Example 5.32 . Show that $\mathrm{L} = \{\mathrm{a^n b^n a b^{n+1}}\}$ is not regular.

Solution :

Step I: Assume that L is regular. Let n be the number of states of the FA accepting L.

Step II: Let w $= \mathrm{a^n b^n a b^{n+1}}$, and $\mathrm{so}|\mathrm{w}| = (3\mathrm{n}+2) > \mathrm{n}$. By using the pumping lemma, we can write $\mathrm{w} = \mathrm{xyz}$, with $\mathrm{w} = \mathrm{xyz}$ with $|\mathrm{xy}| \leq \mathrm{n}$ and $|\mathrm{y}| > 0$

Step III: We want to find a suitable i so that $\mathrm{xy^i z} \notin \mathrm{L}$. The string consists of 'a' and 'b". So, y can be of any of these.

- 1. y contains only "a', and let y $= \mathrm{a^k}$
- 2. y contains only "b', and let y $= \mathrm{b^1}$
- 3. y contains both "a' and "b', and let y $= = \mathrm{b^k a}$

For case I, take $i = 0$.

$$\mathrm{w} = \mathrm{xyz} = \left(\mathrm{a^{n-k}}\right)\left(\mathrm{a^k}\right)\left(\mathrm{b^n a b^{n+1}}\right)$$

$$\mathrm{xy^0 z} = \mathrm{a^{n-k} b^n a b^{n+1}} \text{ which is not in the form } \mathrm{a^n b^n a b^{n+1}}$$

For case II, take $i = 0$.

$$\mathrm{w} = \mathrm{xyz} = \left(\mathrm{a^n b^{n-l}}\right)\left(\mathrm{b^l}\right)\mathrm{ab^{n+1}}$$

$$xy^0 z = a^n b^{n-1} ab^{n+1} \text{ which is not in the form } a^n b^n ab^{n+1}$$

For case III, take $i = 2$

$$\mathrm{w} = \mathrm{xyz} = \mathrm{a^n b^{n-k}}\left(\mathrm{b^k a}\right)^2 \ \mathrm{b^{n+1}}$$

$$= a^n b^n ab^k ab^{n+1} \text{ which is not in the form } a^n b^n ab^{n+1}$$

In all the three cases, there are contradictions and, hence, the language is not regular.

5.34 Show that $L = \{0^n 1^{2n},\ \text{where}\ n \geq 1\}$ is not regular.

Solution:

Step I: Assume that L is regular. Let n be the number of states of the FA accepting L.

Step II: Let $\mathrm{w} = 0^{\mathrm{n}}1^{2\mathrm{n}}$, where $\mathrm{n} \geq 1.|\mathrm{w}| = 3\mathrm{n} > \mathrm{n}$. By using the pumping lemma, we can write

$$\mathrm{w} = \mathrm{xyz}, \text{ with } \mathrm{w} = \mathrm{xyz} \text{ with } |\mathrm{xy}| \leq \mathrm{n} \text{ and } |\mathrm{y}| > 0$$

Step III: We want to find a suitable i so that $\mathrm{xy^iz} \notin \mathrm{L}$. The string consists of ' 0 ' and ' 1 ' y can be any of the following forms

- 1. y consists of only "0', and let $\mathrm{y} = 0^{\mathrm{k}}$
- 2. y consists of only ' 1 , and let $y = 1^{\mathrm{k}}$
- 3. y consists of both ' 0 ' and ' 1 ', and let $y = 0^k 1^{\mathrm{k}}$

For case I, take $i = 0$.

$$\mathrm{w} = \mathrm{xyz} = 0^{\mathrm{n}}1^{2\mathrm{n}} = 0^{\mathrm{n}-k}0^{\mathrm{k}}1^{2\mathrm{n}}$$
$$\mathrm{xy^0z} = 0^{\mathrm{n-k}}1^{2\mathrm{n}},\ \text{as } \mathrm{k} \neq 0, (\mathrm{n}-\mathrm{k}) \neq \mathrm{n}$$

For case II, take $i = 0$

$$\mathrm{w} = \mathrm{xyz} = 0^{\mathrm{n}}1^{2\mathrm{n}} = 0^{\mathrm{n}}1^{\mathrm{k}}1^{2\mathrm{n}-\mathrm{k}}$$
$$\mathrm{xy}^0\mathrm{z} = 0^{\mathrm{n}}1^{2\mathrm{n}-\mathrm{k}},\ \text{as } \mathrm{k} \neq 0, (2\mathrm{n} - \mathrm{k}) \neq \mathrm{n}$$

For case III, take $i = 2$

$$\mathrm{w} = \mathrm{xyz} = 0^{\mathrm{n}}1^{2\mathrm{n}} = 0^{n-k}0^{k}1^{k}1^{2n-k}$$

$$\mathrm{xy}^2\mathrm{z} = 0^{\mathrm{n}}1^{\mathrm{k}}0^{\mathrm{k}}1^{2\mathrm{n}}, \text{ which is not in the form } 0^{\mathrm{n}}1^{2\mathrm{n}}$$

For all the three cases, we are getting contradictions and, therefore, L is not regular.