

2. ماتریس تبدیل را با $N = 5$ و $N = 4$ محاسبه کرده

$$T = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 0.31 - 0.95j & -0.81 - 0.59j & -0.81 + 0.59j & 0.31 + 0.95j \\ 1 & -0.81 - 0.59j & 0.31 + 0.95j & 0.31 - 0.95j & -0.81 + 0.59j \\ 1 & -0.81 + 0.59j & 0.31 - 0.95j & 0.31 + 0.95j & -0.81 - 0.59j \\ 1 & 0.31 + 0.95j & -0.81 + 0.59j & -0.81 - 0.59j & 0.31 - 0.95j \end{bmatrix}$$

(۱)

$$x = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} X[0] \\ X[2] \\ X[3] \\ X[4] \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$X[0] = \frac{1}{2} (1 + 0 + 1 + 0) = 1$$

$$X[1] = \frac{1}{2} (1 + 0 - 1 + 0) = 0$$

$$X[2] = \frac{1}{2} (1 + 0 + 1 + 0) = 1$$

$$X[3] = \frac{1}{2} (1 + 0 - 1 + 0) = 0$$

$$\begin{bmatrix} X[0] \\ X[2] \\ X[3] \\ X[4] \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

(ج)

$$x = \begin{bmatrix} 2 \\ -2 \\ j \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} X[0] \\ X[2] \\ X[3] \\ X[4] \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ j \\ 0 \end{bmatrix}$$

$$X[0] = \frac{1}{2} (2 - 2 + j + 0) = \frac{j}{2}$$

$$X[1] = \frac{1}{2} (2 + 2j - j + 0) = 1 + \frac{j}{2}$$

$$X[2] = \frac{1}{2} (2 + 2 + j + 0) = 2 + \frac{j}{2}$$

$$X[3] = \frac{1}{2} (2 - 2j - j + 0) = 1 - \frac{3j}{2}$$

$$\begin{bmatrix} X[0] \\ X[2] \\ X[3] \\ X[4] \end{bmatrix} = \begin{bmatrix} \frac{j}{2} \\ 1 + \frac{j}{2} \\ 2 + \frac{j}{2} \\ 1 - \frac{3j}{2} \end{bmatrix}$$

(ب)

$$x = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \end{bmatrix} = \frac{1}{\sqrt{5}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 0.31 - 0.95j & -0.81 - 0.59j & -0.81 + 0.59j & 0.31 + 0.95j \\ 1 & -0.81 - 0.59j & 0.31 + 0.95j & 0.31 - 0.95j & -0.81 + 0.59j \\ 1 & -0.81 + 0.59j & 0.31 - 0.95j & 0.31 + 0.95j & -0.81 - 0.59j \\ 1 & 0.31 + 0.95j & -0.81 + 0.59j & -0.81 - 0.59j & 0.31 - 0.95j \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$X[0] = \frac{1}{\sqrt{5}} (1 + 1 + 1 + 1 + 1) = \frac{5}{\sqrt{5}}$$

$$X[1] = \frac{1}{\sqrt{5}} (1 + 0.31 - 0.95j - 0.81 - 0.59j - 0.81 + 0.59j + 0.31 + 0.95j) = 0$$

$$X[2] = \frac{1}{\sqrt{5}} (1 + -0.81 - 0.59j + 0.31 + 0.95j + 0.31 - 0.95j - 0.81 + 0.59j) = 0$$

$$X[3] = \frac{1}{\sqrt{5}} (1 - 0.81 + 0.59j + 0.31 - 0.95j + 0.31 + 0.95j - 0.81 - 0.59j) = 0$$

$$X[4] = \frac{1}{\sqrt{5}} (1 - 0.81 + 0.59j + 0.31 - 0.95j + 0.31 + 0.95j - 0.81 - 0.59j) = 0$$

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \end{bmatrix} = \begin{bmatrix} \frac{5}{\sqrt{5}} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(ت)

$$x = \begin{bmatrix} j \\ j \\ j \\ j \\ j \end{bmatrix}$$

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \\ X[4] \end{bmatrix} = \frac{1}{\sqrt{5}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 0.31 - 0.95j & -0.81 - 0.59j & -0.81 + 0.59j & 0.31 + 0.95j \\ 1 & -0.81 - 0.59j & 0.31 + 0.95j & 0.31 - 0.95j & -0.81 + 0.59j \\ 1 & -0.81 + 0.59j & 0.31 - 0.95j & 0.31 + 0.95j & -0.81 - 0.59j \\ 1 & 0.31 + 0.95j & -0.81 + 0.59j & -0.81 - 0.59j & 0.31 - 0.95j \end{bmatrix} \begin{bmatrix} j \\ j \\ j \\ j \\ j \end{bmatrix}$$

$$X[0] = \frac{1}{\sqrt{5}} (j + j + j + j + j) = \frac{5j}{\sqrt{5}}$$

$$X[1] = \frac{1}{\sqrt{5}} (j + 0.31j + 0.95 - 0.81j - 0.81j - 0.59 + 0.31j - 0.95 + 0.31j - 0.95) = 0$$

$$X[2] = \frac{1}{\sqrt{5}} (j - 0.81j + 0.59 + 0.31j - 0.95 + 0.31j + 0.95 - 0.81j - 0.59) = 0$$

$$X[3] = \frac{1}{\sqrt{5}} (j - 0.81j - 0.59 + 0.31j + 0.95 + 0.31j - 0.95 - 0.81j + 0.59) = 0$$

$$X[4] = \frac{1}{\sqrt{5}} (j - 0.81j - 0.59 + 0.31j + 0.95 + 0.31j - 0.95 - 0.81j + 0.59) = 0$$

$$\begin{bmatrix} X[0] \\ X[2] \\ X[3] \\ X[4] \end{bmatrix} = \begin{bmatrix} \frac{5j}{\sqrt{5}} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$