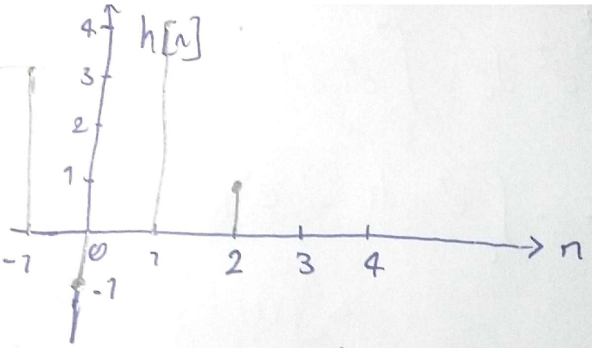
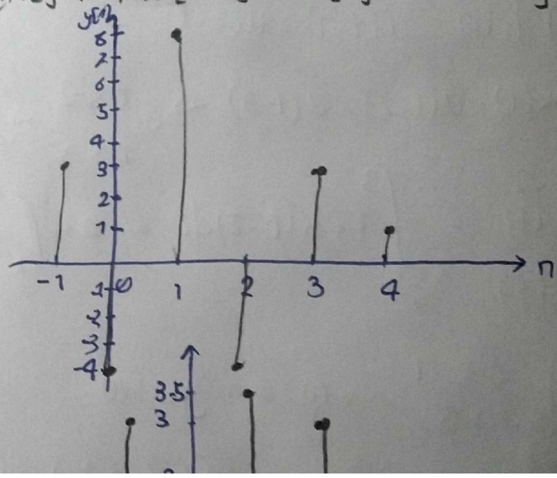
1 - $h[n]$ -



$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k] = \sum_{k=n-2}^{n+1} x[k]\,h[n-k] = 1 \times x[n-2] + 4x[n-1] - x[n] + 3x[n+1]$$

a) $x[n] = \begin{cases} e^{j\pi n} = (-1)^n & 0 \le n < 3 \\ 0 & \text{o.w} \end{cases}$

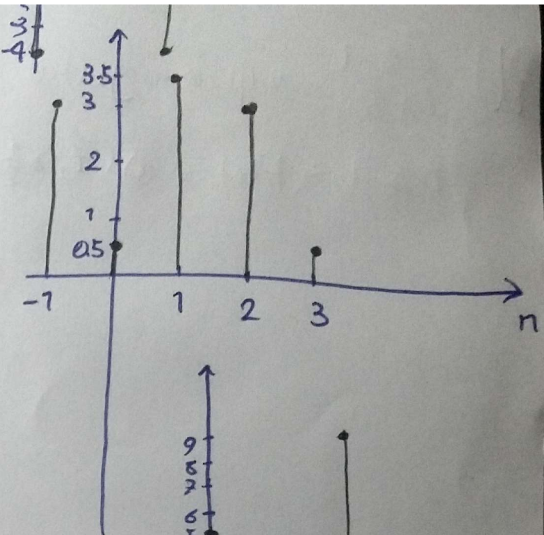
$$y[n] = \begin{cases} 3 & n=-1 \\ -1-3=-4 & n=0 \\ 4+1+3=8 & n=1 \\ 1-4-1=-4 & n=2 \\ -1+4=3 & n=3 \\ 1 & n=4 \end{cases}$$



$$\begin{cases} -1+4 = 3 & n=3 \\ 1 & n=4 \end{cases}$$

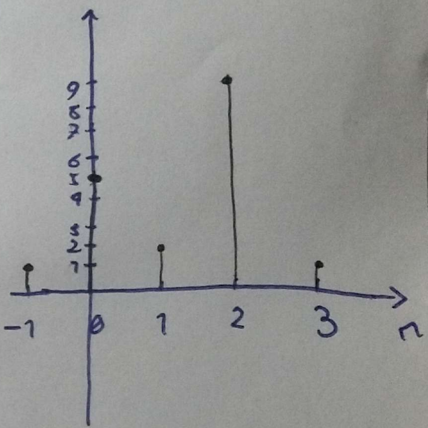
b) $x[n] = \begin{cases} \left(\frac{1}{2}\right)^n & 0 \leq n < 2 \\ 0 & O.W \end{cases}$

$$y[n] = \begin{cases} 3 & n=-1 \\ -1+\frac{3}{2} = \frac{1}{2} = 0.5 & n=0 \\ 4-\frac{1}{2} = 3.5 & n=1 \\ 1+2 = 3 & n=2 \\ \frac{1}{2} = 0.5 & n=3 \end{cases}$$



