Recitation 14 Solutions
October 27, 2011

2. Example 4.17, page 223 in text. See text for solutions.

3. Let X_i denote the number of widgets in the i^{th} box. Then $T = \sum_{i=1}^{N} X_i$.

$$\begin{aligned}
\mathbf{E}[T] &= \mathbf{E}[\mathbf{E}[\sum_{i=1}^{N} X_i|N]] \\
&= \mathbf{E}[\sum_{i=1}^{N} \mathbf{E}[X_i|N]] \\
&= \mathbf{E}[\sum_{i=1}^{N} \mathbf{E}[X]] \\
&= \mathbf{E}[X] \cdot \mathbf{E}[N] = 100.
\end{aligned}$$

and,

$$\begin{aligned}
\text{var}(T) &= \mathbf{E}\left[\text{var}(T|N)\right] + \text{var}\left(\mathbf{E}[T|N]\right) \\
&= \mathbf{E}\left[\text{var}\left(\sum_{i=1}^{N} X_i|N\right)\right] + \text{var}\left(\mathbf{E}\left[\sum_{i=1}^{N} X_i|N\right]\right) \\
&= \mathbf{E}[N\text{var}(X)] + \text{var}(N\mathbf{E}[X]) \\
&= (\text{var}(X))\mathbf{E}[N] + (\mathbf{E}[X])^2 \text{var}(N) \\
&= 16 \cdot 10 + 100 \cdot 16 = 1760.
\end{aligned}$$