MASSACHUSETTS INSTITUTE OF TECHNOLOGY Department of Electrical Engineering & Computer Science 6.041/6.431: Probabilistic Systems Analysis (Fall 2011)

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- 2. Example 4.17, page 223 in text. See text for solutions.
- 3. Let X_i denote the number of widgets in the i^{th} box. Then $T = \sum_{i=1}^{N} X_i$.

$$\mathbf{E}[T] = \mathbf{E}[\mathbf{E}[\sum_{i=1}^{N} X_i | N]]$$

$$= \mathbf{E}[\sum_{i=1}^{N} \mathbf{E}[X_i | N]]$$

$$= \mathbf{E}[\sum_{i=1}^{N} \mathbf{E}[X]]$$

$$= \mathbf{E}[X] \cdot \mathbf{E}[N] = 100.$$

and,

$$\operatorname{var}(T) = \mathbf{E} \left[\operatorname{var}(T|N) \right] + \operatorname{var} \left(\mathbf{E}[T|N] \right)$$

$$= \mathbf{E} \left[\operatorname{var} \left(\sum_{i=1}^{N} X_i | N \right) \right] + \operatorname{var} \left(\mathbf{E} \left[\sum_{i=1}^{N} X_i | N \right] \right)$$

$$= \mathbf{E}[N \operatorname{var}(X)] + \operatorname{var}(N \mathbf{E}[X])$$

$$= (\operatorname{var}(X)) \mathbf{E}[N] + (\mathbf{E}[X])^2 \operatorname{var}(N)$$

$$= 16 \cdot 10 + 100 \cdot 16 = 1760.$$