MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Fall 2011)

Recitation 20 Solutions: November 17, 2011

1. (a) i. True
   ii. False
   iii. False

   There are two recurrent classes, both aperiodic. The probability you end up in either depends on the initial state.

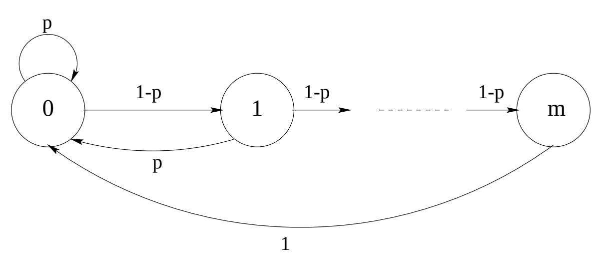
   (b) i. False
   ii. False
   iii. False

   The rightmost three states form a periodic recurrent class, and thus the occupancy probabilities in that class do not converge. The occupancy probabilities in the leftmost four states converge to zero.

   (c) i. True
   ii. True
   iii. True

   The chain consists of one recurrent class and no transient states. One way to see that the recurrent class is aperiodic is to note that from the center state one can reach each state of the chain in exactly four steps.

2. Problem 7.18, page 386 in textbook.
We introduce the states $0, 1, \ldots, m$ and identify them as the number of days the gate survives a crash. The state transition diagram is shown in the figure below.



The balance equations take the form,

$$
\begin{aligned}
\pi_0 &= \pi_0 p + \pi_1 p + \cdots + \pi_{m-1} p + \pi_m \\
\pi_1 &= \pi_0 (1-p) \\
\pi_2 &= \pi_1 (1-p) = \pi_0 (1-p)^2 \\
&\vdots \\
\pi_m &= \pi_0 (1-p)^m
\end{aligned}
$$

These equations together with the normalization equation have a unique solution which gives us the steady-state probabilities of all states. The steady-state expected frequency of gate replacements is the expected frequency of visits to state 0, which by frequency interpretation is given by π_0. Solving the above equations with the normalization equation we get,

$$\begin{aligned} E[\text{frequency of gate replacements}] &= \pi_0 \\ &= \frac{p}{1-(1-p)^{m+1}} \end{aligned}$$

3. Problem 7.13, page 385 in textbook. See online solutions.