۹۶۳۱۰۷۵ علی نوری

1

a) $x(t) = \frac{d}{dt} x(1-t)$

$$\frac{d}{dt} x(t) \xleftrightarrow{F} j\omega X(\omega)$$

$$\frac{d}{dt} x(t+1) \xleftrightarrow{F} j\omega e^{j\omega} X(\omega)$$

$$\frac{d}{dt} x(-t+1) \xleftrightarrow{F} j\omega e^{j\omega} X(-\omega)$$

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b) $x(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

$$\int_{-\infty}^{t} x(\tau)\, d\tau \xleftrightarrow{F} \frac{X(\omega)}{j\omega} + \pi X(0)\, \delta(\omega)$$

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c) $x(t) = x(3-t) + 2x(t)$

$$\left.\begin{aligned} 2x(t) &\xleftrightarrow{F} 2X(\omega) \\ x(3-t) &\xleftrightarrow{F} e^{-3j\omega} X(-\omega) \end{aligned}\right\} \xrightarrow[F]{\oplus} e^{-3j\omega} X(-\omega) + 2X(\omega)$$

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d) $x(t) = \frac{d^2}{dt^2} x(t+2)$

$$\frac{d^2}{dt^2} x(t) \xleftrightarrow{F} (j\omega)^2 X(\omega) = -\omega^2 X(\omega)$$

$$\frac{d^2}{dt^2} x(t+2) \xleftrightarrow{F} -\omega^2 e^{2j\omega} X(\omega)$$

e) $x(t) = t^۲ x(۲t)$

$$x(۲t) \xleftrightarrow{F} \frac{1}{۲} X\left(\frac{\omega}{۲}\right)$$

$$(-jt)^۲ x(۲t) \xleftrightarrow{F} \frac{1}{۲} \frac{d^۲}{d\omega^۲} X\left(\frac{\omega}{۲}\right)$$

$$\Rightarrow t^۲ x(۲t) \xleftrightarrow{F} \frac{-1}{۲} \frac{d^۲}{d\omega^۲} X\left(\frac{\omega}{۲}\right)$$

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f) $x(t) = x(-۲t) * x(۳t)$

$$\left.\begin{array}{l} x(-۲t) \xleftrightarrow{F} \frac{1}{۲} X\left(\frac{-\omega}{۲}\right) \\ x(۳t) \xleftrightarrow{F} \frac{1}{۳} X\left(\frac{\omega}{۳}\right) \end{array}\right\} \xrightarrow{\text{ضرب}} \frac{1}{۶} X\left(\frac{-\omega}{۲}\right) X\left(\frac{\omega}{۳}\right)$$

۲

a) $x(t) = \sin(t) + \cos\left(\frac{\pi}{۲}t + \frac{\pi}{۴}\right)$

$$\sin(t) \xleftrightarrow{F} ۲\pi \times \frac{1}{۲j}\left(\delta(\omega-1) - \delta(\omega+1)\right)$$

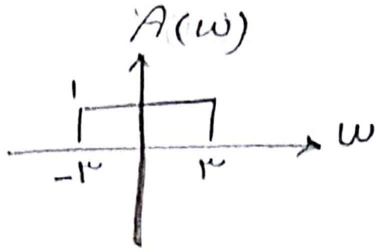
$$\Rightarrow \sin(t) \xleftrightarrow{F} \frac{\pi}{j}\left(\delta(\omega-1) - \delta(\omega+1)\right)$$

$$\cos\left(\frac{\pi}{۲}t + \frac{\pi}{۴}\right) = \underbrace{\frac{1}{۲} e^{-j\frac{\pi}{۴}}}_{a_{-1}} e^{-j\frac{\pi}{۲}t} + \underbrace{\frac{1}{۲} e^{j\frac{\pi}{۴}}}_{a_1} e^{j\frac{\pi}{۲}t}$$

