

Tutorial 1
September 15/16, 2011

1. Let A and B be events such that $A \subset B$. Can A and B be independent?
2. An electrical system consists of identical components that are operational with probability p independently of other components. The components are connected in three subsystems, as shown in the figure. The system is operational if there is a path that starts at point A, ends at point B, and consists of operational components. This is the same as requiring that all three subsystems are operational. What are the probabilities that the three subsystems, as well as the entire system, are operational?

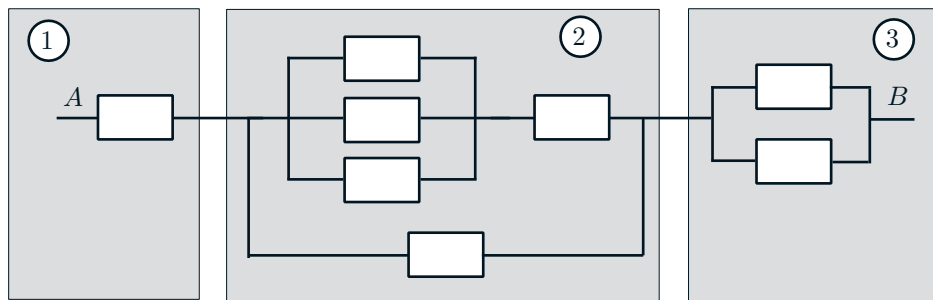


Figure 1: A system of identical components that consists of the three subsystems 1, 2, and 3. The system is operational if there is a path that starts at point A, ends at point B, and consists of operational components.

3. **The Chess Problem.** This year's Belmont chess champion is to be selected by the following procedure. Bo and Ci, the leading challengers, first play a two-game match. If one of them wins both games, he gets to play a two-game *second round* with Al, the current champion. Al retains his championship unless a second round is required and the challenger beats Al in both games. If Al wins the initial game of the second round, no more games are played.

Furthermore, we know the following:

- The probability that Bo will beat Ci in any particular game is 0.6.
- The probability that Al will beat Bo in any particular game is 0.5.
- The probability that Al will beat Ci in any particular game is 0.7.

Assume no tie games are possible and all games are independent.

- (a) Determine the apriori probabilities that
 - i. the second round will be required.
 - ii. Bo will win the first round.
 - iii. Al will retain his championship this year.
- (b) Given that the second round is required, determine the conditional probabilities that

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- i. Bo is the surviving challenger.
 - ii. Al retains his championship.
- (c) Given that the second round was required and that it comprised only one game, what is the conditional probability that it was Bo who won the first round?