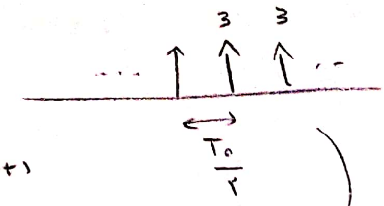
①

ⓐ



$T_0 = \frac{T_0}{2}$ با تناوب

با اندازه 3

$$a_k = \int_{T_0/2} x(t)\, e^{-jkw_0 t}\, dt$$

$$a_k = \frac{1}{T_0}\int_{-T_0/4}^{T_0/4} \underset{3\delta(t)}{x(t)}\, e^{-jkw_0 t}\, dt = 3e^{-jk\frac{4\pi}{T_0}\times 0} = \frac{3}{T_0/2} = \frac{6}{T_0}$$

ⓑ

$$x(t) = 3\cos(\tfrac{3}{2}t) + 2\cos t\,\cos(\tfrac{3}{2}t) \rightarrow 3\cos(\tfrac{3}{2}t) + 2\times\tfrac{1}{2}\left(\cos\tfrac{7}{2}t + \cos\tfrac{1}{2}t\right)$$

$$= \frac{3}{2}\left(e^{j\frac{3}{2}t} + e^{-j\frac{3}{2}t}\right) + \frac{1}{2}\left(e^{j\frac{7}{2}t} + e^{-j\frac{7}{2}t}\right) + \frac{1}{2}\left(e^{j\frac{1}{2}t} + e^{-j\frac{1}{2}t}\right)$$

$w_0 = \frac{1}{2}$

$$\underbrace{+\tfrac{1}{2}}_{a_{-7}} e^{j\frac{1}{2}(-7)t} + \underbrace{\tfrac{3}{2}}_{a_{-3}} e^{+j\frac{1}{2}(-3)t} + \underbrace{\tfrac{1}{2}}_{a_{-1}} e^{j(-1)\frac{1}{2}t} + \underbrace{\tfrac{1}{2}}_{a_{1}} e^{j\frac{1}{2}t} + \underbrace{\tfrac{3}{2}}_{a_{3}} e^{j\frac{1}{2}3t} + \underbrace{\tfrac{1}{2}}_{a_{7}} e^{j\frac{1}{2}7t}$$

مابقی $a_k = 0$ هستند

ⓒ

$$x(t) = e^{j\frac{\pi}{2}t}\left(\cos(\tfrac{\pi}{2}t) + \sin(\tfrac{\pi}{4}t + \tfrac{\pi}{4})\right)$$

$$e^{j\frac{\pi}{2}t}\left(\frac{1}{2}\left(e^{j\frac{\pi}{2}t} + e^{-j\frac{\pi}{2}t}\right) + \frac{1}{2j}\left(e^{j\frac{\pi}{4}t + j\frac{\pi}{4}} - e^{-j\frac{\pi}{4}t - j\frac{\pi}{4}}\right)\right)$$

$$\frac{1}{2}e^{jt\frac{3\pi}{2}} + \frac{1}{2}e^{-jt\frac{\pi}{2}} + \frac{e}{2j}e^{j\frac{\pi}{2}t} - \frac{1}{2j}e^{-j\frac{\pi}{12}t}$$

$$\underbrace{+\tfrac{1}{2}}_{a_{-3}} e^{-j3\frac{\pi}{12}t} + \underbrace{\frac{-e^{j\frac{\pi}{4}}}{2j}}_{a_{+1}} e^{+j\frac{\pi}{12}t} + \underbrace{\frac{e^{j\frac{\pi}{4}}}{2j}}_{a_{5}} e^{+j5\frac{\pi}{12}t} + \underbrace{\tfrac{1}{2}}_{a_{9}} e^{j9\frac{\pi}{12}t}$$

$a_k = 0$ ازسایر k ها

PAYA

Subject :

Year. Month. Day.

d) $x(t) = \cos(2\pi t) + 7\sin(t)$

$\hookrightarrow T = \frac{2\pi}{2\pi} = 1 \qquad T = \frac{2\pi}{1} \qquad \frac{2\pi}{1} \Rightarrow$ گنگ است

نامتناوب است $x(t)$

e) $x[n] = e^{j\frac{\pi}{3}n} \cos\left(\frac{\pi}{8}n\right)$

$w_0 = \frac{\pi}{24} \qquad = \frac{e^{j\frac{\pi}{3}n}}{2}\left(e^{\frac{\pi}{8}nj} + e^{-\frac{\pi}{8}nj}\right)$

