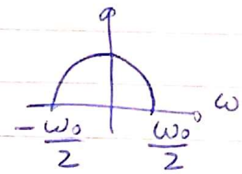
Question 1

Nyquist Rate $= \omega_0 \Rightarrow X(j\omega)$:



a) $x(t) + x(t-5) - x(t+2\sqrt{2})$

$$\left.\begin{aligned} x(t) &\leftrightarrow X(j\omega) \\ x(t-5) &\leftrightarrow e^{-j5\omega} X(j\omega) \\ x(t+2\sqrt{2}) &\leftrightarrow e^{j2\sqrt{2}\omega} X(j\omega) \end{aligned}\right\} \rightarrow X(j\omega)\left[1 + e^{-j5\omega} - e^{j2\sqrt{2}\omega}\right]$$

تاثیری روی Band ندارد ⇐ نرخ نایکوییست ω_0

b) $\frac{d^k}{dt^k} x(t) \quad k \in \mathbb{Z},\ k \geq 2$

$$x(t) \leftrightarrow X(j\omega)$$

$$\frac{d}{dt} x(t) \leftrightarrow j\omega X(j\omega)$$

$$\frac{d^2}{dt^2} x(t) \leftrightarrow (j\omega)^2 X(j\omega)$$

$$\vdots$$

$$\frac{d^k}{dt^k} x(t) \leftrightarrow (j\omega)^k X(j\omega) \rightarrow$$

مثل a

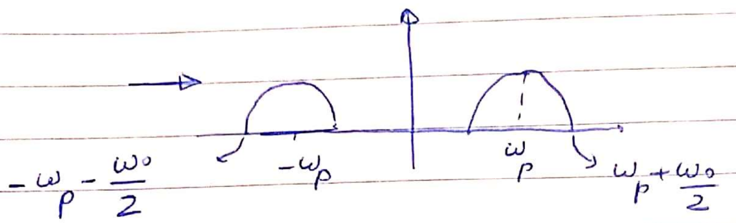
⇐ ω_0

c) $x^2(t) = x(t)\, x(t)$

$$x(t) \cdot x(t) \leftrightarrow \frac{1}{2\pi}\left(X(j\omega) * X(j\omega)\right)$$

باعث دوبرابر شدن Band می‌شود ⇐ $2\omega_0$

d) $x(t)\ \sin(\omega_p t) \longleftrightarrow \frac{1}{2\pi}\left(X(j\omega) * \left(\frac{\pi}{j}\left(\delta(\omega-\omega_p)-\delta(\omega+\omega_p)\right)\right)\right)$



$\Rightarrow$ Nyquist $= 2\omega_p + \omega_0$

Question 2

a) $e^{-6t}u(t) \longleftrightarrow \frac{1}{j\omega+6} \rightarrow$ NOT Band limited

b) $x(t) = 2 + \sin(50\pi t) + \underbrace{\sin(100\pi t)\cos(125\pi t)}$

$\omega = 0$ در impulse داریم

در impulse داریم

$\omega = \pm 25\pi,\ \pm 125\pi$

225

$\omega = \pm 50\pi$ در impulse داریم

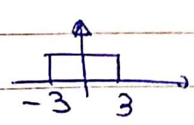
225 225

$\Rightarrow \max\{\pm 50\pi, 0, \pm 25\pi, \pm 125\pi\} = 125\pi = \omega_M$

225 450

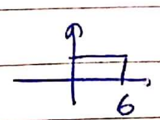
$\Rightarrow$ Nyquist Rate $=$ $2(125\pi) = 250\pi$

c) $x(t) = u(t) - u(t-6)$



$\longleftrightarrow \frac{2\sin(3\omega)}{\omega}$

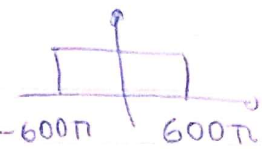
$x(t) =$



$\longleftrightarrow e^{-j\omega 3}\,\frac{2\sin(3\omega)}{\omega}$: NOT Band limited

