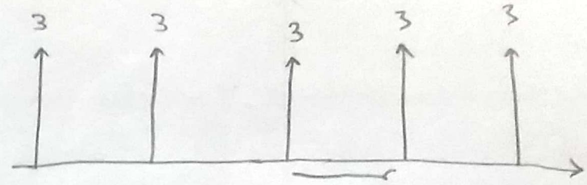
1-a) $x(t) = \sum_{k=-\infty}^{\infty} 3\delta(2t - kT_0)$



$2t - kT_0 = 0 \Rightarrow k = \frac{2t}{T_0} = \frac{t}{\frac{T_0}{2}}$

$\frac{T_0}{2} \rightarrow \omega_0 = \frac{2\pi}{\frac{T_0}{2}} = \frac{4\pi}{T_0}$

$a_m = \frac{1}{\frac{T_0}{2}} \int_{-\frac{T_0}{4}}^{\frac{T_0}{4}} \left(\sum_{k=-\infty}^{\infty} 3\delta(2t - kT_0) \right) \times e^{-jm\frac{4\pi}{T_0}t} \, dt$

$= \frac{6}{T_0} \int_{-\frac{T_0}{4}}^{\frac{T_0}{4}} \left(\sum_{k=-\infty}^{\infty} \delta(2t - kT_0) \right) e^{-jm\frac{4\pi}{T_0}t} \, dt \quad \xrightarrow[\text{در } t=0 \text{ صفر نیست و در } T=0 \text{ مساحت زیر نمودار 1 است}]{\text{تابع زیر انتگرال فقط}} \quad \frac{6}{T_0} \Rightarrow \boxed{a_m = \frac{6}{T_0}}$

b) $x(t) = \cos\frac{3t}{4} \cdot (3 + 2\cos t) = 3\cos\frac{3t}{4} + 2\cos(t) \cdot \cos\left(\frac{3t}{4}\right)$

$= 3\cos\left(\frac{3t}{4}\right) + \cos\left(\frac{t}{4}\right) + \cos\left(\frac{7t}{4}\right) = \frac{3}{2}\left(e^{j\frac{3t}{4}} + e^{-j\frac{3t}{4}}\right) + \frac{1}{2}\left(e^{j\frac{t}{4}} + e^{-j\frac{t}{4}} + e^{j\frac{7t}{4}} + e^{-j\frac{7t}{4}}\right)$

$= \frac{1}{2}e^{j \times 1 \times \frac{t}{4}} + \frac{1}{2}e^{j \times (-1) \times \frac{t}{4}} + \frac{3}{2}e^{j \times 3 \times \frac{t}{4}} + \frac{3}{2}e^{j \times -3 \times \frac{t}{4}} + \frac{1}{2}e^{j \times 7 \times \frac{t}{4}} + \frac{1}{2}e^{j \times -7 \times \frac{t}{4}}$

$a_k = \begin{cases} \frac{1}{2} & k = \pm 1, \pm 7 \\ \frac{3}{2} & k = \pm 3 \\ 0 & \text{O.W} \end{cases}$

c) $x(t) = e^{j\frac{\pi}{4}t}\left(\cos\left(\frac{\pi}{2}t\right) + \sin\left(\frac{\pi}{6}t + \frac{\pi}{4}\right)\right)$

$$= \frac{e^{j\frac{\pi}{4}t}}{2}\left(e^{j\frac{\pi}{2}t} + e^{-j\frac{\pi}{2}t}\right) + \frac{e^{j\frac{\pi}{4}t}}{2j}\left(e^{j\left(\frac{\pi}{6}t + \frac{\pi}{4}\right)} - e^{-j\left(\frac{\pi}{6}t + \frac{\pi}{4}\right)}\right)$$

$$= \frac{1}{2}\left(e^{j \times 9 \times \frac{\pi}{12}t} + e^{j \times -3 \times \frac{\pi}{12}t}\right) + \frac{e^{j\frac{\pi}{4}}}{2j} \times e^{j \times 5 \times \frac{\pi}{12}t} - \frac{e^{-j\frac{\pi}{4}}}{2j} \times e^{j \times 1 \times \frac{\pi}{12}t}$$

$$a_k = \begin{cases} -\frac{e^{-j\frac{\pi}{4}}}{2j} & k = 1 \\ \frac{1}{2} & k = -3, 9 \\ \frac{e^{j\frac{\pi}{4}}}{2j} & k = 5 \\ 0 & \text{O.W} \end{cases}$$

d) $x(t) = \cos(2\pi t) + 7\sin(t) \longrightarrow$ تابع متناوب نیست $\rightarrow$ سری فوریه ندارد.

