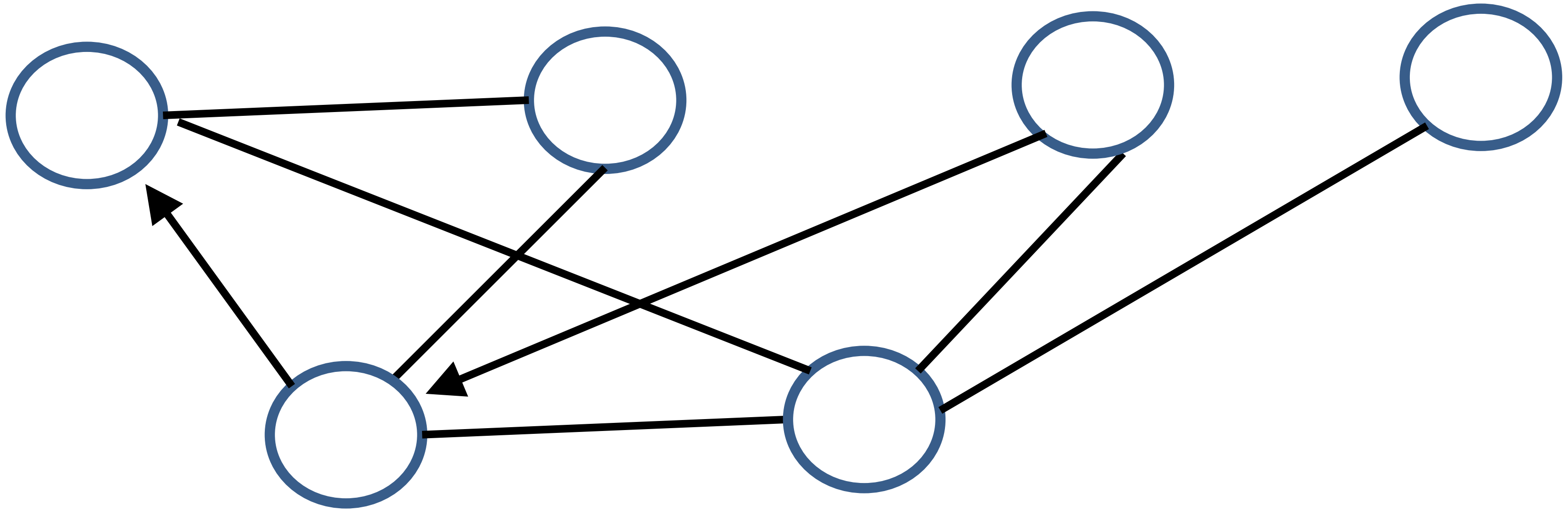
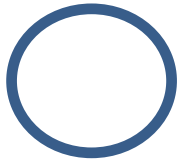
Graph Algorithms

Graphs



Nodes/vertexes: 

Edges: (undirected)

(directed)

Representations of graph G with vertices V and edges E

- V x V adjacency-matrix A: $A_{u, v} = 1 \Leftrightarrow (u, v) \in E$

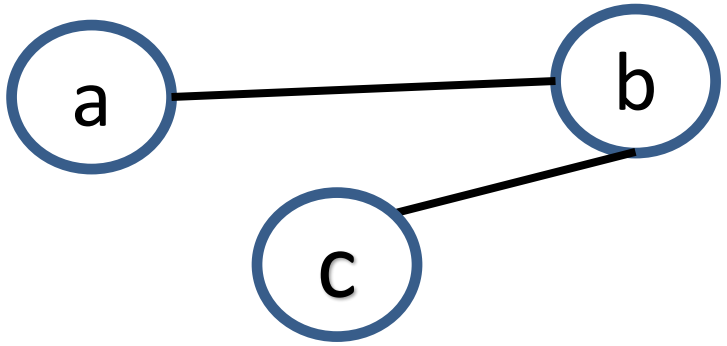
  Size: $|V|^2$

  Better for dense graphs, i.e., $|E| = \Omega(|V|^2)$

- Adjaceny-list, e.g. $(v_1 , v_5), (v_1 , v_{17}), (v_2 , v_3) \ldots$

  Size: O(E)

  Better for sparse graphs, i.e., $|E| = O(|V|)$



	a	**b**	**c**
a	0	1	0
b	1	0	1
c	0	1	0

Adj[a] = b

Adj[b] = a,c

Adj[c] = b

Next we see several algorithms to compute shortest distance

$\delta(u,v)$:= shortest distance from u to v
∞ if v is not reachable from u

Variants include weighted/unweighted, single-source/all-pairs

Algorithms will construct vector/matrix d; we want $d = \delta$

Back pointers π can be computed to reconstruct path

Breadth-first search

Input:

Graph G= (V,E) as adjacency list, and $s \in V$.

Output:

Distance from s to any other vertex

- Discover vertices at distance k before those at distance k+1

Algorithm colors each vertex:

White : not discovered.
Gray : discovered but its neighbors may not be.
Black : discovered and all of its neighbors are too.

```
BFS(G,s)
 For each vertex u ∈ V[G] – {s}
   color[u]:= White; d[u] := ∞; π[u] :=NIL;

 Q:= empty Queue; color[s] := Gray;  d[s] :=0; π[s] :=NIL;

 Enqueue(Q,s)
 While (|Q| > 0) {
  u := Dequeue(Q)        // a vertex with min distance d[u];
  for each v ∈ adj[u]    // checks neighbors
    if color[v] = white {
     color[v] := gray;
     d[v]:= d[u]+1;
     π[v]:=u;
     Enqueue(Q,v)
   }
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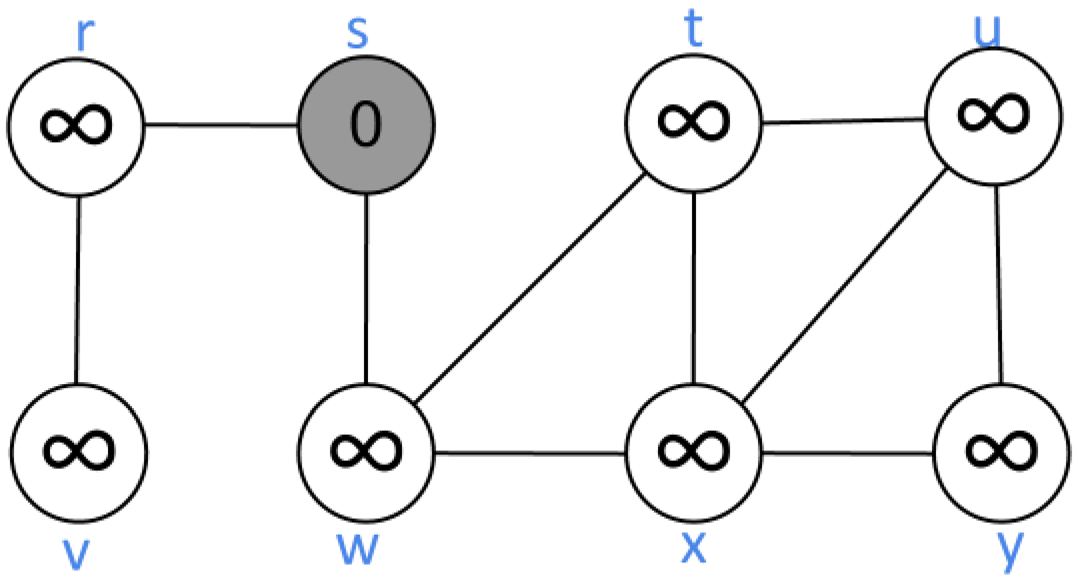

Q	s
d	0

```
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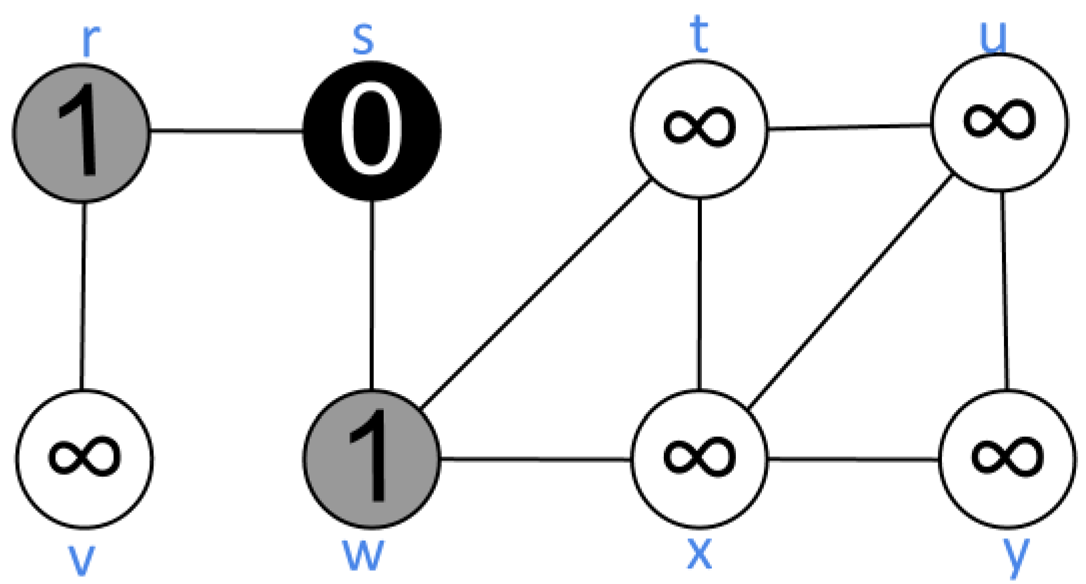

Q	w	r
d	1	1

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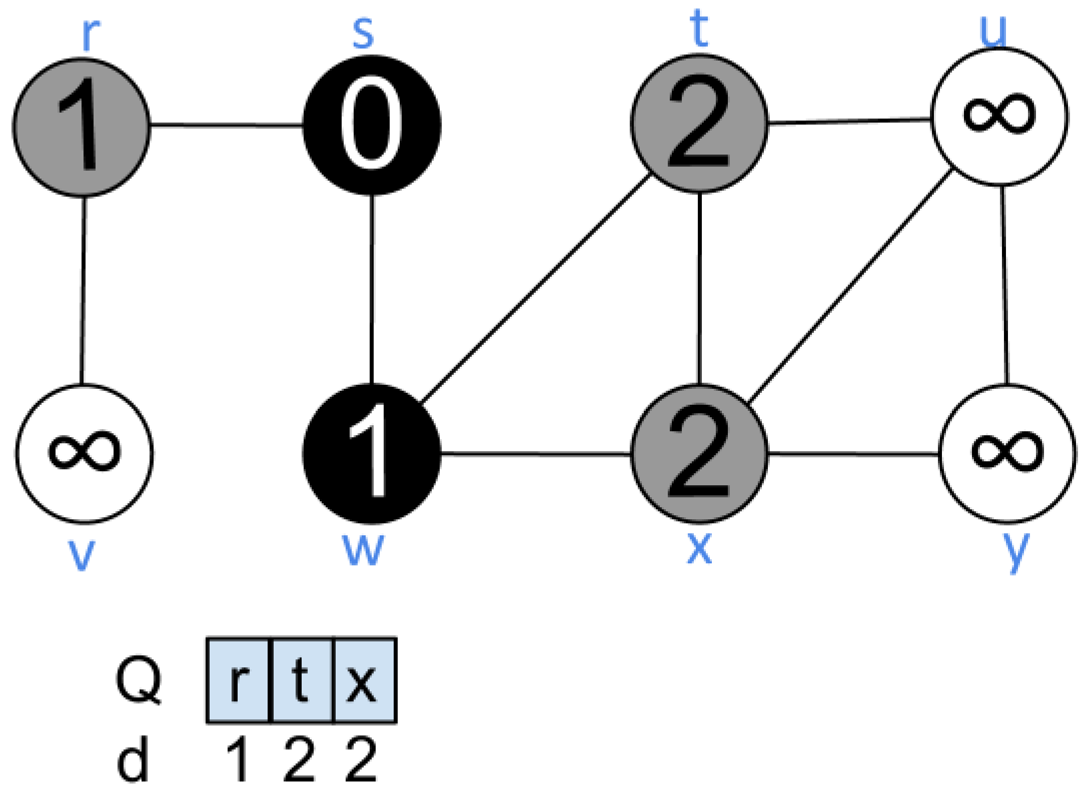

