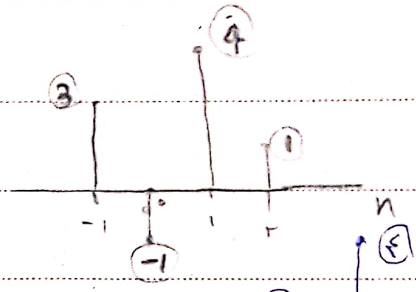
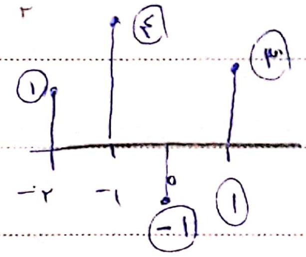
① $x[n] = \delta[n-2]$

$h[n] \longrightarrow$



(a) $\begin{cases} e^{j\pi k} & ۰ \leq n \leq ۳ \\ 0 & o.w \end{cases}$

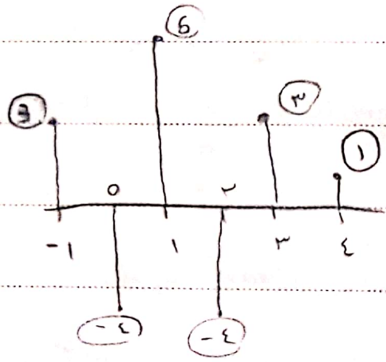
$h(-k)$



$$y[n] = \sum_{k=-\infty}^{+\infty} x[n] \cdot h[n] \longrightarrow \sum_{k=0}^{۳} x[k]\, h[n-k] = \sum_{k=0}^{۳} e^{j\pi k}\, h[n-k]$$

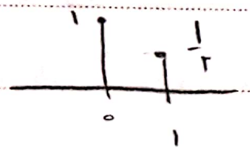
$$e^{j\pi k} = \cos(\pi k) + j\sin(\pi k) = (-1)^k$$

$$y[n] = h[n] - h[n-1] + h[n+2]$$



(b) $x[n] = (\frac{1}{۲})^n (u[n] - u[n-۲])$

$$x[n] = \begin{cases} (\frac{1}{۲})^n & ۰ \leq n < ۲ \\ 0 & o.w \end{cases} \rightarrow x[k] =$$



$n = [-1, 3]$ بازه غیر صفر بودن

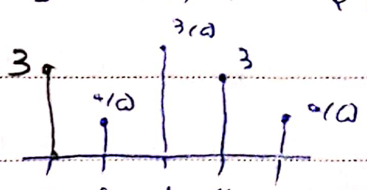
$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]\, h[n-k]$$

$y[-1] = 1 \times ۳ = ۳$ $\quad y[0] = (1 \times -1) + \frac{1}{۲}(۳) = ۰/۵$

$y[1] = 1 \times ۴ + (-1 \times \frac{1}{۲}) = ۳/۵$ $\quad y[۲] = (1 \times 1) + (۴ \times \frac{1}{۲}) = ۳$

$y[۳] = 1 \times \frac{1}{۲} = \frac{1}{۲}$

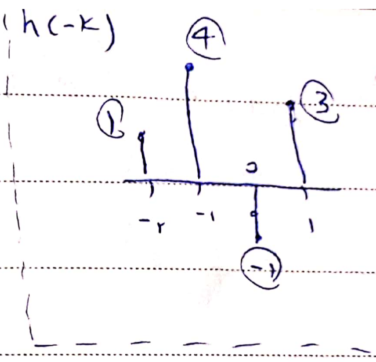
$y[n]$:



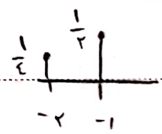
ⓒ $x[n] = ۲^{n} \left(u[n+۲] - u[n] \right)$

$x[k] = \begin{cases} ۲^{k} & -۲ \leq k < ۰ \\ ۰ & o.w \end{cases}$

بازه غیر صفر $[-۳ , ۱]$



$y[+1] = \frac{1}{۴} \times 1 = \frac{1}{۴}$ $\quad y[۰] = \frac{1}{۴} + \frac{1}{۲} \times ۴ = ۲٫۲۵$



