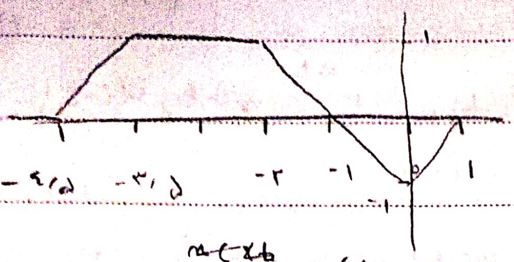
Subject:

Year. Month. Day.

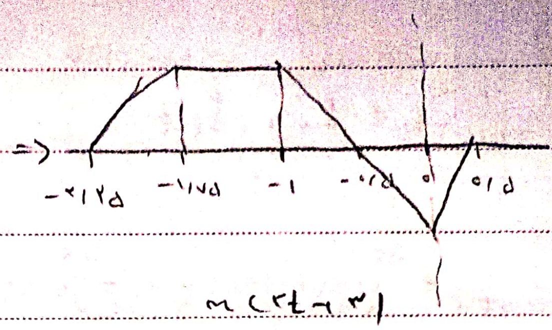
PAYA

1

a

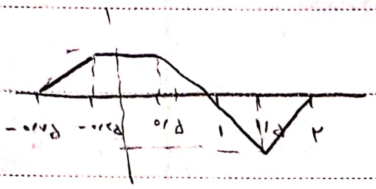


$x(t+3)$

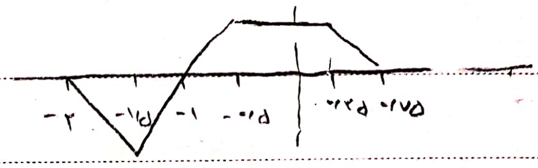


$x(2t+3)$

b $x(-2t)$

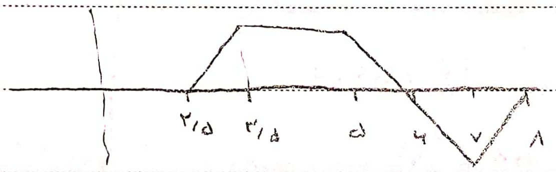


$x(2t)$

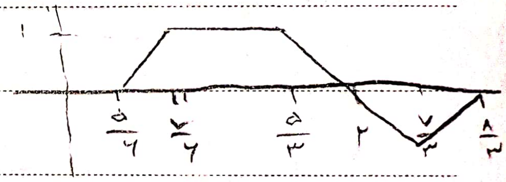


$x(-2t)$

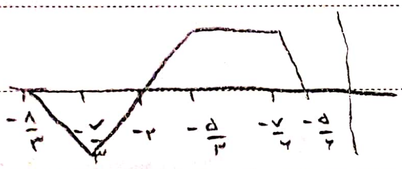
c $x(-3t-4)$



$x(t-4)$

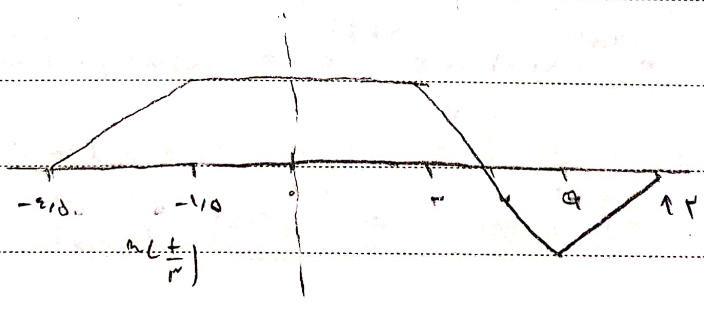


$x(3t-4)$



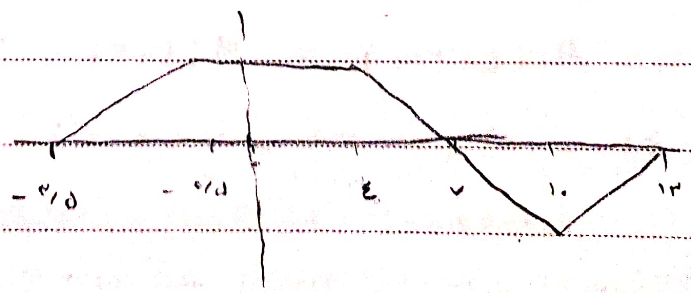
$x(-3t-4)$

d $x\left(\frac{t-1}{3}\right)$



$x\left(\frac{t}{3}\right)$

$x\left(\frac{t-1}{3}\right)$



ex 2

(a) $ev(x(t)) = \frac{f(t)+f(-t)}{2} \qquad f(-t) = e^{5t}\cos(-t)\,u(-t)$

$$ev(x(t)) = \frac{e^{-5t}\cos(t)u(t) + e^{5t}\cos(t)\,u(-t)}{2} \Rightarrow \begin{cases} \frac{e^{-5t}\cos(t)}{2} & t>0 \\ 1 & t=0 \\ \frac{e^{5t}\cos(t)}{2} & t<0 \end{cases}$$

$$od(x(t)) = \frac{e^{-5t}\cos(t)\,u(t) - e^{5t}\cos(t)\,u(-t)}{2} = \begin{cases} \frac{e^{-5t}\cos(t)}{2} & t>0 \\ 0 & t=0 \\ \frac{-e^{5t}\cos(t)}{2} & t<0 \end{cases}$$

(b) $f(-t) = e^{-4|-t|}\cos(-t) = e^{-4|-t|}\cos(t)$

$$ev(x(t)) = \frac{e^{-4|t|}\cos(t) + e^{-4|-t|}\cos t}{2} \Rightarrow \frac{\cos t\,(e^{-4|t|}+e^{-4|t|})}{2} = e^{-4|t|}\cos t$$

خود تابع زوج است

$$od(x(t)) = \frac{e^{-4|t|}\cos(t) - e^{-4|-t|}\cos t}{2} = 0$$

