Massachusetts Institute of Technology
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Fall 2011)

Recitation 13
October 25, 2011

1. Show $\rho(aX+b,Y) = \text{sgn}(a)\rho(X,Y)$, where

$$\text{sgn}(a) = \begin{cases} -1 & \text{if } a < 0, \\ 0 & \text{if } a = 0, \\ 1 & \text{if } a > 0. \end{cases}$$

2. Romeo and Juliet have a date at a given time, and each, independently, will be late by amounts of time, X and Y, respectively, that are exponentially distributed with parameter λ.

   (a) Find the PDF of $Z = X - Y$ by first finding the CDF and then differentiating.

   (b) Find the PDF of Z by using the convolution formula.

3. Problem 4.16, page 248 in text.
   Let X and Y be independent standard normal random variables. The pair (X,Y) can be described in polar coordinates in terms of random variables $R \geq 0$ and $\Theta \in [0, 2\pi]$, so that

$$X = R\cos\Theta, \quad Y = R\sin\Theta.$$

   Show that R and Θ are independent (i.e. show $f_{R,\Theta}(r,\theta) = f_R(r)f_\Theta(\theta)$).

   (a) Find $f_R(r)$.

   (b) Find $f_\Theta(\theta)$.

   (c) Find $f_{R,\Theta}(r,\theta)$.

4. Problem 4.20, page 250 in text. **Schwarz inequality**.
   Show that for any random variables X and Y, we have

$$(\mathbf{E}[XY])^2 \leq \mathbf{E}[X^2]\mathbf{E}[Y^2].$$