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(1)

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} x[n-k]h[k] = \sum_{k=-\infty}^{\infty} \delta[k-2]h[n-k]$$

$$\Rightarrow y[n] = h[n-2] \rightarrow \begin{cases} y[1] = h[-1] = 3 \\ y[2] = h[0] = -1 \\ y[3] = h[1] = 4 \\ y[4] = h[2] = 1 \end{cases} , \quad h[k] = 0 \text{ بقیه}$$

---

a) $$x[n] = e^{j\pi n}\left(u[n] - u[n-3]\right) = \begin{cases} e^{j\pi n} & 0 \le n \le 2 \\ 0 & \text{o.w} \end{cases}$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=0}^{2} (-1)^k h[n-k]$$

$$\Rightarrow y[n] = h[n] - h[n-1] + h[n-2] = \begin{cases} y[-1] = 3 \\ y[0] = -4 \\ y[1] = 8 \\ y[2] = -4 \\ y[3] = 3 \\ y[4] = 1 \end{cases}$$

بقیه yها برابر صفر است.

---

b) $$x[n] = \left(\frac{1}{2}\right)^n \left(u[n] - u[n-2]\right) = \begin{cases} 1 & n = 0 \\ \frac{1}{2} & n = 1 \\ 0 & \text{o.w} \end{cases}$$

$$y[n] = \sum_{k=0}^{1} x[k]h[n-k] = h[n] + \frac{1}{2}h[n-1] = \begin{cases} y[-1] = 3 \\ y[0] = \frac{1}{2} \\ y[1] = \frac{7}{2} \\ y[2] = 3 \\ y[3] = \frac{1}{2} \end{cases}$$

بقیه yها برابر صفر است.

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c) $x[n] = ۲^n (u[n+۲] - u[n]) = \begin{cases} \frac{۱}{۴} & n = -۲ \\ \frac{۱}{۲} & n = -۱ \\ ۰ & o.w \end{cases}$

$$y[n] = \sum_{k=-۲}^{-۱} x[k]h[n-k] = \frac{۱}{۴}h[n+۲] + \frac{۱}{۲}h[n+۱]$$

$$\Rightarrow \begin{cases} y[-۳] = \frac{۳}{۴} \\ y[-۲] = \frac{۵}{۴} \\ y[-۱] = \frac{۱}{۲} \end{cases} \qquad \begin{cases} y[۰] = \frac{۹}{۴} \\ y[۱] = \frac{۱}{۲} \end{cases}$$

جواب برای بقیه n ها برابر صفر است.

---

d) $x[n] = e^{j\pi n} u[n] = \begin{cases} e^{j\pi n} & n \geq ۰ \\ ۰ & n < ۰ \end{cases}$

$$y[۱] = \sum_{k=۰}^{\infty} x[k]h[۱-k] \xrightarrow{-۱ \leq ۱-k \leq ۲} \sum_{k=۰}^{۲} e^{j\pi k} h[۱-k]$$

$$\Rightarrow y[۱] = ۴ + ۱ + ۳ = ۸$$

---

a) $h(t) = u(t)$, $x(t) = e^t$ ②

$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau = \int_{۰}^{\infty} e^{t-\tau} d\tau = -e^{t-\tau}\Big|_{۰}^{\infty}$$

$$\Rightarrow y(t) = e^t$$

---

b) $h(t) = u(t)$, $x(t) = e^t u(t)$

$$y(t) = \int_{-\infty}^{\infty} e^{\tau} u(\tau) u(t-\tau) d\tau = \int_{۰}^{t \geq ۰} e^{\tau} d\tau = e^t - ۱$$

$$\Rightarrow y(t) = e^t - ۱ \quad (t \geq ۰) \Rightarrow y(t) = \begin{cases} e^t - ۱ & t \geq ۰ \\ ۰ & t < ۰ \end{cases}$$

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c) $y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$

$t+2 < -1 \rightarrow t < -3 \rightarrow y(t) = 0$

$-1 \leq t+2 \leq 1 \rightarrow -3 \leq t \leq -1 \rightarrow y(t) = \int_{-1}^{t+2} d\tau = t+3$

$\left.\begin{matrix} 1 \leq t+2 \\ t-3 \leq -1 \end{matrix}\right\} \rightarrow -1 \leq t \leq 2 \rightarrow y(t) = \int_{-1}^{1} d\tau = 2$

$-1 < t-3 < 1 \rightarrow 2 \leq t \leq 4 \rightarrow y(t) = \int_{t-3}^{1} d\tau = 4-t$

$1 \leq t-3 \rightarrow 4 < t \rightarrow y(t) = 0$

---

d) $y(t) = x(t) * \delta(t+\frac{3}{2}) + 2\, x(t) * \delta(t+\frac{5}{2})$

$= x(t+\frac{3}{2}) + 2x(t+\frac{5}{2})$

$\frac{-7}{2} \leq t \leq \frac{-5}{2} \rightarrow 2\left(2(t+\frac{5}{2})+1\right) + 0 = 4t+12$