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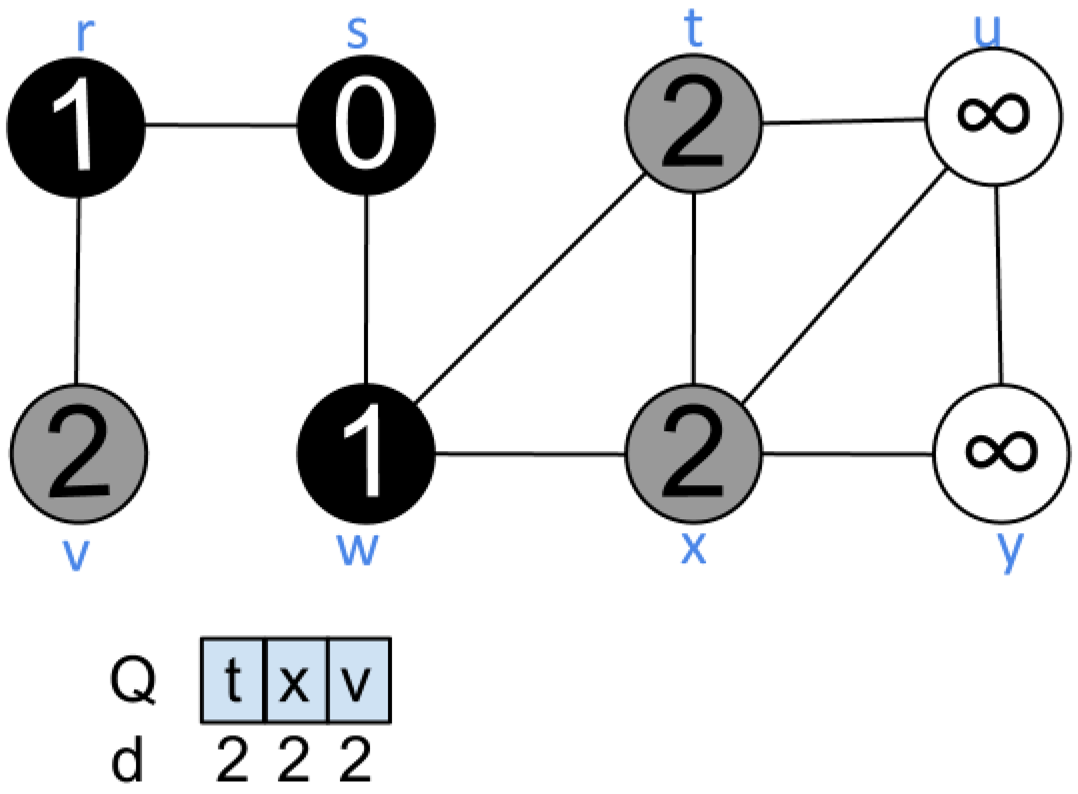

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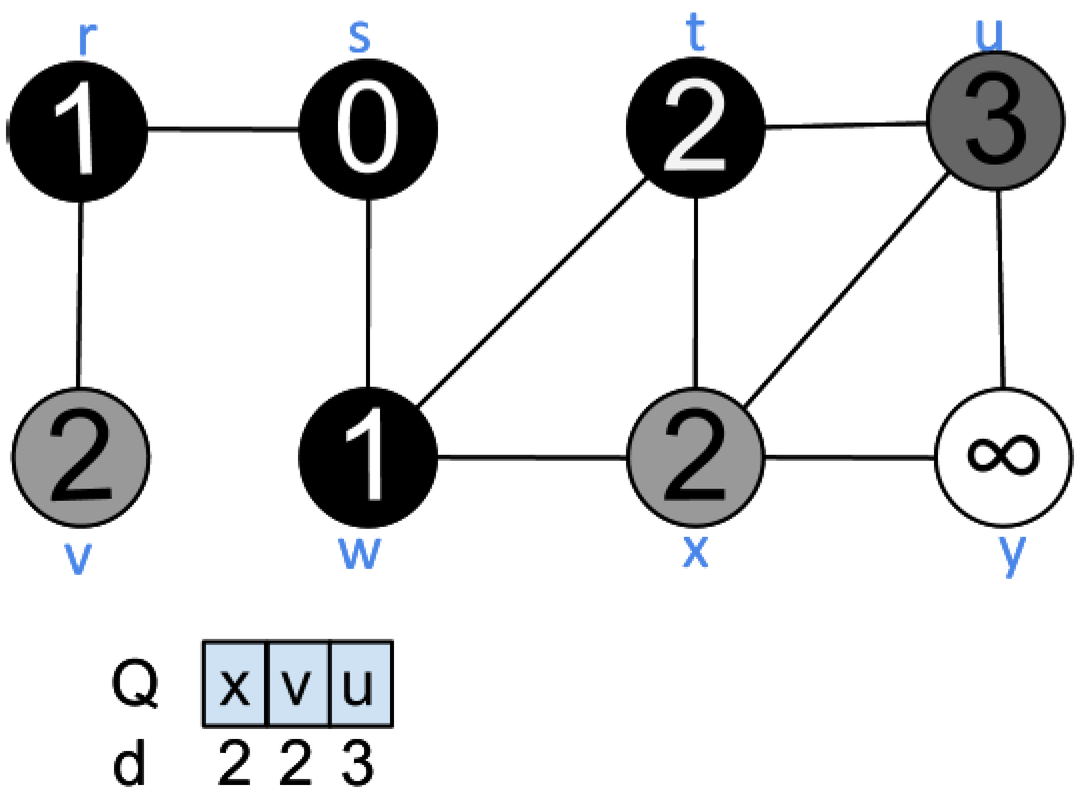

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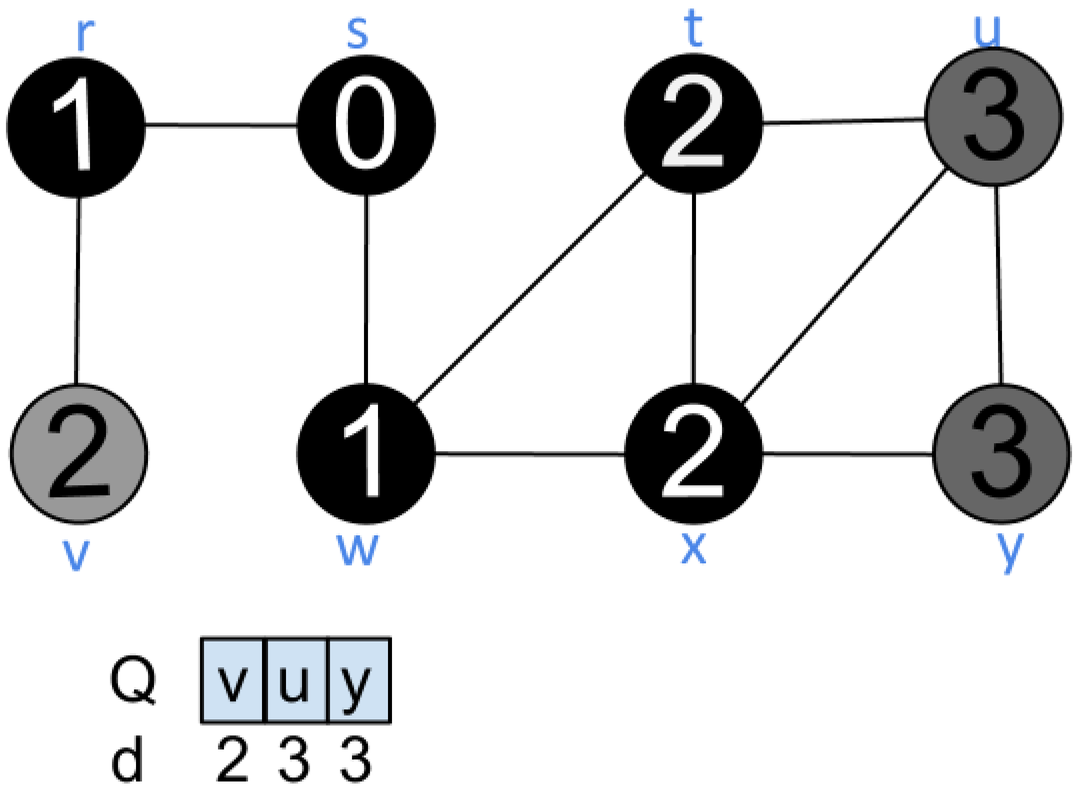

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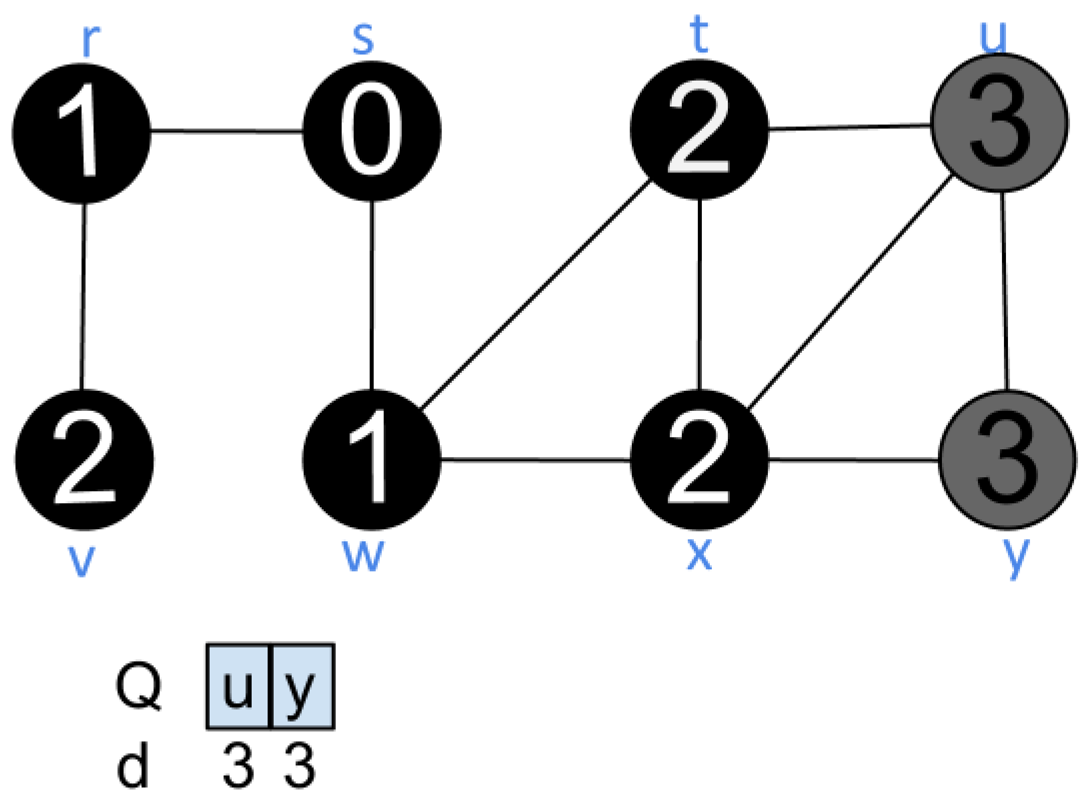

