MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Fall 2011)

Recitation 18
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1. Let $N, X_1, X_2, \ldots$ be independent random variables, where N takes non-negative integer values, and $X_1, X_2, \ldots$ are Bernoulli with parameter p. Let $Y = X_1 + \cdots + X_N$ for positive values of N and let $Y = 0$ when $N = 0$.

   (a) Show that if N is binomial with parameters m and q, then Y is binomial with parameters m and pq.

   (b) Show that if N is Poisson with parameter λ, then Y is Poisson with parameter λp.

2. Let $Y = X_1 + \cdots + X_N$, where the random variables X_i are exponential with parameter λ, and N is geometric with parameter p. Assume that the random variables $N, X_1, X_2, \ldots$ are independent. Show that Y is exponential with parameter λp. *Hint:* Interpret the various random variables in terms of a split Poisson process.

3. We are given the following statistics about the number of children in a typical family in a small village.

   There are 100 families. 10 have no children; 40 have 1; 30 have 2; 10 have 3; 10 have 4.

   (a) If you pick a family at random, what is the expected number of children in that family?

   (b) If you pick a child at random (each child is equally likely), what is the expected number of children in that child's family (including the picked child)?

   (c) Generalize your approach from part (b) to the case where a fraction p_k of the families has k children, and provide a formula.

4. Problem 6.19, page 332 in the textbook.

   (a) Let X_1 and X_2 be independent and exponentially distributed, with parameters λ_1 and λ_2, respectively. Find the expected value of $\max\{X_1, X_2\}$.

   (b) Let Y be exponentially distributed with parameter λ_1. Let Z be Erlang of order 2 with parameter λ_2. Assume that Y and Z are independent. Find the expected value of $\max\{Y, Z\}$.