Trading Algorithms

MEAN REVERTING STRATEGIES

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The Basics of Mean Reversion

- Whether we realize it or not, nature is filled with examples of **mean reversion**. Mean reversion is equally prevalent in the social sciences.

- Is mean reversion also prevalent in financial price series? If so, our lives as traders would be very simple and profitable!

- All we need to do is to buy low (when the price is below the mean), wait for reversion to the mean price, and then sell at this higher price, all day long.

- Alas, most price series are not mean reverting, but are geometric random walks.

- The returns, not the prices, are the ones that usually randomly distribute around a mean of zero.

The Basics of Mean Reversion …

- Those few price series that are found to be mean reverting are called stationary, and we will describe the statistical tests for stationarity.
- **There are not too many prefabricated prices series that are stationary.**
- Fortunately, we can **manufacture** many more mean-reverting price series than there are traded assets because we can often combine two or more individual price series that are not mean reverting into a **portfolio** whose net market value (i.e., price) is mean reverting.
- Those price series that can be combined this way are called **cointegrating**.
- Because of this possibility of artificially creating stationary portfolios, there are numerous opportunities available for mean reversion traders.

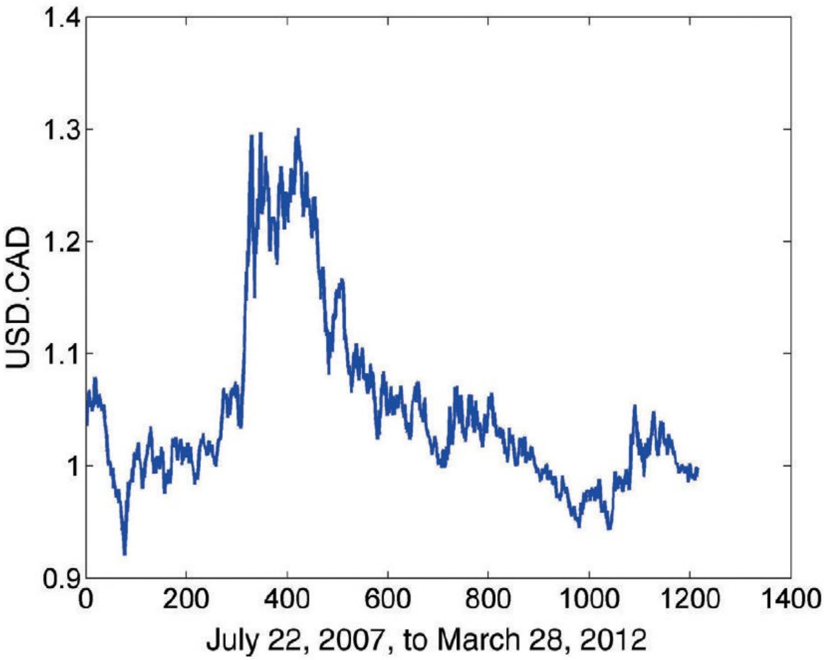
ADF Test

- If a **price series is mean reverting**, then the **current price level** will **tell us something about what the price's next move** will be.
- If the price level is higher than the mean, the next move will be a downward move; if the price level is lower than the mean, the next move will be an upward move. The **ADF test** is based on just this observation.
- We can describe the price changes using a linear model $\Delta y_t = \alpha + \delta y_{t-1} + \sum_{i=1}^{P} \beta_i \Delta y_{t-1} + \varepsilon_t$.
- If the hypothesis $\delta = 0$ can be rejected, that means the next move Δy_t depends on the current level y_{t-1}, and therefore it is not a random walk.
- The test statistic is the regression coefficient δ divided by the standard error of the regression fit, $\frac{\delta}{SE(\delta)}$.
- Notice that since we expect mean regression, $\frac{\delta}{SE(\delta)}$ has to be negative, and it has to be more negative than the critical value for the hypothesis to be rejected.

ADF Test ...

- As an example, we apply ADF test to the **daily close price of USD.CAD** from July 22, 2007 to March 28, 2012. Following is the price series and obviously it is not mean reverting.

- The test statistic for this time series is about −1.84, but the critical value at the 90 percent level is −2.594, so we can't reject the hypothesis that δ is zero. In other words, we can't show that USD.CAD is stationary.



Hurst Exponent and Variance Ratio Test

- Intuitively speaking, a stationary price series means that the prices diffuse from its initial value more slowly than a geometric random walk would. Mathematically, we can determine the nature of the price series by measuring this **speed of diffusion**. The speed of diffusion can be characterized by the variance

$$Var(\tau) = \langle |z(t+\tau) - z(t)|^2 \rangle$$

- where z is the log prices ($z = \log(y)$), τ is an arbitrary time lag, and $\langle \ldots \rangle$ is an average over all t's. For a geometric random walk, we know that

$$\langle |z(t+\tau) - z(t)|^2 \rangle \sim \tau$$

- The notation $\sim$ means that this relationship turns into an equality with some proportionality constant for large τ, but it may deviate from a straight line for small τ.

Hurst Exponent and Variance Ratio Test …

- However, if the (log) price series is **mean reverting** or **trending** (i.e., has positive correlations between sequential price moves), the equation above won't hold. Instead, we can write:

$$\langle |z(t+\tau) - z(t)|^2 \rangle \sim \tau^{2H}$$

- where we have defined the **Hurst exponent H**. **For a price series exhibiting geometric random walk, H = 0.5. But for a mean-reverting series, H < 0.5, and for a trending series, H > 0.5.**

- As H decreases toward zero, the price series is more mean reverting, and as H increases toward 1, the price series is increasingly trending; thus, **H serves also as an indicator for the degree of mean reversion or trendiness.**

- In the previous example, we computed the Hurst exponent for the same currency rate series USD.CAD, and found that H is 0.49, which suggests that the price series is weakly mean reverting.

Hurst Exponent and Variance Ratio Test …

- Because of finite sample size, we need to know the statistical significance of an estimated value of H to be sure whether we can reject the null hypothesis that H is really 0.5.

- This hypothesis test is provided by the **Variance Ratio test**. It simply tests whether the following statistics is equal to 1 or not.

$$\frac{Var\big(z(t)-z(t-\tau)\big)}{\tau\, Var\big(z(t)-z(t-1)\big)}$$

- In the previous example, the outputs are ***h*** and ***pValue***: H_0 means it may be a random walk and H_1 means rejection of the random walk hypothesis. pValue gives the probability that the null (random walk) hypothesis is true.

- We find the pValue for H_0 is 0.367281 for USD.CAD, indicating that there is a 37 percent chance that it is a random walk, so we cannot reject this hypothesis.

Hurst Exponent and Variance Ratio Test …

- The **statistical test** mentioned for mean reversion is very demanding, with its requirements of at least 95 or 90 percent certainty. But in **practical trading**, we can often be profitable with much less certainty.
- We shall find another way to interpret the δ coefficient in $\Delta y_t = \alpha + \delta y_{t-1} + \sum_{i=1}^{P} \beta_i \Delta y_{t-1} + \varepsilon_t$, so that we know whether it is negative enough to make a trading strategy practical, even if we cannot reject the null hypothesis.
- **We shall find that δ is a measure of how long it takes for a price to mean revert.**
- To reveal this new interpretation, it is only necessary to transform the discrete time series equation to a differential form so that the changes in prices become infinitesimal quantities.
- Furthermore, if we ignore the lagged differences in $\Delta y_t = \alpha + \delta y_{t-1} + \sum_{i=1}^{P} \beta_i \Delta y_{t-1} + \varepsilon_t$, then it becomes recognizable in stochastic calculus as the **Ornstein-Uhlenbeck formula** for mean-reverting process:

$$\mathrm{d}y_t = (\alpha + \delta y_{t-1})dt + d\varepsilon$$

Hurst Exponent and Variance Ratio Test …

- The advantage of writing the equation in the differential form is that it allows for an analytical solution for the expected value of y_t.

$$\mathrm{E}(y_t) = y_0 e^{\delta t} - \frac{\alpha}{\delta}\left(1 - e^{\delta t}\right)$$

- Remembering that δ is negative for a mean-reverting process, this tells us that the expected value of the price decays exponentially to the value $-\frac{\alpha}{\delta}$ with the **half-life of decay equals to $-\frac{\log(2)}{\delta}$**.

