2. y consists of only ‘)‘, and let $y =)^k$
3. y consists of both ‘(‘ and ‘)‘ , and let $y = (^k)^k$

For case I , take $i = 0$.

$$w = xyz = (^{n-k}(^k)^n$$

$$xy^0z = (^{n-k})^n \quad \textit{which is not a set of all balanced parenthesis .}$$

For case II , take $i = 0$.

$$w = xyz = (^n)^k)^{n-k}$$

$$xy^0z = (^n)^{n-k} \quad \textit{which is not a set of all balanced parenthesis .}$$

For case III , take $i = 2$.

$$w = xyz = (^{n-k}(^k)^k)^{n-k}$$

$$xy^2z = (^n)^k(^k)^n \quad \textit{which is not a set of all balanced parenthesis .}$$

In all the three cases, there are contradictions and, hence, the language is not regular.

Example 5.34

Show that L = $\{0^n1^{2n}, where n \geq 1\}$ is not regular .

Solution :

Step I : Assume that L is regular. Let n be the number of states of the FA accepting L .

Step II : Let $w = 0^n1^{2n}$, where $n \geq 1$. $|\ w\ | = 3n > n$. By using the pumping lemma, we can write

$$w = xyz, with\ w = xyz\ with|xy| \leq n\ and\ |y| > 0.$$

Step III : We want to find a suitable i so that $xy^iz \notin L$. The string consists of ‘0’ and ‘1’. y can be any of the following forms

1. y consists of only ‘0’, and let $y = 0^k$
2. y consists of only ‘1’, and let $y = 1^k$
3. y consists of both ‘0’ and ‘1’, and let $y = 0^k1^k$

For case I , take $i = 0$.

$$w = xyz = 0^n1^{2n} = 0^{n-k}0^k1^{2n}$$

$$xy^0z = 0^{n-k}1^{2n}, as\ k \neq 0, (n-k) \neq n$$

For case II , take $i = 0$.

$$w = xyz = 0^n1^{2n} = 0^n1^k1^{2n-k}$$

$$xy^0z = 0^n1^{2n-k}, as\ k \neq 0, (2n-k) \neq n$$

For case III , take $i = 2$.

$$w = xyz = 0^n1^{2n} = 0^{n-k}0^k1^k1^{2n-k}$$

$$xy^2z = 0^n1^k0^k1^{2n}, \textit{which is not in the form } 0^n1^{2n}$$

For all the three cases, we are getting contradictions and, therefore, L is not regular.

5.12 Closure Properties of Regular Set

A set is closed (under an operation) if and only if the operation on two elements of the set produces another element of the set. If an element outside the set is produced, then the operation is not closed .