d) $x(t) = \dfrac{\sin(600\pi t)}{\pi t}$

$x(t) = \dfrac{\sin(600\pi t)}{\pi t} \longleftrightarrow$

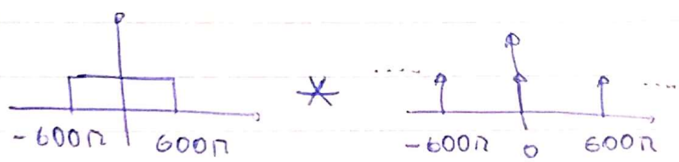


$\omega_M = 600\pi \Rightarrow$ Nyquist Rate $= 1200\pi$

Question 3

* (Invalid Rate) Nyquist $= 600\pi$

$X_p(j\omega) =$



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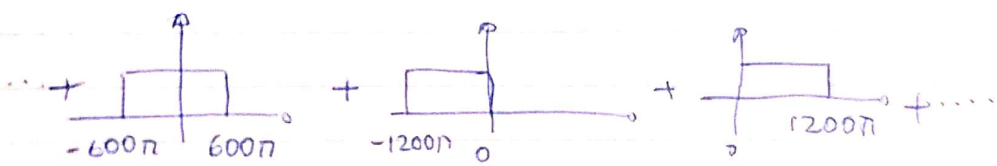

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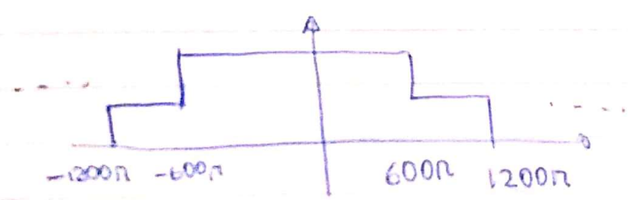
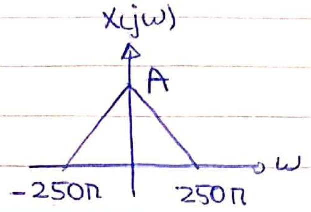


$\Leftarrow X(j\omega)$ قابل بازیابی نیست

Subject
Year Month Date ( )

Question 4

$X(j\omega) =$



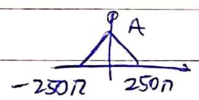
$\omega_M = 250\pi$

a) $x(t)\cos(\omega_c t) \longleftrightarrow \frac{1}{2}\left(X(j(\omega-\omega_c)) + X(j(\omega+\omega_c)) \right)$

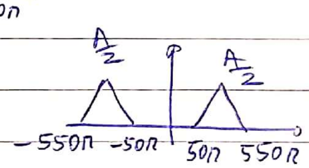
⇐ برای اینکه $X(j(\omega-\omega_c))$ و $X(j(\omega+\omega_c))$ با هم تداخل نداشته باشند باید

$\omega_c \geq \omega_M$ یعنی $\omega_c \geq 250\pi$ باشد ⇐ فرض: $\omega_c = 300\pi$

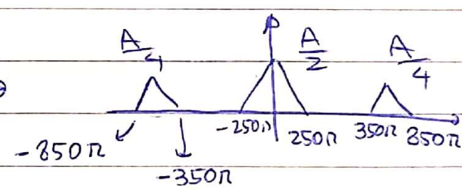
$x(t) \longleftrightarrow$



$x(t)\cos(300\pi t) \longleftrightarrow$



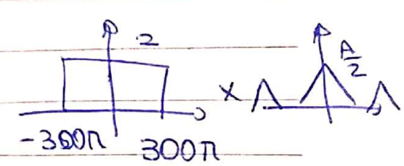
$x(t)\cos(300\pi t)\cos(300\pi t) \longleftrightarrow$



$= P(j\omega)$

b,c) $X(j\omega)$ باید برابر با $P(j\omega)H(j\omega)$ شود

$\Rightarrow$ $250\pi < \text{cutoff} < 350\pi$, gain $= 2$ → مثلا cutoff $= 300\pi$



$= X(j\omega)$

$$5)\ a)\quad X(s) = \int_{-\infty}^{\infty} \left[e^{-2t} u(t) + e^{-3t} u(t) \right] e^{-st}\, dt \quad , \; \underrightarrow{s = \sigma + j\omega}$$

$$= \int_{-\infty}^{\infty} \left[e^{-2t} e^{-\sigma t} u(t) \right] e^{-j\omega t}\, dt + \int_{-\infty}^{\infty} \left[e^{-3t} e^{-\sigma t} u(t) \right] e^{-j\omega t}\, dt$$

