Massachusetts Institute of Technology
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Fall 2011)

Recitation 5
September 22, 2011

1. Derive the multinomial coefficient, which represents the number of different ways we can divide n distinguishable objects into $r \geq 2$ distinct (unordered) subsets $S_1, S_2, \ldots, S_r$, of sizes $n_1, n_2, \ldots, n_r$, where $n_1 + n_2 + \cdots + n_r = n$. The expression

$$\binom{n}{n_1, n_2, \ldots, n_r} = \frac{n!}{n_1! n_2! \cdots n_r!}$$

gives the number of different ways this can be done.

2. A die with r faces is rolled a fixed number of times n, with each roll independent of all other rolls. The kth face has probability p_k of being rolled. Find the probability that the kth face comes up y_k times, where $y_1 + y_2 + \cdots + y_r = n$.

3. The hats of n persons are thrown into a box. The persons then pick up their hats at random (i.e., so that every assignment of the hats to the persons is equally likely). What is the probability that

   (a) every person gets his or her hat back?

   (b) the first m persons who picked hats get their own hats back?

   (c) everyone among the first m persons to pick up the hats gets back a hat belonging to one of the last m persons to pick up the hats?

   Now assume, in addition, that every hat thrown into the box has probability p of getting dirty (independently of what happens to the other hats or who has dropped or picked it up). What is the probability that

   (d) the first m persons will pick up clean hats?

   (e) exactly m persons will pick up clean hats?

4. A candy factory has an endless supply of red, orange, yellow, green, blue, and violet jelly beans. The factory packages the jelly beans into jars of 100 jelly beans each. One possible color distribution, for example, is a jar of 56 red, 22 yellow, and 22 green jelly beans. As a marketing gimmick, the factory guarantees that no two jars have the same color distribution. What is the maximum number of jars the factory can produce?