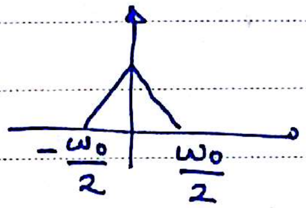
Subject
Date

① Nyquist Rate $= \omega_0$

$\Rightarrow X(j\omega)$:



a) $x(t) + x(t-1)$

$$x(t) \leftrightarrow X(j\omega)$$

$$x(t-1) \leftrightarrow e^{-j\omega} X(j\omega)$$

$$x(t)+x(t-1) \leftrightarrow \underbrace{(1+e^{-j\omega})}_{} X(j\omega)$$

که تأثیری روی Band ندارد

$\Rightarrow$ Nyquist Rate $= \omega_0$

b) $\frac{d}{dt} x(t)$

$$x(t) \leftrightarrow X(j\omega)$$

$$\frac{d}{dt}(x(t)) \leftrightarrow j\omega X(j\omega) \rightarrow \text{just like part a}$$

$\Rightarrow$ Nyquist Rate $= \omega_0$

c) $x^2(t)$

$$x(t) \leftrightarrow X(j\omega)$$

$$x^2(t) = x(t)\,x(t) \leftrightarrow \frac{1}{2\pi}\left(X(j\omega) * X(j\omega)\right)$$

که باعث دوبرابر شدن Band می‌شود

$\Rightarrow$ Nyquist Rate $= 2\omega_0$

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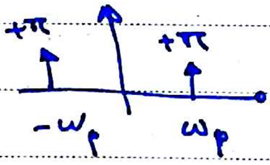
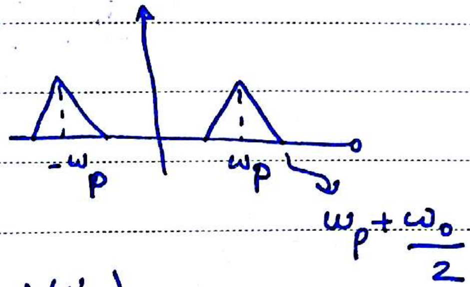
d) $x(t)\cos(\omega_p t)$

$\cos(\omega_p t) \longleftrightarrow$



$\Rightarrow x(t)\cos(\omega_p t) \longleftrightarrow$



$\Rightarrow$ Nyquist Rate $= 2\left(\omega_p + \frac{\omega_0}{2}\right)$

Subject

Date

(2)

a) $x(t) = e^{-5t} u(t) \longleftrightarrow X(j\omega) = \dfrac{1}{j\omega + 5}$

Band-Limited نیست

b) $x(t) = 1 + \cos(100\pi t) + \underbrace{\underbrace{\cos(300\pi t)}_{\substack{2 \text{ impulses} \\ \omega = \pm 300\pi}} \underbrace{\sin(50\pi t)}_{\substack{2 \text{ impulses} \\ \omega = \pm 50\pi}}}$

یک impulse در $\omega = 0$

دو impulse در $\omega = \pm 100\pi$

ضرب در حوزه زمان ← کانولوشن در فرکانس

← باعث ایجاد impulse هایی در نقاط زیر می‌شود

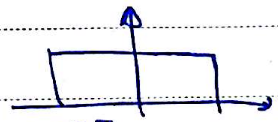
$\omega = -350\pi, -250\pi, 250\pi, 350\pi$

$\max\{0, -100\pi, 100\pi, -350\pi, 350\pi, -250\pi, 250\pi\} = 350\pi$

$\Rightarrow \omega_M = 350\pi$

$\Rightarrow$ Nyquist Rate $= 700\pi = 2(350\pi)$

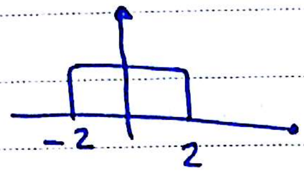
c) $x(t) = \dfrac{\sin(400\pi t)}{\pi t} \longleftrightarrow$



