Q No 1

We have to prove the following identities

a) $5n^2 - 10n = \Theta(n^2)$

We have to prove big-oh and big-~~theeta~~ omega first.

$5n^2 - 10n = O(n^2)$

$5n^2 - 10n \leq cn^2 \quad \forall\ n \geq n_0$

let

$c = 5$

$5n^2 - 10n \leq 5n^2$

let

$n = 1$

$5(1)^2 - 10(1) \leq 5(1)^2$

$5 - 10 \leq 5$

$-5 \leq 5$ ✓

$n = 2$

$5(2)^2 - 10(2) \leq 5(2)^2$

$20 - 20 \leq 20$

$0 \leq 20$ ✓

$n = 5$

$5(5)^2 - 10(5) \le 5(5)^2$

$125 - 50 \le 125$

$75 \le 125$ ✓

$n = 10$

$5(10)^2 - 10(10) \le 5(10)^2$

$500 - 100 \le 500$

$400 \le 500$ ✓

So,

$5n^2 - 10n \le cn^2 \quad \forall\ n \ge n_0$

where

$c = 5$

$n_0 = 1$

$\because 5n^2 - 10n = O(n^2)$

Now:

$5n^2 - 10n = \Omega(n^2)$

$5n^2 - 10n \ge cn^2 \quad \forall\ n \ge n_0$

let

$c = 4$

$5n^2 - 10n \ge 4n^2$

let

$n = 1$

$5(1)^2 - 10(1) \ge 4(1)^2$

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$-5 \geq 4$ X

$n=2$

$5(2)^2 - 10(2) \geq 4(2)^2$

$20 - 20 \geq 16$ X

$n=3$

$5(3)^2 - 10(3) \geq 4(3)^2$

$45 - 30 \geq 36$

$15 \geq 36$ X

$n=4$

$5(4)^2 - 10(4) \geq 4(4)^2$

$80 - 40 \geq 64$

$40 \geq 64$ X

$n=5$

$5(5)^2 - 10(5) \geq 4(5)^2$

$125 - 50 \geq 100$

$75 \geq 100$ X

$n=8$

$5(8)^2 - 10(8) \geq 4(8)^2$

$320 - 80 \geq 256$

$240 \geq 256$ X

$n=10$

$5(10)^2-10(10) \geq 4(10)^2$

$500-100 \geq 400$

$400 \geq 400$ ✓

$n=11$

$5(11)^2-10(11) \geq 4(11)^2$

$605-110 \geq 484$

$495 \geq 484$ ✓

$n=12$

$5(12)^2-10(12) \geq 4(12)^2$

$720-120 \geq 576$

$600 \geq 576$ ✓

$n=15$

$5(15)^2-10(15) \geq 4(15)^2$

$1125-150 \geq 900$

$975 \geq 900$ ✓

So,

$5n^2-10n \geq cn^2 \quad \forall\ n \geq n_0$

where

$c=4$

$n_0=10$

$\therefore 5n^2-10n \geq cn^2 \quad \forall\ n \geq n_0$

$5n^2-10n = \Omega(n^2)$

Now

$5n^2 - 10n = \Theta(n^2)$

$c_1 n \leq 5n^2 - 10n \leq c_2 n \quad \forall n \geq n_0$

where

$c_1 = 4$

$c_2 = 5$

$n_0 = 10$

b) $3n^2 2^n + n\log n = \Theta(n^2 2^n)$

We have to prove big-oh and big omega first

$3n^2 2^n + n\log n = O(n^2 2^n)$

$3n^2 2^n + n\log n \leq c(n^2 2^n) \quad \forall n \geq n_0$

let

$c = 4$

$3n^2 2^n + n\log n \leq 4(n^2 2^n)$

let

$n = 1$

$3(1)^2 2^1 + 1\log 1 \leq 4(1)^2 (2)^1$

$6 \leq 8$ ✓

$n = 2$

$3(2)^2 (2)^2 + 2\log 2 \leq 4(2^2)(2)^2$

$48 + 0.6020 \leq 64$

$48.6020 \leq 64$ ✓

$n = 3$

$3(3)^2(2)^3 + 3\log 3 \leq 4(3)^2(2)^3$

$216 + 1.4313 \leq 288$

$217.4313 \leq 288$ ✓

$n = 5$

$3(5)^2(2)^5 + 5\log 5 \leq 4(5)^2(2)^5$

$2400 + 3.4945 \leq 3200$

$2403.4945 \leq 3200$ ✓

So,

$3n^2 2^n + n\log n \leq cn^2 \quad \forall n \geq n_0$

Where

$c = 4$

$n_0 = 1$

$\because 3n^2 2^n + n\log n = O(n^2 2^n)$

Now

$3n^2 2^n + n\log n = \Omega(n^2 2^n)$

$3n^2 2^n + n\log n \geq cn^2 2^n \; \forall \; n \geq n_0$

let

$c = 2$

$3n^2 2^n + n\log n \geq 2n^2 2^n$

let

$n = 1$

$3(1)^2(2)^1 + 1\log 1 \geq 2(1)^2(2)^1$

$6 \geq 4$ ✓

$n = 2$

$3(2)^2(2)^2 + 2\log 2 \geq 2(2)^2(2)^2$

$3(4)(4) + 2(0.3010) \geq 2(4)(4)$

