Massachusetts Institute of Technology

Department of Electrical Engineering & Computer Science

6.041/6.431: Probabilistic Systems Analysis

(Spring 2010)

Problem Set 9

Due: April 28, 2010

1. Mr. Mean Variance has the only key which locks or unlocks the door to Building 59, the Probability Building. He visits the door once each hour on the hour. When he arrives:

   If the door is unlocked, he locks it with probability 0.3.
   If the door is locked, he unlocks it with probability 0.8.

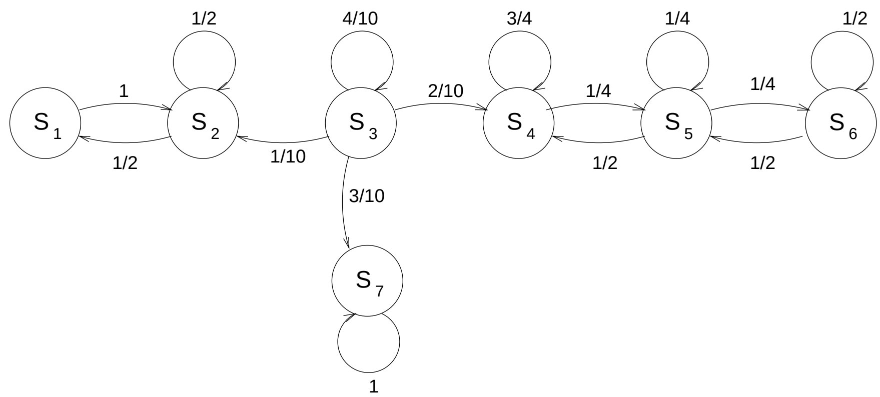
   (a) After he has been on the job several months, is he more likely to lock the door or to unlock it on a randomly selected visit?

   (b) With the process in the steady state, Joe arrived at Building 59 two hours ahead of Harry. What is the probability that they both find the door in the same condition (either both find it locked or both find it unlocked) ?

   (c) Given the door was open at the time Mr. Variance was hired, determine the expected value of the number of visits up to and including the one on which he unlocks the door himself for the first time.

2. Consider two independent Poisson processes with rates $\lambda_A = 1$ and $\lambda_B = 2$. Suppose that we just had a type B arrival (i.e., an arrival from the second process). Find the expected time until we observe a type A arrival (i.e., from the first process) such that the preceding arrival was also of type A.

3. Consider the Markov chain below. For all parts of this problem, the process is in state 3 immediately before the first transition. Be sure to comment on any unusual results.



   (a) Find the variance for J, the number of transitions up to and including the transition on which the process leaves state 3 for the last time.

   (b) Find the expectation for K, the number of transitions up to and including the transition on which the process enters state 4 for the first time.

   (c) Find π_i for $i = 1, 2, \ldots, 7$, the probability that the process is in state i after 10^{10} transitions or explain why these probabilities can't be found.

   (d) Given that the process never enters state 4, find the π_i's as defined in part (c) or explain why they can't be found.

4. Mary loves gambling. She starts out with \$200. She can bet either \$100 or \$200 (assuming she has sufficient funds), and wins with probability p.

   (a) Assume that Mary stops when she runs out of money or has reached \$400, whichever comes first. What is her optimal betting strategy? Here, "optimal" means the strategy that gives her the greatest probability of reaching \$400. And "strategy" means a rule saying how much she should bet when she has \$100, \$200, and \$300. (The amount she bets need not be the same in these three cases.)

   (b) What is the expected number of transitions until she either runs out of money or reaches \$400 for $p = 0.75$ under the optimal strategy?