Massachusetts Institute of Technology

Department of Electrical Engineering & Computer Science

6.041/6.431: Probabilistic Systems Analysis

(Fall 2011)

Recitation 11
October 18, 2011

1. Suppose X_1, X_2, and X_3 are independent exponential random variables, each with parameter λ. Find the PDF of $Z = \max\{X_1, X_2, X_3\}$.

2. (Example 3.13 of the text book, page 165) **Exponential Random Variable is Memoryless.** The time T until a new light bulb burns out is an exponential random variable with parameter λ. Ariadne turns the light on, leaves the room, and when she returns, t time units later, finds that the bulb is still on, which corresponds to the event $A = \{T > t\}$. Let X be the additional time until the bulb burns out. What is the conditional CDF of X, given the event A?

3. Problem 3.23, page 191 in the text.
   Let the random variables X and Y have a joint PDF which is uniform over the triangle with vertices $(0, 0)$, $(0, 1)$, and $(1, 0)$.

   (a) Find the joint PDF of X and Y.

   (b) Find the marginal PDF of Y.

   (c) Find the conditional PDF of X given Y.

   (d) Find $\mathbf{E}[X \mid Y = y]$, and use the total expectation theorem to find $\mathbf{E}[X]$ in terms of $\mathbf{E}[Y]$.

   (e) Use the symmetry of the problem to find the value of $\mathbf{E}[X]$.

4. We have a stick of unit length, and we break it into three pieces. We choose randomly and independently two points on the stick using a uniform PDF, and we break the stick at these points. What is the probability that the three pieces we are left with can form a triangle?