

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Fall 2011)

Recitation 6
September 27, 2011

1. (a) Derive the expected value rule for functions of random variables $\mathbf{E}[g(X)] = \sum_x g(x)p_X(x)$.
(b) Use the expected value rule to derive the mean and variance of a linear function of a random variable $Y = aX + b$.

$$\mathbf{E}[Y] = a\mathbf{E}[X] + b, \quad \text{var}(Y) = a^2\text{var}(X).$$

- (c) Derive $\text{var}(X) = \mathbf{E}[X^2] - (\mathbf{E}[X])^2$
2. A marksman takes 10 shots at a target and has probability 0.2 of hitting the target with each shot, independently of all other shots. Let X be the number of hits.
 - (a) Calculate and sketch the PMF of X .
 - (b) What is the probability of scoring no hits?
 - (c) What is the probability of scoring more hits than misses?
 - (d) Find the expectation and the variance of X .
 - (e) Suppose the marksman has to pay \$3 to enter the shooting range and he gets \$2 dollars for each hit. Let Y be his profit. Find the expectation and the variance of Y .
 - (f) Now let's assume that the marksman enters the shooting range for free and gets the number of dollars that is equal to the square of the number of hits. Let Z be his profit. Find the expectation of Z .
3. 4 buses carrying 148 job-seeking MIT students arrive at a job convention. The buses carry 40, 33, 25, and 50 students, respectively. One of the students is randomly selected. Let X denote the number of students that were on the bus carrying this randomly selected student. One of the 4 bus drivers is also randomly selected. Let Y denote the number of students on his bus.
 - (a) Which of $E[X]$ or $E[Y]$ do you think is larger? Give your reasoning in words.
 - (b) Compute $E[X]$ and $E[Y]$.