LECTURE 13

The Bernoulli process

- **Readings:** Section 6.1

Lecture outline

- Definition of Bernoulli process
- Random processes
- Basic properties of Bernoulli process
- Distribution of interarrival times
- The time of the kth success
- Merging and splitting

The Bernoulli process

- A sequence of independent Bernoulli trials
- At each trial, i:
  - $\mathbf{P}(\text{success}) = \mathbf{P}(X_i = 1) = p$
  - $\mathbf{P}(\text{failure}) = \mathbf{P}(X_i = 0) = 1 - p$
- Examples:
  - Sequence of lottery wins/losses
  - Sequence of ups and downs of the Dow Jones
  - Arrivals (each second) to a bank
  - Arrivals (at each time slot) to server

Random processes

- First view:
  sequence of random variables $X_1, X_2, \ldots$
- $\mathbf{E}[X_t] =$
- $\text{Var}(X_t) =$
- Second view:
  what is the right sample space?
- $\mathbf{P}(X_t = 1 \text{ for all } t) =$
- Random processes we will study:
  - Bernoulli process
    (memoryless, discrete time)
  - Poisson process
    (memoryless, continuous time)
  - Markov chains
    (with memory/dependence across time)

Number of successes S in n time slots

- $\mathbf{P}(S = k) =$
- $\mathbf{E}[S] =$
- $\text{Var}(S) =$

Interarrival times

- T_1: number of trials until first success
  - $\mathbf{P}(T_1 = t) =$
  - Memoryless property
  - $\mathbf{E}[T_1] =$
  - Var$(T_1) =$

- If you buy a lottery ticket every day, what is the distribution of the length of the first string of losing days?

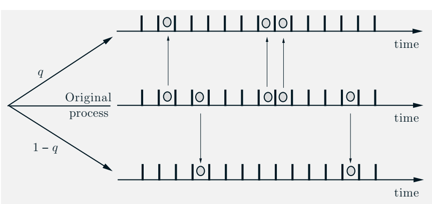
Time of the kth arrival

- Given that first arrival was at time t i.e., $T_1 = t$: additional time, T_2, until next arrival
  - has the same (geometric) distribution
  - independent of T_1

- Y_k: number of trials to kth success
  - $\mathbf{E}[Y_k] =$
  - Var$(Y_k) =$
  - $\mathbf{P}(Y_k = t) =$

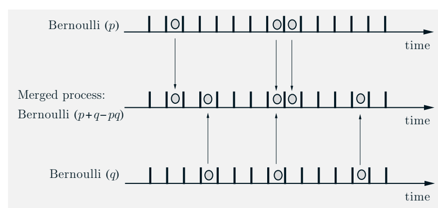
Splitting of a Bernoulli Process

(using independent coin flips)



yields Bernoulli processes

Merging of Indep. Bernoulli Processes



yields a Bernoulli process
(collisions are counted as one arrival)