$48 + 0.6020 \geq 32$

$48.6020 \geq 32$ ✓

$n = 3$

$3(3)^2(2)^3 + 3\log 3 \geq 2(3)^2(2^3)$

$3(9)(8) + 3(0.4771) \geq 2(9)(8)$

$216 + 1.4313 \geq \cancel{216}\ 144$

$217.4313 \geq 144$ ✓

$n = 5$

$3(5)^2(2)^5 + 5\log 5 \geq 2(5)^2(2)^5$

$3(25)(32) + 5(0.6989) \geq 2(25)(32)$

$2400 + 3.4945 \geq 1600$

$2403.4945 \geq 1600$ ✓

So,

$3n^2 2^n + n\log n \geq cn^2 2^n \quad \forall\ n \geq n_0$

where

$n_0 = 1$

$c = 2$

$\because 3n^2 2^n + n\log n = \Omega(n^2 2^n)$

Now

$3n^2 2^n + n\log n = \Theta(n^2 2^n)$

$c_1 n \leq 3n^2 2^n + n\log n \leq c_2 n$

Where

$c_1 = 2$

$c_2 = 4$

$n_0 = 1$

c) $\sum_{i=0}^{n} i^2 = \Theta(n^3)$

As we know:

$$\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{(n^2+n)(2n+1)}{6}$$

$$= \frac{2n^3+n^2+2n^2+n}{6}$$

$$\sum_{i=0}^{n} i^2 = \frac{2n^3+3n^2+n}{6}$$

We have to prove big-oh and big omega first.

$$\frac{2n^3+3n^2+n}{6} = O(n^3)$$

So

$$\frac{2n^3+3n^2+n}{6} \leq cn^3 \quad \forall\, n \geq n_0$$

let

$c = 3$

$$\frac{2n^3+3n^2+n}{6} \leq 3n^3$$

let

$n = 1$

$$\frac{2(1)^3+3(1)^2+1}{6} \leq 3(1)^3$$

$$\frac{2+3+1}{6} \leq 3$$

$$\frac{6}{6} \leq 3$$

$1 \leq 3$ ✓

$n = 2$

$$\frac{2(2)^3 + 3(2)^2 + 2}{6} \leq 3(2)^3$$

$$\frac{16 + 12 + 2}{6} \leq 24$$

$$\frac{30}{6} \leq 24$$

$$5 \leq 24 \checkmark$$

$n = 4$

$$\frac{2(4)^3 + 3(4)^2 + 4}{6} \leq 3(4)^3$$

$$\frac{128 + 48 + 4}{6} \leq 192$$

$$\frac{180}{6} \leq 192$$

$$30 \leq 192 \checkmark$$

$n = 10$

$$\frac{2(10)^3 + 3(10)^2 + 10}{6} \leq 3(10)^3$$

$$\frac{2000 + 300 + 10}{6} \leq 3000$$

$$385 \leq 3000 \checkmark$$

So,

$$\sum_{i=0}^{n} i^2 \leq cn^3 \quad \forall\, n \geq n_0$$

where

$$c = 3$$

$$n_0 = 1$$

$$\therefore \sum_{i=0}^{n} i^2 = O(n^3)$$

Now:

$$\sum_{i=0}^{n} i^2 = \Omega(n^3)$$

$$\frac{2n^3 + 3n^2 + n}{6} \geq cn^3$$

let:

$$c = \frac{1}{3}$$

let

$n = 1$

$$\frac{2(1)^3 + 3(1)^2 + 1}{6} \geq \frac{1}{3}(1)$$

$$\frac{2+3+1}{6} \geq \frac{1}{3} \checkmark$$

$$1 \geq \frac{1}{3} \checkmark$$

$n = 2$

$$\frac{2(2)^3 + 3(2)^2 + 2}{6} \geq \frac{1}{3}(2)^3$$

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$$\frac{16+12+2}{6} \geq \frac{8}{3}$$

$$5 \geq 2.666 \checkmark$$

$n = 3$

$$\frac{2(3)^3 + 3(3)^2 + 3}{6} \geq \frac{1}{3}(3)^3$$

$$14 \geq 9 \checkmark$$

$n = 5$

$$\frac{2(5)^3 + 3(5)^2 + 5}{6} \geq \frac{1}{3}(5)^3$$

$$55 \geq 41.666 \checkmark$$

$n = 10$

$$\frac{2(10)^3 + 3(10)^2 + 10}{6} \geq \frac{1}{3}(10)^3$$

$$385 \geq 333.33 \checkmark$$

So;

$$\sum_{i=0}^{n} i^2 \geq cn^3 \quad \forall \; n \geq n_0$$

where

$$c = \frac{1}{3}$$

$$n_0 = 1$$

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$$\because \sum_{i=0}^{n} i^2 = \Omega(n^3)$$

Now:

$$\sum_{i=0}^{n} i^2 = \Theta(n^3)$$

$$c_1 n \leq \sum_{i=0}^{n} i^2 \leq c_2 n$$

where

$c_1 = \frac{1}{3}$

$c_2 = 3$

$n_0 = 1$

d) $n^4 + 10^7 n^2 = \Theta n^4$

We have to prove big omega and big Oh first.

