## 268 Introduction to Automata Theory, Formal Languages and Computation

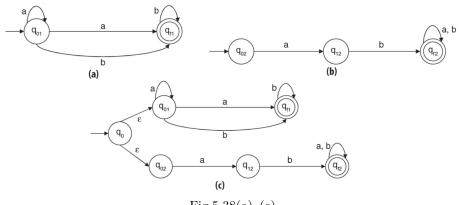


Fig.5.38(a)–(c)

- 3. The initial state of  $M\prime$  is the same as the final state of M ( $M\prime$  is the reverse direction of M)
- 4. The fi nal state of  $M\prime$  is the same as the initial state of M ( $M\prime$  is the reverse direction of M)

Let a string w belong to L, i.e., w  $\varepsilon$  M. So, there is a path from  $q_0$  to F with path value w. By reversing the edges, we get a path from F to  $q_0$  (beginning and final state of M') in M'. The path value is the reverse of w, i.e.,  $w^T$ . So,  $w^T \varepsilon M'$ .

So, the reverse of the string w is regular.

## Example:

Let  $L = ab(a+b)^*$ .

The FA M accepting L is as shown in Fig. 5.39(a). The reverse of the FA  $M\prime$  accepting Lc is shown in Fig. 5.39(b).

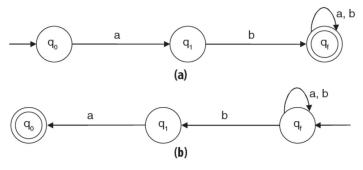


Fig.5.39(a)-(b)

M' accepts  $(a+b)^*ba$  which is reverse of L.

**Theorem 5.5**:If L is regular and L is a subset of  $\Sigma^*$ , prove that  $\Sigma^*-L$  is also a regular set.

As L is regular, there must be an FA,  $M=(Q,\Sigma,\delta,q_0,F)$  , accepting L.Let us construct another DFA

 $M\prime=(Q,\Sigma,\delta,q_0,F\prime)$  where  $F\prime=$  Q–F. So, the two DFA differ only in their final states. A final state of M

is a non-final state of  $M\prime$  and vice versa.

Let us take a string w which is accepted by  $M\prime$ . So, $\delta(q_0,w)\varepsilon F\prime$ , i.e.,  $\delta(q_0,w)\varepsilon(Q-F)$ .