

MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
Department of Electrical Engineering & Computer Science  
**6.041/6.431: Probabilistic Systems Analysis**  
(Fall 2011)

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**Tutorial 3**  
**September 29/30, 2011**

1. Consider a random variable  $X$  such that

$$p_X(x) = \frac{x^2}{a} \text{ for } x \in \{-3, -2, -1, 1, 2, 3\}, \quad \mathbf{P}(X = x) = 0 \text{ for } x \notin \{-3, -2, -1, 1, 2, 3\},$$

where  $a > 0$  is a real parameter.

- (a) Find  $a$ .
- (b) What is the PMF of the random variable  $Z = X^2$ ?

2. **Problem 2.40, page 133 in the textbook**

A particular professor is known for his arbitrary grading policies. Each paper receives a grade from the set  $\{A, A-, B+, B-, B+\}$ , with equal probability, independent of other papers. How many papers do you expect to hand in before you receive each possible grade at least once.

3. The newest invention of the 6.041/6.431 staff is a three-sided die with faces numbered 1, 2, and 3. The PMF for the result of any one roll of this die is

$$p_X(x) = \begin{cases} 1/2, & \text{if } x = 1, \\ 1/4, & \text{if } x = 2, \\ 1/4, & \text{if } x = 3, \\ 0, & \text{otherwise.} \end{cases}$$

Consider a sequence of six independent rolls of this die, and let  $X_i$  be the random variable corresponding to the  $i$ th roll.

- (a) What is the probability that exactly three of the rolls have result equal to 3?
- (b) What is the probability that the first roll is 1, given that exactly two of the six rolls have result of 1?
- (c) We are told that exactly three of the rolls resulted in 1 and exactly three resulted in 2. Given this information, what is the probability that the sequence of rolls is 121212?
- (d) Conditioned on the event that at least one roll resulted in 3, find the conditional PMF of the number of 3's.