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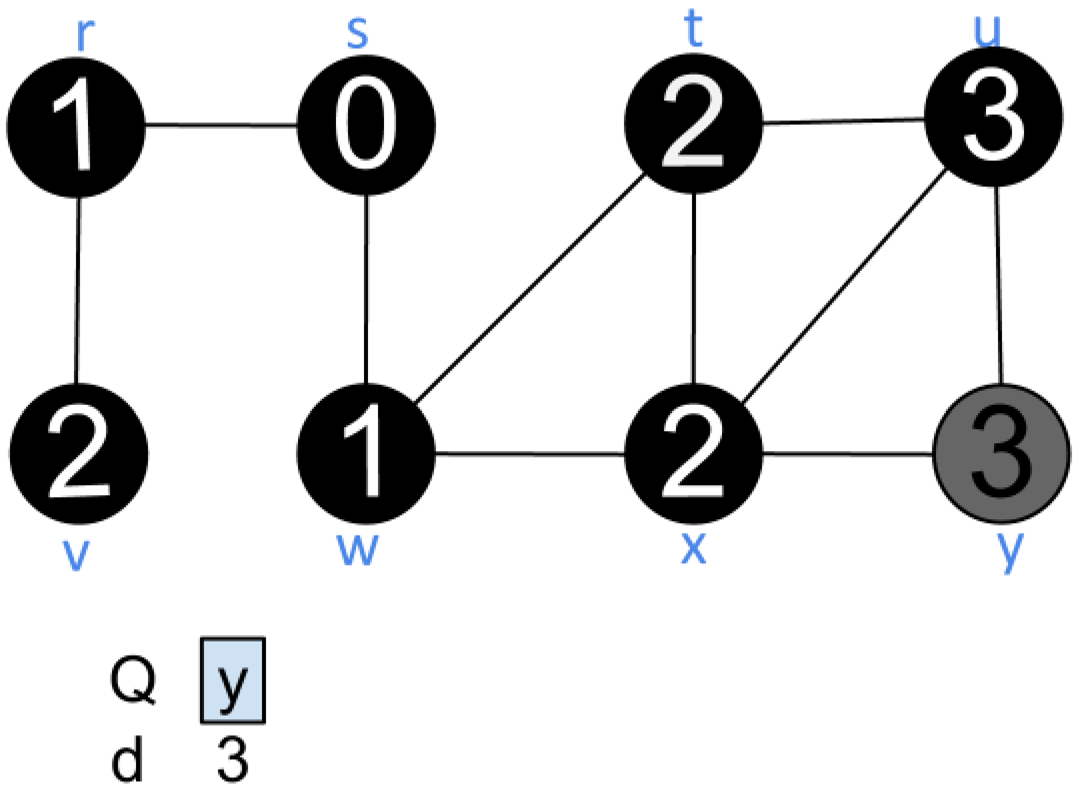

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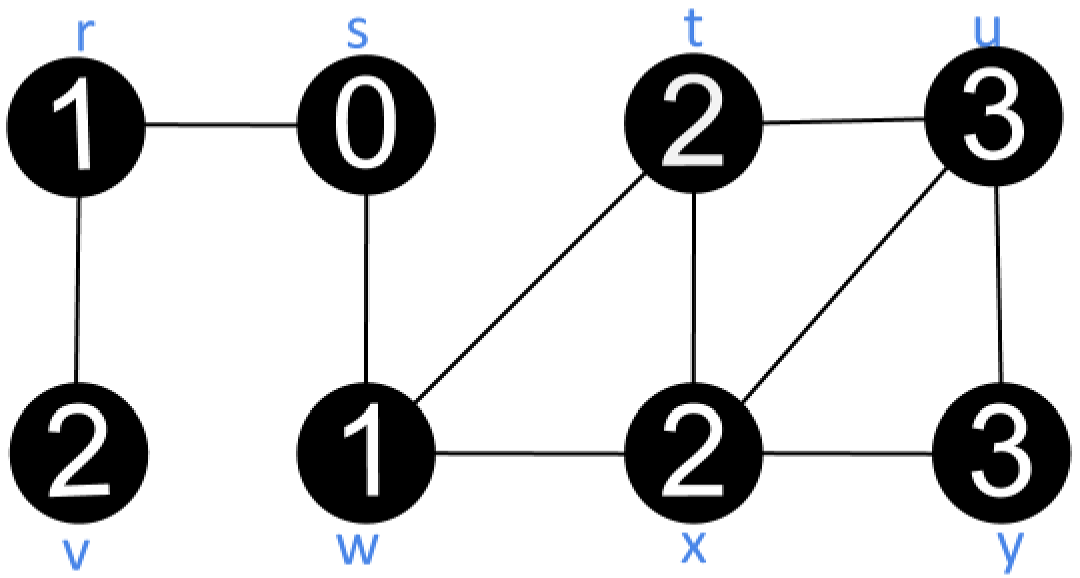


Q

d

Running time of BFS in adjacency-list representation

Recall Enqueue and Dequeue take time ?

Running time of BFS in adjacency-list representation

Recall Enqueue and Dequeue take time $O(1)$

Each edge visited $O(1)$ times.

Main loop costs $O(E)$.

Initialization step costs $O(V)$

Running time $O(V + E)$

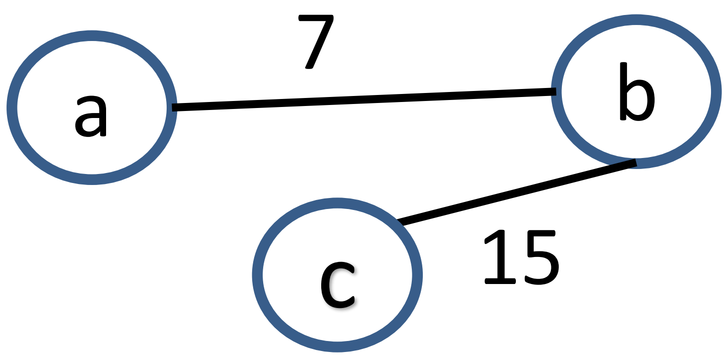
What about space?

Space of BFS

$\Theta(V)$ to mark nodes

Optimal to compute all of d

Next: weighted single-source shortest path



Input: Directed graph $G= (V,E)$, $s \in V$, $w: E \to Z$

Output: Shortest paths from s to all the other vertces

- Note: Previous case was for $w : E \to \{1\}$

- Note: if weights can be negative, shortest paths exist $\Leftrightarrow$ s cannot reach a cycle with negative weight

```
Bellman-Ford(G,w, s)
  d[s] :=0; Set the others to ∞

 Repeat |V| stages:
   for each edge (u,v) ∈ E[G]
       d[v] := min{ d[v], d[u]+w(u,v); }  //relax(u,v)
```

At the end of the algorithm, can detect negative cycles by:

```
for each edge (u,v) ∈ E[G]
   if d[v] > d[u]+w(u,v)
       Return Negative cycle

return No negative cycle
```

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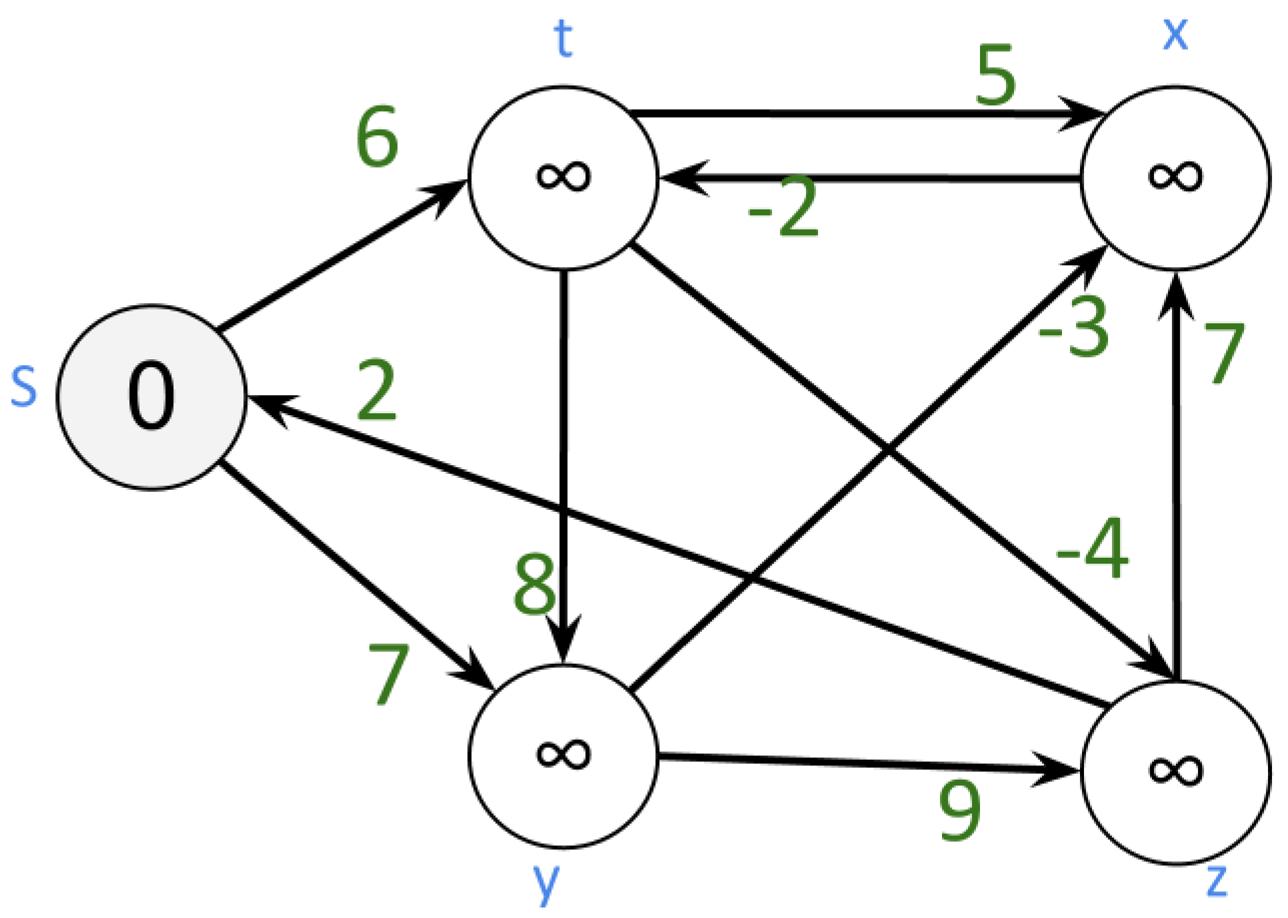

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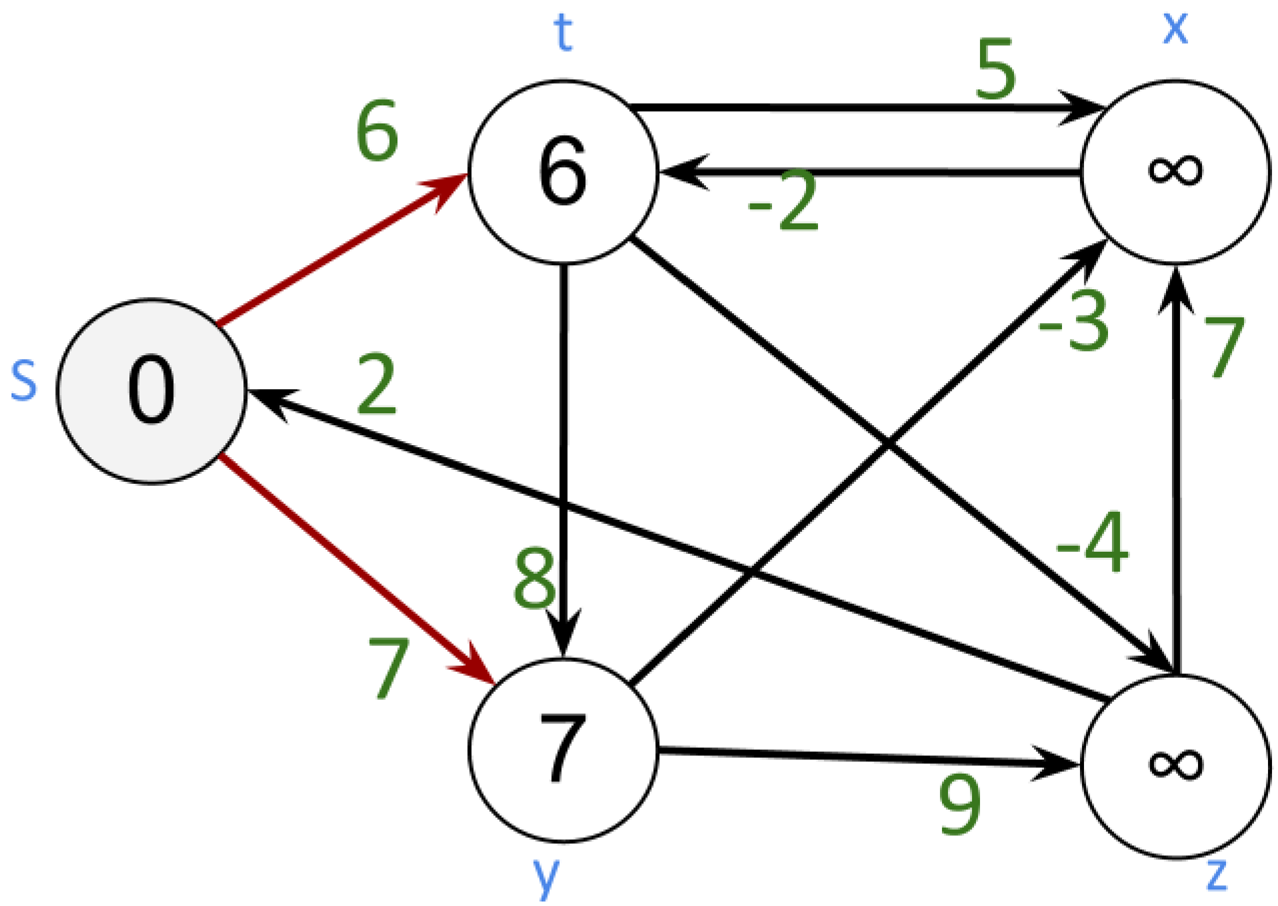

