Massachusetts Institute of Technology
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Spring 2011)

Problem Set 2
Due: February 16, 2011

1. Consider two events with positive probability, A and B. Show that:

$$\mathbf{P}(A \mid B) > \mathbf{P}(A) \qquad \text{implies} \qquad \mathbf{P}(B \mid A) > \mathbf{P}(B).$$

2. **The Chess Problem.** This year's Belmont chess champion is to be selected by the following procedure. Bo and Ci, the leading challengers, first play a two-game match. If one of them wins both games, he gets to play a two-game *second round* with Al, the current champion. Al retains his championship unless a second round is required and the challenger beats Al in both games. If Al wins the initial game of the second round, no more games are played.

   Furthermore, we know the following:

   - The probability that Bo will beat Ci in any particular game is 0.6.
   - The probability that Al will beat Bo in any particular game is 0.5.
   - The probability that Al will beat Ci in any particular game is 0.7.

   Assume no tie games are possible and all games are independent.

   (a) Determine the apriori probabilities that
      i. the second round will be required.
      ii. Bo will win the first round.
      iii. Al will retain his championship this year.

   (b) Given that the second round is required, determine the conditional probabilities that
      i. Bo is the surviving challenger.
      ii. Al retains his championship.

   (c) Given that the second round was required and that it comprised only one game, what is the conditional probability that it was Bo who won the first round?

3. A hunter has two hunting dogs. One day, on the trail of some animal, the hunter comes to a place where the road diverges into two paths. He knows that each dog, independently of the other, will choose the correct path with probability p. The hunter decides to let each dog choose a path, and if they agree, take that one, and if they disagree, to randomly pick a path (i.e., to toss a fair coin and pick the path to the left if the coin comes up heads and the path to the right otherwise). Is his strategy better than just letting one of the two dogs decide on a path?

4. A magnetic tape storing information in binary form has been corrupted. Imagine that you are to make an effort to save as much information as possible. Due to the damage on the tape, you know that there will be errors in the reading. You know that if there was a 0, the probability that you correctly detect it is .9. The probability that you correctly detect a 1 is .85. Given that the each digit is a 1 or a 0 with equal probability, and given that you read in a 1, what is the probability that this is a correct reading?

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5. Anne, Betty, Chloe and Daisy were all friends in school. Subsequently each of the six subpairs meet up once; at each of the six meetings, the pair quarrels with some fixed probability p and otherwise the pair retains a firm friendship. Quarrels take place independently of each other.

   In the future, if any one of the four hears a rumour, she tells it to her firm friends only. Supposing that Anne hears a rumour, what is the probability that:

   (a) Daisy hears it?

   (b) Daisy hears it if Anne and Betty have quarrelled?

   (c) Daisy hears it if Betty and Chloe have quarrelled?

   (d) Daisy hears it if she has quarrelled with Anne?

G1†. A man is saving up to buy a new Jaguar at a cost of N units of money. He starts with a certain amount of money, and tries to win the remainder by the following gamble with his bank manager. He tosses a fair coin repeatedly; if it comes up heads then the manager pays him one unit, but if it comes up tails then he pays the manager one unit. He plays this game repeatedly until one of the two events occurs: either he runs out of money and is bankrupted, or he wins enough to buy the Jaguar. Let p_k denote the probability that the man is eventually bankrupted, where k is the number of units he starts with, and $0 \leq k \leq N$.

   (a) Find a difference equation that expresses p_k in terms of p_{k-1} and/or p_{k+1}.

   (b) Find the boundary conditions for the above difference equation, namely p_0 and p_N.

   (c) Solve the equation for these boundary conditions. (There is only one solution and it is simple.) Hence compute the probability that the man is ultimately bankrupted, as a function of k.

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