$y[-1] = \frac{1}{۴} \times ۴ + \frac{1}{۲}(-1) = ۰٫۵$ $\quad y[-۲] = \frac{1}{۴}(-1) + \frac{1}{۲}(۳) = ۱٫۲۵$

$y[-۳] = \frac{1}{۴} \times ۳ = \frac{۳}{2}$, o.w $y[n] = ۰$

ⓓ $x[n] = e^{j\pi n} u[n]$ $\quad y[1] = ?$

$\begin{cases} e^{j\pi n} = \cos \pi n = (-1)^{n} & n \geq ۰ \\ ۰ & o.w \end{cases}$

$y[1] = (۴ \times 1) + (-1 \times -1) + ۳ \times 1 = ۸$

~~2. (a~~

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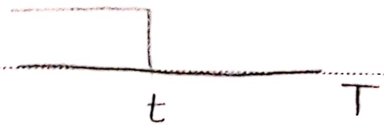
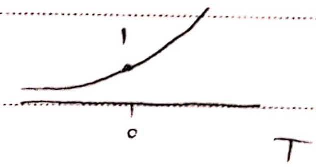
(1) (2)

(a) $h(t) = u(t) \quad x(t) = e^{t} \quad h(t-T) \rightarrow$

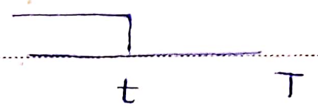


$$y(t) = \int_{-\infty}^{\infty} e^{T} h(t-T)\, dT$$

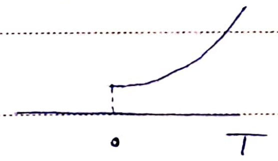


$$y(t) = \int_{-\infty}^{t} e^{T} dt = e^{T}\Big|_{-\infty}^{t} = e^{t}$$

(b) $h(t) = u(t) \quad x(t) = e^{t} u(t) \quad h(t-T)$

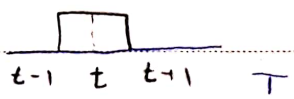


$t < ۰ \qquad y(t) = ۰ \qquad x(T)$



$t \geq ۰ \qquad \int_{۰}^{t} e^{T} dT = e^{t} - ۱$

(c) $h(t-T)$



$۱+t \leq -۲ \rightarrow t \leq -۳ \qquad y(t) = ۰$

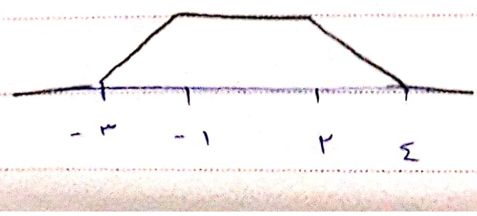
$-۲ \leq t+۱ \leq ۰ \rightarrow -۳ \leq t \leq -۱ \qquad y(t) = \int_{-۲}^{t+۱} dT = t+۳$

$-۲ \leq t-۱,\ t+۱ \leq ۳ \rightarrow -۱ \leq t \leq ۲ \qquad y(t) = \int_{t-۱}^{t+۲} dT = t+۲-t+۱ = ۳$

$t-۱ \leq ۳ \rightarrow t \leq ۴$

$\hookrightarrow \qquad y(t) = \int_{t-۱}^{۳} dT = ۳-t+۱ = ۴-t$

$t-۱ \geq ۳ \rightarrow t \geq ۴ \qquad y(t) = ۰$



(d)

$$y(t) = x(t) * h(t)$$

$$= x(t) * \left(\delta\left(t+\frac{۳}{۲}\right) + ۲\,\delta\left(t+\frac{۵}{۲}\right)\right)$$

$$= \left(x(t) * \delta\left(t+\frac{۳}{۲}\right)\right) + ۲\left(x(t) * \delta\left(t+\frac{۵}{۲}\right)\right)$$

$$y(t) = x\left(t+\frac{۳}{۲}\right) + ۲\,x\left(t+\frac{۵}{۲}\right)$$

$t \leq -\frac{۷}{۲}$ $\qquad y(t) = ۰$

$-\frac{۷}{۲} \leq t \leq -\frac{۵}{۲}$ $\qquad y(t) = ۲\left(۲\left(t+\frac{۵}{۲}\right)+۱\right) = ۴t + ۱۲$