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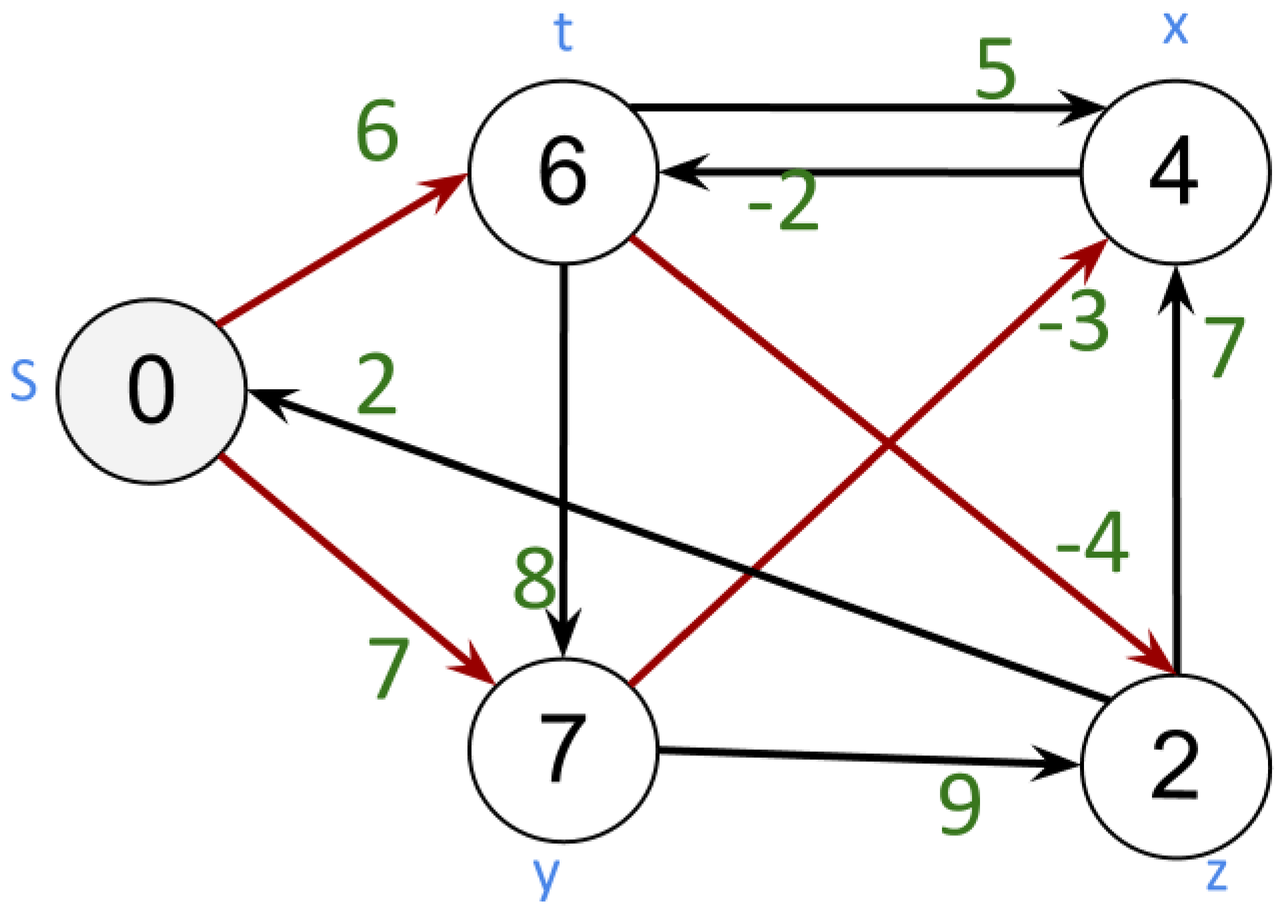

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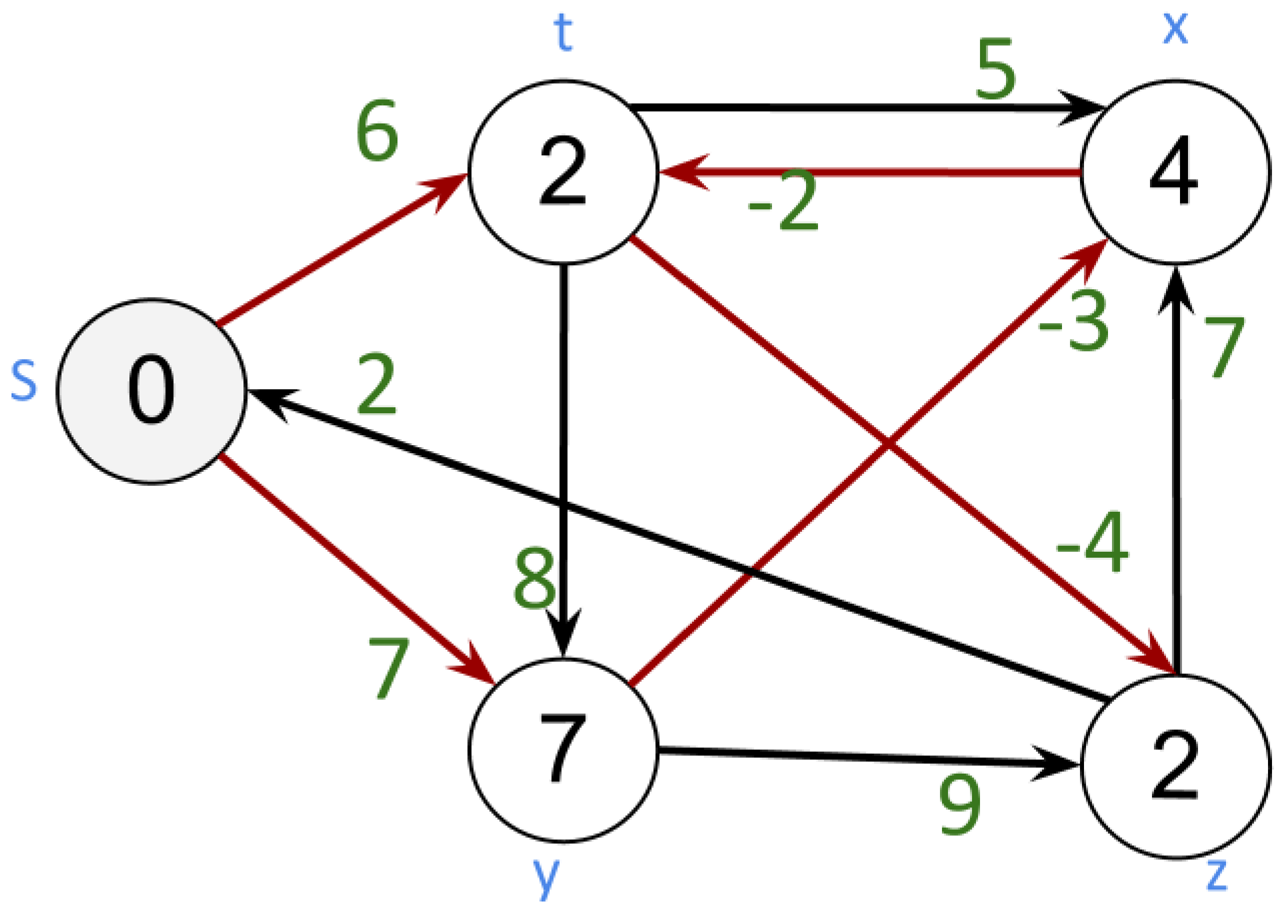

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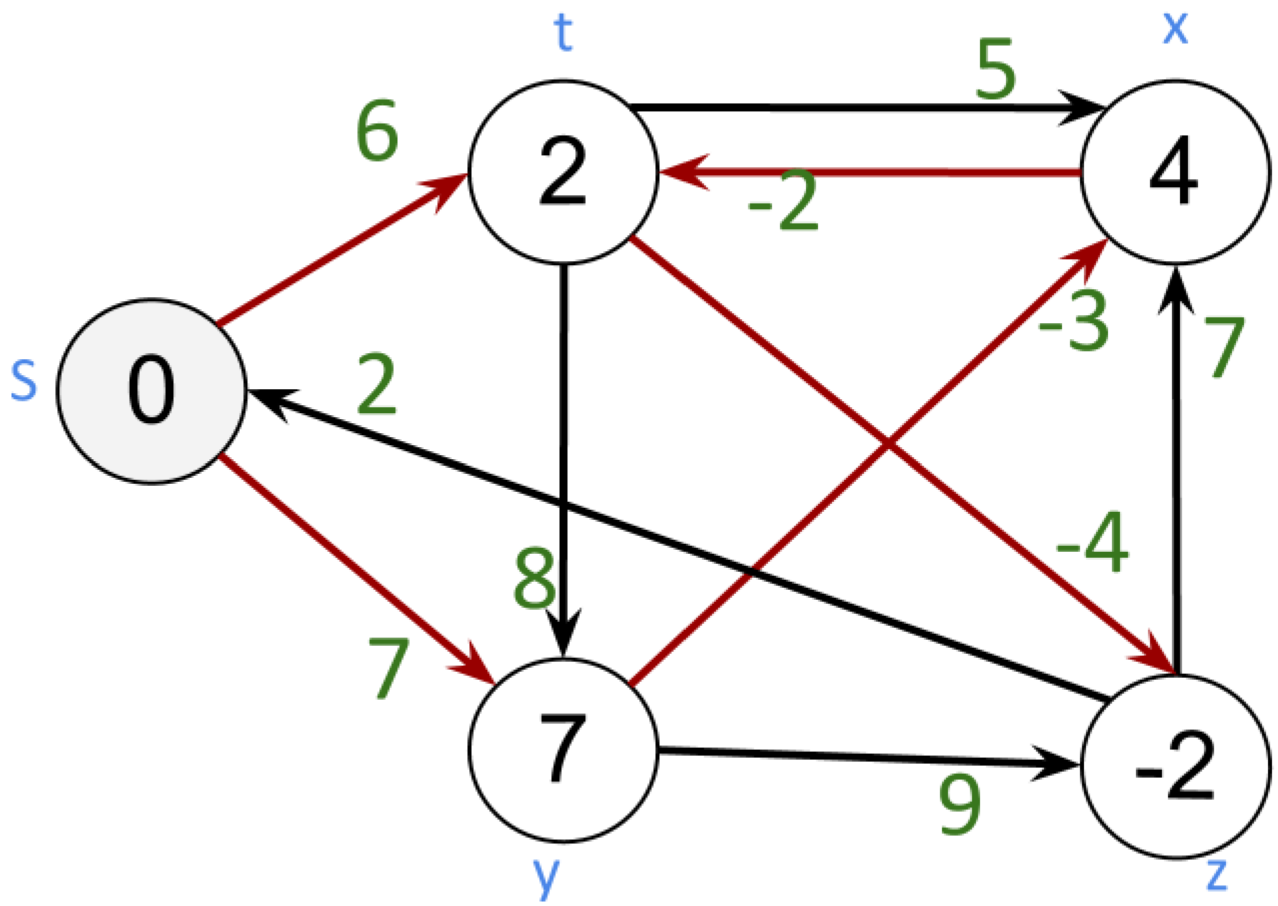

Running time of Bellman-Ford

```
Bellman-Ford(G,w, s)
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```

Time = ??

Running time of Bellman-Ford

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    for each edge $(u,v) \in E[G]$
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Time = $O(|V|.|E|)$

Dijkstra's algorithm

Input: Directed graph $G=(V,E)$, $s \in V$, non-negative $w: E \rightarrow N$

Output: Shortest paths from s to all the other vertices.

