۹۶۳۱۰۷۵ - علی تقوی

1

a) $x_1[n] = x[2-n] - x[1-n]$

$$x[-n+\alpha] \xleftrightarrow{F} e^{-j\omega\alpha} X(-\omega)$$

$$\Rightarrow x[2-n] - x[1-n] \xleftrightarrow{F} e^{-2j\omega} X(-\omega) - e^{-j\omega} X(-\omega)$$

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b) $x_2[n] = n^2 x[n] \xleftrightarrow{F} (j)^2 \dfrac{d^2}{d\omega^2} X(\omega)$

$$= -\frac{d^2}{d\omega^2} X(\omega)$$

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c) $x_3[n] = e^{j\omega_0 n} \sum_{k=-\infty}^{n} x[k]$

$$e^{j\omega_0 n} \xleftrightarrow{F} 2\pi \sum_{l=-\infty}^{\infty} \delta(\omega - \omega_0 - 2\pi l)$$

$$\sum_{k=-\infty}^{n} x[k] \xleftrightarrow{F} \frac{1}{1-e^{-j\omega}} X(\omega) + \pi X(\omega) \sum_{l=-\infty}^{\infty} \delta(\omega - 2\pi l)$$

$$x[n]y[n] \xleftrightarrow{F} \frac{1}{2\pi} \int_{2\pi} X(\theta) Y(\omega - \theta)\, d\theta$$

$$= \int_{2\pi} \left(\sum_{l=-\infty}^{\infty} \delta(\omega - \omega_0 - 2\pi l - \theta) \right) \left(\frac{1}{1-e^{-j\theta}} X(\theta) + \pi X(\theta) \sum_{l=-\infty}^{\infty} \delta(\theta - 2\pi l) \right)$$

a) $x[n] = \left(\frac{1}{۲}\right)^n u[n-۲]$

$$\alpha^n u[n] \xleftrightarrow{F} \frac{1}{1-\alpha e^{-j\omega}}$$

$$\Rightarrow \left(\frac{1}{۲}\right)^{n-۲} u[n-۲] \xleftrightarrow{F} e^{-۲j\omega}\left(\frac{1}{1-\frac{e^{-j\omega}}{۲}}\right)$$

$$\Rightarrow \left(\frac{1}{۲}\right)^{n} u[n-۲] \xleftrightarrow{F} \left(\frac{e^{-۲j\omega}}{۴\left(1-\frac{e^{-j\omega}}{۲}\right)}\right)$$

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b) $x[n] = \frac{(n+1)(n+۲)}{۲}\left(\frac{1}{۴}\right)^n u[n]$

$$= \frac{n(n+1)}{۲}\left(\frac{1}{۴}\right)^n u[n] + \left(\frac{1}{۴}\right)^n u[n]$$

$$\begin{cases} (n+1)\,\alpha^n u[n] \xleftrightarrow{F} \left(\frac{1}{1-\alpha e^{-j\omega}}\right)^۲ \\ \frac{n}{۲}\, x[n] \xleftrightarrow{F} \frac{j}{۲}\frac{d}{d\omega} X(\omega) \end{cases}$$

$$\Rightarrow X(\omega) = \frac{j}{۲}\frac{d}{d\omega}\left(\frac{1}{1-\frac{e^{-j\omega}}{۴}}\right)^۲ + \left(\frac{1}{1-\frac{e^{-j\omega}}{۴}}\right)^۲$$

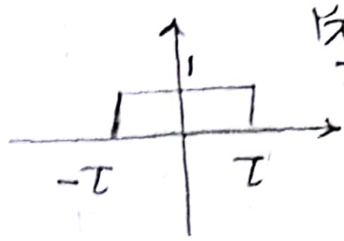
$$= ۸j\frac{d}{d\omega}\left(۴-e^{-j\omega}\right)^{-۲} + ۱۶\left(۴-e^{-j\omega}\right)^{-۲}$$

$$= ۱۶\left(۴-e^{-j\omega}\right)^{-۳} + ۱۶\left(۴-e^{-j\omega}\right)^{-۲}$$

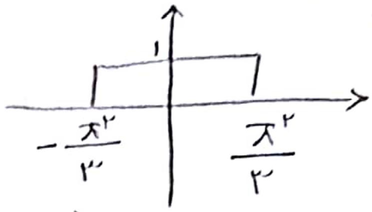
$$= ۱۶\left[\left(\frac{1}{۴-e^{-j\omega}}\right)^۳ + \left(\frac{1}{۴-e^{-j\omega}}\right)^۲\right]$$

c) $x[n] = \text{Sinc}\left(\frac{\pi}{۳}n\right)\text{Cos}\left(\frac{\pi}{۶}n\right)$

$$\frac{\tau}{\pi}\,\text{Sinc}\left(\frac{\tau}{\pi}n\right) \xleftrightarrow{F}$$



$$\Rightarrow \text{Sinc}\left(\frac{\pi}{۳}n\right) \xleftrightarrow{F} \frac{۳}{\pi}\Big($$



$$\Big)$$

$$\text{Cos}\left(\frac{\pi}{۶}n\right) \xleftrightarrow{F} \pi\sum_{l=-\infty}^{\infty}\left(\delta\left(\omega-\frac{\pi}{۶}-۲\pi l\right)+\delta\left(\omega+\frac{\pi}{۶}-۲\pi l\right)\right)$$

$$\Rightarrow X(\omega) = \frac{1}{۲\pi}\Big(\text{کانولوشن متناوب}\Big)$$

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d) $x[n] = u[n+۳] - u[n-۴]$

$$\xleftrightarrow{F}\left(e^{+۳j\omega}-e^{-۴j\omega}\right)\left[\frac{1}{1-e^{-j\omega}}+\sum_{k=-\infty}^{\infty}\pi\,\delta(\omega-۲\pi k)\right]$$

