

Moments

Project: Marker-based Localization

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Goals

- To understand what are moments in image processing.
- To learn about Hu moments.
- To use the `cv2.moments()`, `cv2.HuMoments()` functions in python.

Theory

Working Principle

Introduction

Moments, in image processing, are weighted average of the image pixels' intensities. Weighted average is similar to normal average except some values from the set contribute more to the result than the others. The formula for spatial moments in a raster image is-

$$M_{i,j} = \sum_x \sum_y x^i \cdot y^j \cdot I(x, y)$$

Figure 1: Spatial Moments

Here, $I(x,y)$ can have a value of either 0(background) or 1(foreground).

Using moments, one can obtain several properties of an image such as -

- **Area :**

$$M_{0,0} = \sum_x \sum_y x^0 \cdot y^0 \cdot I(x, y) = \sum_x \sum_y I(x, y)$$

Figure 2: Area

- Centroid :

$$(x_c, y_c) = \{M_{1,0} / M_{0,0}, M_{0,1} / M_{0,0}\}$$

Figure 3: Centroid

where

$$M_{1,0} = \sum_x \sum_y x \cdot I(x, y)$$

$$M_{0,1} = \sum_x \sum_y y \cdot I(x, y)$$

Figure 4:

Central moments

Central moments are derived by reducing the spatial moments with centroid. They are computed in a center referenced frame, in the following manner-

$$\mu_{p,q} = \sum_x \sum_y (x - x_c) \cdot (y - y_c) \cdot I(x, y)$$

Figure 5: Central Moments

However, a disadvantage of spatial and central moments is that they are dependant on the size of the object. So scaling is required. Area of the object is generally used as a scaling factor. So, central normalized moments are obtained by dividing the central moments with powers of area.

$$\eta_{p,q} = \mu_{p,q} / (M_{0,0})^{[(p+q)/2] + 1}$$

Figure 6: Central Normalized Moments

Hu Moments

Normalized moments are invariant to object size. However, they are affected by the orientation of the image. To overcome this, we can use Hu moments. Hu moments are invariant to translation, scaling and rotation.

$$\begin{aligned} I_1 &= \eta_{20} + \eta_{02} \\ I_2 &= (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2 \\ I_3 &= (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \eta_{03})^2 \\ I_4 &= (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2 \\ I_5 &= (\eta_{30} - 3\eta_{12})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] + (3\eta_{21} - \eta_{03})(\eta_{21} + \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] \\ I_6 &= (\eta_{20} - \eta_{02})[(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] + 4\eta_{11}(\eta_{30} + \eta_{12})(\eta_{21} + \eta_{03}) \\ I_7 &= (3\eta_{21} - \eta_{03})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] - (\eta_{30} - 3\eta_{12})(\eta_{21} + \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2]. \end{aligned}$$

Figure 7: Hu Moments

Applications

- Moments can be used to extract properties of an image.
- They are used in pattern recognition.
- Object identification also uses moments.
- Image analysis is done using moments.

Code

Moments

The main function we use to find moments is-

```
cv2.moments(array[, binaryImage])
```

where

- **array** : Raster image (single-channel, 8-bit or floating-point 2D array) or an array (1 times N or N times 1) of 2D points (Point or Point2f).

- **binaryImage** : If it is true, all non-zero image pixels are treated as 1's. The parameter is used for images only.

Hu Moments

The main function we use to find moments is-

```
cv2.HuMoments(m[, hu])
```

where

m : Input moments.

hu : Output Hu invariants.

Now, let us have a look at the code.

```
# Imports
import cv2

# Read image
img = cv2.imread("images/example/example.jpg")

# Convert to grayscale and apply thresholding
gray = cv2.cvtColor(img, cv2.COLOR_BGR2GRAY)
ret, th = cv2.threshold(gray, 127, 255, cv2.THRESH_BINARY)

# Find moments
M = cv2.moments(th, True) #Second parameter is true as th is a binary image

print M

# Find Hu invariant moments
Hu = cv2.HuMoments(M)

print Hu
```

Consider the following image.



Figure 8: Sample Image

When we find moments using `cv2.moments()`, we get this result-

```
{'mu02': 8196768181.423641, 'mu03': 251425068011.5957, 'm11': 28364400148.0, 'nu02': 0.08596224698563278, 'm12': 10288829414586.0, 'mu21': 171710226383.18262, 'mu20': 15356919975.078255, 'nu20': 0.16105315151257488, 'm30': 29489981567745.0, 'nu21': 0.0032406157902109896, 'mu11': 14250377.54253006, 'mu12': -7712212987.141113, 'nu11': 0.00014944847125549946, 'nu12': -0.0001455493927765709, 'm02': 29291682018.0, 'm03': 12192154557202.0, 'm00': 308793.0, 'm01': 80709118.0, 'mu30': -76361934603.83984, 'nu30': -0.0014411470781946316, 'nu03': 0.004745040890197371, 'm10': 108467643.0, 'm20': 53457620373.0, 'm21': 14153920964358.0}
```

Figure 9: Output of moments

The Hu moments calculated will look like this-

```
[[ 2.47015398e-01]
 [ 5.63873328e-03]
 [ 2.57776208e-05]
 [ 6.62883183e-05]
 [-2.53517895e-09]
 [-4.60712574e-06]
 [ 1.03989371e-09]]
```

Figure 10: Output of Hu Moments

Resources

- <http://breckon.eu/toby/teaching/dip/opencv/SimpleImageAnalysisbyMoments.pdf>
- https://en.wikipedia.org/wiki/Image_moment
- http://docs.opencv.org/modules/imgproc/doc/structural_analysis_and_shape_descriptors.html
- http://homepages.inf.ed.ac.uk/rbf/CVonline/LOCAL_COPIES/SHUTLER3/node8.html
- http://opencv-python-tutroals.readthedocs.org/en/latest/py_tutorials/py_imgproc/py_contours/py_con_features