e) $x[n] = e^{j\frac{\pi}{3}n}\cos\left(\frac{\pi}{8}n\right) = \frac{e^{j\frac{8\pi}{24}n}}{2}\left(e^{j\frac{3\pi}{24}n} + e^{-j\frac{3\pi}{24}n}\right)$

$= \frac{1}{2}\left(e^{j\times 11\times\frac{\pi}{24}\times n} + e^{j\times 5\times\frac{\pi}{24}n}\right) \qquad a_k = \begin{cases} \frac{1}{2} & k=5,11 \\ 0 & 1\leq k\leq 48,\ k\neq 5,11 \end{cases}$

f) $x[n] = e^{j\frac{4\pi}{5}n} \qquad T=5 \rightarrow \omega_0 = \frac{2\pi}{5} \rightarrow \sum_{n=0}^{4} a_n e^{j\frac{2\pi}{5}n} = x(t)$

$\Rightarrow a_n = \begin{cases} 1 & n=2 \\ 0 & n\neq 2,\ 0\leq n\leq 4 \end{cases}$

g) $x[n] = \sin\left(\frac{\pi}{2}n\right) + \cos\left(\frac{3\pi}{5}n\right) \rightarrow$ تابع متناوب نیست (در فضای گسسته) $\rightarrow$ سری فوریه ندارد.

h) $x[n] = \sum_{m=-\infty}^{\infty}(-1)^m\left(\delta[n-2m] + \delta[n+3m]\right) \qquad N=12$

$= \sum_{m=-\infty}^{\infty} \delta[n-4m] - \delta[(n-2)-4m] + \delta[n-6m] - \delta[(n-3)-6m]$

Comb $\Rightarrow a_k = \frac{1}{4} + \frac{1}{6} - \left(\underbrace{\frac{1}{4}\times e^{-jk\times\frac{\pi}{2}\times 2}}_{\text{time shift}} + \underbrace{\frac{1}{6}\times e^{-jk\times\frac{\pi}{3}\times 3}}_{\text{time shift}}\right) = \frac{5}{12}\left(e^{-jk\pi} + 1\right)$

2- طبق رابطه پارسوال $P_\infty = \sum_{k=-\infty}^{\infty} |a_k|^2 = \sum_{k=-\infty}^{\infty} 2^{-|k|} = 2\sum_{k=1}^{\infty} 2^{-|k|} + 1 = \boxed{3}$

3- $x[n] = \cos^2\left(\frac{n\pi}{10}\right) = \dfrac{\cos\left(\frac{n\pi}{5}\right)+1}{2} = \frac{1}{2} + \frac{1}{4}\left(e^{j\times\frac{\pi}{5}\times n} + e^{-j\times\frac{\pi}{5}\times n}\right)$

$N = 20$

$a_0 = \frac{1}{2}$, $a_1 = \frac{1}{4}$, $a_{-1} = \frac{1}{4}$, بقیه $= 0$

$P_\infty = \sum_{n=-1}^{18} a_n = \boxed{1}$

4- طبق خاصیت خطی بودن $\Rightarrow a_{k_y} = a_{k_x} - a_{k_1} \Rightarrow a_{k_y} = \begin{cases} 1 + j^k & k = 1, 2, 3 \\ j^k & k = 0 \end{cases}$

$$5-\quad a_k = (-1)^k \frac{\sin\frac{k\pi}{8}}{2k\pi} =$$

$$\omega_0 = \frac{\pi}{2} \Rightarrow T = 4$$

$$\frac{T_1}{T} \times 2\pi = \frac{\pi}{8} \Rightarrow \frac{T_1}{2} = \frac{1}{8} \Rightarrow T = \frac{1}{4} \qquad T_g = 2 \to T_g' = \frac{1}{4}$$

$$\rightarrow k \text{ even} \rightarrow a_{2k} = \frac{\sin\frac{k\pi}{4}}{2k\pi} \rightarrow g(t) = \sum_{k=-\infty}^{\infty} \frac{\sin\frac{k\pi}{4}}{4k\pi} e^{j(2\omega_0)kt}$$

