LECTURE 2

- **Readings:** Sections 1.3 and 1.4

Lecture outline

- Review of probabilistic models
- Conditional probability
- Three important tools:
  - Multiplication rule
  - Total probability theorem
  - Bayes' rule

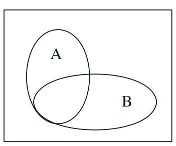
Review of probabilistic models

- **Sample space** Ω of **outcomes**
- **Events** are subsets of Ω
- **Probabilities** assigned to events by $\mathbf{P}(\cdot)$

Axioms:

1. **Nonnegativity:** $\mathbf{P}(A) \geq 0$
2. **Additivity:** If $A \cap B = \emptyset$, then $\mathbf{P}(A \cup B) = \mathbf{P}(A) + \mathbf{P}(B)$
   More generally, if A_1, A_2 ... are disjoint then
   $$\mathbf{P}(A_1 \cup A_2 \cup \cdots) = \mathbf{P}(A_1) + \mathbf{P}(A_2) + \cdots$$
3. **Normalization:** $\mathbf{P}(\Omega) = 1$

Conditional probability



- $\mathbf{P}(A \mid B)$ = probability of A, given that B occurred
  - B is our new universe
- **Definition:** Assuming $\mathbf{P}(B) \neq 0$,
  $$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)}$$

($\mathbf{P}(A \mid B)$ is not defined when $\mathbf{P}(B) = 0$)

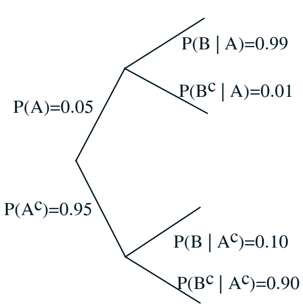
Die roll example

		first roll X			
		1	2	3	4
	1	•	•	•	•
second	2	•	•	•	•
roll Y	3	•	•	•	•
	4	•	•	•	•

- Let B be the event: $\min(X, Y) = 2$
- Let $M = \max(X, Y)$
- $\mathbf{P}(\{M = 1\} \mid B) =$
- $\mathbf{P}(\{M = 2\} \mid B) =$

Models based on conditional probabilities

- Event A: Airplane is flying above
  Event B: Something registers on radar screen

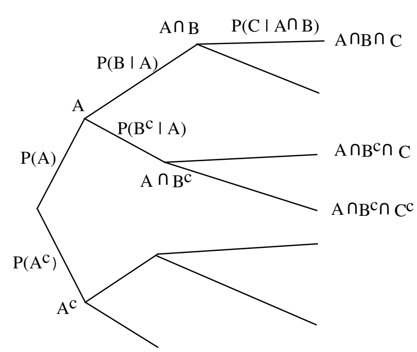


$\mathbf{P}(A \cap B) =$?

$\mathbf{P}(B) =$?

$\mathbf{P}(A \mid B) =$?

Multiplication rule

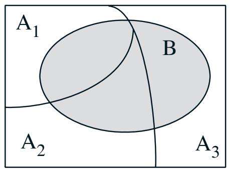


$\mathbf{P}(A \cap B) =$

$\mathbf{P}(A \cap B \cap C) =$

Total probability theorem

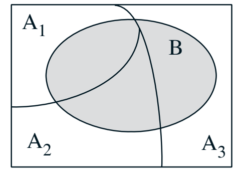
- Partition of Ω into A_1, A_2, A_3 with $\mathbf{P}(B \mid A_i)$ easy for each i



- One way of computing $\mathbf{P}(B)$:

$$\begin{aligned}\mathbf{P}(B) = \quad & \mathbf{P}(A_1)\mathbf{P}(B \mid A_1) \\ + \quad & \mathbf{P}(A_2)\mathbf{P}(B \mid A_2) \\ + \quad & \mathbf{P}(A_3)\mathbf{P}(B \mid A_3)\end{aligned}$$

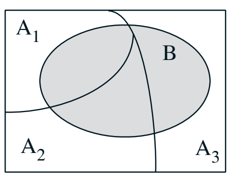
Bayes' rule



- Have "prior" probabilities $\mathbf{P}(A_i)$ and $\mathbf{P}(B \mid A_i)$ for each i
- Want to "update beliefs" based on B:

compute $\mathbf{P}(A_i \mid B)$ for each i

Bayes' rule



$$\begin{aligned}\mathbf{P}(A_i \mid B) &= \frac{\mathbf{P}(A_i \cap B)}{\mathbf{P}(B)} \\ &= \frac{\mathbf{P}(A_i)\mathbf{P}(B \mid A_i)}{\mathbf{P}(B)} \\ &= \frac{\mathbf{P}(A_i)\mathbf{P}(B \mid A_i)}{\sum_j \mathbf{P}(A_j)\mathbf{P}(B \mid A_j)}\end{aligned}$$