علی نظری - ۹۶۳۱۰۷۵

قانون استفاده شده در تمامی موارد

(۱)

$$\left.\begin{array}{l} x(t) \\ w_۰ \to \text{Nyquist rate} \end{array}\right\} \Rightarrow X(w) = ۰ \qquad |w| > \frac{w_۰}{۲}$$

a) $x(t) + x(t-۵) - x(t+۲\sqrt{۳}) = y(t)$

$$\xleftrightarrow{F} X(w) + e^{-۵wj} X(w) - e^{۲\sqrt{۳}wj} X(w)$$

$$= X(w)\left[1 + e^{-۵wj} - e^{۲\sqrt{۳}wj}\right] = Y(w)$$

$$\left.\begin{array}{l} Y(w) = ۰ \\ |w| > \frac{w_۰}{۲} \end{array}\right\} \Rightarrow \begin{array}{l} y(t) \\ w_۰ \to \text{Nyquist rate} \end{array}$$

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b) $y(t) = \frac{d^k}{dt^k} x(t)$

$$\xleftrightarrow{F} (jw)^k X(w) = Y(w)$$

$$\left.\begin{array}{l} Y(w) = ۰ \\ |w| > \frac{w_۰}{۲} \end{array}\right\} \Rightarrow \begin{array}{l} y(t) \\ w_۰ \to \text{Nyquist rate} \end{array}$$

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c) $y(t) = x^{۲}(t)$

$$\Rightarrow Y(w) = \frac{1}{۲\pi}\left[X(w) * X(w)\right] \Rightarrow$$ duty cycle دو برابر می‌شود

$$\left.\begin{array}{l} Y(w) = ۰ \\ |w| > w_۰ \end{array}\right\} \Rightarrow \begin{array}{l} y(t) \\ ۲w_۰ \to \text{Nyquist rate} \end{array}$$

d) $y(t) = x(t)\, Sin(\omega_p t)$

$$\Rightarrow Y(\omega) = \frac{1}{۲\pi} X(\omega) * \frac{\pi}{j}\left[\delta(\omega-\omega_p) - \delta(\omega+\omega_p)\right]$$

$$= \frac{1}{۲j}\left[X(\omega-\omega_p) - X(\omega+\omega_p)\right]$$

$$\left.\begin{array}{l} Y(\omega) = ۰ \\ |\omega-\omega_p| > \frac{\omega_۰}{۲} \\ |\omega+\omega_p| > \frac{\omega_۰}{۲} \end{array}\right\} \Rightarrow \begin{array}{l} Y(\omega) = ۰ \\ |\omega| > \frac{\omega_۰}{۲} + \omega_p \end{array} \Rightarrow \begin{array}{l} y(t) \\ \underline{\omega_۰ + ۲\omega_p \rightarrow rate} \end{array}$$

۲

a) $x(t) = e^{-۴t} u(t) \xleftrightarrow{F} \frac{1}{۴+j\omega} = X(\omega)$

$$X(\omega) = ۰$$
$$|\omega| > ۰$$

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b) $x(t) = 1 + Sin(۵۰\pi t) + Sin(۱۰۰\pi t)\, Cos(۱۲۵\pi t)$

$$\Rightarrow X(\omega) = ۲\pi\delta(\omega) + \frac{\pi}{j}\left[\delta(\omega-۵۰\pi) - \delta(\omega+۵۰\pi)\right]$$
$$+ F\left(\frac{1}{۲}\left(Sin(۲۲۵\pi t) + Sin(-۲۵\pi t)\right)\right)$$

$$= ۲\pi\delta(\omega) + \frac{\pi}{j}\left[\delta(\omega-۵۰\pi) - \delta(\omega+۵۰\pi)\right]$$
$$+ \frac{\pi}{۲j}\left[\delta(\omega-۲۲۵\pi) - \delta(\omega+۲۲۵\pi) - \delta(\omega-۲۵\pi) + \delta(\omega+۲۵\pi)\right]$$

$$X(\omega) = ۰ \qquad \Rightarrow x(t)$$
$$|\omega| > ۲۲۵\pi \qquad rate = ۴۵۰\pi$$

c) $x(t) = u(t) - u(t-۶)$

$$\Rightarrow X(\omega) = \left(\pi\delta(\omega) + \frac{1}{j\omega}\right) - e^{-۶\omega j}\left(\pi\delta(\omega) + \frac{1}{j\omega}\right)$$

$$= \left(1 - e^{-۶\omega j}\right)\left(\pi\delta(\omega) + \frac{1}{j\omega}\right)$$

$X(\omega) = ۰$

$|\omega| > ۰$

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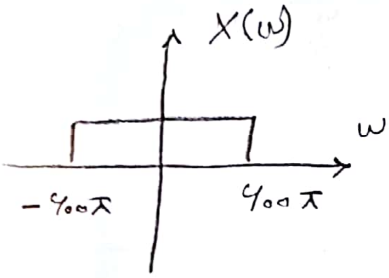
d) $x(t) = \dfrac{\sin(۶۰۰\pi t)}{\pi t}$