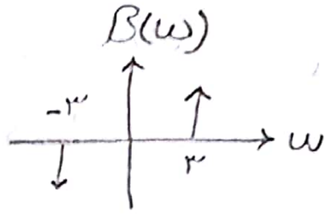
$$\xrightarrow{F} \pi\left(e^{-j\frac{\pi}{۴}} \delta\left(\omega + \frac{\pi}{۲}\right) + e^{j\frac{\pi}{۴}} \delta\left(\omega - \frac{\pi}{۲}\right)\right)$$

b) $x(t) = 2\,\dfrac{\sin^2(3t)}{t}$

$$\frac{\sin(3t)}{\pi t} \xleftrightarrow{F}$$



$$\sin(3t) \xleftrightarrow{F} \frac{\pi}{j}\left(\delta(\omega-3) - \delta(\omega+3)\right)$$



$$2\pi \sin(3t)\,\frac{\sin(3t)}{\pi t} \xleftrightarrow{F} \frac{A(\omega) * B(\omega)}{2\pi}$$

$$= \frac{\pi}{2j\pi}\int_{-\infty}^{\infty}\left[\delta(\eta-3) - \delta(\eta+3)\right] A(\omega-\eta)\,d\eta$$

$$= \frac{1}{2j}\left[A(\omega-3) - A(\omega+3)\right]$$

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c) $x(t) = \left(t^2 e^{-t}\cos(t)\right)u(t)$

$$\cos(t) \xleftrightarrow{F} \pi\left[\delta(\omega-1) + \delta(\omega+1)\right] = A(\omega)$$

$$t^2 e^{-t} \xleftrightarrow{F} 2\left(\frac{1}{j\omega+1}\right)^3 = B(\omega)$$

$$x(t) \xleftrightarrow{F} \frac{1}{2\pi}A(\omega) * B(\omega) = \frac{\pi}{2\pi}\left[B(\omega-1) + B(\omega+1)\right]$$

$$= \left[\left(\frac{1}{j(\omega-1)+1}\right)^3 + \left(\frac{1}{j(\omega+1)+1}\right)^3\right]$$

d) $x(t) = \dfrac{۲t}{۱+t^۲}$

دوگانی $\quad Y(t) \xleftrightarrow{F} ۲\pi y(-\omega)$

$$e^{-a|t|} \xleftrightarrow{F} \frac{۲a}{a^۲+\omega^۲} \Rightarrow e^{-|t|} \longleftrightarrow \frac{۲}{\omega^۲+۱}$$

$$\xrightarrow{\text{دوگانی}} \frac{۲}{۱+t^۲} \xleftrightarrow{F} ۲\pi e^{-|\omega|}$$

$$x(t) = t\left(\frac{۲}{۱+t^۲}\right) \xleftrightarrow{F} ۲\pi j \frac{d}{d\omega} e^{-|\omega|} = -۲\pi j e^{-|\omega|}$$

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e) $x(t) = \displaystyle\int_{-\infty}^{t} \frac{d\tau}{۱+\tau^۲}$

همانند قبل دوگانی : $\dfrac{۱}{۱+t^۲} \xleftrightarrow{F} \pi e^{-|\omega|}$

$$\Rightarrow \int_{-\infty}^{t} \frac{d\tau}{۱+\tau^۲} \xleftrightarrow{F} \frac{\pi e^{-|\omega|}}{j\omega} + \pi^۲ \delta(\omega)$$

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f) $x(t) = e^{-۳|t| + j\frac{\pi}{۴}t}$

$$\left.\begin{aligned} e^{-۳|t|} &\xleftrightarrow{F} \frac{۶}{۹+\omega^۲} \\ e^{j\frac{\pi}{۴}t} &\xleftrightarrow{F} ۲\pi\,\delta\left(\omega - \frac{\pi}{۴}\right) \end{aligned}\right\} \xrightarrow[\div ۲\pi]{\text{Conv}} \frac{۶}{۹+\left(\omega - \frac{\pi}{۴}\right)^۲}$$

g) $x(t) = \begin{cases} 1-t & , -1<t<1 \\ 0 & , \text{o.w} \end{cases}$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt = \int_{-1}^{1} (1-t) e^{-j\omega t} dt$$

