MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Fall 2011)

Recitation 19
November 15, 2011

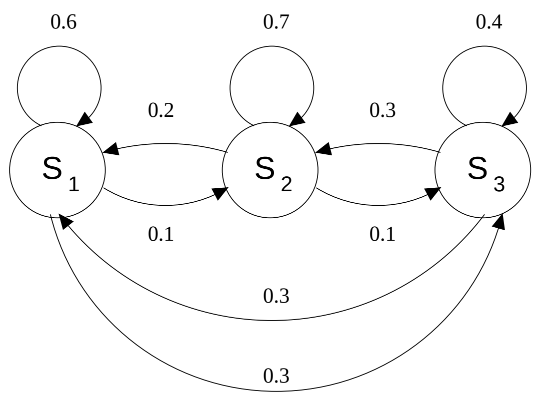
1. There are n fish in a lake, some of which are green and the rest blue. Each day, Helen catches 1 fish. She is equally likely to catch any one of the n fish in the lake. She throws back all the fish, but paints each green fish blue before throwing it back in. Let G_i denote the event that there are i green fish left in the lake.

   (a) Show how to model this fishing exercise as a Markov chain, where $\{G_i\}$ are the states. Explain why your model satisfies the Markov property.

   (b) Find the transition probabilities $\{p_{ij}\}$.

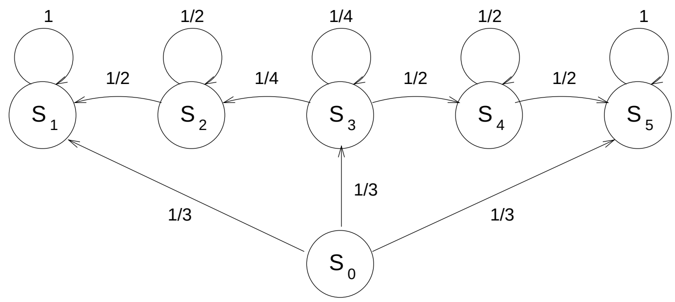
   (c) List the transient and the recurrent states.

2. Problem 5.02, from *Fundamentals of Applied Probability* (Drake).
   Consider the following three-state discrete-transition Markov chain:



   Determine the three-step transition probabilities $r_{11}(3)$, $r_{12}(3)$, and $r_{13}(3)$ both from a sequential sample space and by using the equation $r_{ij}(n+1) = \sum_k r_{ik}(n)p_{kj}$ in an effective manner.

3. Consider the following Markov chain, with states labelled from $s_0, s_1, \ldots, s_5$:



Given that the above process is in state s_0 just before the first trial, determine by inspection the probability that:

(a) The process enters s_2 for the first time as the result of the kth trial.

(b) The process never enters s_4.

(c) The process enters s_2 and then leaves s_2 on the next trial.

(d) The process enters s_1 for the first time on the third trial.

(e) The process is in state s_3 immediately after the nth trial.

(f) An important fact about transient states is that no matter what the initial state is, the probability that the chain is in a transient state n time steps later decreases geometrically. That is, for a Markov chain X_n we can find numbers c and γ with $c > 0$ and $0 < \gamma < 1$, so that

$$q_i(n) \equiv \mathbf{P}(X_n \text{ is transient} | X_0 = i) \leq c\gamma^n, \quad \text{for all } i \text{ and } n \geq 1$$

For the chain in this problem, find values of c and γ that work.