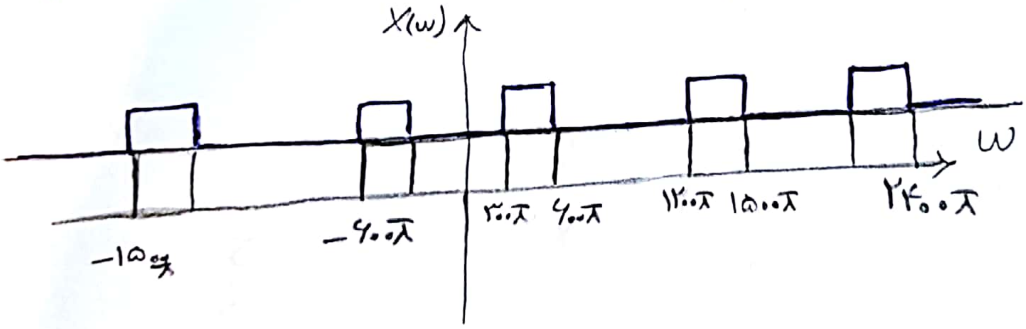
$\overset{F}{\longleftrightarrow}$ 

$\left.\begin{array}{l} X(\omega) = ۰ \\ |\omega| > ۶۰۰\pi \end{array}\right\} \Rightarrow$ $x(t)$ rate $= ۱۲۰۰\pi$

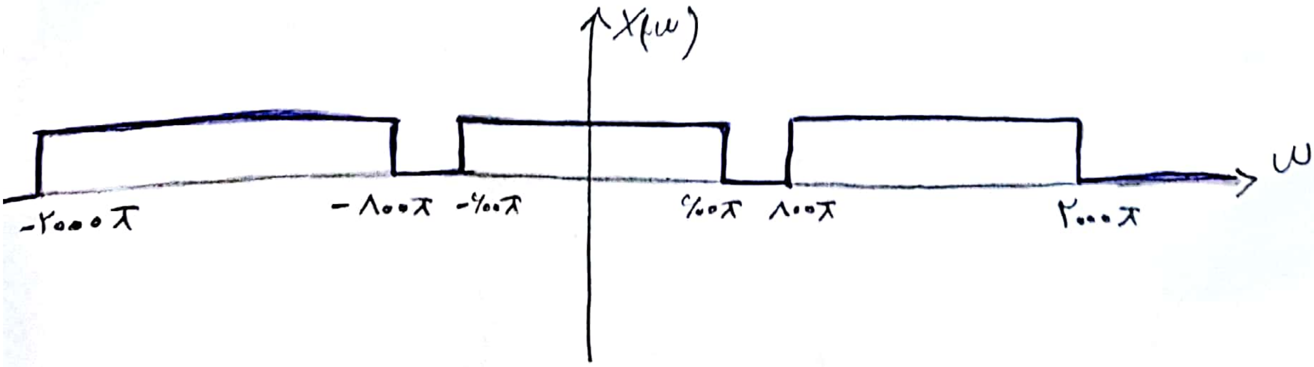
۳

$|\omega| > ۶۰۰\pi \Rightarrow$ valid $\Rightarrow$ rate $= ۱۲۰۰\pi$

a) invalid rate $= ۶۰۰\pi \Rightarrow \omega = ۳۰۰\pi$



b) valid rate $= ۱۲۰۰\pi \Rightarrow \omega = ۸۰۰\pi$



$$|\omega| > ۲۵۰\pi \Rightarrow rate = ۵۰۰\pi$$

$$x_p(t) = x_a(t)\,p(t) = x(t)\cos^{۲}(\omega_c t)$$

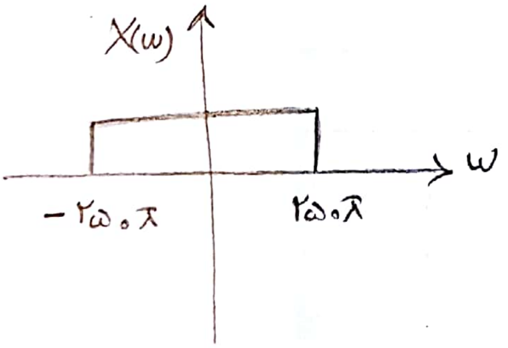
شرط عدم تداخل: $\omega_s > ۲\omega_M$ $\qquad \omega_M = ۲۵۰\pi$