- The coefficient δ is called the speed of mean reversion and **the half-life of the mean-reversion, $t_{1/2}$, is the average time it will take the process to get pulled half-way back to the mean**.

- In particular, the higher the mean-reversion speed is, the smaller is the half-life.

Hurst Exponent and Variance Ratio Test …

- This **connection between a regression coefficient δ** and the half-life of mean reversion is **very useful to traders**.
  - First, if we find that δ is positive, this means the price series is not at all mean reverting, and we shouldn't even attempt to write a mean reverting strategy to trade it.
  - Second, if δ is very close to zero, this means the half-life will be very long, and a mean-reverting trading strategy will not be very profitable because we won't be able to complete many round-trip trades in a given time period.
  - Third, this δ also determines a natural time scale for many parameters in our strategy. For example, if the half life is 20 days, we shouldn't use a **look-back** of 5 days to compute a moving average or standard deviation for a mean-reversion strategy. Often, setting the lookback to equal a small multiple of the half-life is close to optimal, and doing so will allow us to **avoid brute-force optimization** of a free parameter based on the performance of a trading strategy.

Hurst Exponent and Variance Ratio Test …

- We concluded in **the previous example** that the price series USD.CAD is not stationary with at least 90 percent probability.

- But that doesn't necessarily mean we should give up trading this price series using a mean reversion model because most profitable trading strategies do not require such a high level of certainty.

- To determine whether USD.CAD is a good candidate for mean reversion trading, we should determine its half-life of mean reversion.

- The result is **about 115 days**. Depending on your trading horizon, this may or may not be too long.

- But at least we know what look-back to use and what holding period to expect.

Cointegration

- As stated, most financial price series are not stationary or mean reverting.
- But, fortunately, we are not confined to trading those prefabricated financial price series: we can proactively create a portfolio of individual price series so that the market value (or price) series of this portfolio is stationary.
- This is the notion of **cointegration**: **if we can find a stationary linear combination of several nonstationary price series, then these price series are called cointegrated**.
- The most common combination is that of two price series: we long one asset and simultaneously short another asset, with an appropriate allocation of capital to each asset.
- This is the familiar **pairs trading strategy**. But the concept of cointegration easily extends to three or more assets.

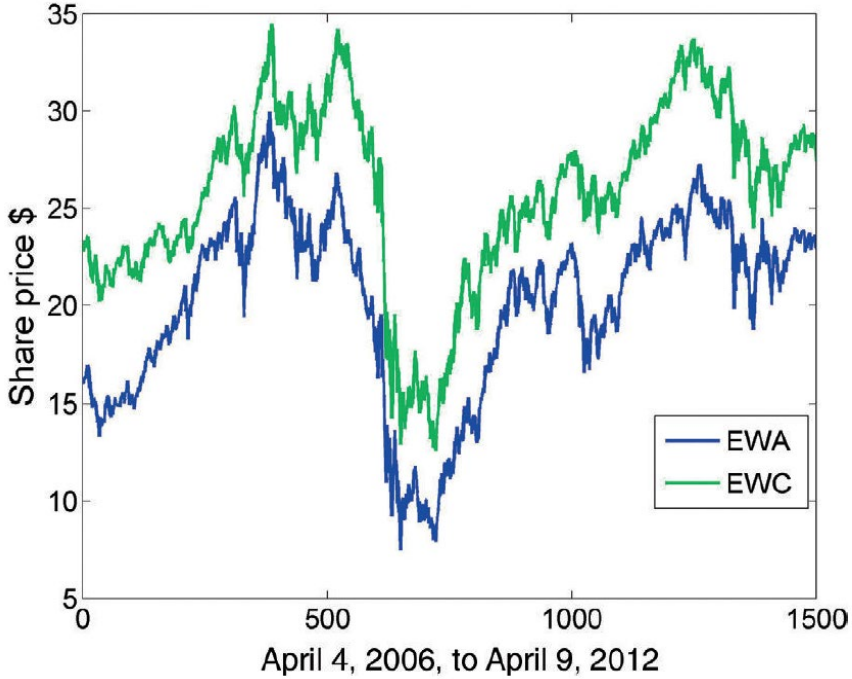
Cointegration …

- Given a number of price series, since we do not know a priori what hedge ratios we should use to combine them to form a stationary portfolio, we have to apply Cointegrated Augmented Dickey-Fuller (CADF) test (for two variables) or Johansen test (for more than two variables).
- The hedge ratio of a particular asset is the number of units of that asset we should be long or short in a portfolio.
- If the asset is a stock, then the number of units corresponds to the number of shares. A negative hedge ratio indicates we should be short that asset.
- Just because a set of price series is cointegrating does not mean that any random linear combination of them will form a stationary portfolio, so **appropriate hedge ratios** should be found.

Cointegration …

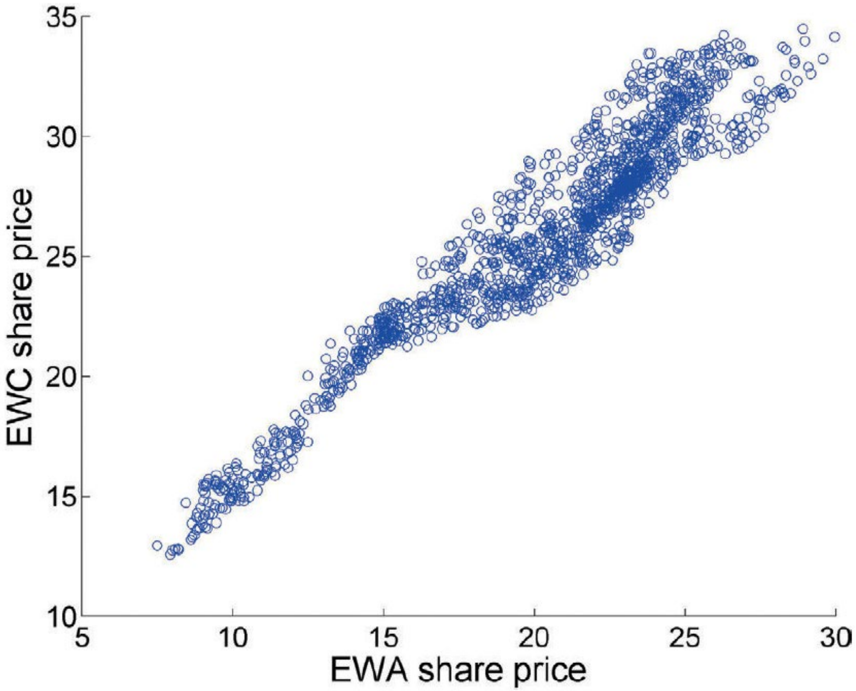
- Following example demonstrates how to use CADF test by applying it to two exchange-traded funds (ETFs) EWA and EWC.
- Both Canadian and Australian economies are commodity based, so they seem likely to cointegrate.

