

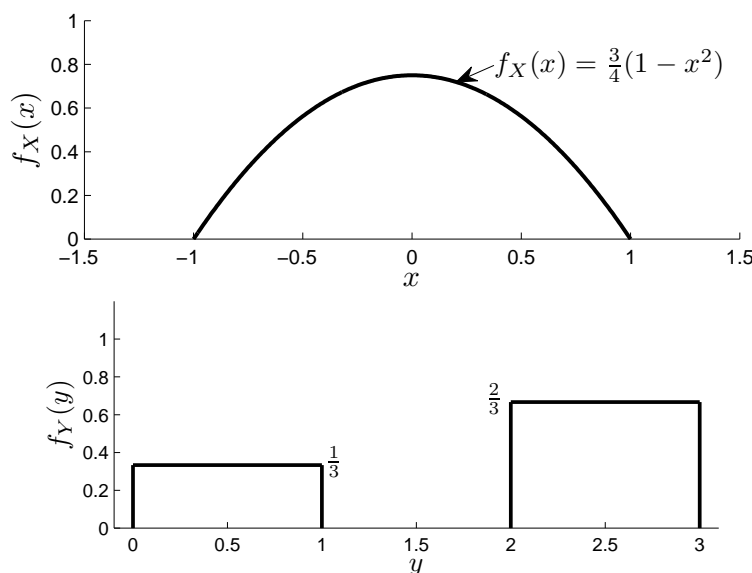
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Fall 2010)

Problem Set 6
Due October 27, 2010

1. Random variables X and Y are distributed according to the joint PDF

$$f_{X,Y}(x,y) = \begin{cases} ax, & \text{if } 1 \leq x \leq 2 \text{ and } 0 \leq y \leq x, \\ 0, & \text{otherwise.} \end{cases}$$

- (a) Evaluate the constant a .
 - (b) Determine the marginal PDF $f_Y(y)$.
 - (c) Determine the conditional expectation of $1/X$ given that $Y = 3/2$.
 - (d) Random variable Z is defined by $Z = Y - X$. Determine the PDF $f_Z(z)$.
2. Let X and Y be two independent random variables. Their probability densities functions are shown below.

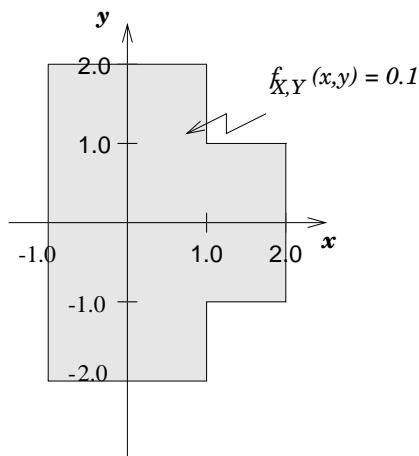


Let $Z = X + Y$. Determine $f_Z(z)$.

3. Consider n independent tosses of a k -sided fair die. Let X_i be the number of tosses that result in i .
- (a) Are X_1 and X_2 uncorrelated, positively correlated, or negatively correlated? Give a one-line justification.
 - (b) Compute the covariance $\text{cov}(X_1, X_2)$ of X_1 and X_2 .

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4. Random variables X and Y have the joint PDF shown below:



- (a) Find the conditional PDFs $f_{Y|X}(y | x)$ and $f_{X|Y}(x | y)$, for various values of x and y , respectively.
 - (b) Find $\mathbf{E}[X | Y = y]$, $\mathbf{E}[X]$, and $\text{var}(X | Y = y)$. Use these to calculate $\text{var}(X)$.
 - (c) Find $\mathbf{E}[Y | X = x]$, $\mathbf{E}[Y]$, and $\text{var}(Y | X = x)$. Use these to calculate $\text{var}(Y)$.
5. The wombat club has N members, where N is a random variable with PMF

$$p_N(n) = p^{n-1}(1-p) \quad \text{for } n = 1, 2, 3, \dots$$

On the second Tuesday night of every month, the club holds a meeting. Each wombat member attends the meeting with probability q , independently of all the other members. If a wombat attends the meeting, then it brings an amount of money, M , which is a continuous random variable with PDF

$$f_M(m) = \lambda e^{-\lambda m} \quad \text{for } m \geq 0.$$

N , M , and whether each wombat member attends are all independent. Determine:

- (a) The expectation and variance of the number of wombats showing up to the meeting.
 - (b) The expectation and variance for the total amount of money brought to the meeting.
- G1[†]. (a) Let $X_1, X_2, \dots, X_n, X_{n+1}, \dots, X_{2n}$ be independent and identically distributed random variables.
 Find

$$\mathbf{E}[X_1 | X_1 + X_2 + \dots + X_n = x_0],$$

where x_0 is a constant.

- (b) Define

$$S_k = X_1 + X_2 + \dots + X_k, 1 \leq k \leq 2n.$$

Find

$$\mathbf{E}[X_1 | S_n = s_n, S_{n+1} = s_{n+1}, \dots, S_{2n} = s_{2n}],$$

where $s_n, s_{n+1}, \dots, s_{2n}$ are constants.