$n^4 + 10^7 n^2 = O(n^4)$

$n^4 + 10^7 n^2 \leq c n^4 \ \forall \ n \geq n_0$

let

$c = 10^6$

$n^4 + 10^7 n^2 \leq 10^6 n^4$

let

$n=1$

$(1)^4 + 10^7(1)^2 \leq 10^6(1)^4$

$1 + 10^7 \leq 10^6$ X

$n=2$

$(2)^4 + 10^7(2)^2 \leq 10^6 \cdot (2)^4$

$16 + 4\times10^7 \leq 16\times10^6$

$16 + 4\times10^7 \leq 1.6\times10^7$ X

$n=3$

$(3)^4 + 10^7(3)^2 \leq 10^6 \cdot (3)^4$

$81 + 9\times10^7 \leq 81\times10^6$

$81 + 9\times10^7 \leq 8.1\times10^7$ X

$n=4$

$(4)^4 + 10^7(4)^2 \leq 10^6(4)^4$

$256 + 16\times10^7 \leq 256\times10^6$

$256 + 16\times10^7 \leq 25.6\times10^7$ ✓

$n=5$

$(5)^4 + 10^7\times(5)^2 \leq 10^6(5)^4$

$625 + 25\times10^7 \leq 625\times10^6$

$625 + 25\times10^7 \leq 62.5\times10^7$ ✓

$n=6$

$(6)^4 + 10^7\times(6)^2 \leq 10^6(6)^4$

$1296 + 36 \times 10^7 \leq 1296 \times 10^6$

$1296 + 36 \times 10^7 \leq 12.96 \times 10^8$ ✓

So;

$$n^4 + 10^6 n^2 \leq c n^4 \quad \forall \, n \geq n_0$$

where

$$c = 10^6$$

$$n_0 = 4$$

Now

$$n^4 + 10^6 n^2 \doteq \Omega(n^4)$$

$n^4 + 10^6 n^2 \geq c n^4 \quad \forall \, n \geq n_0$

let:

$$c = 1$$

$n^4 + 10^6 n^2 \geq 1 n^4 \quad \forall \, n \geq n_0$

let

$n = 1$

$(1)^4 + 10^7 (1)^2 \geq (1)^4$

$1 + 10^7 \geq 1$ ✓

$n = 2$

$(2)^4 + 10^7 (2)^2 \geq (2)^4$

$16 + 4 \times 10^7 \geq 16$

$n=3$

$(3)^4 + 10^7(3)^2 \geq 3^4$

$81 + 9\times10^7 \geq 81$ ✓

$n=5$

$(5)^4 + 10^7 \times 5^2 \geq 5^4$

$1296 + 25\times10^7 \geq 1296$ ✓

So;

$n^4 + 10^7 n^2 \geq cn^4$

where

$c = 1$

$n_0 = 1$

So,

$c_1 n^4 \leq n^4 + 10^7 n^2 \leq c_2 n^4$

where

$c_1 = 1$

$c_2 = 10^6$

$n_0 = 4$

e) $2^{n+5} = \Theta\, 2^n$

We have to prove big-oh and big omega first.

$2^{n+5} = O(2^n)$

$2^{n+5} \leq c\,2^n \quad \forall\ n \geq n_0$

let

$c = 2^6$

let

$n = 1$

$2^{1+5} \leq 2^6 \cdot 2^1$

$2^6 \leq 2^7$ ✓

$n = 2$

$2^{2+5} \leq 2^6 \cdot 2^2$

$2^7 \leq 2^8$ ✓

$n = 3$

$2^{3+5} \leq 2^6 \cdot 2^3$

$2^8 \leq 2^9$ ✓

$n = 10$

$2^{10+5} \leq 2^6 \cdot 2^{10}$

$2^{15} \leq 2^{16}$ ✓

So,

$$2^{n+5} \leq C2^n \quad \forall n \geq n_0$$

where

$n_0 = 1$

$C = 2^6$

$$\therefore 2^{n+5} = O(2^n)$$

Now

$$2^{n+5} = \Omega(2^n)$$

$$2^{n+5} \geq C2^n \quad \forall n \geq n_0$$

let

$C = 2^4$

$$2^{n+5} \geq 2^4 2^n$$

let

$n = 1$

$$2^{1+5} \geq 2^4 . 2^1$$

$$2^6 \geq 2^5 \checkmark$$

$n = 2$

$$2^{2+5} \geq 2^4 \cdot 2^2$$

$$2^7 \geq 2^6 \checkmark$$

$n = 3$

$$2^{3+5} \geq 2^4 \cdot 2^3$$

$$2^8 \geq 2^7 \checkmark$$

$n = 5$

$$2^{5+5} \geq 2^4 \cdot 2^5$$

$$2^{10} \geq 2^9 \checkmark$$

$n = 10$

$$2^{10+5} \geq 2^4 \cdot 2^{10}$$

$$2^{15} \geq 2^{14} \checkmark$$

So,

$$2^{n+5} \geq c2^n \quad \forall \; n \geq n_0$$

where

$$n_0 = 1$$

$$c = 2^4$$

$$\therefore 2^{n+5} = \Omega(2^n)$$

Now,

$$c_1 n \leq 2^{n+5} \leq c_2 n$$

$$c_1 2^n \leq 2^{n+5} \leq c_2 2^n \quad \forall n \geq n_0$$

where

$c_1 = 2^4$

$c_2 = 2^6$

$n_0 = 1$

QNO2

Time of exeation $(t_1) = 0.7\ ms$

input size$(n_1) = 100$

input size $(n_2) = 400$

time of execution $(t_2) = ?$

We have to use following formula.