$\frac{1}{2}e^{nj\frac{11\pi}{24}} + \frac{1}{2}e^{nj\frac{5\pi}{24}}$

$$\begin{cases} \frac{1}{2} & k = 11, 5 \\ 0 & 1 \leq k \leq 24 \ \& \ k \neq 11, 5 \end{cases}$$

f) $x[n] = e^{j\frac{4\pi}{5}n}$

$T_0 = \frac{2\pi}{\frac{4\pi}{5}} = \frac{10\pi}{4} = \frac{5}{2} \rightarrow w_0 = \frac{2\pi}{5}$

$$\begin{cases} a_k = 1 & k = 2 \\ a_k = 0 & \text{o.w in } 1 \leq k \leq 4 \end{cases}$$

g) $x[n] = \sin\left(\frac{\pi}{2}n\right) + \cos\left(\frac{3\pi}{5}n\right) \qquad w_0 = \frac{2\pi}{20} = \frac{\pi}{10}$

$\frac{2\pi}{\frac{\pi}{2}} = 4 \qquad \frac{2\pi}{\frac{3\pi}{5}} = \frac{10}{3} \qquad N = 20$

$\frac{1}{2j}\left(e^{j\frac{\pi}{2}n} - e^{-j\frac{\pi}{2}n}\right) + \frac{1}{2}\left(e^{j\frac{3\pi}{5}n} + e^{-j\frac{3\pi}{5}n}\right) \qquad k =$

$\frac{\pi}{2} = \frac{5\pi}{10} \qquad \frac{3\pi}{5} = \frac{6\pi}{10}$

$a_5 = \frac{1}{2j} \qquad a_{-5} = -\frac{1}{2j} \qquad a_6 = \frac{1}{2} \qquad a_{-6} = \frac{1}{2}$

$\hookrightarrow a_{15} = -\frac{1}{2j} \qquad a_{14} = \frac{1}{2} \qquad a_k = \frac{1}{2} \quad k = 14, 6$

$a_k = \frac{1}{2j} \quad k = 5$

$a_k = \frac{-1}{2j} \quad k = 15$

$a_k = 0 \quad \text{o.w}$

۱۲

(h) $x[n] = \sum_{m=-\infty}^{+\infty} (-1)^m \left(\delta[n-۴m] + \delta[n+۳m] \right) \to N=۱۲$

m زوج : $\sum_{n=-\infty}^{+\infty} -\delta[(n-۲)-۴m] - \delta[(n+۳)-۶m]$ m فرد $\sum_{n=-\infty}^{+\infty} \delta[n-۴m] + \delta[n+۶m]$

$\frac{1}{۴} e^{-۲jk\frac{\pi}{۲}}$ $\quad \frac{۱}{۶} e^{-۳\times jk\frac{\pi}{۳}}$ $\quad a_k = \frac{1}{۴} \quad b_k = \frac{1}{۶}$

$c_k = \frac{1}{۴} + \frac{1}{۶} - \left(\frac{1}{۴} \underbrace{\cancel{e^{-۲jk\frac{\pi}{۲}}}}_{(-1)^k} + \frac{1}{۶} \underbrace{\cancel{e^{-۳jk\frac{\pi}{۳}}}}_{(-1)^k} \right) = \frac{۵}{۱۲}\left(1-(-1)^k\right)$

(۲) $P(t) = \sum_{k=-\infty}^{+\infty} |a_k|^2 = \sum_{k=-\infty}^{+\infty} \left|\frac{1}{2^{|k|}}\right|^2 = 2\sum_{k=1}^{+\infty} 2^{-2|k|} + 1 = \frac{2}{3} + 1 = \frac{5}{3}$

$2^{-2k} \longrightarrow a_1 = \frac{1}{4}$

$q = \frac{1}{4}, \quad \frac{\frac{1}{4}}{1-\frac{1}{4}} = \frac{1}{3}$

(۳) $x[n] = \cos^2\left(\frac{n\pi}{10}\right) \longrightarrow \frac{1+\cos\left(\frac{n\pi}{5}\right)}{2} \longrightarrow N = \frac{2\pi}{\frac{\pi}{5}} = \frac{10}{1}$