a) $$x(t)\cos^{۲}(\omega_c t) = x(t)\left(\frac{1+\cos(۲\omega_c t)}{۲}\right)$$

$$= \frac{1}{۲}x(t) + \frac{1}{۲}x(t)\cos(۲\omega_c t)$$

$$\xleftrightarrow{F} \frac{1}{۲}X(\omega) + \frac{1}{۲\pi}\,\frac{1}{۲}\,X(\omega) * \left[\pi\left(\delta(\omega-۲\omega_c)+\delta(\omega+۲\omega_c)\right)\right]$$

$$= \frac{1}{۲}X(\omega) + \frac{1}{۴}\left(X(\omega-۲\omega_c)+X(\omega+۲\omega_c)\right) = X_p(\omega)$$



$$\begin{cases} ۲\omega_c > ۲(۲۵۰\pi) \Rightarrow \omega_c > ۲۵۰\pi \\ ۲\omega_c < -۲(۲۵۰\pi) \Rightarrow \omega_c < -۲۵۰\pi \end{cases}$$

$$\Rightarrow |\omega_c| > ۲۵۰\pi$$

b) $$\omega_M < \omega_{cutoff} < \omega_s - \omega_M$$

$$\begin{cases} \omega_s = ۲\omega_c \\ |\omega_c| > ۲۵۰\pi \\ \omega_M = ۲۵۰\pi \end{cases} \Rightarrow \begin{cases} ۲۵۰\pi < \omega_{cutoff} < ۲\omega_c - ۲۵۰\pi \\ |\omega_c| > ۲۵۰\pi \end{cases}$$

c) $$\left.\begin{array}{l} \omega_s = \frac{۲\pi}{T} \\ \omega_s = ۲\omega_c \\ T = gain \end{array}\right\} \Rightarrow ۲\omega_c = \frac{۲\pi}{gain} \Rightarrow gain = \frac{\pi}{\omega_c}$$

a) $x(t) = e^{-۲t} u(t) + e^{-۳t} u(t)$

$$\xrightarrow{L} \frac{1}{S+۲} + \frac{1}{S+۳}$$

ROC: $(Re\{S\} > -۲) \cap (Re\{S\} > -۳) \Rightarrow Re\{s\} > -۲$

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b) $x(t) = t e^{-۳|t|} = t\left(e^{-۳t} u(t) + e^{۳t} u(-t)\right)$

$$\xrightarrow{L} \frac{-d}{ds}\left[\frac{1}{S+۳} - \frac{1}{S-۳}\right]$$

$$= \frac{1}{(S+۳)^۲} - \frac{1}{(S-۳)^۲}$$

ROC: $(Re\{s\} > -۳) \cap (Re\{s\} < ۳) \Rightarrow |Re\{s\}| < ۳$

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c) $x(t) = \begin{cases} 1 & ۰ \leq t \leq 1 \\ ۰ & o.w \end{cases} = u(t) - u(t-1)$

$$\xrightarrow{L} \frac{1}{S} - \frac{e^{-S}}{S} = \frac{1-e^{-S}}{S}$$

ROC: $Re\{s\} > ۰$

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d) $x(t) = \delta(۲t) + u(۳t)$

$$\xrightarrow{L} \frac{1}{۲} + \frac{1}{۳}\,\frac{1}{\frac{S}{۳}} = \frac{1}{۲} + \frac{1}{S}$$

ROC: $Re\{S\} > ۰$

a) $\dfrac{1}{s^2+4}$ $\quad Re\{s\}>0$

$$\xrightarrow{L^{-1}} \frac{1}{2}\,Sin(2t)\,u(t)$$

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b) $\dfrac{S}{S^2+4}$ $\quad Re\{S\}<0$

$$\xrightarrow{L^{-1}} -Cos(2t)\,u(-t)$$

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c) $\dfrac{S+1}{(S+1)^2+9}$ $\quad Re\{s\}<-1$

$$\xrightarrow{L^{-1}} \frac{-e^{-t}}{3}\,Sin(3t)\,u(-t)$$

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d) $\dfrac{(S+1)^2}{s^2-S+1}$ $\quad Re\{s\}>\frac{1}{2}$

$$=\frac{(S+1)^2}{\left(S-\frac{1}{2}\right)^2+\frac{3}{4}} \xrightarrow[\text{S-Domain}\ \frac{1}{2}]{\text{shift}} \frac{\left(S+\frac{3}{2}\right)^2}{S^2+\frac{3}{4}} = 1+\frac{3S+\frac{6}{4}}{S^2+\frac{3}{4}}$$

$$=1+3\,\frac{S}{S^2+\frac{3}{4}}+\sqrt{3}\,\frac{\frac{\sqrt{3}}{2}}{S^2+\frac{3}{4}}$$

$$\xrightarrow{L^{-1}} e^{\frac{-t}{2}}\left[\delta(t)+3\,Cos\left(\frac{\sqrt{3}}{2}t\right)u(t)+\sqrt{3}\,Sin\left(\frac{\sqrt{3}}{2}t\right)u(t)\right]$$