/\underline{0^\circ}$

$$\frac{0.2V_m}{80,000} + \frac{0.2V_m - V_o}{32,000 - j16,000} = 0$$

$$\therefore V_o = 0.2V_m + \frac{32,000 - j16,000}{80,000}V_m(0.2) = V_m(0.28 - j0.04)$$

$$\therefore |V_m(0.28 - j0.04)| \leq 10$$

$$\therefore V_m \leq 35.36\text{ V}$$



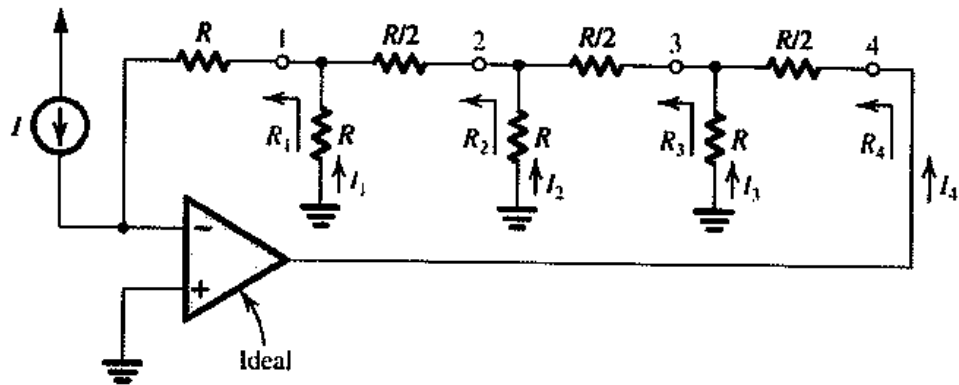
In this case, the first stage is a summer

$$v_a = -\frac{15}{5}(10) - \frac{15}{10}v_o = -30 - 1.5v_o$$

For the second stage,

$$v_o = \left(1 + \frac{6}{2}\right)v_a = 4v_a = 4(-30 - 1.5v_o)$$

$$7v_o = -120 \longrightarrow v_o = -\frac{120}{7} = \underline{\underline{-17.143\text{mV}}}$$

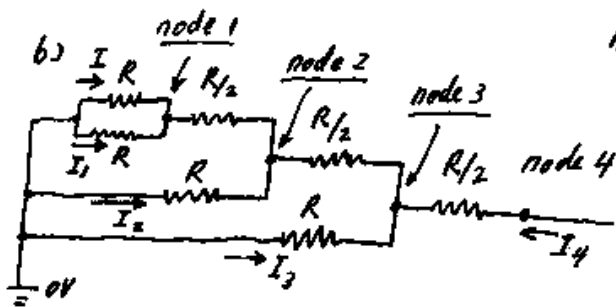


a) looking into node 1:  $R_1 = R$

$$\text{node 2: } R_2 = (R \parallel R) + \frac{R}{2} = R$$

$$\text{node 3: } R_3 = (R_2 \parallel R) + \frac{R}{2} = R$$

$$\text{node 4: } R_4 = (R_3 \parallel R) + \frac{R}{2} = R$$



$$I_1 = I$$

based on current division,

$$I_2 = 2I$$

$$I_3 = 4I$$

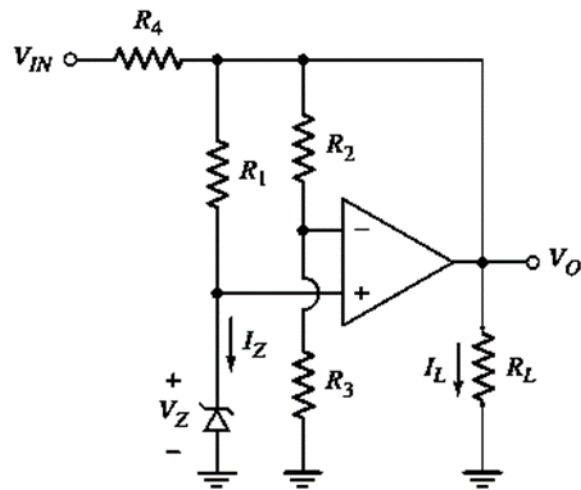
$$I_4 = -8I$$

$$c) \quad V_1 = 0 - IR = -IR$$

$$V_2 = -I_2 R = -2IR$$

$$V_3 = -I_3 R = -4IR$$

$$V_4 = V_3 + \frac{R}{2}(I_4) = -4IR + \frac{R}{2}(-8I) = -8IR$$



$$R_1 = \frac{V_O - V_Z}{I_Z} = \frac{12 - 5.6}{2} = 3.2 \text{ k}\Omega$$

$$\frac{V_O}{V_Z} = \left(1 + \frac{R_2}{R_3}\right) = \frac{12}{5.6} \Rightarrow \frac{R_2}{R_3} = 1.143$$

$$\text{Let } I_R = 2 \text{ mA}, \Rightarrow R_2 + R_3 = \frac{V_O}{I_R} = \frac{12}{2} = 6 \text{ k}\Omega$$

$$\text{Then } 1.143R_3 + R_3 = 6, \Rightarrow R_3 = 2.8 \text{ k}\Omega \text{ and } R_2 = 3.2 \text{ k}\Omega$$

$$\text{Let } I_{R4} = 4 \text{ mA}, R_4 = \frac{V_{IN} - V_O}{I_{R4}} = \frac{15 - 12}{4} = 0.75 \text{ k}\Omega$$