

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Fall 2011)

Tutorial 2
September 22/23, 2011

1. A player is randomly dealt 13 cards from a standard 52-card deck.
 - (a) What is the probability the 13th card dealt is a king?
 - (b) What is the probability the 13th card dealt is the first king dealt?
2. A club is formed by selecting a special person from the group (the club leader) and a number (possibly zero) of additional club members.
 - (a) Find the number of possible club rosters in a group of n people.
 - (b) Show that

$$\sum_{k=1}^n k \binom{n}{k} = n2^{n-1}.$$

3. A fixed set of twenty **distinct** cars park in the same parking lot every day. Ten of these cars are American-made, while the other ten are foreign-made. This parking lot has exactly twenty spaces in one row. When any driver arrives on any day, he or she picks uniformly at random from the available spaces.
 - (a) In how many different ways can the cars line up?
 - (b) What is the probability that on a given day, the cars will park in such a way that they alternate (no two American-made are adjacent and no two foreign-made are adjacent)?
4. A deck of 52 cards is distributed between 4 players (as in bridge).
 - (a) Find the probability that player 1 gets all 13 spades.
 - (b) Find the probability that some player gets all 13 spades.
 - (c) Consider the following two events:
 - i. player 1 gets all 13 spades;
 - ii. player 1 gets the king of hearts.Are these events independent? Are they mutually exclusive?
 - (d) Consider the following 2 events:
 - i. all of player 1's cards are of the same suit;
 - ii. player 1 gets the king of hearts.Are these two events independent?