1- a) $x(t) \xrightarrow{F} X(\omega) \Rightarrow x(-t) \xrightarrow{F} X(-\omega) \Rightarrow x(1-t) \xrightarrow{F} e^{-\omega j} X(-\omega)$

$\Rightarrow \frac{d}{dt} x(1-t) \xrightarrow{F} \boxed{-\omega e^{-\omega j} X(-\omega)}$

b) $x(t) \xrightarrow{F} X(\omega) \Rightarrow \int_{-\infty}^{t} x(\tau)d\tau \xrightarrow{F} \frac{X(\omega)}{\omega} + \pi X(0)\,\delta\left(\frac{\omega}{j}\right)$

c) $x(t) \xrightarrow{F} X(\omega) \Rightarrow x(-t) \xrightarrow{F} X(-\omega) \Rightarrow x(3-t) \xrightarrow{F} e^{-3\omega j} X(-\omega)$

$\hookrightarrow 2x(t) \xrightarrow{F} 2X(\omega)$

$x(3-t) + 2x(t) \xrightarrow{F} e^{-3\omega j} X(-\omega) + 2X(\omega)$

d) $x(t) \xrightarrow{F} X(\omega) \Rightarrow x(t+2) \xrightarrow{F} e^{2\omega j} X(\omega) \Rightarrow \frac{d^2}{dt^2} x(t+2) \xrightarrow{F} \omega^2 e^{2\omega j} X(\omega)$

e) $x(t) \xrightarrow{F} X(\omega) \Rightarrow x(2t) \xrightarrow{F} \frac{1}{2} X\left(\frac{\omega}{2}\right) \Rightarrow t^2 x(2t) \xrightarrow{F} \frac{-1}{2} \frac{d^2}{d\omega^2} X\left(\frac{\omega}{2}\right)$

f) $x(t) \xrightarrow{F} X(\omega) \Rightarrow x(-2t) \xrightarrow{F} \frac{1}{2} X\left(\frac{-\omega}{2}\right)$

$\hookrightarrow x(3t) \xrightarrow{F} \frac{1}{3} X\left(\frac{\omega}{2}\right)$

$\xrightarrow{*} x(-2t) * x(3t) \xrightarrow{F} \frac{1}{6} X\left(\frac{-\omega}{2}\right) \times X\left(\frac{\omega}{3}\right)$

2- a) $\sin(t) \xrightarrow{F} \frac{\pi}{j}\left[\delta(\omega-1) - \delta(\omega+1)\right]$

$$\cos(t) \xrightarrow{F} \underbrace{\pi\left[\delta(\omega-1)+\delta(\omega+1)\right]}_{X(\omega)} \Rightarrow \cos\left(\frac{\pi t}{2}\right) \xrightarrow{F} 2X\left(\frac{2\omega}{\pi}\right) \Rightarrow \cos\left(\frac{\pi t}{2}+\frac{\pi}{4}\right) \xrightarrow{F} 2e^{+\frac{j\omega\pi}{4}} \times X\left(\frac{2\omega}{\pi}\right)$$

$$x(t) \xrightarrow{F} \frac{\pi}{j}\left[\delta(\omega-1)+\delta(\omega+1)\right] + 2e^{+\frac{j\omega\pi}{4}}\left[\delta\left(\frac{2\omega}{\pi}-1\right)+\delta\left(\frac{2\omega}{\pi}+1\right)\right]$$

b) $\sin(3t) \xrightarrow{F} \overbrace{\frac{\pi}{j}\left[\delta(\omega-3) - \delta(\omega+3)\right]}^{X(\omega)}$

$\pi \times \frac{2\sin(3t)}{\pi t} \xrightarrow{F} \underbrace{2\pi \times \left[u(\omega+3) - u(\omega-3)\right]}_{Y(\omega)}$

$$x(t) \xrightarrow{F} \frac{\pi}{j} \times \left(X(\omega) * Y(\omega)\right)$$

$$= \frac{\pi}{j}\left(\cancel{u(\omega)} - u(\omega-6) + u(\omega+6) - \cancel{u(\omega)}\right)$$

c) $2 \times \frac{t^2 e^{-t} u(t)}{2} \xrightarrow{F} \overset{X(\omega)}{\frac{2}{(1+j\omega)^3}}$