$$\text{time of exeation} = \frac{\text{no. of steps}}{\text{computer speed}}$$

a) $T(n) = n$

First we have to find computer speed.

$$0.7 \times 10^{-3}\ s = \frac{n}{\text{comp. speed}}$$

$n = 100$

$$\text{Computer speed} = \frac{100}{0.0007}$$

computer speed = 1.42×10^5 instr./sec

Again using formula to find time of execution for $n = 400$

$$\text{time execution} = \frac{T(n)}{\text{comp. speed}}$$

$$\text{time execution} = \frac{n}{1.42 \times 10^5} \quad \because n = 400$$

$$= \frac{400}{1.42 \times 10^5}$$

$$= 2.81 \times 10^{-3} \text{ sec}$$

So,
Execution time = 2.81 ms for input size $n = 400$.

b) $T(n) = n \lg n$

First we have to find computer speed by using above formula.

$$0.7 \times 10^{-3} \text{ s} = \frac{n \lg n}{\text{Computer speed}}$$

$n = 100$

$$\text{computer speed} = \frac{100 \lg 100}{0.0007s}$$

$$= \frac{664.3856}{0.0007}$$

computer speed $= 9.49 \times 10^5$ instr/sec

Again using formula ~~f~~ to find time execution for $n = 400$

Using formula.

$$\text{time execution} = \frac{T(n)}{\text{comp. speed}}$$

$$= \frac{n \lg n}{9.49 \times 10^5}$$

$n = 400$

$$= \frac{400 \lg 400}{9.49 \times 10^5}$$

$$= \frac{3.45 \times 10^3}{9.49 \times 10^5}$$

$$= 3.64 \times 10^{-3} s$$

So,

Exeation time = 3.64 ms for input size $n = 400$.

c) $T(n) = n^2$

First we have to find computer speed using formula.

$$\text{time execution} = \frac{T(n)}{\text{comp. speed}}$$

$$0.7 \times 10^{-3} = \frac{n^2}{\text{Comp. speed}}$$

$n = 100$

$$\text{Computer speed} = \frac{(100)^2}{0.7 \times 10^{-3}}$$

$$= \frac{10000}{0.7 \times 10^{-3}}$$

Computer speed $= 1.42 \times 10^7$ instr./sec

Again using formula to find execution time for $n = 400$

$$\text{time execution} = \frac{T(n)}{\text{comp.speed}}$$

$$= \frac{n^2}{1.42 \times 10^7} \quad \because n = 400$$

$$= \frac{(400)^2}{1.42 \times 10^7}$$

$$= \frac{160000}{1.42 \times 10^7}$$

$$\text{time execution} = 11.2 \times 10^{-3} \text{ sec}$$

so,

time execution = 11.2 ms for input size n = 400.

d) $T(n) = n^3$

First we have to find computer speed using formula

$$\text{time execution} = \frac{T(n)}{\text{comp.speed}}$$

$$0.7 \times 10^{-3} = \frac{n^3}{\text{comp.speed}} \quad \because n = 1000$$

$$\text{computer speed} = \frac{(100)^3}{0.7\times10^{-3}}$$

$$= \frac{1000000}{0.7\times10^{-3}}$$

$$\text{computer speed} = 1.42\times10^{9}\ \text{instr./sec}$$

Again using formula to find execution time for $n=400$.

$$\text{Time execution} = \frac{T(n)}{\text{comp. speed}}$$

$$= \frac{n^3}{1.42\times10^{9}} \quad \because n=400$$

$$= \frac{(400)^3}{1.42\times10^{9}}$$

$$= \frac{64000000}{1.42\times10^{9}}$$

$$\text{time execution} = 45.07\times10^{-3}\ s$$

So,
Execution time $= 45.07$ ms for input size $n = 400$.

e) $T(n) = 2^n$

First we have to find computer speed by using formula.

$$\text{time execution} = \frac{T(n)}{\text{comp. speed}}$$

$$0.7 \times 10^{-3} = \frac{2^n}{\text{comp. speed}}$$

$\because n = 100$

$$\text{computer speed} = \frac{2^{100}}{0.7 \times 10^{-3}}$$

$$= \frac{1.2676 \times 10^{30}}{0.7 \times 10^{-3}}$$

$$\text{computer speed} = 1.8109 \times 10^{33} \text{ instr./sec}$$

Again using formula to find execution time for $n = 400$

$$\text{time execution} = \frac{T(n)}{\text{comp. speed}}$$

$$\text{time execution} = \frac{2^n}{1.8109 \times 10^{33}}$$

$n = 400$

$$= \frac{2^{400}}{1.8109 \times 10^{33}}$$

$$= \frac{2.5822 \times 10^{120}}{1.8109 \times 10^{33}}$$

time execution $= 1.42 \times 10^{87}$ sec

So,

time execution $= 1.42 \times 10^{84}$ ms
for input size $n = 400$

QNO3

a) $5n^{5/2} + 25n^{2/5}$

We have to find big-oh and prove it.

