Recitation 25
December 8, 2011

1. We are given i.i.d observations $X_1, \cdots, X_n$ that are uniformly distributed over the interval $[\theta, \theta + 1]$.
   (a) Find a ML (Maximum Likelihood) estimate for θ? If there's more than one, find all of them.
   (b) Consider the largest and smallest of the ML estimators. To what values do these two converge?
   (c) Is the largest one consistent? Is the smallest one consistent? What about the others ?

2. **Example 9.4, page 464 in textbook**
   Estimate the mean μ and variance v of a normal distribution using n independent observations $X_1, \ldots, X_n$.

3. **Example 9.8, page 474 of textbook**
   We would like to estimate the fraction of voters supporting a particular candidate for office. We collect n independent sample voter responses $X_1, \ldots, X_n$, where X_i is viewed as a Bernoulli random variable, with $X_i = 1$ if the ith voter supports the candidate. We conducted a poll of 1200 people in North Carolina, and found that 684 were supporting the candidate. We would like to construct a 95% confidence interval for θ, the proportion of people who support the candidate. As we saw in lecture, using the central limit theorem, an (approximate) 95% confidence interval can be defined as

$$\hat{\Theta}^- = \hat{\Theta}_n - 1.96\sqrt{\frac{v}{n}}, \qquad \hat{\Theta}^+ = \hat{\Theta}_n + 1.96\sqrt{\frac{v}{n}}$$

   where $v = \mathrm{var}(X_i)$, and $\hat{\Theta}_n = (X_1 + \ldots + X_n)/n$. Unfortunately, we don't know the value for v. Construct confidence intervals for θ using the following three ways of estimating or bounding the value for v (in each case simply assume that v is equal to the given estimate; note that this is a further approximation in cases (a) and (b)).

   (a)
$$\hat{S}_n^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \hat{\Theta}_n)^2$$

   (b)
$$\hat{\Theta}_n(1 - \hat{\Theta}_n)$$

   (c) The most conservative upper bound for the variance.