$$= \left[\frac{-e^{-j\omega t}}{j\omega} - \frac{e^{-j\omega t}}{\omega^2} + \frac{t e^{-j\omega t}}{j\omega} \right]_{-1}^{1}$$

$$= e^{-j\omega} \left(\frac{-1}{j\omega} - \frac{1}{\omega^2} + \frac{1}{j\omega} \right) - e^{j\omega} \left(\frac{-1}{j\omega} - \frac{1}{\omega^2} - \frac{1}{j\omega} \right)$$

$$= \frac{-e^{-j\omega}}{\omega^2} + e^{j\omega} \left(\frac{2}{j\omega} + \frac{1}{\omega^2} \right)$$

۳

a) $X(\omega) = 3\delta(\omega - 3)$

$$2\pi \delta(\omega - 3) \xleftrightarrow{F^{-1}} e^{3jt}$$

$$\Rightarrow 3\delta(\omega - 3) \xleftrightarrow{F^{-1}} \frac{3}{2\pi} e^{3jt}$$

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b) $X(\omega) = \pi e^{-|\omega|}$

$$e^{-|t|} \xleftrightarrow{F} \frac{2}{1+\omega^2}$$

دوگان

$$\Rightarrow \frac{2}{1+t^2} \xleftrightarrow{F} 2\pi e^{-|\omega|}$$

$$\Rightarrow \frac{1}{1+t^2} \xleftrightarrow{F} \pi e^{-|\omega|} \xleftrightarrow{F^{-1}} \frac{1}{1+t^2}$$

c) $X(\omega) = \dfrac{۷j\omega + ۳۲}{-\omega^{۲} + ۹j\omega + ۲۰}$

$= \dfrac{۷j\omega + ۳۲}{(\omega j + ۴)(\omega j + ۵)} = \dfrac{۴}{\omega j + ۴} + \dfrac{۳}{\omega j + ۵}$

$\xleftrightarrow{F^{-1}} \left(۴e^{-۴t} + ۳e^{-۵t}\right) u(t)$

---

d) $X(\omega) = \begin{cases} \cos(\omega) & , -\pi < \omega < \pi \\ ۰ & , \text{O.W} \end{cases}$

$x(t) = \frac{1}{۲\pi}\int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega = \frac{1}{۲\pi}\int_{-\pi}^{\pi} \cos(\omega) e^{j\omega t} d\omega$

$= \frac{1}{۴\pi}\int_{-\pi}^{\pi} e^{j\omega(t+1)} + e^{j\omega(t-1)} d\omega$

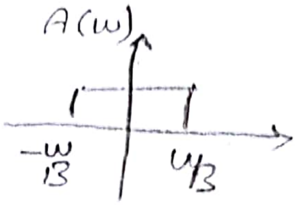
$= \frac{1}{۴\pi}\left[\frac{e^{j\omega(t+1)}}{j(t+1)} + \frac{e^{j\omega(t-1)}}{j(t-1)}\right]_{-\pi}^{\pi}$

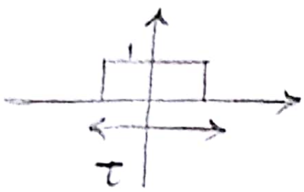
$= \frac{-e^{j\omega t}}{۴\pi(t^{۲}-1)}\left[(t-1)e^{j\omega} + (t+1)e^{-j\omega}\right]_{-\pi}^{\pi}$

$= \frac{-e^{j\omega t}}{۴\pi(t^{۲}-1)}\left[(1-t-t-1) - (-t+1-t-1)\right]$

$= ۰$

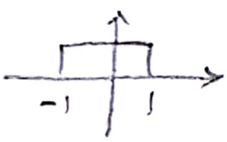
e) $X(\omega) = \dfrac{۲\,Sin(\omega - ۱)}{\omega - ۱} * \dfrac{Sin(۲\omega)}{\omega}$