n	$5n^{5/2}$	$25n^{2/5}$
1	5	25
2	28.28	32.98
3	77.94	38.79
4	160	43.52
5	279.50	47.59
10	1581.13	62.79
15	4357.1	73.85
20	8944.2	82.86
25	15625	90.59
30	24647.1	97.45
35	36235.9	103.64
40	50596.4	109.33

We have seen that $n^{5/2}$ is dominant term so,

$$5n^{5/2} + 25n^{2/5} = O(n^{5/2})$$

$5n^{5/2} + 25n^{2/5} \leq cn^{5/2} \quad \forall n \geq n_0$

let

$c = 6$

$5n^{5/2} + 25n^{2/5} \leq 6n^{5/2}$

let

$n = 1$

$5(1)^{5/2} + 25(1)^{2/5} \leq 6(1)^{5/2}$

$5 + 25 \leq 6$

$30 \leq 6$ X

$n = 2$

$5(2)^{5/2} + 25(2)^{2/5} \leq 6(2)^{5/2}$

$28.28 + 32.98 \leq 33.94$

$61.26 \leq 33.94$ X

$n = 3$

$5(3)^{5/2} + 25(3)^{2/5} \leq 6(3)^{5/2}$

$77.94 + 38.79 \leq 93.53$

$116.73 \leq 93.53$ X

$n=4$

$$5(4)^{5/2} + 25(4)^{2/5} \leq 6(4)^{5/2}$$
$$160 + 43.52 \leq 192$$
$$203.52 \leq 192 \times$$

$n=5$

$$5(5)^{5/2} + 25(5)^{2/5} \leq 6(5)^{5/2}$$
$$279.50 + 47.59 \leq 335.41$$
$$327 \leq 335.41 \checkmark$$

$n=6$

$$5(6)^{5/2} + 25(6)^{2/5} \leq 6(6)^{5/2}$$
$$440.90 + 51.19 \leq 529.08$$
$$492.09 \leq 529.08 \checkmark$$

$n=7$

$$5(7)^{5/2} + 25(7)^{2/5} \leq 6(7)^{5/2}$$
$$648.20 + 54.44 \leq 777.85$$
$$702.64 \leq 777.85 \checkmark$$

So,

$$5n^{5/2} + 25n^{2/5} \leq cn^{5/2} \quad \forall n \geq n_0$$

where

$$c = 6, \; n_0 = 5$$

$$\therefore 5n^{5/2} + 25n^{2/5} = O(n^{5/2})$$

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b) $6\lg n^2 + 2n$

We have to find big-Oh and prove it.

n	$6\lg n^2$	$2n$
1	0	2
2	12	4
3	19.01	6
4	24	8
5	27.86	10
10	39.86	20
15	~~38.68~~ 46.88	30
20	51.86	40
25	55.72	50
30	58.88	60
35	61.55	70
40	63.86	80
45	65.90	90
50	67.72	100

