Recitation 1: Solutions
September 8, 2011

1. Since the events $A \cap B^c$ and $A^c \cap B$ are disjoint, we have, using the additivity axiom,

$$\mathbf{P}((A \cap B^c) \cup (A^c \cap B)) = \mathbf{P}(A \cap B^c) + \mathbf{P}(A^c \cap B).$$

Since $A = (A \cap B) \cup (A \cap B^c)$ is the union of two disjoint sets, we have, again by the additivity axiom,

$$\mathbf{P}(A) = \mathbf{P}(A \cap B) + \mathbf{P}(A \cap B^c),$$

so that

$$\mathbf{P}(A \cap B^c) = \mathbf{P}(A) - \mathbf{P}(A \cap B).$$

Similarly,

$$\mathbf{P}(B \cap A^c) = \mathbf{P}(B) - \mathbf{P}(A \cap B).$$

Therefore,

$$\begin{aligned} \mathbf{P}(A \cap B^c) + \mathbf{P}(A^c \cap B) &= \mathbf{P}(A) - \mathbf{P}(A \cap B) + \mathbf{P}(B) - \mathbf{P}(A \cap B) \\ &= \mathbf{P}(A) + \mathbf{P}(B) - 2\mathbf{P}(A \cap B). \end{aligned}$$

2. Let

A : The event that the randomly selected student is a genius.
B : The event that the randomly selected student loves chocolate.

From the properties of probability laws proved in lecture, we have

$$\begin{aligned} 1 &= \mathbf{P}(A \cup B) + \mathbf{P}((A \cup B)^c) \\ &= \mathbf{P}(A) + \mathbf{P}(B) - \mathbf{P}(A \cap B) + \mathbf{P}(A^c \cap B^c) \\ &= 0.6 + 0.7 - 0.4 + \mathbf{P}(A^c \cap B^c) \\ &= 0.9 + \mathbf{P}(A^c \cap B^c). \end{aligned}$$

Therefore

$$\begin{aligned} &\mathbf{P}(\text{A randomly selected student is neither a genius nor a chocolate lover}) \\ &\quad = \mathbf{P}(A^c \cap B^c) \;=\; 1 - 0.9 \;=\; 0.1. \end{aligned}$$

3. Let c denote the probability of a single odd face. Then the probability of a single even face is $2c$, and by adding the probabilities of the 3 odd faces and the 3 even faces, we get $9c = 1$. Thus, $c = 1/9$. The desired probability is

$$\mathbf{P}(\{1, 2, 3\}) = \mathbf{P}(\{1\}) + \mathbf{P}(\{2\}) + \mathbf{P}(\{3\}) = c + 2c + c = 4c = 4/9.$$

4. See the textbook, Example 1.5, page 13.

G1†. See the textbook, Problem 1.13, page 56.

†Required for 6.431; optional for 6.041