$\cos(t) \xrightarrow{F} \underbrace{\pi\left[\delta(\omega-1)+\delta(\omega+1)\right]}_{Y(\omega)}$

$$x(t) = \frac{2\cancel{\pi}}{2\cancel{\pi}}\left(\frac{1}{(1+j\omega)^3} * \left(\delta(\omega-1)+\delta(\omega+1)\right)\right)$$

$$= \frac{1}{(1+j(\omega-1))^3} + \frac{1}{(1+j(\omega+1))^3}$$

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2-d) $\frac{2 \cdot t}{1^2+t^2} \xrightarrow{\mathcal{F}} \frac{-2j\pi e^{-|w|}}{2} = -j\pi e^{-|w|}$

e) $\frac{1}{1+t^2} \xrightarrow{\mathcal{F}} \pi e^{-|w|} \Rightarrow \int_{-\infty}^{t} \frac{d\tau}{1+\tau} \xrightarrow{\mathcal{F}} \frac{\pi e^{-|w|}}{jw} + \pi^2 \delta(\omega)$

f) $e^{-3|t|} \xrightarrow{\mathcal{F}} \frac{6}{\omega^2+9}$ $\quad u(t) \xrightarrow{\mathcal{F}} \frac{6}{(\omega-\frac{\pi}{6})^2+9}$

$e^{j\frac{\pi}{6}t} \xrightarrow{\mathcal{F}} 2\pi\,\delta(\omega-\frac{\pi}{6})$

g) $\int_{-1}^{1} (1-t)\, e^{-j\omega t}\, dt = \int_{-1}^{1} e^{-j\omega t}\, dt - \int_{-1}^{1} t e^{-j\omega t}\, dt$

t	$e^{-j\omega t}$
1	$\frac{-1}{j\omega} e^{-j\omega t}$
0	$\frac{-1}{\omega^2} e^{-j\omega t}$

$= \frac{-1}{j\omega} e^{-j\omega t} + \frac{t}{j\omega} e^{-j\omega t} - \frac{1}{\omega^2} e^{-j\omega t} \Big|_{-1}^{1}$

$= \left(\frac{-1}{j\omega} + \frac{t}{j\omega} - \frac{1}{\omega^2}\right) e^{-j\omega t} \Big|_{-1}^{1} = \boxed{-\frac{1}{\omega^2} e^{-j\omega} + \left(\frac{2}{j\omega} + \frac{1}{\omega^2}\right) e^{j\omega}}$

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3- a) $\frac{3}{2\pi}\times 2\pi\,\delta(w-3) \xrightarrow{F^{-1}} \boxed{\frac{3}{2\pi}\, e^{+j3t}}$

b) $\frac{\pi}{1}\times e^{-1\times|w|} \xrightarrow{F^{-1}} \boxed{\frac{1}{1^2+t^2}}$

c) $\frac{7jw+32}{\left(\frac{9}{2}+jw\right)^2-\frac{1}{4}} = \frac{3}{5+jw} + \frac{4}{4+jw} \xrightarrow{F^{-1}} \left(3e^{-5t}+4e^{-4t}\right)u(t)$

d) $\frac{1}{2\pi}\int_{-\pi}^{\pi}\cos(w)\,e^{jtw}\,dw = \frac{1}{2\pi}\times\frac{-je^{j\pi t}t+je^{-j\pi t}t}{1-t^2} = \frac{jt}{2\pi(1-t^2)}\left(e^{-j\pi t}+e^{j\pi t}\right)$

$= \frac{jt}{\pi(1-t^2)}\cos(\pi t)$

e) $\frac{2\sin(w-1)}{w-1} \xrightarrow{F^{-1}} x(t) \Rightarrow \frac{2\sin(w)}{w} \xrightarrow{F^{-1}} \frac{1}{2\pi}\times e^{jt}x(t) = u(t+1)-u(t-1)$

$\Rightarrow x(t) = 2\pi e^{jt}\left(u(t+1)-u(t-1)\right)$ $\xrightarrow{\times} \boxed{4\pi e^{jt}\left(u(t+1)-u(t-1)\right)}$

$2\times\frac{\sin(2w)}{2w} \xrightarrow{F^{-1}} 2\left(u(t+2)-u(t-2)\right)$

f) $\frac{\sin^2(2w)}{w^2} \xrightarrow{F^{-1}} x(t) \Rightarrow \frac{\sin^2(w)}{w^2} \xrightarrow{F^{-1}} x(-t) \Rightarrow \frac{\sin^2\left(\frac{w}{2\pi}\right)}{\left(\frac{w}{2\pi}\right)^2} \xrightarrow{F^{-1}} 2\pi x(-2\pi t)$

$\Rightarrow 1\times\mathrm{Sinc}^2\left(\frac{w\times 1}{2\pi}\right) \xrightarrow{F^{-1}} 2\pi x(-2\pi t) = (1-|t|)\left(u(t+1)-u(t-1)\right) \rightarrow \Delta(t/1)$

$\Rightarrow x(t) = \frac{1-\left|\frac{t}{2\pi}\right|}{2\pi}\times\left(u\left(\frac{-t}{2\pi}+1\right)-u\left(\frac{-t}{2\pi}-1\right)\right)$

