6.041 Probabilistic Systems Analysis
6.431 Applied Probability

- Staff
  - Lecturer: Vivek Goyal, vgoyal@mit.edu
  - Recitation instructors: Shay Maymon, Alexandre Megretski (6.431), Paul Njoroge, Alan Willsky
  - Head TA: Shashank Dwivedi, head.ta@mit.edu
  - Other TAs: Pavitra Krishnaswamy, Uzoma Orji, Joongwoo Brian Park
- Pick up **and read** course information handout
- **Turn in recitation and tutorial scheduling form** (last sheet of course information handout)
- Pick up copy of slides
- http://stellar.mit.edu/S/course/6/sp10/6.041/

LECTURE 1

- Text: *Introduction to Probability*, Second Edition, D. P. Bertsekas and J. N. Tsitsiklis, Athena Scientific, 2008
- **Readings:** Sections 1.1, 1.2

Lecture outline

- General course information
- Randomness and probability as a mathematical field
- Probabilistic models
- Axioms of probability

Coursework

– Quiz 1 (March 8, 12:00p-1:00p)	20%
– Quiz 2 (April 7, **7:30p-9:30p**)	28%
– Final exam (scheduled by registrar)	38%
– Weekly homework (best 9 of 10)	9%
– Attendance/participation/enthusiasm in recitations/tutorials	5%

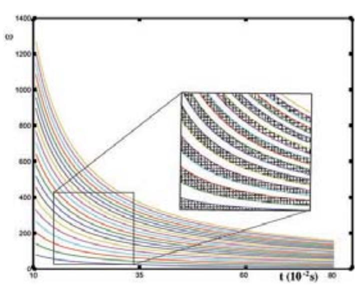
- Pset #1, available on Stellar, due February 10
- **Collaboration policy** described in course info handout

Overview and history

- Goal: be able to reason (quantitatively) about uncertainty
- Applications: all (!) decision making and design (engineering, finance, policy, play calling, . . .)
- Problem of points
  - Luca Pacioli (1446(?)–1517)
  - Niccolò Fontana Tartaglia (1499(?)–1557):

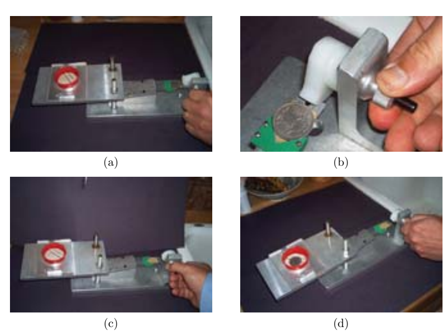
    "The resolution of the question is judicial rather than mathematical, so that in whatever way the division is made there will be cause for litigation."
  - Blaise Pascal (1623–1662) and Pierre de Fermat (1601(?)–1665)
- Jacob Bernoulli (1654–1705), Nicholas Bernoulli (1687–1759), Abraham de Moivre (1667–1754), and Pierre-Simon Laplace (1749–1827)
- Andrey Nikolaevich Kolmogorov (1903–1987)

Coins



Diaconis *et al.* (2007)

Coins



Diaconis *et al.* (2007)

Elements of a probabilistic model

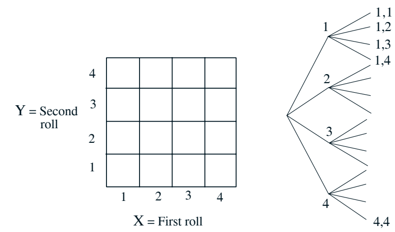
- **Sample space** Ω of all **outcomes** of an experiment
- **Events**: subsets of Ω
- **Probability law** assigning probabilities to events
  Must satisfy Probability Axioms

Sample space

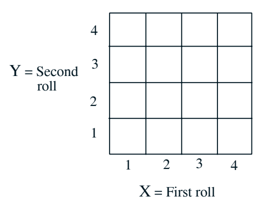
- "List" of possible outcomes
- List must be:
  - Mutually exclusive
  - Collectively exhaustive
- Art: to be at the "right" granularity

Sample space: Discrete examples

- Two rolls of a tetrahedral die
  - Sample space vs. sequential description



Probability law: Example with finite sample space



- Let every possible outcome have probability 1/16
- Probability for any set of outcomes is clear
  - $\mathbf{P}((X,Y) = (1,1)$ or $(X,Y) = (1,2)) =$?
  - $\mathbf{P}(\{X = 1\}) =$?
  - $\mathbf{P}(X + Y$ is odd$) =$?
  - $\mathbf{P}((X,Y) = (1,1)$ or $(X,Y) = (1,1)) =$?

Probability axioms

- **Event:** a subset of the sample space
- Probability is assigned to events

Axioms:

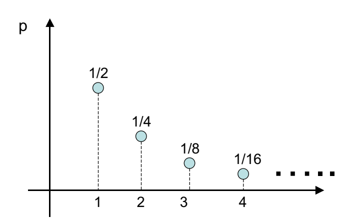
1. **Nonnegativity:** $\mathbf{P}(A) \geq 0$
2. **Additivity:** If $A \cap B = \emptyset$, then $\mathbf{P}(A \cup B) = \mathbf{P}(A) + \mathbf{P}(B)$
   More generally, if A_1, A_2 ... are disjoint then
   $$\mathbf{P}(A_1 \cup A_2 \cup \cdots) \;=\; \mathbf{P}(A_1) + \mathbf{P}(A_2) + \cdots$$
3. **Normalization:** $\mathbf{P}(\Omega) = 1$

Discrete uniform law

- Let all outcomes be equally likely
- Then,
  $$\mathbf{P}(A) = \frac{\text{number of elements of } A}{\text{total number of sample points}}$$
- Computing probabilities $\equiv$ counting
- Defines fair coins, fair dice, well-shuffled decks

Probability law: Ex. w/countably-infinite sample space

- Sample space: $\{1, 2, \ldots\}$
  - We are given $\mathbf{P}(n) = 2^{-n}$, $n = 1, 2, \ldots$
  - Find $\mathbf{P}$(outcome is even)



Continuous sample spaces

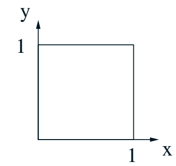
- Suppose the sample space is continuous, e.g. $\Omega = [0, 1]$.
  - Possible probability law: continuous uniform law

$$\mathbf{P}(\text{event}) \;=\; \text{length}(\text{event})$$

  - Technicality: Don't try to say every subset of Ω is an event
  - No troubles with reasonable events
- Consider continuous uniform law on $[0, 3]$
  - $\mathbf{P}([0, 1]) =$?
  - $\mathbf{P}([0, 3]) =$?
  - $\mathbf{P}(1) =$?

Continuous uniform law

- Two "random" numbers in $[0, 1]$.



- **Uniform** law: Probability = Area
  - $\mathbf{P}(y > 2x) =$?

Remember!

- Turn in recitation/tutorial scheduling form **now**
- Check Stellar site very late tonight or early tomorrow for recitation assignments and **attend recitation tomorrow**
- Tutorials start next week