Homwork 4 جواب سوالات

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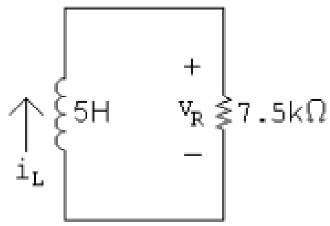
[a] $t > 0$:

$$L_{\text{eq}} = 1.25 + \frac{60}{16} = 5\,\text{H}$$



$$i_L(t) = i_L(0)e^{-t/\tau}\,\text{mA}; \qquad i_L(0) = 2\,\text{A}; \qquad \frac{1}{\tau} = \frac{R}{L} = \frac{7500}{5} = 1500$$

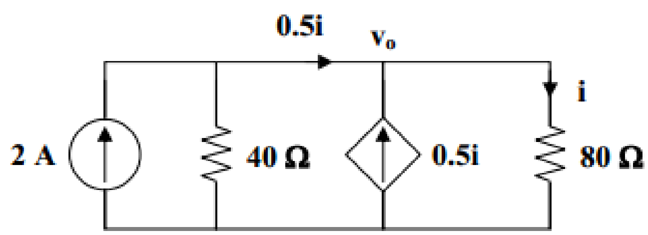
$$i_L(t) = 2e^{-1500t}\,\text{A}, \qquad t \geq 0$$

$$v_R(t) = Ri_L(t) = (7500)(2e^{-1500t}) = 15{,}000e^{-1500t}\,\text{V}, \qquad t \geq 0^+$$

$$v_o = -3.75\frac{di_L}{dt} = 11{,}250e^{-1500t}\,\text{V}, \qquad t \geq 0^+$$

[b] $i_o = \dfrac{-1}{6}\displaystyle\int_0^t 11{,}250e^{-1500x}\,dx + 0 = 1.25e^{-1500t} - 1.25\,\text{A}$

Before t = 0, the circuit has reached steady state so that the capacitor acts like an open circuit. The circuit is equivalent to that shown in Fig. (a) after transforming the voltage source.



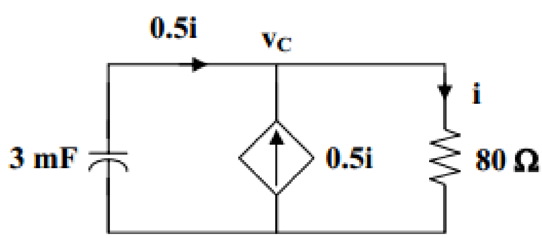
(a)

$$0.5i = 2 - \frac{v_o}{40}, \qquad i = \frac{v_o}{80}$$

Hence, $\frac{1}{2}\frac{v_o}{80} = 2 - \frac{v_o}{40} \longrightarrow v_o = \frac{320}{5} = 64$

$$i = \frac{v_o}{80} = \underline{\mathbf{0.8\ A}}$$

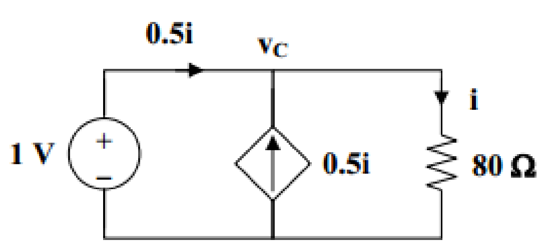
After t = 0, the circuit is as shown in Fig. (b).



(b)

$$v_C(t) = v_C(0)e^{-t/\tau}, \qquad \tau = R_{th}C$$

To find R_{th}, we replace the capacitor with a 1-V voltage source as shown in Fig. (c).



(c)

$$i = \frac{v_C}{80} = \frac{1}{80}, \qquad i_o = 0.5i = \frac{0.5}{80}$$

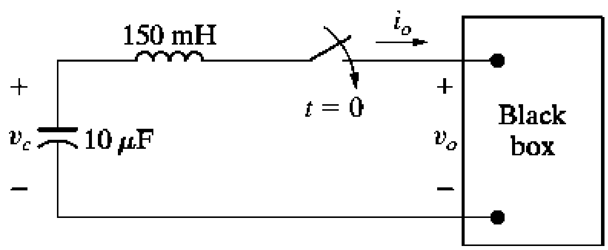
$$R_{th} = \frac{1}{i_o} = \frac{80}{0.5} = 160\,\Omega, \qquad \tau = R_{th}C = 480$$

