Massachusetts Institute of Technology
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Fall 2011)

Recitation 10
October 13, 2011

1. Let Z be a continuous random variable with probability density function

$$f_z(z) = \begin{cases} \gamma(1+z^2), & \text{if } -2 < z < 1, \\ 0, & \text{otherwise.} \end{cases}$$

   (a) For what value of γ is this possible?

   (b) Find the cumulative distribution function of Z.

2. Problem 3.9, pages 186–187 in the text.

   The taxi stand and the bus stop near Al's home are in the same location. Al goes there at a given time and if a taxi is waiting, (this happens with probability 2/3) he boards it. Otherwise he waits for a taxi or a bus to come, whichever comes first. The next taxi will arrive in a time that is uniformly distributed between 0 and 10 minutes, while the next bus will arrive in exactly 5 minutes. Find the CDF and the expected value of Al's waiting time.

3. Let λ be a positive number. The continuous random variable X is called **exponential** with parameter λ when its probability density function is

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & \text{if } x \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

   (a) Find the cumulative distribution function (CDF) of X.

   (b) Find the mean of X.

   (c) Find the variance of X.

   The text on pp. 165-167 introduces the idea of conditioning a continuous random variable on an event, which is nothing different than what we've done previously for discrete random variables. Similarly the total probability theorem applies in an analogous form.

4. Consider a continuous random variable X that is generated in the following manner. We first flip a coin which has a probability of heads of $\frac{3}{4}$. If the flip results in heads, X is drawn from a uniform distribution over the interval $[0, 2]$. If the flip is tails, X is then drawn from a uniform distribution over the interval $[1, 3]$. Provide a labeled sketch of the pdf for X.