We have seen that n is dominant term so

$$6\lg n^2 + 2n = O(n)$$

$6 \lg n^2 + 2n \leq cn \quad \forall\, n \geq n_0$

let

$c = 2.5$

$6 \lg n^2 + 2n \leq (2.5)\, n$

let

$n = 2$

$6 \lg 2^2 + 2(2) \leq 2.5 \times 2$

$12 + 4 \leq 5$ ✗

$n = 3$

$6 \lg 3^2 + 2(3) \leq 2.5\,(3)$

$19.01 + 6 \leq 7.5$ ✗

$n = 4$

$6 \lg 4^2 + 2(4) \leq 2.5\,(4)$

$24 + 8 \leq 10$

$32 \leq 10$ ✗

$n = 5$

$6 \lg (5)^2 + 2(5) \leq 2.5(5)$

$27.86 + 10 \leq 12.5$

$37.86 \leq 12.5$ ✗

$n = 10$

$6\lg(10)^2 + 2(10) \leq 2.5(10)$

$39.86 + 20 \leq 25$

$59.86 \leq 25$ α

$n = 30$

$6\lg(30)^2 + 2(30) \leq 2.5(30)$

$58.88 + 60 \leq 75$

$118.88 \leq 75$ α

$n = 50$

$6\lg 50^2 + 2(50) \leq 2.5(50)$

$67.72 + 100 \leq 125$

$167.72 \leq 125$ α

$n = 100$

$6\lg 100^2 + 2(100) \leq 2.5(100)$

$79.88 + 200 \leq 250$

$279.88 \leq 250$ α

$n = 150$

$6\lg 150^2 + 2(150) \leq 2.5(150)$

$86.74 + 300 \leq 375$

$386.74 \leq 375$

$n=175$

$6\lg 175^2 + 2(175) \leq 2.5(175)$

$89.41+350 \leq 437.5$

$439.41 \leq 437.5$

$n=179$

$6\lg 179^2 + 2(179) \leq 2.5(179)$

$89.80 + 358 \leq 447.5$

$447.8 \leq 447.5$ ✗

$n=180$

$6\lg 180^2 + 2(180) \leq 2.5(180)$

$89.90 + 360 \leq 450$

$449.9 \leq 450$ ✓

$n=181$

$6\lg 181^2 + 2(181) \leq 2.5(181)$

$89.99 + 362 \leq 452.5$

$451.99 \leq 452.5$ ✓

$n=182$

$6\lg 182^2 + 2(182) \leq 2.5(182)$

$90.09 + 364 \leq 455$

$454.09 \leq 455$ ✓

$n = 183$

$6\lg 183^2 + 2(183) \leq 457.5$

$90.18 + 366 \leq 457.5$

$456.18 \leq 457.5$

$n = 185$

$6\lg 185^2 + 2(185) \leq 2.5(185)$

$90.37 + 370 \leq 462.5$

$460.37 \leq 462.5$ ✓

So,

$$6\lg n^2 + 2n = O(n)$$

$$6\lg n^2 + 2n < cn \quad \forall n \geq n_0$$

where

$c = 2.5$

$n_0 = 180$

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c) $3n^4 + n^4 \lg n$

We have to find big-oh first

n	$3n^4$	$n^4 \lg n$
1	3	0
2	48	16
3	243	128.38
4	768	512.
5	1875	1451
10	30000	33219.2
15	151875	197786.3
20	480000	691508.4
25	1171875	1814006.3
30	2430000	3974581.3

We have seen that $n^4 \lg n$ is dominant term so,

$$3n^4 + n^4 \lg n = O(n^4 \lg n)$$

$$3n^4 + n^4 \lg n \leq c n^4 \lg n$$

let

$c = 2.$

$3n^4 + n^4 \lg n \le 2 n^4 \lg n$

let

$n = 1$

$3(1)^4 + (1)^4 \lg 1 \le 2(1)^4 \lg 1$

$3 + 0 \le 0$

$3 \le 0$ ✗

$n = 2$

$3(2^4) + 2^4 \lg 2 \le 2^4 \lg 2 \times 2$

$48 + 16 \le 2 \times 16$

$64 \le 32$ ✗

$n = 4$

$3(4)^4 + 4^4 \lg 4 \le 2 \times 4^4 \lg 4$

$768 + 512 \le 2 \times 512$

$1280 \le 1024$ ✗

$n = 6$

$3(6)^4 + 6^4 \lg 6 \le 2 \times 6^4 \lg 6$

$3888 + 3350.1 \le 67002.2$

$7238.1 \le 67002.2$ ✗

$n = 8$

$3 \times 8^4 + 8^4 \lg 8 \le 2 \times 8^4 \lg 8$

$12288 + 12288 \le 24576$ ✓

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$24576 \leq 24576$ ✓

$n = 9$:

$3 \times 9^4 + 9^4 \lg 9 \leq 2 \times 9^4 \lg 9$

$19683 + 20799.8 \leq 41595.7$

$40482.8 \leq 41595.7$ ✓

$n = 10$

$3 \times 10^4 + 10^4 \lg 10 \leq 2 \times 10^4 \lg 10$

$30000 + 33219.2 \leq 33219.2 \times 2$

$63219.2 \leq 66438$ ✓

So,

$\cancel{6 \lg n^2 + 2n}$

$3n^4 + n^4 \lg n = O(n^4 \lg n)$

$3n^4 + n^4 \lg n \leq c(n^4 \lg n) \; \forall n \geq n_0$

where

$c = 2$

$n_0 = 8$

d) $n(n + 10\sqrt{n})$

We can write as

$= n^2 + 10n\sqrt{n}$

First we have to find big-oh

n	n^2	$10n\sqrt{n}$
1	1	10
2	4	28.28
3	9	51.96
4	16	80
5	25	111.80
10	100	316.22
15	225	580.94
20	400	894.42
25	625	1250
30	900	1643.16
35	1225	2070.6
40	1600	2529.8
45	2025	3018.6
50	2500	3535.5
55	3025	4078.9
60	3600	4647.5
65	4225	5240.4
70	4900	5,856.6

n	n^2	$10n\sqrt{n}$
80	6400	7155.41
90	8100	8538.1
100	10000	10000
110	12100	11536.89
120	14400	13145.34

We have seen that n^2 is dominant term so

$$n(n+10\sqrt{n}) = O(n^2)$$
$$n^2 + 10n\sqrt{n} \leq cn^2 \quad \forall n \geq n_0$$

let $c = 5$

$$n^2 + 10n\sqrt{n} \leq 5n^2$$

let

$n = 1$

$(1)^2 + 10(1)\sqrt{1} \leq 5(1)^2$

$1 + 10 \leq 5$ /

$11 \leq 5$ ✗

$n = 2$

$(2)^2 + 10(2)\sqrt{2} \leq 5(2)^2$

$4 + 28.28 \leq 20$

$32.28 \leq 20$ ✗

$n=4$

$4^2 + 10(4)\sqrt{4} \leq 5(4)^2$

$16 + 80 \leq 80$

$90 \leq 80$ ✗

$n=6$

$(6)^2 + 10(6)\sqrt{6} \leq 5(6)^2$

$36 + 146.96 \leq 180$

$182.96 \leq 180$ ✗

$n=7$

$(7)^2 + 10(7)\sqrt{7} \leq 5(7)^2$

$49 + 185.20 \leq 245$

$234.20 \leq 245$ ✓

$n=8$

$8^2 + 10(8)\sqrt{8} \leq 5(8)^2$

$64 + 226.27 \leq 320$

$290.27 \leq 320$ ✓

$n=9$

$9^2 + 10(9)(\sqrt{9}) \leq 5(9)^2$

$81 + 270 \leq 405$

$351 \leq 405$ ✓

So,

$$n^2 + 10n\sqrt{n} \leq cn^2 \quad \forall n \geq n_0$$

where

$c = 5$

$n_0 = 7$

$$\therefore n(n + 10\sqrt{n}) = O(n^2)$$

e) $\log_4 2^n + \log_4 n^2$

We have to find big-oh first

n	$\log_4 2^n$	$\log_4 n^2$
1	0.5	0
2	1	1
3	1.5	1.58
4	2	2
5	2.5	2.321
10	5	3.32
15	7.5	3.90
20	10	4.3
25	12.5	4.64
30	15	4.90
35	17.5	5.12
40	20	5.321

