

تمرین سری پنجم

$$a) x \cdot (y \oplus z) = xy \oplus xz \rightarrow x \cdot (y \oplus z) \quad \checkmark$$

$$\text{left: } x(y\bar{z} + yz) = xy\bar{z} + xyz$$

$$\text{right: } (\overline{xy} \cdot xz + xy \cdot \overline{xz}) = (\bar{x} + \bar{y})xz + xy(\bar{x} + \bar{z}) = x\bar{y}\bar{z} + xy\bar{z} \quad \checkmark$$

$$b) x + (y \oplus z) = (x + y) \oplus (x + z)$$

$$\text{left: } x + (y\bar{z} + yz) = x + y\bar{z} + yz$$

$$\text{right: } (\overline{x+y})(x+z) + (x+y)(\overline{x+z}) = \bar{x}\bar{y}\bar{z} + \bar{x}y\bar{z} = \bar{x}(y \oplus z)$$

$$c) x \cdot (y \odot z) = xy \odot xz$$

$$\text{left: } x \cdot (y\bar{z} + yz) = xy\bar{z} + xyz \rightarrow x(y \oplus z)$$

$$\text{right: } (\overline{xy})(\overline{xz}) = \bar{x}\bar{y}\bar{z} \rightarrow x(y \oplus z)$$

$$d) x + (y \odot z) = (x + y) \odot (x + z)$$

$$\text{right: } (\overline{x+y})(\overline{x+z}) = \bar{x}\bar{y}\bar{z} \Rightarrow \text{right: } \bar{x}\bar{y}\bar{z} + x + yz = x + y\bar{z} + yz$$

$$\text{left: } x + y\bar{z} + yz$$

(۲)

$$f(A, B, C, D) = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{C}D + BCD + ABC + A\bar{B}$$

$$f = \bar{A}\bar{B}\bar{C}(D + \bar{D}) + \bar{A}(B + \bar{B})\bar{C}D + (A + \bar{A})BCD + ABC(D + \bar{D})$$

(الف)

$$+ A\bar{B}(C + \bar{C})(D + \bar{D})$$

$$f = \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D} + AB\bar{C}D + A\bar{B}C\bar{D}$$

$$+ A\bar{B}CD + AB\bar{C}\bar{D} + A\bar{B}\bar{C}\bar{D} + A\bar{B}C\bar{D} + A\bar{B}\bar{C}D + A\bar{B}C\bar{D}$$

$$\Rightarrow f = \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D} + AB\bar{C}D + A\bar{B}C\bar{D} + A\bar{B}CD + A\bar{B}C\bar{D}$$

$$= \sum (0, 1, 5, 7, 8, 9, 10, 11, 14, 15)$$

$$= m_0 + m_1 + m_5 + m_7 + m_8 + m_9 + m_{10} + m_{11} + m_{14} + m_{15}$$

$$f = \pi(2, 3, 4, 6, 12, 13)$$

$$f \Rightarrow (\bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}B\bar{C}\bar{D} + A\bar{B}C\bar{D} + A\bar{B}C\bar{D})$$

$$\Rightarrow f = (A + B + \bar{C} + D)(A + B + \bar{C} + \bar{D})(A + \bar{B} + C + D)(A + \bar{B} + \bar{C} + D) + (\bar{A} + B + \bar{C} + D)(\bar{A} + B + \bar{C} + \bar{D})$$

1	1	3	2
4	15	7	6
12	13	15	14
18	19	11	16

$$f = \bar{B}\bar{C} + AC + \bar{A}BD$$

(ج)

a) $f = \bar{A}\bar{B}D + \bar{A}CD + \bar{B}CD + \bar{A}B\bar{C}\bar{D}$

b) $g = \bar{A}B\bar{D} + BCD + \bar{B}\bar{C}D + A\bar{B}\bar{D}$

c) $h = B\bar{C} + CD + \bar{A}D$

d) $k = BD + \bar{B}\bar{D} = \overline{B \oplus D} = B \odot D$

e) $j = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{D} + A\bar{C}D + A\bar{B}\bar{D}$

$\Rightarrow j = (A + B + C)(A + \bar{B} + \bar{D})(\bar{A} + \bar{C} + \bar{D})(\bar{A} + B + D)$

f) $k = \bar{B}D + B\bar{C}\bar{D} + \bar{A}CD$

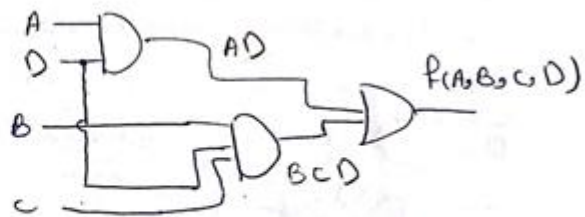
$\Rightarrow k = (B + \bar{D})(\bar{B} + C + D)(A + \bar{C} + \bar{D})$

$$P: \bar{K} = B\bar{C}\bar{D} + \bar{B}C + \bar{A}CD \Rightarrow K = (B+C+D)(B+\bar{C})(A+\bar{C}+\bar{D})$$

#	A	B	C	D	P
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	0
4	0	1	0	0	0
5	0	1	0	1	0
6	0	1	1	0	0
7	0	1	1	1	0
8	1	0	0	0	1
9	1	0	0	1	1
10	1	0	1	0	1
11	1	0	1	1	1
12	1	1	0	0	0
13	1	1	0	1	1
14	1	1	1	0	0
15	1	1	1	1	1

$$P = \Sigma(7, 9, 11, 13, 15)$$

$$P(A, B, C, D) = AD + BCD$$



A	B	C	D	x	y	z	w
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	1	0	0	0	1	1
0	0	1	1	0	0	1	0
0	1	0	0	0	1	1	0
0	1	0	1	0	1	1	1
0	1	1	0	0	1	0	1
0	1	1	1	0	1	0	0
1	0	0	0	1	1	0	0
1	0	0	1	1	1	0	1
1	0	1	0	1	1	1	1
1	0	1	1	1	1	1	0
1	1	0	0	1	0	1	0
1	1	0	1	1	0	1	1
1	1	1	0	1	0	0	1
1	1	1	1	1	0	0	0

$$x(A, B, C, D) = \Sigma(8, \dots, 15)$$

$$y(A, B, C, D) = \Sigma(4, 5, 6, 7, 8, 9, 10, 11)$$

$$z(A, B, C, D) = \Sigma(2, 3, 4, 5, 9, 10, 11, 12)$$

$$w(A, B, C, D) = \Sigma(1, 2, 5, 6, 10, 11, 12, 13)$$

$$x = A$$

$$y = A\bar{B} + \bar{A}B$$

$$z = B\bar{C} + \bar{B}C$$

$$w = \bar{D}C + D\bar{C}$$

$$P(A, B, C, D) = \sum (0, 4, 8, 13, 14, 15) \quad (2)$$

$$= \underbrace{\bar{B}\bar{C}\bar{D} + \bar{A}\bar{C}\bar{D}}_{\bar{C}\bar{D}(\bar{A} + \bar{B})} + \underbrace{ABC + AB\bar{D}}_{AB(C + \bar{D})} = \underbrace{(\bar{C} + \bar{D})(\bar{A} + \bar{B})}_{(\bar{C} + \bar{D} + AB)} + AB(C + \bar{D})$$