- This figure shows the price series of EWA and EWC from April 2006 to 2012, and they look quite cointegrating.



Cointegration …

➢A scatter plot of EWA versus EWC is even more convincing, as the price pairs fall on a straight line.

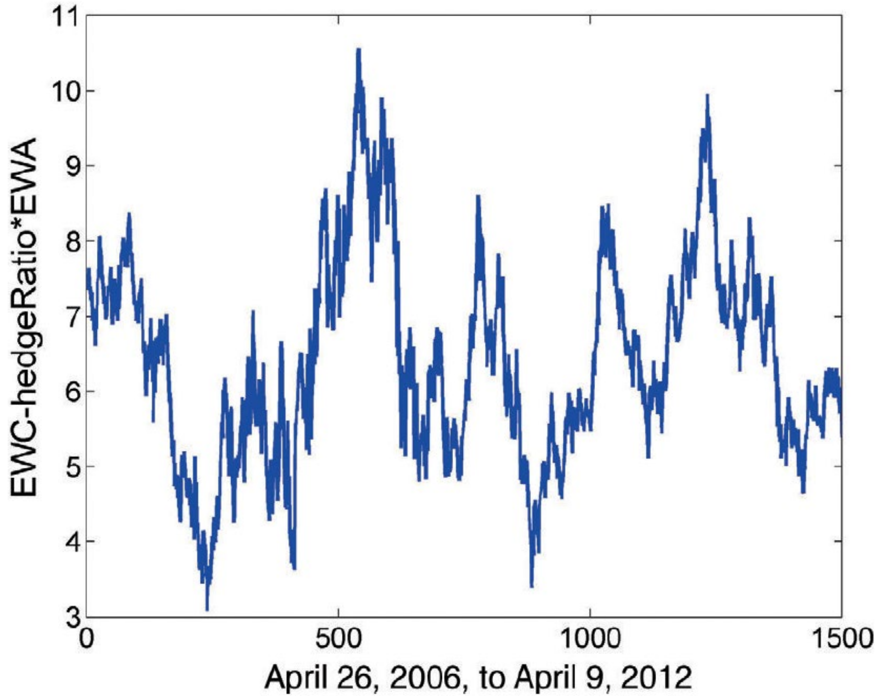


Cointegration …

- After finding hedge ratio, the plot of the residual EWC-hedgeRatio*EWA is depicted and it looks very stationary.

- We find that the ADF test statistic is about –3.64 for the new series, certainly more negative than the critical value at the 95 percent level of –3.359.

- So we can reject the null hypothesis that δ is zero. In other words, EWA and EWC are cointegrating with 95 percent certainty.



A Linear Mean-Reverting Trading Strategy

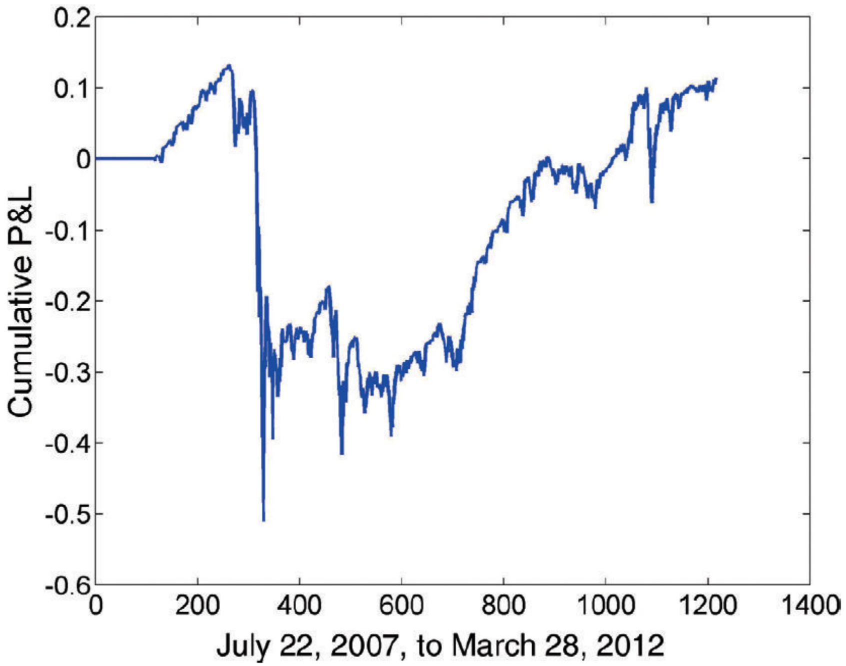
- Once we determine that a price series is mean reverting, and that the half life of mean reversion for a price series short enough for our trading horizon, we can easily trade this price series profitably using a **simple linear strategy**:
  - Determine the normalized deviation of the price from its moving average, and maintain the number of units in this asset negatively proportional to this normalized deviation (a moving Z-score of the last closing price).
  - The lookback period for the moving average and moving standard deviation is set equal to the half life of mean reversion.
- **We usually assume the mean of a price series to be fixed**, in practice it may change slowly due to changes in the economy or corporate management.
- As for the standard deviation, even a stationary price series with $0 < H < 0.5$ has a variance that increases with time, though not as rapidly as a geometric random walk.

A Linear Mean-Reverting Trading Strategy …

➤The cumulative P&L of this simple strategy for the USD.CAD example is plotted in below.

➤Despite the long half-life, the total profit and loss (P&L) manages to be positive, albeit with a large drawdown.

➤Also, there is a **look-ahead bias** involved in this particular example due to the use of in-sample data to find the half-life and therefore the lookback.



A Linear Mean-Reverting Trading Strategy …

- As another example, suppose we have applied the Johansen test on three ETFs EWA-EWC-IGE and formed an stationary portfolio with the best eigenvector from that has a short half-life.
- We can now confidently proceed to backtest our simple linear mean-reverting strategy on this portfolio.
- We accumulate units of the portfolio proportional to the negative Z-Score of the unit **portfolio's price**.
- A unit portfolio is one with shares determined by the Johansen eigenvector; in this case the coefficients are -1.0460, 0.7600, and 0.2233 for EWA, EWC, and IGE, respectively.
- The half-life of 23 days is considerably shorter than the 115 days for USD.CAD, so **we expect a mean reversion trading strategy to work better for this triplet**.

A Linear Mean-Reverting Trading Strategy …

- We find that APR = 12.6 percent with a Sharpe ratio of 1.4 for the strategy.
- Following figure displays the cumulative returns curve of this linear mean-reverting strategy for a stationary portfolio of EWA, EWC, and IGE.
- Obviously, this linear mean-reverting strategy is not a practical strategy, **at least in its simplest version**, as we do not know the maximum capital required.

Pairs Trading using Price Spreads, Log Price Spreads, or Ratios

- In constructing a portfolio for mean reversion trading, we simply use the market value of the unit portfolio as the trading signal.

- This market value or price is just the weighted sums of the constituent price series, where the weights are the hedge ratios we found from linear regression or from the eigenvectors of the Johansen test

$$y = h_1 y_1 + h_2 y_2 + \ldots + h_n y_n$$

- y is, by construction, a stationary time series, and the h_i's tell us the number of shares of each constituent stock (assuming **we are trading a stock portfolio**).

- In the case of just two stocks, this reduces to a spread like this

$$y = y_1 - h y_2$$

Pairs Trading using Price Spreads, Log Price Spreads, or Ratios …

- Suppose instead of price series, we find that the **log of prices** are cointegrating, such that

$$\log(q) = h_1\log(y_1) + h_2\log(y_2) + \ldots + h_n\log(y_n)$$

- is stationary for some set of h's derived from either a regression fit or Johansen's eigenvectors.

