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Why Your Classes Are Larger than "Average"

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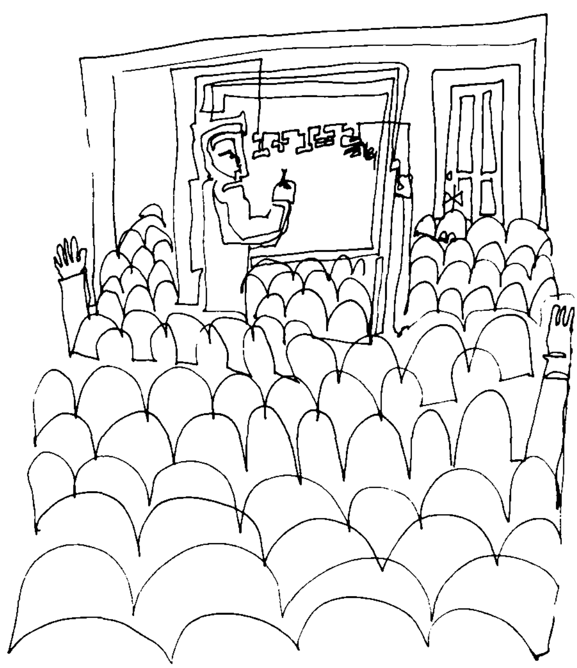
Most schools advertise their "average class size," yet most students find themselves in larger classes most of the time. Here is a typical example.

In the first quarter of the 1980–81 academic year, 111 courses including tutorials, were given at Harvard School of Public Health. These ranged in size from one student to 229. The **average class size**, from the administration's and professors' perspective, was 14.5. The **expected class size** for a typical student was over 78! This huge discrepancy was due to the existence of a few very large classes. Indeed, only three courses had more than 78 students. One enrolled 105, another 171, and there were 229 in Epidemiology.



Given one class of the size of Epidemiology, an expected class size of approximately 78 for a typical student can be achieved in various ways. Four possible configurations for the rest of the classes are: (i) 450 individual tutorials, (ii) 50 courses of size 10, (iii) 25 courses of size 30, (iv) 25 courses of size 50. The administration's "average class size" for these four cases would be 1.5, 14.3 (close to the advertised figure), 38, and 57 respectively.

The discrepancy between average class size and expected class size for a typical student is explained by a simple computation. Suppose we have a population of M individuals divided into N groups, and we let X_i denote the size of the ith group, $1 \leq i \leq N$. Then the expected number of people in a randomly selected group ("average class size") is given by

$$\bar{X} = (\Sigma X_i)/N = M/N,$$

and the expected size of a group containing a randomly selected individual is given by

$$X^* = \Sigma(X_i/M)X_i = (\Sigma X_i^2)/M.$$

Hence

$$\begin{aligned} X^* - \bar{X} &= \left[N\Sigma X_i^2 - (\Sigma X_i)^2\right]/MN \\ &= \left[\frac{N\Sigma X_i^2 - (\Sigma X_i)^2}{N^2}\right]\frac{N}{M} \\ &= \sigma^2/\bar{X}, \end{aligned}$$

where σ^2 is the variance in group sizes.

The difference between the two means $\bar{X}$ and X^* is directly proportional to the variance in sizes of groups and inversely proportional to average group size. It follows that $X^* \geq \bar{X}$, with equality only when all the groups are the same size.

Here are additional examples from everyday life of the differences between $\bar{X}$ and X^*.

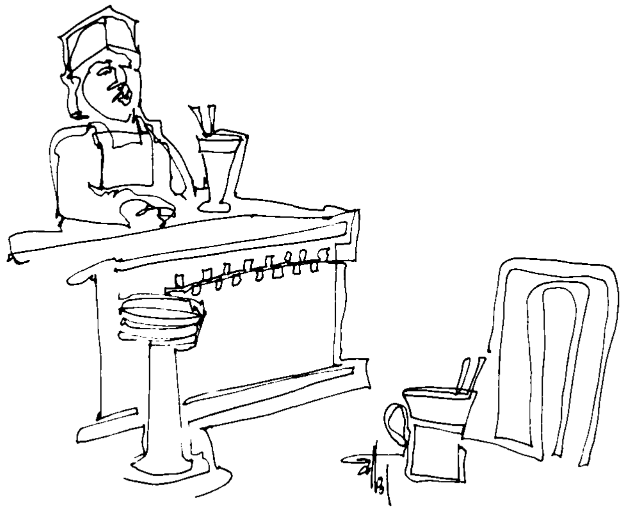
The Nationwide Personal Transportation Survey indicated that average car occupancy ($\bar{X}$) for "home-to-work" trips in metropolitan areas in 1969 was 1.4 people. The table below gives the data.

Number of Occupants	"Home-to-Work" Trips
1	73.5%
2	18.2
3	4.7
4	1.9
5	1.1
6	.5
7	.1

Calculating X^* from these statistics we find that the average number of occupants in the car of a typical commuter was 1.9.

To eliminate most congestion problems in U.S. cities would only require raising the average number of people per car ($\bar{X}$) to 2. This doesn't sound impossible. But suppose this were accomplished by inducing some drivers of single-occupant vehicles to join together in five-person car pools. The percentage of single-occupant cars would need to fall to 58.7%; five-occupant cars would rise to 15.9%. The percentage of people in single-occupant cars would fall below 30%. If X^* is calculated for this situation, one finds that the typical commuter would be in a car carrying more than three people.

I often buy dinner at a fast-food restaurant near my home. Although most customers order "to go," the place is almost always crowded, and I consider it quite a success. One evening about 6:30 I went in and there was no one in line. The manager was serving me, so I asked, "Where is everyone?" "It often gets quiet like this," he said, "even at dinnertime. The customers always seem to come in spurts. Wait fifteen minutes and it will be crowded again." I was surprised that I



had never before seen the restaurant so empty. But I probably shouldn't have been. If I am a typical customer, I am much more likely to be there during one of the spurts, so my estimate of the popularity of the restaurant (X^*) is likely to be much greater than its true popularity ($\bar{X}$).

The average number of people at the beach on a typical *day* will always be less than the average number of people the typical *beach-goer* finds there. This is because there are lots of people at the beach on a crowded day, but few people are ever there when the beach is practically deserted.

If the waiting time at a health clinic increases with the number of patients, the average waiting time for a typical *day* will always be less than the average waiting time for a typical *patient*. This is because there are more patients waiting on those days when the waiting time is especially long.

The expected size of a typical generation will be smaller than the expected number of contemporaries for a randomly chosen individual from one of those generations.

Figures for the population density of any region will understate the actual degree of crowding for the average inhabitant.

This Note distinguishes mathematically between two types of means. It does not report any original findings about human behavior. Yet it does indicate something about perceptions—especially my own. I was surprised at the restaurant. I was also surprised when the courses I took in college were larger than advertised. And I was surprised to realize how many commuters had to carpool to reach an average of even two people per car. If you are similarly surprised by any of these observations, your perceptions and perhaps even your behavior may be affected.

Helpful comments were received from Frederick Mosteller and an anonymous referee.

Kirchhoff's Third Law

The differential equations instructor, confronted with electrical circuit problems, may have trouble remembering names of components and corresponding units. The following mnemonic helps.

Remove the first, middle, and last letters from "Kirchhoff." This leaves triplets IRC and HOF. The former suggests "inductors, resistors, capacitors;" the latter "henries, ohms, farads."

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