c) $x[n] = \begin{cases} 2^n & -2 \leq n < 0 \\ 0 & O.W \end{cases}$

$$y[n] = \begin{cases} 3 & n=-1 \\ -1+6 = 5 & n=0 \\ 4-2 = 2 & n=1 \\ 1+8 = 9 & n=2 \\ 1 & n=3 \end{cases}$$



d) $x[n] = \begin{cases} (-1)^n & n \geq 0 \\ 0 & O.W \end{cases}$

$$y[1] = x[-1] + 4x[0] - x[1] + 3x[2]$$

$$= 0 + 4 + 1 + 3 = \boxed{8}$$

2- a) $y(t) = \int_{-\infty}^{\infty} e^{\tau} u(t-\tau)\, d\tau = \int_{-\infty}^{t} e^{\tau}\, d\tau + \int_{t}^{\infty} e^{\tau} \times 0\, d\tau = e^{\tau}\Big|_{-\infty}^{t} = e^{t}$

b) $\underset{(t \geq 0)}{y(t)} = \int_{0}^{\infty} e^{\tau} u(\tau)\, u(t-\tau)\, d\tau = \int_{0}^{t} e^{\tau} d\tau + \int_{t}^{\infty} e^{\tau} \times 0\, d\tau = e^{\tau}\Big|_{0}^{t} = e^{t} - 1$

$t < 0 \rightarrow u(t) = 0 \rightarrow y(t) = 0$

$$\Rightarrow y(t) = \begin{cases} e^{t-1} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

c) $h(t) = u(t+1) - u(t-1)$

$x(t) = u(t+2) - u(t-3) \rightarrow \begin{matrix} t < -2 \\ \text{or} \\ t \geq 3 \end{matrix} \rightarrow y(t) = 0$

$\underset{(-2 \leq t < 3)}{y(t)} = \int_{-2}^{3} 1 \times h(t-\tau)\, d\tau = 0 + \int_{-1}^{1} 1\, d\tau + 0 = \tau\Big|_{-1}^{1} = 2$

$$\Rightarrow y(t) = \begin{cases} 2 & -2 \leq u < 3 \\ 0 & \text{o.w} \end{cases}$$

d) $\begin{matrix} t < -1 \\ t \geq 3 \end{matrix} \rightarrow u(t) = 0 \rightarrow y(t) = 0$

$t \geq -1 \rightarrow h(t) = \delta(t_1 \geq \frac{2}{2}) + \delta(t_2 \geq \frac{3}{2}) = 0 \rightarrow y(t) = 0$

$\Big\}\ y(t) = 0$

3-

a) Linear: $a x(t_1) + b x(t_2) \rightarrow \frac{d}{dt}(a x(t_1) + b x(t_2)) = a \frac{d}{dt} x(t_1) + b \frac{d}{dt} x(t_2) = a y(t_1) + b y(t_2)$ ✓

TI: $y_2(t) = \frac{d}{dt}(x(t-\tau))$ ✓

$y_1(t-\tau) = \frac{d}{dt}(x(t-\tau))$

b) $\delta(t) \longrightarrow \boxed{L \text{ or } h} \longrightarrow h(t)$

$u(t) \longrightarrow \boxed{L \text{ or } h} \longrightarrow s(t) = u(t) * h(t) = h(t) * u(t) = \int_{-\infty}^{\infty} h(\tau)\, u(t-\tau)\, d\tau$

$\Rightarrow s(t) = \int_{-\infty}^{t} h(\tau)\, d\tau \xRightarrow{\frac{d}{dt}} \frac{d\, s(t)}{dt} = h(t)$

4-(ML) memoryless $\Leftrightarrow$ $h(t) = k\,\delta(t)$ or $h[n] = k\,\delta[n]$

