Problem Set 1
Due: February 11, 2009

1. You flip a fair coin[1] 3 times: determine the probability of the below events. Assume all sequences are equally likely.

   (a) Three heads

   (b) The sequence head, tail, head

   (c) Any sequence with 2 heads and 1 tail

   (d) Any sequence where the number of heads is greater than or equal to the number of tails

2. Express each of the following events in terms of the events A, B and C as well as the operations of complementation, union and intersection:

   (a) at least one of the events A, B, C occurs;

   (b) at most one of the events A, B, C occurs;

   (c) none of the events A, B, C occurs;

   (d) all three events A, B, C occur;

   (e) exactly one of the events A, B, C occurs;

   (f) events A and B occur, but not C;

   (g) either event A occurs or, if not, then B also does not occur.

   In each case draw the corresponding Venn diagrams.

3. Using only probability axioms (page 9) and set algebra show the following:

   (a) If event B is a subset of event A, then $P(B) \leq P(A)$.

   (b) For any two events A and B, $P(A \cup B) \geq P(A) + P(B) - 1$.

4. An experiment consists of picking a student from the set of all students registered on the MIT campus this semester. It is *not* necessary to assume that all students are equally likely to be picked, but you may make this assumption if you like.

   (a) Consider the two events:
   A the student has had four years of high school science (FYS)
   B the student has had calculus in high school
   For any student picked, if the probability that she has had neither FYS nor calculus is 0.3, and the probability that she has missed at least one of the two is 0.8, what is the probability that she has had *exactly* one of the two?

   (b) Let C denote the event that the student is registered in 6.041 this semester, and let events A and B and their probabilities be as in part (a). If students who had at most one of FYS and calculus did not register in 6.041 this semester,

      i. What is $P\left(A^c \cap B^c \cap C^c\right)$?

[1] Unless otherwise stated we will always assume a fair coin has 2 important properties (i) $\mathbf{P}(\text{Heads}) = \mathbf{P}(\text{Tails}) = .5$ (ii) coin flips are independent.

ii. What is the probability that the student picked is not registered in 6.041 *and* has had exactly one of FYS or calculus in high school?

(c) Using the data given in parts (a) and (b), which of the following probabilities

$$P(A \cap B \cap C), P(C), P(A^c \cap B \cap C^c), P(A), P(A^c \cap B^c \cap C), P(A \cup B \cup C^c)$$

can you compute? It is not necessary to actually compute each probability.

G1[†]. Let $A, B, C, A_1, \ldots, A_n$ be some events. Show the following identities. A mathematical derivation is required, but you can use Venn diagrams to guide your thinking.

(a) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$,

(b) $P(\cup_{k=1}^{n} A_k) = P(A_1) + P(A_1^c \cap A_2) + P(A_1^c \cap A_2^c \cap A_3) + \cdots + P(A_1^c \cap \cdots \cap A_{n-1}^c \cap A_n)$.

[†]Required for 6.431; optional for 6.041