Signals and Systems

Assignment 4

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MohammadKhalaji76@gmail.com - JaliliA370@gmail.com
Telegram Channel: @SignalsAndSystems99Spring

Question 1

Let x(t) be a signal with Fourier transform X(ω), express the Fourier transforms of the signals listed below in terms of X(ω).

(a) $x_1(t) = \frac{d}{dt}x(1-t)$

(b) $x_2(t) = \int_{-\infty}^{t} x(\tau)d\tau$

(c) $x_3(t) = x(3-t) + 2x(t)$

(d) $x_4(t) = \frac{d^2}{dt^2}x(t+2)$

(e) $x_5(t) = t^2x(2t)$

(f) $x_6(t) = x(-2t) * x(3t)$

Question 2

Find the Fourier transforms of the following signals.

(a) $x(t) = sin(t) + cos(\frac{\pi}{2}t + \frac{\pi}{4})$

(b) $x(t) = 2\frac{sin^2(3t)}{t}$

(c) $x(t) = (t^2 e^t cos(t))u(t)$

(d) $x(t) = \frac{2t}{1+t^2}$

(e) $x(t) = \int_{-\infty}^{t} \frac{d\tau}{1+\tau^2}$

(f) $x(t) = e^{-3|t|+j\frac{\pi}{6}t}$

(g) $x(t) = \begin{cases} 1-t & ,-1 < t < 1 \\ 0 & ,O.W \end{cases}$

Question 3

Find the inverse Fourier transforms for the following signals.

(a) $X(\omega) = 3\delta(\omega - 3)$

(b) $X(\omega) = \pi e^{-|\omega|}$

(c) $X(\omega) = \frac{7j\omega+32}{-\omega^2+9j\omega+20}$

(d) $X(\omega) = \begin{cases} cos(\omega) & -\pi < \omega < \pi \\ 0 & O.W. \end{cases}$

(e) $X(\omega) = \frac{2sin(\omega-1)}{\omega-1} * \frac{sin(2\omega)}{\omega}$

(f) $X(\omega) = \frac{sin^2(-w)}{w^2}$

(g) $X(\omega) = \frac{d}{d\omega}\left[\frac{sin(\pi\omega)-jcos(\pi\omega)}{1+2j\omega}\right]$

(h) $X(\omega) = \frac{1}{(-1+j\omega)^6}$

Question 4

Consider a stable and causal LTI system whose input x(t) and output y(t) are related by the differential equation:

$$-\frac{d^2}{dt^2}y(t) - 7\frac{d}{dt}y(t) - 10y(t) = 2\frac{d}{dt}x(t) + 13x(t)$$

(a) Find the frequency response of this system.

(b) Find the impulse response of this system.

(c) Given signal $x_1(t) = te^{-t}u(t)$ as input to this system, find the output $y_1(t)$ of the system.

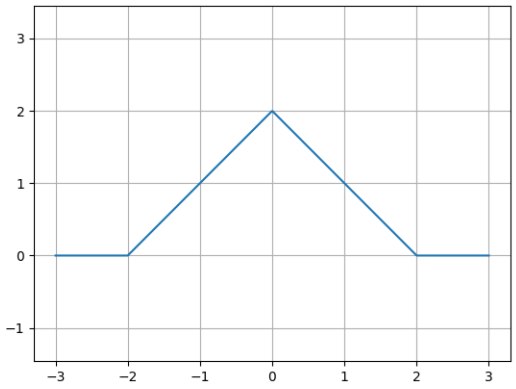
(d) The inverse of this system has impulse response g(t). Find g(t).

(e) Determine a differential equation relating the input and output of the inverse of this system.

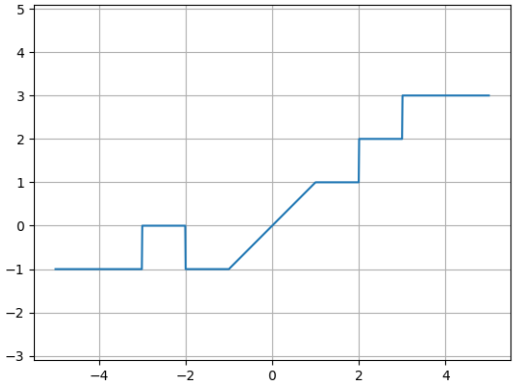
Question 5

Determine the Fourier transforms of the following signals.

(a)



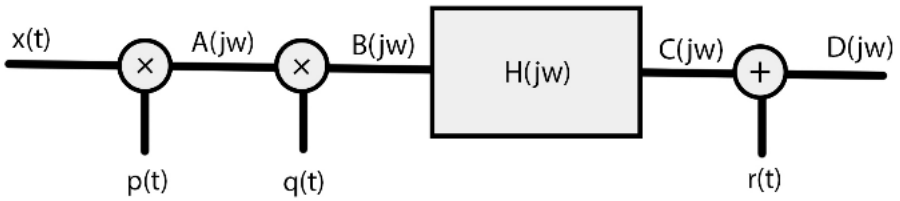
(b)



Hint: Use differentiation property.

Question 6

Consider the following system, determine A(ω), B(ω), C(ω), D(ω).



$$x(t) = \frac{sin(2t)}{\pi t} \qquad p(t) = cos(2t) + jsin(2t)$$
$$q(t) = e^{-2jt}\frac{sin(2t)}{\pi t} \qquad h(t) = \begin{cases} 1, & |t| < 1 \\ 0, & O.W. \end{cases}$$
$$r(t) = \frac{\delta(2t)}{2}$$

note: H(jω), A(jω), B(jω), C(jω) and D(jω) in the figure, denote H(ω), A(ω), B(ω), C(ω) and D(ω) respectively.

Question 7

Use Parseval's relation to determine the numeric values of:

(a) $\int_{-\infty}^{\infty} \frac{d\omega}{1+2j\omega}$

(b) $\int_{-\infty}^{\infty} \frac{d\omega}{(\omega^2+a^2)^2}$

(c) $\int_{-\infty}^{\infty} t^2 \left(\frac{sin(t)}{\pi t}\right)^4 dt$