MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Spring 2010)

Problem Set 3
Due February 24, 2010

1. Count the number of different letter arrangements you can make by changing the order of the letters in the word **aardvark** (count the original word, too).

2. A candy factory has an endless supply of red, orange, yellow, green, blue, black, white, and violet jelly beans. The factory packages the jelly beans into jars in such a way that each jar has 200 beans, equal number of red and orange beans, equal number of yellow and green beans, one more black bean than the number blue beans, and three more violet beans than the number of white beans. One possible color distribution, for example, is a jar of 50 yellow, 50 green, one black, 48 white, and 51 violet jelly beans. As a marketing gimmick, the factory guarantees that no two jars have the same color distribution. What is the maximum number of jars the factory can produce?

3. Vivek offered Dwayne and Star two dollars, on the condition that they decide who gets the money by tossing a fair coin (actually, he even provided the coin and volunteered to do the tossing) until either the total number of tails reaches 20 (in which case Star gets the money) or the total number of heads reaches 20 (in which case Dwayne gets the money). At it turned out, Vivek was so eager to resume his lecture that he terminated the game in the middle. At the time the game was stopped, the count has reached 7 tails to 4 heads. Now, Vivek wants to divide the two dollars between Dwayne and Star in proportion to the probability of their winning, assuming the game would continue starting with the 7 to 4 count. How much money should Dwayne get according to this criterion?

4. A fair m-sided die (with sides numbered 1 through m) is tossed until getting "1" k times (not necessarily in a row). Define random variable N as the total number of tosses required (unless "1" never happens for the kth time, in which case we set $N = -1$). Find the PMF of N.

5. Mary, Tom, and Jerry park their cars in a parking lot that consists of $N > 2$ parking spaces, arranged in a circular fashion and numbered sequentially clock-wise from 1 to N, so that, for example, parking space number 1 is next to parking spaces number 2 and N. Assume that each possible triplet of (different) parking locations is equally likely. Let X be the random variable (taking integer values between 0 and $\frac{N-3}{3}$) defined as the minimum of the three non-negative integers representing the numbers of parking spaces between the cars of Mary, Tom, and Jerry. For example, if $N = 100$ and Mary, Tom, and Jerry have parked their cars in slots number 2, 20, and 97 respectively, then $X = 4$ is the number of parking slots (98, 99, 100, and 1 if listed clock-wise) between the cars of Mary and Jerry. Find the probability mass function of X, and sketch it graphically.

6. The PMF of random variable X is given by $p_X(k) = \frac{2k}{n^2+n}$ for $k \in \{1, 2, \ldots, n\}$. Find the PMF of $Y = f(X)$, where $f(x) = \min\{x, n-x\}$.

7. [**6.431 question**] X and Y are random variables with probability mass functions such that $p_X(0) = p_Y(0) = 1/3$, and $p_X(1) = p_Y(1) = 2/3$. Random variable Z is defined by $Z = X - Y$.
   (a) What are the possible values of $p_Z(0)$?
   (b) Assuming $q = p_Z(0)$ is given, what is the PMF of Z?