9/23 lecture

 probability mass functions • examples

random variables

- expected value
- **Random Variables**

$x_1 = X(omega_1)$

As always, functions can map to the same value, but a single input can't map to more than 1 output

Could be...

Discrete Random Variable, usually use K,L,M for variable

A random variable is a function that maps every point in Omega to a value

Continuous Random Variable, usually use X,Y,Z for variable

Omega is countable

Omega is uncountable

Capital letter variable is the Random Variable (RV) Lowercase letter is the particular value of RV K

Probability Mass Functions

 $P_K(k) = Pr(K=k)$ $SUM(P_K(k) = 1)$ $0 < P_K(k) < 1$

Example

 $P_K(k) = {$?1 | k = a, ..., b?2 | e/w (ELSEWHERE) }

Example PMF of the Bernoulli RV

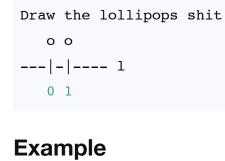
?2 = 0

?1 = 1/(b-a+1)

P -> `1` output

1-P -> `0` output **if** P = 3/4: $P_L(1) = {$

P | 1=1 1-P | 1=0 0 | e/w



M: # of coin flips up to and including 1st H? P -> H 1-P -> T

Draw the fucking tree P -> H_1 1-P -> T_1 -> P -> H2

Get each expression

 $(1-P)^(m-1) * P | m >= 1$ | e/w

-> 1-P -> T2 -> etc

Decays exponentially, called the **Geometric PMF**

 $P_M(1) = P$ $P_M(2) = (1-P) * P$ $P_M(3) = (1-P)^2 * P$ $P_M(m)$ {

Example L: # of Heads in n tosses of a coin

n = 4

 $P_L(1) = P * (1-P)^3 * 4$ $P_L(1) = P^1 * (1-P)^(n-1) * nC1$ This one is called the Binomial PMF

Example

K1: Outcome of 1st roll

2 independent rolls of a fair 4 sided die.

P_L(1)

K2: Outcome of 2nd roll K = K1 + K2

Figure out P_K(k)

 $P_K(2) = 1 * 1/4 * 1/4$ $P_K(3) = 2 * 1/4 * 1/4$ $P_K(4) = 3 * 1/4 * 1/4$ $P_K(5) = 4 * 1/4 * 1/4$

$E(K) = SUM_k(k * PK(k))$ It is a weighted sum. Analogous to the **center of mass**.

you're creating a weighted sum of all values based on how frequently they occur.

Remember, PMF values add up to 1. You're multiplying the value times its frequency, all of which add up to 1, so by adding them all up

Expected Value (mean)

 $P_K(6) = 3 * 1/4 * 1/4$ $P_K(7) = 2 * 1/4 * 1/4$ $P_K(8) = 1 * 1/4 * 1/4$

E of Bernoulli RV $P_L(1) = \{$ P | 1=1

(1-P) * P | m = 1, 2, ...

| e/w

Example

 $P_M(m) = {$

Example

1-P | 1=0

| e/w

$E(M) = SUM_m_1_inf((1-p)^(m-1) * P)$ $SUM_m_1_inf(P_M(m)) = 1$ What?

}

eol