Graph Theory

Definition of a graph

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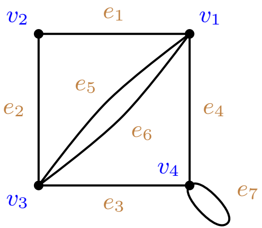

Simple graphs

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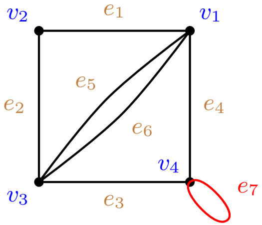

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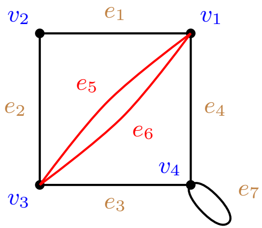

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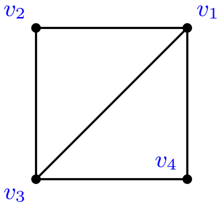

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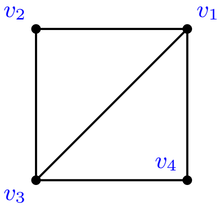

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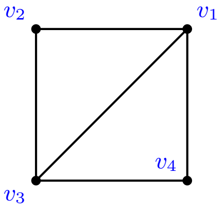

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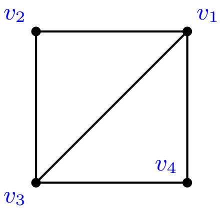

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Note 1.5

In some books, what we defined as a graph is called a multigraph and what we defined as a simple graph is called a graph.

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- The set of neighbors of a vertex v of G, other than v itself, is denoted by $N(v)$ or by $N_G(v)$.
- Similarly, if U is a subset of the vertex set of G, then $N(U)$ is the set of those vertices that are not in U, but are adjacent to a vertex in U.

Isomorphism

Definition 1.7

The graphs $G_1 = (V_1, E_1, \mathcal{I}_1)$ and $G_2 = (V_2, E_2, \mathcal{I}_2)$ are isomorphic, written $G_1 \cong G_2$, if there are bijections $\varphi : V_1 \to V_2$ and $\psi : E_1 \to E_2$ such that $(v, e) \in \mathcal{I}_1$ if and only if $(\varphi(v), \psi(e)) \in \mathcal{I}_2$. Such a pair of bijections is an isomorphism.

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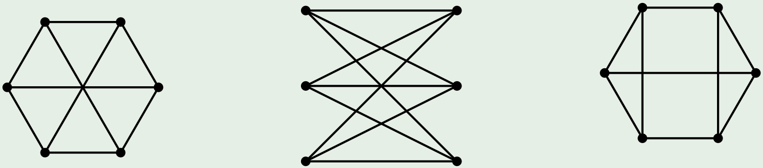
Graph isomorphism problem can be solved in quasi-polynomial time.
There is a constant c and an algorithm that can decide whether two graphs on n vertices are isomorphic or not in at most $2^{\mathcal{O}((\log n)^c)}$ steps.

Isomorphism Example

Example 1.10

Which of the following graphs are isomorphic?



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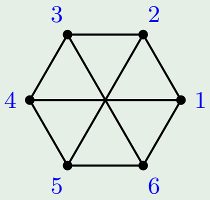

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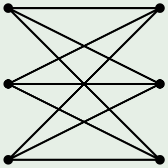
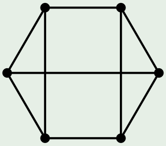


G_2



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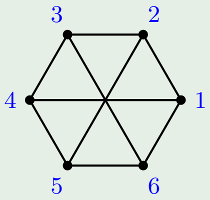

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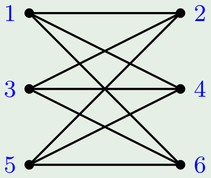
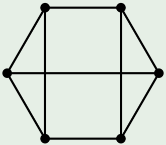


G_2



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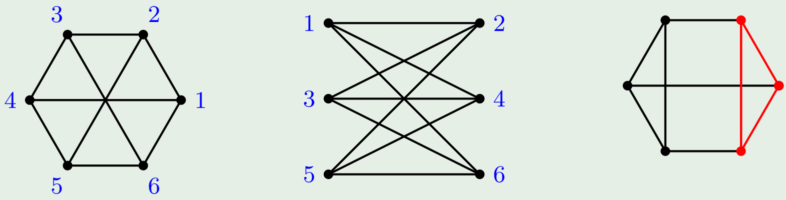

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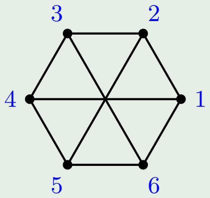

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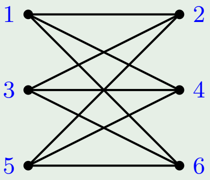

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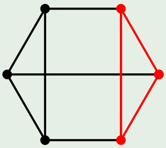

G_3

$$G_1 \cong G_2 \ncong G_3$$

Automorphism

Definition 1.11

An automorphism of a graph is an isomorphism from the graph to itself.

Subgraphs

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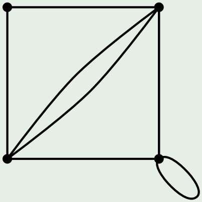
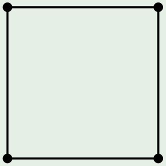
Subgraph Example

Example 1.16



G_1



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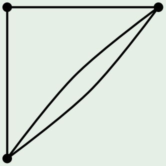

G_3

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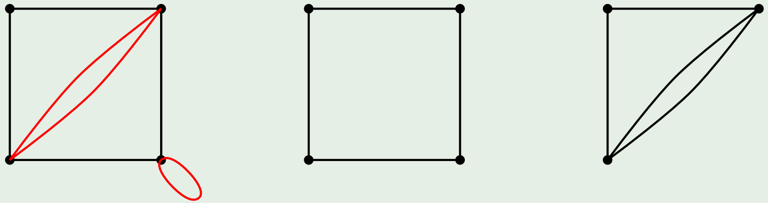

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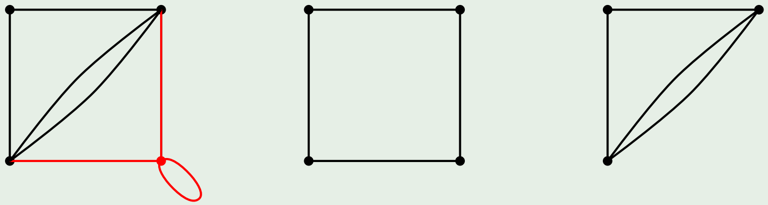

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Conjecture 1.18 (Edge-Reconstruction Conjecture)

Every simple graph on at least four edges is edge-reconstructible.

Walks, Trails, Paths, and Cycles

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- A graph is connected if each pair of its vertices can be connected by a walk (equivalently, a trail or a path).

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- A simple graph is self-complementary if it is isomorphic to its own complement.

Hand-Shaking Lemma

Theorem 1.21 (Hand-Shaking Lemma)

$$\sum_{v \in V(G)} d(v) = 2\|G\|$$

Corollary 1.22

The number of vertices of odd degree is even.