- To find out the properties of the above equation, let's take its first difference in time:

$$\Delta\log(q) = h_1\Delta\log(y_1) + h_2\Delta\log(y_2) + \ldots + h_n\Delta\log(y_n)$$

- Remembering that $\Delta\log(x) = \log\big(x(t)\big) - \log\big(x(t-1)\big) = \log\left(\frac{x(t)}{x(t-1)}\right) \approx \frac{\Delta x}{x}$ for small changes in x ($\Delta x \approx 0$), the right hand side of the equation becomes $h_1\frac{\Delta y_1}{y_1} + h_2\frac{\Delta y_2}{y_2} + \cdots + h_n\frac{\Delta y_n}{y_n}$, which is none other than the returns of a portfolio consisting of the *n* assets with weights h's.

Pairs Trading using Price Spreads, Log Price Spreads, or Ratios …

- However, unlike the hedge ratio h's in price spreads equation, where they referred to the number of shares of each asset, here we can set the market value of each asset to *h*.

- So we can interpret q as the market value of a portfolio of assets with prices $y_1, y_2, \dots, y_n$ and with constant capital weights $h_1, h_2, \dots, h_n$, together with a cash component implicitly included, and this market value will form a stationary time series.

- In order to keep the market value of the portfolio stationary requires a lot of work for the traders, as they **need to constantly rebalance the portfolio**, which is necessitated by using the log of prices.

- The upshot of all these is that mean reversion trading using price spreads is simpler than using log price spreads, but both can be theoretically justified if both price and log price series are cointegrating.

Pairs Trading using Price Spreads, Log Price Spreads, or Ratios …

- In addition to the log prices, the ratio prices y_1/y_2, that many traders favor as the signal for a pair, can be used to form (a short term) stationary time series.
- Suppose price A = \$10 and price B = \$5 initially, so the ratio is 2. After some time, price A increases to \$100 and price B to \$50.
- The spread has gone from \$5 to \$50, and we will probably find that it is not stationary. But, the ratio remains 2, and a mean-reverting strategy that trades based on ratio can be equally effective whether their prices are \$10 versus \$5 or \$100 versus \$50.
- In other words, if your two assets are not really cointegrating but you believe their spread is still mean reverting on a short time frame, then using ratio as an indicator may work better than either price spreads or log price spreads.

Bollinger Bands

- The only **mean-reversal strategy** described so far is the **linear strategy**: simply scale the number of units invested in a stationary unit portfolio to be proportional to the deviation of the market value (price) of the unit portfolio from a moving average.

- This simple strategy is chosen because it is virtually **parameterless**, and therefore least subject to data-snooping bias.

- While this linear strategy is useful for demonstrating whether mean reversion trading can be profitable for a given portfolio, it is not practical because we don’t know beforehand what the maximum capital deployed will be, as there is no limit to the temporary deviation of the price from its average.

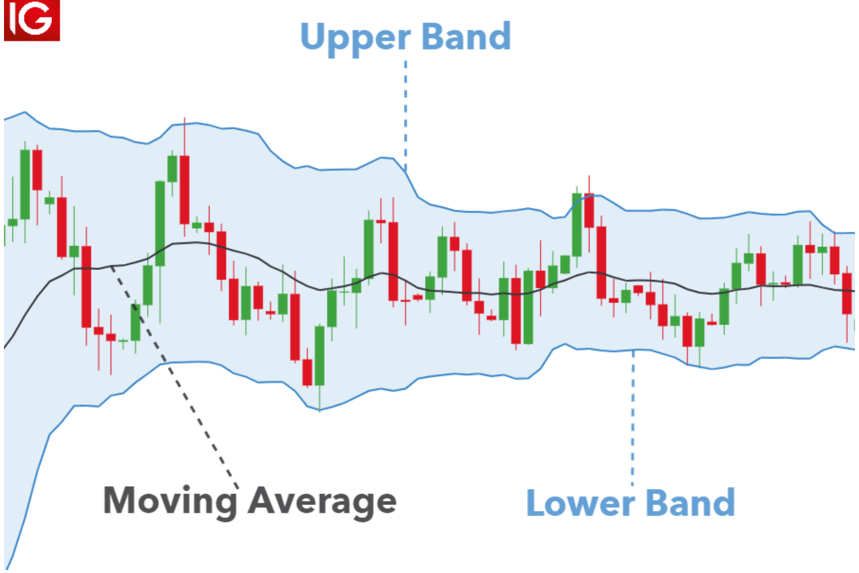
- For practical trading, we can use the **Bollinger band**, where we enter into a position only when the price deviates from the mean.

Bollinger Bands …

- A Bollinger band is a technical analysis tool defined by a set of trendlines plotted two standard deviations (positively and negatively) away from a simple moving average (SMA) of a security's price.

- Bollinger bands are developed by John Bollinger for generating oversold or overbought signals.

- Many traders believe the closer the prices move to the upper band, the more overbought the market, and the closer the prices move to the lower band, the more oversold the market.



Bollinger Bands …

- Here is this Bollinger band formula:

$$\boldsymbol{BOLU = MA(TP, n) + m \times \sigma[TP, n]}$$
$$\boldsymbol{BOLD = MA(TP, n) - m \times \sigma[TP, n]}$$

where

$$BOLU = \text{Upper Bollinger band}$$
$$BOLD = \text{Lower Bollinger band}$$
$$MA = \text{Moving average}$$
$$TP(typical\ price) = \frac{High + Low + Close}{3}$$
$$n = \text{Number of days in smoothing period}$$
$$m = \text{Number of standard deviations}$$
$$\sigma[TP, n] = \text{Standard deviation over last } n \text{ periods of TP}$$

Bollinger Bands …

- For **practical trading**, we can use the Bollinger band, where we enter into a position only when the price deviates by more than *entryZscore* standard deviations from the mean.

- *entryZscore* is a free parameter to be optimized in a training set, and both standard deviation and mean are computed within a look-back period, whose length again can be a free parameter to be optimized, or it can be set equal to the half-life of mean reversion.

- We can exit when the price mean-reverts to *exitZscore* standard deviations from the mean. Note that if *exitZscore* = 0, this means we will exit when the price mean-reverts to the current mean.

- If exitZscore = −entryZscore, we will exit when the price moves beyond the opposite band so as to trigger a trading signal of the opposite sign.

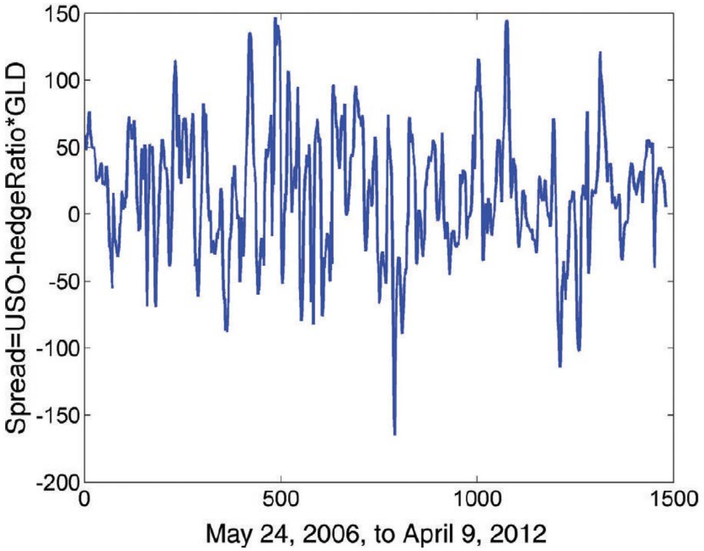
Bollinger Bands …

- In order to illustrate the proficiency of the Bollinger band method, we first apply the simple linear mean-reverting strategy to ETFs GLD and USO, and then improve it by applying the new strategy.