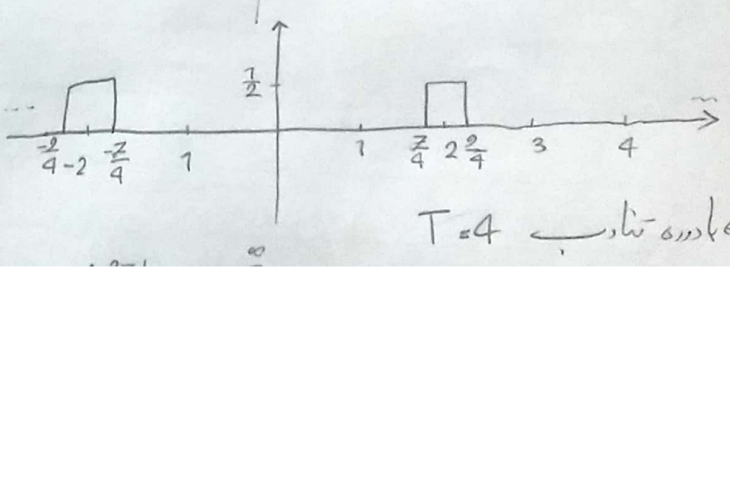
$$\Rightarrow g(t) = \begin{cases} \frac{1}{4} & |t| < \frac{1}{4} \\ 0 & \frac{1}{4} < |t| < 1 \end{cases}$$

$$\rightarrow k \text{ odd} \rightarrow a_{2k+1} = -\frac{\sin\left(\frac{(2k+1)\pi}{8}\right)}{2(2k+1)\pi} \rightarrow h(t) = \sum_{k=-\infty}^{\infty} -\frac{\sin\left(\frac{(2k+1)\pi}{8}\right)}{2\,(2k+1)\pi} e^{j\omega_0(2k+1)t}$$

$$\Rightarrow h(t) = \frac{-1}{2}\left(\sum_{k=-\infty}^{\infty} \frac{\sin\left(\frac{k\pi}{8}\right)}{k\pi} e^{j\frac{\pi}{2}kt} - \frac{1}{2}\sum_{k=-\infty}^{\infty} \frac{\sin\left(\frac{k\pi}{4}\right)}{k\pi} e^{j(\pi)kt}\right)$$

$$= \begin{cases} \frac{-1}{2} & |t| < \frac{1}{4} \\ 0 & \frac{1}{4} < |t| < 2 \end{cases} + \begin{cases} \frac{1}{4} & |t| < \frac{1}{4} \\ 0 & \frac{1}{4} < |t| < 1 \end{cases} = \begin{cases} -\frac{1}{4} & |t| < \frac{1}{4} \\ 0 & \frac{1}{4} < |t| < \frac{7}{4} \\ \frac{1}{4} & \frac{7}{4} < |t| < 2 \end{cases}$$

$$f(t) = h(t) + g(t) = \begin{cases} 0 & |t| < \frac{1}{4} \\ 0 & \frac{1}{4} < |t| < \frac{7}{4} \\ \frac{1}{2} & \frac{7}{4} < |t| < 2 \end{cases}$$



با دوره تناوب $T=4$

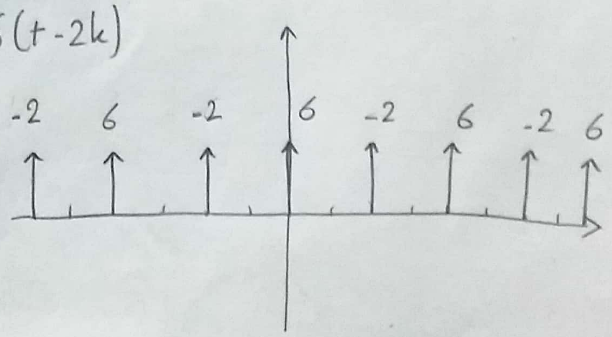
6- $a_k = a_g + a_h$

$$\sum_{k=-\infty}^{\infty} \frac{1}{T} e^{j\frac{2\pi}{T}kt} = \sum_{k=-\infty}^{\infty} \delta(t-kT)$$

$$a_g = 1 \rightarrow g(t) = \sum_{k=-\infty}^{\infty} 4\delta(t-4k)$$

$$a_h = \begin{cases} 0 & \text{زوج } h \\ 1 & \text{فرد } h \end{cases} = 1 - \begin{cases} 1 & \text{زوج } h \\ 0 & \text{فرد } h \end{cases} \Rightarrow g(t) - 2\sum_{k=-\infty}^{\infty} \frac{1}{2} e^{j\pi kt} = g(t) - \sum_{k=-\infty}^{\infty} 2\delta(t-2k)$$

$$f(t) = g(t) + h(t) = \sum_{k=-\infty}^{\infty} 8\delta(t-4k) - 2\delta(t-2k)$$



7- $x(t) = \begin{cases} 0 & -2<t<-1 \\ -1 & -1<t\le -\frac{1}{2} \\ 2t & -\frac{1}{2}<t\le\frac{1}{2} \\ 1 & \frac{1}{2}<t<1 \\ 0 & 1<t<2 \end{cases}$

