Recitation 7
September 29, 2011

1. Consider an experiment in which a fair four-sided die (with faces labeled 0, 1, 2, 3) is thrown once to determine how many times a fair coin is to be flipped. In the sample space of this experiment, random variables N and K are defined by

   - N = the result of the die roll
   - K = the total number of heads resulting from the coin flips

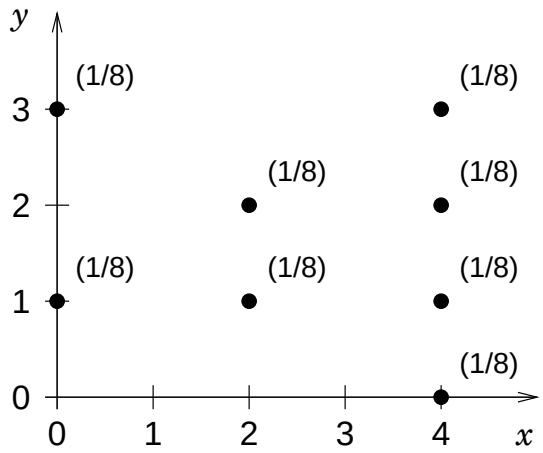
   (a) Determine and sketch $p_N(n)$

   (b) Determine and tabulate $p_{N,K}(n,k)$

   (c) Determine and sketch $p_{K|N}(k \mid 2)$

   (d) Determine and sketch $p_{N|K}(n \mid 2)$

2. Consider an outcome space comprising eight equally likely event points, as shown below:



   (a) Which value(s) of x *maximize(s)* $\mathbf{E}[Y \mid X = x]$?

   (b) Which value(s) of y *maximize(s)* $\text{var}(X \mid Y = y)$?

   (c) Let $R = \min(X, Y)$. Prepare a neat, fully labeled sketch of $p_R(r)$,

   (d) Let A denote the event $X^2 \geq Y$. Determine numerical values for the quantities $\mathbf{E}[XY]$ and $\mathbf{E}[XY \mid A]$.

3. Problem 2.32, page 128 in the text. **D. Bernoulli's problem of joint lives.**
   Consider $2m$ persons forming m couples who live together at a given time. Suppose that at some later time, the probability of each person being alive is p, independently of other persons. At that later time, let A be the number of persons that are alive and let S be the number of couples in which both partners are alive. For any number of total surviving persons a, find $\mathbf{E}[S|A = a]$.