```
Dijkstra(G,w,s)
  d[s] :=0; Set others d to ∞; Q := V

  While (|Q|> 0) {
    u:= extract-remove-min(Q) // vertex with min distance d[u];
    for each v ∈ adj[u]
        d[v] := min{ d[v], d[u]+w(u,v)}    //relax(u,v)
  }
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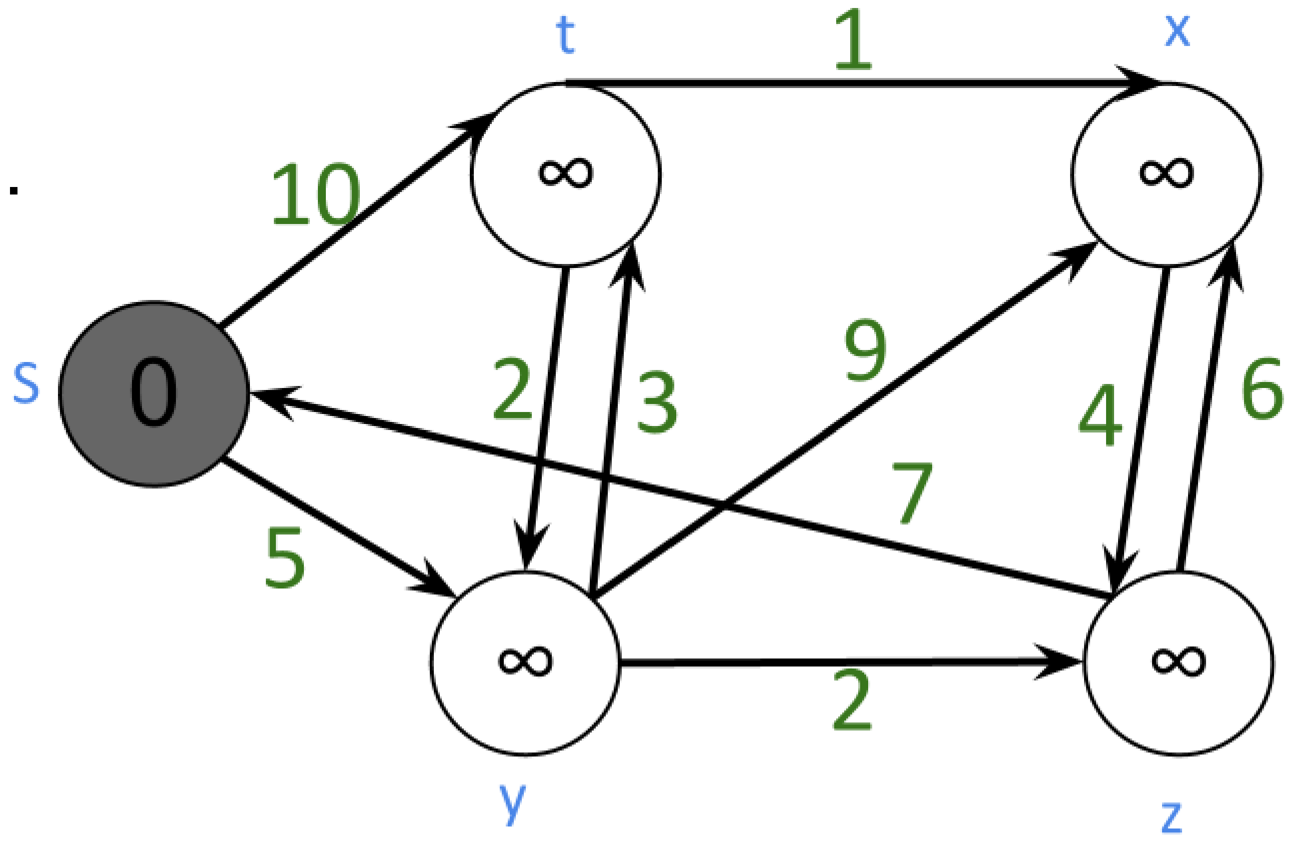
  }

Gray: the extracted u.
Black: not in Q



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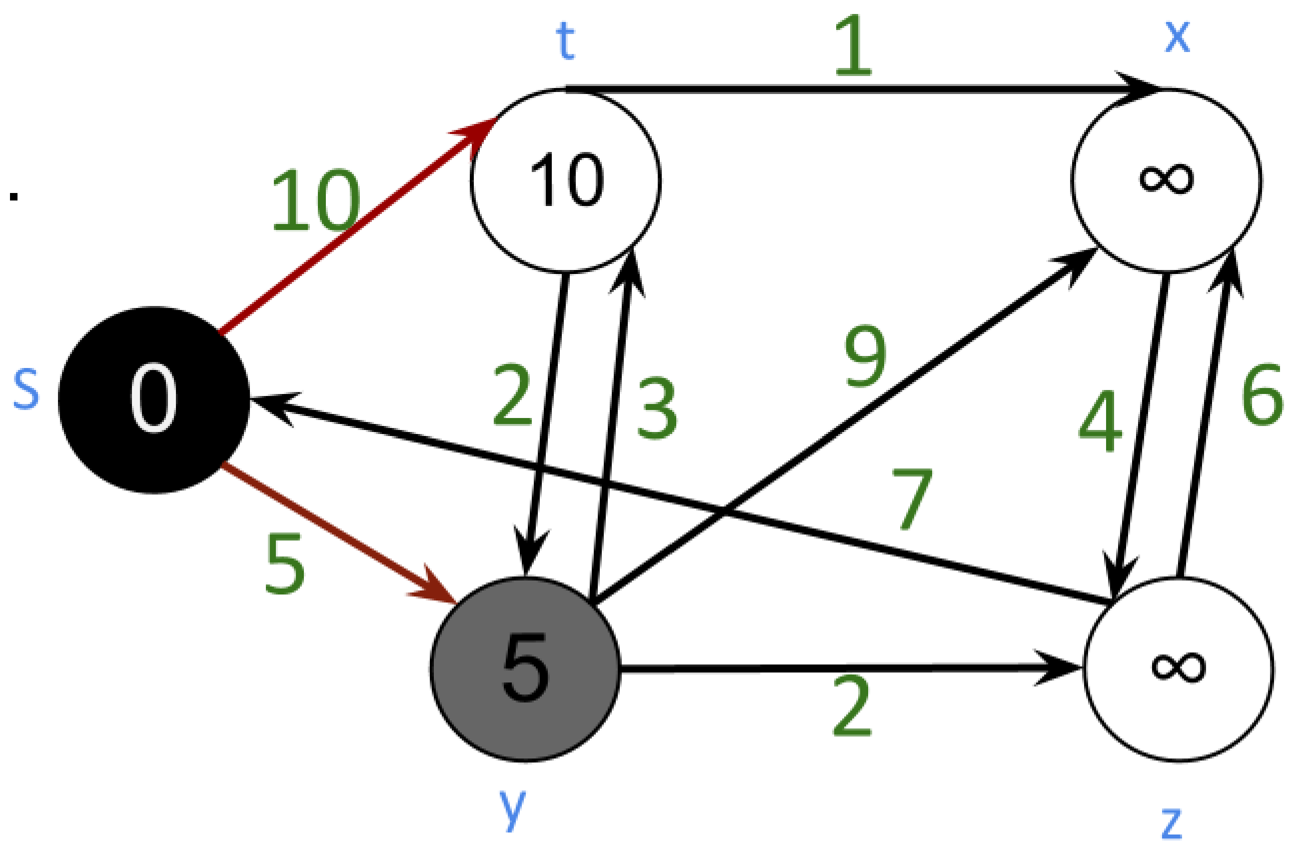

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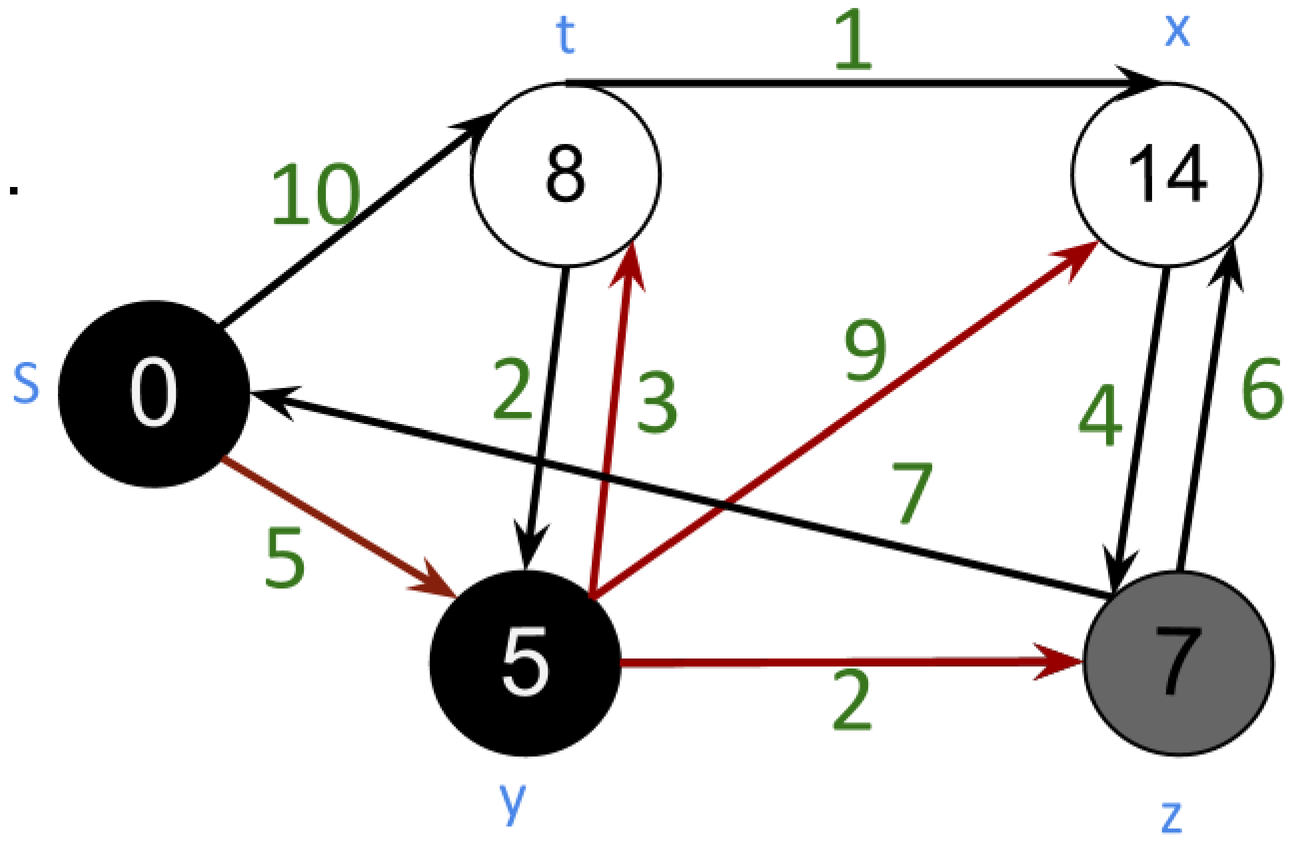

