MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Fall 2011)

Recitation 15
November 1, 2011

1. Saif is a well intentioned though slightly indecisive fellow. Every morning he flips a coin to decide where to go. If the coin is heads he drives to the mall, if it comes up tails he volunteers at the local shelter. Saifs coin is not necessarily fair, rather it possesses a probability of heads equal to q. We do not know q, but we do know it is well-modeled by a random variable Q where the density of Q is

$$f_Q(q) = \begin{cases} 2q & \text{for } 0 \leq q \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

Assume conditioned on Q, each coin flip is independent.

(a) Whats the probability that Saif goes to the local shelter if he flips the coin once?

In an attempt to promote virtuous behavior, Saifs father offers to pay him \$4 every day he volunteers at the local shelter. Define X as Saifs payout if he flips the coin every morning for the next 30 days.

(b) Find var(X).

Let event B be that Saif goes to the local shelter at least once in k days.

(c) Find the conditional density of Q given B, $f_{Q|B}(q)$.

While shopping at the mall, Saif gets a call from his sister Mais. They agree to meet at the Coco Cabana Court yard at exactly 1:30 PM. Unfortunately Mais arrives Z minutes late, where Z is a continuous uniform random variable from zero to 10 minutes. Saif is furious that Mais has kept him waiting, and demands Mais pay him R dollars where $R = \exp(Z + 2)$.

(d) Find Saifs expected payout, $\mathbf{E}[R]$.

2. Each Mac book has a lifetime that is exponentially distributed with parameter λ. The lifetime of Mac books are independent of each other. Suppose you have two Mac books, which you begin using at the same time. Define T_1 as the time of the first laptop failure and T_2 as the time of the second laptop failure.

   (a) Compute $f_{T_1}(t_1)$.

   (b) Let $X = T_2 - T_1$. Compute $f_{X|T_1}(x \mid t_1)$.

   (c) Is X independent of T_1? Give a mathematical justification for your answer.

   (d) Compute $f_{T_2}(t_2)$ and $\mathbf{E}[T_2]$.