Question 1

(a) $x(2t+3)$

$\boxed{\text{Method 1}}$

① $t \leftarrow t+3 \Rightarrow x(t+3)$ (shift left) (3 units)

② $t \leftarrow 2t \Rightarrow x(2t+3)$ (compress) (factor = 2)

$\boxed{\text{Method 2}}$

① $t \leftarrow 2t \Rightarrow x(2t)$ (compress) (factor = 2)

② $t \leftarrow t+\frac{3}{2} \Rightarrow x\left(2\left(t+\frac{3}{2}\right)\right) = x(2t+3)$ (Shift left) ($\frac{3}{2}$ units)

(b) $x(-2t) \rightarrow$ compress & reverse (factor = 2)

(c) $x(-3t-4)$

① $t \leftarrow t-4 \Rightarrow x(t-4)$ (shift right) (4 units)

② $t \leftarrow 3t \Rightarrow x(3t-4)$ (compress) (factor = 3)

③ $t \leftarrow -t \Rightarrow x(-3t-4)$ (reverse)

(d) $x\left(\frac{t-1}{3}\right) = x\left(\frac{1}{3}t - \frac{1}{3}\right)$

① $t \leftarrow t-\frac{1}{3} \Rightarrow x\left(t-\frac{1}{3}\right)$ (shift left) ($\frac{1}{3}$ units)

② $t \leftarrow \frac{1}{3}t \Rightarrow x\left(\frac{1}{3}t-\frac{1}{3}\right)$ (expand) (factor = $\frac{1}{3}$)

Question 2

(a) $x(t) = e^{-5t}\cos(t)\,u(t)$

$\rightarrow x(-t) = e^{5t}\cos(-t)\,u(-t) = e^{5t}\cos(t)\,u(-t)$

$$Ev\{x(t)\} = \frac{x(t)+x(-t)}{2} = \frac{1}{2}\left[\cos(t)\left(e^{-5t}u(t) + e^{5t}u(-t)\right)\right]$$

$$Od\{x(t)\} = \frac{x(t)-x(-t)}{2} = \frac{1}{2}\left[\cos(t)\left(e^{-5t}u(t) - e^{5t}u(-t)\right)\right]$$

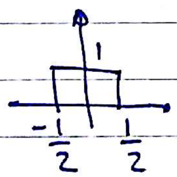
(b) $x(t) = e^{-6|t|}\cos(t)$

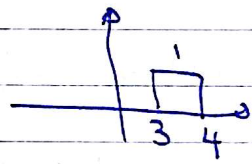
$x(-t) = e^{-6|-t|}\cos(-t) = e^{-6|t|}\cos(t) = x(t)$

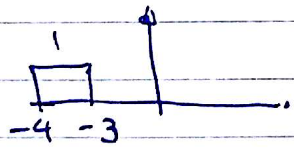
$$\Rightarrow Ev\{x(t)\} = \frac{2x(t)}{2} = x(t) = e^{-6|t|}\cos(t)$$

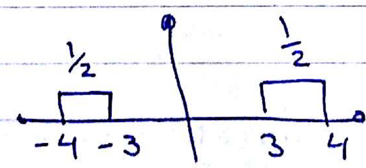
$$Od\{x(t)\} = \frac{0}{2} = 0$$

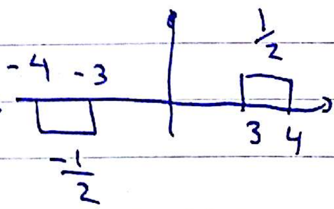
(c) $\Pi(t-3.5) = x(t)$

$\Pi(t) =$ 

$\longrightarrow \Pi(t-3.5) =$ 

$\Rightarrow x(-t) =$ 

$\Rightarrow Ev\{x(t)\} =$ 

$Od\{x(t)\} =$ 

Question 3

(a) $x(t) = \sin^2(4t - \frac{\pi}{4})$

$$\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha) = 1 - 2\sin^2\alpha$$

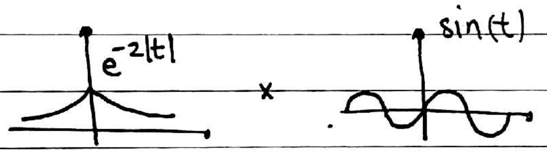
$$\Rightarrow \sin^2(\alpha) = \frac{1}{2}(1 - \cos(2\alpha))$$

$$\Rightarrow \sin^2(4t - \frac{\pi}{4}) = \frac{1}{2}\Big(1 - \underbrace{\cos(8t - \frac{\pi}{2})}_{w(t)}\Big)$$