We have seen that $\log_4 2^n$ is dominant so

$$\log_4 2^n + \log_4 n^2 = O(\log_4 2^n)$$

So,

$$\log_4 2^n + \log_4 n^2 \leq c(\log_4 2^n)$$

let $c = 2$

$$\log_4 2^n + \log_4 n^2 \leq 2(\log_4 2^n)$$

let

$n = 1$

$$\log_4 2^1 + \log_4 1^2 \leq 2(\log_4 2^1)$$

$$0.5 + 0 \leq 2(\cancel{\log_4}\ 0.5)$$

$$0.5 \leq 1 \checkmark$$

$n = 2$

$$\log_4 2^2 + \log_4 2^2 \leq 2(\log_4 2^2)$$

$$1 + 1 \leq 2$$

$$2 \leq 2 \checkmark$$

$n = 3$

$$\log_4 2^3 + \log_4 3^2 \leq 2(\log_4 2^3)$$

$$1.5 + 1.58 \leq 2(1.5)$$

$$3.08 \leq 3 \times$$

(12)

if $n=4$

$\log_4 2^4 + \log_4 4^2 \leq 2(\log_4 2^4)$

$2 + 2 \leq 2(2)$

$4 \leq 4$ ✓

$n=5$

$\log_4 2^5 + \log_4 5^2 \leq 2(\log_4 2^5)$

$2.5 + 2.31 \leq 2(2.5)$

$4.82 \leq 5$ ✓

$n=6$

$\log_4 2^6 + \log_4 6^2 \leq 2(\log_4 2^6)$

$3 + 2.58 \leq 2(3)$

$5.58 \leq 6$ ✓

$n=7$

$\log_4 2^7 + \log_4 7^2 \leq 2(\log_4 2^7)$

$3.5 + 2.80 \leq 2(3.5)$

$6.30 \leq 7$ ✓

$n=8$

$\log_4 2^8 + \log_4 8^2 \leq 2(\log_4 2^8)$

$4 + 3 \leq 2(4)$

$7 \leq 8$ ✓

So,

$$\log_4 2^n + \log_4 n^2 \leq c \cdot \log_4 2^n$$

$$\forall n \geq n_0$$

Where

$n_0 = 4$

$c = 2$

$$\therefore \log_4 2^n + \log_4 n^2 \in O(\log_4 2^n)$$

Q No 4

time execution $(t_1) = 0.6\,ms = 0.0006s$

input size $(n_1) = 100$

time execution $(t_2) = 1\,min = 60\,sec$

input size $n_2 = ?$

a) $T(n) = n$

First we have to find computer speed using formula.

$$\text{time execution}(t_1) = \frac{\text{step count}}{\text{Comp. speed}}$$

$$0.6 \times 10^{-3}s = \frac{n}{\text{comp. speed}}$$

$n = 100$

$$\text{computer speed} = \frac{100}{0.0006}$$

Computer speed $= 1.66 \times 10^5$ instr./sec

Again using formula to find input size when time = 60 sec
Using formula.

$$\text{time execution } (t_2) = \frac{\text{step count}}{\text{Comp. speed}}$$

$$60 \times 1.66 \times 10^5 = \text{step } n$$

$$\Rightarrow n = 10 \times 10^6$$

$$\Rightarrow \boxed{n = 1 \times 10^7}$$

No. of inputs will be 1×10^7 when time execution is 60 sec.

b) $T(n) = n^2$

First we have to find computer speed using formula.

$$\text{time execution} = \frac{\text{step. Count}}{\text{Comp. speed}}$$

$n = 100$

$$0.6 \times 10^{-3} = \frac{(100)^2}{\text{comp. speed}}$$

$$\text{Comp. speed} = \frac{10000}{0.6 \times 10^{-3}}$$

comp. speed = 1.6×10^7 instr./sec

Again using formula to find input size when time is 60 sec

time execution = $\frac{\text{step count}}{\text{comp. speed}}$

$$60 = \frac{n^2}{1.6 \times 10^7}$$

$$\Rightarrow n^2 = 60 \times 1.6 \times 10^7$$
$$n^2 = 1 \times 10^9$$

squaring B.S.

$$n = 31.622 \times 10^3$$

So:

$$n = 31622.7$$

No. of inputs will be $n = 31622.7$ when time is 60 sec for the given algorithm

$$T(n) = n^2$$

c) $T(n) = n^3$

First we have to find computer speed using formula.

$$\text{time execution} = \frac{\text{step count}}{\text{Comp. speed}}$$

$$0.6 \times 10^{-3} = \frac{n^3}{\text{Comp. speed}}$$

$n = 100$

$$\text{Comp. speed} = \frac{(100)^3}{0.6 \times 10^{-3}}$$

$$\text{Comp. speed} = \frac{1000000}{0.6 \times 10^{-3}}$$

$$\text{Comp. speed} = 1.66 \times 10^9 \text{ instr./sec}$$

Again using formula to find input size when time $= 60$ sec
Using formula.

