## MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Department of Electrical Engineering & Computer Science

## 6.041/6.431: Probabilistic Systems Analysis (Fall 2011)

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1. The maximum of a set is upper bounded by z when each element of the set is upper bounded by z. Thus for any positive z,

$$\mathbf{P}(Z \le z) = \mathbf{P}(\max\{X_1, X_2, X_3\} \le z) = \mathbf{P}(X_1 \le z, X_2 \le z, X_3 \le z)$$

$$= \mathbf{P}(X_1 \le z) \mathbf{P}(X_2 \le z) \mathbf{P}(X_3 \le z)$$

$$= (1 - e^{-\lambda z})^3,$$

where the third equality uses the independence of  $X_1$ ,  $X_2$ , and  $X_3$ . Thus,

$$F_Z(z) = \begin{cases} 0, & \text{if } z < 0, \\ (1 - e^{-\lambda z})^3, & \text{if } z \ge 0. \end{cases}$$

Differentiating the CDF gives the desired PDF:

$$f_Z(z) = \begin{cases} 0, & \text{if } z < 0, \\ 3\lambda e^{-\lambda z} (1 - e^{-\lambda z})^2, & \text{if } z \ge 0. \end{cases}$$

- 2. See Example 3.13 in the textbook on page 165.
- 3. Problem 3.23, page 191 in text. See online solutions.
- 4. Problem 3.22, part (i), page 191 in text (see online solution).