Massachusetts Institute of Technology
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Fall 2011)

Recitation 16
November 3, 2011

1. Problem 6.1, page 326 in text.
   Each of n packages is loaded independently onto either a red truck (with probability p) or onto a green truck (with probability $1 - p$). Let R be the total number of items selected for the red truck and let G be the total number of items selected for the green truck.

   (a) Determine the PMF, expected value, and variance of the random variable R.

   (b) Evaluate the probability that the first item to be loaded ends up being the only one on its truck.

   (c) Evaluate the probability that at least one truck ends up with a total of exactly one package.

   (d) Evaluate the expected value and the variance of the difference $R - G$.

   (e) Assume that $n \geq 2$. Given that both of the first two packages to be loaded go onto the red truck, find the conditional expectation, variance, and PMF of the random variable R.

2. Problem 6.6, page 328 in text.

   Sum of a geometric number of independent geometric random variables

   Let $Y = X_1 + \cdots + X_N$, where the random variable X_i are geometric with parameter p, and N is geometric with parameter q. Assume that the random variables $N, X_1, X_2, \cdots$ are independent. Show that Y is geometric with parameter pq. *Hint:* Interpret the various random variables in terms of a split Bernoulli process.

3. Problem 6.4, page 327 in text.
   Consider a Bernoulli process with probability of success in each trial equal to p.

   (a) Relate the number of failures before the rth success (sometimes called a **negative binomial** random variable) to a Pascal random variable and derive its PMF.

   (b) Find the expected value and variance of the number of failures before the rth success.

   (c) Obtain an expression for the probability that the ith failure occurs before the rth success.