

MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
Department of Electrical Engineering & Computer Science  
**6.041/6.431: Probabilistic Systems Analysis**  
(Fall 2011)

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**Recitation 9**  
**October 6, 2011**

1. Assume the following joint PMF for the discrete random variables  $X$  and  $Y$ .

$y = 3$	1/6	1/9	0	1/9
$y = 2$	0	0	2/9	0
$y = 1$	1/6	0	1/9	1/9
	$x = 1$	$x = 2$	$x = 3$	$x = 4$

$p_{X,Y}(x, y)$ , the joint PMF of  $X$  and  $Y$ .

- a. Find the marginal PMF of  $X$ ,  $p_X(x)$ .

Assume the following events:

$$A : Y < 3$$

$$B : X = 1$$

$$C : X = 4$$

- b. Define the event  $D$  as  $D = ((A \cap B^c) \cup C)^c$ . Find  $\mathbf{P}(D)$ .
- c. Find  $\mathbf{E}[Y | A]$ .
- d. Define the event  $E$  as  $E = (B \cup C)$ . Conditioned on  $E$ , are  $X$  and  $Y$  independent? Give a mathematical justification for your answer.
2. As a member of the elite 6.041/6.431 student body you are entitled to shop at the local probabilistic grocery store. This is considered a great privilege. Not unexpectedly choosing fruit is far from deterministic. To obtain a piece of fruit you approach the fruit manager and ask him for a piece of fruit. He will give you an apple with probability  $2/3$  or an orange with probability  $1/3$ . You always receive a piece of fruit after each request, and each request is independent of all other requests.
- a. What is the probability you receive exactly 3 apples in your first 10 requests?
- b. Let  $X$  be the number of requests you make until you receive your first orange, i.e. you receive your first orange on request number  $X$ . What is  $\mathbf{E}[X]$ ?
- c. Given you received 3 apples and 7 oranges in 10 requests, what is the probability that you received exactly 2 apples within the first 5 requests? *For full credit your final answer should contain no summations.*

You proceed to the check out to pay for your fruit. Prices are as follows: each apple is \$3, each orange is \$15. Let  $T$  be your total bill in dollars, assuming you have 10 pieces of fruit.

- d. Find  $\mathbf{E}[T]$
- e. Find  $\text{var}(T)$

You leave the grocery store and get on a bus with your friend Mais. The bus has  $k$  empty seats in a single row, where  $k$  is an integer greater than 2. You and Mais each choose a seat at random. Assume only one person can sit in a seat.

- f. What is the probability you and Mais choose adjacent seats, i.e. you sit next to each other?