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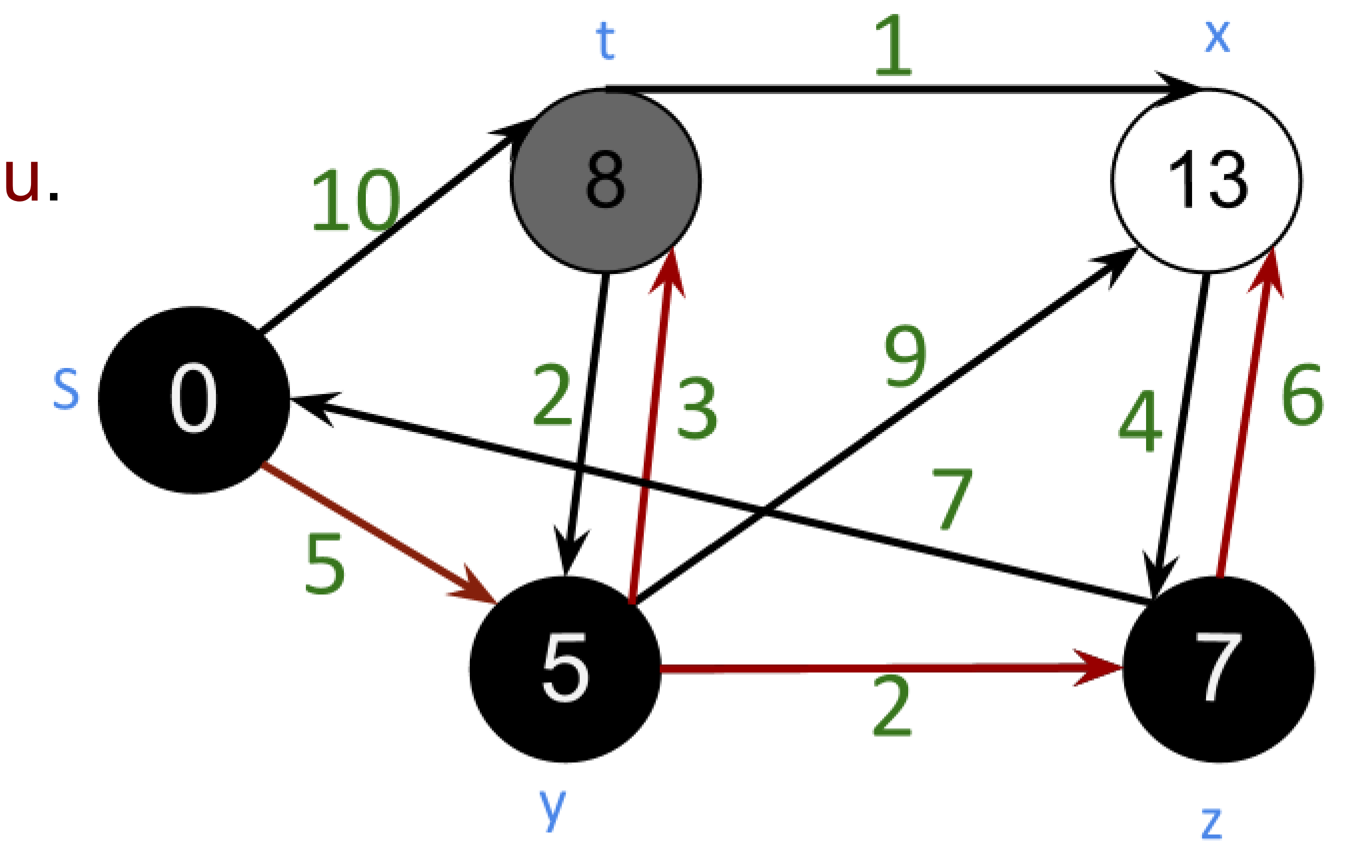

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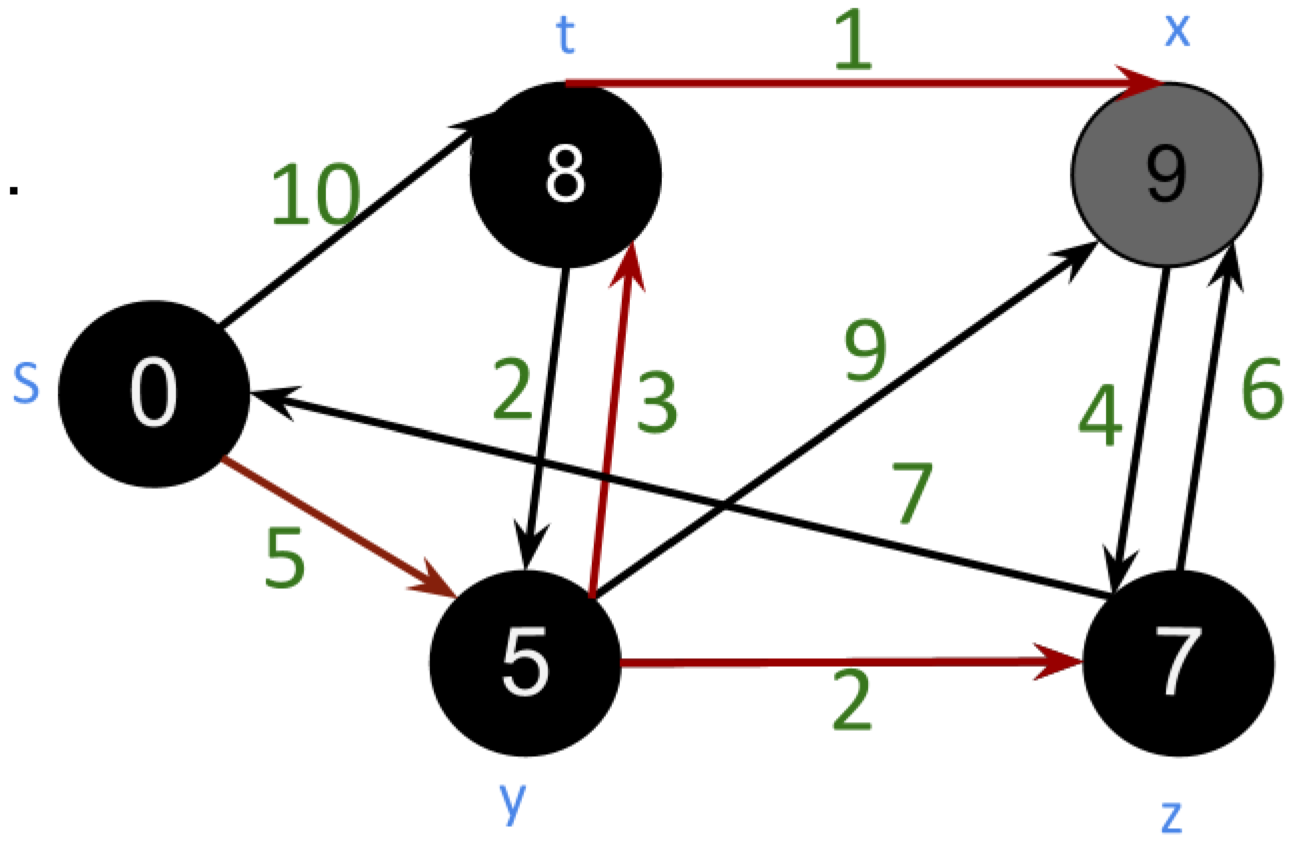

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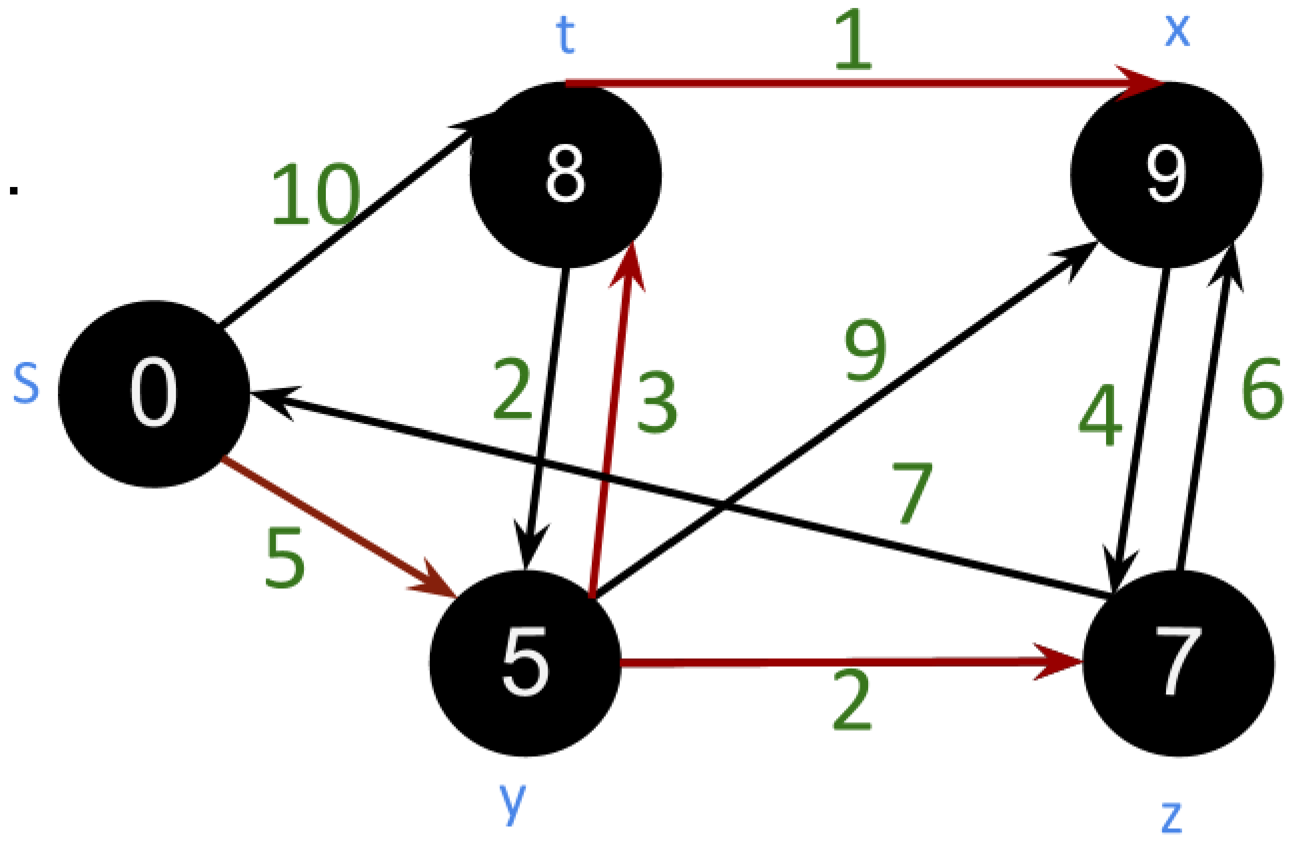

Running time of Dijkstra(G,w,s)

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Running time depends on data structure for Q

Naive implementation, array: Extract-min in time ?

Running time of Dijkstra(G,w,s)

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Running time depends on data structure for Q

Naive implementation, array: Extract-min in time $|V|$

$\Rightarrow$ running time = ?

Running time of Dijkstra(G,w,s)

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Running time depends on data structure for Q

Naive implementation, array: Extract-min in time |V|

$\Rightarrow$ running time = $O(V^2 + E)$

Can we do better?

Running time of Dijkstra(G,w,s)

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Running time depends on data structure for Q

Naive implementation, array: Extract-min in time |V|

$\Rightarrow$ running time = $O(V^2 + E)$

Implement Q with min-heap. Extract-min in time ?

Running time of Dijkstra(G,w,s)

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Running time depends on data structure for Q

Naive implementation, array: Extract-min in time |V|

$\Rightarrow$ running time = $O(V^2 + E)$

Implement Q with min-heap. Extract-min in time O(log V)
$\Rightarrow$ running time = ?

Running time of Dijkstra(G,w,s)

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Running time depends on data structure for Q

Naive implementation, array: Extract-min in time |V|

$\Rightarrow$ running time = $O(V^2 + E)$

Implement Q with min-heap. Extract-min in time O(log V)

$\Rightarrow$ running time =

$$V \log V + E$$

All-pairs shortest paths

Input:

- Directed graph $G = (V,E)$, and $w: E \to R$

Output:

- The shortest paths between all pairs of vertices.

- Run Dijkstra $|V|$ times: $O(V^2 \log V + E V)$ if $w \geq 0$
- Run Bellman-Ford $|V|$ times: $O(V^2 E)$

- Next, simple algorithms achieving time about $|V|^3$ for any w

All-pairs shortest paths

Dynamic programming approach:
$d_{i,j}^{(m)}$ = shortest paths of lengths $\leq m$

$d_{i,j}^{(m)} = \min_k \{ d_{i,k}^{(m-1)} + w(k,j) \}$

(Includes $k = j$, $w(j,j) = 0$)

Compute $|V| \times |V|$ matrix $d^{(m)}$ from $d^{(m-1)}$ in time $|V|^3$.

