**Tutorial 10**
**December 2, 2011**

1. Random variable $X$ is uniformly distributed between $-1.0$ and 1.0. Let $X_1, X_2, \ldots$, be independent identically distributed random variables with the same distribution as $X$. Determine which, if any, of the following sequences (all with $i = 1, 2, \ldots$) are convergent in probability. Give reasons for your answers. Include the limits if they exist.

   (a) $X_i$

   (b) $Y_i = \dfrac{X_i}{i}$

   (c) $Z_i = (X_i)^i$

2. Define $X$ as the height in meters of a randomly selected Canadian, where the selection probability is equal for each Canadian, and denote $\mathbf{E}[X]$ by $h$. Bo is interested in estimating $h$. Because he is sure that no Canadian is taller than 3 meters, Bo decides to use 1.5 meters as a conservative (large) value for the standard deviation of $X$. To estimate $h$, Bo averages the heights of $n$ Canadians that he selects at random; he denotes this quantity by $H$.

   (a) In terms of $h$ and Bo's 1.5 meter bound for the standard deviation of $X$, determine the expected value and standard deviation for $H$.

   (b) Help Bo by calculating a minimum value of $n$ (with $n > 0$) such that the standard deviation of Bo's estimator, $H$, will be less than 0.01 meters.

   (c) Bo would like to be 99% sure that his estimate is within 5 centimeters of the true average height of Canadians. Using the Chebyshev inequality, calculate the minimum value of $n$ that will make Bo happy.

   (d) Redo part (c) using the Central Limit Theorem instead.

   (e) If we agree that no Canadians are taller than three meters, why is it correct to use 1.5 meters as an upper bound on the standard deviation for $X$, the height of any Canadian selected at random?

3. The lifetime of a type-A bulb is exponentially distributed with parameter $\lambda$. Type-B bulbs are identical to Type-A bulbs except for their shorter average lifetime. In particular, the lifetime of type-B bulbs is exponentially distributed with parameter $\mu$, where $\mu > \lambda > 0$. In general, a third of all bulbs are type-B. You found a box full of same type lightbulbs, and you would like to know whether they are of type A or B.

   (a) You observe that the lifetime of one bulb is $T_1 = t_1$, what is the MAP estimate of its type based on this information?

   (b) What is the probability of error of the MAP estimate?

   (c) What is the LMS estimator of $T_2$, the lifetime of another lightbulb from the same box, based on observing $T_1$? Assume that the lifetimes of bulbs are conditionally independent given their type.