Recitation 11 Solutions
October 18, 2011

1. The maximum of a set is upper bounded by z when each element of the set is upper bounded by z. Thus for any positive z,

$$\begin{aligned}\mathbf{P}(Z \leq z) &= \mathbf{P}(\max\{X_1, X_2, X_3\} \leq z) = \mathbf{P}(X_1 \leq z,\, X_2 \leq z,\, X_3 \leq z) \\ &= \mathbf{P}(X_1 \leq z)\,\mathbf{P}(X_2 \leq z)\,\mathbf{P}(X_3 \leq z) \\ &= (1 - e^{-\lambda z})^3,\end{aligned}$$

   where the third equality uses the independence of X_1, X_2, and X_3. Thus,

$$F_Z(z) = \begin{cases} 0, & \text{if } z < 0, \\ (1 - e^{-\lambda z})^3, & \text{if } z \geq 0. \end{cases}$$

   Differentiating the CDF gives the desired PDF:

$$f_Z(z) = \begin{cases} 0, & \text{if } z < 0, \\ 3\lambda e^{-\lambda z}(1 - e^{-\lambda z})^2, & \text{if } z \geq 0. \end{cases}$$

2. See Example 3.13 in the textbook on page 165.

3. Problem 3.23, page 191 in text. See online solutions.

4. Problem 3.22, part (i), page 191 in text (see online solution).