Massachusetts Institute of Technology
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Fall 2011)

Recitation 4: Solutions
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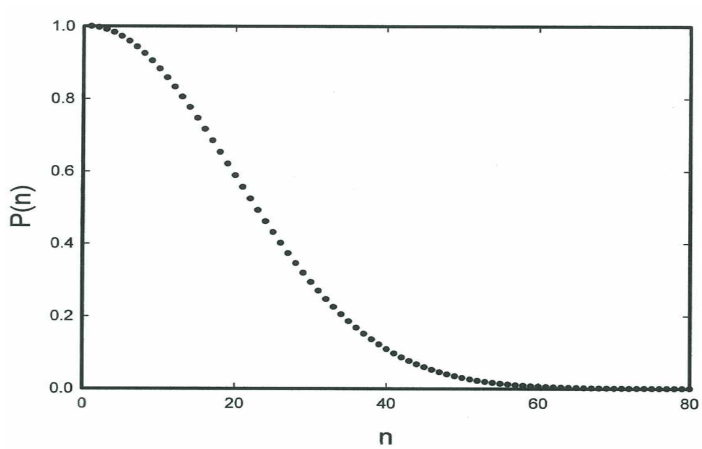
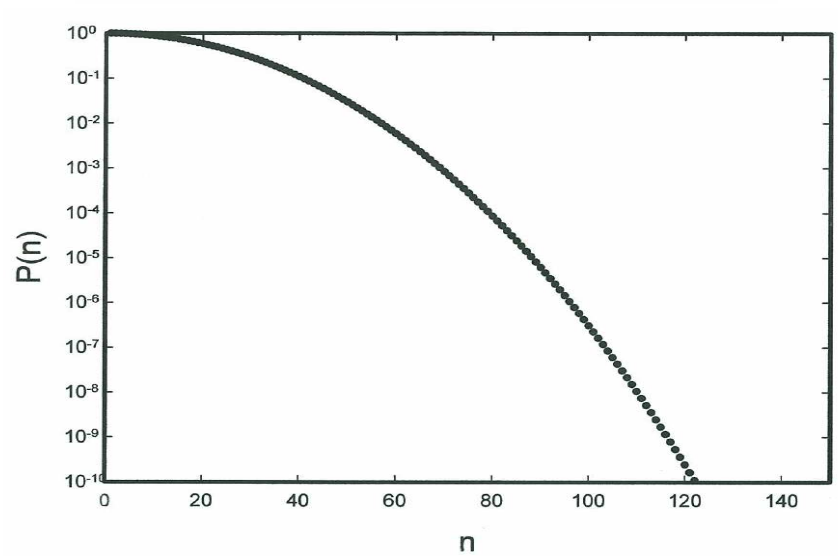
1. The sample space consists of all possible choices for the birthday of each person. Since there are n persons, and each has 365 choices for their birthday, the sample space has 365^n elements. Let us now consider those choices of birthdays for which no two persons have the same birthday. Assuming that $n \leq 365$, there are 365 choices for the first person, 364 for the second, etc., for a total of $365 \cdot 364 \cdots (365 - n + 1)$. Thus,

$$\mathbf{P}(\text{no two birthdays coincide}) = \frac{365 \cdot 364 \cdots (365 - n + 1)}{365^n}.$$

It is interesting to note that for n as small as 23, the probability that there are two persons with the same birthday is larger than 1/2.





2. As we have done before, we will count the number of favorable positions in which we can safely place 8 rooks, and then divide this by the total number of positions for 8 rooks on a 8×8 chessboard. First we count the number of favorable positions for the rooks. We will place the

rooks one by one. For the first rook, there are no constraints, so we have 64 choices. Placing this rook, however, eliminates one row and one column. Thus for our second rook, we can imagine that the illegal column and row have been removed, thus leaving us with a 7×7 chessboard, and thus with 49 choices. Similarly, for the third rook we have 36 choices, for the fourth 25, etc... There are $64 \cdot 63 \cdots 57$ total ways we can place 8 rooks without any restrictions, and therefore the probability we are after is:

$$\frac{64 \cdot 49 \cdot 36 \cdot 25 \cdot 16 \cdot 9 \cdot 4}{\frac{64!}{56!}}.$$

3. The sample space consists of the $\binom{n}{k}$ different ways that we can select k out of the available balls. For the event of interest to occur, we have to select i out of the m red balls, which can be done in $\binom{m}{i}$ ways, and also select $k-i$ out of the $n-m$ balls that are not red, which can be done in $\binom{n-m}{k-i}$ ways. Therefore, the desired probability is

$$\frac{\binom{m}{i}\binom{n-m}{k-i}}{\binom{n}{k}},$$

for $i \geq 0$ satisfying $i \leq m, i \leq k$, and $k-i \leq n-m$. For all other i, the probability is zero.