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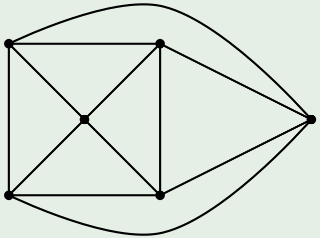

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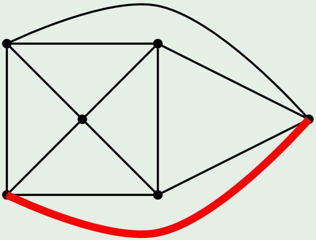

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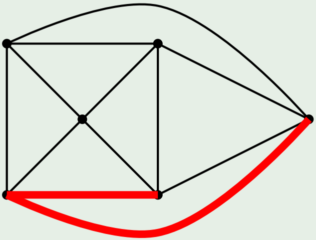

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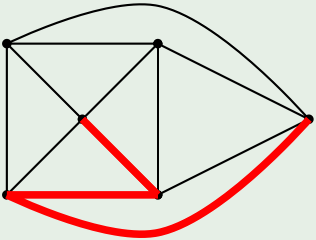

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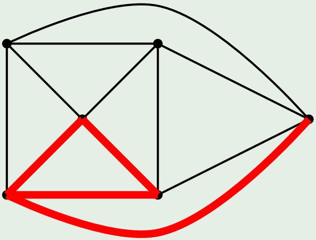

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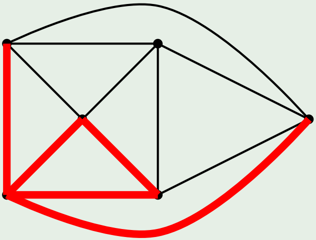

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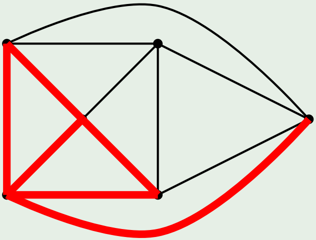

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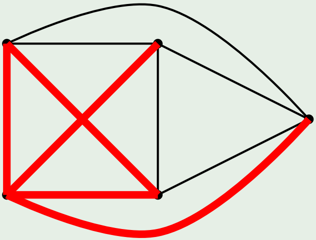

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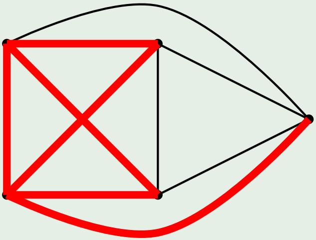

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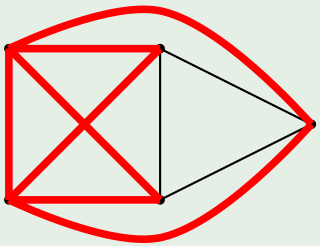

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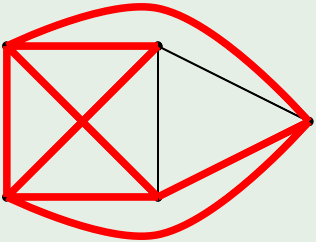

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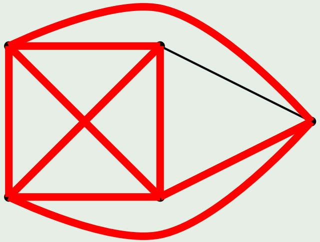

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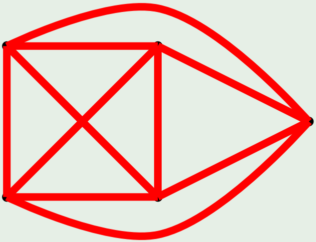

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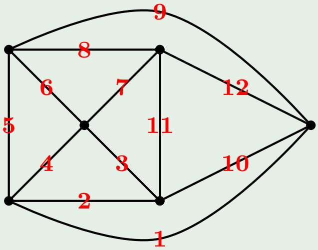

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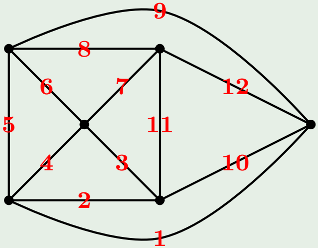

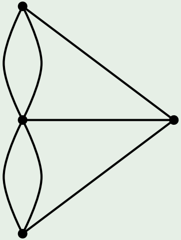

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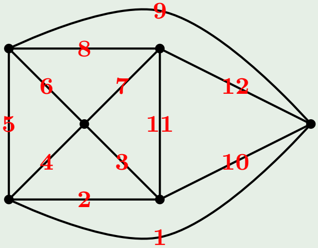

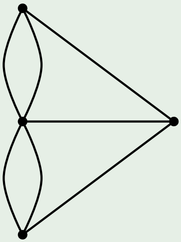

Not Eulerian!

Characterization of Eulerian Graphs

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Let T be a maximal non-trivial trail in some graph G with all degrees even. Since T is maximal, it includes all edges of G incident with its final vertex v. If T is not closed, then the degree of v must be odd, which is impossible. $\square$

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