جواب سوالات Homwork 5

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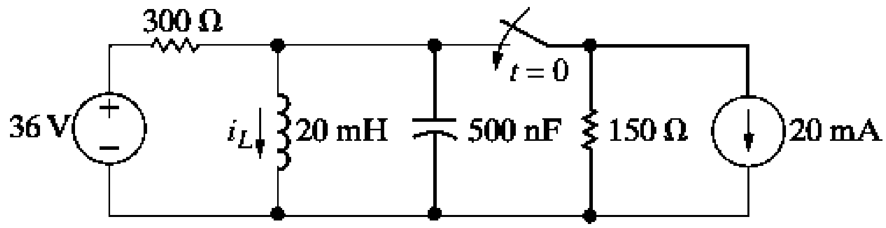
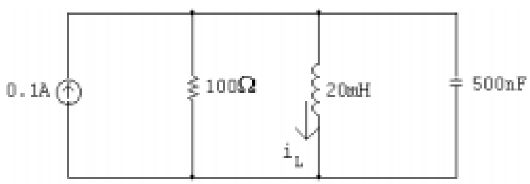


$t < 0$: $\quad i_L(0^-) = \dfrac{36}{300} = 0.12\,\text{A}; \qquad v_C(0^-) = 0\,\text{V}$

The circuit reduces to:



$i_L(\infty) = 0.1\,\text{A}$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(20 \times 10^{-3})(500 \times 10^{-9})}} = 10{,}000 \text{ rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{1}{(100)(500 \times 10^{-9})} = 10{,}000 \text{ rad/s}$$

Critically damped:

$i_L = 0.1 + D_1' t e^{-10{,}000t} + D_2' e^{-10{,}000}$

$i_L(0) = 0.1 + D_2' = 0.12 \qquad \text{so} \qquad D_2' = 0.02$

$\dfrac{di_L}{dt}(0) = D_1' - \alpha D_2' = \dfrac{V_0}{L} \qquad \text{so} \qquad D_1' - (10{,}000)(0.02) = 0$

Solving, $\quad D_1' = 200$

$i_L(t) = 0.1 + 200te^{-10{,}000t} + 0.02e^{-10{,}000t}\,\text{A}, \qquad t \geq 0$

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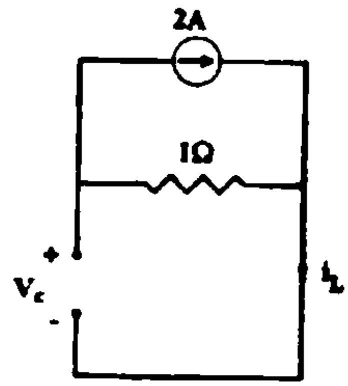
$$\frac{dv_c}{dt}(0^+) = \frac{1}{C} i_c(0^+)$$

