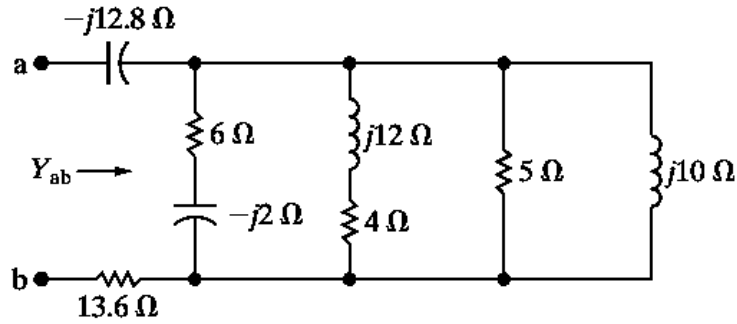


Homwork 6 جواب سوالات

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First find the admittance of the parallel branches

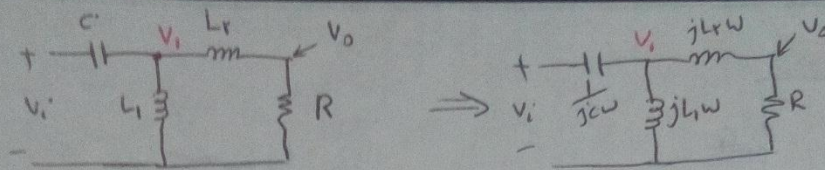
$$Y_p = \frac{1}{6 - j2} + \frac{1}{4 + j12} + \frac{1}{5} + \frac{1}{j10} = 0.375 - j0.125 \text{ S}$$

$$Z_p = \frac{1}{Y_p} = \frac{1}{0.375 - j0.125} = 2.4 + j0.8 \Omega$$

$$Z_{ab} = -j12.8 + 2.4 + j0.8 + 13.6 = 16 - j12 \Omega$$

$$Y_{ab} = \frac{1}{Z_{ab}} = \frac{1}{16 - j12} = 0.04 + j0.03 \text{ S}$$

$$= 40 + j30 \text{ mS} = 50/\underline{36.87^\circ} \text{ mS}$$



در ولتاژ v_1 کحل $\Rightarrow \frac{v_1 - v_i}{\frac{1}{j\omega C}} + \frac{v_1}{j\omega L_i} + \frac{v_1 - v_o}{j\omega L_r} = 0$ (1)

در ولتاژ v_o کحل $\Rightarrow \frac{v_o}{R} + \frac{v_o - v_1}{j\omega L_r} = 0 \rightarrow v_1 = \frac{R + j\omega L_r}{R} v_o$

پس $H = \frac{v_o}{v_i} = \frac{-\omega^2 R L_i C}{R(1 - \omega^2 L_i C) + j\omega(L_i + L_r - \omega^2 L_i L_r C)}$

صورت و مخرج را در مزدوج مخرج ضرب کنیم:

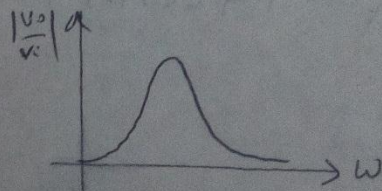
$$\frac{v_o}{v_i} = \frac{-\omega^2 R L_i C [R(1 - \omega^2 L_i C) - j\omega(L_i + L_r - \omega^2 L_i L_r C)]}{R^2(1 - \omega^2 L_i C)^2 + \omega^2(L_i + L_r - \omega^2 L_i L_r C)^2}$$

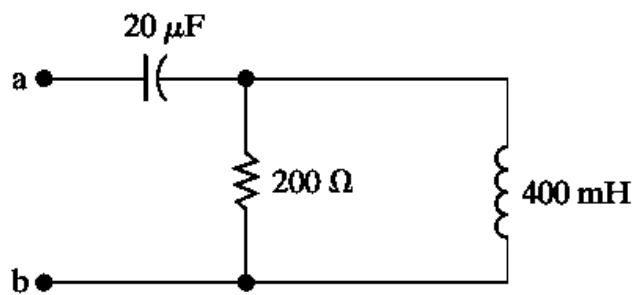
$$\frac{v_o}{v_i} = \frac{-\omega^2 R^2 L_i C(1 - \omega^2 L_i C) + j\omega^2 R L_i C(L_i + L_r - \omega^2 L_i L_r C)}{R^2(1 - \omega^2 L_i C)^2 + \omega^2(L_i + L_r - \omega^2 L_i L_r C)^2}$$

$$\left| \frac{v_o}{v_i} \right| = \frac{\sqrt{[-\omega^2 R^2 L_i C(1 - \omega^2 L_i C)]^2 + [\omega^2 R L_i C(L_i + L_r - \omega^2 L_i L_r C)]^2}}{\sqrt{[R^2(1 - \omega^2 L_i C)^2 + \omega^2(L_i + L_r - \omega^2 L_i L_r C)^2]^2}}$$

