LECTURE 2

- **Readings:** Sections 1.3-1.4

Lecture outline

- Review
- Conditional probability
- Three **important** tools:
  - Multiplication rule
  - Total probability theorem
  - Bayes' rule

Review of probability models

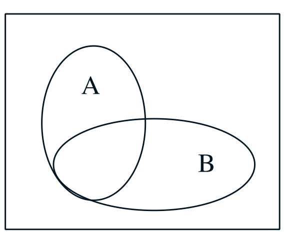
- Sample space Ω
  - Mutually exclusive
    Collectively exhaustive
  - Right granularity
- Event: Subset of the sample space
- Allocation of probabilities to events

1. $\mathbf{P}(A) \geq 0$
2. $\mathbf{P}(\Omega) = 1$
3. If $A \cap B = \emptyset$,
   then $\mathbf{P}(A \cup B) = \mathbf{P}(A) + \mathbf{P}(B)$

3'. If $A_1, A_2, \ldots$ are disjoint events, then:
$\mathbf{P}(A_1 \cup A_2 \cup \cdots) = \mathbf{P}(A_1) + \mathbf{P}(A_2) + \cdots$

- Problem solving:
  - Specify sample space
  - Define probability law
  - Identify event of interest
  - Calculate...

Conditional probability

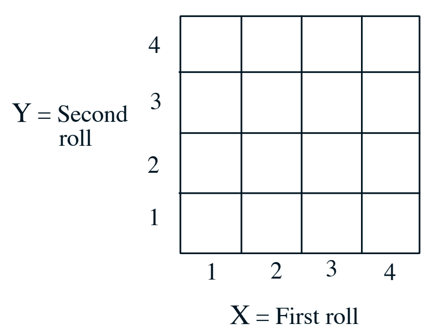


- $\mathbf{P}(A \mid B) =$ probability of A,
  given that B occurred
  - B is our new universe
- **Definition:** Assuming $\mathbf{P}(B) \neq 0$,

$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)}$$

$\mathbf{P}(A \mid B)$ undefined if $\mathbf{P}(B) = 0$

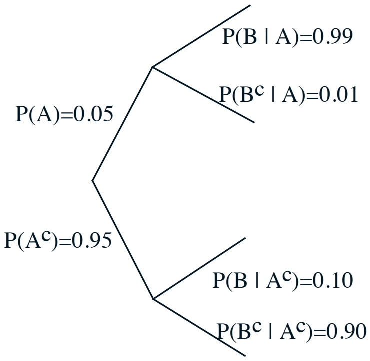
Die roll example



- Let B be the event: $\min(X, Y) = 2$
- Let $M = \max(X, Y)$
- $\mathbf{P}(M = 1 \mid B) =$
- $\mathbf{P}(M = 2 \mid B) =$

Models based on conditional probabilities

- Event A: Airplane is flying above
  Event B: Something registers on radar screen



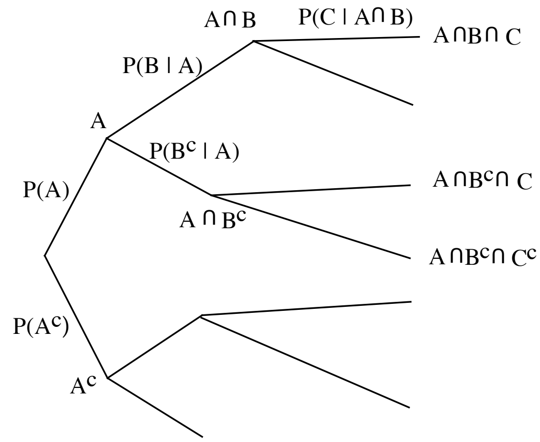
$\mathbf{P}(A \cap B) =$

$\mathbf{P}(B) =$

$\mathbf{P}(A \mid B) =$

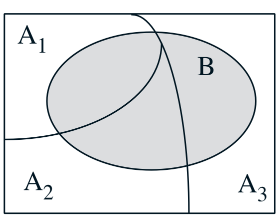
Multiplication rule

$$\mathbf{P}(A \cap B \cap C) = \mathbf{P}(A) \cdot \mathbf{P}(B \mid A) \cdot \mathbf{P}(C \mid A \cap B)$$



Total probability theorem

- Divide and conquer
- Partition of sample space into A_1, A_2, A_3
- Have $\mathbf{P}(B \mid A_i)$, for every i

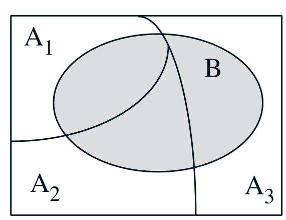


- One way of computing $\mathbf{P}(B)$:

$$\begin{aligned} \mathbf{P}(B) = & \quad \mathbf{P}(A_1)\mathbf{P}(B \mid A_1) \\ & + \mathbf{P}(A_2)\mathbf{P}(B \mid A_2) \\ & + \mathbf{P}(A_3)\mathbf{P}(B \mid A_3) \end{aligned}$$

Bayes' rule

- "Prior" probabilities $\mathbf{P}(A_i)$
  - initial "beliefs"
- We know $\mathbf{P}(B \mid A_i)$ for each i
- Wish to compute $\mathbf{P}(A_i \mid B)$
  - revise "beliefs", given that B occurred



$$\begin{aligned} \mathbf{P}(A_i \mid B) &= \frac{\mathbf{P}(A_i \cap B)}{\mathbf{P}(B)} \\ &= \frac{\mathbf{P}(A_i)\mathbf{P}(B \mid A_i)}{\mathbf{P}(B)} \\ &= \frac{\mathbf{P}(A_i)\mathbf{P}(B \mid A_i)}{\sum_j \mathbf{P}(A_j)\mathbf{P}(B \mid A_j)} \end{aligned}$$