(c) $x(t) = \Pi(t-\frac{3}{2}) = u(t-2) - u(t-4)$

$x(-t) = u(-t-2) - u(-t-4)$

$$ev(x(t)) = \frac{u(t-2) - u(t-4) + [u(-t-2) - u(-t-4)]}{2}$$

$$od(x(t)) = \frac{u(t-2) - u(t-4) - [u(-t-2) - u(-t-4)]}{2}$$

(3)

(a) $x(t) = \sin^{۲}(٤t - \frac{\pi}{۲}) = \frac{۱ - \cos(۸t - \frac{\pi}{۲})}{۲} = \frac{۱}{۲} - \frac{۱}{۲}\cos(۸t - \frac{\pi}{۲})$

$\omega = ۸$ $\qquad T = \frac{۲\pi}{۸} = \frac{\pi}{٤}$

(b) $x(t) = e^{-۲|t|}\sin(t)$

$e^{-۲|t+T|}\sin(t+T) \neq e^{-۲|t|}\sin(t)$ تابع متناوب نیست

(c) $x(t) = e^{۳j(۸t + \frac{\pi}{۳})} = \cos(۲٤t + \pi) + j\sin(۲٤t + \pi)$

$T_۱ = \frac{۲\pi}{۲٤}$ $\qquad T_۲ = \frac{۲\pi}{۲٤}$

$T = \frac{\pi}{۱۲}$ و $\omega = ۲٤$

(d) $x(t) = \sin^{۳}(۵t)$ متناوب است $T = \frac{۲\pi}{۵}$ $\qquad \omega = \frac{۲\pi}{\frac{۲\pi}{۵}} = ۵$

(e) $x[n] = 3\sin(٤\pi n) \Rightarrow N_0 = \frac{۲k\pi}{٤\pi} = \frac{k}{۲} \xrightarrow{k=۲} N = ۱$ $\qquad \omega = ۲\pi$

(f) $x[n] = ۳\sin(٤n) \Rightarrow N_0 = \frac{۲k\pi}{٤} = \frac{k\pi}{۲}$ $k \to$ عدد صحیح نمیشود $\to$ ✗ متناوب

(g) $x[n] = e^{j\frac{n}{۲}} + e^{j\frac{n}{۳}} \rightarrow N_0 = \frac{۲k\pi}{\frac{1}{۳}} \rightarrow N_0 = ۶\pi k \quad k[۶\pi, ۴\pi] = ۱۲\pi k$

$\rightarrow N_0 = \frac{۲k\pi}{\frac{1}{۲}} \rightarrow N_0 = ۴\pi k \quad N_0 = ۱۲\pi k$

