Massachusetts Institute of Technology
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Fall 2011)

Tutorial 5: Solutions

1. (a)

$$\begin{aligned} \mathbf{P}(X \leq 1.5) &= \Phi(1.5) \\ &= 0.9332. \end{aligned}$$

$$\begin{aligned} \mathbf{P}(X \leq -1) &= 1 - \mathbf{P}(X \leq 1) \\ &= 1 - \Phi(1) \\ &= 1 - 0.8413 \\ &= 0.1587. \end{aligned}$$

(b)

$$\begin{aligned} \mathbf{E}\left[\frac{Y-1}{2}\right] &= \frac{1}{2}(\mathbf{E}[Y] - 1) \\ &= 0. \end{aligned}$$

$$\begin{aligned} \text{var}\left(\frac{Y-1}{2}\right) &= \text{var}\left(\frac{Y}{2}\right) \\ &= \frac{1}{4}\text{var}Y \\ &= 1. \end{aligned}$$

Thus, the distribution of $\frac{Y-1}{2}$ is $\mathcal{N}(0,1)$.

(c)

$$\begin{aligned} \mathbf{P}(-1 \leq Y \leq 1) &= \mathbf{P}(\frac{-1-1}{2} \leq \frac{Y-1}{2} \leq \frac{1-1}{2}) \\ &= \Phi(0) - \Phi(-1) \\ &= \Phi(0) - (1 - \Phi(1)) \\ &= 0.3413. \end{aligned}$$

2. (a) We first compute the probability that X is in interval $[n, n+1]$ for an arbitrary nonnegative n.
We could integrate the PDF of X over the given interval but we will use the CDF here. Using the CDF for the exponential random variable,

$$\begin{aligned} p_Y(n) &= \mathbf{P}(n \leq X \leq n+1) \\ &= F_X(n+1) - F_X(n) \\ &= \left(1 - e^{-\lambda(n+1)}\right) - \left(1 - e^{-\lambda n}\right) \\ &= e^{-\lambda n}\left(1 - e^{-\lambda}\right). \end{aligned}$$

(b)

$$
\begin{aligned}
\mathbf{P}(Y \text{ is odd}) &= \sum_{n \text{ odd}} e^{-\lambda n}\left(1-e^{-\lambda}\right) \\
&= \left(1-e^{-\lambda}\right)\sum_{k=0}^{\infty} e^{-\lambda(2k+1)} \\
&= \left(1-e^{-\lambda}\right)e^{-\lambda}\sum_{k=0}^{\infty}\left(e^{-2\lambda}\right)^{k} \\
&= \left(1-e^{-\lambda}\right)e^{-\lambda}\frac{1}{1-e^{-2\lambda}} \\
&= \left(1-e^{-\lambda}\right)e^{-\lambda}\frac{1}{(1-e^{-\lambda})(1+e^{-\lambda})} \\
&= \frac{e^{-\lambda}}{1+e^{-\lambda}}.
\end{aligned}
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3. Problem 3.20, page 191 in text. See online solutions.