$$\frac{di_L}{dt}(0^+) = \frac{1}{L} V_L(0^+)$$

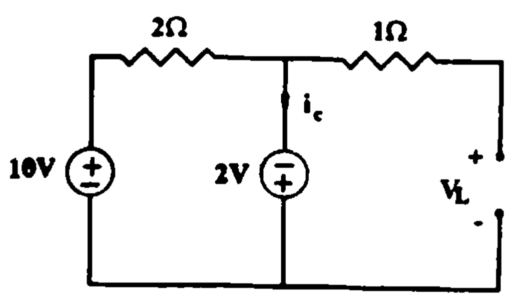
$in \qquad t = 0^-$



$i_L(0) = 0$

$V_c(0) = -2$

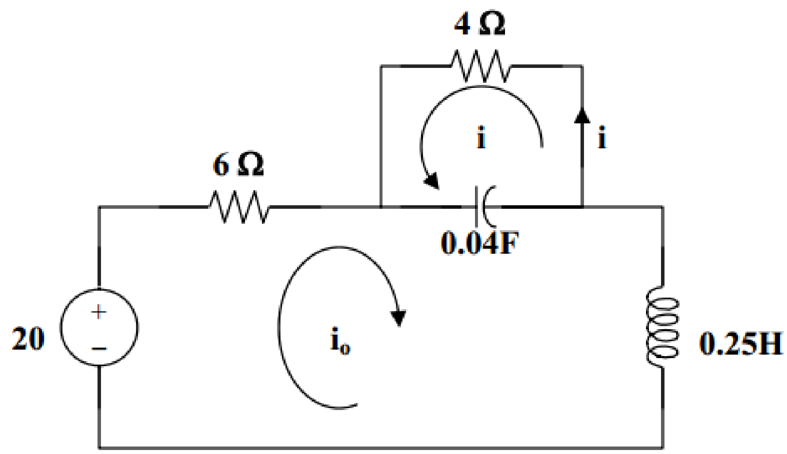
$in \qquad t = 0^+$



$$\begin{matrix} V_L(0^+) = -2V \\ i_C(0^+) = 6A \end{matrix} \Rightarrow \begin{cases} \dfrac{dV_C(0^+)}{dt} = 1 \times 6 = 6 \\ \dfrac{di_L(0^+)}{dt} = \dfrac{1}{1} \times -2 = -2 \end{cases}$$

For $t < 0$, $i(0) = 0$ and $v(0) = 0$.

For $t > 0$, the circuit is as shown below.



Applying KVL to the larger loop,

$$-20 + 6i_o + 0.25di_o/dt + 25\int (i_o + i)dt = 0 \qquad (1)$$

For the smaller loop, $\quad 4i + 25\int (i + i_o)dt = 0$ or $\int (i + i_o)dt = -0.16i \qquad (2)$

Taking the derivative, $\quad 4di/dt + 25(i + i_o) = 0$ or $i_o = -0.16di/dt - i \qquad (3)$

and $\quad di_o/dt = -0.16d^2i/dt^2 - di/dt \qquad (4)$

From (1), (2), (3), and (4), $\quad -20 - 0.96di/dt - 6i - 0.04d^2i/dt^2 - 0.25di/dt - 4i = 0$

Which becomes, $\quad d^2i/dt^2 + 30.25di/dt + 250i = -500$

This leads to, $s^2 + 30.25s + 250 = 0$

$$\text{or } s_{1,2} = \frac{-30.25 \pm \sqrt{(30.25)^2 - 1000}}{2} = -15.125 \pm j4.608$$

This is clearly an underdamped response.

Thus, $i(t) = I_s + e^{-15.125t}(A_1\cos(4.608t) + A_2\sin(4.608t))$A.

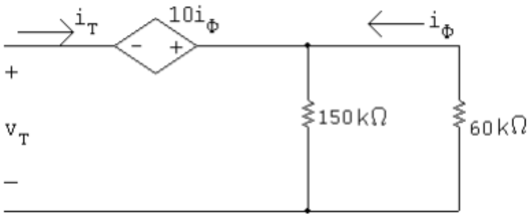
At $t = 0$, $i_o(0) = 0$ and $i(0) = 0 = I_s + A_1$ or $A_1 = -I_s$. As t approaches infinity, $i_o(\infty) = 20/10 = 2A = -i(\infty)$ or $i(\infty) = -2A = I_s$ and $A_1 = 2$.

In addition, from (3), we get $di(0)/dt = -6.25i_o(0) - 6.25i(0) = 0$.

$di/dt = 0 - 15.125\ e^{-15.125t}(A_1\cos(4.608t) + A_2\sin(4.608t)) + e^{-15.125t}(-A_1 4.608\sin(4.608t) + A_2 4.608\cos(4.608t))$. At $t=0$, $di(0)/dt = 0 = -15.125A_1 + 4.608A_2 = -30.25 + 4.608A_2$ or $A_2 = 30.25/4.608 = 6.565$.

This leads to,

$$i(t) = \underline{\mathbf{(-2 + e^{-15.125t}(2\cos(4.608t) + 6.565\sin(4.608t))\ A}}$$



$$v_T = -10i_\phi + i_T\left(\frac{(150)(60)}{210}\right) = -10\frac{-i_T(150)}{210} + i_t\frac{9000}{210}$$

$$\frac{v_T}{i_T} = \frac{1500 + 9000}{210} = 50\,\Omega$$

$$V_o = \frac{4000}{10{,}000}(50) = 20\,\text{V}; \qquad I_o = 0$$

$$i_\text{C}(0) = -i_R(0) - i_\text{L}(0) = -\frac{20}{50} = -0.4\,\text{A}$$

$$\frac{i_\text{C}(0)}{C} = \frac{-0.4}{8\times 10^{-6}} = -50{,}000$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(51.2\times 10^{-3})(8\times 10^{-6})}} = 1562.5 \text{ rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{1}{(2)(50)(8\times 10^{-6})} = 1250 \text{ rad/s}$$