$$\text{time execution} = \frac{\text{step count}}{\text{Comp. speed}}$$

$$60 = \frac{n^3}{1.66 \times 10^9}$$

$\Rightarrow n^3 = 60 \times 1.66 \times 10^9$

$n^3 = 100 \times 10^9$

Taking cube root on B.S

$n = 4.64158 \times 10^3$

so,

$n = 4641.58$

No. of inputs will be $n = 4641.58$ when time execution is 60 sec for given algorithm

$T(n) = n^3$

d) $T(n) = 2^n$

First we have to find computer speed using formula

$$\text{time execution} = \frac{\text{step count}}{\text{comp. speed}}$$

$$0.6 \times 10^{-3} = \frac{2^n}{\text{comp. speed}}$$

$n = 100$

$$\text{comp. speed} = \frac{2^{100}}{0.6 \times 10^{-3}}$$

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$$= \frac{1.26 \times 10^{30}}{0.6 \times 10^{-3}}$$

Comp. speed $= 2.11 \times 10^{33}$ instr./sec

Again using formula to find input size when time $= 60$ sec
Using formula.

time execution $(t_2) = \frac{\text{step count}}{\text{Comp. speed}}$

$$60 \times 2.11 \times 10^{33} = 2^n$$

$$\Rightarrow 2^n = 1.26 \times 10^{35}$$

Taking $\log_2$ on B.S.

$$\log_2 2^n = \log_2 1.26 \times 10^{35}$$

$\because \log_a x^n = n \log$

$$n(1) = 116.609$$

No. of inputs will be 116.609 when time execution $= 60$ sec for given algorithm

$$T(n) = 2^n$$

e) $T(n) = 5^n$

First we have to find computer speed using formula.

$$\text{time execution} = \frac{\text{step count}}{\text{comp. speed}}$$

$n = 100$

$$0.6 \times 10^{-3} = \frac{5^{100}}{\text{comp. speed}}$$

$$\text{comp. speed} = \frac{7.88 \times 10^{69}}{0.6 \times 10^{-3}}$$

$$\text{comp. speed} = 1.314 \times 10^{73} \text{ instr/sec}$$

Again using formula to find (~~step count~~) n when time = 60 sec Using formula.

$$\text{time execution} = \frac{\text{step count}}{\text{comp. speed}}$$

$$60 \times 1.314 \times 10^{73} = 5^n$$

$$\Rightarrow 5^n = 7.88 \times 10^{74}$$

Taking $\log_5$ on B.S.

$$\log_5 5^n = \log_5(7.88 \times 10^{72})$$

$$\because \log_a x^n = n \log_a x$$

$$5(1)n = 107.153$$

$$n = 107.153$$

no. of inputs will be 107.153 when time execution is 60sec for given algorithm

$$T(n) = 5^n$$

Approximate exact step count

The approximate exact step count for all questions will be.

a) 1×10^7
b) 31623
c) 4642
d) 117
e) 107

QNO 8:

$t = 1\ min = 60\ sec$

$n = 1000$

New computer = 1000 times faster

$t = 60\ sec$

$n = ?$

a) $T(n) = n$

using

$$\text{time execution} = \frac{T(n)}{\text{Comp. Speed}}$$

$$60 = \frac{n}{\text{comp. speed}}$$

$$\text{comp. speed} = \frac{1000}{60} = 16.66667$$

New comp. is 1000 times faster so speed of new computer will be

$\text{comp. speed} = 16.667 \times 1000$

$\boxed{\text{comp. speed} = 16666.67}$

Again by formula.

$$\text{time} = \frac{T(n)}{\text{c.speed}}$$

$60 \times 16666.67 = n$

~~1000000000~~

$\boxed{1000000 = n}$

b) $T(n) = n^2$

$$\text{Comp. speed} = \frac{(1000)^2}{60} = 16666.67$$

New speed $= 16666666.67$

$60 \times 16666666.67 = n^2$

$\sqrt{1000000000} = \sqrt{n^2}$

$n = 31622.78 \approx 31622$

c) $T(n) = n^3$

comp. speed $= \frac{(1000)^3}{60} = 16666666.67$

New speed $= 1.66 \times 10^{10}$

$60 \times 1.66 \times 10^{10} = n^3$

$\boxed{10000 = n}$ A.

d) $T(n) = 2^n$

comp. speed $= \frac{2^{1000}}{60} = 1.78 \times 10^{299}$

New speed $= 1.78 \times 10^{302}$

$60 \times 1.78 \times 10^{302} = 2^n$

$10.71 \times 10^{303} = 2^n$

$\log_2$ on B.S

$1.00099 \times 10^3 = n$

$\boxed{n \doteq 1000.9}$

e) $T(n) = 10^n$

comp. speed $= 1.66 \times 10^{998}$

New speed $= 1.66 \times 10^{1001}$

$60 \times 1.66 \times 10^{1001} = 10^n$

$10 \times 10^{1002} = 10^n$

Take $\log_{10}$ on B.S.

$1.003 \times 10^3 = n$

$\Rightarrow \boxed{n = 1003}$