- To do so, first the hedgeRatio in formula “**USO-hedgeRatio*GLD**” should be found using CADF or Johansen tests to form a mean reverting price series. Following is the new series.

- The lookback period is set to near-optimal 20 trading days to adapt to the changing levels of the ETFs over time.

- The reference date is May 24 2006, to April 9, 2012.



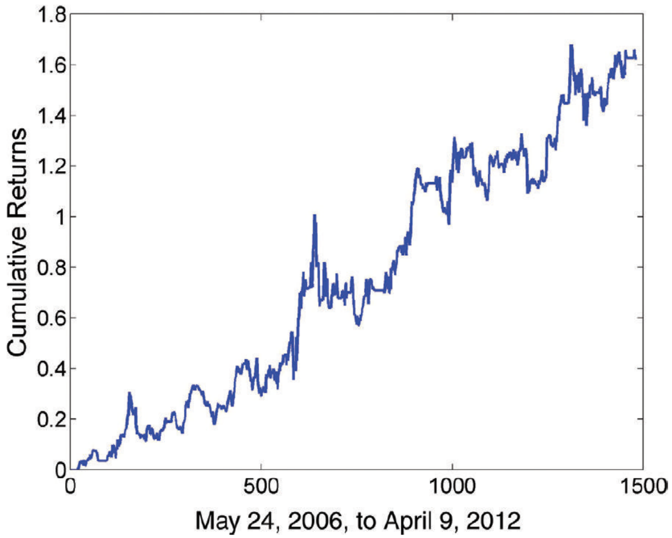
Bollinger Bands …

- In the first strategy, the number of units (shares) of the unit portfolio we should own is set to be the negative Z-Score.

- We obtain an Annual Percentage Rate (APR) of about 10.9 percent and Sharpe ratio of about 0.59 using price spread with a dynamic hedge ratio.

- The second (Bollinger band-based) strategy has an APR = 17.8 percent, and Sharpe ratio of 0.96, quite an improvement from the linear mean reversal strategy.

- This figure shows cumulative returns curve in the Bollinger band strategy.



Scaling-in (Averaging-in)

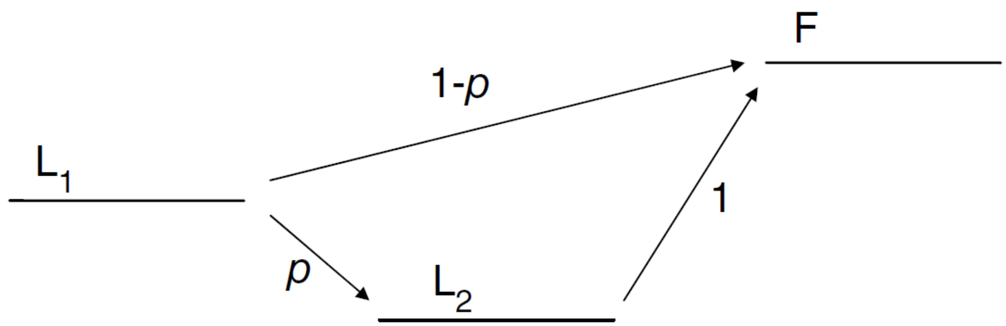
- The notion of scaling into a position with a mean-reverting strategy is familiar to many traders.
- As the price (of an asset, a spread, or a portfolio) deviates further and further from its mean, the potential profit to be reaped from an eventual reversal is also increasing; thus, it makes sense to increase the capital invested.
- This is exactly what our linear mean-reversal strategy does.
- Note also that this type of scaling-in strategies also scale out gradually: **we do not have to wait until the price reverts to its mean before taking profits**.
- The advantage of being able to exit whenever the price reverts by a small increment is that even if the price series is not really stationary and therefore never really reverts to its mean, we can still be profitable by constantly realizing small profits.

Scaling-in (Averaging-in) ...

- An added benefit is that if you are trading large sizes, scaling-in and -out will reduce the market impact of the entry and exit trades.
- If we want to implement scaling-in using Bollinger bands, we can just have multiple entries and exits: for example, entryZscore = 1, 2, 3, ..., N and exitZscore = 0, 1, 2, ..., N − 1.
- Of course, **N is another parameter to be optimized** using a training data set.
- However, **it has been proved that entering or exiting at two or more Bollinger bands is never optimal**; that is, you can always find a single entry/exit level that will generate a higher average return in a backtest.
- This optimal single entry level is named **all-in.**

Scaling-in (Averaging-in) …

- In order to justify the above point, let's say a stock price has recently dropped to a price L1, and you expect it to revert to a higher final price F > L1.
- We have to assume mean reversion to compare averaging-in versus all-in, though there is a probability p that the price will go lower to L2 < L1 before rebounding to F.
- These possibilities are illustrated in the following figure.



Scaling-in (Averaging-in) …

- We have just enough buying power to invest in a total of two shares, whether at prices L1, L2, or F. Let's compare the three different methods of entry:

  I. All-in at L1: We invest all our capital when the price reaches L1, not caring whether it will go lower to L2.

  II. All-in at L2: We wait until the price reaches L2 before investing all our capital. Therefore, we invest nothing and earn zero returns if the price never reaches L2.

  III. Average-in: We invest in one share when the price reaches L1, and in another share if the price reaches L2.

- In all cases, we exit all contracts only when the price reaches F (so no average-out, even if there is average-in).

Scaling-in (Averaging-in) …

- What are the expected profits of each alternative? The expected profits in points are:
  I. 2(F − L1)
  II. 2p(F − L2)
  III. (F − L1) + p(F − L2)
- Obviously, **if p = 0, method I is the most profitable**. **If p = 1, method II is the most profitable**.
- In fact, there is a transition probability $\hat{p} = \frac{F-L1}{F-L2}$ such that if $p < \hat{p}$, method I is more profitable than II, and vice versa if $p > \hat{p}$.
- It is also easy to show that if $p < \hat{p}$, method I is also more profitable than III, and if $p > \hat{p}$, method II is more profitable than III.
- So there is no situation where the average-in strategy is the most profitable one!

Scaling-in (Averaging-in) …

- So does that mean the whole idea of scaling-in/averaging-in has been debunked? Not necessarily.
- Notice the implicit assumption made in this example: **the probability of deviating to L2 before reverting to F is constant throughout time**.
- In real life, we may or may not find this probability to be constant. In fact, volatility is usually not constant, which means that p will not be constant either.
- **In this circumstance, scaling-in is likely to result in a better realized Sharpe ratio if not profits**.
- Another way to put it is that even though you will find that scaling-in is never optimal in-sample, you may well find that it outperforms the all-in method out-of-sample.

Dynamic Linear Regression

- For a pair of truly cointegrating price series, determination of the hedge ratio is quite easy: just take as much historical data as you can find, and use the Johansen test to find the eigenvectors.
- But, as emphasized before, stationarity and cointegration are ideals that few real price series can achieve.
- So how best to estimate the current hedge ratio for a pair of real price series when it can vary with time?
- In all the mean-reverting strategies we have discussed so far, we just took a moving **look-back period** and computed the regression coefficient or Johansen eigenvector over data in that period only.
- This has the **disadvantage** that if the look-back period is short, the deletion of the earliest bar and the inclusion of the latest bar as time moves forward can have an abrupt and artificial impact on the hedge ratio.