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e) $x[n] = \text{Sin}^۲\left[\frac{\pi}{۸}n\right] + \text{Cos}\left[\frac{\pi}{۸}n\right] + \text{Cos}^۲\left[\frac{\pi}{۸}n\right] = \text{Cos}\left[\frac{\pi}{۸}n\right]$

$$\xleftrightarrow{F}\pi\sum_{l=-\infty}^{\infty}\left(\delta\left(\omega-\frac{\pi}{۸}-۲\pi l\right)+\delta\left(\omega+\frac{\pi}{۸}-۲\pi l\right)\right)$$

۳

a) $$X(\omega) = \frac{۱}{۱-e^{-j\omega}} \; \frac{۲}{۲-e^{-j\omega}} + ۲\pi\,\delta(\omega)$$

$$\xleftrightarrow{F^{-1}} \sum_{k=-\infty}^{n} \left(\frac{۱}{۲}\right)^{k} u[k] + ۱$$

b) $$X(\omega) = \sum_{k=۰}^{\infty} e^{-j\omega k}$$

$$\xleftrightarrow{F^{-1}} \sum_{k=۰}^{\infty} \delta[n-k]$$

c) $$X(\omega) = \frac{۲-۱۲e^{-j\omega}}{۱-۱۲e^{-j\omega}+۳۵e^{-۲j\omega}}$$

$$= \frac{۲(۱-۶e^{-j\omega})}{(۱-۵e^{-j\omega})(۱-۷e^{-j\omega})} = \frac{۲}{(۱-۵e^{-j\omega})(۱-۷e^{-j\omega})(۱-۶e^{-j\omega})^{-۱}}$$

(۴)

a) $Y(\omega) + \frac{1}{۲} e^{-j\omega} Y(\omega) = X(\omega)$

$$\Rightarrow H(\omega) = \frac{1}{1 + \frac{e^{-j\omega}}{۲}}$$

b) $h[n] = \left(\frac{-1}{۲}\right)^n u[n]$

c) $y[n] = h[n] * x[n] = \sum_{k=۰}^{\infty} u[n-k] \left(\frac{-1}{۲}\right)^k \left(\frac{+1}{۲}\right)^{n-k}$

$$= \sum_{k=۰}^{n} (-1)^k \left(\frac{1}{۲}\right)^n = \begin{cases} \left(\frac{1}{۲}\right)^n & k \text{ زوج} \\ ۰ & k \text{ فرد} \end{cases}$$

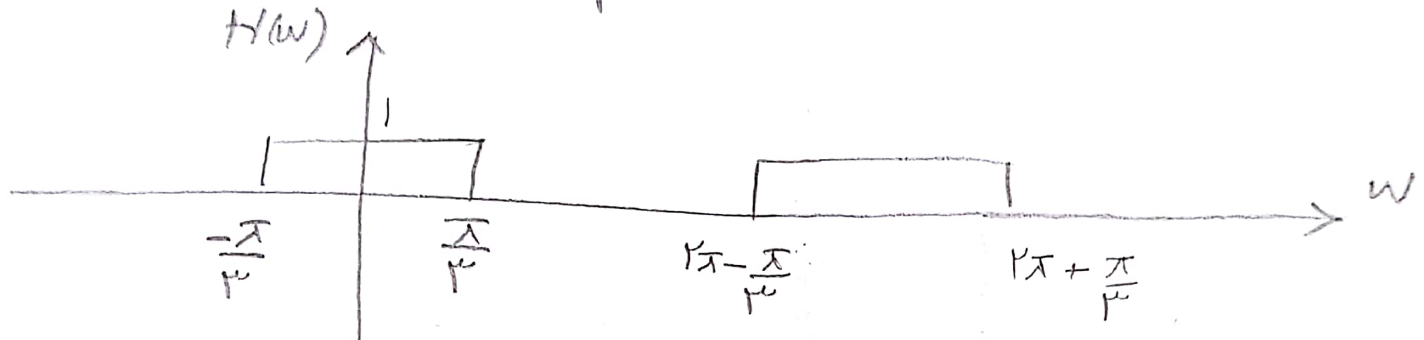
d) $y[n] = h[n] + \frac{1}{۲} h[n-1]$

$$= \left(\frac{-1}{۲}\right)^n u[n] + \frac{1}{۲}\left(\frac{-1}{۲}\right)^{n-1} u[n-1]$$

۵

a) $H(w) = \begin{cases} ۱ & |w| < \frac{\pi}{۳} \\ ۰ & \frac{\pi}{۳} < |w| < \pi \end{cases}$

با تناوب ۲π



(6)

$$x[n] = \cos\left[\frac{\pi}{3}n\right] + j\sin\left[\frac{2\pi}{3}n\right]$$

$$= \cos\left[\frac{\pi}{3}n\right] + j\sin\left[\frac{\pi}{3}n\right] = e^{\frac{\pi}{3}jn}$$

a) $$X(\omega) = 2\pi \sum_{l=-\infty}^{\infty} \delta\left(\omega - \frac{\pi}{3} - 2\pi l\right)$$

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b) $$H_1(\omega) = \begin{cases} 1 & 0 < \left|\omega + \frac{\pi}{2}\right| < \frac{\pi}{6} \\ 0 & \frac{\pi}{6} < \left|\omega + \frac{\pi}{2}\right| < \pi \end{cases}$$ تناوب 2π
