LECTURE 15

- **Readings:** Finish Section 6.2

Lecture outline

- Big Screw
- Quiz 2
- Review of Poisson process
- Merging and splitting
- Random incidence

Big Screw

- Alpha Phi Omega Institute Screw Contest
- Criteria
- Teach for America `www.teachforamerica.org`
- Donations are anonymous

Quiz 2

- Wednesday, April 7, 7:30pm-9:30pm, 50-340
- No lecture on Wednesday
- Coverage is through Bernoulli processes

Review: Poisson process with rate λ

- Defining characteristics:
  - **Time homogeneity**: $P(k, \tau)$
  - **Independence**
  - **Small interval probabilities**: For very small δ,

$$P(k, \delta) \approx \begin{cases} 1 - \lambda\delta, & \text{if } k = 0; \\ \lambda\delta, & \text{if } k = 1; \\ 0, & \text{if } k > 1. \end{cases}$$

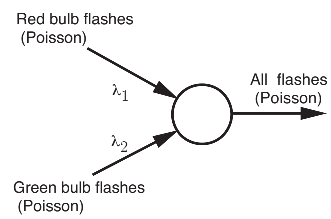
- N_τ is a Poisson random variable with parameter $\lambda\tau$:

$$P(k, \tau) = \frac{(\lambda\tau)^k e^{-\lambda\tau}}{k!}, \qquad k = 0, 1, \ldots$$

- Independent exponential interarrival times
- Time of kth arrival: Erlang of order k

Adding (merging) Poisson processes

- Merging indep. Poisson **processes** gives a Poisson process



- By-product: Sum of independent Poisson **random variables** is a Poisson random variable

Example: Three similar light bulbs

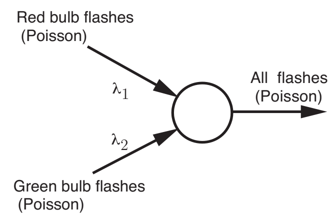
- Three light bulbs have independent lifetimes

$$X_1, X_2, X_3 : \quad \text{exponential with parameter } \lambda$$

- Install all three. Let T be the time when the last of the three burns out. Find $\mathbf{E}[T]$ and var(T).

Merging Poisson processes (again)

- Merging indep. Poisson **processes** gives a Poisson process



- What is the probability that the next arrival comes from the first process?

Example: Two dissimilar light bulbs

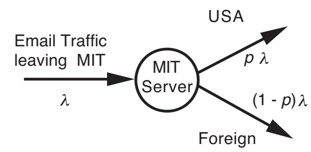
- Two light bulbs have independent lifetimes

X_1 : exponential with parameter λ_1
X_2 : exponential with parameter λ_2

- Install both. Let T be the time when the last burns out. Find $\mathbf{E}[T]$.

Splitting of a Poisson process

- Suppose email traffic through server is a Poisson process and destinations are independent



- Each output stream is Poisson. Why?

Example: Geometric number of exponential RVs

- Let $Y = X_1 + X_2 + \cdots X_N$ where

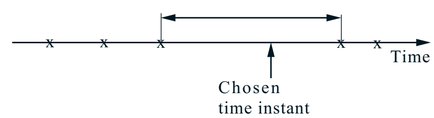
each X_i : exponential with parameter λ
N : geometric with parameter p

and N, X_1, X_2, ... are independent

- What is the distribution of Y?

Random incidence in Poisson process

- Poisson process that has been running forever
- Show up at some "random time" ("arbitrary")



- What is the distribution of the length of the chosen inter-arrival interval?

Random incidence in Bernoulli process

- You watch Ali Farokmanesh shoot free throws on several days. On average, the length of the streak of made free throws when you walk into the gym is 10. Assuming free throws are independent, what is his probability of making any one free throw?

Recitation 5, Problem 1

- 4 buses carrying 148 job-seeking MIT students arrive at a job convention. The buses carry 40, 33, 25, and 50 students, respectively.
- One of the students is selected randomly ...
- One of the buses is selected randomly ...