$-\frac{۵}{۲} \leq t \leq -\frac{۳}{۲}$ $\qquad y(t) = \left(۲\left(t+\frac{۳}{۲}\right)+۱\right) + ۲\left(۲\left(t+\frac{۵}{۲}\right)+۱\right)$

$-\frac{۳}{۲} \leq t \leq -\frac{۱}{۲}$ $\qquad y(t) = \left(۲\left(t+\frac{۳}{۲}\right)+۱\right) + ۲\left(\ln\left(t+\frac{۵}{۲}\right)\right)$

$-\frac{۱}{۲} \leq t < \frac{۱}{۲}$ $\qquad y(t) = \ln\left(t+\frac{۳}{۲}\right) + ۲e^{t+\frac{۵}{۲}}$

$\frac{۱}{۲} \leq t < \frac{۳}{۲}$ $\qquad y(t) = e^{t+\frac{۳}{۲}}$

$\frac{۳}{۲} \leq t$ $\qquad y(t) = ۰$

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(3) (a) Linear: $x_1(t) \to y_1(t) = \frac{d(x_1(t))}{dt}$

$x_2(t) \to y_2(t) = \frac{d(x_2(t))}{dt}$

$a x_1(t) + b x_2(t) \longrightarrow \frac{d}{dt}(a x_1(t) + b x_2(t)) = a\frac{d x_1(t)}{dt} + b\frac{d(x_2(t))}{dt}$ ✓

TI =

$x_1(t)\ y_1(t) = \frac{d(x_1(t))}{dt}$

$y_2(t-T) = \frac{d(x_1(t-T))}{dt}$ ✓ TI

$y_1(t-T) = \frac{d(x_1(t-T))}{dt}$

(b) $h(t) = \frac{ds(t)}{dt}$

$s(t) = u(t) * h(t) = h(t) * u(t) = \int_{-\infty}^{\infty} h(t)\,u(t-T)\,dT$

$s(t) = \int_{-\infty}^{t} h(t)\,dT$

$\frac{d()}{d(t)}$

$\frac{ds(t)}{dt} = h(t)$

(7)

(a) $h(t) = e^{-4|t|}$ $\quad h(2) = e^{-8} \neq 0$ memoryless (✗)

$h(-3) = k e^{-12} \neq 0$ casual (✗)

$$\int_{-\infty}^{+\infty} e^{-4|t|} dt = 2\int_{0}^{+\infty} e^{-4t} dt = -\frac{2}{4} e^{-4t}\Big|_0^{\infty} = \frac{1}{2}$$ stable (✓)

(b) $h(t) = te^{-t}u(t)$ $\quad h(2) = 2e^{-2} \neq 0$ memoryless (✗)

$$h(t) = \begin{cases} 0 & t<0 \\ te^{-t} & t\geq 0 \end{cases}$$ casual (✓)

$$\int_{-\infty}^{\infty} |te^{-t}u(t)| dt = \int_0^{\infty} te^{-t} dt = -te^{-t} - e^{-t}\Big|_0^{\infty} = 1$$ stable (✓)

(c) $h(t) = \cos(2\pi t)u(t+1)$

$h(2) = \cos(4\pi) = 1 \neq 0$ memoryless (✗)

$h(-1) = \cos(-2\pi) = 1$ casual (✗)

$$\int_{-\infty}^{\infty} |\cos(2\pi t)u(t+1)| dt = \int_{-1}^{\infty} \cos 2\pi t \, dt = \frac{1}{2\pi}\sin 2\pi t\Big|_{-1}^{\infty} = 0$$

stable (✓)

