Massachusetts Institute of Technology

Department of Electrical Engineering & Computer Science

6.041/6.431: Probabilistic Systems Analysis

(Fall 2011)

Tutorial 8

November 3/4, 2011

1. Problem 6.3, page 326, of the text.

   A computer system carries out tasks submitted by two users. Time is divided into slots. A slot can be idle, with probability $p_I = 1/6$, and busy with probability $p_B = 5/6$. During a busy slot, there is probability $p_{1|B} = 2/5$ (respectively, $p_{2|B} = 3/5$) that a task from user 1 (respectively, 2) is executed. We assume that events related to different slots are independent.

   (a) Find the probability that a task from user 1 is executed for the first time during the 4th slot.

   (b) Given that exactly 5 out of the first 10 slots were idle, find the probability that the 6th idle slot is 12.

   (c) Find the expected number of slots up to and including the 5th task from user 1.

   (d) Find the expected number of busy slots up to and including the 5th task from user 1.

   (e) Find the PMF, mean, and variance of the number of tasks from user 2 until the time of the 5th task from user 1.

2. Alice and Bob alternate playing at the casino table. (Alice starts and plays at odd times $i = 1, 3, \ldots$; Bob plays at even times $i = 2, 4, \ldots$.) At each time i, the net gain of whoever is playing is a random variable G_i with the following PMF:

$$p_G(g) = \begin{cases} \frac{1}{3}, & g = -2, \\ \frac{1}{2}, & g = 1, \\ \frac{1}{6}, & g = 3, \\ 0, & \text{otherwise.} \end{cases}$$

   Assume that the net gains at different times are independent. We refer to an outcome of -2 as a "loss."

   (a) They keep gambling until the first time where a loss by Bob immediately follows a loss by Alice. Write down the PMF of the total number of rounds played. (A round consists of two plays, one by Alice and then one by Bob.)

   (b) Write down the PMF for Z, defined as the time at which Bob has his third loss.

   (c) Let N be the number of rounds until each one of them has won at least once. Find $\mathbf{E}[N]$.

3. Suppose there are n papers in a drawer. You draw a paper and sign it, and then, instead of filing it away, you place the paper back into the drawer. If any paper is equally likely to be drawn each time, independent of all other draws, what is the expected number of papers that you will draw before signing all n papers? You may leave your answer in the form of a summation.