Dynamic Linear Regression …

- We face the same problem if we use moving averages or moving standard deviations to calculate the current mean and standard deviation of a price series.
- In all cases, we may be able to improve the estimate by using a weighting scheme that gives more weight to the latest data, and less weight to the earlier data, **without an arbitrary cutoff point**.
- The familiar exponential moving average (EMA) is one such weighting scheme.
- Also, dynamically updating the hedge ratios using the Kalman filter that avoids the problem of picking a weighting scheme arbitrarily can be exploited.

Difficulties of Trading Stock Pairs

- In theory, we can form pairs of stocks belonging to any sector and expect them to cointegrate due to their exposure to many common economic factors. Their number is large, so diversification is easy.
- In practice, though, there are some serious difficulties with applying these generic techniques to trading stocks and ETFs.
- We find that in the **short term**, most stocks exhibit mean-reverting properties under normal circumstances.
- **Normal circumstance means there isn't any news on the stock, or fundamental changes**.
- This is despite the fact that stock prices follow geometric random walks over the long term.
- We will build a strategy to exploit this short-term, or "seasonal," mean reversion.

Difficulties of Trading Stock Pairs …

- **Index arbitrage** is another familiar mean reversion strategy. In this case, we are counting on the **cointegration** of stocks versus futures or stocks versus ETFs.
- In addition to the familiar time series mean reversion to which we have devoted all our attention so far, there is the phenomenon of cross-sectional mean reversion, which is prevalent in baskets of stocks.
- Recall that in time series mean reversion, the prices are reverting to a mean determined by their own historical prices, while cross-sectional mean reversion means that the cumulative returns of the instruments in a basket will revert to the cumulative return of the basket.
- **This additional type of mean reversion makes creating any sort of mean-reverting strategy for stocks even easier**.

Difficulties of Trading Stock Pairs …

- Because of this ease of finding mean-reverting patterns, the stock market attracts a large number of traders, often called statistical arbitrageurs, to exploit such patterns.
- As a result, the returns in such strategies have generally decreased, so new tricks and ideas should be taught to benefit them.
- If we test the daily price series of individual stocks, they almost never meet the definition of stationarity as defined earlier.
- The **geometric random walk** describes their behaviors fairly well: **once they walked away, they seldom returned to their starting points**.
- Even if you pair them up in some sensible way, they are seldom cointegrating **out-of-sample**.

Difficulties of Trading Stock Pairs …

- We emphasize out-of-sample because it is quite easy to find cointegrating stock pairs in any chosen period of time, but they can just as easily lose cointegration in the subsequent out-of-sample period.
- The reason for this difficulty is that the fortunes of one company can change very quickly depending on management decisions and the competition.
- The fact that two companies are in the same industry sector does not guarantee that they will be subjected to the same fortune.
- The upshot is that it is difficult to be consistently profitable in trading a single pair of stocks using a mean-reverting strategy unless you have a fundamental understanding of each of the companies and **can exit a position in time before bad news** on one of them becomes public.

Difficulties of Trading Stock Pairs …

- In mean reverting strategies on stocks and ETFs, usually, the small profits gained by the good pairs have been completely overwhelmed by the large losses of the pairs that have gone bad.

- Other than these fundamental problems with stock pairs trading, there is another technical difficulty named **short-sale constraint**.

- It is particularly dangerous for a stock pair that involves shorting a hard-to-borrow stock, because even if your position is ultimately profitable, you may be forced to liquidate it at the most unprofitable and inopportune time.

- This difficulty vanishes in currencies and futures and short-sale constraint is only considered when trading stocks and ETFs.

Difficulties of Trading Stock Pairs …

- The one advantage of trading ETF pairs instead of stock pairs is that, once found to be cointegrating, ETF pairs are less likely to fall apart in out-of-sample data.
- That is because the fundamental economics of a basket of stocks changes much more slowly than that of a single company.
- For example, since both Australia and Canada are commodity-based economies, EWA and EWC are good candidates for cointegration tests.
- The pair selection process for ETFs is quite easy: we need to find ETFs that are exposed to common economic factors.
- **Besides country ETFs, sector ETFs are another fertile ground for finding cointegrated instruments.**
- For example, the gold fund GLD versus the gold miners fund GDX is a good example.

Intraday Mean Reversion: Buy-on-Gap Model

- Stock prices follow geometric random walks, as many financial scholars have tirelessly reminded us.
- But this is true only if we test their price series for mean reversion strictly at regular intervals (such as using their daily closes).
- Our job as traders is to find special conditions, or special periods, such that mean reversion occurs with regularity, while at the same time avoiding data-snooping bias.
- As the following strategy shows, there may indeed be seasonal mean reversion occurring at the intraday time frame even for stocks.

Intraday Mean Reversion: Buy-on-Gap Model …

- The rules for the strategy are:

  1) Select all stocks **near the market open** whose returns from their previous day's lows to today's opens are lower than one standard deviation. The standard deviation is computed using the daily close-to-close returns of the last 90 days. These are the stocks that **gapped down**.

  2) Narrow down this list of stocks by requiring their open prices to be higher than the 20-day moving average of the closing prices.

  3) Buy the 10 stocks within this list that have the lowest returns from their previous day's lows. If the list has fewer than 10 stocks, then buy the entire list.

  4) Liquidate all positions at the market close.

Intraday Mean Reversion: Buy-on-Gap Model …

- The rationale for this strategy is that on days when the stock index futures are down before the open, certain stocks suffer disproportionately due to panic selling at the open.
- But once this panic selling is over, the stock will gradually appreciate over the course of the day.
- Rule 2 is often very useful in mean-reverting strategies: it is basically a momentum filter superimposed on a mean-reverting strategy, a technique that we reprise often.
- Usually, **those stocks that dropped "just a little" have a better chance of reversal than those that dropped "a lot"** because the latter are often the ones that have negative news such as poor earnings announcements.

Intraday Mean Reversion: Buy-on-Gap Model …

- Drops caused by negative news are less likely to revert.
- Furthermore, the fact that a stock is higher than a long-term moving average attracts selling pressure from larger players such as long-only funds, whose trading horizons tend to be longer.
- This demand for liquidity at the open may exaggerate the downward pressure on the price, but **price moves due to liquidity demands are more likely to revert when such demands vanish than price moves due to a shift in the fundamental economics of the stock**.
- We apply this strategy to S&P500 from May 11, 2006 to April 24, 2012.

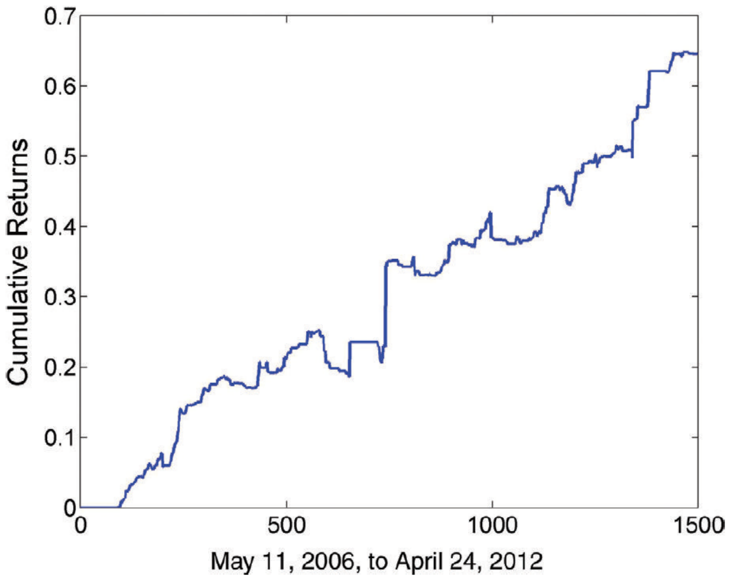
Intraday Mean Reversion: Buy-on-Gap Model ...