$T=4$

$\rightarrow \omega_0 = \frac{\pi}{2}$

$$\rightarrow a_{k_x} = \frac{1}{4}\left[-\int_{-1}^{-\frac{1}{2}} e^{-j\frac{\pi}{2}kt} + 2\int_{-\frac{1}{2}}^{\frac{1}{2}} t e^{-j\frac{\pi}{2}kt} + \int_{\frac{1}{2}}^{1} e^{-j\frac{\pi}{2}kt}\right]$$

$$\Rightarrow a_{k_x} = \frac{1}{4}\left[\frac{1}{j\frac{\pi}{2}k} e^{-j\frac{\pi}{2}kt}\Big|_{-1}^{-\frac{1}{2}} - 2\left(\frac{te^{-j\frac{\pi}{2}kt}}{j\frac{\pi}{2}k} + \frac{e^{-j\frac{\pi}{2}kt}}{(j\frac{\pi}{2}k)^2}\Big|_{-\frac{1}{2}}^{\frac{1}{2}}\right) - \frac{1}{j\frac{\pi}{2}k} e^{-j\frac{\pi}{2}kt}\Big|_{\frac{1}{2}}^{1}\right]$$

$$= \frac{1}{2j\pi k}\left[e^{\frac{j\pi k}{4}} - e^{\frac{j\pi k}{2}} - e^{\frac{-j\pi k}{4}} - \frac{4e^{\frac{-j\pi k}{4}}}{j\pi k} - e^{\frac{j\pi k}{4}} + \frac{4e^{\frac{+j\pi k}{4}}}{j\pi k} - e^{\frac{-j\pi k}{2}} + e^{\frac{-j\pi k}{4}}\right]$$

$$= \frac{1}{2j\pi k}\left[-\left(e^{\frac{j\pi k}{2}} + e^{\frac{-j\pi k}{2}}\right) + \frac{4}{j\pi k}\left(e^{\frac{j\pi k}{4}} - e^{\frac{-j\pi k}{4}}\right)\right]$$

$$= \frac{1}{2j\pi k}\left[-2\cos\left(\frac{\pi k}{2}\right) + \frac{8}{\pi k}\sin\left(\frac{\pi k}{4}\right)\right] = \frac{1}{j\pi k}\left[\frac{4\sin(\frac{\pi k}{4})}{\pi k} - \cos\left(\frac{\pi k}{2}\right)\right] \quad k\neq 0$$

$k=0 \rightarrow a_0 = 0$

$$a_{k_y} = H(k\omega_0)a_{k_x} = H\left(\frac{k\pi}{2}\right)a_{k_x}$$

$-1\le\frac{k\pi}{2}\le 1 \Rightarrow -2\le k\pi\le 2 \Rightarrow k=0$ $\rightarrow$ [illegible] $\frac{\infty}{4}$! $\rightarrow$ $k = \ldots, -8, -4, 0, 4, 8, \ldots$

$$\Rightarrow a_{k_y} = \begin{cases} a_{k_x} & k=4m \\ 0 & k\neq 4m \end{cases} \qquad y(t) = \sum_{k=-\infty}^{\infty} a_{k_y} e^{j\omega_0 kt} = \sum_{k=-\infty}^{\infty} a_{4k_x} e^{j(4\omega_0)kt}$$

$$= \sum_{\substack{k=-\infty \\ k\neq 0\ (k=0\rightarrow a_0=0)}}^{\infty} \frac{1}{4j\pi k}\left[\frac{\overset{0}{\sin(\pi k)}}{\pi k} - \overset{1}{\cos(2\pi k)}\right] e^{j2\pi kt} = \boxed{\sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \frac{-1}{4j\pi k} e^{j2\pi kt}}$$

8- خیر زیرا برخی فرکانس ها بدون توجه به مقادیر اولیه‌شان 0 می‌شوند ← نمی‌توان این عملیات را وارون کرد

9- $T_g = 4 \to T'_g = \frac{1}{2}$

$$g(t) = \begin{cases} \frac{1}{4} & |t| < \frac{1}{2} \\ 0 & \frac{1}{2} < |t| < 2 \end{cases}$$

$$x(t) = f(t) * g(t) \to a_k = a_{k_f} \times (a_{k_g} \times 4)$$

$$\to f(t) = \sum_{k=-\infty}^{\infty} j^k e^{\frac{j\pi t}{2}k} = \left(1 + je^{\frac{j\pi t}{2}} - e^{j\pi t} - je^{\frac{3j\pi t}{2}}\right) \sum_{k=-\infty}^{\infty} 1 \times e^{j2\pi t k} = \left(1 + je^{\frac{j\pi t}{2}} - e^{j\pi t} - e^{\frac{3j\pi t}{2}}\right) \times \sum_{k=-\infty}^{\infty} \delta(t-k)$$