$\Rightarrow \omega_M = 400\pi \Rightarrow$ Nyquist Rate $= 800\pi$

Subject
Date

d) $x(t) = u(t) - u(t-4)$



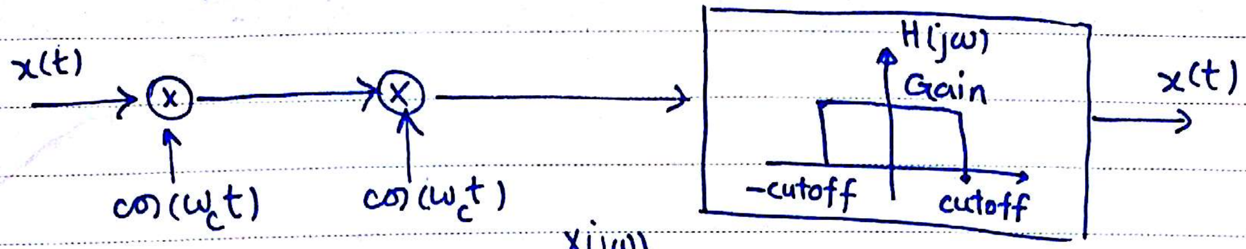
$\longleftrightarrow \quad \dfrac{2\sin(2\omega)}{\omega}$

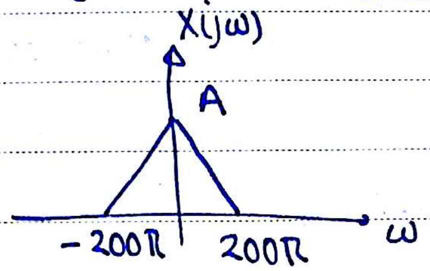
shift $\longrightarrow \quad x(t) \longleftrightarrow e^{-j\omega 2} \, \dfrac{2\sin(2\omega)}{\omega}$

ليس Band-limited

Subject
Date

3





a) ضرب کردن $x(t)$ در $\cos(\omega_c t)$ باعث روابط زیر می شود :

$$x(t) \longleftrightarrow X(j\omega)$$

$$x(t)\cos(\omega_c t) \longleftrightarrow \frac{1}{2}\left(X(j(\omega-\omega_c)) + X(j(\omega+\omega_c))\right)$$

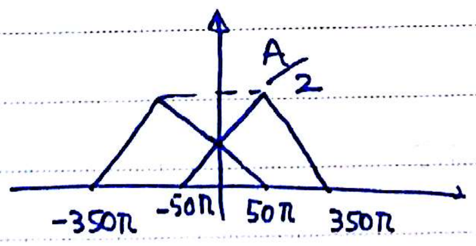
برای اینکه $X(j(\omega-\omega_c))$ و $X(j(\omega+\omega_c))$ باهم overlap نداشته باشند باید $\omega_c \geq \omega_M$ باشد یعنی $\omega_c \geq 200\pi$

بررسی حالات نادرست

* $\boxed{\omega_c < 200\pi} \xrightarrow{\text{Example}} \omega_c = 150\pi$

$$\rightarrow \frac{1}{2}\left(X(j(\omega-150\pi)) + X(j(\omega+150\pi))\right)$$

$$=$$

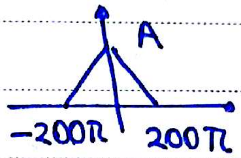


Subject
Date

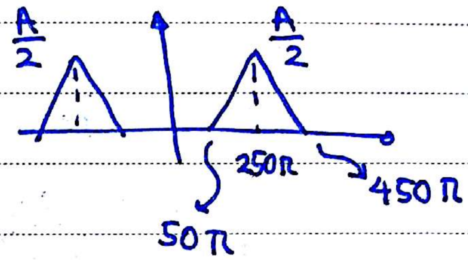
پس مقدار $\omega_c = 250\pi$ را در نظر می‌گیریم که شرط $\omega_c > \omega_M$ رعایت شود

b و c)

