Massachusetts Institute of Technology

Department of Electrical Engineering & Computer Science

6.041/6.431: Probabilistic Systems Analysis

(Spring 2010)

Problem set 6
Due March 29, 2010

1. X and Y are jointly Gaussian random variables with zero mean and unit variance, such that $\mathbf{E}[XY] = 0.5$. Find the PDF of $|X + Y|$.

2.

6.041: Random variables X, Y are independent and uniformly distributed over the interval $[0, 1]$. Find the CDF of $W = \max\{X, Y\} - X$.

6.431: Random variables X, Y, Z are independent and uniformly distributed over the interval $[0, 1]$. Find the CDF of $W = \max\{X, Y\} - \min\{Y, Z\}$.

3. Find a function h such that $\mathbf{E}[X \mid (X-1)^2] = h(X)$, where $X \sim N(0, 1)$.

4. Z is a random point on the plane which is distributed uniformly over the rectangle with vertices $(1, 1)$, $(0, 2)$, $(2, 4)$, and $(3, 3)$. Let X, Y be the random variables defined as the coordinates of Z. We are interested in finding functions $h: \mathbf{R} \mapsto \mathbf{R}$ which produce good estimates $\hat{X} = h(Y)$ of X given Y. Find a function $h_{NL}: \mathbf{R} \mapsto \mathbf{R}$ which minimizes $J(h(\cdot)) = \mathbf{E}[|X - h(Y)|^2]$ over the set of all continuous functions $h: \mathbf{R} \mapsto \mathbf{R}$. What is the value of $J(h_{NL})$?

5. John is participating in a 6.041 magic ritual. He is given an unfair coin with random probability Q of "tail" distributed uniformly between 0 and 1. John tosses the coin two times. For every "tail" toss, he is given a light bulb with exponentially distributed lifetime (parameter $\lambda = 1$). In addition, he is given one such bulb for just participating in the ritual (so he ends up with one, two, or three light bulbs). John turns all light bulbs on simultaneously. Let T be the time until the first of these bulbs burns out. Find $\mathbf{E}[T]$ and $\mathrm{var}(T)$.