Massachusetts Institute of Technology
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Fall 2011)

Recitation 12
October 20, 2011

1. Let Q be a continuous random variable with PDF

$$f_Q(q) \;=\; \begin{cases} 6q(1-q), & \text{if } 0 \le q \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

   This Q represents the probability of success of a Bernoulli random variable X, i.e.,

$$\mathbf{P}(X = 1 \mid Q = q) \;=\; q.$$

   Find $f_{Q|X}(q|x)$ for $x \in \{0, 1\}$ and all q.

2. (Example 3.20 on page 180 of the text). A binary signal S is transmitted, and we are given that $\mathbf{P}(S = 1) = p$ and $\mathbf{P}(S = -1) = 1 - p$. The received signal is $Y = N + S$, where N is normal noise, with zero mean and unit variance, independent of S. What is the probability that $S = 1$, as a function of the observed value y of Y?

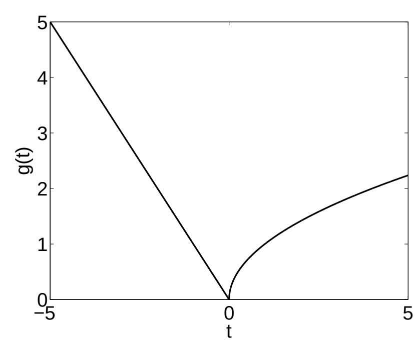
3. Let X have the normal distribution with mean 0 and variance 1, i.e.,

$$f_X(x) \;=\; \frac{1}{\sqrt{2\pi}} e^{-x^2/2}.$$

   Also, let $Y = g(X)$ where

$$g(t) \;=\; \begin{cases} -t, & \text{for } t \le 0; \\ \sqrt{t}, & \text{for } t > 0, \end{cases}$$

   as shown to the right.



   Find the probability density function of Y.

4. Let X and Y be independent random variables that are uniformly distributed on the interval [0, 1]. What is the PDF of the random variable $Z = X/(1 + Y)$?