$\alpha^2 < \omega_0^2$ so the response is underdamped

$$\omega_d = \sqrt{1562.5^2 - 1250^2} = 937.5$$

$$v_o = B_1 e^{-1250t}\cos 937.5t + B_2 e^{-1250t}\sin 937.5t$$

$$v_o(0) = B_1 20\,\text{V}$$

$$\frac{dv_o}{dt}(0) = -\alpha B_1 + \omega_d B_2 = \frac{i_\text{C}(0)}{C}$$

$$\therefore \qquad -1250(20) + 937.5B_2 = -50{,}000 \qquad \text{so} \qquad B_2 = -26.67$$

$$v_o = 20e^{-1250t}\cos 937.5t - 26.67e^{-1250t}\sin 937.5t\ \text{V}, \qquad t \geq 0$$

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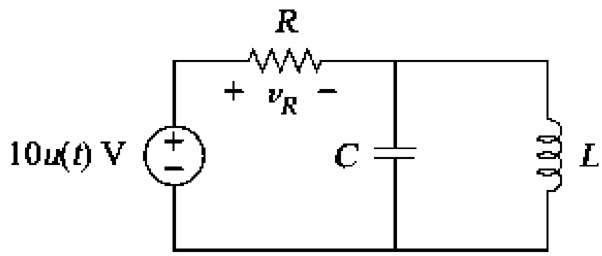


This is a parallel RLC circuit as evident when the voltage source is turned off.

$$\alpha = 1/(2RC) = (1)/(2x3x(1/18)) = 3$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{2x1/18} = 3$$

Since $\alpha = \omega_o$, we have a critically damped response.

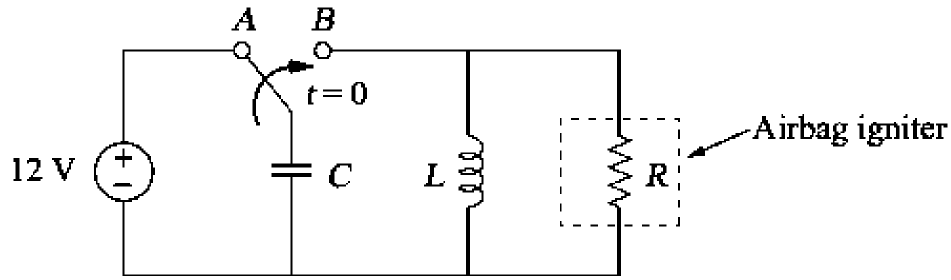
$$s_{1,2} = -3$$

Let v(t) = capacitor voltage

Thus, $v(t) = V_s + [(A + Bt)e^{-3t}]$ where $V_s = 0$

But $-10 + v_R + v = 0$ or $v_R = 10 - v$

Therefore v_R = $\underline{\mathbf{10 - [(A + Bt)e^{-3t}]}}$ where A and B are determined from initial conditions.



The voltage across the igniter is $v_R = v_C$ since the circuit is a parallel RLC type.

$$v_C(0) = 12, \text{ and } i_L(0) = 0.$$

$$\alpha = 1/(2RC) = 1/(2x3x1/30) = 5$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{60x10^{-3}x1/30} = 22.36$$

$\alpha < \omega_o$ produces an underdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -5 \pm j21.794$$

$$v_C(t) = e^{-5t}(A\cos 21.794t + B\sin 21.794t) \quad (1)$$

$$v_C(0) = 12 = A$$

$$dv_C/dt = -5[(A\cos 21.794t + B\sin 21.794t)e^{-5t}]$$

$$+ 21.794[(-A\sin 21.794t + B\cos 21.794t)e^{-5t}] \qquad (2)$$

$$dv_C(0)/dt = -5A + 21.794B$$

But, $dv_C(0)/dt = -[v_C(0) + Ri_L(0)]/(RC) = -(12 + 0)/(1/10) = -120$

Hence, $-120 = -5A + 21.794B$, leads to $B\ (5x12 - 120)/21.794 = -2.753$

At the peak value, $dv_C(t_o)/dt = 0$, i.e.,

$$0 = A + B\tan 21.794t_o + (A21.794/5)\tan 21.794t_o - 21.794B/5$$

$$(B + A21.794/5)\tan 21.794t_o = (21.794B/5) - A$$

$$\tan 21.794t_o = [(21.794B/5) - A]/(B + A21.794/5) = -24/49.55 = -0.484$$

Therefore, $21.7945t_o = |-0.451|$

$t_o = |-0.451|/21.794 =$ $\underline{\textbf{20.68 ms}}$