Massachusetts Institute of Technology
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Fall 2009)

Problem Set 2
Due September 23, 2009

1. Suppose that two fair six-sided dice are rolled. Assume that all possible outcomes are equally likely.

   (a) Find the probability that "doubles" were rolled.

   (b) Given that the roll resulted in a sum 4 or less, find the conditional probability that "doubles" were rolled.

2. You have a fair five-sided die. The sides of the die are numbered from 1 to 5. Each die roll is independent of all others, and all faces are equally likely to come out on top when the die is rolled. Suppose you roll the die twice.

   (a) Let event A to be "the total of two rolls is 10", event B be "at least one roll resulted in 5", and event C be "at least one roll resulted in 1".

      i. Is event A independent of event B?

      ii. Is event A independent of event C?

   (b) Let event D be "the total of two rolls is 7", event E be "the difference between the two roll outcomes is exactly 1", and event F be "the second roll resulted in a higher number than the first roll".

      i. Are events E and F independent?

      ii. Are events E and F independent given event D?

3. Oscar has lost his dog in either forest A (with a priori probability 0.4) or in forest B (with a priori probability 0.6).

   On any given day, if the dog is in A and Oscar spends a day searching for it in A, the conditional probability that he will find the dog that day is 0.25. Similarly, if the dog is in B and Oscar spends a day looking for it there, the conditional probability that he will find the dog that day is 0.15.

   The dog cannot go from one forest to the other. Oscar can search only in the daytime, and he can travel from one forest to the other only at night.

   (a) In which forest should Oscar look to maximize the probability he finds his dog on the first day of the search?

   (b) Given that Oscar looked in A on the first day but didn't find his dog, what is the probability that the dog is in A?

   (c) If Oscar flips a fair coin to determine where to look on the first day and finds the dog on the first day, what is the probability that he looked in A?

   (d) If the dog is alive and not found by the Nth day of the search, it will die that evening with probability $\frac{N}{N+2}$. Oscar has decided to look in A for the first two days. What is the probability that he will find a live dog for the first time on the second day?

4. The local widget factory is having a blowout widget sale. Everything must go, old and new. The factory has 1000 old widgets, and 1500 new widgets in stock. The problem is that 15% of the old widgets are defective, and 5% of the new ones are defective as well. You can assume that widgets are selected at random when an order comes in. You are the first customer since the sale was announced.

   (a) You flip a fair coin once to decide whether to buy old or new widgets. You order two widgets of the same type, chosen based on the outcome of the coin toss. What is the probability that they will both be defective?

   (b) Given that both widgets turn out to be defective, what is the probability that they were old widgets?

5. In bin 1, there are two red and one white balls. In bin 2, there are one red and two white balls. You reach into bin 1, take a ball at random (so each ball is equally likely to be picked) and toss it into bin 2. Then, you take a ball at random from bin 2, and toss it into bin 1.

   After this exchange, you pick a bin at random and take a ball, again at random, out of it. Given that the ball is red, what is the probability that you picked bin 1?

6. In solving this problem, feel free to browse problems 43-45 in Chapter 1 of the text for ideas. If you need to, you may quote the results of these problems.

   (a) Suppose that A, B, and C are independent. Use the definition of independence to show that A and $B \cup C$ are independent.

   (b) Prove that if $A_1, \ldots, A_n$ are independent events, then

$$\mathbf{P}(A_1 \cup A_2 \cup \ldots \cup A_n) = 1 - \prod_{i=1}^{n}(1 - \mathbf{P}(A_i)).$$