(d) $h(t) = \frac{\sin(t)}{t} u(t)$

$h(\frac{\pi}{4}) = \frac{\frac{\sqrt{2}}{2}}{\frac{\pi}{4}} = \frac{2\sqrt{2}}{\pi} \neq 0$ memoryless (✗)

$$h(t) = \begin{cases} \frac{\sin(t)}{t} & t\geq 0 \\ 0 & o.w \end{cases}$$ casual (✓) $\quad \int_{-\infty}^{\infty} |\frac{\sin t}{t} u(t)| dt = \int_0^{\infty} |\frac{\sin t}{t}| dt$

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(e) $h[n] = (\frac{1}{2})^n u[-n]$

$h(-1) = 2 \neq 0$ memoryLess (✗) Casual (✗)

$\sum_{k=-\infty}^{\infty} (\frac{1}{2})^k u[-k] = \sum_{k=0}^{\infty} 2^k = \infty$ ~~stay~~ stable (✗)

(f) $h[n] = \delta[2n]$

$\delta[2n] = \delta[n] \rightarrow$ memoryLess (✓)

Casual (✓)

$\sum_{-\infty}^{\infty} \delta[2n] = 1$ stable (✓)

(g) $h[n] = \cos(\frac{\pi}{2}n)\, u[n+1]$

$\cos(\frac{\pi}{2}n) =$ $h(2) = \cos(\pi) = -1 \neq 0$ memoryLess (✗)

$u[n+1] = 0$ به ازای n های منفی ، $h(-1) = \cos(-\frac{\pi}{2}) = 0 \rightarrow$ casual (✓)

$\sum_{n=-\infty}^{\infty} |\cos(\frac{\pi}{2}n) u[n+1]| = \sum_{n=-1}^{\infty} \cos(\frac{\pi}{2}n) = 0$ stable (✓)

(h) $h[n] = e^{2n} u[n] \rightarrow h[2] = e^4 \neq 0$ memoryLess (✗)

$u[n] \xrightarrow{n<0} = 0$ Casual (✓)

$\sum_{n=-\infty}^{\infty} |e^{2n} u[n]| = \sum_{n=0}^{\infty} e^{2n} = \infty$ stable (✗)

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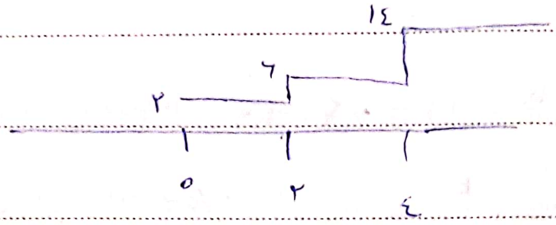
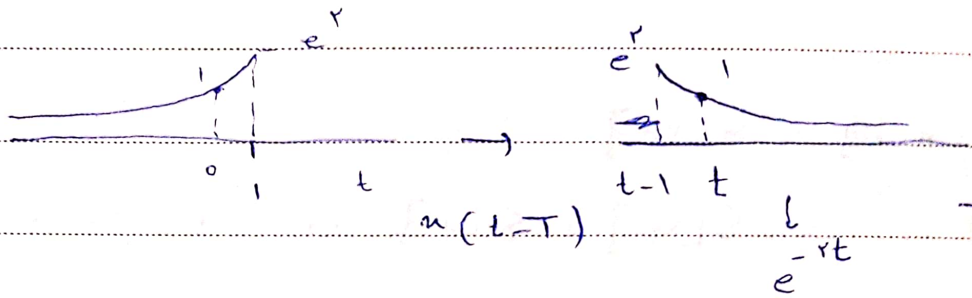
(5) $h(t) =$



$x(t) =$



$$h(t) * x(t) = \int_{-\infty}^{+\infty} h(T)\, x(t-T)\, dT =$$

$t-1 \geq 4 \rightarrow t \geq 5 \qquad \int_{t-1}^{\infty} 14 e^{-2T}\, dT$

$2 \leq t-1 \leq 4 \qquad 3 \leq t \leq 5 \qquad \int_{t-1}^{4} 6 e^{-2T}\, dT + \int_{4}^{\infty} 14 e^{-2T}\, dT$