$x(t)$ is periodic if $w(t)$ is.

$$w(t) = \cos(8t - \frac{\pi}{2}) \qquad T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{8} = \frac{\pi}{4}$$

(b) $x(t) = e^{-2|t|}\sin(t)$



= periodic? No

(c) $x(t) = e^{3j(8t + \frac{\pi}{3})} = e^{j(24t + \pi)}$

$$\omega_0 = 24 \Rightarrow T_0 = \frac{2\pi}{24} = \frac{\pi}{12} \quad ✓$$

(d) $x(t) = \sin^3(5t)$

$$\sin(3\alpha) = 3\sin(\alpha) - 4\sin^3(\alpha)$$

$$\Rightarrow \sin^3(\alpha) = \frac{1}{4}\big(3\sin(\alpha) - \sin(3\alpha)\big)$$

$$\Rightarrow x(t) = \sin^3(5t) = \frac{1}{4}\Big(\underbrace{3\sin(5t)}_{x_1} - \underbrace{\sin(15t)}_{x_2}\Big)$$

$$T_{01} = \frac{2\pi}{5} \qquad T_{02} = \frac{2\pi}{15} \quad \rightarrow T_0 = \mathrm{lcm}\left(\frac{2\pi}{5}, \frac{2\pi}{15}\right) = \frac{2\pi}{5} = 0.4\pi$$

Subject
Year Month Date ( )

(e) $x[n] = 3\sin(4\pi n)$

$$\frac{2\pi}{\omega_0} = \frac{2\pi}{4\pi} = \frac{1}{2} \longrightarrow \text{Rational} \Rightarrow N_0 = 1$$

(f) $x[n] = 3\sin(4n)$

$$\frac{2\pi}{\omega_0} = \frac{2\pi}{4} = \frac{\pi}{2} \longrightarrow \text{Not Rational} \Rightarrow \text{Not Periodic}$$

(g) $x[n] = e^{\frac{jn}{2}} + e^{\frac{jn}{3}}$

Neither $e^{\frac{jn}{2}}$ nor $e^{\frac{jn}{3}}$ is periodic

$\Rightarrow$ Not Periodic

(h) $x[n] = e^{jn\frac{\pi}{2}} + e^{jn\frac{\pi}{3}}$

$$\left.\begin{array}{l} \text{Period}_1: \frac{2\pi}{\omega_0} = \frac{2\pi}{\pi/2} = \frac{4}{1} \Rightarrow N_{01} = 4 \\ \text{Period}_2: \frac{2\pi}{\omega_0} = \frac{2\pi}{\pi/3} = \frac{6}{1} \Rightarrow N_{02} = 6 \end{array}\right\} N_0 = \text{lcm}(4,6) = 12$$

Subject
Year Month Date ( )

Question 4

(a) $y(t) = e^{x(t)}$

- Memoryless : Yes
- Causal : Yes
- Stable : Yes, BIBO applies
- TI : Yes

$input = x_1(t) \Rightarrow y_1(t) = e^{x_1(t)}$

$input = x_2(t) = x_1(t-t_0) \Rightarrow y_2(t) = e^{x_1(t-t_0)}$

$y_1(t-t_0) = e^{x_1(t-t_0)} \Rightarrow$ Yes

- Linear : No

$x_1(t) \rightarrow y_1(t) = e^{x_1(t)}$

$x_2(t) \rightarrow y_2(t) = e^{x_2(t)}$

$x_3(t) = ax_1(t) + bx_2(t) \rightarrow y_3(t) = e^{x_3(t)} = e^{ax_1(t)+bx_2(t)}$

$= e^{ax_1(t)} e^{bx_2(t)}$

$\neq ay_1(t) + by_2(t)$

(b) $y(t) = \cos(3t)\, x(t)$

- Memoryless : Yes
- Causal : Yes
- Stable : Yes, BIBO applies
- TI : No

$input = x_1(t) \Rightarrow y_1(t) = \cos(3t)\, x_1(t)$

$input = x_1(t-t_0) \Rightarrow y_2(t) = \cos(3t)\, x_1(t-t_0)$

$y_1(t-t_0) = \cos(3(t-t_0))\, x_1(t-t_0) \Rightarrow$ No

- Linear : Yes

$x_1(t) \rightarrow y_1(t) = \cos(3t)\, x_1(t)$

$x_2(t) \rightarrow y_2(t) = \cos(3t)\, x_2(t)$

$ax_1(t) + bx_2(t) \rightarrow y_3(t) = \cos(3t)\, x_3(t) = \cos(3t)\,(ax_1 + bx_2)$