متناوب نیست ← عدد صحیح وجود ندارد

(h) $x[n] = e^{j\frac{n\pi}{۲}} + e^{j\frac{n\pi}{۳}}$

$\hookrightarrow N_0 = \frac{۲k\pi}{\frac{\pi}{۳}} = ۶k \quad [۶k, ۴k] = ۱۲k \xrightarrow{k=۱} N = ۱۲$

$\hookrightarrow N_0 = \frac{۲k\pi}{\frac{\pi}{۲}} = ۴k \quad \omega = \frac{\pi}{۶}$

(i) $x[n] = \cos(\frac{\pi}{۸} n^۲) \rightarrow N_0 = \dfrac{۲k\pi}{\frac{\pi}{۸}} = ۱۶k$?

(j) $x[n] = (-1)^n \cos(\frac{۲\pi}{۵} n) \rightarrow \dfrac{۲k\pi}{\frac{۲\pi}{۵}} = ۵k \xrightarrow{k=۱} N_۲ = ۵$

$\hookrightarrow N_۱ = ۲$

$N = [۲, ۵] = ۱۰$

$\omega = \frac{\pi}{۵}$

(k) $x(t) = \sum_{n=-\infty}^{\infty} e^{-(۲t-n)} = e^{-۲t} \sum_{n=-\infty}^{\infty} e^{n} = \infty \rightarrow$ تابع متناوب می‌باشد

∞ به ازای تمام مقادیر t — تناوب اصلی نداریم

Year. Month. Day.

(4)

(a) $y(t) = e^{x(t)}$ $\rightarrow$ memoryless ✓ $y(t) \rightarrow e^{x(t)}$ در هر لحظه تنها به سیگنال ورودی همان لحظه بستگی دارد

$x_1(t) \rightarrow y_1(t) = e^{x_1(t)}$ Causal ✓ $\rightarrow$ خروجی به سیگنال حال بستگی دارد

$x_2(t) = x_1(t-t_0) \rightarrow y_2(t) = e^{x_1(t-t_0)}$ ①

$y_1(t-t_0) = e^{x_1(t-t_0)}$ ② ① و ② $\rightarrow$ برابرند پس time invariant ✓

stable ✓ $\rightarrow$ اگر $x(t)$ کراندار باشد $e^{x(t)}$ نیز کراندار است

$x_1(t) \rightarrow y_1(t) = e^{x_1(t)}$

$x_2(t) \rightarrow y_2(t) = e^{x_2(t)}$

$x_3(t) = a x_1(t) + b x_2(t) \rightarrow y_3(t) = e^{x_3(t)} = e^{a x_1(t) + b x_2(t)}$

$y_3 \neq a y_1(t) + b y_2(t)$

Linear ✗

(b) $y(t) = \cos(3t)\, x(t)$ memoryless ✓ $\rightarrow x(t)$ [illegible]

Linear (x)

(b) $y(t) = \cos(3t)\, x(t)$ memory Less ✓ → $x(t)$ ~ فقط على اللحظة
↳ ليست دالة حال

casual ✓ → $x(t)$ ~ حالية

Stable ✓ → $\cos(3t)$ → محدود $x(t)$ → محدود

time invariant: (x)

$x_1(t) \rightarrow y_1(t) = \cos 3t\, x_1(t)$

$x_2(t) = x_1(t - t_0) \rightarrow y_2(t) = \cos 3t\, x_1(t - t_0)$ (1)   (1) $\neq$ (2)

$y_1(t - t_0) = \cos 3(t - t_0)\, x_1(t - t_0)$ (2)

Linear: ✓

$x_1(t) \Rightarrow y_1(t) = \cos 3t\, x_1(t)$

$x_2(t) \Rightarrow y_2(t) = \cos 3t\, x_2(t)$

$x_3(t) = a X_1(t) + b X_2(t) \Rightarrow y_3(t) = \cos 3t\,(a x_1(t) + b x_2(t))$

