Recitation 23
December 1, 2011

1. **Examples 8.2, 8.7, and 8.12 in the textbook**
   Romeo and Juliet start dating, but Juliet will be late on any date by a random amount X, uniformly distributed over the interval $[0, \theta]$. The parameter θ is unknown and is modeled as the value of a random variable Θ, uniformly distributed between zero and one hour.

   (a) Assuming that Juliet was late by an amount x on their first date, how should Romeo use this information to update the distribution of Θ?

   (b) How should Romeo update the distribution of Θ if he observes that Juliet is late by $x_1, \ldots, x_n$ on the first n dates? Assume that Juliet is late by a random amount $X_1, \ldots, X_n$ on the first n dates where, given θ, $X_1, \ldots, X_n$ are uniformly distributed between zero and θ and are conditionally independent.

   (c) Find the MAP estimate of Θ based on the observation $X = x$.

   (d) Find the LMS estimate of Θ based on the observation $X = x$.

   (e) Calculate the conditional mean squared error for the MAP and the LMS estimates. Compare your results.

2. Let Θ be a Bernoulli random variable that indicates which of the two hypotheses is true, $\mathbf{P}(\Theta = 1) = p$. Under the null hypothesis ($\Theta = 0$), the random variable X is uniformly distributed over the interval $[0, 1]$. Under the alternative hypothesis ($\Theta = 1$), the PDF of X is given by

$$f_{X|\Theta}(x \mid 1) = \begin{cases} 2x & \text{if } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Find the MAP rule for deciding which of the two hypotheses is true based on $X = x$. Give the decision rule for $p = 2/3$, $p = 1/2$, $p = 1/3$.

   (b) For this question only, assume that $p = 2/3$. Find the probability of error for the MAP decision rule given that the null hypothesis is true.

   (c) Find the probability of error for the MAP decision rule as a function of p.