Network dynamics III: Opinion dynamics

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Abstract

We study opinion dynamics in complex networks. Each node represents an individual holding a binary opinion, and the edges between nodes represent social interactions through which opinions can be influenced. Using a hybrid model constructed from majority dynamics and voter dynamics, we analyze the evolution of dynamics through Erdos Renyi random networks. As a result, we seethe importance of stochasticity and choosing pairwise relationships for reaching to consensus.

1 Introduction

A popular approach adopted in social opinion evolution and opinion formation is to model social opinions into a binary system. The majority model and the voter model are two of the most widely used models in opinion dynamics research. In the classic majority model, each individual adopts an opinion following the simple majority of her or his immediate neighbors. In the voter model, individuals update their opinions by randomly selecting one of their neighbors and adopting that opinion. In the following, we will use a hybrid model combining elements of both the classic majority model and the voter model to explore the effects of random pairwise interactions relative to local consensus on the dynamics of opinion evolution.

2 Model

As a hybrid dynamic, each node on a network can either update its opinion by adopting the neighbors' majority opinion or by randomly choosing a neighbor's opinion. The probability of q determines whether to use the majority or the voter dynamics on each node. The focal node takes the opinion of the majority of its neighbors with probability q and adopts a random neighbor's opinion with probability $1 - q$.

3 Results

3.1 Examples

First, we look at three examples with q equal to 0, 0.5, and 1 to see general differences between a pure majority dynamics and a pure voter model and a dynamics equally choosing between these two.

3.1.1 Pure Voter Dynamics ($q = 0$)

When we just have voter dynamics, Fig.1, as it focuses on random pairwise interactions to choose the opinion of one of the neighbors, we see that it is possible for one opinion population (blue) to grow near consensus and then decrease and reach consensus on the other opinion (pink). It shows focusing on pairwise interactions makes it possible for relatively unpopular opinions to grow into consensus.

3.1.2 Pure Majority Dynamics ($q = 1$)

As an example for $q = 0.5$ equal probability for both dynamics, Fig.3, we see a slight growth in blue at the beginning but as we go forward in time pink grows and takes the majority, and then with a few steps it reaches the full population. When we have a majority of opinions in the network the majority dynamics can be relatively fast for reaching consensus (in a dense network).

3.1.3 Equal Probability ($q = 0.5$)

In this example (Fig.2) we see a pure majority dynamics, and as we expect in a few iterations we reach to consensus.

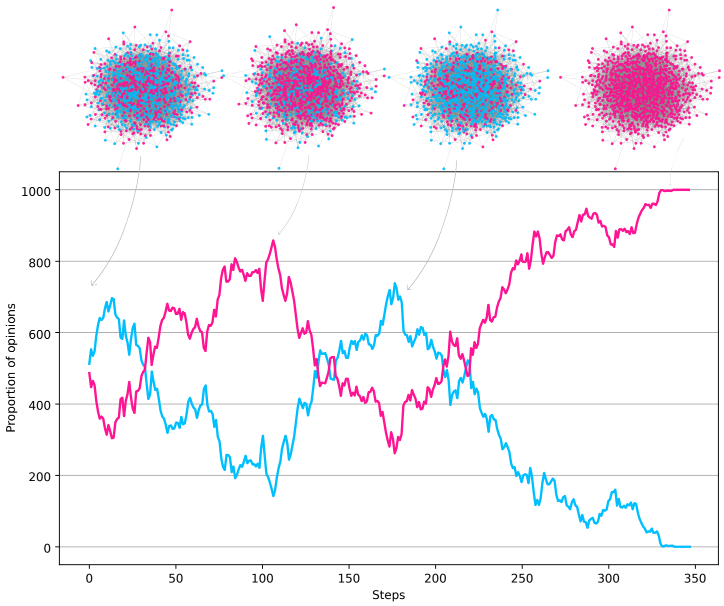

Figure 1: Pure voter model. Opinions are pink and blue. The initial configuration is with equal probability for pink and blue. Despite blue being the majority on the third network visualization, in the end, the system reaches a consensus on pink.

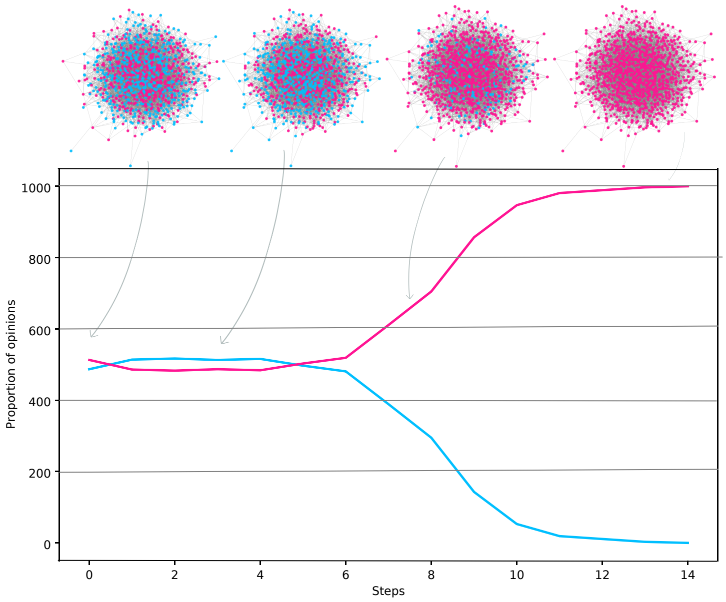

Figure 2: Equal chance for the majority dynamics and the voter dynamics. Opinions are pink and blue. The initial configuration is with equal probability for pink and blue. Despite the slight growth of blue at first pink wins.