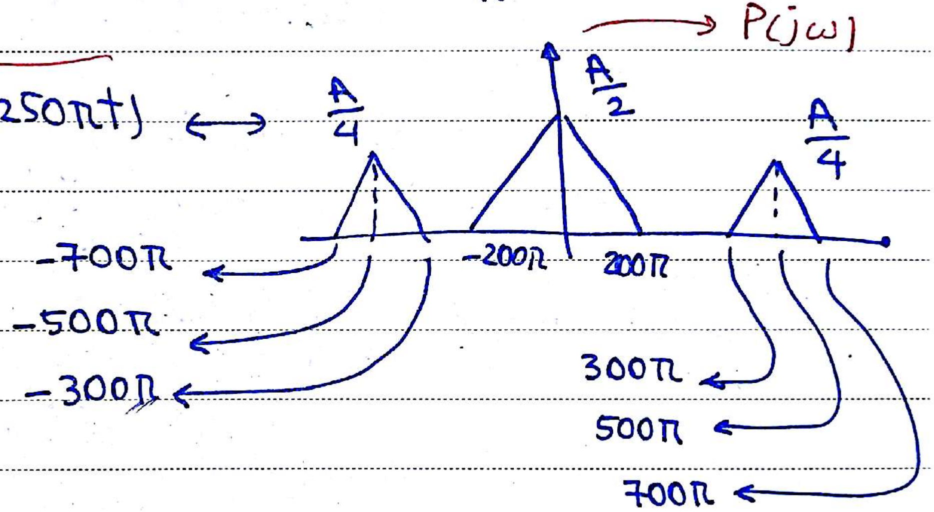
$x(t) \longleftrightarrow$



$x(t)\cos(250\pi t) \longleftrightarrow$



$\overbrace{x(t)\cos(250\pi t)\cos(250\pi t)}^{p(t)} \longleftrightarrow$



← برای اینکه خروجی نهایی $x(t)$ باشد، باید $H(j\omega)P(j\omega) = X(j\omega)$ باشد.

با بررسی نمودار $X(j\omega)$ مشخص است که gain برای $H(j\omega)$ باید

برابر با 2 و cutoff آن میان 200π و 300π باشد

$\longrightarrow$ gain $= 2$

$200\pi <$ cutoff $< 300\pi$

Subject

Date

(4) a) $x[n] = 1 + \cos\left(\frac{n\pi}{2}\right) + \sin(n\pi)$

$\sin(n\pi) = 0 \Rightarrow x[n] = 1 + \cos\left(\frac{n\pi}{2}\right)$

متناوب با $N=4$

$x[n] = 1 + \frac{1}{2}\left(e^{jn\frac{\pi}{2}} + e^{-jn\frac{\pi}{2}}\right)$

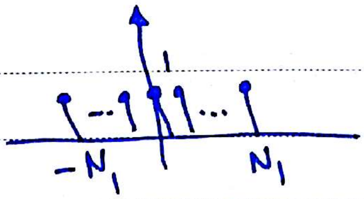
$\longrightarrow a_0 = a_4 = a_8 = a_{12} = \cdots = 1$

$a_1 = a_5 = a_9 = a_{13} = \cdots = \frac{1}{2}$

$a_{-1} = a_3 = a_7 = a_{11} = \cdots = \frac{1}{2}$

Subject
Date

b) $x[n] =$



(one period)

$$a_k = \frac{1}{N} \sum_{n=-N_1}^{N_1} 1 \times e^{-jk\frac{2\pi}{N}n}$$

$$\xrightarrow{m=n+N_1} a_k = \frac{1}{N} \sum_{m=0}^{2N_1} e^{-jk\left(\frac{2\pi}{N}\right)(m-N_1)}$$

$$= \frac{1}{N} e^{jk\frac{2\pi}{N}N_1} \sum_{m=0}^{2N_1} e^{-jk\frac{2\pi}{N}m} \longrightarrow \text{دنباله هندسی}$$

$$\Rightarrow a_k = \frac{1}{N} e^{jk\frac{2\pi}{N}N_1} \left(\frac{1 - e^{-jk\frac{2\pi(2N_1+1)}{N}}}{1 - e^{-jk\frac{2\pi}{N}}} \right)$$

$= \dots$ فاکتورگیری ...

$$= \frac{1}{N} \frac{\sin\left(2\pi k \frac{N_1+\frac{1}{2}}{N}\right)}{\sin\left(\pi \frac{k}{N}\right)} \qquad k \neq k'N$$

و

$$a_k = \frac{2N_1+1}{N} \qquad k = k'N$$

PAPCO

Subject
Date

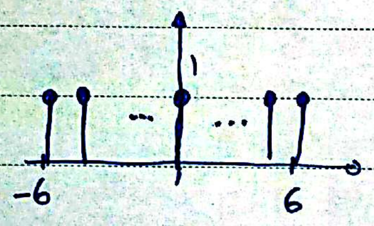
c) $N=5 \Rightarrow \omega_0 = \dfrac{2\pi}{5}$

$$a_k = \frac{1}{N}\sum_{n=\langle N\rangle} x[n]\, e^{-jk\omega_0 n} \Rightarrow a_k = \frac{1}{5}\sum_{n=0}^{4} x[n]\, e^{-jk\frac{2\pi}{5}n}$$

$$\Rightarrow a_k = \frac{1}{5}\left[0 + 1\left(e^{-jk\frac{2\pi}{5}}\right) + 2\left(e^{-jk\frac{4\pi}{5}}\right) + 3\left(e^{-jk\frac{6\pi}{5}}\right) + 4\left(e^{-jk\frac{8\pi}{5}}\right)\right]$$

