دانشگاه صنعتی امیرکبیر
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Signals and Systems

Assignment 4

Fall 2019 - Group 1

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Question 0

Given that $x(t)$ has the Fourier Transform $X(j\omega)$, express the Fourier Transforms of the following signals in terms of $X(j\omega)$:

(a) $x_1(t) = x(2-t) + x(-3-t)$

(b) $x_2(t) = x(2t-4)$

(c) $x_3(t) = \frac{d^2}{dt^2}x(t+1)$

(d) $x_4(t) = tx(t-1)$

(e) $x_5(t) = \int_{-\infty}^{t} x(\tau)d\tau$

Question 1

Determine the Fourier Transform of the following signals:

(a) $x(t) = 1 + cos(6\pi t + \frac{\pi}{8})$

(b) $x(t) = (te^{-2t} sin(4t))u(t)$

(c) $x(t) = t\left(\frac{sin(t)}{\pi t}\right)^2$

(d) $x(t) = \frac{4t}{(1+t^2)^2}$

(e) $x(t) = \frac{sin(t-2\pi)}{\pi(t-2\pi)}$

Question 2

Determine the Inverse Fourier Transform of the following signals:

(a) $X(j\omega) = 2\delta(\omega + 6)$

(b) $X(j\omega) = \frac{7j\omega+46}{-\omega^2+13j\omega+42}$

(c) $X(j\omega) = \pi e^{-3|\omega|}$

Question 3

The input and the output of a stable and causal LTI system are related by the differential equation

$$\frac{d^2}{dt^2}y(t) + 6\frac{d}{dt}y(t) + 8y(t) = 2x(t)$$

(a) Find the impulse response of this system.

(b) Determine $y(t)$ if $x(t) = te^{-2t}u(t)$

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Question 4

A causal and stable LTI system S has the frequency response

$$H(j\omega) = \frac{j\omega + 4}{6 - \omega^2 + 5j\omega}$$

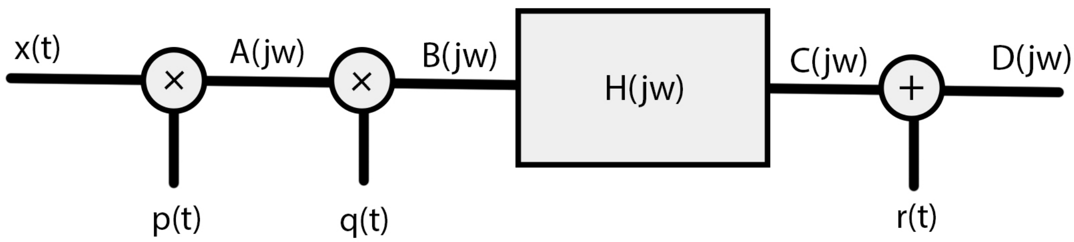
(a) Determine a differential equation relating to the input $x(t)$ and output $y(t)$ of S.

(b) Determine the impulse response $h(t)$ of S.

(c) Determine the output $y(t)$ when $x(t) = e^{-4t}u(t) - te^{-4t}u(t)$.

Question 5

Consider the following system, determine $A(j\omega), B(j\omega), C(j\omega), D(j\omega)$. Do not confuse the addition sign with multiplication!



$$x(t) = \frac{sin(\pi t)}{\pi t}$$

$$p(t) = \frac{sin(2\pi t)}{\pi t}$$

$$q(t) = cos(3\pi t)$$

$$H(j\omega) = 2\Big(u(\omega + 3\pi) - u(\omega - 3\pi)\Big)$$

$$r(t) = \frac{sin(\pi t)}{\pi t}$$

Question 6

Using Parseval's energy equation, prove the following equality:

$$\int_{-\infty}^{+\infty} \left(\frac{sin(\omega)}{\omega}\right)^2 d\omega = \pi$$