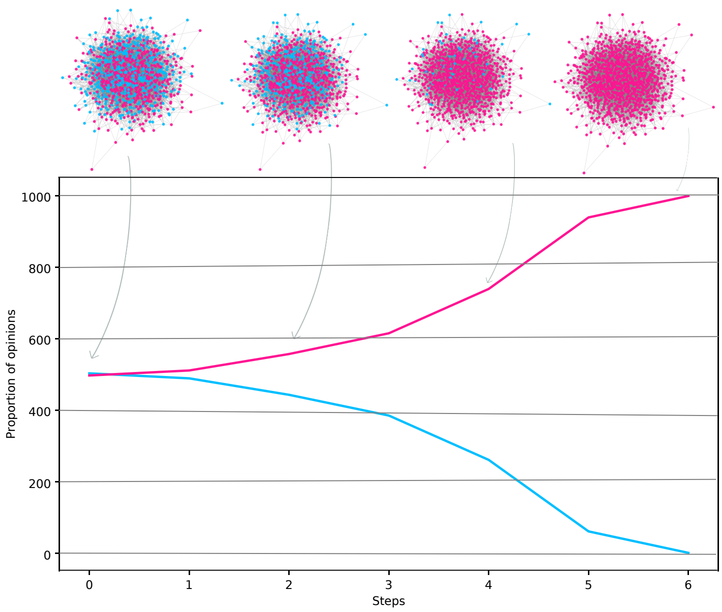

Figure 3: Pure majority dynamics. Opinions are pink and blue. The initial configuration is with equal probability for pink and blue. The system reaches to consensus on pink in a few steps.

3.2 Consensus Rate

Now, we want to see the fraction of samples reaching consensus in each q in an ensemble of ER networks. For each q between zero and one, we generate M (ensemble size) ER random networks with random opinion configuration with equal and independent probability of individuals getting opinions zero or one.

3.2.1 Dense Network

For dense ER networks with $p = 0.01$ (edge probability) and $N = 1000$ (Fig.4), the consensus rate always remains near one. This is because of the high edge density of the ER graphs with $p = 0.01$, which enables majority dynamics to reach consensus. A small fraction of samples are not reaching consensus. In these samples, we always have more than one connected component, which makes it impossible for the opinions to interact and reach a consensus between the components.

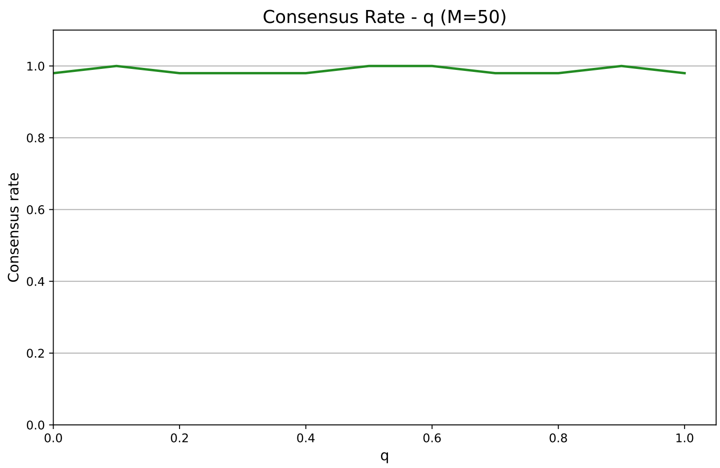

Figure 4: For each q we run the dynamics in ($M = 50$) different ER networks with different initial configurations and $p = 0.01$. Almost all runs reach to a consensus.

3.2.2 Sparse Networks

For sparse ER networks with $p = 0.002$ (edge probability) and $N = 1000$ (Fig.5), the consensus rate decreases as we increase the probability of choosing the majority dynamics over the voter dynamics (q). Here we just look at the biggest connected component of the ER network, as it is impossible for disconnected nodes to reach consensus with the giant component. As we know, for pure majority dynamics ($q = 1$) on sparse ER networks, we do not reach consensus; therefore, as we expect, the consensus drops for higher q values.