$\dfrac{Sin(\omega_B t)}{\pi t} \overset{F}{\longleftrightarrow}$ 



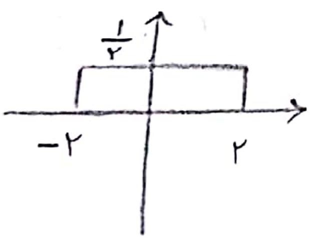
$\overset{F}{\longleftrightarrow} B(\omega) = \tau\, Sinc\left(\dfrac{\tau\omega}{۲\pi}\right) = \dfrac{۲\,Sin\left(\frac{\omega\tau}{۲}\right)}{\omega}$

$\xrightarrow{\tau = ۲} \dfrac{۲\,Sin(\omega)}{\omega}$



$\overset{F}{\longleftrightarrow} \dfrac{۲\,Sin(\omega)}{\omega}$

$\Rightarrow [u(t+۱) - u(t-۱)] \times e^{j\omega} \overset{F}{\longleftrightarrow} \dfrac{۲\,Sin(\omega - ۱)}{\omega - ۱}$



$\overset{F}{\longleftrightarrow} \dfrac{Sin(۲\omega)}{\omega}$

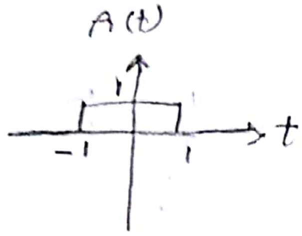
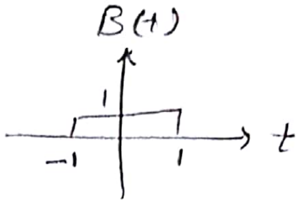
$\Rightarrow \dfrac{۱}{۲}[u(t+۲) - u(t-۲)] \overset{F}{\longleftrightarrow} \dfrac{Sin(۲\omega)}{\omega}$

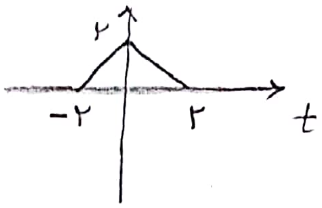
$\xRightarrow{\text{ضرب}} x(t) = \dfrac{۱}{۲} e^{j\omega} [u(t+۱) - u(t-۱)]$

f) $X(\omega) = \dfrac{\sin^2(-\omega)}{\omega^2} = \dfrac{\sin(-\omega)}{\omega} \times \dfrac{\sin(-\omega)}{\omega}$

$$\frac{\sin(-\omega)}{\omega} \xleftrightarrow{F^{-1}} \frac{-1}{2}\left[u(t+1) - u(t-1)\right] = A(t) = B(t)$$

$$\xRightarrow{\text{ضرب}} x(t) = A(t) * B(t)$$

$\frac{-1}{2}$  $*$ $\frac{-1}{2}$ 

$= \frac{1}{4}$ 

$$\Rightarrow x(t) = \begin{cases} \frac{1}{4}(t+2) & -2 \leq t < 0 \\ \frac{1}{4}(-t+2) & 0 \leq t \leq 2 \\ 0 & \text{o.w} \end{cases}$$

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g) $X(\omega) = \dfrac{d}{d\omega}\left[\dfrac{\sin(\pi\omega) - j\cos(\pi\omega)}{1 + 2j\omega}\right]$

$$= \frac{1}{2j}\,\frac{d}{d\omega}\left[\frac{e^{j\pi\omega}}{\frac{1}{2} + j\omega}\right]$$

$* \; \delta(t+\pi) \xleftrightarrow{F} e^{j\pi\omega}$

$* \; e^{-\frac{t}{2}} u(t) \xleftrightarrow{F} \dfrac{1}{\frac{1}{2} + j\omega}$