- This strategy has an APR of 8.7 percent and a Sharpe ratio of 1.5.

- The cumulative returns curve is depicted in the following figure.

- The long-only nature of the strategy also presents some risk management challenges.

- Finally, the number of stocks traded each day is quite small, which means that the strategy does not have a large capacity.

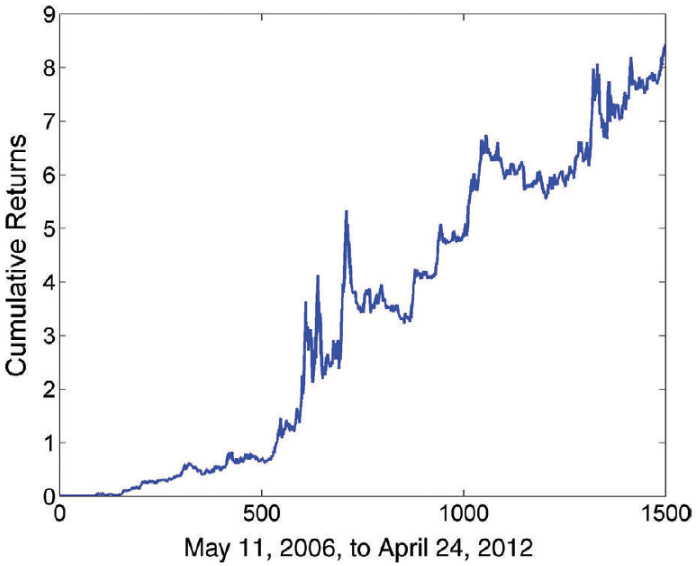


Intraday Mean Reversion: Buy-on-Gap Model …

- The question is how we can use open prices to determine the trading signals for entry at the open and be filled at the official open prices.
  - The short answer is, of course: We can't!
- We can, however, use the preopen prices to determine the trading signals.
- The signals thus determined will not exactly match the ones determined by the actual open prices, but the **hope** is that the difference will not be so large as to wipe out the returns. We can call this difference ***signal noise***.

Intraday Mean Reversion: Buy-on-Gap Model …

- What about the mirror image of this strategy? Can we **short** stocks that gap up a standard deviation but are still lower than their 20-day moving average?
  - Yes, we can.
- The APR is 46 percent and the Sharpe ratio is 1.27 over the same period.
- Despite the seemingly higher return than the long-only strategy, the short-only one does have steeper drawdown, as shown in the following figure, and it suffered from the same short-sale constraint pitfall discussed before.



Intraday Mean Reversion: Buy-on-Gap Model ...

- This strategy is actually quite well known among traders, and there are **many variations** on the same theme.
- For example, you can obviously trade both the long-only and short-only versions simultaneously.
- Or you can trade a hedged version that is long stocks but short stock index futures.
- You can buy a larger number of stocks, but restricting the number of stocks within the same sector.
- You can extend the buying period beyond the market open.
- **But the important message is: Price series that do not exhibit mean reversion when sampled with daily bars can exhibit strong mean reversion during specific periods.**
- This is seasonality at work at a short time scale.

Arbitrage between an ETF and Its Component Stocks

- The strategy index arbitrage refers to the trades on the difference in value between a portfolio of stocks constituting an index and the futures on that index.
- If the stocks are weighted in the same way used to construct the index, then the market value of the portfolio will cointegrate very tightly with the index futures.
- **Maybe too tightly**—unfortunately, this is such a well-known strategy that the difference in market values has become extremely small.
- All but the most sophisticated traders can profit from this strategy, and it most certainly needs to be traded intraday, perhaps at high frequency.
- In order to increase this difference, we can select only a subset of the stocks in the index to form the portfolio.

Arbitrage between an ETF and Its Component Stocks …

- The same concept can be applied to the arbitrage between a portfolio of stocks constituting an ETF and the ETF itself.
- In this case, we choose just a proper subset of the constituent stocks to form the portfolio.
- One selection method is to just pick all the stocks that cointegrate individually with the ETF.
- We demonstrate the method by using the most famous ETF of all: SPY.
- We pick **one year of data** (in this example, January 1, 2007, to December 31, 2007) as a **training set** and look for all the stocks that cointegrate with SPY with at least 90 percent probability using the Johansen test.

Arbitrage between an ETF and Its Component Stocks …

- Then we form a portfolio of these stocks (totally 98 stocks) with equal capital on each stock, and confirm using the Johansen test again that this long-only portfolio still cointegrates with SPY.
- **This step is necessary because an arbitrary assignment of equal capital weight to each stock does not necessarily produce a portfolio price series that cointegrates with that of SPY, even if each of the constituent stocks is cointegrating with SPY**.
- We are using log price in this second test because we expect to rebalance this portfolio every day so that the capital on each stock is constant.
- After confirming cointegration, we can then backtest our simple linear mean reversion strategy.

Arbitrage between an ETF and Its Component Stocks …

- As the Johansen test was performed on the log prices, the hedge ratios on the stocks or SPY represent **dollar capital allocation, not number of shares**.

- We then apply the linear mean reversion strategy on this portfolio over the test period January 2, 2008, to April 9, 2012.

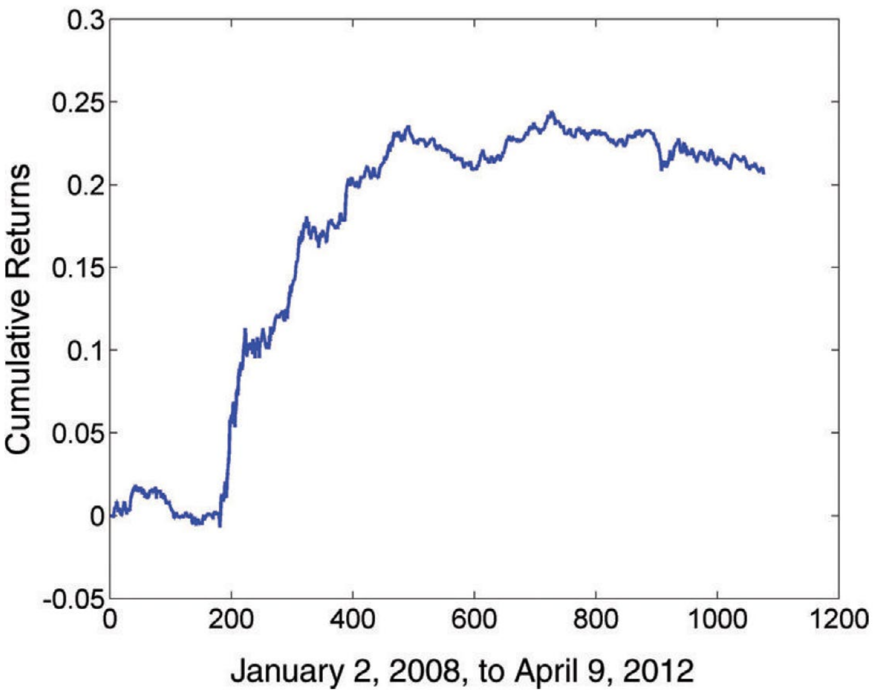
- We have fixed the look-back used for calculating the moving average and standard deviations of the portfolio market value to be 5.

- The **APR of this strategy is 4.5** percent, and the **Sharpe ratio is 1.3**.

Arbitrage between an ETF and Its Component Stocks …

- As you can see from the cumulative returns chart below, the performance decreases as time goes on, partly because we have not retrained the model periodically to select new constituent stocks with new hedge ratios.