$P = \frac{1}{N} \sum_{k<N>} |x[n]|^2 \longrightarrow \frac{1}{10}\sum_{n=0}^{9} \left|\frac{1+\cos\left(\frac{n\pi}{5}\right)}{2}\right|^2 = \frac{1}{10}\sum_{n=0}^{9} \frac{1+\cos\left(\frac{n\pi}{5}\right)}{2}$

قرینه

$\frac{1}{10}\left(5 + \cos\left(\frac{4\pi}{5}\right) + \cos\frac{7\pi}{5} + \cos\frac{8\pi}{5} + \cos\frac{9\pi}{5}\right) = \frac{5}{10} \longleftarrow \frac{1}{10}\left(5 + \sum_{n=0}^{9} \frac{\cos\left(\frac{n\pi}{5}\right)}{2}\right)$

قرینه

از $\cos(0)$ تا $\cos(2\pi)$ نفی

(۴) $y[n] = \underbrace{x[n]}_{\text{خروجی سیستم}} - 1$

a_0 تمایزی $\longrightarrow a_0 = 1 + j$ $\qquad a_k = 1 + j^k$

اثر می‌گذارد $\qquad b_0 = 2 - 1 = j$

ضرایب با b_k

$P = \sum_{k=0}^{3} |b_k|^2 = (1)^2 + (\sqrt{2})^2 + (0)^2 + (\sqrt{2})^2 = 1+2+0+2 = 5$

(۵) $\omega_0 = \frac{\pi}{2} \qquad T = 4$

$\frac{\pi}{4} = \omega T_1 \longrightarrow \frac{1}{4} \qquad a_k = \frac{\sin(\omega k T_1)}{k\pi}$ موج مربعی

$a_k = (-1)^k \frac{\sin\frac{k\pi}{8}}{2k\pi} \xrightarrow{\text{k زوج}} a_{2k} = \frac{\sin\left(\frac{\pi k}{4}\right)}{4k\pi} =$

$\xrightarrow{\text{موج مربعی}} e(t) = \begin{cases} \frac{1}{4} & |t| < \frac{1}{4} \\ 0 & \frac{1}{4} < |t| < 1 \end{cases} \longrightarrow$ با دوره تناوب ۲

$\xrightarrow{\text{k فرد}} a_{2k+1} = \frac{-\sin\left((2k+1)\frac{\pi}{8}\right)}{2(2k+1)\pi}$

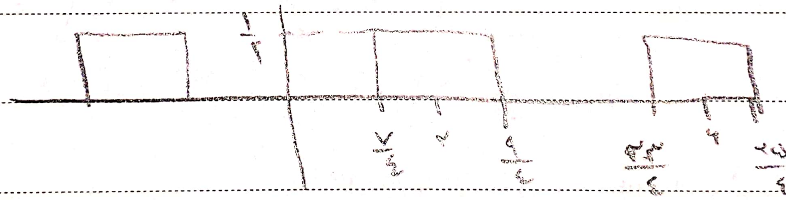
5

$\omega_0 = \frac{\pi}{۲}$, $T = ۴ \Rightarrow \frac{k\pi}{۸} = k\frac{\pi}{۲}T_1 \rightarrow T_1 = \frac{1}{۴} \rightarrow b_k = \frac{\sin k\frac{\pi}{۴}}{k\pi}$

$f(t)$

$(-1)^k \xrightarrow[\text{راهنمای}]{\text{براساس}} e^{jk\omega_0 t_0} \rightarrow e^{jk\frac{\pi}{۲}t_0} = e^{jk\pi} \longrightarrow t_0 = ۲$ time shifting

$y(t) \rightarrow c_k = (-1)^k \frac{\sin k\frac{\pi}{۴}}{k\pi} \rightarrow y(t) = \frac{1}{۲} f(t-۲)$