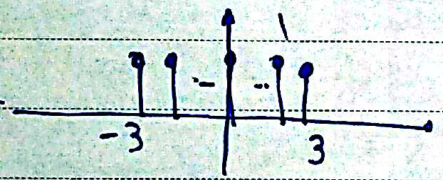
d) $N=20$

$x_1[n] =$



$$\xleftrightarrow{FS} \quad a_k = \begin{cases} k \neq 20k' : & \dfrac{1}{20}\,\dfrac{\sin\left(2\pi k \frac{6.5}{20}\right)}{\sin\left(\pi \frac{k}{20}\right)} \\[2ex] k = 20k' : & \dfrac{13}{20} \end{cases}$$

$x_2[n] =$



$$\xleftrightarrow{FS} \quad b_k = \begin{cases} k \neq 20k' : & \dfrac{1}{20}\,\dfrac{\sin\left(2\pi k \frac{3.5}{20}\right)}{\sin\left(\pi \frac{k}{20}\right)} \\[2ex] k = 20k' : & \dfrac{7}{20} \end{cases}$$

$$x[n] = x_1[n] - 3x_2[n] \xleftrightarrow{FS} \boxed{a_k - 3b_k}$$

e) $N=8 \Rightarrow \omega_0 = \dfrac{\pi}{4}$

$$a_k = \frac{1}{8}\sum_{n=0}^{7} x[n]\, e^{-jk\frac{\pi}{4}n} \xrightarrow[\text{باز کنید}]{} \underbrace{\frac{1}{8}\left[4 + 3e^{-jk\frac{\pi}{4}} + \dots + 3e^{-jk\frac{7\pi}{4}}\right]}_{=a_k}$$

PAPCO

Subject

Date

⑤ a)

$$x[n] \longleftrightarrow a_k$$

$$\xrightarrow{\text{shift}} x[n+3] \longleftrightarrow e^{-jk\frac{2\pi}{N}(-3)} a_k = e^{jk\frac{6\pi}{N}} a_k$$

$$\xrightarrow{\text{Reverse}} x[-n+3] \longleftrightarrow e^{-jk\frac{6\pi}{N}} a_{-k} = c_k$$

$$\xrightarrow{\text{conjugate}} x^*[-n+3] \longleftrightarrow c^*_{-k}$$

$$= e^{-jk\frac{6\pi}{N}} a^*_k$$

b) $x[n] \longleftrightarrow a_k$

$$\Rightarrow a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk\frac{2\pi}{N}n}$$

$$\Rightarrow a_{k-M} = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-j(k-M)\frac{2\pi}{N}n}$$

$$= \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{jM\frac{2\pi}{N}n} e^{-jk\frac{2\pi}{N}n}$$

$$\Rightarrow e^{jM\frac{2\pi}{N}n} x[n] \longleftrightarrow a_{k-M}$$

از آنجا که $(-1)^n = \cos(n\pi)$ و $\cos(n\pi) = e^{jn\pi}$ ، اگر $M = \frac{N}{2}$ باشد داریم :

$$e^{jn\pi} x[n] \longleftrightarrow a_{k-\frac{N}{2}} \Rightarrow (-1)^n x[n] \longleftrightarrow a_{k-\frac{N}{2}}$$

P4PCO

Subject
Date

(6) $x[n]$ Real & Odd

خواص : $x[n] \longleftrightarrow a_k$

$x^*[n] \longleftrightarrow a^*_{-k}$

$x[-n] \longleftrightarrow a_{-k}$

$\rightarrow x[n]$ real $\Rightarrow x[n] = x^*[n] \Rightarrow a_k = a^*_{-k}$

$\rightarrow x[n]$ odd $\Rightarrow x[n] = -x[-n] \Rightarrow a_k = -a_{-k}$

$\left.\begin{matrix} \\ \\ \end{matrix}\right\} \begin{matrix} a^*_{-k} = -a_{-k} \\ a^*_k = -a_k \end{matrix}$

$\Leftarrow$ پس a_k کاملا موهومی و فرد است $\Leftarrow$ $a_0 = 0$

x Discrete $\Rightarrow a_k$ periodic with $N=9$

$$\left.\begin{matrix} \Rightarrow a_{19} = a_{10} = a_1 = j \\ a_{20} = a_{11} = a_2 = 2j \\ a_{21} = a_{12} = a_3 = 6j \end{matrix}\right\} \xrightarrow{a_k \text{ odd}} \left\{\begin{matrix} a_{-1} = -j \\ a_{-2} = -2j \\ a_{-3} = -6j \end{matrix}\right.$$