$\frac{-5}{2} < t < \frac{-3}{2} \rightarrow \left(2(t+\frac{3}{2})+1\right) + (4t+12) = 6t+16$

$\frac{-3}{2} \leq t < \frac{-1}{2} \rightarrow \left(2(t+\frac{3}{2})+1\right) + 2\ln(t+\frac{5}{2}) = 2t + 2\ln(t+\frac{5}{2}) + 4$

$\frac{-1}{2} \leq t < \frac{1}{2} \rightarrow \ln(t+\frac{3}{2}) + 2e^{(t+\frac{5}{2})}$

$\frac{1}{2} \leq t < \frac{3}{2} \rightarrow e^{(t+\frac{3}{2})}$

$O.W \rightarrow 0$

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۳

a) Linear: $x_1(t) \rightarrow y_1(t) = \frac{d}{dt}(x_1(t))$

$x_2(t) \rightarrow y_2(t) = \frac{d}{dt}(x_2(t))$

$\Rightarrow a x_1(t) + b x_2(t) \rightarrow \frac{d}{dt}\left(a x_1(t) + b x_2(t)\right)$

$= \frac{d}{dt}(a x_1(t)) + \frac{d}{dt}(b x_2(t))$

$= a\frac{d}{dt}(x_1(t)) + b\frac{d}{dt}(x_2(t))$ ✓ خطی است

TI:

$x_1(t) \rightarrow y_1(t) = \frac{d}{dt}(x_1(t))$

$x_2(t) = x_1(t-t_0) \rightarrow y_2(t) = \frac{d}{dt}(x_1(t-t_0))$ ✓ TI است

$y_1(t-t_0) = \frac{d}{dt}(x_1(t-t_0))$

---

b) $s(t) = u(t) * h(t) = h(t) * u(t) = \int_{-\infty}^{\infty} h(\tau) u(t-\tau) d\tau$

$= \int_{-\infty}^{t} h(\tau) d\tau \xRightarrow{\text{مشتق}} h(t) = \frac{d}{dt} s(t)$

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(۴)

memoryless $\xrightarrow{n \neq 0}$ $h[n] = 0$

causal $\xrightarrow{n<0}$ $h[n] = 0$

stable $\longrightarrow$ $\sum_{k=-\infty}^{\infty} |h[k]| < \infty$

a) $h(t) = e^{-4|t|}$

$h(1) = e^{-4} \neq 0 \xrightarrow{\times}$ با حافظه

$h(-1) = e^{-4} \neq 0 \xrightarrow{\times}$ علّی نیست

$$\int_{-\infty}^{\infty} \left| e^{-4|t|} \right| dt = 2\int_{0}^{\infty} e^{-4t}\, dt = \frac{1}{2} \xrightarrow{\checkmark} \text{پایدار}$$

---

b) $h(t) = te^{-t}u(t)$

$h(1) = e^{-1} \neq 0 \xrightarrow{\times}$ با حافظه

$h(t<0) = 0 \xrightarrow{\checkmark}$ علّی است

$$\int_{-\infty}^{\infty} \left| te^{-t}u(t) \right| dt = \int_{0}^{\infty} te^{-t}\, dt = \left(-te^{-t} - e^{-t}\right)\Big|_{0}^{\infty} = 1 \xrightarrow{\checkmark} \text{پایدار}$$

---

c) $h(t) = \cos(2\pi t)\, u(t+1)$

$h(1) = 1 \neq 0 \xrightarrow{\times}$ با حافظه

$h(-1) = 1 \neq 0 \xrightarrow{\times}$ علّی نیست

$$\int_{-\infty}^{\infty} \left| \cos(2\pi t)\, u(t+1) \right| dt = \int_{-1}^{\infty} |\cos(2\pi t)|\, dt = \infty \xrightarrow{\times} \text{ناپایدار}$$

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d) $h(t) = \frac{\sin(t)}{t} u(t)$

$h(2) \neq 0 \xrightarrow{\times}$ باحافظه

$h(t<0) = 0 \xrightarrow{\checkmark}$ علی است

$\int_{-\infty}^{\infty} \left| \frac{\sin(t)}{t} u(t) \right| dt = \int_{0}^{\infty} \frac{|\sin(t)|}{t} dt = \frac{\pi}{2} \xrightarrow{\checkmark}$ پایدار

---

e) $h[n] = (\frac{1}{2})^n u[-n]$

$h(-1) = 2 \neq 0 \xrightarrow{\times}$ باحافظه

$h(-1) = 2 \neq 0 \xrightarrow{\times}$ علی نیست

$\sum_{k=-\infty}^{\infty} \left| (\frac{1}{2})^n u[-n] \right| = \sum_{k=-\infty}^{0} (\frac{1}{2})^n = \infty \xrightarrow{\times}$ ناپایدار

