

Lecture 10/2

Day before the test oh fuck

- Joint PMFs
- Mean of functions of multiple Random Variables
- Variance for functions of multiple RVs

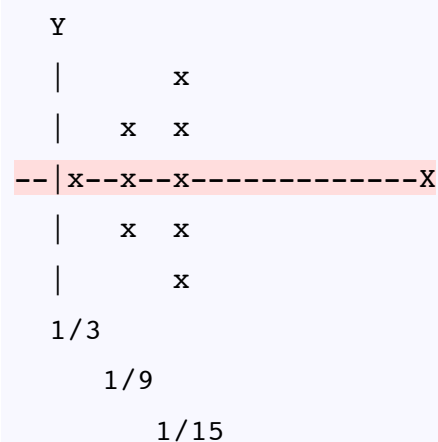
Joint PMF

For problems with two independent variables, like taking the sample of the class’s weights and heights,

$$P_{x,y}(x,y) = \Pr(X=x \wedge Y=y)$$

Example:

Plot some shit



$$0 \leq P_{x,y}(x,y) \leq 1$$

$$\sum_x (\sum_y P_{x,y}(x,y)) \text{ should equal } 1$$

So...what is:

$$P_x(1) = ?$$

If you sum over all the Y’s, then you’ll have the total value of X

So

$$P_x(1) = P_{x,y}(1,-1) + P_{x,y}(1,0) + P_{x,y}(1,1) = \sum_y (P_{x,y}(1,y))$$

So..

$$P_x(x) = \sum_y (P_{x,y}(x,y)) \text{ is the Marginal PMF for } x$$

$$P_y(y) = \sum_x (P_{x,y}(x,y)) \text{ is the Marginal PMF for } y$$

So what is this?

$$P_{X|Y}(x|y) = \Pr(X=x \mid Y=y)$$

You can break this down with BAYEeEeS + intersection equivalence shit

$$P(A|B) = \Pr(A \wedge B) / \Pr(B)$$

So

$$P_{X|Y}(x|y) = \Pr(X=x \mid Y=y) \quad \lll \text{remember } \Pr(A \text{ and } B) = \Pr(A) * \Pr(A \mid B)$$

$$= \Pr(X=x \wedge Y=y) / \Pr(Y=y) \quad \lll \text{and because } P_{x,y}(x,y) = \Pr(X=x \wedge Y=y),$$

$$= P_{x,y}(x,y) / P_y(y)$$

$$\text{hinges on the fact that } P_y(y) > 0$$

So we can find

$$P_x(x) = ?$$

$$= \sum_y (P_{x,y}(x,y))$$

What you’re doing here is “flattening” against an axis. This case you’re flattening to the X axis, so at each possible point you sum the probabilities. It ends up being (1/3, 1/3, 1/3)

$$P_y(y) = ?$$

$$= \sum_x (P_{x,y}(x,y))$$

$$= 1$$

Flattening against Y axis, so you get (3/45, 8/45, 23/45, 8/45, 3/45).

$$P_{x|y}(x|y=0) = ?$$

$$\text{Remember that } P_{X|Y}(x|y) = P_{x,y}(x,y) / P_y(y)$$

$$\text{So you divide the intersection by } P_y(0)$$

$$\text{AKA normalize the points in your conditioned universe (ie } y=0) \text{ so that the } P_{x|y} \text{ sum} = 1$$

Independence

$$P_{x|y}(x|y) = P_x(x)$$

As before, when the conditional probability is equal to the individual probability, then the events are independent

So

$$P_{x,y}(x,y) / P_y(y)$$

$$= P_x(x) \Rightarrow P_{x,y}$$

To test for independence do a quick example

$$P_{x,y}(0,2) = P_x(0) * P_y(2) ?$$

If so then independent

If not then dependent

Expected Value of functions of multiple random variables

$$E(X+Y) = \sum_x (\sum_y ((x+y) * P_{x,y}(x,y))) = \sum_x (\sum_y (x * P_{x,y}(x,y) + y * P_{x,y}(x,y))) = \sum_x (\sum_y (P_{x,y}(x,y) * x) + \sum_y (P_{x,y}(x,y) * y)) = \sum_x (x * \sum_y (P_{x,y}(x,y))) + \sum_y (y * \sum_x (P_{x,y}(x,y))) = \sum_x (x * P_x(x)) + \sum_y (y * P_y(y))$$

$$E(X+Y) = E(X) + E(Y)$$

example

Mean of the binomial PMF L: # of successes in n independent trials
$$E(L) = \sum_l (l * \text{combination}(n, l) * P^l * (1-p)^{(n-l)}) < \text{wtf?}$$

$$K_i \{ 1 \text{ if success } 0 \text{ if failure } \}$$

$$P \rightarrow 1$$

$$1-P \rightarrow 0$$

$$E(K_i) = P \cdot L = K_1 + K_2 + \dots + K_n \Rightarrow E(L) = np$$

What the fuck just happened