So, $xy^2z = a^{n-k}a^{2k}ba^n = a^{n+k}ba^n$ *or* $a^nba^{2k}a^{n-k} = a^nba^{n+k}$.
As $k \neq 0, n+k \neq n$. Therefore, $xy^2z \notin L$.
This is a contradiction, and so L is not regular .
For Case II , let us take i = k . So, $xy^kz = a^n\ b^k\ a^n$.This is not in the form a^nba^n.Therefore , $xy^kz \notin L$.
This is a contradiction, and so L is not regular .

Example 5.32

Show that L = $\{a^nb^nab^{n+1}\}$ is not regular .

Solution :
Step I : Assume that L is regular. Let n be the number of states of the FA accepting L .
Step II : Let w =$a^nb^nab^{n+1}$, and so $|w| = (3n+2) > n$. By using the pumping lemma, we can write w = xyz ,with w=xyz with$|xy| \leq$ n and $|y| > 0$.
Step III : We want to find a suitable i so that $xy^iz \notin L$ .The string consists of 'a' and 'b'. So, y can be of any of these .

1. y contains only 'a', and let y = a^k
2. y contains only 'b', and let y = b^l
3. y contains both 'a' and 'b', and let y = b^ka

For case I, take $i = 0$.

$$w = xyz = (a^{n-k})(a^k)(b^nab^{n+1})$$
$$xy^0z = a^{n-k}b^nab^{n+1} \text{ which is not in the form } a^nb^nab^{n+1}.$$

For case II, take $i = 0$.

$$w = xyz = (a^nb^{n-1})(b^1)ab^{n+1}$$
$$xy^0z = a^nb^{n-1}ab^{n+1} \text{ which is not in the form } a^nb^nab^{n+1}.$$

For case III, take $i = 2$.

$$w = xyz = a^nb^{n-k}(b^ka)^2b^{n+1}$$
$$= a^nb^na\,b^kab^{n+1} \text{ which is not in the form } a^nb^nab^{n+1}.$$

In all the three cases,there are contradictions and,hence,the language is not regular.

Example 5.33

Show that the language containing the set of all balanced parenthesis is not

Solution :
Step I : Assume that L is regular. Let n be the number of states of the FA accepting L .
Step II : Let w = (((. . . .(). . . .)))=$(^n)^n$. | w |=2$n > n$. By using the pumping lemma,we can writew=xyz ,with w=xyz with | xy | $\leq$ n and | y | > 0.
Step III : We want to find a suitable i so that $xy^iz \notin$ L .The string consists of '(' and ')'. So, y can be of any of these

1. y consists of only '(', and let $y = (^k$