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3 g) $\frac{d}{dw}\left[\frac{\sin(\pi w) - j\cos(\pi w)}{1+2jw}\right] \xrightarrow{\mathcal{F}^{-1}} u(t) \Rightarrow \frac{\cos(\pi w) - j\sin(\pi w)}{1+2jw} \xrightarrow{\mathcal{F}^{-1}} \frac{u(t)}{t}$

$\Rightarrow e^{-jw(\pi)} \times \frac{1}{\frac{1}{2}+jw} \xrightarrow{\mathcal{F}^{-1}} \frac{2u(t)}{t} = \delta(t+\pi) * e^{-\frac{t}{2}} u(t)$

$\Rightarrow \boxed{u(t) = \frac{t}{2} e^{-\frac{t+\pi}{2}} u(t+\pi)}$

h) $\frac{1}{(1+jw)^6} \xrightarrow{\mathcal{F}^{-1}} \boxed{\frac{t^5}{5!} \times e^{-t} u(t)}$

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4- $y(t) \xrightarrow{F} Y(w) = X(w)H(w)$

$x(t) \xrightarrow{F} X(t) \quad , \quad h(t) \xrightarrow{F} H(w)$

$$-(jw)^2 Y(w) - 7(jw)Y(w) - 10Y(w) = 2(jw)X(w) + 13X(w)$$

$$\Rightarrow \underbrace{Y(w)}_{H(w)\cancel{X(w)}} \times (w^2 - 7jw - 10) = \cancel{X(w)}(2jw + 13) \Rightarrow \boxed{H(w) = \frac{2jw + 13}{w^2 - 7jw - 10}}$$ (a

b) $$\frac{-(2jw+13)}{-w^2 + 7jw + 10} = \frac{-(2jw+13)}{(\frac{7}{2} + jw)^2 - \frac{9}{4}} = \frac{-3}{2 + jw} + \frac{1}{5 + jw}$$

$$\xrightarrow{F^{-1}} \boxed{h(t) = (-3e^{-2t} + e^{-5t})u(t)}$$

c) $$y_1 = \int_0^t (t-\tau) \times (-3e^{-3t+\tau} + e^{-6t+\tau}) d\tau = t(-3e^{-3t} + e^{-6t}) \int_0^t e^{\tau} d\tau - (-3e^{-3t} + e^{-6t}) \int_0^t \tau e^{\tau} d\tau$$

τ	e^{τ}
1	e^{τ}
0	e^{τ}

$\to (\tau e^{\tau} + e^{\tau})$

$$= (-3e^{-3t} + e^{-6t}) \times \left[\cancel{te^t} - t - \cancel{te^t} + e^t - 1\right]$$

$$= \boxed{(-3e^{-3t} + e^{-6t})(e^t - t - 1)}$$

d) $$H(w)G(w) = 1 \Rightarrow G(w) = \frac{w^2 - 7jw - 10}{2jw + 13} = \frac{-1}{2}jw - \frac{1}{4} - \frac{\frac{27}{8}}{jw + \frac{13}{2}}$$

$$\xrightarrow{F^{-1}} g(t) = \frac{-1}{2}\delta'(t) - \frac{1}{4}\delta(t) - \frac{27}{8}e^{-\frac{13t}{2}}u(t)$$

e) $$Y(w) = X(w)G(w) \Rightarrow 8Y(w) = -4jwX(w) - 2X(w) - \frac{27X(w)}{jw + \frac{13}{2}}$$

$$\Rightarrow 8jwY(w) + 52Y(w) = -4(jw)^2X(w) - 28(jw)X(w) - 40X(w)$$

$$\Rightarrow \boxed{8\frac{d}{dt}y(t) + 52y(t) = -4\frac{d^2}{dt^2}x(t) - 28\frac{d}{dt}x(t) - 40x(t)}$$

5- a)

$$x(t) = \begin{cases} t+2 & -2 \le t \le 0 \\ -t+2 & 0 < t \le 2 \\ 0 & \text{o.w} \end{cases}$$

$$X(w) = \int_{-\infty}^{\infty} x(t) e^{-jwt} dt = \int_{-2}^{0} (t+2) e^{-jwt} dt + \int_{0}^{2} (-t+2) e^{-jwt} = \int_{-2}^{0} t e^{-jwt} dt - \int_{0}^{2} t e^{-jwt} dt + 2\int_{-2}^{2} e^{-jwt} dt$$

t	e^{-jwt}
1	$\frac{1}{jw} e^{-jwt}$
0	$\frac{-1}{w^2} e^{-jwt}$

$$= -e^{-jwt}\left(\frac{t}{jw} + \frac{1}{w^2}\right)\Bigg|_{-2}^{0} + e^{-jwt}\left(\frac{t}{jw} + \frac{1}{w^2}\right)\Bigg|_{0}^{2} - \frac{2e^{-jwt}}{jw}\Bigg|_{-2}^{2}$$