(C) causal $\Leftrightarrow$ $h(t) = 0 \quad t<0$ or $h[n] = 0 \quad n<0$

(S) stable $\Leftrightarrow$ $\int_{-\infty}^{\infty} |h(t)|\,dt < \infty$ or $\sum_{-\infty}^{\infty} |h[n]| < \infty$

a) $h(t) = e^{-4|t|}$ $\boxed{ML \times}$ $h(-1) \neq 0 \rightarrow \boxed{C \times}$

$$\int_{-\infty}^{\infty} |e^{-4|t|}|\,dt = 2\int_{0}^{\infty} e^{-4t}\,dt = \frac{-1}{2} e^{-4t}\Big|_0^{\infty} = \frac{+1}{2} < \infty \quad \boxed{S \checkmark}$$

b) $h(t) = t e^{-t} u(t)$ $\boxed{ML \times}$ $h(t<0) = 0 \rightarrow \boxed{C \checkmark}$

$$\int_{-\infty}^{\infty} |t e^{-t} u(t)|\,dt = \int_{0}^{\infty} t e^{-t}\,dt = -te^{-t} - e^{-t}\Big|_0^{\infty} = +1 < \infty \rightarrow \boxed{S \checkmark}$$

c) $h(t) = \cos(2\pi t)\, u(t+1)$ $\boxed{ML \times}$ $h(-1) \neq 0 \rightarrow \boxed{C \times}$

$$\int_{-\infty}^{\infty} |\cos(2\pi t)\, u(t+1)|\,dt = \int_{-1}^{\infty} |\cos(2\pi t)|\,dt = \infty \rightarrow \boxed{S \times}$$

d) $h(t) = \frac{\sin(t)}{t} u(t)$ $\boxed{ML \times}$ $h(t<0) = 0 \rightarrow \boxed{C \checkmark}$

$$\int_{-\infty}^{\infty} \left| \frac{\sin(t)}{t} u(t) \right| dt = \int_{0}^{\infty} \frac{|\sin(t)|}{t} dt$$

e) $h[n] = (\frac{1}{2})^n u[n]$ $\boxed{ML \times}$ $h(t<0) \neq 0$ $\boxed{C \times}$

$$\sum_{n=-\infty}^{\infty} (\frac{1}{2})^n u[n] = \sum_{-\infty}^{0} 2^{-n} = \infty \quad \boxed{S \times}$$

f) $h[n] = \delta[2n]$ $\boxed{ML \times}$ $h[n<0] = 0 \rightarrow \boxed{C \checkmark}$

$$\sum_{-\infty}^{\infty} |\delta[2n]| = 1 \rightarrow \boxed{S \checkmark}$$

g) $h[n] = \cos(\frac{\pi}{2} n) u[n+1]$ $\boxed{ML \times}$ $h[-1] = 0$, $h[n<-1] = 0 \rightarrow \boxed{C \checkmark}$

$$\sum_{-\infty}^{\infty} \left| \cos(\frac{\pi}{2} n) u[n+1] \right| = \sum_{-1}^{\infty} \left| \cos(\frac{\pi}{2} n) \right| = 0+1+0+1+0+1+\dots = \infty \quad \boxed{S \times}$$

$-\infty$

h) $h[n] = e^{2n} u[n]$ $\boxed{ML \times}$ $h[n<0] = 0 \to \boxed{C \checkmark}$

$$\sum_{-\infty}^{\infty} |e^{2n} u[n]| = \sum_{0}^{\infty} e^{2n} = \infty \quad \boxed{S \times}$$

5-

$$u(t) = \begin{cases} e^{2t} & t \leq 1 \\ 0 & t > 1 \end{cases} \qquad h(t) = \begin{cases} 0 & t < 0 \\ 2 & 0 \leq t < 2 \\ 6 & 2 \leq t < 4 \\ 14 & 4 \leq t \end{cases} \rightarrow u(t) * h(t)$$

فقط در بازه $0 \leq t \leq 1$ می‌تواند صفر نباشد

$$\underset{0 \leq t \leq 1}{y(t)} = \int_{-\infty}^{1} e^{2\tau} h(t-\tau)\,d\tau = 2\int_{-\infty}^{1} e^{2\tau} u(t-\tau)\,d\tau + 4\int_{-\infty}^{1} e^{2\tau} u(t-\tau-2)\,d\tau + 8\int_{-\infty}^{1} e^{2\tau} u(t-\tau-4)\,d\tau$$

$$\underset{\text{جزء به جزء}}{=} 2\left(\frac{e^{2\tau}}{2} u(t-\tau)\Big|_{-\infty}^{1} - \int_{-\infty}^{1} \frac{e^{2\tau}}{2} \delta(t-\tau)\,d\tau\right) + 4\left(\frac{e^{2\tau}}{2} u(t-\tau-2)\Big|_{-\infty}^{1} - \int_{-\infty}^{1} \frac{e^{2\tau}}{2} \delta(t-\tau-2)\,d\tau\right)$$