$\xrightarrow{\text{کانولوشن}} e^{-\frac{(t+\pi)}{2}} u(t+\pi)$

$$(-jt)\, y(t) \xleftrightarrow{F} \frac{d}{d\omega} Y(\omega)$$

$$\Rightarrow x(t) = \frac{-t}{2}\, e^{-\frac{(t+\pi)}{2}} u(t+\pi)$$

h) $X(w) = \dfrac{1}{(1+jw)^{۶}}$

$$\xleftrightarrow{F^{-1}} \frac{t^{۵}}{۵!} e^{-t} u(t)$$

۴

$$y(t) = h(t) * x(t) \longleftrightarrow Y(w) = H(w)X(w)$$

a) $H(w) = ?$

$$-(jw)^{۲} Y(w) - ۷ jw Y(w) - ۱۰ Y(w) = ۲jwX(w) + ۱۳X(w)$$

$$Y(w)\left(w^{۲} - ۷jw - ۱۰\right) = X(w)\left(۲jw + ۱۳\right)$$

$$\Rightarrow H(w) = \frac{۲jw + ۱۳}{w^{۲} - ۷jw - ۱۰}$$

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b) $$H(w) = \frac{۲jw + ۱۳}{-(۲ + wj)(۵ + wj)}$$

$$= \frac{-۳}{۲ + wj} + \frac{1}{۵ + wj}$$

$$e^{-at} u(t) \xleftrightarrow{F} \frac{1}{a + jw}$$

$$\Rightarrow h(t) = \left(-۳e^{-۲t} + e^{-۵t}\right) u(t)$$

c)

$$y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} (t-\tau)u(t-\tau)u(t)\left(-3e^{-3t+\tau} + e^{-6t+\tau}\right)d\tau$$

$$= \int_{0}^{t} (t-\tau)\left(-3e^{-3t+\tau} + e^{-6t+\tau}\right)d\tau$$

$$= \left(-3e^{-3t} + e^{-6t}\right)\int_{0}^{t} (t-\tau)e^{\tau}\,d\tau$$

$$= \left(-3e^{-3t} + e^{-6t}\right)\left[te^{\tau} - \tau e^{\tau} + e^{\tau}\right]_{0}^{t}$$

$$= \left(-3e^{-3t} + e^{-6t}\right)\left(e^{t} - t - 1\right)$$

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d) $H(\omega)G(\omega) = 1 \Rightarrow G(\omega) = \dfrac{1}{H(\omega)}$

$$= \frac{\omega^2 - 7j\omega - 10}{2j\omega + 13} = \frac{-\left(\omega j + \frac{13}{2}\right)\left(\omega j + \frac{1}{2}\right) - \frac{27}{4}}{2\left(j\omega + \frac{13}{2}\right)}$$

$$= -\frac{1}{2}\omega j - \frac{1}{4} - \frac{\frac{27}{8}}{j\omega + \frac{13}{2}}$$

$$\Rightarrow g(t) = \frac{-1}{2}\frac{d}{dt}\delta(t) - \frac{1}{4}\delta(t) - \frac{27}{8}e^{-\frac{13}{2}t}u(t)$$

$$Y(w) = X(w)\,G(w)$$

$$= \left(\frac{-۱}{۲} wj - \frac{۱}{۴} - \frac{\frac{۲۷}{۸}}{jw + \frac{۱۳}{۲}}\right) X(w)$$