(6) $T = 4$

$$a_k \begin{cases} 1 & k \text{ زوج} \\ 2 & k \text{ فرد} \end{cases}$$

در این a_k ها ۲ عدد ثابت است پس قطار $\delta(t) = \sum_{-\infty}^{+\infty} \delta(t-kT)$ استفاده شود

$$a_k = \frac{1}{T}$$

$$x(t) = g(t) + h(t)$$

$$g(t) \downarrow a_k \qquad h(t) \downarrow b_k$$

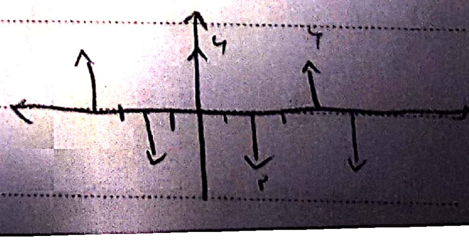
$$a_k = 1 \longrightarrow g(t) = \sum_{k=-\infty}^{+\infty} T \times \delta(t-kT) \xrightarrow{T=4} 4 \sum_{k=-\infty}^{+\infty} \delta(t-4k)$$

$$b_k \Rightarrow \begin{cases} 0 & \text{زوج} \\ 1 & \text{فرد} \end{cases} \rightarrow \underbrace{1}_{a_k} - \underbrace{\begin{cases} 1 & \text{زوج} \\ 0 & \text{فرد} \end{cases}}_{c_k} \rightarrow T = 2 = 2 \sum_{k=-\infty}^{+\infty} \delta(t-2k)$$

$$g(t) - 2 \sum_{k=-\infty}^{+\infty} \delta(t-2k) \overset{FS}{\Longleftrightarrow} b_k$$

$$x(t) = g(t) + g(t) - 2 \sum_{k=-\infty}^{+\infty} \delta(t-2k)$$

$$x(t) = \sum_{k=-\infty}^{+\infty} 8\delta(t-4k) - 2\delta(t-2k)$$



⑦ $$y(t) = \sum_{k=-\infty}^{+\infty} \underbrace{a_k H(k\omega_0)}_{a_{ky}} e^{jk\omega_0 t} \, dt$$

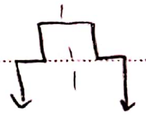
$$a_k = ? \longrightarrow a_k = \frac{1}{T}\int_T x(t) e^{-jk\omega_0 t} \, dt$$

$$= \frac{1}{4}\left(-\int_{-1}^{-\frac{1}{2}} e^{-j\frac{k\pi}{2}kt} + 2\int_{-\frac{1}{2}}^{\frac{1}{2}} t e^{-j\frac{\pi}{2}k}\right)$$

قطارهای ضربه → هم   به مرتبه   انتگرال

به صورت ساده سازی $a_k = \frac{1}{j\pi k}\left(\frac{4\sin\left(\frac{\pi k}{4}\right)}{\pi k} - \cos\left(\frac{\pi k}{2}\right)\right)$

برای بدست آوردن حاصل این رابطه اگر از $x(t)$ دو بار مشتق بگیریم

 → از یک تابع ضربه و قطارهای تشکیل شده

که اگر از یک پالس دوبار انتگرال بگیریم $a_0 = \frac{ST}{T} = \frac{S(\text{تابع مد})}{4} = 0$

به صورت بالا در میاید

$$a_{ky} = H(k\omega_0)\, a_{kx} \rightarrow H\left(\frac{k\pi}{2}\right) a_{kx}$$

$$-1 \le \frac{k\pi}{2} \le 1 \longrightarrow k = 0 \xrightarrow{T=0} \ldots, -8, -4, 0, 4, 8, \ldots$$

$$a_{ky} = \begin{cases} a_{kx} & k = 4\ell \\ 0 & k \ne 4\ell \end{cases} \rightarrow \sum_{\substack{k=-\infty \\ k\ne 0}}^{+\infty} \frac{1}{4j\pi k}\left[\frac{\sin(\pi k)}{\pi k} - \cos(2\pi k)\right] e^{j2\pi kt}$$

⑧ وارون پذیر نیست زیرا در بسیاری از فرکانس‌ها بدون توجه به مقدار آن‌ها تبدیل به صفر می‌شود پس وارون پذیر نمی‌باشد

(9) $x_1(t) \longrightarrow c_k = \dfrac{\sin k\omega_0 T_1}{k\pi}$

$T = 4 \,,\; \omega_0 = \dfrac{\pi}{2} \,,\; T_1 = \dfrac{1}{2} \qquad y(t) \to b_k = j^k c_k = e^{jk\frac{\pi}{2}} c_k$

$y(t) = x_1(t-1) \,,\quad c_0 = \dfrac{1}{2}$

$a_0 = 0 \,,\quad x(t) = y(t) - \dfrac{1}{2} \;\to\; x(t) = x_1(t-1) - \dfrac{1}{2}$