$\Rightarrow d^{|V|}$ computables in time $|V|^4$

How to speed up?

All-pairs shortest paths

Note:

$d_{i,j}^{(m)} = \min_k \{ d_{i,k}^{(m-1)} + w(k,j) \}$

Is just like matrix multiplication: $d^{(m)} = d^{(m-1)} W$,

except $+ \rightarrow \min$

$\times \rightarrow +$

Like matrix multiplication, this is associative. So, instead of doing $d^{|V|} = (\ldots)W)W)W$ can do ?

All-pairs shortest paths

Note:

$d_{i,j}^{(m)} = \min_k \{ d_{i,k}^{(m-1)} + w(k,j) \}$

Is just like matrix multiplication: $d^{(m)} = d^{(m-1)} W$,

except $+ \rightarrow \min$

$x \rightarrow +$

Like matrix multiplication, this is associative. So,

instead of doing $d^{|V|} = (...)W)W)W$ can do repeated squaring:

Compute $d^{(2)} = W^2$

$d^{(4)} =$?

All-pairs shortest paths

Note:

$d_{i,j}^{(m)} = \min_k \{ d_{i,k}^{(m-1)} + w(k,j) \}$

Is just like matrix multiplication: $d^{(m)} = d^{(m-1)} W$,
except $+ \rightarrow \min$
$\quad\quad x \rightarrow +$

Like matrix multiplication, this is associative. So,
instead of doing $d^{|V|} = (...)W)W)W$ can do repeated squaring:

Compute $d^{(2)} = W^2$
$\quad\quad d^{(4)} = d^{(2)} \times d^{(2)} = W^2 \times W^2$
$\quad\quad d^{(8)} = $?

All-pairs shortest paths

Note:

$d_{i,j}^{(m)} = \min_k \{ d_{i,k}^{(m-1)} + w(k,j) \}$

Is just like matrix multiplication: $d^{(m)} = d^{(m-1)} W$,
except $+ \rightarrow \min$
$\quad x \rightarrow +$

Like matrix multiplication, this is associative. So,
instead of doing $d^{|V|} = (\ldots)W)W)W$ can do repeated squaring:

Compute $d^{(2)} = W^2$

$d^{(4)} = d^{(2)} \times d^{(2)} = W^2 \times W^2$

$d^{(8)} = d^{(4)} \times d^{(4)}$

...

To get $d^{|V|}$ need ?

All-pairs shortest paths

Note:

$d_{i,j}^{(m)} = \min_k \{ d_{i,k}^{(m-1)} + w(k,j) \}$

Is just like matrix multiplication: $d^{(m)} = d^{(m-1)} W$,
except $+ \rightarrow \min$
$\quad\quad x \rightarrow +$

Like matrix multiplication, this is associative. So,
instead of doing $d^{|V|} = (...)W)W)W$ can do repeated squaring:

Compute $d^{(2)} = W^2$

$d^{(4)} = d^{(2)} \times d^{(2)} = W^2 \times W^2$

$d^{(8)} = d^{(4)} \times d^{(4)}$

...

To get $d^{|V|}$ need log $|V|$ multiplications only $\Rightarrow$? time

All-pairs shortest paths

Note:

$d_{i,j}^{(m)} = \min_k \{ d_{i,k}^{(m-1)} + w(k,j) \}$

Is just like matrix multiplication: $d^{(m)} = d^{(m-1)} W$,
except $+ \rightarrow \min$
$\quad\quad x \rightarrow +$

Like matrix multiplication, this is associative. So,
instead of doing $d^{|V|} = (...)W)W)W$ can do repeated squaring:

Compute $d^{(2)} = W^2$

$d^{(4)} = d^{(2)} x\ d^{(2)} = W^2 x\ W^2$

$d^{(8)} = d^{(4)} x\ d^{(4)}$

...

To get $d^{|V|}$ need log |V| multiplications only $\Rightarrow$ $|V|^3 \log |V|$ time

The Floyd-Warshall algorithm

A more clever dynamic programming algorithm

Before, $d_{i,j}^{(m)}$ = shortest paths of lengths $\leq m$

Next: $d_{i,j}^{(m)}$ = shortest paths from i to j such that
all INTERMEDIATE vertices are $\leq m$

$d^{(0)} = W$

$d^{(m)} = ???$

The Floyd-Warshall algorithm

A more clever dynamic programming algorithm

Before, $d_{i,j}^{(m)}$ = shortest paths of lengths ≤ m

Next: $d_{i,j}^{(m)}$ = shortest paths from i to j such that
all INTERMEDIATE vertices are ≤ m

$d^{(0)} = W$

$$d^{(m)}_{i,j} = \begin{cases} w(i,j) \\ \min \left(d^{(m-1)}_{i,j} \, , \, d^{(m-1)}_{i,m} + d^{(m-1)}_{m,j} \right) & \text{if } m \geq 1. \end{cases}$$

Floyd-Warshall(W)

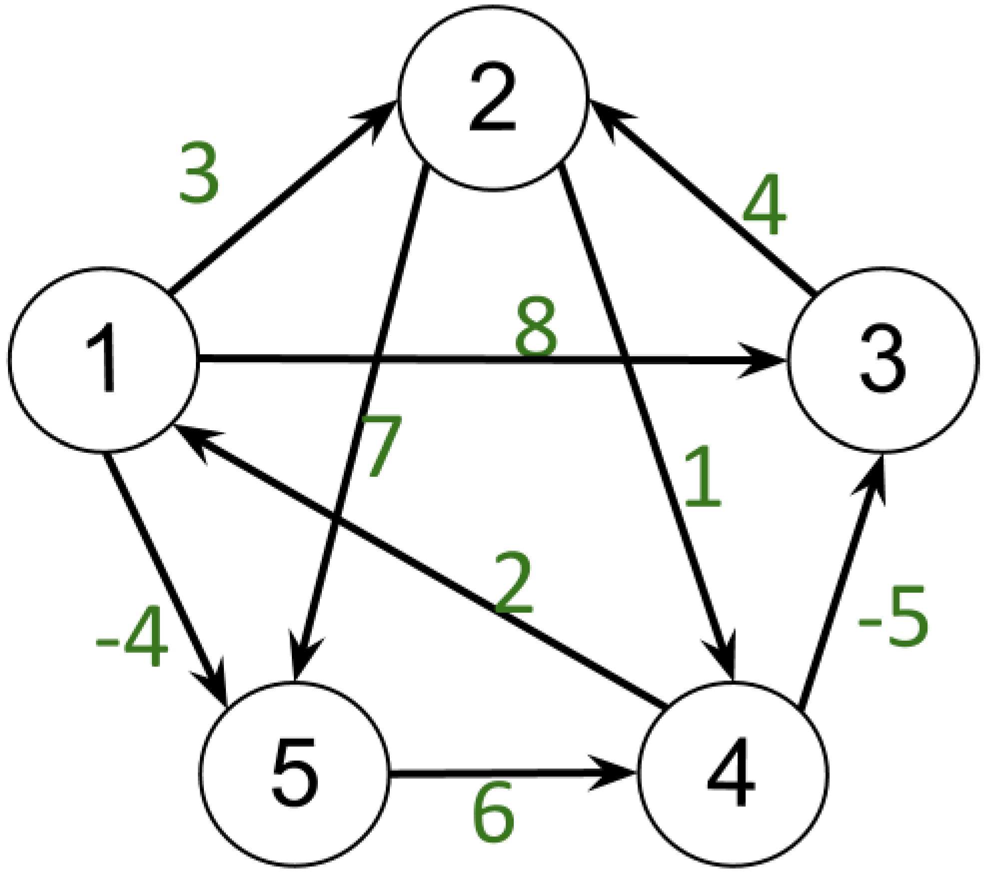
$D^{(0)} := W;$

for m = 1 to n
  for every i,j :
    $d^{(m)}_{i,j} = \min\left(d^{(m-1)}_{i,j}\ ,\ d^{(m-1)}_{i,m} + d^{(m-1)}_{m,j}\right)$

Return $D^{(n)}$

Time $\Theta(|V|^3)$

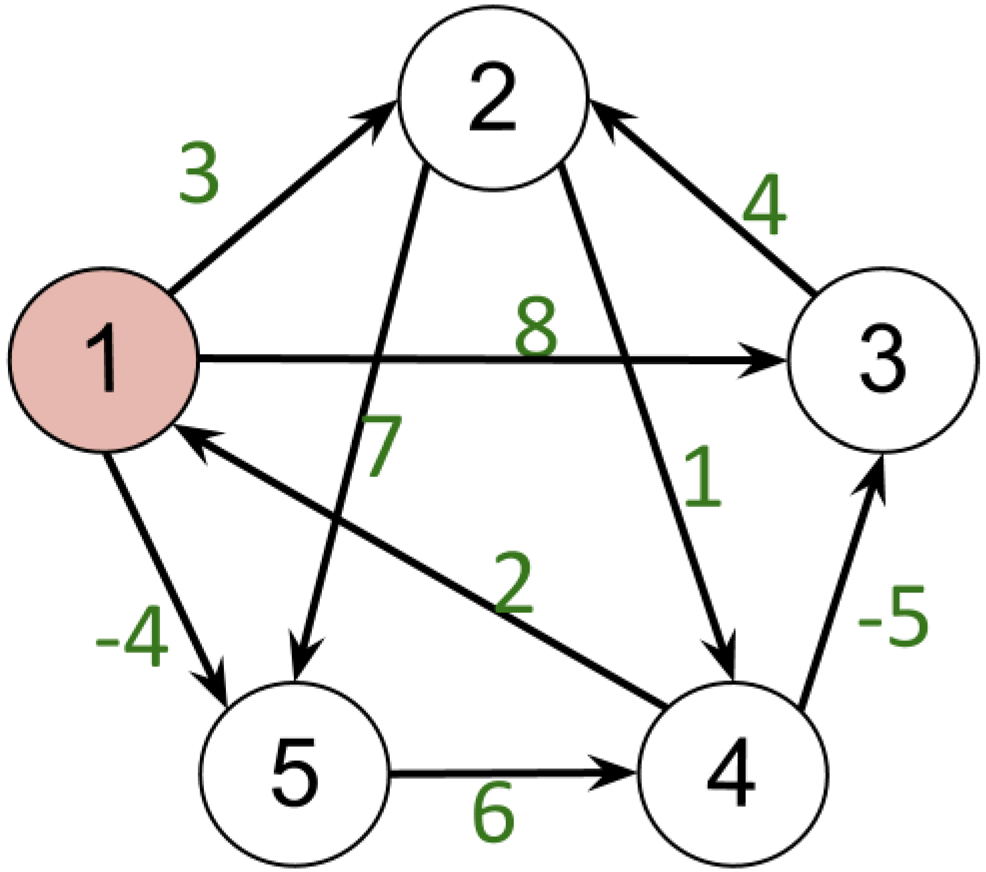
The Floyd-Warshall Example



$$d^{(0)} = \begin{pmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & \infty & -5 & 0 & \infty \\ \infty & \infty & \infty & 6 & 0 \end{pmatrix}$$