$$+ 8\left(\frac{e^{2\tau}}{2} u(t-\tau-4)\Big|_{-\infty}^{1} - \int_{-\infty}^{1} \frac{e^{2\tau}}{2} \delta(t-\tau-4)\,d\tau\right) = 2\left(\frac{e^{2}}{2} u(t-1) - \frac{e^{2t}}{2}\right) + 4\left(\frac{e^{2}}{2} u(t-3) - \frac{e^{2t}}{2}\right)$$

$$+ 8\left(\frac{e^{2}}{2} u(t-5) - \frac{e^{2t}}{2}\right)$$

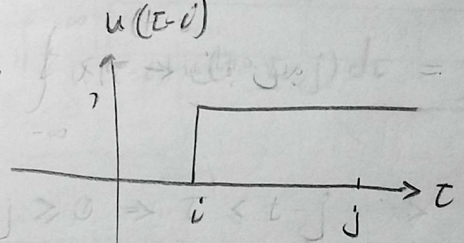
$$6-\quad h_{eq}(t) = h_1(t) + h_2(t) * \left(\underbrace{h_3(t) + h_4(t)}\right) = h_1(t) + \underbrace{h_2(t) * h_3(t)}_{h_5} + \underbrace{h_2(t) * h_4(t)}_{h_6}$$

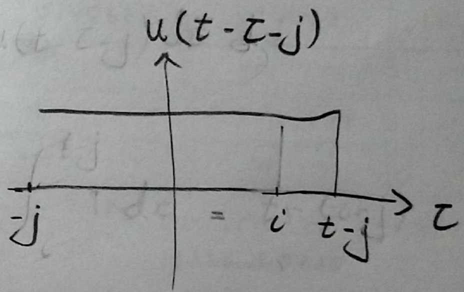
$$h_6(t) = u(t-1) - u(t-2)$$

$$h_5(t) = u(t)*u(t) + u(t-1)*u(t-1) - 2\,u(t)*u(t-1) = t\,u(t) + (t-2)\,u(t-2) - 2(t-1)\,u(t-1)$$

$$u(t-i) * u(t-j) \Rightarrow$$

$$(i \le j)$$





$$t-j \ge i \Rightarrow t \ge i+j \rightarrow \int_i^{t-j} 1\, d\tau = t-(i+j)$$

$$t < i+j \rightarrow 0 \quad (\text{هم‌پوشانی ندارند})$$

$$\rightarrow u(t-i) * u(t-j) = \boxed{t-(i+j)\,u(t-(i+j))}$$

$$h_{eq} = e^{-t}u(t) + t\,u(t) + (t-2)\,u(t-2) - 2(t-1)\,u(t-1) + u(t-1) - u(t-2)$$

7 - $x[n] * h[n] = h[n] * x[n] = \sum_{-\infty}^{\infty} \alpha^N u[N]\, x[n-N] = \sum_{0}^{\infty} \alpha^N x[n-N] = y[n]$

$x[n] * h[n-1] = h[n-1] * x[n] = \sum_{-\infty}^{\infty} \alpha^{N-1} u[N-1]\, x[n-N] = \sum_{1}^{\infty} \alpha^{N-1} x[n-N]$

$= \frac{1}{\alpha} \sum_{1}^{\infty} \alpha^N x[n-N] = \frac{1}{\alpha}\left(y[n] - x[n]\right)$

$x[n] * \left(h[n] - h[n-1]\right) = x[n] * h[n] - x[n] * h[n-1]$

$\Rightarrow x[n+1] \rightarrow \boxed{h} \rightarrow \frac{1}{\alpha}\left(y[n+1] - x[n+1]\right)$

$\quad \rightarrow y[n+1]$

$\alpha\, y[n+1] = y[n+1] - x[n+1]$

$\Rightarrow y[n+1] = \frac{-x[n+1]}{\alpha - 1} \Rightarrow \boxed{y[n] = \frac{x[n]}{1-\alpha}}$

8- a) $h_1(t) * h_2(t) = e^{-t}u(t) * \delta(t) + e^{-t}u(t) * \delta'(t) = e^{-t}u(t) - e^{-t}u(t) + e^{-t}\delta(t)$

$= e^{-t}\delta(t) = \boxed{\delta(t)}$

$\begin{cases} 1 & t=0 \\ 0 & t\neq 0 \end{cases}$

b) $h_1[n] * h_2[n] = h_2[n] * h_1[n] = u[n] * \delta[n] - u[n] * \delta[n-1]$

$= u[n] - u[n-1] = \boxed{\delta[n]}$

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