- In a more complete backtest, we can add this dynamic updating of the hedge ratios.



Cross-Sectional Mean Reversion

- In mean reversion trading based on cointegration, we form a portfolio with a fixed set of instruments and with either a fixed number of shares or a fixed dollar capital for each instrument.

- This fixed number may be determined by fiat, linear regression, the Johansen test, or constrained optimization.

- But there is no reason why the portfolio has to consist of the same fixed set of instruments or the same weightings over this set of instruments every day.

- For many portfolio stock-trading strategies, the hedge comes precisely from the intelligent daily selection or reweighting of stocks.

Cross-Sectional Mean Reversion …

- In this type of so-called **cross-sectional mean reversion strategy**, the individual stock (and this type of strategy most commonly involves stocks, not futures or currencies) price does not necessarily revert to its own historical mean.

- **Rather, the focus is on their short-term relative returns**.

- **In most cases**, the relative returns are computed as a stock's return minus the average returns of all the stocks in a particular universe.

- So we expect the underperformance of a stock to be followed by overperformance, and vice versa.

- Since we are measuring only **relative return**, it is **quite possible that we will short a stock even though its previous (absolute) return is negative**, as long as **it is not as negative as the average return across all stocks in the universe**.

Cross-Sectional Mean Reversion …

- One interesting feature of cross-sectional strategies is that, in contrast to time series strategies, we should not expect profits from every individual stock, as some of them may serve as hedges on some days.

- **Rather, profits can be obtained only in the aggregate across all the stocks.**

- Let's define a new strategy; we invest in every stock from some favorite index such as S&P 500, S&P 1500, and so on, but with different capital allocation per stock.

- Near the market close of each day, we will determine the long or short capital w_i allocated to the i^{th} stock as

$$w_i = -\frac{r_i - \langle r_j \rangle}{\sum_k \left| r_k - \langle r_j \rangle \right|}$$

- where r_i is the daily return of the i^{th} stock, and $\langle r_j \rangle$ is the average daily return of all the stocks in the index.

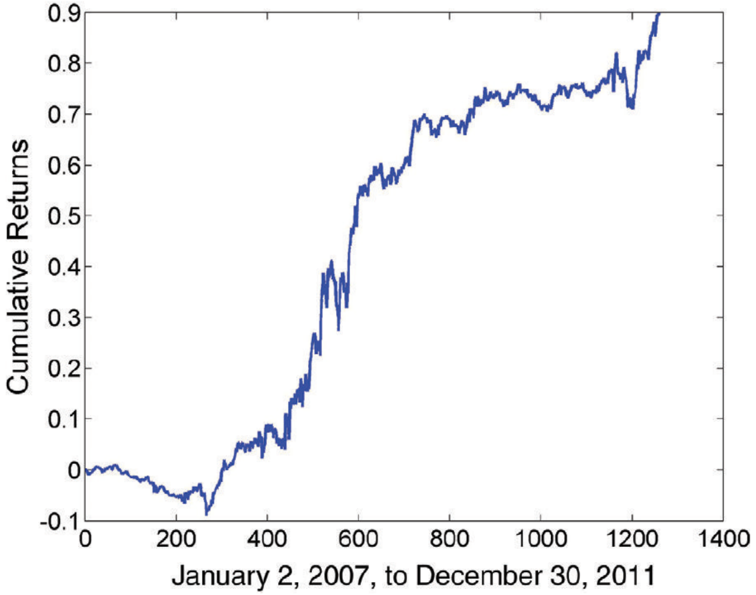
Cross-Sectional Mean Reversion …

- In other words, if a stock has a very positive return relative to its peers, we will short lots of it, and if it has a very negative return relative to its peers, we will buy lots of it.

- Note that we always invest the same total gross capital of $1 to the portfolio every day because of the normalization factor in the denominator.

- This strategy shows an APR of 13.7 percent and Sharpe ratio of 1.3 from January 2, 2007, to December 30, 2011, even if we backtest on the SPX.

- The cumulative returns are plotted in the following figure.



Cross-Sectional Mean Reversion …

- We can enhance the returns of this strategy by using the return from the previous close to today's open to determine the weights for entry at the open.
- All the positions will be liquidated at the market close, thus turning it into an intraday strategy.
- **The APR and Sharpe ratio over the same period are 73 percent and 4.7, respectively, in the new version**.
- Despite such seemly stellar performance, the open-to-close version suffers from a few drawbacks that the close-to-close version does not have.
- First, the transaction costs (not included in our backtests) will be doubled, because we are trading twice a day instead of just once a day.
- Second, since this strategy also has to use open prices to determine the trading signals for entry at the open, it is subject to the same trading **signal noise** mentioned in the Buy-on-Gap Model.

Cross-Sectional Mean Reversion …

- There are possibly other variables (also called factors) that are better at predicting cross-sectional mean reversion of stock prices than the relative returns.
- One popular variable that traders use to rank stocks is the price-earnings (P/E) ratio, where the earnings may be that of the last quarter, or they may be projected earnings estimated by the analysts or the companies themselves.
- Stock prices will drift toward a new equilibrium value if there are earning announcements or estimates changes.
- So a stock that experiences a positive change in earnings estimates will likely enjoy a positive return, and we should not expect the price to mean-revert if this return is in line with the percent change in its earnings estimates.
- We can therefore avoid shorting such a stock if we use P/E ratio to rank the stocks.

Pros and Cons of Mean-Reverting Strategies

- It is often fairly easy to construct mean-reverting strategies because we are not limited to trading instruments that are intrinsically stationary.
- We can pick and choose from a great variety of cointegrating stocks and ETFs to create our own stationary, mean-reverting portfolio.
- Besides the plethora of choices, there is often a good fundamental story behind a mean-reverting pair.
- Why does EWA cointegrate with EWC? That's because both the Canadian and the Australian economies are dominated by commodities.
- Why does GDX cointegrate with GLD? That's because the value of gold-mining companies is very much based on the value of gold.
- This **availability of fundamental reasoning** is in contrast to many momentum strategies whose only justification is that there are investors who are slower than we are in reacting to the news.

Pros and Cons of Mean-Reverting Strategies …

- Another advantage of mean-reverting strategies is that they span a **great variety of time scales**.
- At one extreme, market-making strategies rely on prices that mean-revert in a matter of seconds.
- At the other extreme, fundamental investors invest in undervalued stocks for years and patiently wait for their prices to revert to their fair value.
- The short end of the time scale is particularly beneficial to traders like ourselves, since a short time scale means a higher number of trades per year, which in turn translates to higher statistical confidence and higher Sharpe ratio for our backtest and live trading, and ultimately higher compounded return of our strategy.

Pros and Cons of Mean-Reverting Strategies …

- Unfortunately, it is because of the seemingly high consistency of mean reverting strategy that may lead to its eventual downfall.
- This high consistency often lulls traders into overconfidence and overleverage as a result.
- When a mean-reverting strategy suddenly breaks down, perhaps because of a fundamental reason that is discernible only in hindsight, it often occurs when we are trading it at maximum leverage after an unbroken string of successes.
- So the rare loss is often very painful and sometimes catastrophic.
- Hence, **risk management** for mean reverting is particularly important, and particularly difficult since the usual stop losses cannot be logically deployed.

Pros and Cons of Mean-Reverting Strategies …

- **Mean reversion strategies** profit from calm markets but lose money in crises.
- **Momentum strategies** profit from crises but lose money in calm markets.
- **Combining both** creates volatility **all-weather** portfolio -> volatility neutral portfolio