Massachusetts Institute of Technology

Department of Electrical Engineering & Computer Science

6.041/6.431: Probabilistic Systems Analysis

(Fall 2011)

Tutorial 9

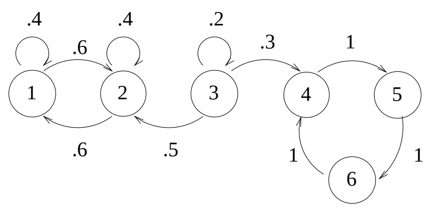
November 18, 2011

1. Problem 7.1, page 380 in textbook.
   The times between succesive customer arrivals at a facility are independent and identically distributed random variables with the following PMF:

$$p(k) = \begin{cases} 0.2, & k = 1 \\ 0.3, & k = 3 \\ 0.5, & k = 4 \\ 0, & \text{otherwise.} \end{cases}$$

   Construct a four-state Markov chain model that describes the arrival process. In this model, one of the states should correspond to the times when an arrival occurs.

2. The Markov chain shown below is in state 3 immediately before the first trial.



   (a) Indicate which states, if any, are recurrent, transient, and periodic.
   (b) Find the probability that the process is in state 3 after n trials.
   (c) Find the expected number of trials up to and including the trial on which the process leaves state 3.
   (d) Find the probability that the process never enters state 1.
   (e) Find the probability that the process is in state 4 after 10 trials.
   (f) Given that the process is in state 4 after 10 trials, find the probability that the process was in state 4 after the first trial.

3. The MIT football team's performance in any given game is very much correlated to its morale. In fact, if the team has won the past two games, then it has a 0.7 probability of winning the next game. If it lost the last game but won before that, it has a 0.4 probability of winning. If it won its last game but lost before that it has a 0.5 probability of winning, and finally if it lost the last two games it has only a 0.2 probability of winning the next game. Assume that the above details the complete correlation between the history of victories and defeats, and the future performance.

   (a) Define with a Markov chain that models the above process. Remember that the process must have the Markov property.
   (b) Find the long run probability that the MIT football team will win its next game.