$$\Rightarrow ۸\,Y(w) = -۴wjX(w) - ۲X(w) - \frac{۲۷X(w)}{jw + \frac{۱۳}{۲}}$$

$$\Rightarrow ۸jw\,Y(w) + ۵۲\,Y(w) = ۴w^۲X(w) - ۲۶wjX(w)$$

$$-۲jwX(w) - ۱۳X(w) - ۲۷X(w)$$

$$\Rightarrow ۸jw\,Y(w) + ۵۲\,Y(w) = ۴w^۲X(w) - ۲۸wjX(w) - ۴۰X(w)$$

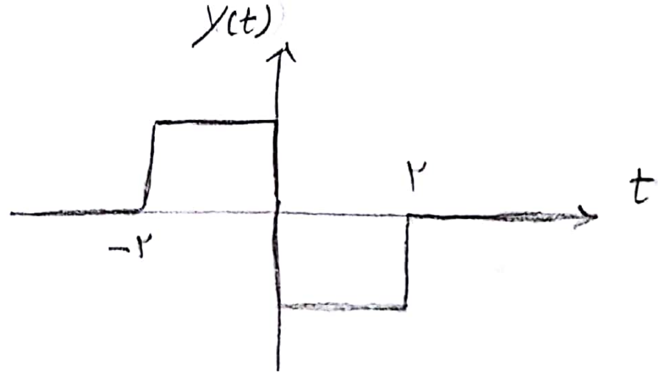
$$\Rightarrow ۸\frac{d}{dt}y(t) + ۵۲\,y(t) = -۴\frac{d^۲}{dt^۲}x(t) - ۲۸\frac{d}{dt}x(t) - ۴۰x(t)$$

$$X(w) = ?$$

۵

a)

$$y(t) = \frac{d}{dt}x(t)$$



$$Y(w) = jw\,X(w)$$

$$y(t) = \Pi_۲(t+۱) - \Pi_۲(t-۱)$$

$$\Pi_\tau(t) \xleftrightarrow{F} ۲\frac{\sin\left(\frac{w\tau}{۲}\right)}{w}$$

$$\Rightarrow Y(w) = ۲e^{jw}\frac{\sin(w)}{w} - ۲e^{-jw}\frac{\sin(w)}{w}$$

$$\Rightarrow X(w) = \frac{۲\sin(w)}{jw^۲}\left(e^{jw} - e^{-jw}\right) = \left(\frac{۲\sin(w)}{w}\right)^۲$$

b)
$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$= \int_{-\infty}^{-3} (-1) e^{-j\omega t} dt + \int_{-2}^{-1} (-1) e^{-j\omega t} dt + \int_{-1}^{1} t e^{-j\omega t} dt$$

$$+ \int_{1}^{2} e^{-j\omega t} dt + \int_{2}^{3} 2 e^{-j\omega t} dt + \int_{3}^{\infty} 3 e^{-j\omega t} dt$$

یکی از این انتگرال ها واگرا است $\Leftarrow$ $X(\omega)$ موجود نمی باشد.

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(۶)

$$x(t) = \frac{\sin(2t)}{\pi t} \qquad p(t) = e^{2jt}$$

$$q(t) = e^{-2jt} \frac{\sin(2t)}{\pi t} \qquad r(t) = \frac{1}{2}\delta(2t)$$

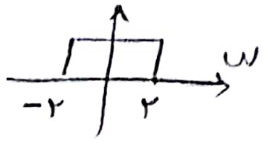
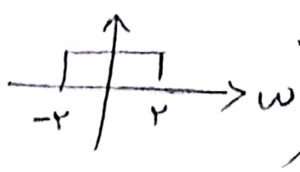
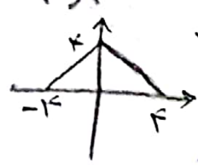
$$h(t) = \begin{cases} 1 & |t|<1 \\ 0 & \text{o.w} \end{cases}$$

$$a(t) = x(t)p(t) = e^{2jt}\frac{\sin(2t)}{\pi t} = \frac{2e^{2jt}}{\pi} \frac{\sin(\frac{2t}{\pi}\pi)}{\frac{2t}{\pi}\pi}$$

$$\Rightarrow a(t) = \frac{2e^{2jt}}{\pi} \operatorname{Sinc}\left(\frac{2t}{\pi}\right) = e^{2jt} \frac{2}{\pi} \operatorname{Sinc}\left(\frac{2}{\pi} t\right)$$

$$\xleftrightarrow{F} \Pi_4(\omega-2) = u(\omega) - u(\omega-4) = A(\omega)$$

$$b(t) = a(t)q(t) = \left(\frac{\sin(2t)}{\pi t}\right)^2 \xleftrightarrow{F} \frac{\Pi_4(\omega) * \Pi_4(\omega)}{2\pi}$$

