

# 10/7 lecture

- Variance and covariance
- uncorrelated and correlated RVs
- relation to statistical independence

## Variance

```
sigma_k^2 = E[(k-E(k))^2]
           = E(k^2 + E(k)^2 - 2*E(k)*k)
           = E(k^2) + E[E(k)] - 2*E[E(k)*k]
           = E(k^2) + E(k)^2 - 2*E(k)^2
var(k) = sigma_k^2 = E(k^2) - E(k)^2
```

This is called the second moment of k.

## Chebyshev’s Inequality

The probability that a random variable is away from its mean more than a certain amount c is equal to the variance / c

$$P(|K-E[k]| > c) \leq \text{sigma\_k}^2 / c$$

Takeaway: there’s an upper bound on how far you can go from the mean- depends on what you’re looking at and the variance.

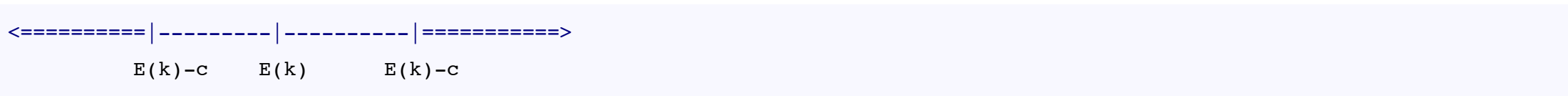
### Example: K = alpha (constant)

```
var(K) = 0

P(veer from mean by any amount c) = 0
```

## Derivation

$(k-E(k))^2$  IS  $c^2$   
Plug this into the equation for variance, take it out of the sums  
What you are left with is the summed PMF less than c and greater than c....which is the *total probability* that k is farther than c from the mean



So the variance has a direct correlation with the likelihood you’re away from the mean.

## Properties of variance

### How to find var(alpha \* K) ?

$$\begin{aligned} \text{var}(\alpha * K) &= E(k_{\text{hat}}^2) - E(k_{\text{hat}})^2 && \text{(you can take the constant out of Expected value)} \\ &= \alpha^2 [E(k^2) - E(k)^2] \end{aligned}$$

Takeaway: Variance varies with the square of K

### How to find var(alpha+ K) ?

The alpha cancels with itself when you subtract squares.

Takeaway: It doesn’t affect the variance when you shift your distribution

## What about two RVs?

```
E(K+L) = E(K) + E(L)
var(K+L) = ???

var(K+L) = E[(K+L-E(K)-E(L))^2]
         = E[(K-E(K))+(L-E(L))^2]
         magic
         = sigma_k^2 + sigma_L^2 + 2*cov(K,L)
```

cov(K,L): covariance: a measure of how the two RVs move relative to each other.

cov(K,L) > 0 is positively correlated  
cov(K,L) < 0 is negatively correlated  
cov(K,L) = 0 is uncorrelated

$$\begin{aligned} \text{cov}(K,L) &:= E[(K-E(K))(L-E(L))] \\ &= E[(KL - K * E(L) - L * E(K) + E(K) * E(L))] \\ &= E(KL) - E(K) * E(L) - E(L) * E(K) + E(K) * E(L) && \text{when you have E(E(K)) it cancels the inner, so equals E(K)} \\ &= E(KL) - E(K) * E(L) \\ &= \text{var}(K) \end{aligned}$$

When K,L are statistically independent, then K,L uncorrelated  
When K,L uncorrelated, hard to prove that also statistically independent.