$y_3(t) = a y_1(t) + b y_2(t)$

c) $y(t) = \int_{-\infty}^{2t} x(\tau)\, d\tau$

memoryLess ✗ به زمان های دیگر علاوه بر t هم نیاز دارد

causal ✗ برای زمان t_0 به مقدارها تا $2t_0$ نیاز دارد

stable ✗ $x(t) = c \quad \int_{-\infty}^{2t} c\, d\tau = \infty$

time-invariant: ✗

$x_1(t) \Rightarrow y_1(t) = \int_{-\infty}^{2t} x_1(\tau)\, d\tau$

$x_2(t) = x_1(\underbrace{t - t_0}_{\alpha}) \Rightarrow y_2(t) = \int_{-\infty}^{2t} x_2(t)\, dt = \int_{-\infty}^{2t} x_1(\alpha)\, d\alpha$ ۲

$y_1(t - t_0) = \int_{-\infty}^{2(t-t_0)} x_1(\tau)\, d\tau$ ۱

$۱ \neq ۲$

Linear: ✓ انتگرال تابعی خطی است

$\int (a x_1(\tau) + b x_2(\tau))\, dt = \underbrace{a \int x_1(\tau) + b \int x_2(\tau)}_{a y_1(t) + b y_2(t)}$

d) y(...

Linear: ✓ انتگرال تابعی خطیست

$$\int (a x_1(\tau) + b x_2(\tau))\, d\tau = a \underbrace{\int x_1(\tau) + b \int x_2(\tau)}_{a\, y_1(t) + b\, y_2(t)}$$

d) $y(t) = \sin(x(t))$ memoryLess ✓ Causal ✓ stable ✓ تابع محدود sin

time-invariant: ✓

$x_1(t) \to y_1(t) = \sin(x_1(t))$

$x_2(t) = x_1(t - t_0) \Rightarrow y_2(t) = \sin(x_1(t - t_0))$ ① ① = ②

$y_1(t - t_0) = \sin(x_1(t - t_0))$ ①

Linear: ✗

$x_1(t) \to y_1(t) = \sin(x_1(t))$

$x_2(t) \to y_2(t) = \sin(x_2(t))$

$x_3(t) \Rightarrow a x_1(t) + b x_2(t) \longrightarrow y_3(t) = \sin(x_3(t)) = \sin(a x_1(t) + b x_2(t))$

$y_3 \neq a \sin(x_1(t)) + b \sin(x_2(t))$

(e) $y[n] = x[\{n-1\}]$ memory less (x) Causal (x)

stable (✓) $a < x[n] < b \rightarrow a' < y[n] < b'$

time invarition (x)

$y[n] = x_1[\{n-1\}]$

x_2 : $y_2[n] = x_1[\{n+1+d\}]$ ① ① ≠ ②

$y_1[n+d] = x_1[\{n+\{d\}+1\}]$ ②

Linear: (✓)

$y_1[n] = x_1[\{n+1\}]$ $y_2[n] = x_2[\{n+1\}]$

$a y_1[n] + b y_2[n] = a x_1[an+1] + b x_2[an+1]$

$a x_1[\{n+1\}] + b x_2[\{n+1\}) = a y_1[n] + b y_2[n$

۴ $y[n] = (n-۲)\,x[n]$ memoryLess ✓ Causal ✓

Stable ✗ میتونه ابد باشه

tim - invarition ✗ $y_1[n] = n-۲\,x_1[n]$

$x_۲[n] = x_1[n-n_0] \longrightarrow y_۲[n] = n-۲\,x_۲[n] = n-۲ \times x_1[n-n_0]$

$y_1[n-n_0] = \big((n-n_0)-۲\big) \times x_1[n-n_0]$ ۲

$۱ \neq ۲$

Linear ✓ $y_1[n] = (n-۲)\,x_1[n]$

$y_۲[n] = (n-۲)\,x_۲[n]$

$x_۳ = a x_1[n] + b x_۲[n] \longrightarrow y_۳[n] = (n-۲)\,(a x_1[n] + b x_۲[n])$