$\frac{1}{2\pi}\Big($  $*$  $\Big) = \frac{1}{2\pi}\Big($  $\Big)$

$$\Rightarrow B(\omega) = \begin{cases} \frac{t+4}{2\pi} & -4 \leq t < 0 \\ \frac{-t+4}{2\pi} & 0 \leq t \leq 4 \end{cases}, \quad \text{بقیه جاها} = 0$$

$$c(t) = b(t) * h(t) \longleftrightarrow C(\omega) = B(\omega)H(\omega)$$

$$h(t) = \Pi_2(t) \Rightarrow H(\omega) = \frac{2\,\text{Sin}(\omega)}{\omega}$$

$$C(\omega) = \begin{cases} \dfrac{(t+4)\,\text{Sin}(\omega)}{\omega\pi} & -4 \leqslant t \leqslant 0 \\ \dfrac{(-t+4)\,\text{Sin}(\omega)}{\omega\pi} & 0 \leqslant t \leqslant 4 \end{cases} \quad , \quad \text{بقیه جاها} = 0$$

$$D(\omega) = C(\omega) + R(\omega)$$

$$r(t) = \frac{1}{2}\delta(2t) \xleftrightarrow{F} \frac{1}{4}$$

$$\Rightarrow D(\omega) = C(\omega) + \frac{1}{4} = \begin{cases} C(\omega) + \frac{1}{4} & -4 \leqslant t \leqslant 0 \\ C(\omega) + \frac{1}{4} & 0 \leqslant t \leqslant 4 \\ \frac{1}{4} & \text{o.w} \end{cases}$$

$$\int_{-\infty}^{\infty} |x(t)|^2\, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2\, d\omega$$ (۷)

a) $\int_{-\infty}^{\infty} \left(\frac{1}{a^2+\omega^2}\right)^2 d\omega$

$$X(\omega) = \frac{1}{a^2+\omega^2} \xleftrightarrow{F^{-1}} x(t) = \frac{1}{2a} e^{-a|t|}$$

$$\Rightarrow \text{جواب} = 2\pi \int_{-\infty}^{\infty} \left| \frac{e^{-a|t|}}{2a} \right|^2 dt$$

$$= \frac{2\pi}{4a^2} \times 2 \int_{0}^{\infty} e^{-2at}\, dt = \frac{-\pi}{2a^3}$$

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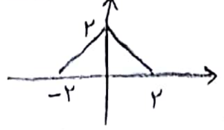
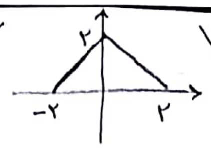
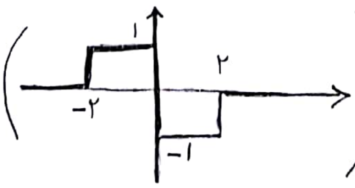
b) $\int_{-\infty}^{\infty} \left(t\left(\frac{\text{Sin}(t)}{\pi t}\right)^2\right)^2 dt$

$$\frac{\text{Sin}(t)}{\pi t} \longleftrightarrow \Pi_2(\omega) \Rightarrow \left(\frac{\text{Sin}(t)}{\pi t}\right)^2 \longleftrightarrow \frac{1}{2\pi}\Pi_2(\omega) * \Pi_2(\omega)$$

$$= \frac{1}{2\pi}\Bigg($$  $$\Bigg)$$

$$\Rightarrow t\left(\frac{\text{Sin}(t)}{\pi t}\right)^2 \xleftrightarrow{F} j\frac{d}{d\omega}\Bigg(\frac{1}{2\pi}\Bigg($$  $$\Bigg)\Bigg)$$

$$= \frac{j}{2\pi}\Bigg($$  $$\Bigg)$$

$$\Rightarrow \text{جواب} = \frac{j}{4\pi^2}\int_{-\infty}^{\infty}\Bigg($$  $$\Bigg)^2 d\omega = \frac{j}{\pi^2}$$