$\omega \rightarrow 0 \quad \left| \frac{v_o}{v_i} \right| = 0$

$\omega \rightarrow \infty \quad \left| \frac{v_o}{v_i} \right| = 0$





$$\begin{aligned}
 \text{[a]} \quad \frac{1}{j\omega C} + R \parallel j\omega L &= \frac{1}{j\omega C} + \frac{j\omega RL}{j\omega L + R} \\
 &= \frac{j\omega L + R - \omega^2 RLC}{j\omega C(j\omega L + R)} \\
 &= \frac{(R - \omega^2 RLC + j\omega L)(-\omega^2 LC - j\omega RC)}{(-\omega^2 LC + j\omega RC)(-\omega^2 LC - j\omega RC)}
 \end{aligned}$$

The denominator in the expression above is purely real; set the imaginary part of the numerator in the above expression equal to zero and solve for ω :

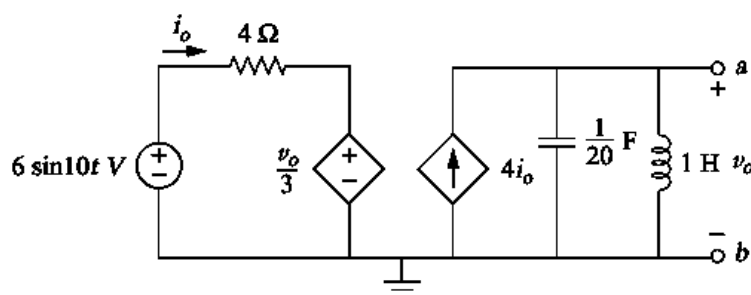
$$-\omega^3 L^2 C - \omega R^2 C + \omega^3 R^2 C^2 L = 0$$

$$-\omega^2 L^2 - R^2 + \omega^2 R^2 LC = 0$$

$$\omega^2 = \frac{R^2}{R^2 LC - L^2} = \frac{200^2}{200^2(0.4)(20 \times 10^{-6}) - (0.4)^2} = 250,000$$

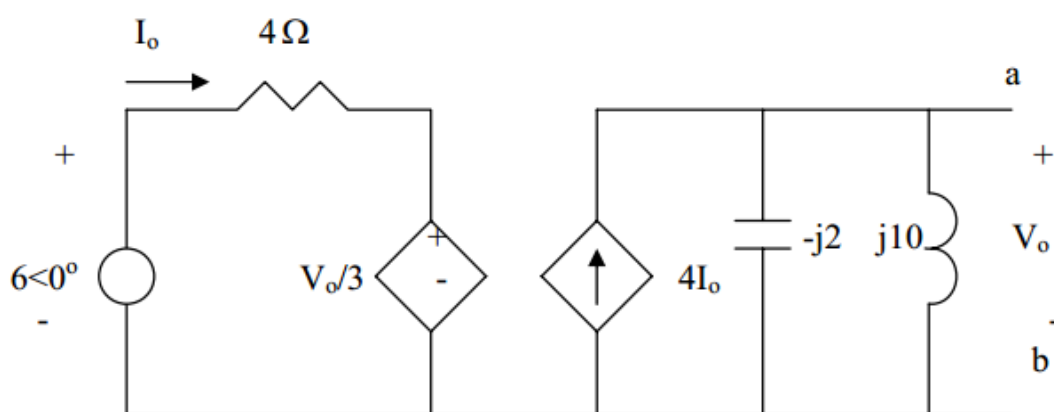
$$\therefore \quad \omega = 500 \text{ rad/s}$$

$$\text{[b]} \quad Z_{ab}(500) = -j100 + \frac{(200)(j200)}{200 + j200} = 100 \Omega$$



$$\begin{aligned}
 1\text{H} &\longrightarrow j\omega L = j10 \times 1 = j10 \\
 \frac{1}{20}\text{F} &\longrightarrow \frac{1}{j\omega C} = \frac{1}{j10 \times \frac{1}{20}} = -j2
 \end{aligned}$$

We obtain V_{Th} using the circuit below.