---

f) $h[n] = \delta[2n]$

$h[n \neq 0] = 0 \xrightarrow{\checkmark}$ بی‌حافظه

$h[n<0] = 0 \xrightarrow{\checkmark}$ علی است

$\sum_{k=-\infty}^{\infty} |\delta[2n]| = 1 \xrightarrow{\checkmark}$ پایدار

g) $h[n] = \cos(\frac{\pi}{2}n)u[n+1]$

$h[4] = 1 \neq 0 \xrightarrow{\times}$ باحافظه

$h[-1] = 0 \xrightarrow{\checkmark}$ علّی است

$$\sum_{k=-\infty}^{\infty} \left| \cos(\frac{\pi}{2}n)u[n+1] \right| = \infty \xrightarrow{\times} \text{ناپایدار}$$

---

h) $h[n] = e^{2n}u[n]$

$h[2] = e^{4} \neq 0 \xrightarrow{\times}$ باحافظه

$h[n<0] = 0 \xrightarrow{\checkmark}$ علّی است

$$\sum_{k=-\infty}^{\infty} \left| e^{2n}u[n] \right| = \infty \xrightarrow{\times} \text{ناپایدار}$$

---

۵) $x(t) = e^{2t}u(1-t)$, $h(t) = 2u(t) + 4u(t-2) + 8u(t-4)$

$$x(t) = \begin{cases} e^{2t} & t \leq 1 \\ 0 & t > 1 \end{cases}$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\Big(2u(t-\tau) + 4u(t-\tau-2) + 8u(t-\tau-4)\Big)d\tau$$

$$= \int_{-\infty}^{t} 2x(\tau)d\tau + \int_{-\infty}^{t-2} 4x(\tau)d\tau + \int_{-\infty}^{t-4} 8x(\tau)d\tau$$

$$= e^{2t}\Big|_{-\infty}^{t} + 2e^{2t}\Big|_{-\infty}^{t-2} + 4e^{2t}\Big|_{-\infty}^{t-4} \quad , \quad t \leq 1$$

$$= e^{2t} + 2e^{2t-4} + 4e^{2t-8}$$

(4)

$$y(t) = x(t) * \Big(h_1(t) + \big(h_2(t) * (h_3(t) + h_4(t)) \big) \Big)$$

$$= x(t) * \big(h_1(t) + h_2(t) * h_3(t) + h_2(t) * h_4(t) \big)$$

$$= x(t) * h_{eq}(t)$$

$$h_1(t) = e^{-t} u(t)$$

$$h_2(t) * h_3(t) = [u(t) - u(t-1)] * [u(t) - u(t-1)] = y_{23}(t)$$

$$\Rightarrow y_{23}(t) = \begin{cases} t < 0 \rightarrow 0 \\ 0 \leqslant t \leqslant 1 \longrightarrow \int_0^t d\tau = t \\ 0 \leqslant t-1 \leqslant \longrightarrow 1 \leqslant t \leqslant 2 = \int_{t-1}^{1} d\tau = 2 - t \\ 1 < t-1 \rightarrow t > 2 \longrightarrow 0 \end{cases}$$

$$h_2(t) * h_4(t) = [u(t) - u(t-1)] * \delta(t-1) = u(t-1) - u(t-2)$$

$$\Rightarrow h_{eq}(t) = e^{-t} u(t) + y_{23}(t) + [u(t-1) - u(t-2)]$$

$$\Rightarrow h_{eq}(t) = \begin{cases} t < 0 \longrightarrow 0 \\ 0 \leqslant t < 1 \longrightarrow e^{-t} + t \\ 1 \leqslant t < 2 \longrightarrow e^{-t} + (2 - t) + 1 = e^{-t} - t + 3 \\ 2 \leqslant t \longrightarrow e^{-t} \end{cases}$$

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(۷)

$$x[n]*h[n] = h[n]*x[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k] = y[n]$$

$$\Rightarrow y[n] = \sum_{k=0}^{\infty} \alpha^k x[n-k] \quad \text{(I)}$$

$$h[n-1]*x[n] = \sum_{k=1}^{\infty} \alpha^{k-1} x[n-k] = \frac{1}{\alpha}\sum_{k=1}^{\infty} \alpha^k x[n-k]$$

$$= \frac{1}{\alpha}\left(y[n] - x[n]\right) \quad \text{(II)}$$

$$\text{(I)} = \text{(II)} \Rightarrow y[n] = \frac{1}{\alpha}\left(y[n] - x[n]\right)$$

$$\Rightarrow y[n] = \frac{x[n]}{1-\alpha}$$

---

(۸)

a) $h_1(t) * h_2(t) = \delta(t)$

$$\Rightarrow [e^{-t}u(t)] * [\delta(t) + \delta'(t)] = (e^{-t}u(t)) * \delta(t) + (e^{-t}u(t)) * \delta'(t)$$

$$= e^{-t}u(t) + \left(-e^{-t}u(t) + e^{-t}\delta(t)\right) = e^{-t}\delta(t) = \delta(t)$$

معکوس هم اند ✓

---

b) $h_1[n] * h_2[n] = \delta[n]$

$$\Rightarrow (\delta[n] - \delta[n-1]) * u[n] = u[n] - u[n-1] = \delta[n]$$

معکوس هم اند. ✓