$d^{(0)}$ = adjacency matrix with diagonal 0

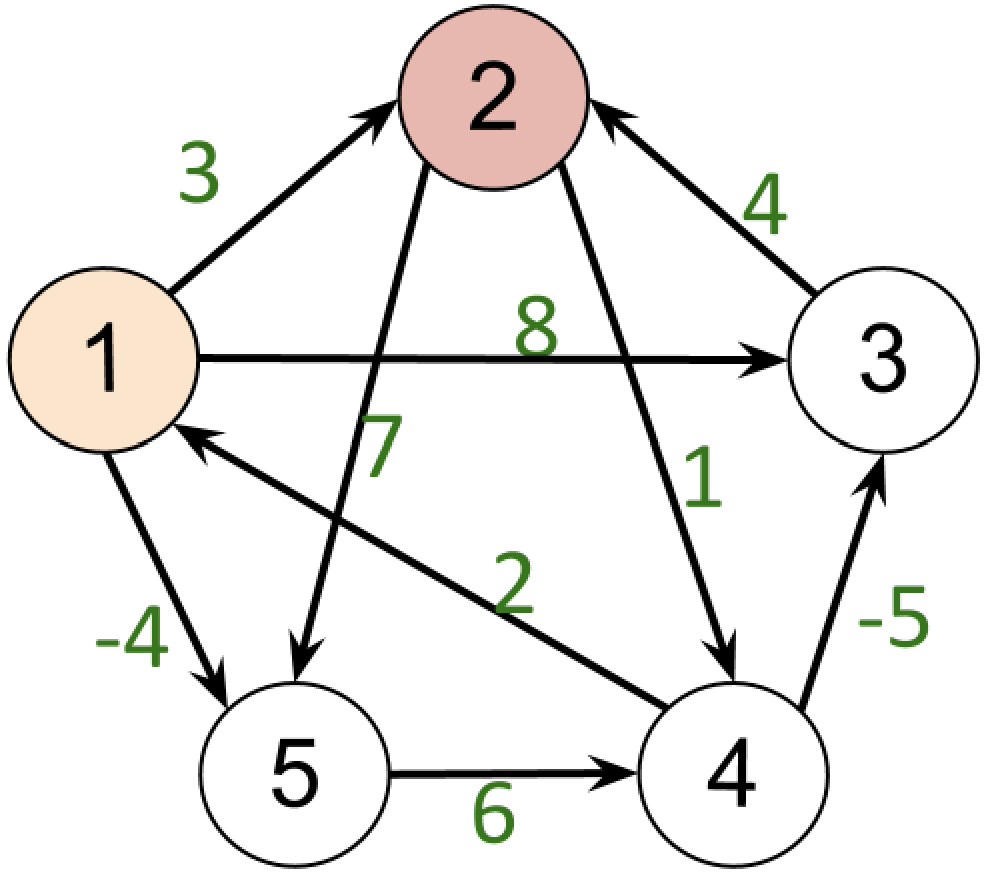
The Floyd-Warshall Example



$$d^{(1)} = \begin{pmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & 5 & -5 & 0 & -2 \\ \infty & \infty & \infty & 6 & 0 \end{pmatrix}$$

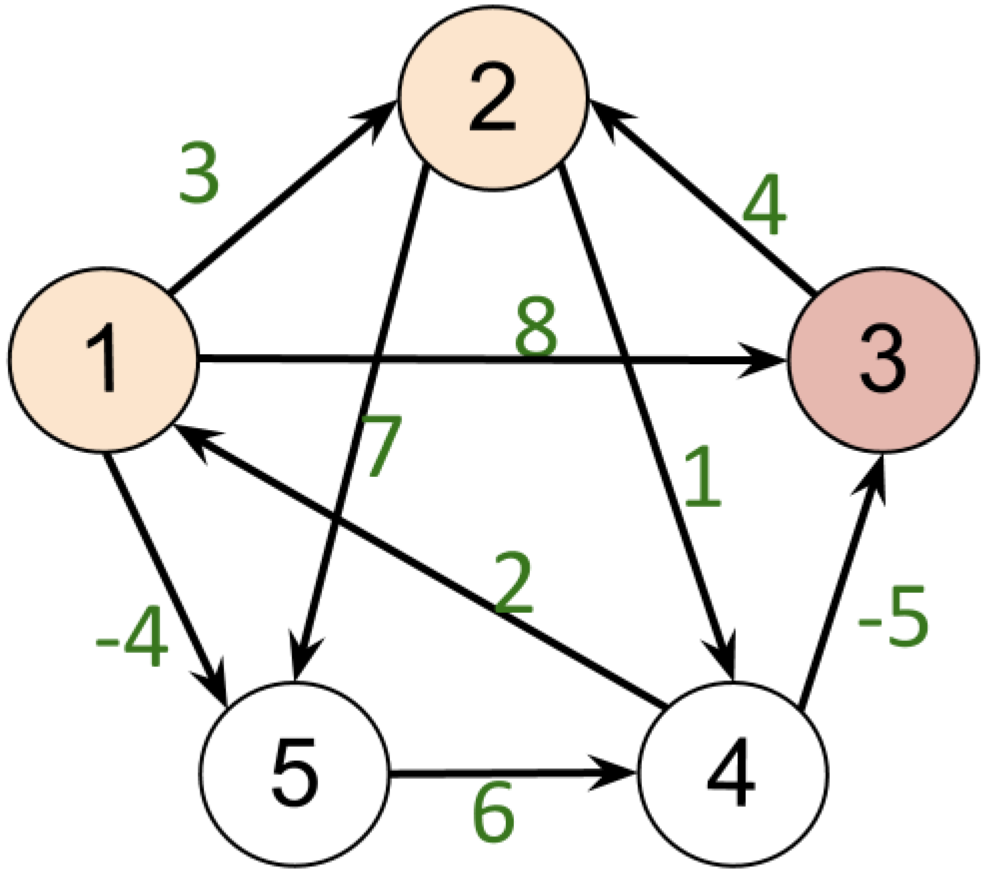
Entries $d_{(4,2)}$ and $d_{(4,5)}$ updated

The Floyd-Warshall Example



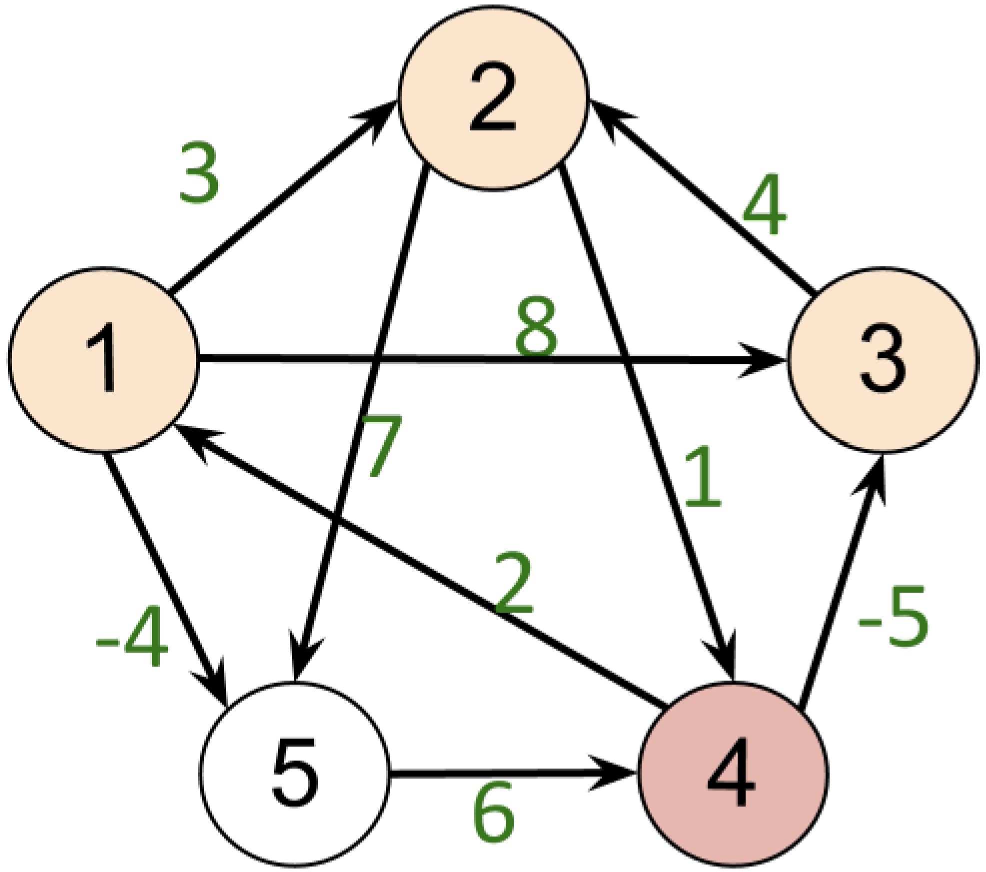
$$d^{(2)} = \begin{pmatrix} 0 & 3 & 8 & 4 & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & 5 & 11 \\ 2 & 5 & -5 & 0 & -2 \\ \infty & \infty & \infty & 6 & 0 \end{pmatrix}$$

The Floyd-Warshall Example



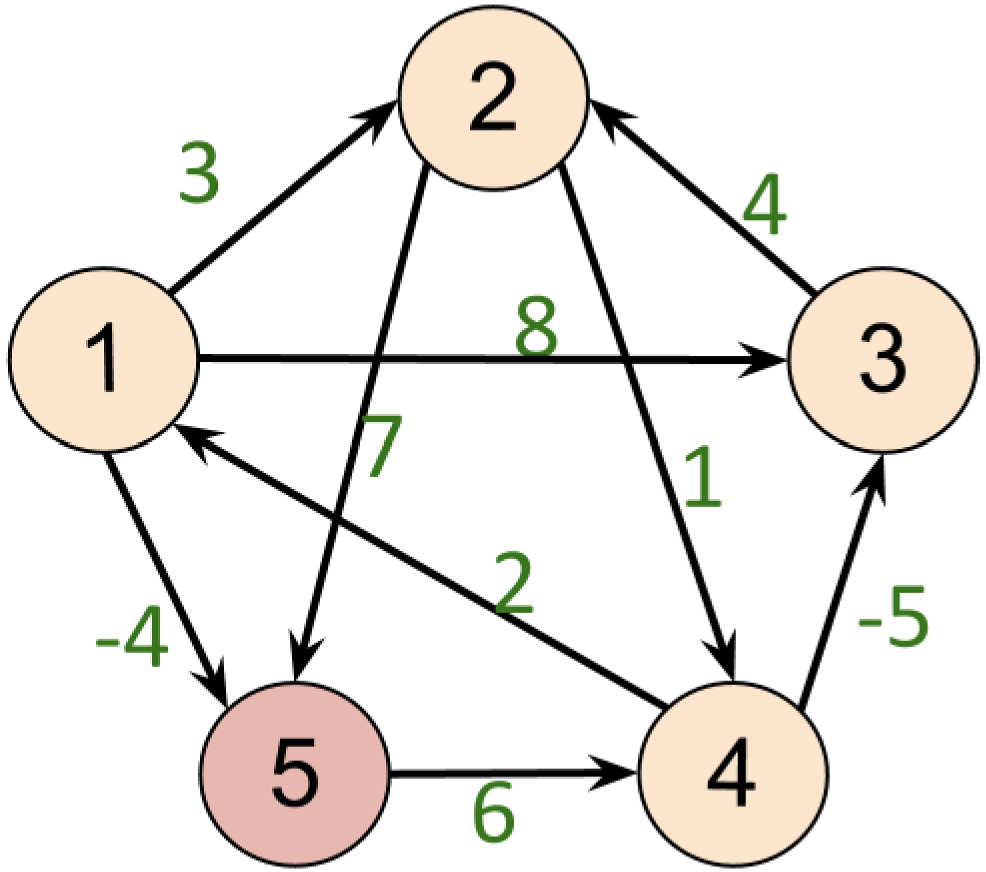
$$d^{(3)} = \begin{pmatrix} 0 & 3 & 8 & 4 & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & 5 & 11 \\ 2 & -1 & -5 & 0 & -2 \\ \infty & \infty & \infty & 6 & 0 \end{pmatrix}$$

The Floyd-Warshall Example



$$d^{(4)} = \begin{pmatrix} 0 & 3 & -1 & 4 & -4 \\ 3 & 0 & -4 & 1 & -1 \\ 7 & 4 & 0 & 5 & 3 \\ 2 & -1 & -5 & 0 & -2 \\ 8 & 5 & 1 & 6 & 0 \end{pmatrix}$$

The Floyd-Warshall Example



$$d^{(5)} = \begin{pmatrix} 0 & 1 & -3 & 2 & -4 \\ 3 & 0 & -4 & 1 & -1 \\ 7 & 4 & 0 & 5 & 3 \\ 2 & -1 & -5 & 0 & -2 \\ 8 & 5 & 1 & 6 & 0 \end{pmatrix}$$

Note: Matrix multiplication/ Floyd Warshall allow for $w < 0$

If $w \geq 0$, can repeat Dijkstra. Time: $O(V^2 \log V + VE) = O(|V|^3)$

Floyd Warshall is easier and has better constants