$$j10 // (-j2) = \frac{j10(-j2)}{j10 - j2} = -j2.5$$

$$V_o = 4I_o \times (-j2.5) = -j10I_o \quad (1)$$

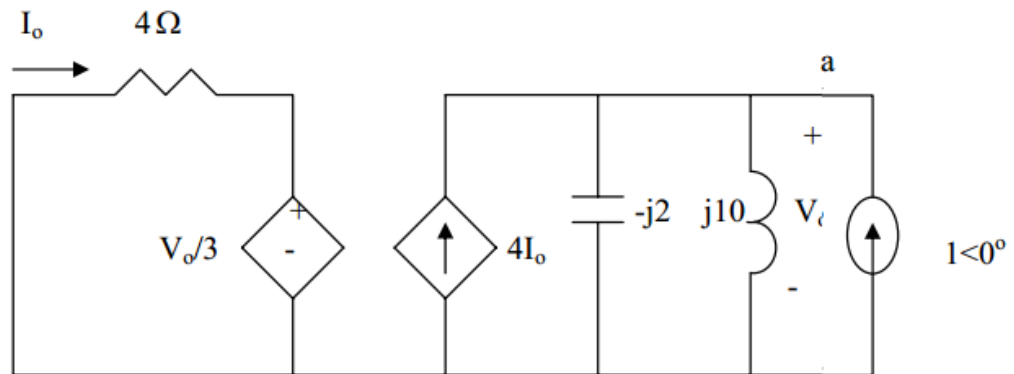
$$-6 + 4I_o + \frac{1}{3}V_o = 0 \quad (2)$$

Combining (1) and (2) gives

$$I_o = \frac{6}{4 - j10/3}, \quad V_{Th} = V_o = -j10I_o = \frac{-j60}{4 - j10/3} = 11.52 \angle -50.19^\circ$$

$$\underline{v_{Th} = 11.52 \sin(10t - 50.19^\circ)}$$

To find R_{Th} , we insert a 1-A source at terminals a-b, as shown below.



$$4I_o + \frac{1}{3}V_o = 0 \quad \longrightarrow \quad I_o = -\frac{V_o}{12}$$

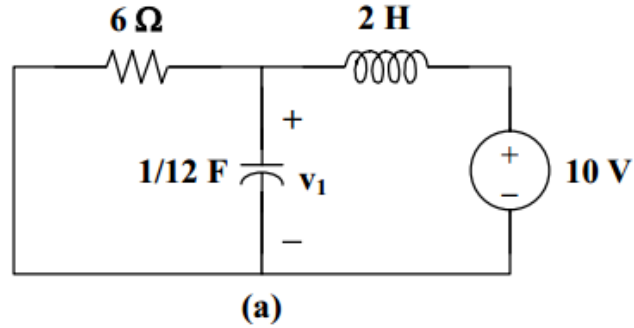
$$1 + 4I_o = \frac{V_o}{-j2} + \frac{V_o}{j10}$$

Combining the two equations leads to

$$V_o = \frac{1}{0.333 + j0.4} = 1.2293 - j1.4766$$

$$Z_{Th} = \frac{V_o}{1} = 1.2293 - j1.4766$$

Let $v_o = v_1 + v_2 + v_3$, where v_1 , v_2 , and v_3 are respectively due to the 10-V dc source, the ac current source, and the ac voltage source. For v_1 consider the circuit in Fig. (a).



The capacitor is open to dc, while the inductor is a short circuit. Hence,

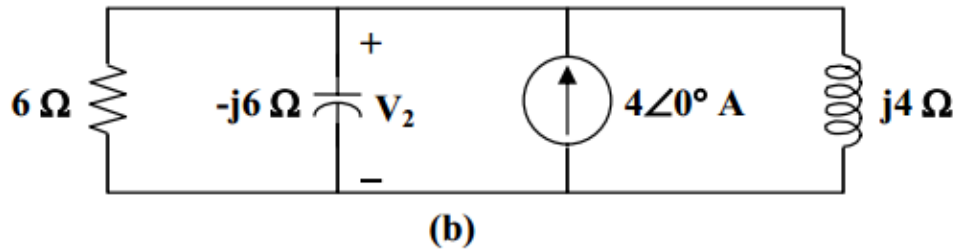
$$v_1 = 10 \text{ V}$$

For v_2 , consider the circuit in Fig. (b).

$$\omega = 2$$

$$2 \text{ H} \longrightarrow j\omega L = j4$$

$$\frac{1}{12} \text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2)(1/12)} = -j6$$



