Massachusetts Institute of Technology
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Fall 2011)

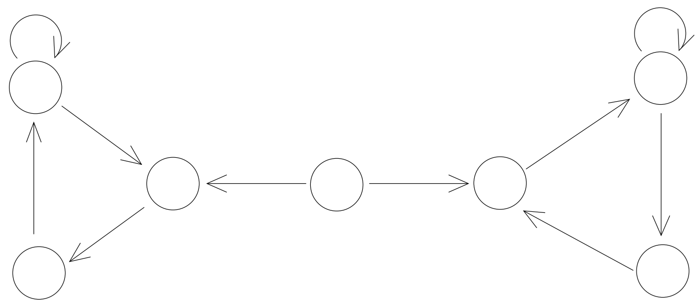
Recitation 20: November 17, 2011

1. For any finite Markov chain, we define

$$r_{ij}(n) \equiv \mathbf{P}\left(X_n = j | X_0 = i\right).$$

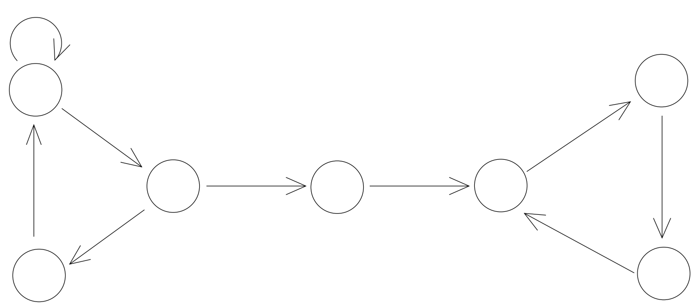
We define π_j as the steady state probability of being in state j, **provided $r_{ij}(n)$ converges as $n \to \infty$ and the value to which it converges does not depend on the starting state i.** The arrows correspond to positive single-step transition probabilities. Determine if each statement is true or false for each of the chains below.

(a)



i. $r_{ij}(n)$ converges to a limit as $n \to \infty$ for each pair of states (i, j).
ii. Statement (i) is true and $\lim_{n\to\infty} r_{ij}(n) = \pi_j$ (not varying with i) for each pair of states (i, j).
iii. Statement (ii) is true and $\pi_j > 0$ for each state j.

(b)



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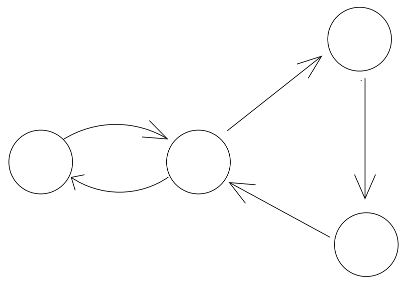
(c)



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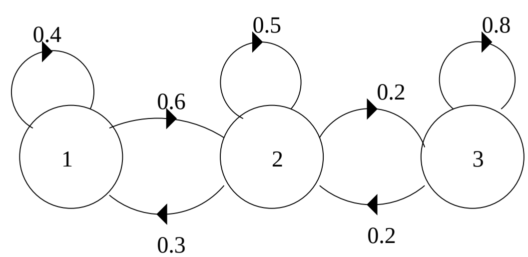
2. Problem 7.18, page 386 in textbook.
The parking garage at MIT has installed a card operated gate, which, unfortunately, is vulnerable to absent-minded faculty and staff. In particular, in each day a car crashes the gate with probability p, in which case a new gate must be installed. Also a gate that has survived for m days must be replaced as a matter of periodic maintenance. What is the steady-state expected frequency of gate replacements?

3. Problem 7.13, page 385 in textbook.
Consider the Markov chain below. Let us refer to a transition that results in a state with a higher (respectively, lower) index as a birth (respectively, death). Calculate the following quantities, assuming that when we start observing the chain, it is already in steady-state.



(a) For each state i, the probability that the current state is i.
(b) The probability that the first transition we observe is a birth.
(c) The probability that the first change of state we observe is a birth.
(d) The conditional probability that the process was in state 2 before the first transition that we observe, given that this transition was a birth.
(e) The conditional probability that the process was in state 2 before the first change of state that we observe, given that this change of state was a birth.
(f) The conditional probability that the first observed transition is a birth given that it resulted in a change of state.
(g) The conditional probability that the first observed transition leads to state 2, given that it resulted in a change of state.