$= ay_1 + ay_2$

(c) $y(t) = \int_{-\infty}^{2t} x(\tau)\, d\tau$

- Memoryless : No, $y(1)$ needs $x(2)$
- Causal : No, $y(1)$ needs $x(2)$
- Stable : No, if $x(t)=1 \Rightarrow y(0) = \int_{-\infty}^{0} x(\tau)\, d\tau = \infty$
- TI : No

$$\text{input} = x_1(t) \Rightarrow y_1(t) = \int_{-\infty}^{2t} x_1(\tau)\, d\tau$$

$$\text{input} = x_2(t) = x_1(t-t_0) \Rightarrow y_2(t) = \int_{-\infty}^{2t} x_1(\tau - t_0)\, d\tau$$

$$y_1(t-t_0) = \int_{-\infty}^{2(t-t_0)} x_1(\tau)\, d\tau \longrightarrow \text{No}$$

- Linear : Yes

$$x_1(t) \to y_1(t) = \int_{-\infty}^{2t} x_1(\tau)\, d\tau$$

$$x_2(t) \to y_2(t) = \int_{-\infty}^{2t} x_2(\tau)\, d\tau$$

$$ax_1 + bx_2 \to y_3(t) = \int_{-\infty}^{2t} (ax_1(\tau) + bx_2(\tau))\, d\tau = ay_1(t) + by_2(t) \checkmark$$

(d) $y(t) = \sin(x(t))$

- Memoryless : Yes
- Causal : Yes
- Stable : Yes : $-1 \le y(t) \le +1$
- TI : Yes

$$\text{input} = x_1(t) \to y_1(t) = \sin(x_1(t))$$

$$\text{input} = x_2(t) = x_1(t-t_0) \to y_2(t) = \sin(x_1(t-t_0))$$

$$y_1(t-t_0) = \sin(x_1(t-t_0)) \checkmark$$

- Linear : No

$$\sin(ax_1(t) + bx_2(t)) \neq a\sin(x_1(t)) + b\sin(x_2(t))$$

(e) $y[n] = x[4n+1]$

- Memoryless : No, $y[0] = x[1]$
- Causal : No, $y[0] = x[1]$
- Stable : Yes, BIBO applies
- TI : No

$$\text{input} = x_1[n] \Rightarrow y_1[n] = x_1[4n+1]$$
$$\text{input} = x_2[n] = x_1[n-n_0] \Rightarrow y_2[n] = x_2[4n+1] = x_1[4n+1-n_0]$$

$$y_1[n-n_0] = x_1[4(n-n_0)+1]$$

- Linear : Yes

$$x_1[n] \longrightarrow y_1[n] = x_1[4n+1]$$
$$x_2[n] \longrightarrow y_2[n] = x_2[4n+1]$$
$$x_3[n] = ax_1[n] + bx_2[n] \longrightarrow y_3[n] = x_3[4n+1] = ax_1[4n+1] + bx_2[4n+1]$$
$$= ay_1[n] + by_2[n]$$

(f) $y[n] = (n-2)\,x[n]$

- Memoryless : Yes
- Causal : Yes.
- Stable : ~~Yes, BIBO applies~~

No, if $x[n] = 1 \rightarrow y[\infty] = \infty \Rightarrow$ Not Bounded

- TI : No

$$\text{input} = x_1[n] \Rightarrow y_1[n] = (n-2)\,x_1[n]$$
$$\text{input} = x_2[n] = x_1[n-n_0] \Rightarrow y_2[n] = (n-2)\,x_2[n] = (n-2)\,x_1[n-n_0]$$

$$y_1[n-n_0] = (n-n_0-2)\,x_1[n-n_0]$$

- Linear : Yes

$$x_1[n] \longrightarrow y_1[n] = (n-2)\,x_1[n]$$
$$x_2[n] \longrightarrow y_2[n] = (n-2)\,x_2[n]$$
$$x_3[n] = ax_1[n] + bx_2[n] \longrightarrow y_3[n] = (n-2)\big(ax_1[n] + bx_2[n]\big) = ay_1[n] + by_2[n]$$

(g) $y[n] = x[n] \sum_{k=-\infty}^{+\infty} \delta[n-(2k+1)] = \begin{cases} x[n] & n \text{ is odd} \\ 0 & \text{o.w} \end{cases}$

- Memoryless : Yes
- Causal : Yes
- Stable : Yes, BIBO applies
- TI : No

$\text{input} = x_1[n] \Rightarrow y_1[n] = x_1[n] \sum_{k=-\infty}^{+\infty} \delta[n-(2k+1)]$

$\text{input} = x_2[n] = x_1[n-n_0] \Rightarrow y_2[n] = x_1[n-n_0] \sum_{k=-\infty}^{+\infty} \delta[n-(2k+1)]$

$y_1[n-n_0] = x_1[n-n_0] \sum_{k=-\infty}^{+\infty} \delta[n-n_0-(2k+1)]$

اگر n_0 فرد باشد ممکن است این سیستم، اشتباهاً TI درنظر گرفته شود.