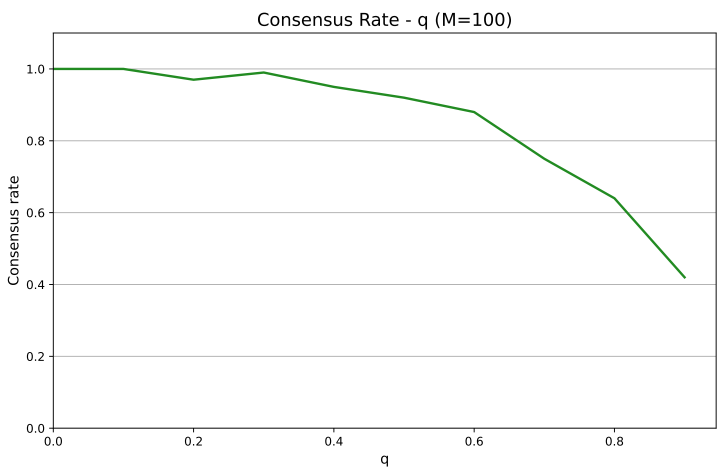

Figure 5: For each q we run the dynamics in ($M = 100$) different ER networks with different initial configurations and $p = 0.002$. Increasing q the consensus rate drops as the majority model does not reach consensus on sparse networks.

3.3 Average Steps to Reach Consensus

Using the ER network ensembles above, we now look at the average number of time steps it took to reach consensus at each q.

3.3.1 Dense Network

For dense ER networks with $p = 0.01$ (edge probability) and $N = 1000$ (Fig.6), the average steps to consensus drops fast as we decrease the probability of voter dynamics $(1 - q)$. The stochastic nature of the voter model does not allow the dynamics to easily reach the consensus steady state; therefore, as q increases, we see a lower number of average steps needed to reach consensus.

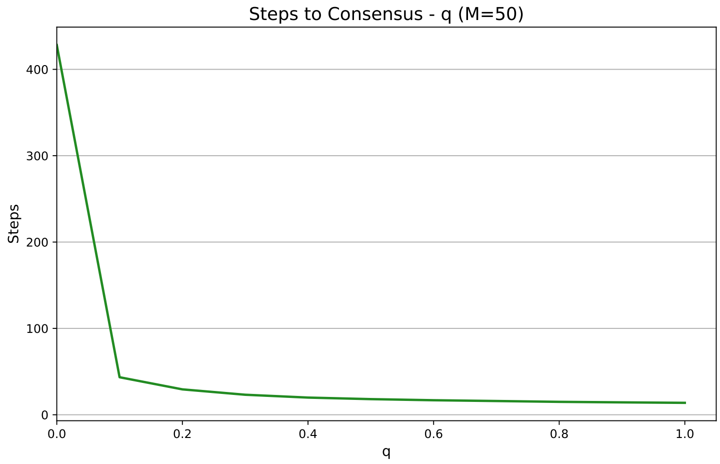

Figure 6: For each q we run the dynamics in $(M = 50)$ different ER networks with different initial configurations and $p = 0.01$. Average steps decrease fast by lowering the probability of the vote dynamics.

3.3.2 Sparse Networks

For sparse ER networks with $p = 0.002$ (edge probability) and $N = 1000$ (Fig.7), as q increases, the average steps to consensus first drops and rises again when q reaches one. The first drop we see when increasing q (lower probability for voter dynamics) is what we expected and saw in (Fig.6), but the increase at the end is because the majority dynamics makes it harder to reach consensus in sparse networks at large q.

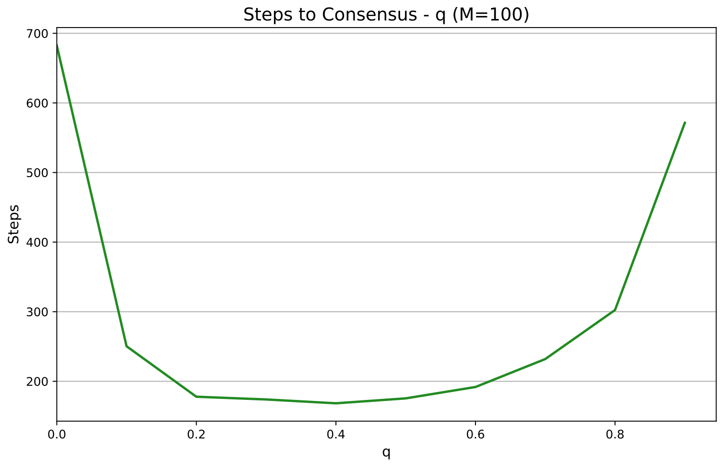

Figure 7: For each q we run the dynamics in $(M = 50)$ different ER networks with different initial configurations and $p = 0.01$. The increase at the q near one is because the majority model does not reach consensus in sparse networks.

4 Conclusion

By using the hybrid model implementing both voter and majority dynamics we have been able to change the ratio of dynamics' focus on pairwise and group interactions. We see by increasing the importance of pairwise relations relative to group interactions the system generally has more chance to reach a consensus although it may take a long time, furthermore, we see it gets probable to reach a consensus on the minority opinion by increasing the chance of voter dynamics (more focus on pairwise interactions).