$$v_C(0) = 64\text{ V}$$

$$v_C(t) = 64e^{-t/480}$$

$$0.5i = -i_C = -C\frac{dv_C}{dt} = -3\left(\frac{1}{480}\right)64e^{-t/480}$$

$$i(t) = \underline{0.8e^{-t/480}\ u(t)A}$$



$$
\begin{aligned}
v_c &= -\frac{1}{10\times 10^{-6}}\left(\int_0^t 0.2e^{-800x}\,dx - \int_0^t 0.04e^{-200x}\,dx\right) + 5 \\
&= 25(e^{-800t}-1) - 20(e^{-200t}-1) + 5 \\
&= 25e^{-800t} - 20e^{-200t} \ \text{V} \\
v_L &= 150\times 10^{-3}\frac{di_o}{dt} \\
&= 150\times 10^{-3}(-160e^{-800t} + 8e^{-200t}) \\
&= -24e^{-800t} + 1.2e^{-200t} \ \text{V} \\
v_o &= v_c - v_L \\
&= (25e^{-800t} - 20e^{-200t}) - (-24e^{-800t} + 1.2e^{-200t}) \\
&= 49e^{-800t} - 21.2e^{-200t} \ \text{V},\ t > 0
\end{aligned}
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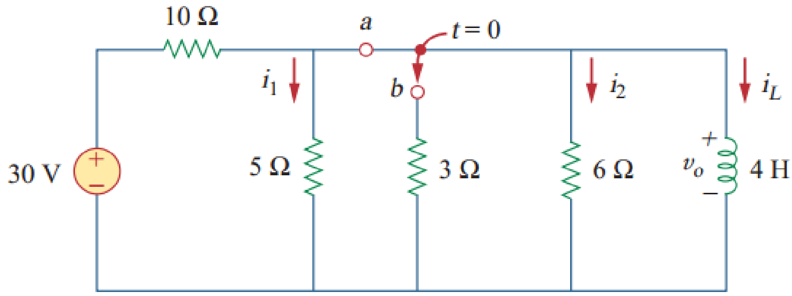


(a) When the switch is in position A, the 5-ohm and 6-ohm resistors are short-circuited so that

$$\underline{i_1(0) = \quad i_2(0) = v_o(0) = 0}$$

but the current through the 4-H inductor is $i_L(0) = 30/10 = 3$A.

(b) When the switch is in position B,

$$R_{Th} = 3//6 = 2\Omega, \qquad \tau = \frac{L}{R_{Th}} = 4/2 = 2\sec$$

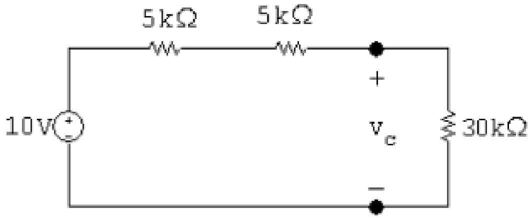
$$i_L(t) = i_L(\infty) + [i_L(0) - i_L(\infty)]e^{-t/\tau} = 0 + 3e^{-t/2} = \underline{3e^{-t/2}\ \mathrm{A}}$$

(c) $i_1(\infty) = \dfrac{30}{10+5} = \underline{2\ \mathrm{A}}, \qquad i_2(\infty) = -\dfrac{3}{9} i_L(\infty) = \underline{0\ \mathrm{A}}$

$$v_o(t) = L\frac{di_L}{dt} \quad \longrightarrow \quad \underline{v_o(\infty) = 0\ \mathrm{V}}$$

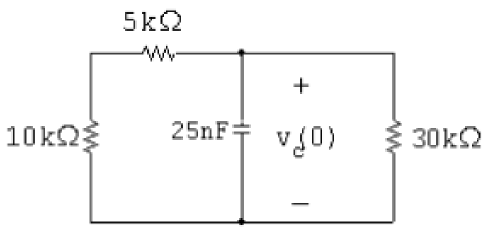
Note that for $t > 0$, $v_o = (10/15)v_c$, where v_c is the voltage across the 25 nF capacitor. Thus we will find v_c first.

$t < 0$



$$v_c(0) = \frac{30}{40}(10) = 7.5\,\text{V}$$

$0 \le t \le 0.2\,\text{ms}$:



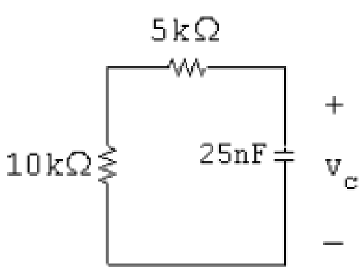
$$\tau = R_eC, \qquad R_e = 15{,}000\|30{,}000 = 10\,\text{k}\Omega$$

$$\tau = (10 \times 10^3)(25 \times 10^{-9}) = 0.25\,\text{ms}, \qquad \frac{1}{\tau} = 4000$$

$$v_c = 7.5e^{-4000t}\,\text{V}, \qquad t \ge 0$$

$$v_c(0.2\,\text{ms}) = 7.5e^{-0.8} = 3.37\,\text{V}$$

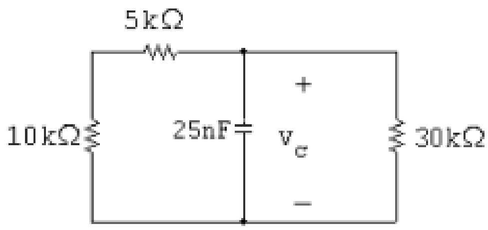
$0.2\,\text{ms} \le t \le 0.8\,\text{ms}$:



$\tau = (15 \times 10^3)(2.5 \times 10^{-9}) = 375\,\mu\text{s}, \qquad \frac{1}{\tau} = 2666.67$

$v_c = 3.37e^{-2666.67(t-200\times 10^{-6})}\,\text{V}$

$0.8\,\text{ms} \leq t <$:



$\tau = 0.25\,\text{ms}, \qquad \frac{1}{\tau} = 4000$

$v_c(0.8\,\text{ms}) = 3.37e^{-2666.67(800-200)10^{-6}} = 3.37e^{-1.6} = 0.68\,\text{V}$

$v_c = 0.68e^{-4000(t-0.8\times 10^{-3})}\,\text{V}$

$v_c(1\,\text{ms}) = 0.68e^{-4000(1-0.8)10^{-3}} = 0.68e^{-0.8} = 0.306\,\text{V}$

$v_o = (10/15)(0.306) = 0.204\,\text{V}$

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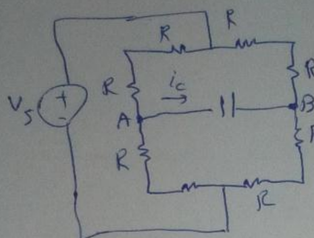
با توجه به اینکه دو منبع مستقل داریم از جمع آثار استفاده می‌کنیم :

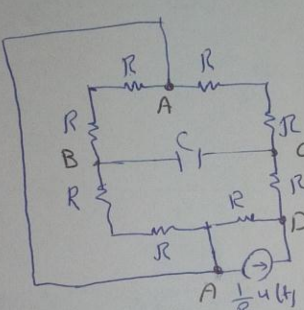
از منبع ولتاژ V_s



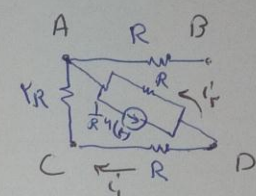
با توجه به اینکه بین دو گره A و B پل وتستون تشکیل شده است ولتاژ گره‌های A و B برابر بوده و جریان i_c برابر صفر خواهد بود

پس ولتاژ V_s تاثیری در جریان i_c نخواهد داشت .

از منبع جریان $\frac{1}{R}u(t)$



مدار در نهایت



$$V_C(\infty) = V_B - V_C$$

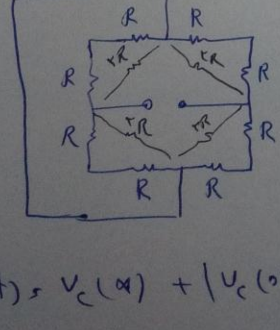
از مقاومت R بین گره‌های A و B جریانی نمی‌گذرد پس $\frac{1}{R}u(t)$ بین مقاومتهای دیگر مدار تقسیم می‌شود :
u(t) در نهایت برابر ۱ است

$$V_A = V_B \quad , \quad i_1 = \frac{R\times(\frac{1}{R}u(t))}{R+R+2R} = \frac{1}{4}$$

$$V_A - V_C = -2R\left(\frac{1}{4}\right) = -\frac{1}{2} \qquad \boxed{V_C(\infty) = -\frac{1}{2}}$$

چون قبل از t=0 منبعی برای شارژ خازن وجود ندارد $\boxed{V_C(0) = 0}$

محاسبه τ : منابع مستقل صفر ←



$$R_{eq} = (2R \| 2R + 2R \| 2R) = 2R$$

$$\tau = 2RC$$

$$v_c(t) = v_c(\infty) + (v_c(0) - v_c(\infty))e^{-t/\tau} = -\frac{1}{4}\left(1 - e^{-\frac{t}{2RC}}\right)$$

$$i_c(t) = C\frac{dv_c(t)}{dt} = C\left(-\frac{1}{4}\times\frac{1}{2RC}\right)e^{-\frac{t}{2RC}} = -\frac{1}{4R}e^{-\frac{t}{2RC}}$$

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