- Linear : Yes

(h) $y[n] = \sum_{k=-\infty}^{n} x[k+3] \xrightarrow[\text{قاعده تلسکوپی}]{k \leftarrow k-3} y[n] = \sum_{k=-\infty}^{n+3} x[k]$

- Memoryless : No, $y[0] = x[3] + x[2] + x[1] + \cdots$
- Causal : No, 〃
- Stable : No, if $x[n] = 1$, $y[0] = \sum_{k=-\infty}^{3} 1 = -\infty$

- TI : Yes

$\text{input} = x_1[n] \Rightarrow y_1[n] = \sum_{k=-\infty}^{n+3} x_1[k]$

$\text{input} = x_2[n] = x_1[n-n_0] \Rightarrow y_2[n] = \sum_{k=-\infty}^{n+3} x_1[k-n_0]$

$y_1[n-n_0] = \sum_{k=-\infty}^{n-n_0+3} x_1[k] \xleftarrow[\text{تلسکوپی}]{k \leftarrow k-n_0} \sum_{k=-\infty}^{n+3} x_1[k-n_0]$

- Linear : Yes, $\sum$ is linear

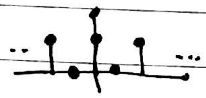
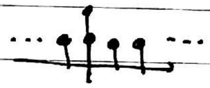
$\sum a x_1 + b x_2 = a \sum x_1 + b \sum x_2 = a y_1 + b y_2$

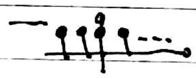
Question 5

(a) $y(t) = \frac{d}{dt} x(t) \rightarrow$ Not Invertible

$x_1(t) = 5t+3$
$x_2(t) = 5t+4 \longrightarrow y_1(t) = y_2(t) = 5$

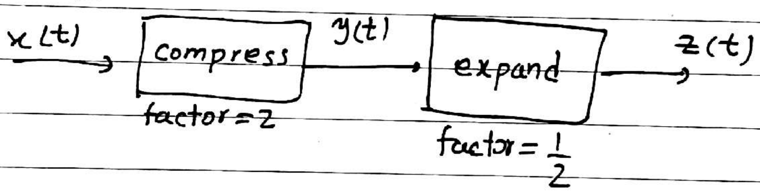
(b) $y[n] = x[2n] \rightarrow$ Not Invertible

$x_1[n]$  $y_1[n] =$ 

$x_1[n] = 1$ $y_1[n] =$ 

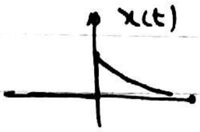
(c) $y(t) = x(2t)$ — Invertible

$z(t) = y(\frac{t}{2})$



Question 6

(a) $x(t) = e^{-8t}u(t)$



$$E_\infty = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \int_0^{\infty} e^{-16t} dt = \frac{-1}{16}(0-1) = \frac{1}{16}$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{+T} |x(t)|^2 dt = \lim_{T\to\infty}\frac{1}{2T}\int_0^T e^{-16t}dt$$

$$= \frac{-1}{16}\lim_{T\to\infty}\frac{1}{2T}(e^{-16T}-1) = \frac{0}{\infty} = 0$$

(b) $x(t) = e^{j(3t+\frac{\pi}{8})}$

$$|x(t)|^2 = \sqrt{\cos^2(\dots)+\sin^2(\dots)} = 1$$

$$E_\infty = \int_{-\infty}^{+\infty} dt = \infty$$

$$P_\infty = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} dt = \lim_{T\to\infty}\frac{1}{2T}2T = 1$$

(c) $x[n] = (\frac{1}{3})^n u[n]$

$$E_\infty = \sum_{n=-\infty}^{+\infty} |x[n]|^2 = \sum_{n=0}^{\infty}(\frac{1}{3})^{2n} = \sum_{n=0}^{\infty}(\frac{1}{9})^n$$

$$= 1 + \frac{1}{9} + \frac{1}{81} + \dots$$

$$= \frac{1}{1-\frac{1}{9}} = \frac{9}{8}$$

$$P_\infty = \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{+N}|x[n]|^2 = \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=0}^{N}(\frac{1}{9})^n = \frac{9/8}{\infty} = 0$$