Massachusetts Institute of Technology

Department of Electrical Engineering & Computer Science

6.041/6.431: Probabilistic Systems Analysis

(Fall 2011)

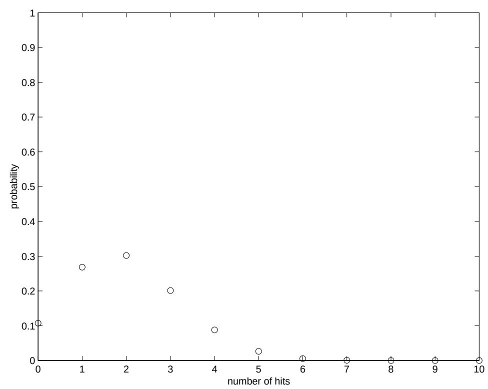
Recitation 6 Solutions
September 27, 2011

1. (a) See derivation in textbook pp. 84-85.
   (b) See derivation in textbook p. 86.
   (c) See derivation in textbook p. 87.

2. (a) X is a Binomial random variable with $n = 10$, $p = 0.2$. Therefore,

$$p_X(k) = \binom{10}{k} 0.2^k \, 0.8^{10-k}, \qquad \text{for } k = 0, \ldots, 10$$

and $p_X(k) = 0$ otherwise.



   (b) $\mathbf{P}(\text{No hits}) = p_X(0) = (0.8)^{10} = \boxed{0.1074}$
   (c) $\mathbf{P}(\text{More hists than misses}) = \sum_{k=6}^{10} p_X(k) = \sum_{k=6}^{10} \binom{10}{k} 0.2^k \, 0.8^{10-k} = \boxed{0.0064}$
   (d) Since X is a Binomial random variable,

$$\mathbf{E}[X] = 10 \cdot 0.2 = \boxed{2} \qquad \text{var}(X) = 10 \cdot 0.2 \cdot 0.8 = \boxed{1.6}$$

   (e) $Y = 2X - 3$, and therefore

$$\mathbf{E}[Y] = 2\mathbf{E}[X] - 3 = \boxed{1} \qquad \text{var}(Y) = 4\text{var}(X) = \boxed{6.4}$$

   (f) $Z = X^2$, and therefore

$$\mathbf{E}[Z] = \mathbf{E}[X^2] = (\mathbf{E}[X])^2 + \text{var}(X) = \boxed{5.6}$$

3. (a) We expect $\mathbf{E}[X]$ to be higher than $\mathbf{E}[Y]$ since if we choose the student, we are more likely to pick a bus with more students.
   (b) To solve this problem formally, we first compute the PMF of each random variable and then compute their expectations.

$$p_X(x) = \begin{cases} 40/148 & x = 40 \\ 33/148 & x = 33 \\ 25/148 & x = 25 \\ 50/148 & x = 50 \\ 0 & \text{otherwise.} \end{cases}$$

and $\mathbf{E}[X] = 40\,\frac{40}{148} + 33\,\frac{33}{148} + 25\,\frac{25}{148} + 50\,\frac{50}{148} = 39.28$

$$p_Y(y) = \begin{cases} 1/4 & y = 40, 33, 25, 50 \\ 0 & \text{otherwise.} \end{cases}$$

and $\mathbf{E}[Y] = 40\,\frac{1}{4} + 33\,\frac{1}{4} + 25\,\frac{1}{4} + 50\,\frac{1}{4} = 37$
Clearly, $\mathbf{E}[X] > \mathbf{E}[Y]$.