6.041 Probabilistic Systems Analysis
6.431 Applied Probability

- Staff:
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  - Other TAs: Aliaa Atwi, Jimmy Li, Jagdish Ramakrishnan
- Pick up **and read** course information handout
- **Turn in recitation and tutorial scheduling form** (last sheet of course information handout)
- Pick up copy of slides
- `http://stellar.mit.edu/S/course/6/fa11/6.041/`

Coursework

– Quiz 1 (October 12, 12:05-12:55pm)	17%
– Quiz 2 (November 2, 7:30-9:30pm)	30%
– Final exam (scheduled by registrar)	40%
– Weekly homework (best 9 of 10)	10%
– Attendance/participation/enthusiasm in recitations/tutorials	3%

- Pset #1, available on Stellar, due September 14
- **Collaboration policy** described in course info handout
- Text: *Introduction to Probability*, 2nd Edition, D. P. Bertsekas and J. N. Tsitsiklis, Athena Scientific, 2008
  Read the text!

LECTURE 1

- **Readings:** Sections 1.1, 1.2

Lecture outline

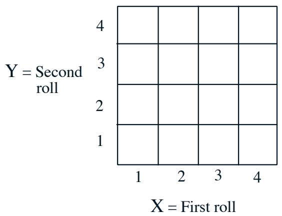
- Probability as a mathematical framework for reasoning about uncertainty
- Probabilistic models
  - sample space
  - probability law
- Axioms of probability
- Simple examples

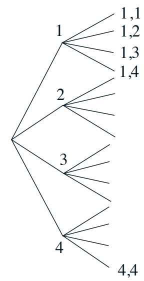
Sample space Ω

- "List" (set) of possible outcomes
- List must be:
  - Mutually exclusive
  - Collectively exhaustive
- Art: to be at the "right" granularity

Sample space: Discrete example

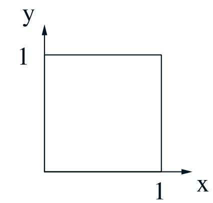
- Two rolls of a tetrahedral die
  - Sample space vs. sequential description





Sample space: Continuous example

$\Omega = \{(x, y) \mid 0 \leq x, y \leq 1\}$



Probability axioms

- **Event:** a subset of the sample space
- Probability is assigned to events

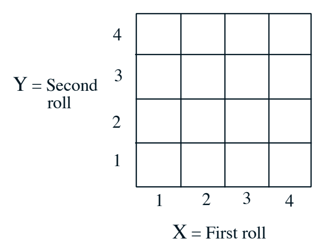
Axioms:

1. **Nonnegativity:** $\mathbf{P}(A) \geq 0$
2. **Normalization:** $\mathbf{P}(\Omega) = 1$
3. **Additivity:** If $A \cap B = \varnothing$, then $\mathbf{P}(A \cup B) = \mathbf{P}(A) + \mathbf{P}(B)$

- $$\begin{aligned} \mathbf{P}(\{s_1, s_2, \ldots, s_k\}) &= \mathbf{P}(\{s_1\}) + \cdots + \mathbf{P}(\{s_k\}) \\ &= \mathbf{P}(s_1) + \cdots + \mathbf{P}(s_k) \end{aligned}$$

- Axiom 3 needs strengthening
- Do weird sets have probabilities?

Probability law: Example with finite sample space



- Let every possible outcome have probability 1/16
  - $\mathbf{P}((X, Y)$ is (1,1) or (1,2)$) =$
  - $\mathbf{P}(\{X = 1\}) =$
  - $\mathbf{P}(X + Y$ is odd$) =$
  - $\mathbf{P}(\min(X, Y) = 2) =$

Discrete uniform law

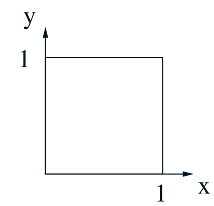
- Let all outcomes be equally likely
- Then,

$$\mathbf{P}(A) = \frac{\text{number of elements of } A}{\text{total number of sample points}}$$

- Computing probabilities $\equiv$ counting
- Defines fair coins, fair dice, well-shuffled decks

Continuous uniform law

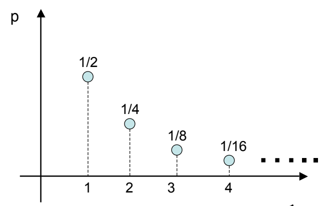
- Two "random" numbers in $[0,1]$.



- **Uniform** law: Probability = Area
  - $\mathbf{P}(X + Y \leq 1/2) = ?$
  - $\mathbf{P}(\,(X,Y) = (0.5, 0.3)\,)$

Probability law: Ex. w/countably infinite sample space

- Sample space: $\{1, 2, \ldots\}$
  - We are given $\mathbf{P}(n) = 2^{-n}$, $n = 1, 2, \ldots$
  - Find $\mathbf{P}$(outcome is even)



$$\mathbf{P}(\{2,4,6,\ldots\}) \;=\; \mathbf{P}(2) + \mathbf{P}(4) + \cdots = \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} + \cdots = \frac{1}{3}$$

- Countable additivity axiom (needed for this calculation): If $A_1, A_2, \ldots$ are disjoint events, then:

$$\mathbf{P}(A_1 \cup A_2 \cup \cdots) = \mathbf{P}(A_1) + \mathbf{P}(A_2) + \cdots$$

Remember!

- Turn in recitation/tutorial scheduling form **now**
- Check Stellar site very late tonight or early tomorrow for recitation assignments and **attend recitation tomorrow**
- Tutorials start next week