Tutorial 4
October 6/7, 2011

1. Let X and Y be independent random variables. Random variable X has mean μ_X and variance σ_X^2, and random variable Y has mean μ_Y and variance σ_Y^2. Let $Z = 2X - 3Y$. Find the mean and variance of Z in terms of the means and variances of X and Y.

2. The joint PMF of the random variables X and Y is given by the following table:

$y = 3$	c	c	$2c$
$y = 2$	$2c$	0	$4c$
$y = 1$	$3c$	c	$6c$
	$x = 1$	$x = 2$	$x = 3$

   (a) Find the value of the constant c.
   (b) Find $p_Y(2)$.
   (c) Consider the random variable $Z = YX^2$. Find $\mathbf{E}[Z \mid Y = 2]$.
   (d) Conditioned on the event that $X \neq 2$, are X and Y independent? Give a one-line justification.
   (e) Find the conditional variance of Y given that $X = 2$.

3. Consider 10 independent tosses of a biased coin with a probability of heads of p.

   (a) Let A be the event that there are 6 heads in the first 8 tosses. Let B be the event that the 9th toss results in heads. Show that events A and B are independent.
   (b) Find the probability that there are 3 heads in the first 4 tosses and 2 heads in the last 3 tosses.
   (c) Given that there were 4 heads in the first 7 tosses, find the probability that the 2nd head occurred during the 4th trial.
   (d) Find the probability that there are 5 heads in the first 8 tosses and 3 heads in the last 5 tosses.