

# Signals and Systems

## Assignment 5

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### Question 1

Let  $x[n]$  be a signal with Fourier transform  $X(\omega)$ , express the Fourier transforms of the signals listed below in terms of  $X(\omega)$ .

(a)  $x_1[n] = x[2 - n] - x[1 - n]$

(b)  $x_2[n] = n^2 x[n]$

(c)  $x_3[n] = e^{j\omega_0 n} \sum_{k=-\infty}^n x[k]$

## Question 2

Find the Fourier transforms of the following signals.

(a)  $x[n] = \left(\frac{1}{2}\right)^n u[n-2]$

(b)  $x[n] = \frac{(n+1)(n+2)}{2} \left(\frac{1}{4}\right)^n u[n]$

(c)  $x[n] = \text{sinc}\left(\frac{\pi}{3}n\right)\cos\left(\frac{\pi}{6}n\right)$

(d)  $x[n] = u[n+3] - u[n-4]$

(e)  $x[n] = \sin^2\left[\frac{\pi}{8}n\right] + \cos\left[\frac{\pi}{8}n\right] + \cos^2\left[\frac{\pi}{8}n\right]$

### Question 3

Find the inverse Fourier transforms for the following signals.

(a)  $X(\omega) = \frac{1}{1-e^{-j\omega}} \frac{2}{2-e^{-j\omega}} + 2\pi\delta(\omega)$

(b)  $X(\omega) = \sum_{k=0}^{\infty} e^{-j\omega k}$

(c)  $X(\omega) = \frac{2-12e^{-j\omega}}{1-12e^{-j\omega}+35e^{-2j\omega}}$

(d)  $X(\omega) = \cos^2(\omega) + \sin^2(3\omega)$

## Question 4

A causal LTI system is described by the difference equation:

$$y[n] + \frac{1}{2}y[n-1] = x[n]$$

- (a) Find the frequency response of this system.
- (b) Find the impulse response of this system.
- (c) Determine system's output given input  $x[n] = (\frac{1}{2})^n u[n]$ .
- (d) Determine system's output given input  $x[n] = \delta[n] + \frac{1}{2}\delta[n-1]$ .

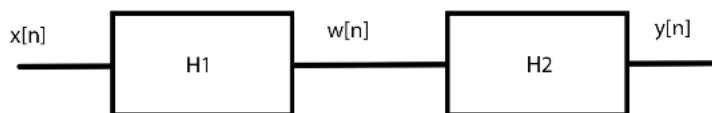
## Question 5

Consider an LTI system with impulse response  $h[n] = \frac{\sin(\frac{\pi}{3}n)}{\pi n}$ .

- (a) Find the frequency response of the system and plot it.
- (b) Find and plot the system's output to input  $x[n] = \sin(\frac{2\pi}{5}n)$ .
- (c) Find and plot the system's output to input  $x[n] = (-1)^n \cos(\frac{3\pi}{4}n)$ .

## Question 6

Consider the following system.



Where:

$$x[n] = \cos\left[\frac{\pi}{3}n\right] + j\sin\left[\frac{2\pi}{3}n\right]$$

$$h_1[n] = e^{j\frac{\pi}{2}n} \frac{\sin\left[\frac{\pi}{6}n\right]}{\pi n}$$

$$H_2(\omega) = e^{-j\omega} - 2e^{-2j\omega}$$

- (a) Determine  $X(\omega)$ .
- (b) Determine  $H_1(\omega)$ .
- (c) Determine  $W(\omega)$ .
- (d) Determine  $w[n]$ .
- (e) Determine  $h_2[n]$ .
- (f) Determine  $y[n]$ .
- (g) Determine  $Y(\omega)$ .
- (h) Determine  $H_{eq}(\omega)$ .
- (i) Determine  $h_{eq}[n]$ .

## Matlab Question 1

Consider the signal  $x(t) = \cos(20\pi t) + \sin(10\pi t)$ .

- (a) Sample  $x(t)$  with sampling frequency 40Hz to generate discrete-time signal  $x_1[n]$ .
- (b) Use fft to compute Fourier transform of  $x_1[n]$ . Plot the result.

## Matlab Question2

Compute the Fourier transform of signal  $x(t) = \frac{\sin(3\pi t)}{\sin(\frac{\pi}{2}t)}$  using fft and plot the result. Use an appropriate sampling rate.