$y_۳[n] = a y_1[n] + b y_۲[n]$

$$\text{(g)}\quad y[n] = x[n] \underbrace{\overbrace{\sum_{k=-\infty}^{\infty} \delta[n-(2k+1)]}^{g}}_{} \neq$$

memoryless ✓ Causal ✓

stable ✓

تابع g با مقادیر ... ضرب ... مثبت و مقادیر ... اند ... کرده است

Time-invariation: ✗

$x_1[n] \rightarrow y_1[n] = x_1[n]\, g$

$x_2[n] = x_1[n-n_0] \rightarrow y_2[n] = x_1[n-n_0]\, g$ ①

$y_1[n-n_0] = x_1[n-n_0] \sum_{k=-\infty}^{\infty} n-n_0-(2k+1)$ ②

① $\neq$ ② $\rightarrow$ برای مقادیر متفاوت n_0

Linear: ✓ $x_1[n] \Rightarrow y_1[n] = x_1[n]\, g$

$x_2[n] \Rightarrow y_2[n] = x_2[n]\, g$

$x_3[n] = a x_1[n] + b x_2[n] \longrightarrow y_3[n] = (a x_1[n] + b x_2[n])\, g$

$y_3[n] = a\, y_1[n] + b\, y_2[n]$

(h) $y[n] = \sum_{k=-\infty}^{n} x[k+3]$ memoryless ✗ Causal ✗ $\Rightarrow$ از ورودی که برای n آینده است استفاده می‌شود

stable ✗ $x[n] = C \rightarrow n-(-\infty)-1$

$y[n] = \infty$ بی‌کران می‌شود.

time-invariation: ✓

$y_1[n] = \sum_{k-\infty}^{n} x_1[k+3]$

$y_2[n] = \sum_{k-\infty}^{n} x_1[k+3-n_0] = \sum_{k=-\infty}^{n-n_0} x_1[k+3]$ (1)

(1) = (2)

$y_1[n-n_0] = \sum_{k=-\infty}^{n-n_0} x[k+3]$ (2)

Linear

$y_1[n] = \sum_{-\infty}^{n} x_1[k+3]$

$y_2[n] = \sum_{-\infty}^{k} x_2[k+3]$

$x_3 = a x_1[n] + b x_2[n] \rightarrow y_3 = \sum_{-\infty}^{n} a x_1[3+k] + b x_2[3+k]$

$y_3 = a\, y_1[n] + b\, y_2[n]$

5

a) $y(t) = \frac{dx(t)}{dt}$ ✗

مثال نقض

$x_1(t) = x^۲$ $y_1(t) = ۲x$ $y_1 = y_۲$ $x_1 \neq x_۲$

$x_۲(t) = x^۲ + 1$ $y_۲(t) = ۲x$

b) $y[n] = x[۲n]$ ✗ مثال نقض

$x_1[n] = 1$ $y_1 = 1$ $y_1 = y_۲$

$x_۲[n] = (-1)^n$ $y_۲ = 1$ $x_1 \neq x_۲$

c) $y(t) = x(۲t)$ معکوس پذیر است

$y_2^{-1}(t) = x(t/۲)$

(4) (a) $x(t) = e^{-8t}u(t)$

$$E_\infty = \lim_{T\to\infty}\int_{-T}^{T}|x(t)|^2dt \longrightarrow \lim_{T\to\infty}\int_{0}^{T} e^{-16t} = -\frac{1}{16}e^{-16t}\Big|_0^\infty = \frac{1}{16}$$

$P_\infty = 0 \longrightarrow$ انرژی محدود

(b)

$$x(t) = e^{j(3t+\frac{\pi}{8})} \qquad |x(t)| = \cos^2(3t+\frac{\pi}{8}) + \sin^2(3t+\frac{\pi}{8}) = 1$$

$$E_\infty = \lim_{T\to\infty}\int_{-T}^{T}1\,dt = \lim_{T\to\infty}(T-(-T)) = \lim_{T\to\infty}2T = \infty$$

$$P_\infty = \lim_{T\to\infty}\frac{\int_{-T}^{T}1\,dt}{2T} = \lim_{T\to\infty}\frac{2T}{2T} = 1$$

(c) $x[n] = (\frac{1}{3})^n u[n]$

$$E_\infty = \lim_{N\to\infty}\sum_{n=-N}^{+N}\left((\tfrac{1}{3})^n u[n]\right)^2 = \lim_{N\to\infty}\sum_{n=0}^{\infty}(\tfrac{1}{3})^{2n} = 1 + \frac{1}{9} + \cdots = \frac{1}{1-\frac{1}{9}} = \frac{9}{8}$$

$P_\infty =$ انرژی محدود ، توان برابر صفر باشد $\longrightarrow \lim_{N\to\infty}\frac{E_\infty}{2N+1} = \frac{c}{\infty} = 0$