$$= \frac{-1}{w^2} + e^{2jw}\left(\frac{1}{w^2} - \frac{2}{jw}\right) + e^{-2jw}\left(\frac{2}{jw} + \frac{1}{w^2}\right) - \frac{1}{w^2} - \frac{2e^{-2jw}}{jw} + \frac{2e^{2jw}}{jw}$$

$$= \frac{-2}{w^2} + \frac{1}{w^2}\left(e^{j2w} + e^{-j2w}\right) - \frac{4}{jw}\left(e^{j2w} - e^{-j2w}\right)$$

$$= \boxed{\frac{-2}{w^2} + \frac{2}{w^2}\cos(2w) - \frac{8}{w}\sin(2w)}$$

b)
$$x(t) = \begin{cases} -1 & t < -3 \\ 0 & -3 < t < -2 \\ -1 & -2 < t < -1 \\ t & -1 < t < 1 \\ 1 & 1 < t < 2 \\ 2 & 2 < t < 3 \\ 3 & 3 < t \end{cases}$$

$$X(w) = \int_{-\infty}^{\infty} x(t) e^{-jwt} dt = -\int_{-\infty}^{-3} e^{-jwt} dt - \int_{-2}^{-1} e^{-jwt} dt + \int_{-1}^{1} t e^{-jwt} dt + \int_{1}^{2} e^{-jwt} dt + 2\int_{2}^{3} e^{-jwt} + 3\int_{3}^{\infty} e^{-jwt}$$

انتگرال واگراست ← $X(w)$ موجود نیست

c) $x(t) = e^{j2t}$

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انتگرال واگراست ← $X(w)$ موجود نیست

6) $p(t) = e^{j2t}$

$$\underbrace{\frac{\sin(2t)}{\pi t}}_{x(t)}\ \underbrace{e^{j2t}}_{p(t)} \xrightarrow{\mathcal{F}} u(w) - u(w-4) = A(jw)$$

$$x(t)p(t)q(t) = \left(\frac{\sin(2t)}{\pi t}\right)^2 \xrightarrow{\mathcal{F}} \big(u(w+2) - u(w-2)\big) * \big(u(w+2) - u(w-2)\big) = B(jw)$$

$$B(jw) = \int_{-2}^{2} u(x-w+2) - u(x-w-2)\, dx \overset{w>0}{=} \int_{w-2}^{2} 1\, dx = \begin{cases} 4-w & 0<w\leq 4 \\ 0 & 4\leq w \end{cases}$$

$$h(t) = \begin{cases} 1 & |t|<1 \\ 0 & |t|>1 \end{cases} \xrightarrow{\mathcal{F}} H(jw) = \frac{2\sin w}{w} \Rightarrow C(jw) = \begin{cases} 4 \times \frac{2\sin w}{w} - 2\sin w & 0<w\leq 4 \\ 0 & 4<w \end{cases}$$

$$r(t) = \frac{\delta(2t)}{2} \xrightarrow{\mathcal{F}} \frac{1}{2} \times \frac{1}{2} \times 1 = \frac{1}{4} \Rightarrow R(jw) = \frac{1}{4}$$

$$\Rightarrow D(jw) = C(jw) + R(jw) = \begin{cases} \frac{8\sin w}{w} - 2\sin w + \frac{1}{4} & 0<w\leq 4 \\ \frac{1}{4} & 4<w \end{cases}$$

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7 - a) $\frac{1}{2a} \times \frac{2a}{\omega^2+a^2} \xrightarrow[a>0]{\mathcal{F}^{-1}} \frac{1}{2a} e^{-a|t|} = x(t)$ $\quad \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$ رابطه پارسوال

$$\frac{1}{4a^2}\int_{-\infty}^{\infty} e^{-2a|t|} dt = \frac{1}{2a^2}\int_{0}^{\infty} e^{-2at} dt = \frac{1}{2a^2} \times \frac{-1}{2a} \times e^{-2at}\Big|_0^{\infty} = \boxed{\frac{1}{4a^3}} = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

b) $= \left(\frac{\sin(t)}{\pi t}\right)^2 \xrightarrow{\mathcal{F}} \begin{cases} 0 & \omega < -2 \\ 2\omega - 4 & -2 < \omega < 0 \\ -2\omega + 4 & 0 < \omega < 2 \\ 0 & 2 < \omega \end{cases}$

$$\Rightarrow t\left(\frac{\sin(t)}{\pi t}\right)^2 \xrightarrow{\mathcal{F}} \begin{cases} 0 & \omega < -2 \\ 2j & -2 < \omega < 0 \\ -2j & 0 < \omega < 2 \\ 0 & 2 < \omega \end{cases} = X(t)$$

$$\int_{-\infty}^{\infty} |X(t)|^2 d\omega = \int_{-2}^{2} 4\, d\omega = \boxed{16}$$