هرکدام از انتگرال ها در واقع تبدیل فوریه سیگنال داخل براکت ها هستند. تبدیل

فوریه آن ها را هم بلدیم:

$$e^{-2t} u(t) \xleftrightarrow{L} \frac{1}{2+\sigma+j\omega} \quad , \; 2+\sigma > 0$$

$$e^{-3t} u(t) \xleftrightarrow{L} \frac{1}{3+\sigma+j\omega} \quad , \; 3+\sigma > 0$$

$$\Rightarrow X(s) = \frac{1}{s+2} + \frac{1}{s+3} \quad , \; \text{RoC}: Re\{s\} > -2 \wedge Re\{s\} > -3 = Re\{s\} > -2$$

$$b)\quad t e^{-3|t|} = \begin{cases} t e^{-3t} , & t \geq 0 \\ t e^{3t} , & t < 0 \end{cases} = t e^{-3t} u(t) + t e^{3t} u(-t)$$

$$X(s) = \int_{-\infty}^{\infty} \left[t e^{-3t} u(t) + t e^{+3t} u(-t) \right] e^{-st}\, dt \quad , \; \underrightarrow{s = \sigma + j\omega}$$

$$= \int_{-\infty}^{\infty} \left[t e^{-3t} e^{-\sigma t} u(t) \right] e^{-j\omega t}\, dt + \int_{-\infty}^{\infty} \left[t e^{3t} e^{-\sigma t} u(-t) \right] e^{-j\omega t}\, dt$$

$$t e^{-3t} u(t) \xleftrightarrow{L} \frac{1}{(3+\sigma+j\omega)^2} \quad , \; 3+\sigma > 0$$

$$t e^{3t} u(-t) \xleftrightarrow{L} \frac{-1}{(3-\sigma-j\omega)^2} \quad , \; 3-\sigma > 0$$

5) b) $\Rightarrow X(s) = \frac{1}{(s+3)^2} - \frac{1}{(s-3)^2}$, RoC: $-3 < Re\{s\} < 3$

c) $X(s) = \int_{-\infty}^{\infty} u(t) e^{-st} dt = \int_{0}^{1} e^{-st} dt = \frac{e^{-st}}{-s} \Big|_0^1$

$= -\frac{e^{s}}{s} + \frac{1}{s} = \frac{1-e^{s}}{s}$, RoC: s-plane

d) $\delta(t) \xleftrightarrow{L} 1 \Rightarrow \delta(2t) \xleftrightarrow{L} \frac{1}{2}$, RoC: s-plane

$u(t) \xleftrightarrow{L} \frac{1}{s} \Rightarrow u(3t) \xleftrightarrow{L} \frac{1}{3} \frac{1}{\frac{s}{3}}$, RoC: $Re\{s\} > 0$

$\Rightarrow X(s) = \frac{1}{2} + \frac{1}{s}$, RoC: $Re\{s\} > 0$

6) a) $\sin(2t)u(t) \xleftrightarrow{L} \frac{2}{s^2+4}$, $Re\{s\} > 0$

$\Rightarrow \frac{\sin(2t)u(t)}{2} \xleftrightarrow{L} \frac{1}{s^2+4}$, $Re\{s\} > 0$

b) $\cos(2t)u(t) \xleftrightarrow{L} \frac{s}{s^2+4}$, $Re\{s\} > 0$

$\Rightarrow -\cos(2t)u(-t) \xleftrightarrow{L} \frac{s}{s^2+4}$, $Re\{s\} < 0$

1)c) $e^{-t}\cos 3t\, u(t) \overset{L}{\longleftrightarrow} \dfrac{s+1}{(s+1)^2+9}$, $Re\{s\} > -1$

$\Rightarrow -e^{-t}\cos 3t\, u(-t) \overset{L}{\longleftrightarrow} \dfrac{s+1}{(s+1)^2+9}$, $Re\{s\} < -1$

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d) $\dfrac{s^2+2s+1}{s^2-s+1} = \dfrac{s^2-s+1+3s}{s^2-s+1} = 1 + \dfrac{3s - \frac{3}{2} + \frac{3}{2}}{\left(s-\frac{1}{2}\right)^2+\frac{3}{4}}$

$= 1 + 3\dfrac{s-\frac{1}{2}}{\left(s-\frac{1}{2}\right)^2+\frac{3}{4}} + \sqrt{3}\dfrac{\frac{\sqrt{3}}{2}}{\left(s-\frac{1}{2}\right)^2+\frac{3}{4}}$

$\updownarrow L \qquad \updownarrow L \qquad \updownarrow L$

$\delta(t) + e^{t/2}\cos\left(\frac{\sqrt{3}}{2}t\right)u(t) + \sqrt{3}\, e^{t/2}\sin\left(\frac{\sqrt{3}}{2}t\right)u(t)$