دانشگاه صنعتی امیرکبیر
( پلی تکنیک تهران )

# Signals and Systems

Assignment 2

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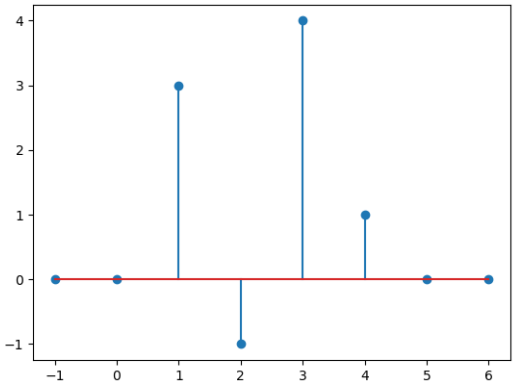
## Question 1

Output of an LTI system given input $x[n] = \delta[n-2]$ is depicted below:




(a) Determine system's output y[n] given input $x[n] = e^{j\pi n}(u[n] - u[n-3])$.

(b) Determine system's output y[n] given input $x[n] = (\frac{1}{2})^n(u[n] - u[n-2])$

(c) Determine system's output y[n] given input $x[n] = 2^n(u[n+2] - u[n])$

(d) If system's output is y[n] for input $x[n] = e^{j\pi n}u[n]$, determine y[1].

## Question 2

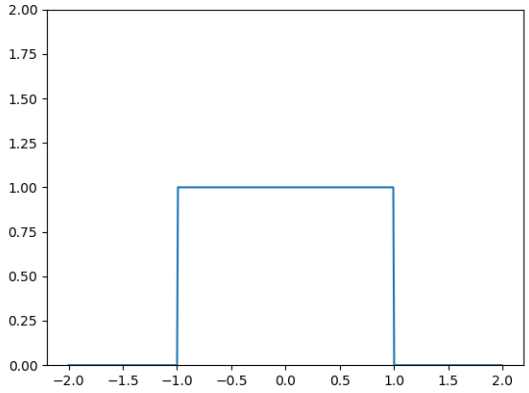
For each pair of impulse response and input given below, determine LTI system's output:

(a) $h(t) = u(t)$ , $x(t) = e^t$

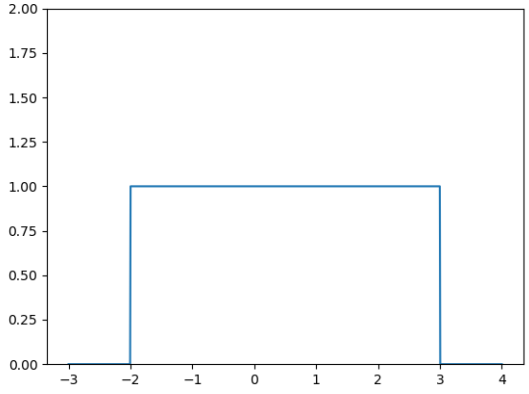
(b) $h(t) = u(t)$ , $x(t) = e^t u(t)$

(c)

h(t)




x(t)

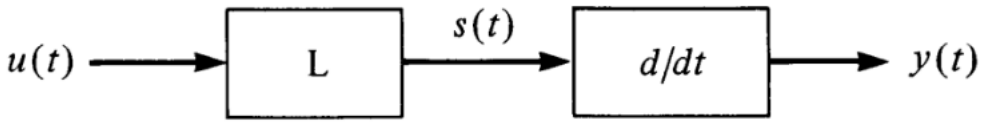


(d)

$$x(t) = \begin{cases} 2t+1 & , -1 \le t < 1 \\ ln(t) & , 1 \le t < 2 \\ e^t & , 2 \le t < 3 \\ 0 & , O.W. \end{cases} \qquad , \qquad h(t) = \delta(t + \frac{3}{2}) + 2\delta(t + \frac{5}{2})$$

# Question 3

Consider the cascade of two systems shown below:




(a) The system on the right is called a differentiator. Prove it is an LTI system.

(b) Assuming the system on the left (L) is also an LTI system with impulse response h(t) and step response s(t), Prove:

$$h(t) = \frac{ds(t)}{dt}$$

**note:** *Step response of a system, is the system's output given unit step as input.*
**hint:** Use commutative property of convolution

## Question 4

For each of the following impulse responses, determine whether the corresponding LTI system is memoryless, causal and stable. Justify your answers.

(a) $h(t) = e^{-4|t|}$

(b) $h(t) = te^{-t}u(t)$

(c) $h(t) = cos(2\pi t)u(t+1)$

(d) $h(t) = \frac{sin(t)}{t}u(t)$

(e) $h[n] = (\frac{1}{2})^n u[-n]$

(f) $h[n] = \delta[2n]$

(g) $h[n] = cos(\frac{\pi}{2}n)u[n+1]$

(h) $h[n] = e^{2n}u[n]$

## Question 5

Using convolution integral, determine the output y(t) of a system where:

$$x(t) = e^{2t}u(1-t)$$

$$h(t) = 2u(t) + 4u(t-2) + 8u(t-4)$$

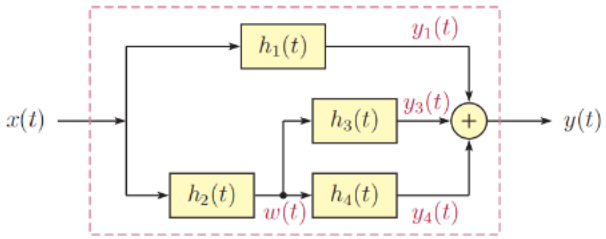
## Question 6

Consider cascaded interconnection of LTI systems shown below, where:

$$h_1(t) = e^{-t}u(t)$$

$$h_2(t) = h_3(t) = u(t) - u(t-1)$$

$$h_4(t) = \delta(t-1)$$




Determine the impulse response $h_{eq}(t)$ of the equivalent system.

## Question 7

Consider an LTI system with impulse response $h[n] = \alpha^n u[n]$, Determine the equation by which the system's input x[n] is related to its output y[n].
**hint1:** $x[n] * h[n] = y[n] \quad \Rightarrow \quad x[n] * h[n-1] = ?$
**hint2:** $u[n] - u[n-1] = ?$

## Question 8

For each pair of impulse responses below, show the corresponding systems are inverses of each other:

(a) $h_1(t) = e^{-t}u(t) \quad , \quad h_2(t) = \delta(t) + \delta'(t)$

(b) $h_1[n] = \delta[n] - \delta[n-1] \quad , \quad h_2[n] = u[n]$

## Matlab Question

Use Matlab to compute the output of an LTI system with input:
$x[n] = u[n+2] - 2u[n-12]$
and impulse response:
$h[n] = 0.9^n(u[n-2] - 2u[n-4])$