$0 \leq t-1 \leq 2 \qquad 1 \leq t \leq 3 \qquad \int_{t-1}^{2} 2 e^{-2T}\, dT + \int_{2}^{4} 6 e^{-2T}\, dT + \int_{4}^{\infty} 14 e^{-2T}\, dT$

$t-1 \leq 0 \qquad t \leq 1 \qquad \int_{0}^{2} 2 e^{-2T}\, dT + \int_{2}^{4} 6 e^{-2T}\, dT + \int_{4}^{\infty} 14 e^{-2T}\, dT$

⑥ $y(t) = x(t) * h_{eq}(t)$ $\qquad h_2 = h_3 = \delta$

$h_4 = \delta(t-1)$

$y(t) = y_1(t) + y_3(t) + y_4(t)$

$y_1(t) = x(t) * h_1(t)$

$w(t) = x(t) * h_2(t)$

$y_3(t) = w(t) * h_3(t) = (x(t) * h_2(t)) * h_3(t)$

$y_4(t) = w(t) * h_4(t) = (x(t) * h_2(t)) * h_4(t)$

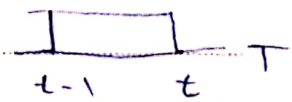
$y(t) = x(t) * h_1(t) + (x(t) * h_2(t)) * h_3(t) + (x(t) * h_2(t)) * h_4(t)$

$y(t) = x(t) * \Big(h_1(t) + (h_2(t) * h_3(t)) + (h_2(t) * h_4(t))\Big)$

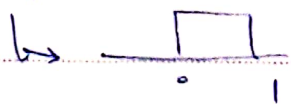
$\boxed{h_{eq}} = e^{-t}u(t) + G(t) + u(t-1) - u(t-2)$

$h_{eq} = h_2(t) * h_4(t) = h_2(t) * \delta(t-1) = h_2(t-1) = u(t-1) - u(t-2)$

$G(t) = h_2(t) * h_3(t) \rightarrow \int_{-\infty}^{\infty} h_2(\tau)\, h_3(t-\tau)\, d\tau$ $\qquad h_3(t-\tau)$



$h_2(t)$

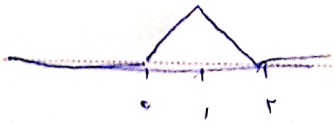


$t < 0 \qquad G(t) = 0$

$0 \leq t \leq 1 \qquad G(t) = \int_0^t d\tau = t$

$1 \leq t \leq 2 \qquad G(t) = \int_{t-1}^{1} d\tau = 2 - t$

$G(t)$:



$t > 2 \qquad G(t) = 0$

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(7) $$h[n] * x[n] = \sum_{k=0}^{\infty} \alpha^{k} x[n-k] = y[n]$$

$$h[n-1] * x[n] = \sum_{k=1}^{\infty} \alpha^{k} x(n-k) = \frac{1}{\alpha}\left(y[n] - x[n] \right)$$

$$\alpha\, y[n+1] = y[n+1] - x[n+1]$$

$$y[n+1] = \frac{-x[n+1]}{\alpha - 1} \Rightarrow y[n] = \frac{x[n]}{1-\alpha}$$

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8 (a) $h_1 = e^{-t} u(t)$ $\qquad h_2(t) = \delta(t) + \delta'(t)$

if $h_{1(t)} * h_{2(t)} = \delta(t)$ $\xrightarrow{\text{invertable}}$

$$h_{1(t)} * h_2(t) = \underbrace{\left(e^{-t} u(t) * \delta(t)\right)}_{e^{-t} u(t)} + \underbrace{\left(e^{-t} u(t) * \delta'(t)\right)}_{-e^{-t} u(t) + e^{-t} \delta(t)}$$

$$= e^{-t} \delta(t) \longrightarrow \begin{cases} 1 & t = 0 \\ 0 & t \neq 0 \end{cases} = \delta(t)$$

invertable ✓

(b) $U[n] * (\delta[n] - \delta[n-1]) = \underbrace{U[n]\,\delta[n]}_{U[n]} - U[n]\,\delta[n-1]$

$U[n] - U[n-1] = \delta[n]$