Applying nodal analysis,

$$4 = \frac{V_2}{6} + \frac{V_2}{-j6} + \frac{V_2}{j4} = \left(\frac{1}{6} + \frac{j}{6} - \frac{j}{4} \right) V_2$$

$$V_2 = \frac{24}{1 - j0.5} = 21.45 \angle 26.56^\circ$$

Hence,

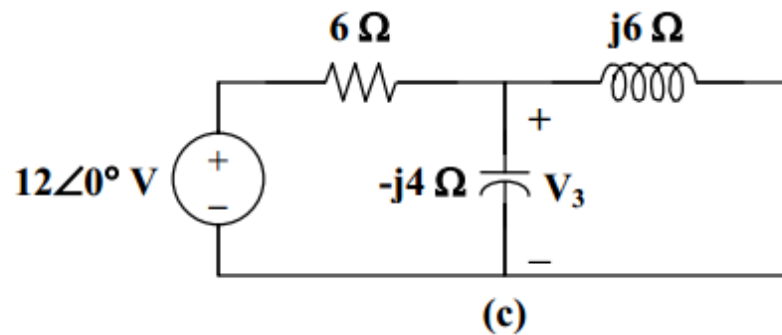
$$v_2 = 21.45 \sin(2t + 26.56^\circ) \text{ V}$$

For v_3 , consider the circuit in Fig. (c).

$$\omega = 3$$

$$2 \text{ H} \longrightarrow j\omega L = j6$$

$$\frac{1}{12} \text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(3)(1/12)} = -j4$$



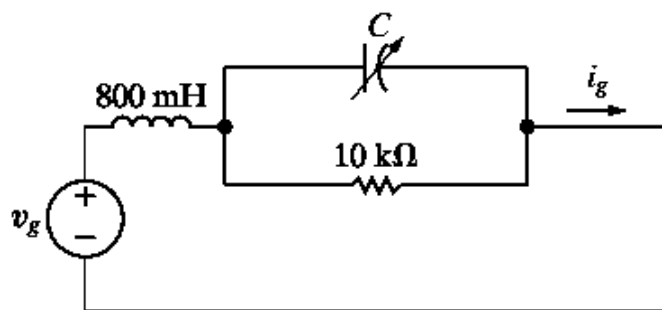
At the non-reference node,

$$\frac{12 - V_3}{6} = \frac{V_3}{-j4} + \frac{V_3}{j6}$$

$$V_3 = \frac{12}{1 + j0.5} = 10.73 \angle -26.56^\circ$$

Hence, $v_3 = 10.73 \cos(3t - 26.56^\circ) \text{ V}$

Therefore, $v_o = \underline{10 + 21.45 \sin(2t + 26.56^\circ) + 10.73 \cos(3t - 26.56^\circ) \text{ V}}$



$$\begin{aligned}
 \text{[a]} \quad Z_p &= \frac{\frac{R}{j\omega C}}{R + (1/j\omega C)} = \frac{R}{1 + j\omega RC} \\
 &= \frac{10,000}{1 + j(5000)(10,000)C} = \frac{10,000}{1 + j50 \times 10^6 C} \\
 &= \frac{10,000(1 - j50 \times 10^6 C)}{1 + 25 \times 10^{14} C^2} \\
 &= \frac{10,000}{1 + 25 \times 10^{14} C^2} - j \frac{5 \times 10^{11} C}{1 + 25 \times 10^{14} C^2}
 \end{aligned}$$

$$j\omega L = j5000(0.8) = j4000$$

$$\therefore 4000 = \frac{5 \times 10^{11} C}{1 + 25 \times 10^{14} C^2}$$

$$\therefore 10^{14} C^2 - 125 \times 10^6 C + 1 = 0$$

$$\therefore C^2 - 5 \times 10^{-8} C + 4 \times 10^{-16} = 0$$

Solving,

$$C_1 = 40 \text{ nF} \qquad C_2 = 10 \text{ nF}$$

$$[\mathbf{b}] \quad R_e = \frac{10,000}{1 + 25 \times 10^{14} C^2}$$

$$\text{When } C = 40 \text{ nF} \quad R_e = 2000 \, \Omega;$$

$$\mathbf{I}_g = \frac{80/\underline{0^\circ}}{2000} = 40/\underline{0^\circ} \text{ mA}; \quad i_g = 40 \cos 5000t \text{ mA}$$

$$\text{When } C = 10 \text{ nF} \quad R_e = 8000 \, \Omega;$$

$$\mathbf{I}_g = \frac{80/\underline{0^\circ}}{8000} = 10/\underline{0^\circ} \text{ mA}; \quad i_g = 10 \cos 5000t \text{ mA}$$